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Backtesting VaR Models: A Two-Stage Procedure

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Abstract

Academics and practitioners have extensively studied Value-at-Risk (VaR) to propose a unique risk management technique that generates accurate VaR estimations for long and short trading positions. However, they have not succeeded yet as the developed testing frameworks have not been widely accepted. A two-stage backtesting procedure is proposed in order a model that not only forecasts VaR but also predicts the loss beyond VaR to be selected. Numerous conditional volatility models that capture the main characteristics of asset returns (asymmetric and leptokurtic unconditional distribution of returns, power transformation and fractional integration of the conditional variance) under four distributional assumptions (normal, GED, Student-t, and skewed Student-t) have been estimated to find the best model for three financial markets (US stock, gold and dollar-pound exchange rate markets), long and short trading positions, and two confidence levels. By following this procedure, the risk manager can significantly reduce the number of competing models.

Keywords: Backtesting, Value-at-Risk, Expected Shortfall, Volatility Forecasting, Arch Models.

JEL: C22, C52, G15.

1. Introduction

The need of major financial institutions to measure their risk started in 1970s after an increase in financial instability. Baumol (1963) first attempted to estimate the risk that financial institutions faced. He proposed a measure based on standard deviation adjusted to a confidence level parameter that reflects the user's attitude to risk. However, this measure is not different from the widely known Value-at-Risk (VaR), which refers to a portfolio's worst outcome that is likely to occur at a given confidence level. According to the proposal of Basle Committee, the VaR methodology can be used by financial institutions to calculate capital charges in respect of their financial risk and, hence, academics and practitioners were triggered to find the best-performing risk management technique. However, even now, the results are conflicting and confusing.

Giot and Laurent (2003a) calculated VaR for long and short equity trading positions and proposed the APARCHⁱ model with skewed Student-t conditionally distributed innovations (APARCH-skT) as it had the best overall performance in terms of the proportion of failure test. In a similar study, Giot and Laurent (2003b) suggested the same model to the risk managers, even if a simpler model (ARCH-skT) generated accurate VaR forecasts. Huang and Lin (2004) argued that for the Taiwan Stock Index Futures, the APARCH model under the normal (Student-t) distribution must be used by risk managers at the lower (higher) confidence level.

Although the APARCH model comprises several volatility specifications, its superiority has not been proved by all researchers. Angelidis and Degiannakis (2005) opined that “a risk manager must employ different volatility techniques in order to forecast accurately VaR for long and short trading positions”, whereas Angelidis et al. (2004) argued that “the Arch structure that produces the most accurate VaR forecasts is different for every portfolio”. Furthermore, Guermat and Harris (2002) applied an exponentially weighted likelihood model in three equity portfolios (US, UK, and Japan) and proved its superiority to the GARCH model under the normal and the Student-t distributions in terms of two backtesting measures (unconditional and conditional coverage). Moreover, Degiannakis (2004) studied the forecasting performance of various risk models to estimate the one-day-ahead realized volatility and the daily VaR. He proposed the fractional integrated APARCH model with skewed Student-t conditionally distributed innovations (FIAPARCH-skT) that efficiently captures the main characteristics of the empirical distribution. Focusing only on VaR forecasts, So and Yu (2006) argued, on the other hand, that it was more important to model the fat tailed underlying distribution than the fractional integration of the volatility process. The two papers, one by Degiannakis (2004) and the other by So and Yu (2006), among many others, highlight that different volatility techniques must be applied for different purposes.

Contrary to the contention of the previous authors that the most flexible models generate the most accurate VaR forecasts, Brooks and Persaud (2003) pointed out that the simplest ones, such as the historical average of the variance or the autoregressive volatility model, achieve an appropriate out-of-sample coverage rate. Similarly, Bams et al. (2005) argued that complex (simple) tail models often lead to overestimation (underestimation) of VaR.

VaR, however, has been criticized on two grounds. On the one hand, Taleb (1997) and Hoppe (1999) argued that the underlying statistical assumptions are violated because they could not capture many features of the financial markets (e.g. intelligent agents). Under the same framework, many researchers (see for example Beder, 1995 and Angelidis et al., 2004) showed that different risk management techniques produced different VaR forecasts and therefore, these risk estimates might be imprecise.

On the other hand, even if VaR is useful for financial institutions to understand the risk they face, it is now widely believed that VaR is not the best risk measure. Artzner et al. (1997, 1999) showed that it is not necessarily sub-additive, i.e., the VaR of a portfolio may be greater than the sum of individual VaRs and therefore, managing risk by using it may fail to automatically stimulate diversification. Moreover, it does not indicate the size of the potential loss, given that this loss exceeds VaR. To remedy these shortcomings, Delbaen (2002) and Artzner et al. (1997) introduced the Expected Shortfall (ES), which equals the expected value of the loss, given that a VaR violation occurred. Furthermore, Basak and Shapiro (2001) suggested an alternative risk management procedure, namely limited expected losses based risk management (LEL-RM), that focuses on the expected loss also when (and if) losses occur. They substantiated that the proposed procedure generates losses lower than what VaR-based risk management techniques generate.

ES is the most attractive coherent riskⁱⁱ measure and has been studied by many authors (see Acerbi et al. 2001; Acerbi, 2002; and Inui and Kijima, 2005). Yamai and Yoshida (2005) compared the two measures—VaR and ES—and argued that VaR is not reliable during market turmoil as it can mislead investors, whereas ES can be a better choice overall. However, they pointed out that gains on efficient management by using the ES measure are substantial whenever its estimation is accurate. In other cases, they advise the market practitioners to combine the two measures.

The study sheds light on the issue of volatility forecasting under risk management environment and on the evaluation procedure of various risk models. It compares the performances of the most well known risk management techniques for different markets (stock exchanges, commodities, and exchange rates) and trading positions. Specifically, it estimates the VaR and the ES by using 11 ARCH volatility specifications under four distributional assumptions, namely normal, Student-t, skewed Student-t, and generalized error distribution. We investigated 44 models

following a two-stage backtesting procedure to assess the forecasting power of each volatility technique and to select one model for each financial market. In the first stage, to test the statistical accuracy of the models in the VaR context, we examined whether the average number of violations is statistically equal to the expected one and whether these violations are independently distributed. In the second stage, we employed standard forecast evaluation methods by comparing the returns of a portfolio with the ES, whenever a violation occurs.

The results of this paper are important for many reasons. VaR summarizes the risk exposure of an investor in just one number, and therefore portfolio managers can interpret it quite easilyⁱⁱⁱ. Yet, it is not the most attractive risk measure. On the other hand, the use of ES in evaluating the risk models can be rewarding. Currently, however, most researchers judge the models only by calculating the average number of violations. Moreover, even if the risk managers hold both long and short trading positions to hedge their portfolios, most of the research has been applied only on long positions. Therefore, it is interesting in investigating if a model can capture the characteristics of both tails simultaneously.

This study, to best of our knowledge, is the first that estimates VaR and ES^{iv} for three different markets simultaneously and therefore, we can infer if these markets share common features in risk management framework. Therefore, we combined the most well-known and concurrent parametric models with four distributional assumptions to find out which model has the best overall performance. Even though we did not include all ARCH specifications available in the literature, we estimated the models that captured the most important characteristics of the financial time series and those that were already used or were extensions of specifications that were implemented in similar studies. Finally, we employed a two-stage procedure to investigate the forecasting power of each volatility technique and to guide on VaR model selection process. Following this procedure, the selected risk model predicts VaR accurately and minimizes, if a VaR violation occurs, the difference between the realized and the expected loss. In contrast to this, earlier research focused mainly on the unconditional and conditional coverage of the models.

To summarize, this study juxtaposes the performance of the most well-known parametric techniques, and shows that for each financial market, there is a small set of models that accurately estimate VaR for both long and short trading positions and two confidence levels. Moreover, contrary to the findings of the previous research, the more flexible models do not necessarily generate the most accurate risk forecasts, as a simpler specification can be selected regarding two dimensions: (a) distributional assumption and (b) volatility specification. For distributional assumption, standard normal^v or GED is the most appropriate choice depending on the financial asset, trading position, and confidence level. Besides the distributional choice, asymmetric volatility

specifications perform better than symmetric ones, and in most cases, fractional integrated parameterization of volatility process is necessary.

The rest of the paper is organized as follows: Section 2 describes the ARCH models and presents the calculation of VaR and ES, whereas section 3 describes the evaluation framework of VaR and ES forecasts. Section 4 presents preliminary statistics for the dataset, explains the estimation procedure, and presents the results of the empirical investigation. Section 5 presents the conclusions.

2. ARCH Volatility Models

To fix notation, let $\{y_t\}_{t=0}^T = \{\ln(p_t/p_{t-1})\}_{t=0}^T$ refer to the continuously compounded return series, where p_t is the closing price at trading day t . The return series follows the stochastic process:

$$\begin{aligned} y_t &= \mu_t + \varepsilon_t \\ \mu_t &= c_0(1 - c_1) + c_1 y_{t-1} \\ \varepsilon_t &= z_t \sigma_t \\ z_t &\stackrel{i.i.d.}{\sim} f[E(z_t) = 0, V(z_t) = 1; \theta] \\ \sigma_t^2 &= g(I_{t-1}), \end{aligned} \tag{1}$$

where $E(y_t | I_{t-1}) \equiv \mu_t(\theta)$ denotes the conditional mean, given the information set available at time $t-1$, I_{t-1} , $\{\varepsilon_t\}_{t=0}^T$ is the innovation process with unconditional variance $V(\varepsilon_t) = \sigma^2$ and conditional variance $V(\varepsilon_t | I_{t-1}) \equiv \sigma_t^2(\theta)$, $f(\cdot)$ is the density function of $\{z_t\}_{t=0}^T$, $g(\cdot)$ is any of the functional forms presented in Table 1 and θ is the vector of the unknown parameters.

[Insert Table 1 about here]

We take into consideration the following conditional volatility specifications: GARCH(p, q) of Bollerslev (1986), EGARCH(p, q) of Nelson (1991), TARCH(p, q) of Glosten et al. (1993), APARCH(p, q) of Ding et al. (1993), IGARCH(p, q) of Engle and Bollerslev (1986), FIGARCH(p, q) of Baillie et al. (1996), FIGARCHC(p, q) of Chung (1999), FIEGARCH(p, q) of Bollerslev and Mikkelsen (1996), FIAPARCH(p, q) of Tse (1998), FIAPARCHC(p, q) of Chung (1999), and HYGARCH(p, q) of Davidson (2004). To summarize, the selected volatility models include, besides others, the simplest GARCH model as also the most complex ones, such as FIAPARCHC and HYGARCH. All the selected models reflect the most recent developments in financial forecasting.

Similarly, the chosen density functions of $\{z_t\}_{t=0}^T$ are widely applied in finance. In seminal Engle's (1982) paper, the standard normal density function was used:

$$f(z_t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_t^2}{2}}. \quad (13)$$

However, as the empirical distribution of financial assets is fat-tailed, Bollerslev (1987) introduced the Student-t distribution:

$$f(z_t; \nu) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\pi(\nu-2)}} \left(1 + \frac{z_t^2}{\nu-2}\right)^{-\frac{\nu+1}{2}}, \quad (14)$$

where $\Gamma(\cdot)$ is the gamma function. As ν tends to infinity, the Student-t tends to the normal distribution. Since Student-t is not the only fat tailed distribution available, we also considered the generalized error distribution (GED), which is more flexible than the Student-t as it can include both fat and thin tailed distributions. It was introduced by Subbotin (1923) and applied in ARCH framework by Nelson (1991). Its density function is given in the following equation:

$$f(z_t; \nu, \lambda) = \frac{\nu \exp(-0.5|z_t/\lambda|^\nu)}{\lambda 2^{(1+1/\nu)} \Gamma(\nu^{-1})}, \quad (15)$$

where $\lambda \equiv \sqrt{2^{-2/\nu} \Gamma(\nu^{-1}) \Gamma(3\nu^{-1})}$ and $\nu > 0$ are the tail-thickness parameters (i.e. for $\nu = 2$, z_t is standard normally distributed and for $\nu < 2$, the distribution of z_t has thicker tails than the normal distribution). Finally, given that both the long and short trading positions are important, the skewed Student-t distribution^{vi} is also applied:

$$f(z_t; \nu, g) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)\sqrt{\pi(\nu-2)}} \left(\frac{2s}{g+g^{-1}}\right) \left(1 + \frac{s z_t + m}{\nu-2} g^{-d_t}\right)^{-\frac{\nu+1}{2}}, \quad (16)$$

where g is the asymmetry parameter, $\nu > 2$ denotes the degrees of freedom of the distribution, $\Gamma(\cdot)$ is the gamma function, $d_t = 1$ if $z_t \geq -m/s$, and $d_t = -1$ otherwise. $s = \sqrt{g^2 + g^{-2} - m^2 - 1}$ and $m = \Gamma((\nu-1)/2)\sqrt{(v-2)}(\Gamma(\nu/2)\sqrt{\pi})^{-1}(g - g^{-1})$ are the standard deviation and the mean, respectively.

Having estimated the vector of the unknown parameters, it is straightforward to calculate VaR using the following equation:

$$VaR_{t+1|t} = \mu_{t+1|t} + F(a; \theta^{(t)}) \sigma_{t+1|t}, \quad (17)$$

where $\mu_{t+1|t}$ and $\sigma_{t+1|t}$ are the conditional forecasts of the mean and the standard deviation at time $t+1$, given the information at time t , and $F(a; \theta^{(t)})$ is the a^{th} quantile of the assumed distribution, which is computed based on the vector of parameters estimated at time t .

As we have already mentioned, ES is defined as the conditional expected loss, given a VaR violation. Specifically, for long trading positions, it is calculated as

$$ES_{t+1|t} = E(y_{t+1} | (y_{t+1} \leq VaR_{t+1|t})). \quad (18)$$

In particular, ES is a probability-weighted average of tail loss and therefore, to calculate it, we follow Dowd (2002) who suggested that for any distributional assumption “slice the tail into a large number κ of slices, each of which has the same probability mass, estimate the VaR associated with each slice and take the ES as the average of these VaRs”. To implement this approach, we set $\kappa = 5000$ to increase the accuracy.

3. Evaluate VaR and ES Forecasts

Having presented various risk management techniques, we now discuss their formal statistical evaluation. Given that VaR is never observed, not even after violation, we have to first calculate it and then rank the risk models by examining the statistical properties of the forecasts. Specifically, in the first stage, a model is deemed adequate only if it has not been rejected by both the unconditional and the independence hypotheses. The first hypothesis examines if the average number of violations is statistically equal to the expected one and the second hypothesis if these violations are independent. However, risk managers who use these tests cannot rank the *adequate* models, because a model with greater p -value is not superior to its competitors and, hence, cannot be the best-performing model.

We extended the forecast evaluation approach of Lopez (1999) and Sarma et al. (2003) as the ES was introduced in the second stage by creating a loss function that calculated the difference between the actual and the expected loss when a violation occurred. For all the best-performing models of the first stage, we implemented Hansen’s (2005) superior predictive ability (SPA) test to evaluate their differences statistically. As Yamai and Yoshioka (2005) pointed out, the two risk measures must be combined and hence, under the proposed backtesting framework, the selected models not only calculate VaR accurately but also minimize the difference between the actual loss and the ES.

3.1. First Stage Evaluation

The most widely used test, developed by Kupiec (1995), examines whether the observed exception rate is statistically equal to the expected one. Under the null hypothesis that the model is adequate, the appropriate likelihood ratio statistic is:

$$LR_{uc} = 2\ln\left(\left(1 - \frac{N}{\tilde{T}}\right)^{\tilde{T}-N} \left(\frac{N}{\tilde{T}}\right)^N\right) - 2\ln\left((1 - \rho)^{\tilde{T}-N} \rho^N\right) \sim X_1^2, \quad (19)$$

where N is the number of days over a period \tilde{T} that a violation occurred and ρ is the desired coverage rate. Therefore, the risk model is rejected if it generates too many or too few violations, but based on it, the risk manager can accept a model that generates dependent exceptions.

Christofersen (1998) proposed a more elaborate criterion, which simultaneously examines if (i) the total number of failures is equal to the expected one and (ii) the VaR failure process is independently distributed. The appropriate likelihood ratio test of the first hypothesis is given by equation (19) and that of the second one by the following equation:

$$LR_{in} = 2\left(\ln\left((1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}}\right) - \ln\left((1 - \pi_0)^{n_{00}+n_{10}} \pi_0^{n_{01}+n_{11}}\right)\right) \sim X_1^2, \quad (20)$$

where n_{ij} is the number of observations with value i followed by j , for $i, j = 0, 1$ and $\pi_{ij} = \frac{n_{ij}}{\sum_j n_{ij}}$

are the corresponding probabilities. $i, j = 1$ denotes that a violation has been made, whereas $i, j = 0$ indicates the opposite, which implies that the process of VaR failures must be spread over the entire sample^{vii}. The main advantage of using these two tests is that the risk managers can reject a VaR model that generates too few or too many *clustered* violations and thereby identify the reason for its failure. However, they cannot rank the models based only on the p -values of these tests.

3.2. Second Stage Evaluation

The statistical adequacy of the VaR forecasts is obtained by the previous backtesting tests: the unconditional coverage (equation 19) and the independence test (equation 20). If a model is not rejected, it forecasts VaR accurately. However, in most cases, more than one model can be deemed as adequate and hence, the risk manager cannot select a unique risk management technique.

To overcome this shortcoming of the backtesting measures, Lopez (1999) proposed a forecast evaluation framework based on a loss function. The loss function enables the researcher to rank the models and specify a utility function that accommodates the specific concerns of the risk manager. Specifically, he suggested the following loss function:

$$\Psi_{t+1} = \begin{cases} 1 + (VaR_{t+1|t} - y_{t+1})^2 & \text{if violation occurs} \\ 0 & \text{else,} \end{cases} \quad (21)$$

which accounts for the magnitude of the tail losses $\left((VaR_{t+1|t} - y_{t+1})^2\right)$ and adds a score of one whenever a violation is observed. The model that minimizes the total loss $\sum_{t=1}^T \Psi_t$, is preferred to other models.

Nevertheless, his approach has two drawbacks. First, if the risk management techniques are not filtered by the aforementioned unconditional or conditional coverage procedures, a model that does not generate any violation is deemed as the most adequate as $\Psi_{t+1} = 0$. Second, the return, y_{t+1} , should be better compared with the ES measure and not with the VaR, as VaR does not give any indication about the size of the expected loss. Therefore, with these limitations in mind, in order to judge the models in the second stage, we computed for each model i the mean absolute error (MAE), $\tilde{T}^{-1} \sum_{t=1}^{\tilde{T}} \Psi_{1,t}^{(i)}$, and the mean squared error (MSE), $\tilde{T}^{-1} \sum_{t=1}^{\tilde{T}} \Psi_{2,t}^{(i)}$, where:

$$\Psi_{1,t+1}^{(i)} = \begin{cases} |y_{t+1} - ES_{t+1|t}^{(i)}| & \text{if violation occurs} \\ 0 & \text{else,} \end{cases} \quad (22)$$

and

$$\Psi_{2,t+1}^{(i)} = \begin{cases} (y_{t+1} - ES_{t+1|t}^{(i)})^2 & \text{if violation occurs} \\ 0 & \text{else.} \end{cases} \quad (23)$$

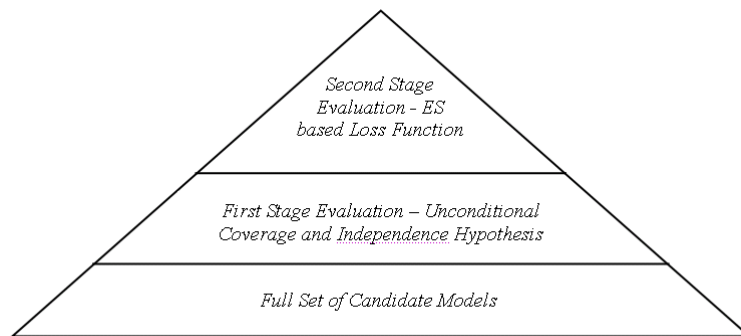
According to the two-stage backtesting procedure, the best performing model will (i) calculate VaR accurately, as it will satisfy the prerequisite of correct unconditional and conditional coverage and (ii) forecast the expected loss, given a VaR violation, as it minimizes the total loss value, $\sum_{t=1}^{\tilde{T}} \Psi_{1,t}^{(i)}$.

The statistical significance of the volatility forecasts is investigated by using Hansen's (2005) Superior Predictive Ability (SPA) test. For $X_{l,t}^{(i^*,i)} = \Psi_{l,t}^{(i^*)} - \Psi_{l,t}^{(i)}$, the null hypothesis, that the benchmark model i^* is not outperformed by competing models i , for $i = 1, \dots, M$, is investigated against the alternative hypothesis that the benchmark model is inferior to one or more of the competing models. The null hypothesis, $E\left(X_{l,t}^{(i^*,1)} \dots X_{l,t}^{(i^*,M)}\right)' \leq 0$, is tested with the statistic

$$T_l^{SPA} = \max_{i=1, \dots, M} \frac{\sqrt{M} \bar{X}_{l,i}}{\sqrt{Var(\sqrt{M} \bar{X}_{l,i})}}, \text{ where } \bar{X}_{l,i} = \tilde{T}^{-1} \sum_{t=1}^{\tilde{T}} X_{l,t}^{(i^*,i)}. \text{ The estimation of } Var(\sqrt{M} \bar{X}_{l,i}) \text{ and the } p\text{-}$$

values of the T_l^{SPA} statistic are obtained by using the stationary bootstrap of Politis and Romano (1994).

Under the proposed backtesting environment, the risk manager achieves three goals: forecasts VaR accurately and thus satisfies the prerequisites of the Basel Committee for Banking Supervision; selects one model or a family of models among various candidates following a statistical inference procedure; and finally knows in advance the amount that may be needed if a VaR violation occurs, and therefore is better prepared to face the future losses by forecasting ES accurately. The next figure briefly demonstrates the two-stage backtesting procedure. In the first stage, the investor can work with fewer than the available models by applying the two tests (equations 19 and 20). In the next stage, according to the developed loss functions (equations 22 and 23), the ES measure is used to evaluate statistically the best-performing models.



4. Empirical Analysis

To evaluate all the available volatility models, we generated out-of-sample VaR and ES forecasts for S&P500 equity index, Gold Bullion \$ per Troy Ounce commodity and US dollar/British pound exchange rate, obtained from Datastream for the period April 4th 1988 to April 5th 2005. The daily prices, the log-returns, and the autocorrelations for the absolute log-returns are presented in Figure 1. Volatility clustering is clearly visible and suggests the presence of heteroskedasticity. The absolute log-returns are significantly positive serial autocorrelated over long lags, whereas the sample autocorrelations decrease too fast at the first lags; at higher lags however, the decrease becomes slower, indicating the long-memory property of volatility process and the need to use fractionally integrated volatility specifications.

[Insert Figure 1 about here]

In the conditional volatility specifications, we choose to set $p = q = 1$, given that in the majority of empirical volatility forecasting studies, the order of one lag has proven to work effectively. Based on a $\tilde{T} = 3000$ rolling sample, we generated $\tilde{T} = 1435^{\text{viii}}$ out-of-sample forecasts (the parameters are re-estimated each trading day) to calculate the 95% and 99% VaR and ES values for long and short trading positions. The parameters of the models were estimated using the G@RCH (Laurent and Peters, 2002) package of Ox (Doornik, 2001).

[Insert Table 2 about here]

[Insert Table 3 about here]

[Insert Table 4 about here]

The MAE and MSE (equations 22 and 23), the average values of VaR and ES, the exception rates, and the p -values of the two backtesting measures are presented in Tables 2 to 4 for all the models that survived the first evaluation (equations 19 and 20)^{ix}.

Irrespective of the volatility models and the financial assets, ARCH specifications under the Student-t distribution and its corresponding skewed version overestimate VaR at both confidence levels. A similar observation was made in several earlier studies (see Guermat and Harris, 2002 and Billio and Pelizzon, 2000 among others). Even at a 99% confidence level, they did not show any major improvement, as the average realized exception rates were significantly lower than the expected ones. The introduction of the asymmetry parameter (g) in the underlying distribution did not make any significant difference. In most cases, they were overestimated, mainly because $\ln(g)$ was close to zero and therefore, the two distributions in the VaR context, were similar^x.

For each financial asset appears to be a different model that forecasts VaR accurately. So and Yu (2006) favored different models for stock indexes and exchange rates. Specifically, for stock indexes, they favored an asymmetric specification and for exchange rates, a symmetric function was preferred.

In our dataset and for the S&P500 index, five models (FIEGARCH-N, EGARCH-N, APARCH-N, TARCH-N, and FIGARCH-GED) generate adequate VaR forecasts, as the p -values of the backtesting measures are greater than 10% for both confidence levels and both trading positions. Even if the more complex models generate, in some cases, the most accurate VaR forecasts (i.e. FIEGARCH-GED for 95% confidence level and long trading position), they do not give the best overall performance. This finding is in line with that of Brooks and Persaud (2003) but not with the argument of Mittnik and Paolella (2000) that more general ARCH structures are needed. Highlighting this conclusion is the observation that the IGARCH-GED model generates exception rates that are close to the expected ones only for the short trading positions, whereas it is rejected for the long trading positions, because either the model generates clustered violations or the model misestimates the *true* VaR. As far as the underlying distribution is concerned, there are indications that standard normal is the best overall choice, as four out of five models are normally distributed.

The GED and normal distribution are the best overall choices for Gold. Between the two, GED is considered more appropriate for the commodity market. For example, if the risk manager is interested only in the higher confidence level and for short trading positions, he/she should use the GED distribution. Any other model would generate inaccurate risk forecasts. To summarize, five

models (GARCH-GED, IGARCH-GED, FIAGARCH-GED, FIAGARCHC-GED, and FIAPARCHC-GED) generated accurate predictions for both confidence levels and both trading positions. The risk manager can select any of these models, irrespective of the trading position, and, hence, satisfy the requirements of the Basel Committee.

For \$/£ exchange rate, the choice of the most appropriate distribution is not straightforward, even if the Student-t and skewed Student-t distributions are rejected. For long (short) trading position and at 99% confidence level, the best overall distribution is the GED (normal), whereas for the other two cases, the results are mixed. EGARCH under the normal distribution appears to have the best overall performance, as only this model generates adequate VaR forecasts for long and short trading positions and for both confidence levels. At the lower confidence level and for long (short) trading position, the exception rate of the model equals to 4.67% (4.25%), whereas the corresponding rates at the higher confidence level are 1.39% (0.91%). Furthermore, according to the two loss functions, the EGARCH under the normal distribution model is always ranked first except for the higher confidence level and the long trading position. Therefore, it is plausible to consider this model as the most appropriate, as it forecasts VaR accurately for the two trading positions and confidence levels.

The difference among the VaR models cannot be evaluated statistically as neither the greatest p -value of the backtesting criteria nor the lowest value of the loss functions indicates the superiority of a model. Therefore, to evaluate the reported differences statistically, we implemented the SPA test taking the following as benchmark models: FIEGARCH-N, EGARCH-N, APARCH-N, TARCH-N, and FIGARCH-GED for S&P500, GARCH-GED, IGARCH-GED, FIGARCH-GED, FIGARCHC-GED, and FIAPARCHC-GED for Gold and EGARCH-N for US dollar to British pound. These models predicted VaR accurately for all cases (long and short trading positions, and at 95% and 99% confidence levels).

[Insert Table 5 about here]

Table 5 presents the p -values of the SPA test for the null hypothesis that the benchmark model i^* is not outperformed by its competing models. For example, in S&P500 index, the benchmark model (FIEGARCH-N) has superior forecasting ability, as the p -value of the test is greater than 10% in any case. All other benchmark models are outperformed, at least in one case, and therefore, there are indications that among the various candidate techniques only one survived the proposed evaluation framework. In the case of Gold, the GARCH-GED and the IGARCH-GED models are not outperformed by their competitors, whereas at least for 95% confidence level and short trading position, FIGARCH-GED, FIGARCHC-GED, and FIAPARCHC-GED models do not generate significantly better forecasts. Finally, for the US \$ to UK £ exchange rate, the forecasting ability of EGARCH-N model is superior to those of other models. Also, it must be noted that the

evaluation of the models is robust to the choice of the used loss function, because irrespective of the measurement method, we select the same models as the most appropriate

According to the two-stage backtesting procedure, the risk manager has two choices: (a) to select one model for each trading position and each confidence level from those models that have not been rejected by the backtesting measures and (b) to use the model that forecasts VaR accurately for both trading positions and both confidence levels. Naturally, the second choice is better, because it reduces the complexity and computational costs. Consequently, the researcher focuses only on one model for each financial asset. Moreover, by employing the two-stage backtesting procedure, the researcher evaluates statistically the differences between the models, and selects, in most cases, only one volatility specification.

In summary, only some models can forecast VaR accurately in all cases. Specifically, in the case of S&P500 index, the FIEGARCH-N generates adequate forecasts for both confidence levels and both trading positions, whereas in the case of Gold, two models (GARCH-GED and IGARCH-GED) give the best overall performance. Lastly, for the US \$ to UK £ exchange rate, EGARCH-N is considered the best specification.

5. Conclusions

The performance of the most recently developed risk management techniques is examined. The paper proposes a two-step backtesting procedure where in the first step all the models, which are rejected by univariate VaR backtests, are discarded, whereas, in the second step a multivariate superior predictive ability test is run taking as a benchmark model each of the models which were not rejected in the first step for any of 95% long/short and 99% long/short VaRs. Specifically, for S&P500 equity index, Gold commodity and US \$ to UK £ exchange rate, the VaR and ES were computed for two confidence levels (95% and 99%) and for two trading positions. We investigated whether the models forecast accurately the expected number of violations, generate independent violations, and predict the ES. As Hansen (2005) rightly suggested, a filtering procedure must be accounted for the full data exploration, before a legitimate statement of the statistical differences among the candidate models. The reduction of the under consideration models was achieved because the evaluation was made in two stages. In the first stage, the framework developed by Kupiec (1995) and Christofersen (1998) was implemented and in the second, the SPA hypothesis testing was applied.

As multiple risk management techniques exhibit unconditional and conditional coverage, the utility function of risk management must be brought into picture to evaluate statistically the differences among the adequate VaR models. Since an investor is also interested in the loss, given a

VaR violation, we introduce the ES to the loss function. According to the SPA test, the risk manager can select, for each financial asset, a model that forecasts both the risk measures accurately. Therefore, the number of under consideration techniques is reduced to a smaller set of competing models.

Different volatility models achieve accurate VaR and ES forecasts for each dataset. In summary, the proposed models are the following:

Market	Model
S&P500	FIEGARCH-N
Gold Bullion \$ per Troy Ounce	GARCH-GED/ IGARCH-GED
US dollar / British pound	EGARCH-N

Although the most appropriate conditional volatility models are not the same for the three financial assets, they share some common characteristics. The normal distribution is often a better choice than more highly parameterized distributions. The Student-t and skewed Student-t distributions overestimate the *true* VaR. Asymmetry in volatility specification is inevitable, as all the selected models incorporate some form of asymmetry, whereas fractional integration is also important in forecasting VaR and ES. This is potentially important as the normal is more parsimonious than the other distributions and so this finding suggests that risk managers may be able to focus less on the appropriate distributional assumptions and more on the appropriate functional form or degree of long-memory. These findings are interesting in their own right and important for risk management as they help to narrow the field of empirically relevant VaR models.

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Figures and Tables

Table 1. Conditional volatility model specifications.

Model		Eq.
GARCH	$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$	(2)
EGARCH	$\ln \sigma_t^2 = \omega(1 - \beta) + (1 + \alpha L) \left(\gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \gamma_2 \left(\left \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right - E \left \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right \right) \right) + \beta \ln \sigma_{t-1}^2$	(3)
TARCH	$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta \sigma_{t-1}^2$	(4)
APARCH	$\sigma_t^\delta = \omega + \alpha \left(\varepsilon_{t-1} - \gamma \varepsilon_{t-1} \right)^\delta + \beta \sigma_{t-1}^\delta$	(5)
IGARCH	$\sigma_t^2 = \omega + \alpha \varepsilon_{t-1}^2 + (1 - \alpha) \sigma_{t-1}^2$	(6)
FIGARCH	$\sigma_t^2 = \omega + \left(1 - \beta L - (1 - aL)(1 - L)^d \right) \varepsilon_t^2 + \beta \sigma_{t-1}^2$	(7)
FIGARCHC	$\sigma_t^2 = \sigma^2(1 - \beta) + \left(1 - \beta L - (1 - aL)(1 - L)^d \right) (\varepsilon_t^2 - \sigma^2) + \beta \sigma_{t-1}^2$	(8)
FIEGARCH	$\ln \sigma_t^2 = \omega(1 - \beta) + \left((1 + \alpha L)(1 - L)^{-d} \right) \left(\gamma_1 \frac{\varepsilon_{t-1}}{\sigma_{t-1}} + \gamma_2 \left(\left \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right - E \left \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right \right) \right) + \beta \ln \sigma_{t-1}^2$	(9)
FIAPARCH	$\sigma_t^\delta = \omega + \left(1 - \beta L - (1 - aL)(1 - L)^d \right) \left(\varepsilon_t - \gamma \varepsilon_t \right)^\delta + \beta \sigma_{t-1}^\delta$	(10)
FIAPARCHC	$\sigma_t^\delta = \sigma^2(1 - \beta) + \left(1 - \beta L - (1 - aL)(1 - L)^d \right) \left(\left(\varepsilon_t - \gamma \varepsilon_t \right)^\delta - \sigma^2 \right) + \beta \sigma_{t-1}^\delta$	(11)
HYGARCH	$\sigma_t^2 = \omega + \left(1 - \beta L - (1 - aL) \left(1 + \zeta \left((1 - L)^d - 1 \right) \right) \right) \varepsilon_t^2 + \beta \sigma_{t-1}^2$	(12)

GARCH = Generalized ARCH, EGARCH = Exponential GARCH, TARCH = Threshold ARCH, APARCH = Asymmetric Power ARCH, IGARCH = Integrated ARCH, FIGARCH = Fractionally Integrated GARCH, FIGARCHC = Chung's FIGARCH, FIEGARCH = Fractionally Integrated EGARCH, FIAPARCH = Fractionally Integrated APARCH, FIAPARCHC = Chung's FIAPARCH, HYGARCH = Hyperbolic GARCH. In TARCH model, $d_t = 1$ if $\varepsilon_t < 0$ and $d_t = 0$ otherwise.

Table 2. The S&P500 case. Column 1 presents the models that have not been rejected by the backtesting criteria (unconditional coverage and the independence test). Columns 2 and 3 present the values of the MAE and the MSE loss functions multiplied by 10^3 (in parentheses the ranking of the models is presented). The average values of the VaR and ES forecasts are presented in 4th and 5th columns, respectively. The percentage of violations is presented in 6th column, whereas the 7th and 8th columns present the Kupiec's and Christofersen's p-values, respectively.

Model	MAE (Rank)	MSE (Rank)	Av.Var	Av.ES	Rate	Kupiec	Chr/sen
Panel A. Long Position - 95% VaR							
FIEGARCH-GED	19.209(1)	18.642(1)	-1.964	-2.664	4.18%	14.35%	14.41%
EGARCH-N	19.868(2)	24.350(12)	-1.848	-2.324	5.16%	78.62%	54.10%
FIEGARCH-N	20.028(3)	21.554(3)	-1.879	-2.365	5.16%	78.62%	27.33%
TARCH-N	20.195(4)	24.638(13)	-1.830	-2.302	5.30%	61.00%	32.97%
APARCH-N	20.230(5)	23.944(10)	-1.870	-2.352	5.23%	69.59%	58.03%
HYGARCH-N	20.269(6)	23.767(9)	-1.894	-2.389	4.95%	92.75%	43.06%
FIAPARCH-N	20.681(7)	25.742(14)	-1.890	-2.377	5.09%	88.00%	50.28%
FIAPARCHC-N	21.112(8)	27.365(15)	-1.942	-2.441	4.46%	33.93%	50.20%
IGARCH-N	21.473(9)	24.213(11)	-1.883	-2.374	5.23%	69.59%	30.07%
FIAPARCHC-GED	21.537(10)	22.817(5)	-1.967	-2.668	4.25%	18.19%	79.73%
HYGARCH-GED	21.598(11)	22.799(4)	-1.907	-2.616	4.88%	83.15%	17.99%
EGARCH-GED	21.833(12)	21.407(2)	-1.952	-2.659	4.53%	40.64%	54.01%
FIGARCHC-N	22.221(13)	27.486(17)	-1.837	-2.317	5.37%	52.95%	36.03%
TARCH-GED	22.279(14)	22.944(7)	-1.856	-2.534	5.09%	88.00%	50.28%
APARCH-GED	22.376(15)	22.903(6)	-1.901	-2.588	4.88%	83.15%	74.62%
FIAPARCH-GED	22.388(16)	23.726(8)	-1.912	-2.591	4.81%	73.75%	70.33%
FIGARCH-N	23.691(17)	28.718(18)	-1.799	-2.269	5.64%	27.19%	49.82%
FIGARCH-GED	25.598(18)	27.420(16)	-1.820	-2.494	5.71%	22.43%	13.61%
Panel B. Long Position - 99% VaR							
APARCH-GED	3.938(1)	4.635(2)	-3.015	-3.651	0.63%	12.75%	73.60%
EGARCH-GED	4.383(2)	3.914(1)	-3.097	-3.751	0.70%	22.22%	70.78%
GARCH-GED	4.711(3)	5.412(3)	-3.003	-3.658	0.63%	12.75%	73.60%
FIAPARCH-GED	4.855(4)	6.221(4)	-3.014	-3.637	0.63%	12.75%	73.60%
FIEGARCH-N	5.450(5)	7.381(5)	-2.672	-3.066	0.91%	71.59%	62.58%
FIGARCH-GED	6.322(6)	8.158(6)	-2.913	-3.540	0.77%	35.40%	68.01%
HYGARCH-N	6.456(7)	10.304(8)	-2.701	-3.103	1.12%	66.73%	54.79%
APARCH-N	6.813(8)	10.057(7)	-2.656	-3.046	0.98%	92.57%	59.93%
FIAPARCHC-N	6.836(9)	12.936(11)	-2.756	-3.161	0.98%	92.57%	12.42%
EGARCH-N	6.965(10)	10.323(9)	-2.625	-3.011	1.05%	86.41%	57.33%
TARCH-N	7.487(11)	10.782(10)	-2.600	-2.983	1.18%	49.45%	52.30%

Table 2. Continued

Model	MAE (Rank)	MSE (Rank)	Av.Var	Av.ES	Rate	Kupiec	Chr/sen
Panel C. Short Position - 95% VaR							
APARCH-N	15.106(1)	9.702(1)	1.921	2.402	4.53%	40.64%	17.32%
EGARCH-N	16.357(2)	12.342(5)	1.902	2.378	4.53%	40.64%	17.32%
FIEGARCH-N	16.541(3)	12.115(4)	1.951	2.438	4.25%	18.19%	79.73%
TARCH-N	17.550(4)	11.554(2)	1.890	2.362	4.81%	73.75%	12.08%
IGARCH-GED	17.715(5)	12.351(6)	1.972	2.683	4.11%	11.15%	28.27%
GARCH-N	17.790(6)	15.400(10)	1.948	2.429	4.32%	22.71%	22.29%
GARCH-GED	18.061(7)	12.732(7)	1.949	2.650	4.18%	14.35%	26.16%
FIGARCHC-N	18.267(8)	14.966(9)	1.944	2.424	4.32%	22.71%	65.03%
APARCH-GED	18.314(9)	11.614(3)	1.956	2.643	4.39%	27.95%	20.52%
EGARCH-GED	19.484(10)	13.706(8)	1.993	2.699	4.11%	11.15%	71.09%
FIGARCH-N	20.041(11)	16.353(12)	1.906	2.376	4.81%	73.75%	41.42%
FIGARCHC-GED	21.789(12)	16.574(13)	1.936	2.621	4.39%	27.95%	61.29%
TARCH-GED	23.007(13)	16.134(11)	1.917	2.595	5.02%	97.59%	33.23%
FIGARCH-GED	23.649(14)	17.524(14)	1.904	2.579	4.95%	92.75%	78.97%
Panel D. Short Position - 99% VaR							
APARCH-N	1.968(1)	0.963(1)	2.707	3.097	0.77%	35.40%	68.01%
IGARCH-GED	2.669(2)	1.600(5)	3.124	3.790	0.63%	12.75%	73.60%
FIGARCH-GED	2.726(3)	1.564(4)	2.997	3.624	0.63%	12.75%	73.60%
TARCH-N	2.747(4)	1.200(2)	2.661	3.044	0.98%	92.57%	59.93%
FIGARCHC-GED	2.829(5)	1.702(6)	3.046	3.682	0.63%	12.75%	73.60%
FIEGARCH-N	2.874(6)	1.436(3)	2.745	3.139	0.98%	92.57%	59.93%
GARCH-GED	3.212(7)	1.745(7)	3.084	3.739	0.77%	35.40%	68.01%
EGARCH-N	3.380(8)	2.441(10)	2.679	3.065	0.98%	92.57%	59.93%
IGARCH-N	3.473(9)	2.002(8)	2.786	3.184	1.05%	86.41%	57.33%
HYGARCH-N	3.600(10)	2.231(9)	2.811	3.212	0.98%	92.57%	59.93%
FIAPARCH-N	3.702(11)	2.959(13)	2.740	3.135	0.98%	92.57%	59.93%
FIGARCHC-N	3.828(12)	2.622(12)	2.727	3.116	1.05%	86.41%	57.33%
GARCH-N	4.337(13)	2.597(11)	2.733	3.124	1.18%	49.45%	52.30%
FIGARCH-N	4.683(14)	3.263(14)	2.673	3.055	1.25%	35.16%	49.87%

Table 3. The Gold Bullion \$ per Troy Ounce case. Column 1 presents the models that have not been rejected by the backtesting criteria (unconditional coverage and the independence test). Columns 2 and 3 present the values of the MAE and the MSE loss functions multiplied by 10^3 (in parentheses the ranking of the models is presented). The average values of the VaR and ES forecasts are presented in 4th and 5th columns, respectively. The percentage of violations is presented in 6th column, whereas the 7th and 8th columns present the Kupiec's and Christofersen's p-values, respectively.

Model	MAE (Rank)	MSE (Rank)	Av.Var	Av.ES	Rate	Kupiec	Chr/sen
Panel A. Long Position - 95% VaR							
FIGARCHC-N	18.342 (1)	17.047 (8)	-1.426	-1.785	4.11%	11.15%	71.09%
FIGARCH-N	18.581 (2)	17.317 (9)	-1.425	-1.783	4.11%	11.15%	76.84%
EGARCH-N	20.169 (3)	21.477 (10)	-1.436	-1.801	4.18%	14.35%	75.38%
FIAPARCHC-GED	20.296 (4)	15.772 (6)	-1.415	-2.054	4.18%	14.35%	75.38%
APARCH-GED	20.311 (5)	15.128 (2)	-1.450	-2.113	4.11%	11.15%	71.09%
TARCH-GED	20.355 (6)	15.094 (1)	-1.464	-2.133	4.11%	11.15%	71.09%
FIGARCHC-GED	20.430 (7)	15.652 (3)	-1.394	-2.027	4.39%	27.95%	88.53%
FIGARCH-GED	20.657 (8)	15.710 (4)	-1.397	-2.030	4.39%	27.95%	88.53%
GARCH-GED	21.165 (9)	15.753 (5)	-1.438	-2.104	4.32%	22.71%	65.03%
IGARCH-GED	21.272 (10)	16.043 (7)	-1.442	-2.109	4.32%	22.71%	65.03%
Panel B. Long Position - 99% VaR							
EGARCH-GED	2.012 (1)	0.916 (1)	-2.692	-3.431	0.70%	22.22%	70.78%
TARCH-GED	2.463 (2)	1.469 (2)	-2.540	-3.224	0.70%	22.22%	70.78%
FIEGARCH-GED	3.011 (3)	1.626 (3)	-2.659	-3.388	0.70%	22.22%	70.78%
FIAPARCH-GED	3.040 (4)	1.862 (4)	-2.479	-3.136	0.84%	52.11%	65.27%
FIAPARCHC-GED	3.136 (5)	1.926 (5)	-2.442	-3.091	0.84%	52.11%	65.27%
HYGARCH-GED	3.292 (6)	1.927 (6)	-2.509	-3.187	0.91%	71.59%	62.58%
GARCH-GED	3.299 (7)	2.452 (8)	-2.508	-3.191	0.77%	35.40%	68.01%
IGARCH-GED	3.412 (8)	2.563 (10)	-2.514	-3.200	0.77%	35.40%	68.01%
FIGARCH-GED	3.565 (9)	2.214 (7)	-2.415	-3.061	0.91%	71.59%	62.58%
APARCH-GED	4.019 (10)	2.811 (11)	-2.516	-3.194	0.84%	52.11%	65.27%
FIGARCHC-GED	4.077 (11)	2.505 (9)	-2.412	-3.057	0.98%	92.57%	59.93%
TARCH-N	6.038 (12)	5.005 (12)	-2.120	-2.428	1.25%	35.16%	49.87%
APARCH-N	6.317 (13)	5.158 (13)	-2.100	-2.406	1.32%	23.99%	47.50%
IGARCH-N	6.440 (14)	5.685 (15)	-2.093	-2.396	1.32%	23.99%	47.50%
GARCH-N	6.742 (15)	5.788 (16)	-2.086	-2.387	1.39%	15.71%	45.19%
FIAPARCH-N	6.859 (16)	6.215 (17)	-2.075	-2.376	1.25%	35.16%	49.87%
HYGARCH-N	6.860 (17)	5.635 (14)	-2.085	-2.386	1.39%	15.71%	45.19%

Table 3. Continued

Model	MAE (Rank)	MSE (Rank)	Av.Var	Av.ES	Rate	Kupiec	Chr/sen
Panel C. Short Position - 95% VaR							
FIEGARCH-GED	21.571 (1)	31.570 (1)	1.523	2.230	4.18%	14.35%	73.22%
APARCH-N	22.838 (2)	39.708 (15)	1.480	1.857	4.53%	40.64%	52.08%
TARCH-N	23.160 (3)	39.281 (14)	1.492	1.872	4.67%	56.09%	59.93%
IGARCH-N	23.802 (4)	42.209 (19)	1.454	1.828	4.67%	56.09%	59.93%
HYGARCH-N	23.940 (5)	38.789 (13)	1.448	1.820	4.95%	92.75%	76.78%
FIAPARCH-N	23.964 (6)	37.328 (5)	1.455	1.826	4.88%	83.15%	72.46%
FIAPARCHC-N	23.992 (7)	38.619 (12)	1.430	1.794	4.88%	83.15%	72.46%
EGARCH-N	24.157 (8)	45.752 (21)	1.439	1.803	4.53%	40.64%	52.08%
GARCH-N	24.287 (9)	42.504 (20)	1.449	1.821	4.81%	73.75%	68.20%
APARCH-GED	24.547 (10)	37.366 (6)	1.450	2.113	4.74%	64.69%	64.02%
GARCH-GED	24.576 (11)	38.025 (11)	1.439	2.104	4.88%	83.15%	72.46%
IGARCH-GED	24.672 (12)	37.849 (10)	1.442	2.109	4.88%	83.15%	72.46%
TARCH-GED	24.781 (13)	36.966 (3)	1.464	2.133	4.74%	64.69%	91.62%
FIEGARCH-N	25.078 (14)	41.954 (18)	1.382	1.738	4.81%	73.75%	34.85%
HYGARCH-GED	25.455 (15)	36.201 (2)	1.444	2.106	5.02%	97.59%	81.16%
FIAPARCH-GED	26.332 (16)	37.076 (4)	1.437	2.084	5.09%	88.00%	85.58%
FIGARCHC-N	26.620 (17)	41.097 (17)	1.397	1.755	5.71%	22.43%	75.06%
FIGARCH-N	26.706 (18)	41.083 (16)	1.396	1.754	5.71%	22.43%	75.06%
FIGARCH-GED	27.028 (19)	37.510 (8)	1.396	2.030	5.44%	45.51%	92.20%
FIGARCHC-GED	27.081 (20)	37.477 (7)	1.393	2.026	5.44%	45.51%	92.20%
FIAPARCHC-GED	27.445 (21)	37.599 (9)	1.415	2.054	5.30%	61.00%	98.94%
Panel D. Short Position - 99% VaR							
FIAPARCH-GED	7.017 (1)	13.462 (1)	2.478	3.136	1.05%	86.41%	14.57%
FIAPARCHC-GED	8.003 (2)	14.782 (2)	2.442	3.091	1.12%	66.73%	16.92%
FIGARCH-GED	8.131 (3)	15.040 (3)	2.415	3.060	1.05%	86.41%	14.57%
GARCH-GED	8.162 (4)	17.210 (6)	2.508	3.191	0.98%	92.57%	12.42%
IGARCH-GED	8.176 (5)	16.925 (5)	2.514	3.199	0.98%	92.57%	12.42%
EGARCH-GED	8.635 (6)	20.424 (7)	2.692	3.431	0.91%	71.59%	10.44%
FIGARCHC-GED	8.909 (7)	15.308 (4)	2.411	3.056	1.18%	49.45%	19.44%

Table 4. The US \$ to UK £ case. Column 1 presents the models that have not been rejected by the backtesting criteria (unconditional coverage and the independence test). Columns 2 and 3 present the values of the MAE and the MSE loss functions multiplied by 10^3 (in parentheses the ranking of the models is presented). The average values of the VaR and ES forecasts are presented in 4th and 5th columns, respectively. The percentage of violations is presented in 6th column, whereas the 7th and 8th columns present the Kupiec's and Christofersen's p-values, respectively.

Model	MAE (Rank)	MSE (Rank)	Av.Var	Av.ES	Rate	Kupiec	Chr/sen
Panel A. Long Position - 95% VaR							
EGARCH-N	8.402 (1)	2.606 (1)	-0.886	-1.112	4.67%	56.09%	61.97%
FIEGARCH-N	8.757 (2)	2.762 (2)	-0.895	-1.123	4.74%	64.69%	44.41%
IGARCH-N	9.172 (3)	2.824 (3)	-0.864	-1.084	5.09%	88.00%	68.64%
FIAPARCHC-GED	9.976 (4)	2.876 (4)	-0.833	-1.167	5.44%	45.51%	50.29%
FIGARCHC-N	9.987 (5)	3.173 (7)	-0.834	-1.047	5.44%	45.51%	20.36%
FIAPARCH-N	10.150 (6)	3.282 (12)	-0.831	-1.043	5.64%	27.19%	40.80%
HYGARCH-GED	10.181 (7)	3.059 (5)	-0.842	-1.180	5.37%	52.95%	53.72%
FIGARCH-N	10.265 (8)	3.288 (13)	-0.835	-1.048	5.57%	32.61%	17.04%
HYGARCH-N	10.400 (9)	3.362 (15)	-0.830	-1.041	5.57%	32.61%	17.04%
FIAPARCHC-N	10.568 (10)	3.410 (18)	-0.823	-1.033	5.78%	18.32%	68.94%
GARCH-N	10.570 (11)	3.391 (16)	-0.832	-1.045	5.64%	27.19%	77.13%
FIGARCHC-GED	10.581 (12)	3.260 (11)	-0.835	-1.170	5.51%	38.72%	47.00%
FIGARCH-GED	10.584 (13)	3.230 (10)	-0.844	-1.183	5.44%	45.51%	50.29%
TARCH-N	10.592 (14)	3.393 (17)	-0.834	-1.047	5.71%	22.43%	72.99%
EGARCH-GED	10.644 (15)	3.161 (6)	-0.903	-1.272	4.67%	56.09%	61.97%
FIAPARCH-GED	10.694 (16)	3.218 (9)	-0.842	-1.180	5.51%	38.72%	47.00%
IGARCH-GED	10.777 (17)	3.212 (8)	-0.863	-1.213	5.16%	78.62%	64.74%
GARCH-GED	11.255 (18)	3.360 (14)	-0.843	-1.181	5.51%	38.72%	85.65%
TARCH-GED	11.646 (19)	3.482 (19)	-0.845	-1.183	5.64%	27.19%	77.13%
APARCH-GED	12.052 (20)	4.656 (20)	-0.830	-1.161	5.71%	22.43%	72.99%
Panel B. Long Position - 99% VaR							
FIGARCHC-GED	1.340 (1)	0.360 (1)	-1.376	-1.701	0.77%	35.40%	68.01%
IGARCH-GED	1.397 (2)	0.381 (4)	-1.428	-1.769	0.70%	22.22%	70.78%
GARCH-GED	1.453 (3)	0.382 (5)	-1.389	-1.716	0.84%	52.11%	65.27%
FIAPARCH-GED	1.488 (4)	0.377 (3)	-1.388	-1.715	0.84%	52.11%	65.27%
EGARCH-GED	1.545 (5)	0.402 (6)	-1.499	-1.858	0.70%	22.22%	70.78%
HYGARCH-GED	1.546 (6)	0.407 (7)	-1.388	-1.714	0.84%	52.11%	65.27%
FIGARCH-GED	1.569 (7)	0.411 (8)	-1.391	-1.719	0.84%	52.11%	65.27%
EGARCH-N	1.572 (8)	0.364 (2)	-1.254	-1.437	1.39%	15.71%	45.19%
TARCH-GED	1.616 (9)	0.433 (9)	-1.391	-1.718	0.91%	71.59%	62.58%
FIEGARCH-GED	1.658 (10)	0.543 (11)	-1.586	-1.975	0.63%	12.75%	73.60%
FIAPARCHC-GED	1.669 (11)	0.464 (10)	-1.373	-1.696	0.77%	35.40%	68.01%
APARCH-GED	2.020 (12)	1.171 (12)	-1.364	-1.684	0.91%	71.59%	62.58%

Table 4. Continued

Model	MAE (Rank)	MSE (Rank)	Av.Var	Av.ES	Rate	Kupiec	Chr/sen
Panel C. Short Position - 95% VaR							
EGARCH-N	7.754(1)	2.839(1)	0.894	1.120	4.25%	18.19%	79.73%
IGARCH-N	8.369(2)	3.057(5)	0.872	1.092	4.53%	40.64%	54.01%
FIGARCHC-N	8.499(3)	3.046(4)	0.842	1.055	4.81%	73.75%	70.33%
TARCH-N	8.639(4)	3.013(2)	0.844	1.057	4.88%	83.15%	74.62%
FIGARCH-N	8.926(5)	3.107(8)	0.843	1.056	4.95%	92.75%	78.97%
HYGARCH-N	8.950(6)	3.107(9)	0.838	1.050	5.02%	97.59%	83.37%
GARCH-N	8.966(7)	3.085(7)	0.842	1.055	5.02%	97.59%	83.37%
FIAPARCH-N	9.016(8)	3.220(14)	0.839	1.051	4.95%	92.75%	78.97%
FIAPARCHC-N	9.083(9)	3.245(15)	0.830	1.040	5.23%	69.59%	96.72%
FIAPARCH-GED	9.273(10)	3.013(3)	0.847	1.185	4.46%	33.93%	22.58%
APARCH-N	9.545(11)	3.344(16)	0.823	1.031	5.37%	52.95%	94.40%
FIGARCH-GED	9.659(12)	3.084(6)	0.848	1.187	4.60%	48.04%	57.94%
HYGARCH-GED	9.693(13)	3.116(10)	0.846	1.184	4.67%	56.09%	61.97%
FIGARCHC-GED	9.818(14)	3.153(12)	0.839	1.174	4.74%	64.69%	66.11%
GARCH-GED	10.029(15)	3.134(11)	0.848	1.186	4.60%	48.04%	57.94%
TARCH-GED	10.033(16)	3.183(13)	0.850	1.188	4.53%	40.64%	54.01%
FIAPARCHC-GED	10.345(17)	3.427(19)	0.837	1.172	4.88%	83.15%	39.66%
IGARCH-GED	10.376(18)	3.392(17)	0.867	1.217	4.46%	33.93%	50.20%
APARCH-GED	10.668(19)	3.407(18)	0.835	1.166	4.88%	83.15%	39.66%
Panel D. Short Position - 99% VaR							
EGARCH-N	1.922(1)	0.623(1)	1.262	1.446	0.91%	71.59%	10.44%
FIEGARCH-N	1.999(2)	0.650(2)	1.273	1.458	0.98%	92.57%	12.42%
TARCH-N	2.175(3)	0.695(3)	1.192	1.364	1.05%	86.41%	14.57%
IGARCH-N	2.246(4)	0.730(5)	1.231	1.410	1.05%	86.41%	14.57%
GARCH-N	2.256(5)	0.701(4)	1.189	1.362	1.12%	66.73%	16.92%
HYGARCH-N	2.501(6)	0.786(6)	1.184	1.355	1.25%	35.16%	22.13%
FIAPARCH-N	2.511(7)	0.852(9)	1.185	1.357	1.12%	66.73%	16.92%
FIGARCH-N	2.585(8)	0.816(7)	1.190	1.363	1.25%	35.16%	22.13%
APARCH-N	2.614(9)	0.849(8)	1.162	1.330	1.25%	35.16%	22.13%
FIAPARCHC-N	2.647(10)	0.908(11)	1.172	1.343	1.25%	35.16%	22.13%
FIGARCHC-N	2.753(11)	0.877(10)	1.189	1.362	1.32%	23.99%	25.00%

Table 5. The p-values of the SPA test for the null hypothesis that the benchmark model is not outperformed by its competing models.

Loss Function	Long Position 95% VaR	Long Position 99% VaR	Short Position 95% VaR	Short Position 99% VaR
S&P500				
(Benchmark Model: FIEGARCH-N)				
MAE	0.81390	0.12790	0.61030	0.26370
MSE	0.34780	0.17080	0.28250	0.38550
(Benchmark Model: EGARCH-N)				
MAE	0.89750	0.08000 ^b	0.59720	0.20970
MSE	0.32300	0.11730	0.33850	0.43500
(Benchmark Model: APARCH-N)				
MAE	0.87810	0.04360 ^a	0.97800	0.89900
MSE	0.34690	0.11450	0.99820	0.99050
(Benchmark Model: TARCH-N)				
MAE	0.88740	0.02690 ^a	0.16050	0.48620
MSE	0.35010	0.12440	0.07160 ^b	0.69910
(Benchmark Model: FIGARCH-GED)				
MAE	0.01340 ^a	0.09190 ^b	0.00600 ^a	0.79410
MSE	0.06970 ^b	0.19970	0.00330 ^a	0.77080

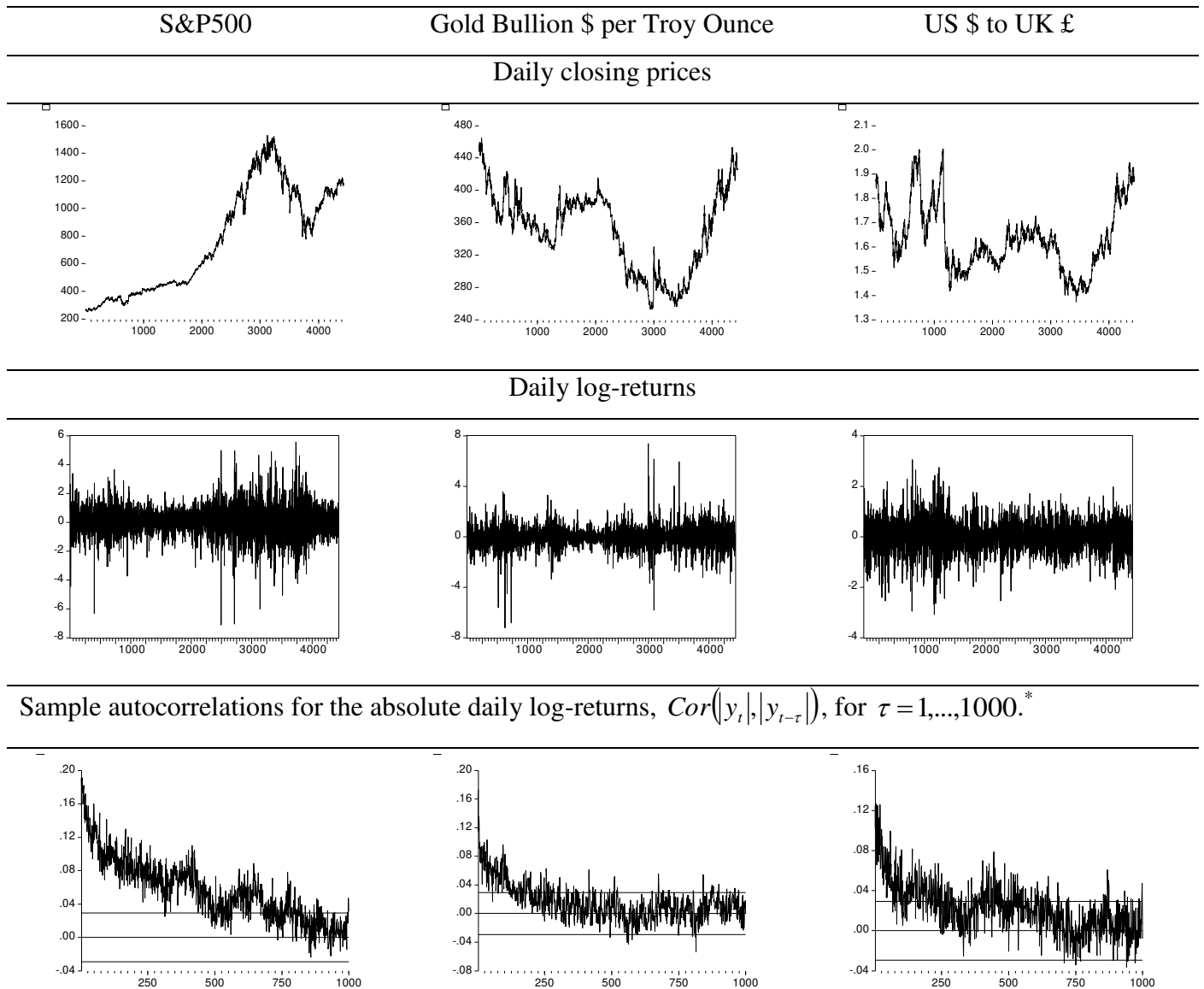
Table 5. Continued

Loss Function	Long Position 95% VaR	Long Position 99% VaR	Short Position 95% VaR	Short Position 99% VaR
Gold Bullion \$ per Troy Ounce (Benchmark Model: GARCH-GED)				
MAE	0.36750	0.34000	0.34680	0.56090
MSE	0.84510	0.32590	0.43570	0.34920
(Benchmark Model: IGARCH-GED)				
MAE	0.31350	0.35870	0.32110	0.53350
MSE	0.23210	0.32260	0.43330	0.43680
(Benchmark Model: FIGARCH-GED)				
MAE	0.26480	0.21560	0.06820 ^b	0.38710
MSE	0.81200	0.11440	0.20400	0.47350
(Benchmark Model: FIGARCHC-GED)				
MAE	0.32250	0.11250	0.06840 ^b	0.10370
MSE	0.84020	0.07850 ^b	0.19470	0.28370
(Benchmark Model: FIAPARCHC-GED)				
MAE	0.37010	0.41260	0.02870 ^a	0.12790
MSE	0.83110	0.37380	0.05430 ^b	0.05330 ^b
US \$ to UK £ (Benchmark Model: EGARCH-N)				
MAE	0.95180	0.72780	0.96560	0.78310
MSE	0.97730	0.89700	0.97300	0.91270

^a Indicates that the null hypothesis is rejected at 5% level of significance.

^b Indicates that the null hypothesis is rejected at 10% level of significance.

Figure 1. Daily closing prices, log-returns and the lag 1 through 1000 autocorrelations for the absolute log-returns from April 4th, 1988 through April 5th, 2005.



* The vertical lines present the 95% confidence interval of no serial dependence, $\pm 1.96/\sqrt{T}$.

ⁱMittnik and Paoella (2000) also used the APARCH model to accommodate the time varying skewness of the exchange rate market.

ⁱⁱA coherent risk measure is defined as the one that satisfies the following four properties: (a) sub-additivity, (b) homogeneity, (c) monotonicity, and (d) risk-free condition. These are described in the following equations: (a) $\rho(x) + \rho(y) \leq \rho(x + y)$, (b) $\rho(tx) = t\rho(x)$, (c) $\rho(x) \geq \rho(y)$ if $x \leq y$, and (d) $\rho(x + n) = \rho(x) - n$. For more details on coherent risk measures, see Artzner et al. (1997).

ⁱⁱⁱAs the 4:15 report JP Morgan did.

^{iv}Bali and Theodosiou (2006) suggested either the TS-GARCH, proposed by Taylor (1986) and Schwert (1989), or the EGARCH model, introduced by Nelson (1991), as the VaR and ES measures were estimated accurately.

^vThis is potentially important as it suggests that risk managers must focus less on the appropriate distributional assumptions and more on the appropriate volatility functional form.

^{vi}The skewed Student-t distribution was introduced by Fernandez and Steel (1998) and was applied by Lambert and Laurent (2000) in ARCH framework. Moreover, Kuester et al. (2006) argued that compared to the normal distribution, substantial improvement in predicting VaR was achieved when asymmetrical fat tailed distribution was used.

^{vii}The log-likelihood ratio statistic of the combined hypothesis is computed as:

$$LR_{cc} = -2 \ln \left((1 - \rho)^{T-N} \rho^N \right) + 2 \ln \left((1 - \pi_{01})^{n_{00}} \pi_{01}^{n_{01}} (1 - \pi_{11})^{n_{10}} \pi_{11}^{n_{11}} \right) \sim X_2^2$$
, under the null hypothesis of an independence failure process with failure probability ρ .

^{viii} $T = \tilde{T} + \check{T}$.

^{ix}We set the cut off point to 10% to ensure that the successful models will neither over nor underestimate the *true* VaR and the sequence of violations will be independent. Detailed results for all the models are available upon request.

^xThe rolling parameters are available upon request.