Search and Segregation

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Abstract

Consumers' willingness to pay for an identical product, e.g. as caused by differences in local income or tastes, may differ greatly across locations. Yet, while a large literature examines consumers' optimal price and product-search behavior under various market configurations, the equilibrium effects of such consumer segregation remain unexplored. To this end, I study a stylized model in which two local monopolistic markets differ in size and their consumers' willingness to pay. After observing their native market's price, a subset of flexible consumers may travel to the other market at positive cost, hoping for a bargain. I show that as long as the proportion of flexible high-valuation consumers is not too large, active and directed search to the lower-valuation market will occur in equilibrium. If the higher-valuation market is relatively large in size, complex mixed-strategy pricing emerges in equilibrium. For regulators, increasing the fraction of flexible consumers tends to be more effective than manipulating search costs.

Keywords: Consumer Search, Segregation, Clustering, Mixed-Strategy Pricing, Asymmetric Market Structure, Active Search

JEL Classification: D43, D83, L11, L13

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1 Introduction

Consumers' characteristics often vary considerably across geographically close locations. For example, many empirical studies document that income tends to be highly segregated in urban areas—the rich rarely locate door-to-door with the poor.[1] Similarly, the local average income and purchasing power differ greatly between many neighboring countries, such that cross-border shopping is a widely observed phenomenon.[2] Alternatively, even with identical income, consumers' tastes may be heterogeneous due to differences in their composition (young families vs. pensioners, students vs. employees, etc.). But also the usage or consumption of a physically identical good may be more costly or generate less gross utility for consumers in one location than another, independently of where they purchased it (for example, because country/state A has different taxation levels than country/state B—cars come to mind—or because the required infrastructure is less developed in one location than another).

All of the above examples have two things in common. The first is that consumers in one local market may have a higher (average) willingness to pay for a given good than consumers in a nearby, distinct local market. The second is that due to geographical proximity, at least some consumers may find it optimal to travel from their native market to another if they perceive its price level to be lower.

Although there is a large theoretical literature dealing with (often heterogeneous) consumers' optimal search behavior for low prices (or good product matches) and firms' equilibrium response,[3] the specific form of consumer heterogeneity analyzed in this paper remains unexplored: local clustering and segregation of consumers resulting in heterogeneous demand characteristics across submarkets. The question is then how the presence of such consumer heterogeneity affects firms' equilibrium pricing and consumers' purchasing be-

[1]See e.g. Bischoff and Reardon (2013) and Florida and Mellander (2015) for recent reports on income segregation in major U.S. metropolitan areas.

[2]See e.g. "Swiss Shoppers Storm German Border Towns," Spiegel Online, 2011. A relatively recent survey of the vast economic literature on cross-border shopping is given by Leal et al. (2010).

[3]Seminal papers in the consumer search literature include Diamond (1971) and Stahl (1989) (sequential search for homogeneous products), Wolinsky (1986) and Anderson and Renault (1999) (sequential search for differentiated products), and Burdett and Judd (1983) (fixed-sample-size search for homogeneous products). Baye et al. (2006) provide a detailed survey of theoretical and empirical studies on price dispersion in homogeneous-goods markets. A detailed literature review, with particular emphasis on research dealing with consumer search under asymmetric market configurations, can be found below.

havior, given that consumers do not observe prices outside their local market. If consumers have to incur a strictly positive cost of accessing an outside market, under which circumstances would they still be willing to search[4]? How does this affect firms' pricing strategy, and is there scope for beneficial policy intervention?

In order to answer these questions, I study the following stylized setting. There are two spatially separated markets, each home to an identical local monopolist producing a homogeneous good. The markets differ in size and their local consumers' willingness to pay for the good. Hence, in the absence of a link between the two markets, each firm would charge the local monopoly price. However, there *is* a link between the markets, such that a subset of "flexible" consumers may travel to the other market at strictly positive cost. The flexible consumers can be thought of as sophisticated consumers who are aware of the (from their perspective) outside market's existence and its characteristics. At the same time, they do not face prohibitively high costs of accessing the outside market, for example because they own a car, do not have high opportunity costs of time, or enjoy shopping. In contrast, all consumers who are not flexible are captive to their local firm. Then, given that the flexible consumers' search costs are not prohibitively high, the following main results are obtained.

The first is that if a large fraction of consumers in the high-valuation market is flexible, paradoxically no search occurs in the unique equilibrium of the game. This is because the firm in the high-valuation market (henceforth called "H") finds its local flexible consumers too important to lose, and optimally charges a sufficiently low price that discourages them from leaving towards its rival in the low-valuation market (henceforth called "L"). Although the markets are segregated, there is a strong link between them due to a large fraction of flexible consumers. Consequently, H's pricing is fully disciplined by L's existence, and search does not take place.

The second major result is that if the fraction of flexible consumers in the high-valuation market is sufficiently small *and* at the same time the high-valuation market is not too large relative to the low-valuation one, in the unique equilibrium of the game, each firm sets its

[4]Throughout the paper, I will often speak of "search costs" when referring to consumers' costs of reaching an outside market and obtaining a price quote from there. Of course, a large part of these costs may be comprised of physical transport costs and/or opportunity costs of time. Still, the important property justifying the terminology is that prices outside the local market are not observable *before* incurring the transport cost. In Section 5, I consider a model variant in which prices are perfectly observable ex ante, such that the costs of accessing an outside market can be thought of as pure transport costs. See further below in the introduction for a more detailed motivation.

price equal to its local consumers' valuation, and the high-valuation market's flexible consumers travel to the low-valuation market and purchase there with certainty. L has no incentive to increase its price, as this would drive out its local consumers with a lower willingness to pay. At the same time, H has no incentive to discourage its local flexible consumers from searching, as it would have to decrease its price by too much. This situation is reminiscent of cross-border shopping and other forms of directed travel in which consumers exploit local price differences. Instead of choosing prices that are low enough to retain all local consumers, firms in a higher-income country may accept that some consumers will purchase abroad, and tailor their prices towards local consumers who are less mobile—be it due to opportunity costs, travel costs, lack of information, or other reasons.

The third main result is that, if the high-valuation market is relatively large (and the proportion of flexible consumers is not too high), a pure-strategy equilibrium fails to exist, and complex mixed-strategy equilibria with active search emerge. The intuition is as follows. If H chose the local monopoly price and the flexible high-valuation consumers were to search, expecting a low price, L would find it optimal to maximally exploit these consumers by (almost) charging the price of its rival, despite driving out its (relatively small mass of) local consumers. However, this would undermine the high-valuation consumers' incentive to search in the first place, and even if they searched, this would give firm H a reason to undercut. This tension can only be resolved by mixed-strategy pricing in which L probabilistically exploits incoming searchers by pricing above its local consumers' valuation, but then sometimes does not sell at all because it is priced out by its rival. The latter occurs because with positive probability, H engages in a sale that may beat L's high-range prices. Remarkably, if market H is very large compared to L, a novel and second type of mixed-strategy equilibrium emerges in which H sometimes engages in a deep sale, which altogether discourages its local flexible consumers from searching. This is necessary to sufficiently reduce L's incentive to price above its local consumers' valuation.

Considering the above findings, my model contributes to the theory of search (for homogeneous products) by unifying several plausible properties of search markets that are relatively uncommon in the literature. The first is that, although all consumers face a strictly positive search cost, the famous Diamond (1971) paradox[5] does not (always) arise: H may choose prices well below its local monopoly price in equilibrium. This does not only occur when H finds it optimal to prevent its flexible consumer group from searching due to competitive pressure by L, but also as an equilibrium response to firm L's attempt to exploit incoming searchers. While the former rationale is well-understood in different setups (see e.g. Reinganum (1979)—compare with the literature discussion below), the latter is, to the best of my knowledge, novel. The second property is that active search may emerge, as H may price above its flexible consumers' reservation price in equilibrium. I argue that this stems from an interaction of search-cost heterogeneity and spatial heterogeneity in tastes, and point out that the *combination* of these two is necessary to generate search in the model. In contrast, standard sequential search models such as Stahl (1989) and Janssen et al. (2005) induce an endogenous reservation price above which no firm prices in equilibrium, preventing active search.[6] Third, and again to the best of my knowledge, the model is the first which can simultaneously generate spatial *and* temporal price dispersion in equilibrium. The price dispersion is spatial, in the sense of Salop and Stiglitz (1977) and Reinganum (1979), because L charges prices that are on average lower than H's. On the other hand, if no pure-strategy equilibrium exists, the price dispersion is also temporal, in the sense of e.g. Shilony (1977), Varian (1980) and Rosenthal (1980), because in equilibrium, both firms sample prices randomly from overlapping supports. Hence, complex sales patterns arise in which H sometimes engages in promotions which may beat L's price, or which may altogether discourage its local flexible consumers from shopping around.

After discussing the different types of equilibria that arise in the baseline model, I turn to a welfare analysis. I identify two potential sources of welfare loss in the market: wasteful travel expenditures undertaken by searching high-valuation consumers, and deadweight loss created by dropout low-valuation consumers. While the former occurs whenever the fraction of flexible high-valuation consumers is not too large (otherwise, H fights for its flexible consumers and the social first-best is achieved), the latter only occurs if a pure-strategy equi-

[5]Roughly speaking, the Diamond paradox says that if all consumers face positive search costs and search sequentially, every firm in a symmetric oligopoly must charge the monopoly price in equilibrium.

[6]Two exceptions are given by Stahl (1996) and Chen and Zhang (2011). In both of these papers, search-cost heterogeneities across consumers with positive search costs may lead some consumers to search actively in equilibrium. However, the respective models require that consumers' search cost distribution extends down to zero (with a positive mass at zero in the case of Chen and Zhang) in order to generate price dispersion.

librium fails to exist. In that case, firm L prices above its local consumers' valuation with positive probability in equilibrium. This finding also endogenizes an empirical regularity that has been widely documented, namely that poorer consumer groups tend to find it more difficult to access certain product markets (see e.g. Somekh (2012, 2015) and the references therein). In my model, I show that, if the high-valuation (high-income) market is relatively large in size, the firm in the low-valuation (low-income) market may consider it optimal to (probabilistically) exclude its local consumers from purchasing. This is because higher rents can be extracted from less price sensitive (or more wealthy) shoppers coming from outside.

Regulators aiming to improve market efficiency or consumer surplus may try to manipulate consumers' search costs or alter the fraction of potentially searching consumers. I establish that increases or decreases in these variables have no clear-cut effects, such that even a reduction of search costs or an increase in competition through the fraction of flexible consumers may backfire. However, one main result is that boosting the fraction of potential searchers may be less risky, as once a certain threshold is reached, both total social welfare and consumer surplus will be maximized. This is caused by the competitive pressure that H faces for its (then large) segment of flexible consumers, which forces it to price aggressively and discourage search. Importantly, this result continues to hold when considering extensions to downward-sloping individual demand.

Finally, in some markets the assumption that firm L's price is not observed by H-market consumers is clearly violated. Moreover, it is *a priori* unclear how much of the models' results are driven by unobservable prices outside consumers' local markets (and their ensuing search problem), rather than pure transport costs in a perfect-information setting.[7] After the main analysis, I thus consider a variation of the baseline model in which the flexible consumers costlessly observe all prices, while they still need to incur strictly positive travel costs in order to purchase outside of their home market. This has several advantages. First, it allows for a characterization of the resulting pricing equilibria and market outcomes when all prices are observable, which is arguably more realistic for certain market environments. Second, it provides a robustness check of the baseline model's findings with respect to the described change in information structure. And third, it helps to disentan-

[7]Note also that comparative statics with respect to search costs may alternatively be interpreted as comparative statics with respect to transport costs, while prices outside the local market remain unobservable.

gle the different effects of search and transport costs on market outcomes. I find that the model's complexity increases significantly if there is a large fraction of flexible consumers: the baseline model's pure-strategy equilibrium without search breaks down, and three new types of mixed-strategy equilibria emerge. At the same time, L-market consumers unambiguously benefit relative to the baseline model when there are many flexible consumers, while total social welfare is reduced. Hence, regulations that improve consumers' access to price information may increase consumer surplus at the cost of aggregate welfare.

The remainder of this article is organized as follows. The paragraph below discusses the related literature in more detail. In Section 2, the model setup is introduced. The different equilibria of the baseline game are analyzed in Section 3. Section 4 is concerned with welfare and regulation. An extension to perfect information is provided in Section 5. Section 6 concludes and points out some potential directions for future research. Technical proofs related to the existence of all characterized equilibria are relegated to Appendix A. Appendix B establishes uniqueness of the baseline model's equilibria.

Related Literature This article ties into the literature on price dispersion and consumer search under asymmetric market configurations. An important early contribution was given by Narasimhan (1988), who extends Varian's (1980) classic model of sales (where firms have symmetric loyal consumer bases, and compete in prices for a perfectly price-sensitive mass of "shoppers") to the case of asymmetric shares of loyal consumers across (duopolistic) firms. However, in contrast to the present work, consumers' willingness to pay is symmetric, and (sequential) search is ruled out, as consumers are either perfectly informed about all prices, or are fully captive to their preferred firm.

Kocas and Kiyak (2006) extend Narasimhan (1988)'s model to oligopoly and allow for differences in willingness to pay across firms. Similar to Narasimhan, sequential search is ruled out. Moreover, their model differs from the present contribution because there is no local clustering of consumers with different valuations: each available product is valued the same by all consumers, although product quality may vary across firms. Instead, I study a situation in which products are homogeneous, while consumers are segregated and differentiated with respect to their willingness to pay.

Reinganum (1979) allows for sequential search, but generates price dispersion through marginal-cost heterogeneity across a continuum of firms, combined with elastic consumer demand. In her model, high-cost firms charge consumers' reservation price, whereas low-cost firms set their monopoly price. Contrary to the present contribution, consumers are homogeneous, and no active search arises in equilibrium.

Extending Reinganum's (1979) model, Benabou (1993) admits heterogeneity in consumers' search costs on top of firms' heterogeneity in marginal costs. In his model, low-cost firms charge their monopoly price, while all others are disciplined by consumers' (active) search. There may also be a bunching of prices for certain segments of marginal costs. Clearly, search is driven by different forces than in the present article, as consumers differ in their search costs, but not in their valuations. Moreover, there is a continuum of firms, and mixed-strategy pricing does not occur.

Rajiv et al. (2002) provide a complex marketing model in which differentiated consumers, both with respect to firm loyalty and their valuation for product quality, may search across vertically differentiated retailers. In their model, search only occurs if at least one firm advertises its price, and consumers are not segregated. Moreover, the authors' focus lies on firms' equilibrium frequency of advertising prices and their depth of promotional discounts.

Close in spirit is a recent paper by Astorne-Figari and Yankelevich (2014), who consider a setup in which duopolistic competitors differ in their number of local (captive) consumers.[8] As in my model, these consumers do not directly observe the outside firm's price, but may obtain this information at positive cost. In the unique equilibrium, both firms play mixed strategies, but the price distribution of the firm with the larger mass of local consumers first-order stochastically dominates that of its rival. The major difference to the present work is that price dispersion is driven by an atom of shoppers, rather than by a local heterogeneity in consumers' willingness to pay. Proper search does not occur in equilibrium, and eliminating the atom of shoppers leads to the Diamond result. Moreover, the firm with lower average prices cannot have an incentive to exploit incoming searchers, as non-local consumers with positive search cost never visit it.

[8] See also Astorne-Figari and Yankelevich (2011) for a more detailed, earlier working paper version.

Other related papers that explicitly account for market asymmetries in a search framework are given by Burdett and Smith (2010) and Kuniavsky (2014). In Burdett and Smith (2010), one dominant firm with a continuum of retail outlets competes with a fringe mass of atomistic sellers, and consumers employ a noisy search technology in the spirit of Burdett and Judd (1983). Kuniavsky (2014) extends the standard sequential search model of Stahl (1989) to allow for heterogeneously sized sellers (where sellers with more outlets have a higher probability of being sampled first). In both of these papers, price dispersion is driven by supply-side heterogeneities, rather than market segregation and the resulting differences in local demand characteristics.

Since all consumers in my model face positive search costs, yet prices are dispersed in equilibrium, the paper also relates to a small literature on resolving the Diamond paradox under strictly positive search costs. Examples include Bagwell and Ramey (1992), who resolve the paradox by consumers making repeat purchases, and Rhodes (2015), who avoids the problem by considering multi-product retailers.

Finally, since the flexible consumers in my model may find it optimal to exploit local price differences, this article is also related to a literature on (third-degree) price discrimination with costly arbitrary, see e.g. Aguirre and Paz Espinosa (2004), Marchand et al. (2000), Anderson and Ginsburgh (1999), and Wright (1993).

2 Model Setup

Consider the following market. There are two spatially separated local submarkets H ("high valuation") and L ("low valuation") that host one risk-neutral firm each, labeled and indexed by their locations. The firms compete in prices p_H, p_L and sell a single homogeneous product that is offered in their respective market only. The firms' identical, constant unit costs are normalized to zero.

A total mass $\alpha \in (0,1)$ of consumers live in H, whereas the remaining mass $1-\alpha$ live in L. The consumers' valuations for the homogeneous product are identical within the local markets. That is, all consumers that live in H have unit demand up to a maximum valuation of v_H, whereas all consumers that live in L have unit demand up to a lower maximum valuation of $v_L < v_H$.

In the baseline model, each consumer only observes the price posted by the firm in her home market. However, some consumers are flexible in the sense that they can travel to the other market at positive cost, purchasing there if the observed price is lower. For expositional simplicity, I assume that the L-market consumers are fully captive in the sense that they will never visit H. Given p_L, they either buy directly (if $p_L \leq v_L$), or not at all.[9] In contrast, some consumers in H have the possibility to search. Being heterogeneous with respect to their search behavior, a fraction $1-\beta$ of H-consumers is captive as well. Given p_H, they either buy directly (if $p_H \leq v_H$), or not at all. On the other hand, a fraction β of H-consumers are (potential) searchers: at a travel cost $s \in (0, v_H - v_L)$,[10] they can visit market L and return, purchasing there if the observed price is lower. In all of what follows, I will refer to these potentially searching consumers as *flexible* H-consumers. Note that in the model, searching consumers have to return to their home market after observing the other firm's price. This setup is both natural and consistent with the usual assumption of *free recall* in search models.

The timing of the game is as follows. First, firms H and L simultaneously choose their prices p_H and p_L, which are then fixed for the rest of the game. Second, each consumer observes her home market's price, and all captive consumers buy immediately as long as the observed price does not exceed their valuation. Third, the mass $\alpha\beta$ of flexible H-consumers observe p_H, form (potentially probabilistic) beliefs about firm L's price p_L, and optimally decide whether to search L, purchase directly at H, or drop out of the market.[11] If they do not visit market L, they purchase at H, provided that $p_H \leq v_H$. If they visit market L, they

[9] This assumption does not affect any of the results and is only made to streamline the model setup. In Appendix B, I show that, as long as the L-market consumers' search costs are bounded away from zero, they will never search in equilibrium, irrespective of their search-cost distribution.

[10] For $s \geq v_H - v_L$, the unique equilibrium of the game is given by the uninteresting case in which H prices at v_H, L prices at v_L, and no consumers search. On the other hand, while the subsequent equilibrium characterization also fully applies to the case where $s = 0$, some of the resulting equilibria require the flexible H-consumers to play the weakly dominated strategy of not always searching initially. And precisely in these cases, there is equilibrium multiplicity, because another equilibrium exists in which the flexible H-consumers *do* always search initially. In fact, the model extension to perfect information in Section 5 fully characterizes these additional equilibria when setting $s = 0$. And conversely, when there is no equilibrium multiplicity, the equilibria of the baseline model coincide with those of the perfect-information framework. Further details are available from the author upon request.

[11] They might also randomize among some subset of these options in case of indifference, but it turns out that this not relevant in equilibrium. See also footnote 13 below.

incur the travel cost s, observe L's price p_L, and optimally buy at the cheaper firm (given that its price does not exceed their valuation).[12,13]

The solution concept I employ is a "strong" variant of perfect Bayesian equilibrium in the spirit of Fudenberg and Tirole (1991, Section 6). As is usual, the flexible H-consumers' beliefs need to be consistent with firm L's pricing in equilibrium. Furthermore, the solution concept imposes some restraints on firms' signaling abilities. In particular, it entails a "no-signaling-what-you-do-not-know" property: Even if firm H chooses a price p'_H that is never played in equilibrium, the flexible H-consumers' beliefs about firm L's price are not affected, since firm H has no more information about firm L's price than these consumers and hence cannot signal anything about it. For the present model, this has the same consequences, but is weaker than assuming passive beliefs.[14]

Figure 1 provides a graphical summary of the considered market structure. In the next section, I proceed to solve for the equilibrium of the described game given the parameters v_H, v_L, α, β, and s.[15]

3 Equilibrium Analysis

The game's different types of equilibria are characterized by the following sequence of propositions.[16]

[12] Note that it is assumed throughout that consumers purchase with certainty whenever they are indifferent between purchasing or dropping out of the market. Indeed, this is pinned down in any equilibrium where such indifference may arise with positive probability.

[13] Some further words on tie-breaking. Note that apart from the situation where $p_H = v_H$, the flexible H-consumers may be indifferent between some of their available actions under two different circumstances: (i) they may observe a price p_H that makes them indifferent between purchasing directly at H or searching market L, given their beliefs about firm L's price, and (ii) after having searched market L, it may turn out that $p_L = p_H$, such that they are indifferent between from where to buy. For (i), it is clear that in any equilibrium where this is relevant (i.e., indifference occurs with positive probability), all flexible H-consumers must buy directly at H. If they did not (such that they purchased at L with strictly positive probability), firm H would have a profitable deviation by reducing its price marginally and breaking the indifference. (This argument also relies on the fact that firm H makes a positive profit in equilibrium, which is evident due to its mass $\alpha(1-\beta)$ of locked-in consumers.) For (ii), it turns out that the tie-breaking rule is not determined in equilibrium, as such ties arise with zero probability in any equilibrium of the game.

[14] I thank Régis Renault and an anonymous referee for stressing this fact.

[15] Clearly, either v_H, v_L or s can be normalized to some arbitrary constant, e.g., $v_H = 1$ (such that v_L and s can be expressed as fractions of v_H). For expositional reasons, I will not do so throughout the paper.

[16] All existence proofs can be found in Appendix A, while uniqueness of the baseline model's equilibria is established in Appendix B.

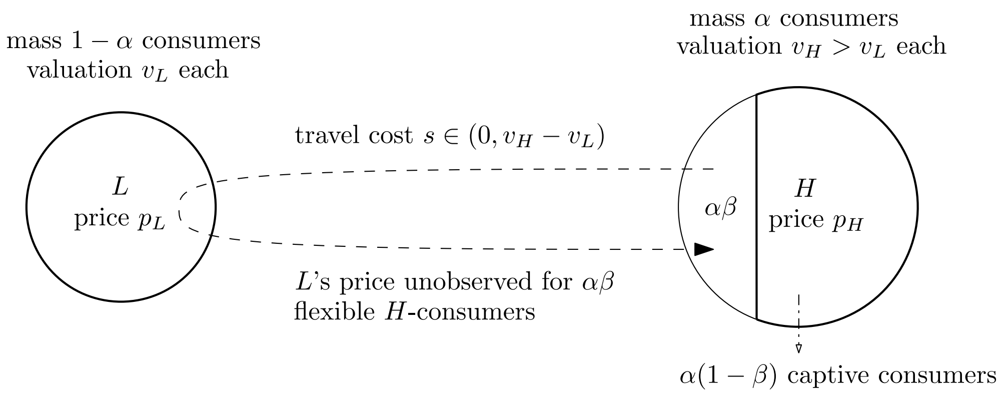

Figure 1: Depiction of the analyzed market.

Proposition 1. *If* $\beta > \overline{\beta} := 1 - \frac{v_L+s}{v_H} \in (0,1)$*, the unique equilibrium of the game is in pure strategies such that* $p_H^* = v_L + s \in (v_L, v_H)$*,* $p_L^* = v_L$*, and all* $\alpha\beta$ *flexible* H*-consumers purchase in* H*.* H*'s equilibrium profit is given by* $\Pi_H^* = (v_L+s)\alpha$*, whereas* L*'s equilibrium profit is given by* $\Pi_L^* = v_L(1-\alpha)$.

The intuition to Proposition 1 is straightforward: if sufficiently many H-consumers are flexible, H finds it worthwhile to fight for these consumers and discourage them from searching. The optimal way for H to achieve this is by charging the maximal markup over L's price which deters the flexible H-consumers from searching: $p_L^* + s$. Note moreover that $p_L^* < v_L$ cannot be part of an equilibrium. If it was, H would either find it optimal to charge $p_L^* + s < v_H$ (if p_L^* is sufficiently close to v_L) or the highest possible price v_H (if p_L^* is small). In either case, L could achieve a higher profit by increasing its price a little, as this would not decrease its demand. Hence, for a large β, the only possible equilibrium is such that $p_L^* = v_L$, $p_H^* = v_L + s$, and no search occurs.

Proposition 2. *If* $\beta < \overline{\beta}$ *and* $\alpha \leq \underline{\alpha}(\beta) := \frac{v_L}{\beta(v_H - v_L) + v_L} \in (\alpha_{min}, 1)$*, where* $\alpha_{min} = \frac{v_H v_L}{v_H^2 - (v_L+s)(v_H - v_L)} \in (0,1)$*, the unique equilibrium of the game is in pure strategies such that* $p_H^{**} = v_H$*,* $p_L^{**} = v_L$*, and all* $\alpha\beta$ *flexible* H*-consumers search and purchase in* L*.*[17] H*'s equilibrium profit is given by* $\Pi_H^{**} = v_H\alpha(1-\beta)$*, whereas* L*'s equilibrium profit is given by* $\Pi_L^{**} = v_L(1-\alpha+\alpha\beta)$.

[17] In the non-generic case where $\beta = \overline{\beta}$, given that $\alpha \leq \underline{\alpha}(\overline{\beta}) = \alpha_{min}$, the equilibria of Propositions 1 and 2 coexist (see Figure 2 below for an illustration). This is because for $\beta = \overline{\beta}$, H is indifferent between discouraging its local flexible consumers from searching (by pricing at $v_L + s$) or maximally exploiting its captive consumers while letting go of its flexible consumers (by pricing at v_H). Note that firm L's expected profit is strictly higher if the equilibrium of Proposition 2 is played.

Intuitively, $\beta < \overline{\beta}$ is simply the converse of the condition in Proposition 1: if sufficiently few H-consumers are flexible, H would not even find it worthwhile to fight for them if L priced at v_L deterministically. Instead, H prefers to fully exploit its captive consumers by pricing at v_H, and accepts that all its local flexible consumers will search and buy at the other firm.

Note that this logic is the key driving force which leads to active search in equilibria of the game. In typical search models, active search can be generated by taste and product heterogeneity or search-cost heterogeneity. In my model, it is the *combination* of taste and search-cost heterogeneity that induces active search. To see this, note that each factor alone would be insufficient to do so: If the consumers in market H were homogeneous in search costs (with common costs $s > 0$), firm H would clearly not be willing to let them go in any equilibrium, so the spatial heterogeneity and clustering of consumers' valuations would be unable to induce search. On the other hand, if consumers' willingness to pay was identical across submarkets, search-cost heterogeneity would also be insufficient to generate search, as both firms would simply set their price equal to consumers' (common) valuation. But taken together, consumers' difference in valuations may create a wedge in local monopoly prices that would lead to a directed outflow of flexible high-valuation consumers, and this outflow is indeed not prevented by firm H if the fraction of flexible H-consumers is not too large.

Although $\beta \leq \overline{\beta}$ is necessary to generate the above pure-strategy equilibrium with active search, it is not sufficient. A further requirement is that the size of market H is not too large, $\alpha \leq \underline{\alpha}(\beta)$, which rules out that L has a profitable deviation. The logic behind this is as follows. Clearly, given that H prices at v_H deterministically and does not fight for its flexible consumers, an expectation of $p_L^* = v_L$ by the flexible H-consumers would induce them to search. But then, if the H-market is sufficiently important in size (α is large), L no longer finds it optimal to charge v_L. Namely, rather than to also serve its own local consumers at this low price, L would prefer to exploit the flexible consumers' beliefs (of finding $p_L^* = v_L$ in L) and charge them the highest possible price (v_H) for which they do not return to H. This is the case if $\alpha > \underline{\alpha}(\beta)$.

The outlined incentive to exploit incoming searchers and the tension to resolve it is what generates the mixed-strategy equilibria which will be discussed below. Importantly,

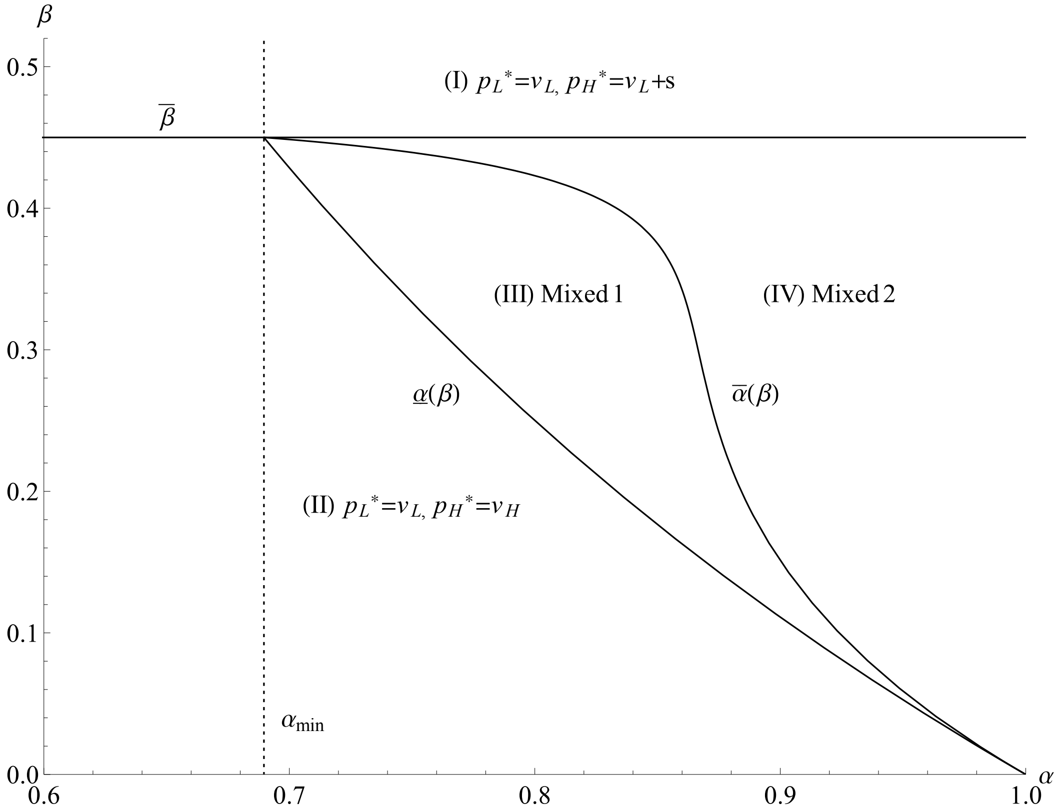

Figure 2: Equilibrium regions for $v_H = 200$, $v_L = 100$, $s = 10$.

these equilibria also entail active search by the flexible H-consumers (at least with positive probability in the case of Proposition 4). Figure 2 illustrates the different equilibrium regions in (α, β)-space.

Proposition 3. *If* $\beta < \overline{\beta}$ *and* $\alpha \in (\underline{\alpha}(\beta), \overline{\alpha}(\beta)]$, *where* $\overline{\alpha}(\beta) := \frac{v_L}{(1-\beta)\left\{v_L + \frac{v_H\beta[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}\right\}} \in (\underline{\alpha}(\beta), 1)$,[18] *the unique equilibrium of the game is in mixed strategies such that*[19]

- *H samples prices continuously from the interval $[\underline{p}, v_H)$, where $\underline{p} = \frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta} \in (v_L, v_H)$, following the CDF $F_H(p_H) = 1 - \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(\frac{v_L}{p_H}\right)$. Moreover, H prices at v_H with probability $q_H^* := \frac{v_L(1-\alpha+\alpha\beta)}{v_H\alpha\beta} \in (0, 1)$.*

- *L prices at v_L with probability $q_L^* := \frac{1}{\beta} - \frac{v_H\alpha(1-\beta)}{v_L(1-\alpha+\alpha\beta)} \in (0, 1)$. Moreover, L samples prices continuously from the interval $[\underline{p}, v_H)$ following the CDF $F_L(p_L) = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{p_L}\right)$.*

[18] While $\overline{\alpha}(\beta)$ always falls in this range (with $\overline{\alpha}(0) = 1$ and $\overline{\alpha}(\overline{\beta}) = \underline{\alpha}(\overline{\beta}) = \alpha_{min}$), it can be non-monotonic in β for certain combinations of v_H, v_L and s.

[19] Note that unlike the case where $\beta = \overline{\beta}$, there is no multiplicity of equilibria for $\alpha = \underline{\alpha}(\beta)$. This is because the equilibria of Propositions 2 and 3 coincide for $\alpha \to \underline{\alpha}(\beta)$, as can easily be shown.

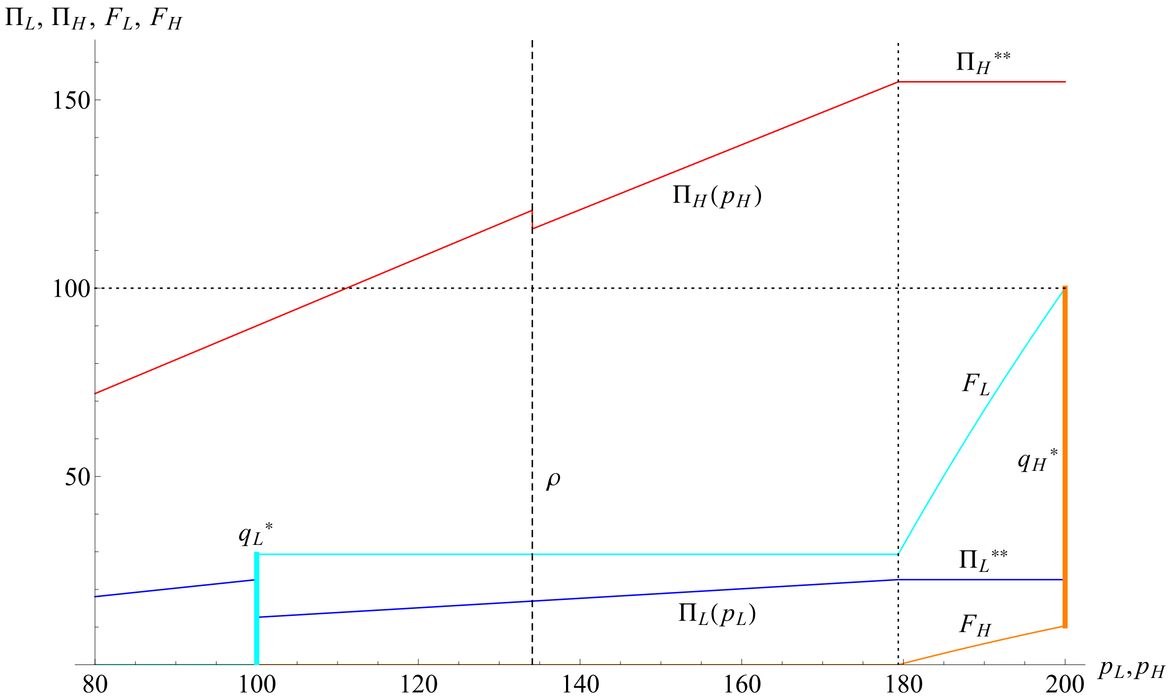

Figure 3: Expected firm profits and equilibrium CDFs for $v_H = 200$, $v_L = 100$, $s = 10$, $\alpha = 0.9$, $\beta = 0.14$. The vertical axis can be interpreted both as monetary units (for $\Pi_L(p_L)$ and $\Pi_H(p_H)$) and percentage points (for q_L^*, q_H^*, $F_L(.)$, $F_H(.)$).

- *As $\underline{p} > \rho$, where the flexible H-consumers' reservation price ρ solves $q_L^*(\rho - v_L) = s$, all $\alpha\beta$ flexible H-consumers search initially. However, they return with probability $1 - F_L(p_H)$, as in those cases L charges a higher price than H.*
- *As in the case of Proposition 2, H's equilibrium profit is given by Π_H^{**}, whereas L's equilibrium profit is given by Π_L^{**}.*

Figure 3 provides a graphical example of an equilibrium of the characterized type.

The intuition to Proposition 3 is as follows. Because the H-market is large compared to L ($\alpha > \underline{\alpha}(\beta)$), firm L would no longer find it optimal to charge v_L if the flexible H-consumers searched (after facing $p_H = v_H$ and a belief of $p_L = v_L$), as it would strictly prefer to exploit these consumers' beliefs by charging v_H instead. However, this cannot be an equilibrium, because (a) given $p_L = v_H$, the flexible H-consumers would clearly prefer not to search, and (b) even if these consumers searched, H would have a profitable deviation by marginally undercutting v_H (say, by pricing at $v_H - \varepsilon$), which would lead all flexible H-consumers to return to H after observing $p_L = v_H$. Consequently, L would also have a profitable deviation of pricing marginally below $v_H - \varepsilon$, and so on. This cycle of best responses gives rise to the mixed-strategy equilibrium characterized in the proposition: both L and H price at their local consumers' valuation with positive probability mass, but they also "fight" for the flexible H-consumers in those cases where L prices above v_L.[20] In some sense, in order to mitigate L's incentive to always exploit the searchers, H alters its strategy in such a way that it becomes harder for L to sell to the searching H-consumers if it prices above v_L. H achieves this by spreading positive probability mass on some interval below v_H, implying that L is indifferent between choosing v_L or any price larger than v_L that lies in that interval.

Since firm L charges prices higher than v_L with positive probability in equilibrium, this implies that low-valuation consumers are excluded from buying probabilistically. Hence, the characterized equilibrium is in line with the empirical finding that low-income consumers tend to suffer from poor access to certain product markets, as discussed in the introduction. This continues to hold for the last type of equilibrium to be characterized below.

Proposition 4. *If* $\beta < \overline{\beta}$ *and* $\alpha \in (\overline{\alpha}(\beta), 1)$, *the unique equilibrium of the game is in mixed strategies such that*[21]

- *H prices at the flexible H-consumers' reservation price* $\rho^* := v_H(1-\beta)$ *with probability* $q^*_{H,\rho} := 1 - \frac{1-\alpha}{\alpha\beta}\left(\frac{v_L[v_H(1-\beta)-v_L-\beta s]}{[v_H(1-\beta)-v_L]^2+v_L\beta s}\right) \in (0,1)$. *Moreover, H samples prices continuously from* $[\underline{p}, v_H)$, *where* $\underline{p} = \frac{v_H(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s} \in (\rho^*, v_H)$, *following the CDF* $F_H(p_H) = 1 - q^*_{H,v_H}\left(\frac{v_H}{p_H}\right) = 1 - \frac{1-\alpha}{\alpha\beta}\left(\frac{(1-\beta)v_L[v_H(1-\beta)-v_L]}{[v_H(1-\beta)-v_L]^2+v_L\beta s}\right)\frac{v_H}{p_H}$. *Finally, H prices at* v_H *with probability* $q^*_{H,v_H} := \frac{1-\alpha}{\alpha\beta}\left(\frac{(1-\beta)v_L[v_H(1-\beta)-v_L]}{[v_H(1-\beta)-v_L]^2+v_L\beta s}\right) \in (0,1)$.
- *L prices at v_L with probability* $q^*_{L,v_L} := \frac{s}{v_H(1-\beta)-v_L} \in (0,1)$. *Moreover, L samples prices continuously from* $[\underline{p}, v_H)$ *following the CDF* $F_L(p_L) = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{p_L}\right)$.
- *As H prices at the flexible H-consumers' reservation price* ρ^* *with positive probability* $q^*_{H,\rho}$, *these consumers will only search if H prices at or above* $\underline{p} > \rho^*$, *which happens*

[20] The technical reason why a pure-strategy equilibrium may break down is that there may be discontinuities in firms' best response functions. For example, if $\beta < \overline{\beta}$ and $\alpha > \underline{\alpha}(\beta)$, firm L's best response to $p_H > v_L$ and (for simplicity) a strategy of always searching by the flexible H-consumers is $p^*_L = v_L$ for $p_H \leq \frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta}$ and $p^*_L = p_H - \varepsilon$ for larger p_H. Similarly, firm H's best response to $p_L \leq v_H$ and a strategy of always searching by the flexible H-consumers is $p^*_H = v_H$ for $p_L \leq v_H(1-\beta)$ and $p^*_H = p_L - \varepsilon$ for higher p_L. Due to these discontinuities, no pure-strategy equilibrium exists in the parameter region in question.

[21] Note again that unlike the case where $\beta = \overline{\beta}$, there is no multiplicity of equilibria for $\alpha = \overline{\alpha}(\beta)$. This is because the equilibria of Propositions 3 and 4 coincide for $\alpha \to \overline{\alpha}(\beta)$, as can easily be verified. On the other hand, the equilibria of Propositions 1 and 4 coexist if $\beta = \overline{\beta}$ and $\alpha > \overline{\alpha}(\overline{\beta}) = \alpha_{min}$ (see Figure 2 above for an illustration). As in the case where Propositions 1 and 2 coexist (if $\beta = \overline{\beta}$ and $\alpha \leq \alpha_{min}$), L makes a strictly higher profit if the equilibrium of Proposition 4 is played.

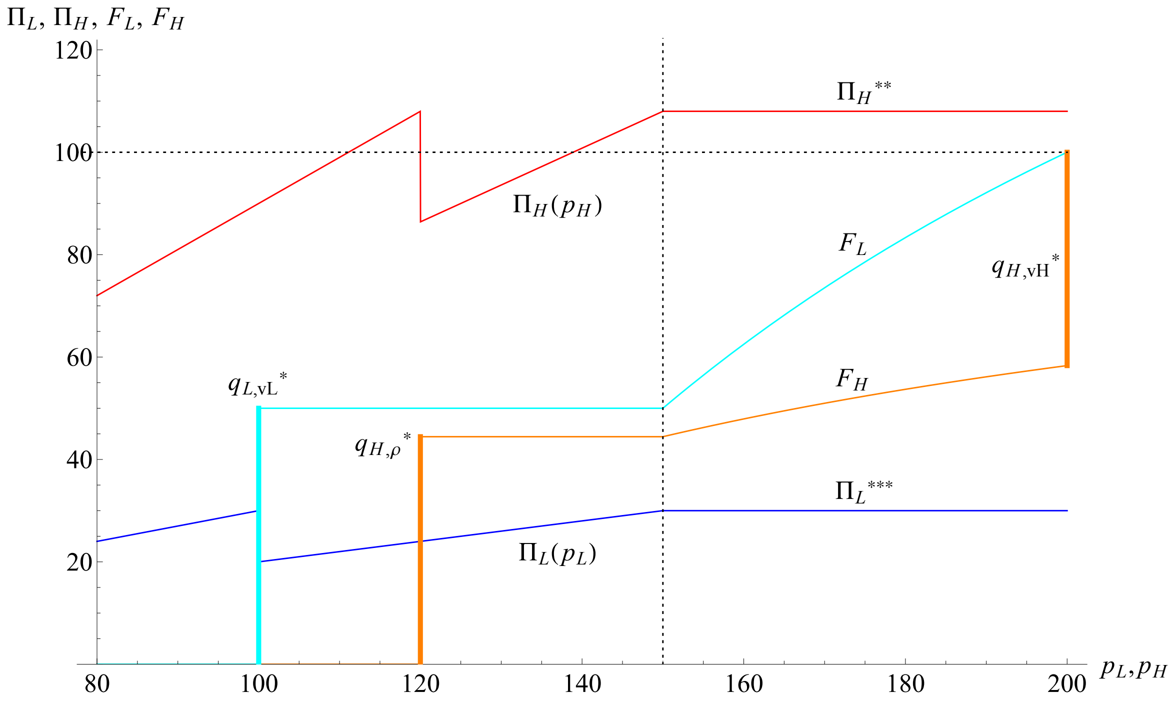

Figure 4: Expected firm profits and equilibrium CDFs for $v_H = 200$, $v_L = 100$, $s = 10$, $\alpha = 0.9$, $\beta = 0.4$. The vertical axis can be interpreted both as monetary units (for $\Pi_L(p_L)$ and $\Pi_H(p_H)$) and percentage points (for q^*_{L,v_L}, $q^*_{H,\rho}$, q^*_{H,v_H}, $F_L(.)$, $F_H(.)$).

with probability $1-q^*_{H,\rho}$. *However, given that H prices at* $p_H \in [\underline{p}, v_H)$, *they return with probability* $1-F_L(p_H)$, *as in those cases L charges a higher price than H.*

- *H's equilibrium profit is given by* Π^{**}_H, *whereas L's equilibrium profit is given by* $\Pi^{***}_L := \frac{(1-\alpha)(1-\beta)v_H v_L[v_H(1-\beta)-v_L]}{[v_H(1-\beta)-v_L]^2+v_L\beta s}$.

Again, Figure 4 depicts a graphical example of an equilibrium of the characterized type.

The intuition to Proposition 4 is similar to that of Proposition 3. The crucial difference is that for a very large α, the H-market is so important relative to L that firm L would always want to price above its local consumers' valuation if the flexible H-consumers searched with certainty. Indeed, note that since firm H can guarantee to make a profit of $v_H\alpha(1-\beta)$ by pricing at v_H, in equilibrium firm H may never price below $v_H(1-\beta)$ $(> v_L)$, as it would make a lower profit than $v_H\alpha(1-\beta)$ even if it always sold to the flexible H-consumers. In turn, if the flexible H-consumers searched with certainty, L could guarantee to attract them by pricing at $v_H(1-\beta)-\varepsilon$, making a profit arbitrarily close to $v_H(1-\beta)\alpha\beta$. This profit strictly exceeds firm L's profit when pricing at v_L, that is, $v_L(1-\alpha+\alpha\beta)$, for α sufficiently close to one.[22] Hence, for very large α, L can only be made indifferent between charging v_L or exploiting the searching H-consumers if the flexible H-consumers do not always search initially. H achieves this by putting positive probability mass on the flexible H-consumers' reservation price $\rho^* = v_H(1-\beta)$, such that L cannot even be certain to exploit the flexible H-consumers if it prices at $\underline{p}$, the lowest price in its pricing range above v_L.

Note that the equilibrium of Proposition 4, particularly the pricing strategy of firm H, is consistent with empirical evidence that retail price distributions tend to be bimodal, with prices fluctuating between a "regular" high price and a low "sales" price, and little mass between (see Hosken and Reiffen (2004), Pesendorfer (2002)). The present model provides a complementary explanation to that of Garcia et al. (2015), who generate a two-point price distribution by introducing costly retailer search for manufacturers' offers.

4 Welfare and Regulation

In this section, I first pin down the expressions for total social welfare and consumer surplus that arise in the different equilibrium regions of the model. I proceed to argue which parameters may be potential targets for policy intervention, and discuss how changes in these parameters affect total and consumer welfare. Finally, I briefly consider what would happen if demand was price elastic.

Welfare. Since the consumers have inelastic demand up to a maximum valuation of v_H in H (where a total mass α of consumers reside) and up to v_L in L (where the remaining $1-\alpha$ consumers reside), it is obvious that the maximal surplus which can be achieved in the whole market is given by

$$\overline{W} := \alpha v_H + (1-\alpha)v_L. \tag{1}$$

Considering the different equilibria which were outlined in Section 3, there are two possible sources of welfare loss in the market. First, wasteful travel expenditures to the extent of

[22] Precisely, this is true for $\alpha > \frac{v_L}{(1-\beta)(v_L+v_H\beta)} \in (\overline{\alpha}(\beta), 1)$. Although this value is close to $\overline{\alpha}(\beta)$ for small β (e.g., 0.9084 vs. 0.9045 for $v_H = 200$, $v_L = 100$, $s = 10$, $\beta = 0.14$), there is always a non-empty range of α's where the mixed-strategy equilibrium of Proposition 3 breaks down due to more subtle reasons. Indeed, the mixed-strategy equilibrium of Proposition 3 breaks down for $\alpha > \overline{\alpha}(\beta)$ because firm H, and not firm L, has a profitable deviation (i.e., to price at the flexible H-consumers' reservation price with certainty).

$\alpha\beta s$ can be incurred if the $\alpha\beta$ flexible H-consumers search. And second, the L-market surplus of $(1-\alpha)v_L$ is lost in those cases where L prices above v_L, as this leads all L-consumers to drop out of the market. The following proposition then follows straightforwardly from Propositions 1 to 4.

Proposition 5. *The expected total welfare in the market is given by*[23]

$$W := \begin{cases} \alpha v_H + (1-\alpha)v_L & \text{if } \beta > \overline{\beta} \\ \alpha v_H - \alpha\beta s + (1-\alpha)v_L & \text{if } \beta < \overline{\beta} \text{ and } \alpha \leq \underline{\alpha}(\beta) \\ \alpha v_H - \alpha\beta s + q_L^*(1-\alpha)v_L & \text{if } \beta < \overline{\beta} \text{ and } \alpha \in (\underline{\alpha}(\beta), \overline{\alpha}(\beta)] \\ \alpha v_H - (1-q_{H,\rho}^*)\alpha\beta s + q_{L,v_L}^*(1-\alpha)v_L & \text{if } \beta < \overline{\beta} \text{ and } \alpha \in (\overline{\alpha}(\beta), 1). \end{cases} \tag{2}$$

As consumers' demand is inelastic, the aggregate expected consumer surplus for each parameter region can easily be calculated as $CS = W - \Pi_H^{[*]} - \Pi_L^{[*]}$, where $\Pi_i^{[*]}$ denotes the equilibrium profit of firm $i \in \{H, L\}$ in the respective parameter region. Clearly, with inelastic demand, L-market consumers never make any positive surplus, as firm L never prices below v_L in equilibrium.

Proposition 6. *The aggregate expected consumer welfare (derived exclusively by H-market consumers) is given by*[24,25]

$$CS := \begin{cases} \alpha(v_H - v_L - s) & \text{if } \beta > \overline{\beta} \\ \alpha\beta(v_H - v_L - s) & \text{if } \beta < \overline{\beta} \text{ and } \alpha \leq \underline{\alpha}(\beta) \\ \alpha\beta(v_H - v_L - s) - (1-q_L^*)(1-\alpha)v_L & \text{if } \beta < \overline{\beta} \text{ and } \alpha \in (\underline{\alpha}(\beta), \overline{\alpha}(\beta)] \\ \alpha\beta(v_H - v_L - s) - (1-q_{L,v_L}^*)(1-\alpha)v_L & \\ \qquad + \; q_{H,\rho}^*\alpha\beta(v_L + s) & \text{if } \beta < \overline{\beta} \text{ and } \alpha \in (\overline{\alpha}(\beta), 1). \end{cases} \tag{3}$$

[23] If $\beta = \overline{\beta}$, the expected total welfare depends on which equilibrium is played. It is $\alpha v_H + (1-\alpha)v_L$ if H plays $v_L + s$, whereas it is $\alpha v_H - \alpha\beta s + (1-\alpha)v_L$ if H plays v_H (for $\alpha \leq \underline{\alpha}(\overline{\beta})$), or $\alpha v_H - (1-q_{H,\rho}^*)\alpha\beta s + q_{L,v_L}^*(1-\alpha)v_L$ if H plays its (mixed) equilibrium strategy of Proposition 4 (for $\alpha > \underline{\alpha}(\overline{\beta}) = \overline{\alpha}(\overline{\beta})$).

[24] Again, if $\beta = \overline{\beta}$, the equilibrium consumer welfare depends on which equilibrium is played. See footnote 23 above for details.

[25] In order to obtain the expression for the last case, note that $\Pi_L^{***} = v_L[1 - \alpha + \alpha\beta(1 - q_{H,\rho}^*)]$.

Comparative Statics and Regulation. The analyzed model is governed by five parameters: Consumers' maximal willingness to pay in the high and low-valuation market (v_H and v_L), the distribution of consumers across markets (α), the search cost of flexible high-valuation consumers (s), and the share of flexible consumers in the high-valuation market (β). Note however that the first three of these are rather deep structural parameters that cannot easily be targeted by policymakers. For example, if the model is used to describe firms' equilibrium pricing and consumers' shopping patterns across the neighborhoods of a city, neither the composition of consumers, nor their different willingness to pay (e.g., as caused by differences in average income or preferences) can easily be manipulated.

On the other hand, both the search friction of consumers traveling across spatially separated submarkets (s), as well as the fraction of (flexible) consumers that are aware of this opportunity and do not find it prohibitively costly to do so (β), may potentially be influenced by policy.[26] For instance, in order to decrease s, the local administration could build new roads or walkways, improve the public transportation system, or provide amenities such as parking spaces, public toilets, and temporary childcare facilities. Of course, s may also be increased through opposing measures.[27] Likewise, the fraction of flexible (high-valuation) consumers β may be increased by an informational campaign that educates consumers about the possibility and attractiveness of purchasing in an outside market (with on average lower prices), infrastructural measures such as the connection of formerly remote areas, or investment in services like a shuttle-bus line for consumers without cars.

While policy-relevant, analyzing the impact of changes in s or β on total social welfare and consumer surplus turns out to be a complex task. First, since four different equilibrium regions arise in the model, changes in s and β may lead to transitions across equilibrium regions, which, in turn, may have conflicting comparative statics. For example, while total social welfare is maximal and independent of s in region I (where no search takes place), it

[26] Note that both s and β provide information about the high-valuation consumers' search cost distribution. Indeed, the present model has an identical outcome to the model variation where a fraction β of high-valuation consumers has a search cost of s, while all remaining high-valuation consumers have search costs that exceed $v_H - v_L$. In a more general model allowing for an arbitrary search-cost distribution, policy-induced shifts of this distribution could be analyzed.

[27] As one referee put it, a regulator may want to build a really tall wall between markets in order to discourage search. While this may indeed maximize market efficiency in the present setup with inelastic demand and wasteful search frictions, the right balance needs to be struck if consumers' interests are also to be considered. See below for a more detailed discussion.

strictly decreases in s in regions II and III, while it can be shown that it strictly increases in s in region IV.[28] At the same time, consumer surplus strictly decreases in s for regions I to III, while it strictly increases in s for region IV.[29] Second, even within equilibrium regions, the comparative statics of social welfare and consumer surplus may not be monotone. Precisely, an increase in β has an ambiguous effect on social welfare in region III,[30] while its effect on consumer surplus is ambiguous in both regions III and IV.[31] An example of the comparative statics of total social welfare and consumer surplus with respect to s and β is provided in Figure 5.

Although it is thus apparent that no clear-cut comparative statics with respect to s and β can be provided, some policy recommendation can still be given. Namely, it turns out that for all parameter values, there is a critical s' and β' such that for all $s \geq s'$ (keeping β fixed), or for all $\beta \geq \beta'$ (keeping s fixed), total social welfare is maximized. Moreover, consumer surplus is maximized for a single value of s in this range, namely the lower bound $s = s'$, whereas it is maximized for all $\beta \geq \beta'$. These findings are highlighted in the following proposition.[32]

[28] That total social welfare strictly decreases in s in regions II and III follows from the fact that the flexible H-consumers always search initially in the corresponding equilibria (creating a wasteful search friction of $\alpha\beta s$), while firm L's probability of sampling v_L (such that no deadweight loss is created) is independent of s. In region IV, an increase in s increases both firm L's equilibrium frequency of charging v_L and firm H's equilibrium frequency of charging the flexible H-consumers' reservation price ρ (such that no wasteful search friction is incurred). It can be shown that the welfare gains induced by this always dominate the welfare loss stemming from higher search costs in those cases where firm H prices above ρ. A proof is available from the author upon request.

[29] The latter effect is surprising and stems from the fact that both firm L's equilibrium probability of charging v_L and firm H's equilibrium probability of charging ρ (the lowest price in its support) strictly increase in s in the relevant region (see also the previous footnote related to social welfare). This is always sufficient to offset the higher travel cost incurred by searching high-valuation consumers (in those cases where H prices above ρ). A proof is once again available from the author upon request.

[30] The intuition for this is that countervailing effects are at play. Since all flexible H-consumers search initially in region III, an increase in their absolute number through β leads to an unambiguous increase in the wasteful search friction $\alpha\beta s$ that is incurred. On the other hand, there are parameter constellations within region III under which firm L's equilibrium probability of sampling v_L increases in β, which reduces the (expected) deadweight loss that stems from dropout low-valuation consumers. For some of these parameter constellations, the latter positive effect on welfare dominates.

[31] Again, these surprising results are caused by firms' equilibrium responses. In region III (IV), consumer welfare may decline because an increase in β may lead firm L (H) to reduce its equilibrium probability of charging v_L (ρ).

[32] For the proposition, it is assumed that equilibrium type I (without search) is played whenever $\beta = \overline{\beta}$ (which is the unique equilibrium for any $\beta > \overline{\beta}$). The statement for social welfare is obvious (region I without any welfare losses is reached if and only if $s \geq s'$ or $\beta \geq \beta'$). The statement for consumer surplus follows because all H-consumers are able to buy at the lowest possible effective price $v_L + s$ (firm L may never price below v_L in any equilibrium) if and only if equilibrium type I is played. If s is allowed to vary, this price is lowest for $s = s'$.

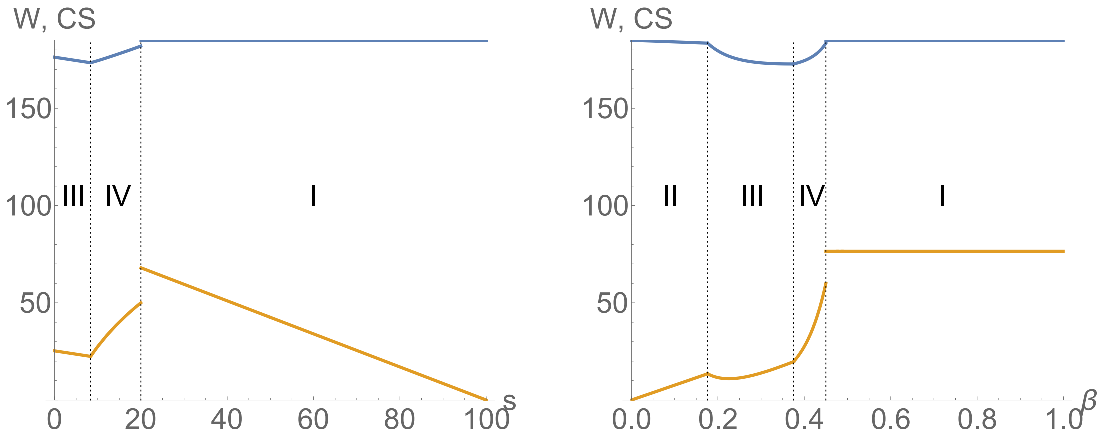

Figure 5: Expected total social welfare (blue) and consumer surplus (orange) as functions of s (left panel) and β (right panel). The parameters used are $v_H = 200$, $v_L = 100$, $\alpha = 0.85$, as well as $\beta = 0.4$ (left panel) and $s = 10$ (right panel). The different equilibrium regions that are reached are separated by dotted lines.

Proposition 7. *For any combination of parameters v_H, v_L, α and β, total social welfare is maximized if and only if $s \geq s' := \max\{v_H(1-\beta) - v_L, 0\}$, while consumer surplus is maximized if and only if $s = s'$. Likewise, for any combination of parameters v_H, v_L, α and s, both total social welfare and consumer surplus are maximized if and only if $\beta \geq \beta' := \overline{\beta}$.*

A regulator that aims to maximize total social welfare *and* consumer surplus (which seems sensible, given that both objectives can be achieved simultaneously) thus faces two options. First, it may try to find the "sweet spot" for the search cost s such that it is just high enough that H finds it optimal to fight for its local flexible consumers by pricing at $v_L + s$ (and not charging v_H or sampling prices from $[\underline{p}, v_H]$, with $\underline{p} > v_L + s$), but not higher than that. If $v_H(1-\beta) - v_L \leq 0$, then this is the case even for $s = 0$, such that a regulator that is (also) concerned for consumer surplus should aim to reduce consumers' search costs as much as possible. If instead $v_H(1-\beta) - v_L > 0$, the optimal s is positive, and hence a regulator may even have an incentive to artificially increase s (making the outside market less accessible, levying taxes, etc.) if s is low initially. In reality, a regulator may find it hard to justify such measures (which, after all, may have severe negative consequences for society that are not captured in the model), and even in the context of the model, it runs a risk of harming consumers (compare with Figure 5, left panel). Hence, it seems that influencing s through policy measures should be undertaken cautiously: If there is no search but only competitive pressure (region I), then reducing s may increase consumer welfare, but there is a risk that firms in a high-valuation market may respond by increasing their prices in order to exploit locked-in consumers—which, at the same time, induces wasteful search frictions. On the other hand, if there is search initially, increasing s may restore the first-best, but this may also harm consumers when done incorrectly, face resistance from the population, and have adverse consequences that are not part of this analysis.

The second option a regulator may have is to raise the fraction of flexible high-valuation consumers β. Although marginal increases in β may still have a (moderately) negative effect on welfare and consumer surplus (compare with Figure 5, right panel), there is no longer a danger of "overshooting": Once a certain threshold is surpassed ($\beta \geq \overline{\beta}$), full market efficiency is restored, and consumers' surplus is maximized. The intuition is that a sufficiently large fraction of non-loyal high-valuation consumers is able to fully discipline the behavior of the local incumbent, even though no search takes place. This is because, by these consumers' (correct) expectations of finding $p_L = v_L$ in market L, firm H cannot afford to charge a higher price than $v_L + s$, as this would turn away a sizable chunk of its demand. In other words, efficiency is restored precisely because H faces a *threat* of search by a large fraction of its local consumers, to which it responds by pricing aggressively (and thus, the threat does not materialize).

Note that there can be circumstances where increasing the fraction β of *potentially* searching consumers is not very costly: Indeed, it may suffice to inform (a larger fraction of) the population that searching is a viable strategy. On the other hand, especially reductions in s could require infrastructural measures that are prohibitively expensive. Thus, promoting the *possibility* of search may often be more beneficial than actually inducing search through a physical reduction in search costs.

Downward Sloping Demand The showcased model focuses on inelastic consumer demand in order to build intuition and keep the analysis tractable. Of course, this comes at the cost of realism, such that it would be desirable to allow for (a) downward-sloping individual demand and/or (b) heterogeneous consumer valuations within submarkets. While providing a full equilibrium analysis is challenging for both of these variations (and lies somewhat beyond the scope of the paper), the following result prevails under similar assumptions as in the baseline model:[33] If the fraction of flexible consumers in the market with a higher local monopoly price is sufficiently large, there will be no search in equilibrium, as the local incumbent finds it optimal to discourage these consumers from searching by pricing aggressively. In contrast, the firm in the neighboring low-valuation market charges the local monopoly price. In the setup with downward-sloping individual demand, it can be shown that this equilibrium is constrained-efficient, as no wasteful search frictions are incurred, and deadweight loss is minimized. Indeed, increasing β should generally be (even) more desirable under elastic demand, as giving a larger fraction of consumers access to low prices will tend to reduce deadweight loss.

5 Perfect Information Setting

In contrast to the specification of the main model, consumers may sometimes be well-informed about the prices charged by firms in a distinct regional market. For example, a consumer considering cross-border shopping for some expensive good (e.g. a new car) will likely try to obtain price quotes *before* traveling to the outside market.[34] Moreover, as motivated in the introduction, it seems worthwhile to check the baseline model's robustness with respect to changes in the information structure, and to contrast the different effects of search costs (in the sense of information-acquisition costs) and pure travel costs on market outcomes. While an exhaustive comparison between all variables of interest (equilibrium pricing strategies, firm profits, consumer surplus and total social welfare) across the different model setups would be beyond the scope of this paper, some selected key differences will be highlighted.

To this end, I consider a variation of the main model in which flexible H-market consumers can perfectly observe the price posted by firm L. However, they still have to incur a travel cost s in order to access market L. Hence, in the corresponding perfect-information framework, flexible H-consumers find it optimal to purchase from L whenever $p_L < p_H - s$ ($\leq v_H - s$). All other market parameters (α, β, v_H and v_L) keep the same interpretation as in the main model. Importantly, in order to facilitate a comparison with the baseline setup,

[33] Details are available from the author upon request.

[34] Clearly, the rise of price-comparison platforms on the internet, as well as the tendency of many firms to post prices online, has greatly facilitated such a practice.

I still assume that all L-market consumers are fully locked in to their local firm, and that $s < v_H - v_L$.

Note that in this adaptation, it still holds that for a small fraction β of flexible H-consumers, firm H would not even be willing to fight for these consumers if firm L charged v_L deterministically. The corresponding critical value of β is again given by $\overline{\beta} = 1 - \frac{v_L+s}{v_H}$. Hence, for $\beta \leq \overline{\beta}$, one might expect that similar types of equilibria emerge as in the baseline model. However, for $\beta > \overline{\beta}$, firm H finds it no longer sufficient to price at $v_L + s$ in order to optimally discourage its local flexible consumers from searching. In contrast to the baseline model, where firm L could not credibly convey that it may ever set a price below v_L, this is now clearly possible by simply setting an arbitrary price $p_L < v_L$. As it turns out, this variation introduces substantial additional complexity when the fraction of flexible H-consumers β is large. In what follows, I will separately consider the cases $\beta \leq \overline{\beta}$ and $\beta > \overline{\beta}$.

Proposition 8. *Suppose that the flexible H-consumers observe p_L, and that $\beta \leq \overline{\beta}$. Then there exist three types of pricing equilibria.*

(Pure) If α is small, $\alpha \leq \underline{\alpha}'(\beta) := \frac{v_L}{\beta(v_H - v_L) + v_L - \beta s}$, where $\underline{\alpha}'(\beta) \in (\underline{\alpha}(\beta), 1)$, there is a pure-strategy equilibrium in which $p_H^* = v_H$, $p_L^* = v_L$, and all flexible H-consumers purchase in L. L-market consumers are always served in this equilibrium. Firm H makes a profit of Π_H^{**}, while firm L makes a profit of Π_L^{**}.

(Mixed I) If α is intermediate, $\alpha \in (\underline{\alpha}'(\beta), \overline{\alpha}'(\beta)]$, where $\overline{\alpha}'(\beta) := \frac{v_L}{v_L(1-\beta) + \beta[v_H(1-\beta) - s]} \in (\underline{\alpha}'(\beta), 1)$, the following constitutes an equilibrium:

- *H samples prices continuously from the interval $[\underline{p}_H, v_H)$, where $\underline{p}_H = \frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta} + s > v_L + s$, following the CDF $F_H(p_H) = 1 - \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(\frac{v_L}{p_H - s}\right)$. Moreover, H prices at v_H with probability $q_H = \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(\frac{v_L}{v_H - s}\right) \in (0, 1)$.*

- *L prices at v_L with probability $q_L = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{v_L\left(\frac{1-\alpha+\alpha\beta}{\alpha\beta}\right) + s}\right) \in [0, 1)$. Moreover, L samples prices continuously from the interval $[\underline{p}_H - s, v_H - s)$ following the CDF $F_L(p_L) = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{p_L + s}\right)$.*

- *L-market consumers are only sometimes served in this equilibrium.*

- *H's equilibrium profit is given by Π_H^{**}, whereas L's equilibrium profit is given by Π_L^{**}.*

(Mixed II) If α is large, $\alpha > \overline{\alpha}'(\beta)$, the following constitutes an equilibrium:

- *H samples prices continuously from the interval $[\underline{p}_H, v_H)$, where $\underline{p}_H = v_H(1-\beta) > v_L + s$, following the CDF $F_H(p_H) = 1 - \frac{v_H(1-\beta)-s}{p_H - s}$. Moreover, H prices at v_H with probability $q_H = \frac{v_H(1-\beta)-s}{v_H - s} \in (0,1)$.*
- *L samples prices continuously from $[\underline{p}_H - s, v_H - s)$ following the CDF $F_L(p_L) = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{p_L+s}\right)$.*
- *L-market consumers are never served in this equilibrium.*
- *H's equilibrium profit is given by Π_H^{**}, whereas L's equilibrium profit is given by $\Pi_L = [v_H(1-\beta) - s]\alpha\beta$.*

Clearly, the pure-strategy equilibrium in which $p_L^* = v_L$ and $p_H = v_H^*$ is identical to the one in the baseline model with imperfectly informed consumers. Again, firm H has no incentive to retain its local flexible consumers by pricing at $v_L + s$, as their fraction is too small. In the baseline model, firm L's best deviation was to price at v_H in order to maximally exploit the incoming searchers from H, who could not observe this deviation. In the new framework, firm L's best deviation is to increase its price only to $v_H - s$ $(-\varepsilon)$, which is the highest price for which it can still attract the flexible consumers from H, given $p_H = v_H$. Of course, this implies that the parameter region for which (v_L, v_H) can be supported in equilibrium increases ($\underline{\alpha}' > \underline{\alpha}$).

But again, there is some critical value of α, $\alpha = \underline{\alpha}'$, above which firm L has a profitable deviation. This is to charge the highest possible price $v_H - s$ $(-\varepsilon)$ that still attracts the flexible consumers from H, even though this drives out its local consumers. As clearly, firm H would want to counteract this by marginally undercutting in turn, there is a cycle of best responses which rules out a pure-strategy equilibrium. Once again, two types of mixed-strategy equilibria emerge.

The first, for intermediate α, has a similar structure to the mixed-strategy equilibrium of the baseline model with intermediate α. In this equilibrium, both firms choose their local monopoly price with positive probability, but they also draw prices from two convex regions above v_L. The bounds of these regions are now such that the *effective* prices paid by flexible H-consumers, p_H when buying at H and $p_L + s$ when buying at L, overlap. Hence, if H

does not choose its mass point at v_H, it samples prices from an interval $[\underline{p}_H, v_H)$, while if L does not choose its mass point at v_L, it samples prices from the corresponding interval $[\underline{p}_H - s, v_H - s)$ (with $\underline{p}_H - s > v_L$).

If instead α is very large, firm L no longer finds it worthwhile to cater to its local consumers. While H's equilibrium strategy is still to price at v_H with positive probability and otherwise to draw prices from some interval $[\underline{p}_H, v_H)$, firm L now only draws prices from $[\underline{p}_H - s, v_H - s)$ (with $\underline{p}_H - s > v_L$), which deterministically excludes its local consumers. This differs qualitatively from the baseline model, where even as $\alpha \to 1$, the L-market consumers keep being served with strictly positive probability. Hence, with observable prices, poor consumer groups may find it even harder to access certain product markets. Another qualitative difference is that the "quasi-bimodal" equilibrium price distribution of Proposition 4 (where, with positive probability, firm H offers a deep discount by pricing at the flexible H-consumers' reservation price) does not emerge in the perfect-information setting. Information-acquisition costs are therefore a necessary prerequisite to induce bimodal price distributions in the context of the present model.

Overall, the qualitative results of the baseline model are thus robust to the change in information structure if there is a small fraction of flexible consumers and the relative size of market H is low (pure-strategy equilibrium) or intermediate (first type of mixed-strategy equilibrium), but not if its large. I now turn to the case where there is a large fraction of flexible consumers, $\beta > \overline{\beta}$.

Proposition 9. *Suppose that the flexible H-consumers observe p_L, and that $\beta > \overline{\beta}$. Then, the pure-strategy equilibrium of Proposition 1 does not exist anymore. Instead, there exist three types of mixed pricing equilibria.*

(Mixed III) If α is small, $\alpha \leq \tilde{\alpha}(\beta) := \frac{v_L - [v_H(1-\beta)-s]}{v_L-(1-\beta)[v_H(1-\beta)-s]}$ (which is always the case for β sufficiently close to 1, $\beta \geq \hat{\beta} := 1 - \frac{s}{v_H} \in (\overline{\beta}, 1)$), where $\tilde{\alpha}(\beta) > 0$, the following constitutes an equilibrium:

- *L samples prices continuously from the interval $[\underline{p}_L, v_L)$, where $\underline{p}_L = \frac{v_L(1-\alpha)}{1-\alpha+\alpha\beta}$, following the CDF $F_L(p_L) = \frac{1-\frac{\underline{p}_L+s}{p_L+s}}{\beta}$. Moreover, L prices at v_L with probability $q_L = 1 - \frac{1-\frac{\underline{p}_L+s}{v_L+s}}{\beta} \in (0,1)$.*

- *H samples prices continuously from the interval* $[\underline{p}_L + s, v_L + s)$*, following the CDF* $F_H(p_H) = 1 - \frac{1-\alpha}{\alpha\beta}\left(\frac{v_L}{p_H - s} - 1\right)$.

- *L-market consumers are always served in this equilibrium.*

- *H's equilibrium profit is given by* $\Pi_H = \left(\frac{v_L(1-\alpha)}{1-\alpha+\alpha\beta} + s\right)\alpha$*, whereas L's equilibrium profit is given by* $\Pi_L = v_L(1-\alpha)$.

(Mixed IV) If β *is not too large,* $\beta < \hat{\beta}$*, and* α *is intermediate,* $\alpha \in (\tilde{\alpha}(\beta), \hat{\alpha}(\beta)]$*, where* $\hat{\alpha}(\beta) := \frac{v_L - [v_H(1-\beta)-s]\left(1-\frac{v_L}{v_H - s}\right)}{v_L - (1-\beta)[v_H(1-\beta)-s]\left(1-\frac{v_L}{v_H-s}\right)} > \tilde{\alpha}(\beta)$*, the following constitutes an equilibrium:*

- *H samples prices continuously from the interval* $[\underline{p}_H, v_L + s)$*, where* $\underline{p}_H = v_H(1-\beta)$*, following the CDF* $F_H(p_H) = \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(1 - \frac{\underline{p}_H - s}{p_H - s}\right)$*. Moreover, H prices at* v_H *with probability* $q_H = 1 - \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(1 - \frac{\underline{p}_H - s}{v_L}\right) \in (0,1)$.

- *L samples prices continuously from the interval* $[\underline{p}_H - s, v_L)$ *following the CDF* $F_L(p_L) = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{p_L + s}\right)$*. Moreover, L prices at* v_L *with probability* $q_L = \frac{1-\beta}{\beta}\left(\frac{v_H}{v_L + s} - 1\right) \in (0,1)$.

- *L-market consumers are always served in this equilibrium.*

- *H's equilibrium profit is given by* Π_H^{**}*, whereas L's equilibrium profit is given by* $[v_H(1-\beta) - s](1-\alpha+\alpha\beta)$.

(Mixed V) If β *is not too large,* $\beta < \hat{\beta}$*, and* α *is large,* $\alpha > \hat{\alpha}(\beta)$*, the following constitutes an equilibrium:*

- *H samples prices continuously from the interval* $[\underline{p}_H, v_L + s)$*, where* $\underline{p}_H = v_H(1-\beta)$*, following the CDF* $F_{H,1}(p_H) = \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(1 - \frac{\underline{p}_H - s}{p_H - s}\right)$*. Furthermore, H samples prices continuously from the interval* $[\hat{p}_H, v_H)$*, where* $\hat{p}_H = \frac{[v_H(1-\beta)-s](1-\alpha+\alpha\beta)}{\frac{v_H(1-\beta)-s}{v_L}(1-\alpha+\alpha\beta) - (1-\alpha)} + s > v_L + s$*, following the CDF* $F_{H,2}(p_H) = 1 - (1-\alpha+\alpha\beta)\left[\frac{v_H(1-\beta)-s}{(p_H - s)\alpha\beta}\right]$*. Finally, H has a mass point of size* $q_H = (1-\alpha+\alpha\beta)\left[\frac{v_H(1-\beta)-s}{(v_H - s)\alpha\beta}\right] \in (0,1)$ *on* v_H.

- *L samples prices continuously from* $[\underline{p}_H - s, v_L)$ *following the CDF* $F_{L,1}(p_L) = \frac{1}{\beta} - \frac{1-\beta}{\beta}\left(\frac{v_H}{p_L + s}\right)$*. L has a mass point of size* $q_L = \frac{1-\beta}{\beta}\left(\frac{v_H}{v_L + s}\right) - \frac{1-\beta}{\beta}\frac{v_H}{\hat{p}_H} \in (0,1)$ *on* v_L*. Finally, L samples prices continuously from* $[\hat{p}_H - s, v_H - s)$ *following the CDF* $F_{L,2}(p_L) = F_{L,1}(p_L)$.

- *L-market consumers are only sometimes served in this equilibrium.*
- *H's equilibrium profit is given by Π_H^{**}, whereas L's equilibrium profit is given by* $[v_H(1-\beta)-s](1-\alpha+\alpha\beta)$.

It is thus apparent that when the flexible H-consumers are endowed with information about firm L's pricing, this drastically changes the equilibrium outcome, given that there are many flexible consumers in the market. Indeed, irrespective of the size distribution of markets, the original pure-strategy equilibrium fails to exist, and in every emerging mixed-strategy equilibrium, both firms compete actively for the mass $\alpha\beta$ of flexible H-consumers. However, the type of mixed-strategy equilibrium depends crucially on the size of α. If α is not too high such that market L is relatively large compared to market H ($\alpha \leq \hat{\alpha}(\beta)$), firm L never prices above its local consumers' valuation in equilibrium. This is because serving its local captives is more profitable than extracting high rents from incoming flexible H-consumers. Therefore, competition for these consumers takes place at prices below $p_L = v_L$ and below $p_H = v_L + s$. Interestingly, for a sufficiently low α ($\alpha \leq \tilde{\alpha}(\beta) < \hat{\alpha}(\beta)$), firm H always competes aggressively for its local flexible consumers, and never finds it optimal to price at v_H in order to maximally exploit its captive consumers. If instead α is large ($\alpha > \hat{\alpha}(\beta)$), L prices above v_L with positive probability in equilibrium, as selling at a high price to the large mass of flexible H-consumers becomes attractive. In turn, firm H also spreads some probability mass in the corresponding price region (on top of pricing at v_H with positive probability).

Figure 6 showcases the different equilibrium regions of the perfect-information framework in (α, β)-space.

I conclude this section with some words on total welfare and consumer surplus in the perfect-information framework. Note first that with perfect information, the social first-best can no longer be achieved, as wasteful search activities take place with positive probability in every equilibrium region. Hence, regulators striving to maximize market efficiency should be cautious when implementing policies that improve consumers' access to price information from outside markets, as this may backfire when promoting excessive search.[35]

[35] Of course, with downward-sloping demand, there would typically be a trade-off between inducing wasteful search frictions and reducing deadweight loss (by increasing competition), such that no clear-cut recommendation can be given.

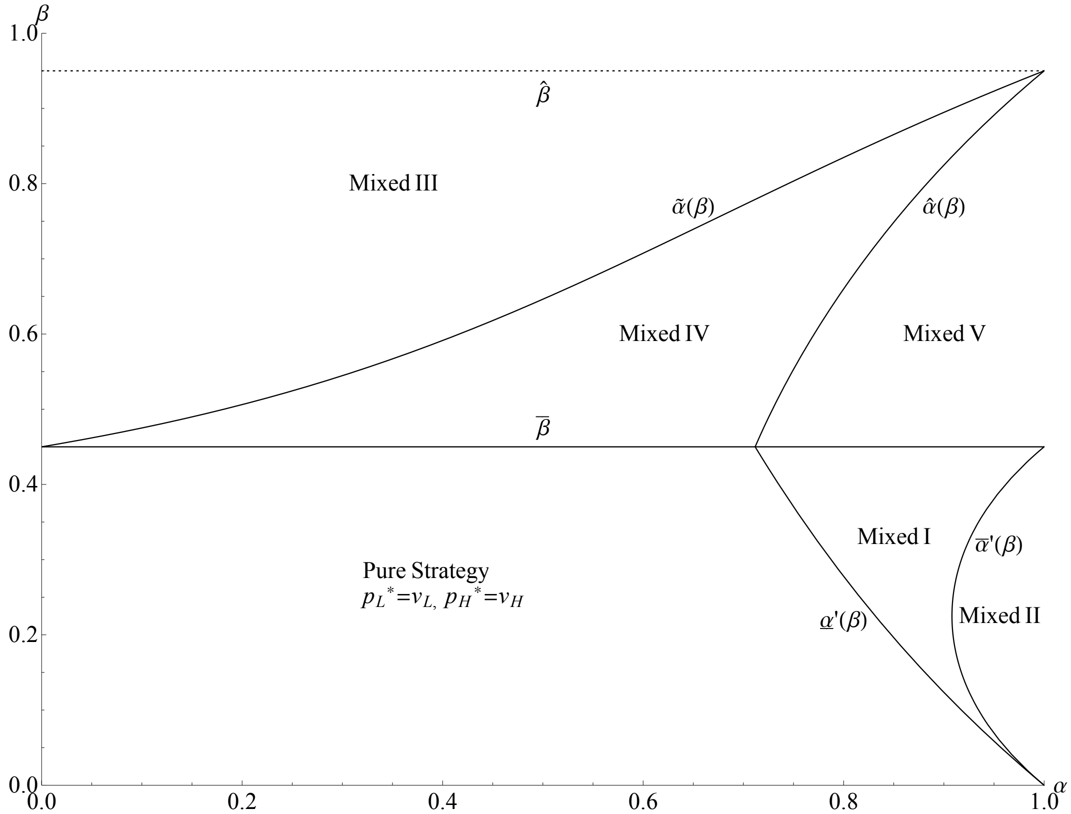

Figure 6: Equilibrium regions with observable p_L for $v_H = 200$, $v_L = 100$, $s = 10$.

On the other hand, making price information available to consumers tends to intensify price competition, such that consumers in both markets may benefit from it. In particular, with inelastic demand, L-market consumers strictly benefit from making their local market's price visible whenever $\beta > \overline{\beta}$, as in this case they start to make a positive surplus with strictly positive probability, which was not possible in the baseline model with imperfect information. Consequently, better price information may improve consumer surplus at the cost of total social welfare.

6 Conclusion

I have analyzed a market configuration in which consumers' price-search behavior is driven by segregation and a local difference in willingness to pay. In the model, two spatially separated firms simultaneously set prices, where initially, each firm's price is only observed by its local consumer base. However, a group of "flexible" high-valuation consumers may search the non-local market at strictly positive cost, hoping to find a lower price. Inner-city shopping across neighborhoods, or cross-border shopping, are examples of this.

An important feature of this model is that active search may occur in equilibrium, which is driven by an interplay of taste and search-cost heterogeneity. Paradoxically, active search only emerges if the fraction of flexible consumers in the high-valuation market is sufficiently *low*, as otherwise, the local incumbent prefers to price aggressively and thereby discourage its flexible consumers from searching. If active search occurs, the size distribution of markets is crucial for determining the equilibrium outcome. If the proportion of high-valuation consumers is sufficiently large, non-trivial mixed-strategy pricing results: the firm in the low-valuation market sometimes charges high prices in order to exploit incoming searchers (while at the same time excluding its local consumer base), whereas its rival in the high-valuation market sometimes offers discounts which may beat these exploitative prices. If the market imbalance is severe enough, the firm in the high-valuation market offers a deep discount with positive probability, which altogether discourages its local flexible consumers from searching.

Two sources of welfare loss may arise in the market: A wasteful search friction incurred by searching high-valuations consumers, and deadweight loss induced by dropout low-valuation consumers. While the comparative statics of total social welfare and consumer surplus with respect to the flexible consumers' search costs and their fraction in the population are complex, regulators may want to focus on boosting the latter, as both welfare and consumer surplus are maximized once a certain threshold is surpassed. It can be shown that this result also prevails for the case of downward-sloping individual demand.

Lastly, I consider a variation of the baseline model in which consumers are endowed with perfect information, yet still face positive travel costs. If the fraction of flexible high-valuation consumers is large, this induces new types of mixed-strategy equilibria, reduces social welfare, but increases the surplus of low-valuation consumers. Increasing competition through endowing consumers with price information may thus promote consumer surplus at the cost of overall efficiency.

The model can be extended in several dimensions. For example, it would be desirable to allow for a more general (i.e., continuous) search-cost distribution. Preliminary calculations have revealed that the main qualitative features of the characterized equilibria remain intact:

active search can still only occur if not too many high-valuation consumers are tempted to search, and also the same tensions lead to a breakdown of pure-strategy equilibrium. Alternatively, the assumption of homogeneous consumer valuations within submarkets could be relaxed (e.g., consumers' valuations could be normally distributed with two different means across submarkets). Whether such a setup would remain tractable and give rise to qualitatively similar results is an open question for future research. Finally, it would be worthwhile to allow for multiple firms within submarkets, which may still be able to reap positive profits due to local differentiation. With perfect price information and symmetric firms within submarkets, it is conjectured that similar results to a model with downward-sloping individual demand would arise. Indeed, it is straightforward to construct corresponding pure-strategy equilibria.

Overall, the presented framework is a first attempt to model consumers' search across markets that are segregated by location and local demand characteristics. Due to its relevance for important phenomena such as cross-neighborhood or cross-border shopping, its rich predictions, and the numerous possibilities for extensions, it is hoped that fruitful and diverse applications will arise.

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Appendix A: Technical Proofs

Proof of Proposition 1. (Existence) The proposed equilibrium implies the stated firm profits of Π_H^* and Π_L^*. Given that H prices at $v_L + s$ and the flexible H-consumers do not search, L cannot do better than to price at v_L (as pricing higher than v_L would induce all L-consumers to exit the market, and pricing lower than v_L induces no search, as it is unobserved by the flexible H-consumers). Firm H's best deviation is to increase its price to v_H, lose all $\alpha\beta$ flexible H-consumers, but fully exploit its captive consumers. This gives rise to a maximal deviation profit of $\Pi_H^{dev^*} = v_H\alpha(1-\beta)$, which does not exceed Π_H^* if $\beta \geq \overline{\beta}$. $\square$

Proof of Proposition 2. (Existence) The proposed equilibrium implies the stated firm profits of Π_H^{**} and Π_L^{**}. Each firm has a unique optimal deviation to this. First, H can reduce its price to $v_L + s$, discourage the $\alpha\beta$ flexible H-consumers from leaving, and make an optimal deviation profit of $\Pi_H^{dev^{**}} = (v_L + s)\alpha$. By the reverse logic of Proposition 1, this is not profitable if $\beta \leq \overline{\beta}$. Second, L can increase its price to v_H, lose all L-consumers who drop out of the market, but maximally charge the $\alpha\beta$ searching H-consumers. This gives rise to an optimal deviation profit of $\Pi_L^{dev^{**}} = v_H\alpha\beta$, which does not exceed Π_H^{**} if $\alpha \leq \underline{\alpha}(\beta)$. $\square$

Proof of Proposition 3. (Existence) In order for the proposed strategy-combination to form an equilibrium, it is necessary that each price that is sampled by a given firm with positive probability (probability density) must yield the same, maximal expected profit. Furthermore, all equilibrium objects need to be well-behaved (e.g., mass points must fall in the range $[0,1]$). In what follows, I will prove that this is satisfied for the characterized strategies.[36]

Note first that $\underline{p} > v_L$. Suppose now that the flexible H-consumers always search initially (the respective condition will be verified further below), and start with firm L. We thus have that $\Pi_L(v_L) = \Pi_L^{**}$, while for $p_L \in [\underline{p}, v_H]$,

$$\Pi_L(p_L) = p_L\alpha\beta(1 - F_H(p_L)) = \Pi_L^{**}.$$

Moreover, given H's pricing strategy, it is clearly the case that $\Pi_L(p_L) < \Pi_L^{**}$ for $p < v_L$ and $p \in (v_L, \underline{p})$.

Consider firm H next. For $p_H \in [\underline{p}, v_H]$, we have that

$$\Pi_H(p_H) = p_H\alpha(1 - \beta F_L(p_H)) = \Pi_H^{**}.$$

H's best deviation is to price at the flexible consumers' reservation price ρ and thereby prevent the flexible H-consumers from searching. (Assuming still that $\rho < \underline{p}$, ρ thus solves $q_L^*(\rho - v_L) = s$, i.e., $\rho = \frac{s}{q_L^*} + v_L$.) This deviation is not profitable if $\rho\alpha \leq \Pi_H^{**}$, which implies

$$v_L + \frac{s}{q_L^*} - v_H(1-\beta) \leq 0. \tag{4}$$

[36] For a more detailed constructive proof, see Obradovits (2015).

To see this, note first that q_L^* is strictly decreasing in α. Hence, as Proposition 3 requires that $\alpha \le \overline{\alpha}(\beta) = \frac{v_L}{(1-\beta)\left\{v_L + \frac{v_H\beta[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}\right\}}$, q_L^* is bounded below by

$$\frac{1}{\beta} - \frac{v_H(1-\beta)}{v_L\left(\frac{1}{\alpha}-1+\beta\right)}\Bigg|_{\alpha=\overline{\alpha}(\beta)}.$$

Simplifying this expression in a straightforward manner, it follows that $q_L^* \ge \frac{s}{v_H(1-\beta)-v_L}$, which confirms that inequality (4) is indeed satisfied.

I proceed to check that all equilibrium objects are well-behaved. Clearly, $q_H^* > 0$ and $\underline{p} > v_L$, while $q_H^* < 1$ and $\underline{p} < v_H$ both follow from $\alpha > \underline{\alpha}(\beta)$. Also $q_L^* < 1$ follows immediately from $\alpha > \underline{\alpha}(\beta)$. In order to show that $q_L^* > 0$, I use the above finding that $q_L^* \ge \frac{s}{v_H(1-\beta)-v_L}$, where $\frac{s}{v_H(1-\beta)-v_L} > 0$ follows from $\beta < \overline{\beta}$. It is also easy to check that $F_H(\underline{p}) = 0$, $F_H(v_H) = 1 - q_H^*$, $F_L(\underline{p}) = q_L^*$, $F_L(v_H) = 1$, and that both CDFs are strictly increasing in prices.

It still needs to be verified that the flexible H-consumers will always search initially, i.e., $q_L^*(\underline{p} - v_L) > s$. As it was already shown that $q_L^* \ge \frac{s}{v_H(1-\beta)-v_L}$, the above inequality is certainly fulfilled if $\frac{s}{v_H(1-\beta)-v_L}\left(\underline{p} - v_L\right) > s$, which implies $\frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta} > v_H(1-\beta)$. Because the LHS of this inequality strictly decreases in α, it is straightforward to show that this is indeed the case if $\alpha \le \overline{\alpha}(\beta)$ and $\beta < \overline{\beta}$.

Lastly, it remains to prove the claim that $\overline{\alpha}(\beta) \in (\underline{\alpha}(\beta), 1)$ whenever $\beta < \overline{\beta}$. For $\overline{\alpha}(\beta) = \frac{v_L}{(1-\beta)\left\{v_L + \frac{v_H\beta[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}\right\}} > \frac{v_L}{\beta(v_H-v_L)+v_L} = \underline{\alpha}(\beta)$, a straightforward manipulation shows that this is indeed the case (for $\beta < \overline{\beta}$). On the other hand, the inequality $\overline{\alpha}(\beta) < 1$ can be reduced to $[(1-\beta)v_H - v_L]^2 + v_L\beta s > 0$ if $\beta > 0$, which is always satisfied. For $\beta = 0$, it holds that $\overline{\alpha}(\beta) = 1$. □

Proof of Proposition 4. (Existence) Again, in order for the proposed strategy-combination to form an equilibrium, it is necessary that each price that is sampled by a given firm with positive probability (probability density) must yield the same, maximal expected profit. Furthermore, all equilibrium objects need to be well-behaved (e.g., mass points must fall in the range $[0,1]$). As in the proof of Proposition 3, I will show that this is satisfied for the characterized strategies.[37]

[37] A more detailed constructive proof can again be found in Obradovits (2015).

Note first that $\rho^* \in (v_L, \underline{p})$, as follows from $\beta < \overline{\beta}$. Furthermore, ρ^* indeed constitutes the flexible H-consumers' reservation price, as $q^*_{L,v_L}(\rho^* - v_L) = s$. Hence, given H's specified strategy, the flexible H-consumers visit market L with probability $1 - q^*_{H,\rho}$.

If firm L prices at v_L, its profit is given by

$$\Pi_L(v_L) = v_L(1 - \alpha + \alpha\beta(1 - q^*_{H,\rho})) = \Pi_L^{***}.$$

If instead L chooses some price $p_L \in [\underline{p}, v_H]$, it also makes an expected profit of

$$\Pi_L(p_L) = p_L \alpha\beta(1 - F_H(p_L)) = \Pi_L^{***}.$$

Moreover, given H's pricing strategy, clearly $\Pi_L(p_L) < \Pi_L^{***}$ for $p_L < v_L$ and $p_L \in (v_L, \underline{p})$.

Consider firm H next. For $p = \rho^*$, we have that

$$\Pi_H(\rho^*) = \rho^* \alpha = \Pi_H^{**}.$$

If instead H chooses some price $p_H \in [\underline{p}, v_H]$, it follows that

$$\Pi_H(p_H) = p_H \alpha(1 - \beta F_L(p_H)) = \Pi_H^{**}.$$

Moreover, given L's pricing strategy, clearly $\Pi_H(p_H) < \Pi_H^{**}$ for $p_H < \rho^*$ and $p_H \in (\rho^*, \underline{p})$.

I finally check that all equilibrium objects are well-behaved. Observe first that $q^*_{L,v_L} \in (0,1)$ follows directly from $\beta < \overline{\beta}$. Note next that $q^*_{H,\rho} > 0$ is equivalent to $\underline{p} > \frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta}$. As $\underline{p} = \frac{v_H(1-\beta)[v_H(1-\beta)-v_L]}{v_H(1-\beta)-v_L-\beta s}$ does not depend on α while $\frac{v_L(1-\alpha+\alpha\beta)}{\alpha\beta}$ strictly decreases in α, the inequality is hardest to fulfill for the boundary level $\overline{\alpha}(\beta)$. Indeed, after a straightforward calculation, it turns out that the RHS equals the LHS for $\alpha = \overline{\alpha}(\beta)$. Hence, for every $\alpha > \overline{\alpha}(\beta)$ (as required by the proposition), it is in fact the case that $q^*_{H,\rho} > 0$. It is further immediate that $q^*_{H,v_H} > 0$. Note also that $q^*_{H,\rho} + q^*_{H,v_H} = 1 - \frac{1-\alpha}{\alpha\beta}\left(\frac{v_L\beta[v_H(1-\beta)-v_L-s]}{[v_H(1-\beta)-v_L]^2+v_L\beta s}\right)$, which is clearly less than one for $\beta < \overline{\beta}$. Lastly, it is straightforward to verify that $F_H(\underline{p}) = q^*_{H,\rho}$, $F_H(v_H) = 1 - q^*_{H,v_H}$, $F_L(\underline{p}) = q^*_{L,v_L}$, $F_L(v_H) = 1$, and that both CDFs are strictly increasing in prices. □

Proof of Proposition 8. Pure strategy: The proposed equilibrium implies the stated firm profits of Π_H^{**} and Π_L^{**}. Each firm has a unique optimal deviation to this. First, H can reduce its price to $v_L + s$, discourage the $\alpha\beta$ flexible H-consumers from leaving, and make an optimal deviation profit of $\Pi_H^{dev} = (v_L + s)\alpha$. This is not profitable due to $\beta \leq \overline{\beta}$. Second, L can increase its price to $v_H - s$, lose all L-consumers who drop out of the market, but maximally charge the $\alpha\beta$ flexible-H-consumers (for $p_L > v_H - s$, they would not purchase from L).

This gives rise to an optimal deviation profit of $\Pi_L^{dev} = (v_H - s)\alpha\beta$, which does not exceed Π_H^{**} if $\alpha \leq \underline{\alpha}'(\beta)$.

<u>Mixed I:</u> Note first that $\underline{p}_H > v_L + s$, which implies that $\Pi_L(v_L) = \Pi_L^{**}$. On the other hand, for $p_L \in [\underline{p}_H - s, v_H - s)$,

$$\Pi_L(p_L) = p_L\alpha\beta(1 - F_H(p_L + s)) = \Pi_L^{**}.$$

Clearly, $\Pi_L(p_L) < \Pi_L^{**}$ for $p_L < v_L$, $p_L \in (v_L, \underline{p}_H - s)$, and $p_L > v_H - s$.

Next, for $p_H \in [\underline{p}_H, v_H]$, we have that

$$\Pi_H(p_H) = p_H\alpha[1 - \beta F_L(p_H - s)] = \Pi_H^{**},$$

and clearly $\Pi_H(p_H) < \Pi_H^{**}$ for $p_H < \underline{p}_H$.

It remains to check that all equilibrium objects are well-behaved. Observe first that $q_L \geq 0$ follows from $\alpha \leq \overline{\alpha}'(\beta)$, whereas $q_L < 1$ follows from $\alpha > \underline{\alpha}'(\beta)$. Moreover, $q_H > 0$ is immediate, while $q_H < 1$ follows from $\alpha > \underline{\alpha}'(\beta)$. Also $\underline{p}_H < v_H$ is implied by $\alpha > \underline{\alpha}'(\beta)$. Finally, it is easy to check that $F_L(\underline{p}_H - s) = q_L$, $F_L(v_H - s) = 1$, $F_H(\underline{p}_H) = 0$, $F_H(v_H) = 1 - q_H$, and that both CDFs are strictly increasing in prices.

<u>Mixed II:</u> Note first that $\underline{p}_H \geq v_L + s$ due to $\beta \leq \overline{\beta}$, which implies that $\Pi_L = \Pi_L(\underline{p}_H - s) = (\underline{p}_H - s)\alpha\beta$. Also for $p_L \in (\underline{p}_H - s, v_H - s)$, it holds that

$$\Pi_L(p_L) = p_L[\alpha\beta(1 - F_H(p_L + s))] = \Pi_L.$$

Clearly, $\Pi_L(p_L) < \Pi_L$ for $p_L \in (v_L, \underline{p}_H - s)$ and $p_L > v_H - s$. On the other hand, L's best possible deviation is to price at $v_L < \underline{p}_H - s$ and also serve its local consumers, which gives a maximal deviation profit of $\Pi_L^{dev} = v_L(1 - \alpha + \alpha\beta)$. It is however easy to check that $\Pi_L^{dev} < \Pi_L$ due to $\alpha > \overline{\alpha}'(\beta)$.

Next, for $p_H \in [\underline{p}_H, v_H]$, we have that

$$\Pi_H(p_H) = p_H\alpha[1 - \beta F_L(p_H - s)] = \Pi_H^{**},$$

and clearly $\Pi_H(p_H) < \Pi_H^{**}$ for $p_H < \underline{p}_H$.

It remains to check that all equilibrium objects are well-behaved. Observe first that $q_H > 0$ follows from $\beta \leq \overline{\beta}$, whereas $q_H < 1$ is immediate. Also $\underline{p}_H < v_H$ is immediate. Finally, it is easy to check that $F_L(\underline{p}_H - s) = 0$, $F_L(v_H - s) = 1$, $F_H(\underline{p}_H) = 0$, $F_H(v_H) = 1 - q_H$, and that both CDFs are strictly increasing in prices.

$\square$

Proof of Proposition 9. Mixed III: First, for $p_L \in [\underline{p}_L, v_L]$, we have that

$$\Pi_L(p_L) = p_L[1-\alpha+\alpha\beta(1-F_H(p_L+s))] = v_L(1-\alpha) = \Pi_L.$$

Clearly, $\Pi_L(p_L) < \Pi_L$ for $p_L < \underline{p}_L$ and $p_L > v_L$.

Next, for $p_H \in [\underline{p}_L + s, v_L + s)$, it holds that

$$\Pi_H(p_H) = p_H\alpha[1-\beta F_L(p_H-s)] = \alpha(\underline{p}_L + s) = \Pi_H,$$

and clearly $\Pi_H(p_H) < \Pi_H$ for $p_H < \underline{p}_L + s$. H's best possible deviation above $v_L + s$ is to price at v_H for a maximal deviation profit of $\Pi_{dev} = v_H\alpha(1-\beta)$. However, this is not profitable due to $\alpha \leq \tilde{\alpha}(\beta)$ (and it is easy to check that $\beta \geq \hat{\beta}$ is sufficient for this).

It remains to check that all equilibrium objects are well-behaved. Observe first that $q_L > 0$ can be simplified to $v_L + s > \frac{v_L\alpha}{1-\alpha+\alpha\beta}$. Since the RHS of this inequality is strictly increasing in α, it is hardest to satisfy for $\alpha = \tilde{\alpha}(\beta)$. Inserting $\tilde{\alpha}(\beta)$ and simplifying yields the requirement $v_L + s > \frac{v_L - [v_H(1-\beta)-s]}{\beta}$, which is indeed true due to $s < v_H - v_L$. On the other hand, $q_L < 1$ can be reduced to $\underline{p}_L < v_L$, which is evidently the case. Finally, it is easy to check that $F_L(\underline{p}_L) = 0$, $F_L(v_L) = 1 - q_L$, $F_H(\underline{p}_L + s) = 0$, $F_H(v_L + s) = 1$, and that both CDFs are strictly increasing in prices.

Mixed IV: Note first that $\Pi_H(v_H) = \Pi_H^{**}$. Also for $p_H \in [\underline{p}_H, v_L + s)$, it holds that

$$\Pi_H(p_H) = p_H\alpha[1-\beta F_L(p_H-s)] = \Pi_H^{**}.$$

Clearly, $\Pi_H(p_H) < \Pi_H^{**}$ for $p_H < \underline{p}_H$ and $p_H \in [v_L + s, v_H)$.

Next, for $p_L \in [\underline{p}_H - s, v_L]$, it holds that

$$\Pi_L(p_L) = p_L[1-\alpha+\alpha\beta(1-F_H(p_L+s))] = (1-\alpha+\alpha\beta)(\underline{p}_H - s) = \Pi_L,$$

and clearly $\Pi_L(p_L) < \Pi_L$ for $p_L < \underline{p}_H - s$. L's best possible deviation above v_L is to price at $v_H - s$ for a maximal deviation profit of $\Pi_{dev} = (v_H - s)\alpha\beta q_H = (v_H - s)\alpha\beta\left[1 - \frac{1-\alpha+\alpha\beta}{\alpha\beta}\left(1 - \frac{p_H - s}{v_L}\right)\right]$. However, this is not profitable due to $\alpha \leq \hat{\alpha}(\beta)$.

It remains to check that all equilibrium objects are well-behaved. Observe first that $q_H > 0$ follows from $\alpha > \tilde{\alpha}(\beta)$, whereas $q_H < 1$ follows from $\beta > \overline{\beta}$. Moreover, $q_L > 0$ follows from $s < v_H - v_L$, whereas $q_L < 1$ follows from $\beta > \overline{\beta}$. It is also easy to see that

$\underline{p}_H < v_L + s$ due to $\beta > \overline{\beta}$. Finally, one can check that $F_L(\underline{p}_H - s) = 0$, $F_L(v_L) = 1 - q_L$, $F_H(\underline{p}_H) = 0$, $F_H(v_L + s) = 1 - q_H$, and that both CDFs are strictly increasing in prices.

<u>Mixed V:</u> Note first that $\Pi_H(v_H) = \Pi_H^{**}$. Also for $p_H \in [\underline{p}_H, v_L + s)$, it holds that

$$\Pi_H(p_H) = p_H \alpha[1 - \beta F_{L,1}(p_H - s)] = \Pi_H^{**}.$$

And again for $p_H \in [\hat{p}_H, v_H)$, we have that

$$\Pi_H(p_H) = p_H \alpha[1 - \beta F_{L,2}(p_H - s)] = \Pi_H^{**}.$$

Clearly, $\Pi_H(p_H) < \Pi_H^{**}$ for $p_H < \underline{p}_H$ and $p_H \in [v_L + s, \hat{p}_H)$.

Next, for $p_L \in [\underline{p}_H - s, v_L]$, it holds that

$$\Pi_L(p_L) = p_L[1 - \alpha + \alpha\beta(1 - F_{H,1}(p_L + s))] = (1 - \alpha + \alpha\beta)(\underline{p}_H - s) = \Pi_L,$$

while also for $p_L \in [\hat{p}_H - s, v_H - s)$,

$$\Pi_L(p_L) = p_L \alpha\beta(1 - F_{H,2}(p_L + s)) = \Pi_L.$$

Once more, we clearly have that $\Pi_L(p_L) < \Pi_L$ for $p_L < \underline{p}_H - s$, $p_L \in (v_L, \hat{p}_H - s)$, and $p_L \geq v_H - s$.

It remains to check that all equilibrium objects are well-behaved. Note first that $\hat{p}_H > v_L + s$ follows from direct calculation, using the fact that the denominator of $\hat{p}_H$ is strictly positive. The latter is implied by $\alpha > \tilde{\alpha}(\beta)$, which is true since by assumption $\alpha > \hat{\alpha}(\beta)$, and it is straightforward to verify that $\hat{\alpha}(\beta) > \tilde{\alpha}(\beta)$. Likewise, $\hat{p}_H < v_H$ follows straightforwardly from $\alpha > \hat{\alpha}(\beta)$.

Next, $q_H > 0$ follows directly from $\beta < \hat{\beta}$, as assumed. On the other hand, the requirement $q_H < 1$ can be transformed to the condition $\alpha > \check{\alpha}(\beta) := \frac{v_H(1-\beta)-s}{(1-\beta)[v_H(1-\beta)-s]+(v_H-s)\beta}$. A straightforward calculation reveals that this is indeed implied by $\alpha > \hat{\alpha}(\beta)$ and $\beta > \overline{\beta}$ (namely, it holds that $\hat{\alpha}(\beta) > \check{\alpha}(\beta)$ for $\beta > \overline{\beta}$). That $q_L > 0$ is a direct implication of $\hat{p}_H > v_L + s$, as established above. Moreover, $q_L < 1$ follows immediately from $\beta > \overline{\beta}$ and $\hat{p}_H < v_H$, where the latter has already been established.

Finally, it is easy to verify that $F_{L,1}(\underline{p}_H - s) = 0$, $q_L = F_{L,2}(\hat{p}_H - s) - F_{L,1}(v_L)$, $F_{L,2}(v_H - s) = 1$, $F_{H,1}(\underline{p}_H) = 0$, $F_{H,2}(v_H) = 1 - q_H$, and that all CDFs are strictly increasing in prices.

$\square$

Appendix B: Uniqueness of Baseline Equilibria

In what follows, I will show that the equilibria of Propositions 1 to 4 are unique, and that this even holds when consumers in market L have the ability to search market H (at strictly positive cost). To allow for the latter, assume that consumers' search costs in market L follow an arbitrary distribution $G(s) = \mathbb{P}(\tilde{s} \leq s)$, with lower bound $s_L > 0$.

Lemma 1. *No firm will ever price below v_L in any equilibrium.*

Proof. Denote the lower support bound of firm i's $\in \{L, H\}$ pricing strategy by $\underline{p}_i$, such that it is to be shown that $\underline{p}_i \geq v_L$. To see this, assume to the contrary that $\underline{p}_i < v_L$. Then, consumers who observe a price in the range $[\underline{p}_i, \underline{p}_i + \min\{s, s_L\})$ (where $\underline{p}_i + \min\{s, s_L\} < v_H$ by $\underline{p}_i < v_L$ and $v_L + s < v_H$) will never find it optimal to search, as their respective search cost would surely exceed any price saving. In turn, instead of pricing at or slightly above $\underline{p}_i$, firm i could profitably deviate by transferring this probability mass to a point close below $\underline{p}_i + \min\{s, s_L\}$ (if $\underline{p}_i + \min\{s, s_L\} \leq v_L$) or to v_L (if $\underline{p}_i + \min\{s, s_L\} > v_L$), as doing so would not decrease its demand. $\square$

Corollary 1. *L-market consumers will never search in equilibrium.*

Proof. Since $\underline{p}_i \geq v_L$ due to Lemma 1, it can never be profitable to search for L-market consumers, as their expected surplus when doing this would be negative due $s_L > 0$. $\square$

I proceed to prove uniqueness by a sequence of claims. In order to make the argument less tedious, I henceforth assume that flexible H-consumers who are indifferent between searching and purchasing at H will always do the latter, as motivated in the model setup.

(A) Pure-strategy equilibria

Claim 1. *In any pure-strategy equilibrium, $p_L^* = v_L$.*

Proof. Due to Lemma 1, it remains to be shown that $p_L^* > v_L$ cannot be part of any pure-strategy equilibrium. Suppose to the contrary that it does (and hence, that L cannot sell to its local consumers). Then it must hold that $p_L^* + s < v_H$. Otherwise, the flexible H-consumers would not even search L after observing $p_H = v_H$, implying that L would not attract any consumers (hence, L could profitably deviate by charging v_L). Given a pure p_L^*

(with $p_L^* + s < v_H$), there is no need for H to randomize, such that one of the following two options maximizes its profit. First, if β is sufficiently large, H finds it optimal to price at $p_L^* + s$ and discourage its local flexible consumers from searching. But this cannot be part of an equilibrium, as L would face zero demand (and could again profitably deviate to v_L). Second, if β is not that large, H finds it optimal to charge v_H. This fully exploits its local captive consumers, but due to $p_L^* + s < v_H$, the flexible H-consumers would all search and purchase at L. However, this cannot be part of an equilibrium either, as then L would have a profitable deviation by increasing its price to $v_H - \varepsilon$. As this is unobserved by the flexible H-consumers, they would still search L and purchase there. Hence, a pure-strategy equilibrium in which L charges a higher price than v_L can be ruled out. □

Claim 2. *There are only two possible pure-strategy equilibria. First, if and only if* $\beta \geq \overline{\beta} = 1 - \frac{v_L+s}{v_H}$, *the pair* $p_L^* = v_L$, $p_H^* = v_L + s$ *constitutes an equilibrium. Second, if and only if* $\beta \leq \overline{\beta}$ *and* $\alpha \leq \underline{\alpha}(\beta) = \frac{v_L}{\beta(v_H - v_L) + v_L}$, *the pair* $p_L^* = v_L$, $p_H^* = v_H$ *constitutes an equilibrium.*

Proof. From Claim 1 it is known that $p_L^* = v_L$ in any pure-strategy equilibrium. Given this price and the flexible H-consumers' optimal search behavior, it is easy to check that H finds it strictly optimal to charge $v_L + s$ if $\beta > \overline{\beta}$, while it finds it strictly optimal to charge v_H if $\beta < \overline{\beta}$ (for $\beta = \overline{\beta}$, H is indifferent). In the first case, since the flexible H-consumers are discouraged from searching, L cannot have a profitable deviation, as it already maximally exploits its local consumers. In the second case, since $v_H - v_L > s$ by assumption, all $\alpha\beta$ flexible H-consumers find it optimal to search and buy at L. However, given this, L may have a profitable deviation by increasing its price to $v_H - \varepsilon$ (for ε sufficiently small), driving out its local consumers, but fully exploiting the incoming searchers. It is easy to check that this is not the case if and only if $\alpha \leq \underline{\alpha}(\beta)$. □

(B) Mixed-strategy equilibria

In what follows, denote the lower support bound of firm i's pricing strategy, where $i \in \{L, H\}$, by $\underline{p}_i$ and the upper support bound by $\overline{p}_i$, with $\overline{p}_i > \underline{p}_i$.

Claim 3. $\underline{p}_L = v_L$.

Proof. Suppose to the contrary that $\underline{p}_L \in (v_L, v_H)$.[38] Then the L-consumers will never buy in equilibrium. Moreover, it has to hold that $\underline{p}_H \geq \min\{\underline{p}_L + s, v_H\}$, as H can already guarantee to discourage its flexible consumers from searching for this price. Hence, pricing any lower cannot be optimal (unless $\underline{p}_L + s > v_H$, in which case v_H should be chosen). But in turn, L could profitably transfer all probability mass at and close above $\underline{p}_L$ in the direction of $\min\{\underline{p}_L + s, v_H\}$, which contradicts the assertion that $\underline{p}_L$ can be L's lower support bound. $\square$

Claim 4. *L must have a mass point at v_L.*

Proof. Given $\underline{p}_L = v_L$, it must again hold that $\underline{p}_H \geq v_L + s$ (see the proof of Claim 3 above). In turn, L cannot find it optimal to price close above v_L, as transferring this probability mass to a price close below $v_L + s$ would not decrease its demand. Hence, there is certainly a hole in L's equilibrium price distribution close above v_L, such that L must have a mass point at v_L due to $\underline{p}_L = v_L$ (see Claim 3). $\square$

Claim 5. $\overline{p}_H = v_H$.

Proof. Suppose to the contrary that $\overline{p}_H \in (v_L + s, v_H)$.[39] Then there are two possibilities. First, suppose that the flexible H-consumers search if they observe $\overline{p}_H$. Then, since it must hold that $\overline{p}_L \leq \overline{p}_H$ (otherwise, L would not make any sales for prices where $p_L > \overline{p}_H$, implying zero profits), H could make a higher profit by pricing at v_H instead of $\overline{p}_H$, as this would not lose any additional consumers. Hence this cannot be part of an equilibrium. Second, suppose that the flexible H-consumers do not search at $\overline{p}_H$. If they strictly prefer not to search, H would have a profitable deviation by pricing slightly higher. If the flexible H-consumers are indifferent between searching and not searching for $p_H = \overline{p}_H$, it follows that H should concentrate all probability mass at $\overline{p}_H$ (as it makes no sense to charge any price lower than $\overline{p}_H$ if the latter already discourages the flexible H-consumers from searching). That is, in the respective equilibrium, H would charge some deterministic price $p_H^* \in (v_L + s, v_H)$, and the flexible H-consumers would all stay at H (being indifferent between searching and not searching). But then, L's dominant action would be to charge v_L with full probability mass

[38]Clearly, $\underline{p}_L = v_H$ cannot be part of an equilibrium, as this would never generate search from H, implying zero profits by L.

[39]Since $\underline{p}_L = v_L$, it follows that $\underline{p}_H \geq v_L + s$. For a mixed-strategy equilibrium, it also cannot hold that $\underline{p}_H = \overline{p}_H = v_L + s$ (such that H plays a pure strategy), as this would induce L to play the pure strategy v_L, giving rise to a pure-strategy equilibrium. Hence $\overline{p}_H > v_L + s$ must hold.

in order to maximally exploit its local consumers. In turn, since the hypothesized p_H^* is larger than $v_L + s$, the flexible H-consumers *should* optimally search: a contradiction. $\square$

From now on, let $\underline{p}'_L$ denote the lower support bound of L's pricing strategy for prices that strictly exceed v_L. Furthermore, let ρ denote the flexible H consumers' reservation price, i.e. the price which makes them indifferent between visiting L and purchasing directly at H.

Claim 6. $\rho < v_H$.

Proof. Suppose to the contrary that $\rho \geq v_H$. Then the flexible H-consumers would not even search for $p_H = v_H$, which implies $p_H^* = v_H$ and $p_L^* = v_L$. But then, $\rho = v_L + s < v_H$, a contradiction. $\square$

Claim 7. $\rho \in (v_L + s, \underline{p}'_L]$.

Proof. First, $\rho > v_L + s$ follows directly from $\underline{p}_L = v_L$ (see Claim 3) and the fact that L does not put full probability mass on v_L (if it did, a pure-strategy equilibrium would result). Hence, in order to make the flexible H-consumers indifferent between searching L and purchasing at H, choosing a price slightly larger than $v_L + s$ is sufficient for H. Second, in order to establish that $\rho \leq \underline{p}'_L$, suppose to the contrary that $\rho > \underline{p}'_L$. But then, the positive probability mass that L puts in the range $[\underline{p}'_L, \rho)$ could profitably be transferred to ρ, as the flexible H-consumers will not search anyway if H samples a price that is weakly lower than ρ (hence, charging $p_L = \rho$ already beats all the prices H may set which induce search). $\square$

Claim 8. *H must have a mass point at v_H.*

Proof. Suppose to the contrary that $\overline{p}_H = v_H$ (as established by Claim 5), but H has no mass point at v_H. Then there exists some $d > 0$ such that H puts a probability mass of less than $\frac{v_L(1-\alpha)}{v_H} < 1$ in the interval $[v_H - d, v_H]$. This in turn implies that L will not find it optimal at all to sample prices $p_L \geq v_H - d$, i.e., it must hold that $\overline{p}_L \leq v_H - d$. This is because by pricing in that interval, L's profit is bounded above by $v_H * \Pr\{\tilde{p_H} \geq v_H - d\} < v_H * \frac{v_L(1-\alpha)}{v_H} = v_L(1-\alpha)$, where the latter profit could be guaranteed if L priced at v_L. Because of this, H also cannot find it optimal to put any probability mass in the interval $(v_H - d, v_H)$.[40] But

[40] As $\rho \leq \underline{p}'_L \leq \overline{p}_L$ (where the first inequality follows from Claim 7), any price that H samples in $(v_H - d, v_H)$ would induce search by the flexible H-consumers, which leads them to leave H with certainty (due to $\overline{p}_L \leq v_H - d$). Therefore, H strictly prefers to sample v_H in order to maximally exploit its captive consumers.

if H has no mass point at v_H, this leads to a contradiction, as it would then follow that $\overline{p}_H \leq v_H - d$. □

Claim 9. *L cannot have a mass point at v_H.*

Proof. Note that H prices at v_H with strictly positive probability due to Claim 8, and that the flexible H-consumers always search at this price due to Claim 6. Then, if L also had a mass point at v_H, ties would arise with positive probability. In turn, no matter how the flexible H-consumers' indifference was resolved, at least one firm would have an incentive to undercut v_H marginally and break the indifference in its favor. □

Claim 10. *If H samples ρ in equilibrium, it must be the case that H has a mass point at ρ, and that there is no probability mass below ρ or immediately above ρ.*

Proof. First, it is clear that H will not put any probability mass below ρ, as already pricing at ρ ensures that all H-consumers will stay in H. Moreover, since L has a mass point at v_L (see Claim 4), pricing marginally above ρ entails a discrete loss for H (since the flexible H-consumers will search and find a price of v_L with positive probability). Hence, H cannot put any probability mass immediately above ρ. Then, the fact that H can only sample ρ directly (and not any prices very close to ρ) implies that H must have a mass point at ρ if ρ is sampled at all in equilibrium. □

Claim 11. *H cannot put any probability mass in $(\rho, \underline{p}'_L)$.*[41]

Proof. Suppose to the contrary that this was the case. Then any price p_H in that interval would lead to search by the flexible H-consumers. However, because L will only sell to these searching consumers if it prices at v_L (due to $p_H < \underline{p}'_L$), H has a profitable deviation to transfer all of its probability mass in $(\rho, \underline{p}'_L)$ to a price arbitrarily close to $\underline{p}'_L$. □

From now on, let $\underline{p}'_H$ denote the lower support bound of H's pricing strategy for prices that strictly exceed ρ.

Claim 12. $\underline{p}'_L = \underline{p}'_H =: \underline{p}$.

[41]Recall that $\rho \leq \underline{p}'_L$ due to Claim 7.

Proof. Claim 11 already established that $\underline{p}'_H \geq \underline{p}'_L$. It remains to show that it cannot be the case that $\underline{p}'_H > \underline{p}'_L$. To see this, suppose that the relation holds. But then, due to $\underline{p}'_L \geq \rho$ (see Claim 7), the flexible H consumers will always search when H does not price at ρ (if it does so at all), and thus L could profitably deviate by transferring all of its probability mass in the interval $[\underline{p}'_L, \underline{p}'_H)$ to a price arbitrarily close below $\underline{p}'_H$. $\square$

Claim 13. $\overline{p}_L = \overline{p}_H = v_H$.

Proof. The second equality is given by Claim 5. To establish the first, suppose to the contrary that $\overline{p}_L < \overline{p}_H = v_H$. Then clearly, because $\rho \leq \overline{p}_L$ (as follows from Claim 7), H will not find it optimal to put any probability mass in $(\overline{p}_L, v_H)$, as this is strictly dominated by pricing at v_H (and at least fully exploiting its captive consumers). But in turn, it cannot be optimal for L to sample $\overline{p}_L$, as this will only win H's flexible consumers if H prices at v_H. Hence, by deviating to $v_H - \varepsilon$, L could increase its profit. $\square$

Claim 14. *Neither H nor L can have a mass point in* $[\underline{p}, v_H)$.

Proof. Suppose to the contrary that H (L) does have a mass point at some price $\hat{p} \in [\underline{p}, v_H)$. Then there must exist some $d > 0$ such that L (H) will never find it optimal to price in $(\hat{p}, \hat{p} + d]$, as L's (H's) profit drops discontinuously at $\hat{p}$. But then, H (L) should not have a mass point at $\hat{p}$ in the first place, as pricing closer to $\hat{p} + d$ would give the firm a strictly higher profit. $\square$

Claim 15. *If one firm puts no probability mass in some interval* $[a, b] \subset [\underline{p}, v_H)$*, the other firm cannot do so either.*

Proof. Suppose to the contrary that only one firm puts positive probability mass in $[a, b]$. But then, it must have a mass point at b, as pricing anywhere in $[a, b)$ gives the firm a strictly lower expected profit than pricing at b. However, this contradicts Claim 14. $\square$

Claim 16. *The firms cannot have any holes in their pricing range above* $\underline{p}$.

Proof. Suppose they do. Then, examine the lowest of such holes, and denote its infimum by $z > \underline{p}$. Clearly, as the firms can have no mass points in $[\underline{p}, v_H)$ due to Claim 14, it cannot be optimal for either firm to price at z and close below, as shifting this probability mass towards the top of the lowest hole yields a strictly higher expected profit. In particular, this is always possible due to Claim 15. $\square$

To sum up, the above claims establish the following. First, only two pure-strategy equilibria exist. These are given by $p_L^* = v_L$ and $p_H^* = v_L + s$ (for $\beta \geq \overline{\beta}$) and $p_L^* = v_L$, $p_H^* = v_H$ (for $\beta \leq \overline{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$). Second, any mixed-strategy equilibrium must satisfy the following: (i) no firm ever prices below v_L, (ii) L has a mass point on v_L, (iii) H has a mass point on v_H, (iv) if H samples the flexible H-consumers' reservation price ρ, it must have a mass point on it (and it holds that $v_L < \rho \leq \underline{p}_L'$), (v) there can be no other mass points, (vi) both L and H spread probability mass over a common interval $[\underline{p}, v_H]$, and (vii) this interval does not contain any holes.

It is now straightforward to check that each of the four equilibria in Propositions 1 to 4 fulfills these criteria. Moreover, it can be verified that only the specified equilibria in their respective parameter ranges do indeed constitute equilibria.

More specifically, Proposition 1 states that whenever $\beta > \overline{\beta}$, the unique equilibrium of the game is such that $p_L^* = v_L$, $p_H^* = v_L + s$, and no consumers search. The candidate equilibria of Propositions 2 to 4 must all fail, as in each of these, firm H's expected profit is given by $v_H\alpha(1-\beta)$, which, for $\beta > \overline{\beta}$, is strictly worse than what H could achieve by deviating to $v_L + s$ (and thereby, discouraging its local consumers from searching).

Proposition 2 states that whenever $\beta < \overline{\beta}$ and $\alpha \leq \underline{\alpha}(\beta)$, the unique equilibrium of the game is such that $p_L^* = v_L$ and $p_H^* = v_H$. Clearly, the equilibrium of Proposition 1 must fail, because for $\beta < \overline{\beta}$, H finds discouraging its local consumers from searching by pricing at $v_L + s$ to be dominated by pricing at v_H. The equilibrium of Proposition 3 must fail, for example because $\alpha \leq \underline{\alpha}(\beta)$ implies $q_H^* \geq 1$ (with strict equality for $\alpha < \underline{\alpha}(\beta)$). The equilibrium of Proposition 4 must also fail, as $\alpha \leq \underline{\alpha}(\beta)$ together with $\beta < \overline{\beta}$ implies that $q_{H,\rho}^* < 0$.

Proposition 3 claims that there is a unique mixed-strategy equilibrium with mass points on v_H (by H) and v_L (by L) if $\beta < \overline{\beta}$ and $\alpha \in (\underline{\alpha}(\beta), \overline{\alpha}(\beta)]$. Again, the pure-strategy equilibrium of Proposition 1 must fail because discouraging its local consumers from searching by pricing at $v_L + s$ is not profitable for H when $\beta > \overline{\beta}$. The pure-strategy equilibrium of Proposition 2 must also fail, as $\alpha > \underline{\alpha}(\beta)$ implies that L could profitably deviate from $p_L^* = v_L$ to charging v_H. The equilibrium of Proposition 4 must fail, as $\alpha \leq \overline{\alpha}(\beta)$ implies $q_{H,\rho}^* \leq 0$.

Proposition 4 says that there is a unique mixed-strategy equilibrium with mass points on v_H and $\rho^* = v_H(1-\beta)$ (by H) and v_L (by L) if $\beta < \overline{\beta}$ and $\alpha > \overline{\alpha}(\beta)$. Clearly, the equilibria of Propositions 1 and 2 must fail due to the same reasons as given above (in particular, the equilibrium of Proposition 2 can be ruled out because $\alpha > \overline{\alpha}(\beta)$ implies $\alpha > \underline{\alpha}(\beta)$ for $\beta < \overline{\beta}$). Lastly, the equilibrium of Proposition 3 must fail because of the following. In this candidate equilibrium, L charges v_L with probability $q_L^* = \frac{1}{\beta} - \frac{v_H\alpha(1-\beta)}{v_L(1-\alpha+\alpha\beta)}$, while both L and H sample prices from an identical continuous interval $[\underline{p}, v_H]$, with $\underline{p} = \frac{v_L(1-\alpha+\alpha\beta)}{v_H\alpha\beta} > v_L$. Suppose that q_L^* is well-behaved, such that it falls in the range $(0,1)$ (if this is not the case, the considered equilibrium fails anyway). Now there are three situations to consider. In parameter constellations where $q_L^*(\underline{p} - v_L) > s$, the flexible H-consumers would even find it optimal to search L if firm H charged its lowest equilibrium price $\underline{p}$. A potentially profitable deviation by H is then to charge a price $\rho' < \underline{p}$ that satisfies $q_L^*(\rho' - v_L) = s$ (i.e., $\rho' = \frac{s}{q_L^*} + v_L$), which is sufficient to discourage the flexible H-consumers from searching. This gives H a deviation profit of $\rho'\alpha = \left(\frac{s}{q_L^*} + v_L\right)\alpha$, compared to the candidate equilibrium's profit of $v_H\alpha(1-\beta)$. Hence, the equilibrium of Proposition 3 fails to exist if $\frac{s}{q_L^*} + v_L > v_H(1-\beta)$. Since q_L^* is strictly decreasing in α and $q_L^* = \frac{s}{v_H(1-\beta)-v_L}$ for $\alpha = \overline{\alpha}(\beta)$, it is easy to check that this is indeed the case if $\alpha > \overline{\alpha}(\beta)$. For parameter constellations where $q_L^*(\underline{p} - v_L) < s$ (such that the flexible H-consumers would not want to search when facing a price of $\underline{p}$ or slightly higher), the candidate equilibrium of Proposition 3 also breaks down. This is because firm H could sample (slightly) higher prices than $\underline{p}$ without losing any demand, implying a profitable deviation. Finally, if it holds exactly that $q_L^*(\underline{p} - v_L) = s$, the flexible H-consumers would not search if H charges exactly $\underline{p}$, but they would do so for any higher price in the firm's pricing domain. It follows that H would have a profitable deviation by transferring probability mass from close above $\underline{p}$ to $\underline{p}$.