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# Multivariate modelling of 10-day-ahead VaR and dynamic correlation for worldwide real estate and stock indices

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### Abstract

The Basel Committee regulations require the estimation of Value-at-Risk at 99% confidence level for a 10-trading-day-ahead forecasting horizon. The paper provides a multivariate modelling framework for multi-period VaR estimates for leptokurtic and asymmetrically distributed real-estate portfolio returns. The purpose of the paper is to estimate accurate 10-day-ahead 99% VaR forecasts for real estate markets along with stock markets for seven countries across the world (USA, UK, GERMANY, JAPAN, AUSTRALIA, HONG KONG and SINGAPORE) following the Basel Committee requirements for financial regulation.

A fourteen-dimensional multivariate Diag-VECH model for seven equity indices and their relative real estate indices is estimated. We evaluate the VaR forecasts over a period of two weeks in calendar time, or 10 trading days, and at 99% confidence level based on the Basle Committee on Banking Supervision requirements.

The Basel regulations require 10-day-ahead 99% VaR forecasts. This is the first study that provides successful evidence for 10-day-ahead 99% VaR estimations for real estate markets. Additionally, we provide evidence that there is a statistically significant relationship between the magnitude of the 10-day-ahead 99%VaR and the level of dynamic correlation for real estate and stock market indices; a valuable recommendation for risk managers who forecast risk across markets.

Risk managers, investors and financial institutions require dynamic multi-period VaR forecasts that will take into account properties of financial time series. Such accurate dynamic forecasts lead to successful decisions for controlling market risks.

## 1. Introduction

Value-at-Risk (VaR) models calculate the maximum loss for a portfolio of financial instruments at a pre-specified time and level of confidence. VaR exhibits the attractive property of summarizing market risks in one single number. VaR at a given probability level $(1-p)$ is the predicted amount of financial loss over a given time horizon. For $y_t$ denoting the log-returns over the time horizon from $t-1$ to $t$, and for $y_t \sim N(\mu_t, \sigma_t^2)$, the VaR at time $t$, for $(1-p)$ probability level, is the value $VaR_t^{(1-p)}$ that satisfies the condition; $p = P(y_t \leq VaR_t^{(1-p)}) = \frac{1}{\sigma_t \sqrt{2\pi}} \int_{-\infty}^{VaR_t^{(1-p)}} \exp\left(-\frac{1}{2}\left(\frac{y_t - \mu_t}{\sigma_t}\right)^2\right) dy_t$ .

For a reference text, see Choudhry (2006), Gregoriou (2009), and Jorion (2006).

Risk managers, investors and financial institutions need a dynamic VaR forecast that will take into account properties such as volatility clustering, leverage effect[1], asymmetric and leptokurtic (both conditional and unconditional) distribution of log-returns. Accurate dynamic forecasts of VaR thresholds could lead to successful decisions for controlling market risks.

Literature has been trying to find which method provides more accurate VaR forecasts. Bauwens *et al*. (2006), Brooks and Persand (2003) and Kuester *et al.* (2006), among others, propose the use of univariate conditional volatility models for VaR computation as there are no gains in forecasting accuracy when using multivariate models. Christoffersen (2009) argues that univariate models are more appropriate if the purpose is VaR computation, whereas multivariate models are more suitable for risk management (i.e. portfolio selection). Also, Cheong (2011), estimates multivariate volatility models for quantifying the cross-market risk and hedging among the energy markets, and Santos *et al.* (2009) conduct a comparative analysis of the predictive performance for one-step-ahead VaR obtained with both Monte Carlo simulations and with real market data and provided evidence that the multivariate ARCH models outperformed competing univariate models. Santos' *et al.* (2009) results, based on the backtesting analysis established by the Basel II Accord, indicate that multivariate models delivered lower levels of daily capital requirements in comparison to univariate models.

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[1] Changes in returns tend to be negatively correlated with changes in volatility.

McAleer and da Veiga (2008a) state that multivariate ARCH models provide superior VaR forecasts than their nested univariate counterparts as they model the relationship between subsets of the portfolio and allow for scenario and sensitivity analyses.

The importance of real estate securities for investors and economists has grown remarkably during the last decade since they overcome many of the drawbacks related to direct real estate investment, such as high unit value, transactions costs and illiquidity of properties[2]. Since the prices of real estate securities are exposed not only to the performance of the direct real estate market but also to the volatility of the stock markets, there is an extensive research interest in the relationship between securitized real estate and stock markets (He *et al.*, 2008). Clayton and MacKinnon (2003) Yang *et al.* (2012) and Liow and Ibrahim (2010), among others, have examined the relationship between stock and securitized real estate markets. Clayton and MacKinnon (2003) showed that "the REIT market went from being driven largely by the same economic factors that drive large cap stocks through the 1970s and 1980s to being more strongly related to both small cap stock and real estate-related factors in the 1990s", whereas Yang *et al.* (2012) estimated a DCC ARCH model to S&P500, US corporate bonds, and their real estate counterparts, REITs and CMBS, and provided evidence for asymmetric volatilities and correlations in CMBS and REITs. Liow and Ibrahim (2010) indicate that there are significant volatility co-movements between the international securitized real estate and stock markets either in the long-run or in the short-run. However, the long-run volatility relationships are stronger.

The Basel regulations (2006, 2009) require the estimation of VaR for 99% confidence level and for a 10-trading-day-ahead forecasting horizon. Rossignolo *et al.* (2012) offer a detailed theoretical description of Basel's Committee Regulatory framework, whereas Degiannakis *et al.* (2012, 2013) provide an outline for the estimation of multi-period VaR forecasts.

The contribution of this paper in the literature is twofold. Firstly, we generate accurate 10-day-ahead 99%VaR for real estate market indices which reflect the Basel Committee requirements for financial regulation. This is the first study that provides

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[2]Hoesli and Reka (2011) and He *et al.* (2008) provide information regarding high unit value and illiquidity of properties.

successful evidence for 10-day-ahead VaR estimations for real estate markets. In order to produce VaR estimations, we use a multivariate ARCH model with time varying correlations in contrast to other studies which consider multivariate models with constant conditional correlations, i.e. Brooks and Persand (2003), McAleer and da Veiga (2008b). Furthermore, this paper is the first approach which models the volatility and VaR estimates simultaneously for real estate and stock markets from USA, Europe and Asia-Pacific over a period of more than 20 years. Secondly, we provide evidence that there is a statistically significant relationship between the magnitude of 10-day-ahead 99%VaR and the level of dynamic correlation for real estate and stock market indices.

This paper provides an efficient approach to forecasting market risk. An underestimation of market risk may impact on the profitability of banks firstly directly through higher capital charges, secondly, through damage of banks and of other participants in financial markets reputation, and thirdly, through the imposition of a more stringent external model to forecast the VaR thresholds (see McAleer and da Veiga, 2008a). On the other hand, an overestimation of market risk may force banks to keep too much of regulatory capital leading to a cost on profits. Moreover, an accurate estimation of 10-day-ahead VaR is crucial when there are liquidity crises where investors might be unable to liquidate their wealth for a 10-trading-day period. Furthermore, the paper gives evidence on local dynamic correlations between the securitized real estate markets and stock markets for seven countries across the world. The results are significant, on the grounds of an international investor or a portfolio manager, since real estate securitized assets usually contribute to portfolio diversification (see Liow and Ibrahim, 2010). This could urge an investor or a risk manager, who wants to forecast market risk across the above markets, to take into account not only the lagged VaR estimations within a market but also the effects from one market to another.

The rest of the paper is structured as follows: Section 2 provides a brief literature review; Section 3 offers the background on the empirical data; Section 4 outlines the methodology used in this study and presents the technique of estimating the 10-day-ahead 99%VaR; Section 5 analyses the empirical results; Section 6 concludes the paper and refers to some important implications.

## 2. Literature Review

There are not a lot of studies that examine the VaR estimation in real estate markets. Jin and Ziobrowski (2011) focus on downside market risk in residential housing for ten US metropolitan regions, for the period 1991-2007. They indicate that the conditional VaR models provide a more conservative estimate of market risk in comparison to modern portfolio theory (MPT), whereas the traditional risk measure based on a longer time series is less influenced by short-term extremes. He *et al.* (2008) propose the wavelet denoising ARMA-GARCH approach for estimating VaR in Chinese real estate market. Their data cover the closing prices for the Shanghai real estate index for the period from April, 1993 to December, 2007. They find that the wavelet denoising ARMA-GARCH model for VaR estimation has superior performance compared to the traditional ARMA-GARCH model.

A number of studies have focused on the relationship among securitized real estate markets worldwide, as well as among securitized real estate markets and general financial markets. Liow and Ibrahim (2010) evaluate the pattern and degree of international securitized real estate market volatility linkages, as well as their co-movements with the stock markets from the viewpoint of an international investor considering the mean-variance framework to portfolio management. They use monthly data from twelve countries worldwide for the period 1984-2006. A C-GARCH model was used to decompose the temporal variation of twelve securitized real estate market volatilities into long-run (*permanent*) and short-run (*transitory*) components. They use a factor analysis technique to summarize the *permanent* and *transitory* volatility dynamics into latent factors and demonstrate significant volatility co-movements between real estate and stock markets' *permanent* and *transitory* components suggesting that real estate markets are integrated within the international stock markets. Clayton and MacKinnon (2003), using data from the US market for the period 1979 to 1998, show that the REIT market was driven largely by the same economic factors that drive large cap stocks through the 1970s and 1980s. On the other hand, REIT returns were strongly related to both small cap stock and real estate-related factors in the 1990s. Yang *et al.* (2012) find that USREIT returns exhibit stronger asymmetric volatilities than stock, bond and Commercial Mortgage-Backed Securities returns. They also provide evidence that

REIT and stock returns have significant asymmetries in conditional correlation which implies less hedging potential of REITs to stock market investors during the US market downturn. Michayluk *et al.* (2006) look at the asymmetric volatility transmission, the correlation and the return dynamics between the US and the UK securitized real estate markets. They demonstrate that real estate markets have significant interaction on a daily basis when synchronously priced data are utilized. Moreover, significant asymmetric effects were found on both volatility and correlation dynamics between the markets and, more specifically, it was evident that daily foreign news from the US can influence UK volatility. Hoesli and Reka (2011) investigate the relationships between securitized real estate and stock markets in a national level, as well as between local and global securitized real estate markets. They use data for the period 1990-2010 from the US, the UK and the Australia markets. Based on an asymmetric t-BEKK model, they find that the strongest volatility spillovers between securitized real estate and stock markets exist in the US. They argue that the three national markets influence more the volatility of global market than vice versa. Additionally, using the copula theory and a structural break test, they found financial contagion between the US and the UK securitized real estate markets since the beginning of 2007.

## 3. Data Description

We use seven (USA, UK, GERMANY, JAPAN, AUSTRALIA, HONG KONG and SINGAPORE) international real estate indices obtained from FTSE EPRA/NAREIT[3] which include listed/securitized companies that have their core business in real estate activities (REITs and non-REITs), as well as seven major stock market indices for the same countries (S&P500, FTSE100, DAX30, NIKKEI225, ALL ORDINARIES AUS, HANG SENG and WISNGP). The dataset for stock indices was obtained from Bloomberg® database except from the Singapore (WISNGP) index which was obtained from the DataStream® database services. The sample covers the period of January 1990-

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[3]According to FTSE EPRA/NAREIT (2011) "A real estate investment trust (REIT) is a publicly traded real estate company that owns and may manage investment-grade commercial or residential real estate. REITs provide investors with a liquid and cost efficient way to earn the investment returns typically available from direct real estate investment. To qualify as a REIT, a real estate company must satisfy certain requirements set forth by each Government legislation, including the yearly distribution to its shareholders of at least 90% of its taxable income. In return for distributing most or all of its taxable income, the company pays no corporate tax on the distributed income. Non-REIT property companies are those which are either domiciled in countries without REIT legislation or have chosen not to opt for REIT status".

September 2011 period and incorporates 5,640 daily observations for each asset. All data are expressed in US dollars.

Table 1 gives the descriptive statistics for the securitized real estate and stock market log-returns. Japan stock market provides the lowest mean (-0.00027), whereas Hong Kong stock markets the highest (0.00033). Minima vary between -0.21700 (Germany real estate index) and -0.08875 (DAX 30). Maxima vary between 0.06069 (ALL ORDINARIES AUS) and 0.23101 (Singapore real estate index). The historical standard deviations range between 0.927% for Australia stock market and 2.194% for Japan real estate market. All stock markets are domestically less volatile compared to their respective real estate markets. The skewness is negative for the majority of the fourteen returns series. Only the Hong Kong real estate market and the two markets of Singapore are positively skewed. The fourteen log-return series are leptokurtic. The kurtosis appears to be the largest for the USA real estate (31.27625), as well as the Singapore (12.67160) and Hong Kong (12.68444) stock markets. The normality assumption is violated, at any reasonable level of significance, for all the fourteen log-return series (Jarque-Bera statistics are available upon request).

[Insert Table    1     about here]


## 4. Methodology

### 4.1. Model Description

Let us denote $\{\mathbf{y}_t\}$, the $(n \times 1)$ vector containing the log-returns for the $n = 14$ indices, at day $t$. A multivariate model for the $n$ discrete time real-valued stochastic processes can be expressed as $\mathbf{y}_t = \mathbf{B}'\mathbf{x}_t + \boldsymbol{\varepsilon}_t$, where $\mathbf{B}$ is a matrix of parameters to be estimated, $\mathbf{x}_t$ denotes the vector of explanatory variables included in $I_{t-1}$ (the information set at time $t-1$) and $\boldsymbol{\varepsilon}_t$ represents the vector of the innovation process. In order to model the time varying property of volatility of $\mathbf{y}_t$, the innovation process is defined as $\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2}\mathbf{z}_t$, where, $\mathbf{z}_t$ is an identically and independently distributed vector process such that $E(\mathbf{z}_t) = \mathbf{0}$ and $E(\mathbf{z}_t \mathbf{z}_t') = \mathbf{I}$. For $f(.)$ stating the conditional density function of $\boldsymbol{\varepsilon}_t$, we define the distribution of the innovation process as $\boldsymbol{\varepsilon}_t \mid I_{t-1} \sim f[0, \mathbf{H}_t]$.

Hence, $\mathbf{H}_t$ denotes the conditional variance covariance matrix of the $\mathbf{y}_t$, or $V_{t-1}(\mathbf{y}_t) \equiv \mathbf{H}_t$. Moreover, the $\mathbf{H}_t$ is defined as a function, $g(.)$, of the lagged $\mathbf{H}_t$ and $\boldsymbol{\varepsilon}_t$, or $\mathbf{H}_t = g(\mathbf{H}_{t-1}, \mathbf{H}_{t-2},..., \boldsymbol{\varepsilon}_{t-1}, \boldsymbol{\varepsilon}_{t-2},...)$. For more information about multivariate ARCH models, see to Andresen *et al.* (2006) and Xekalaki and Degiannakis (2010).

Bollerslev *et al.* (1988) introduce a positive definite multivariate ARCH model named Diag-VECH model, which ensures the positive definiteness of the conditional variance covariance matrix and requires the estimation of fewer parameters compared to other specifications such as the multivariate-GARCH, the BEKK, the VECH, the unrestricted Diag-VECH, etc. The model is estimated for two multivariate density functions: the normal distribution, as well as the Student-*t* density function.

We estimate the fourteen-dimensional multivariate Diag-VECH model for the seven equity indices and their relative real estate indices. The $y_{i,t}$ denotes the daily log-returns for $i = 1,2,...,n$ indices, where $n = 14$. The Diag-VECH model is estimated in the form:

$$
\mathbf{y}_t = \begin{pmatrix} \beta_1 & \beta_2 & ... & \beta_n \end{pmatrix}' + \boldsymbol{\varepsilon}_t
$$
$$
\boldsymbol{\varepsilon}_t = \mathbf{H}_t^{1/2} \mathbf{z}_t
$$
$$
\mathbf{z}_t \sim f(\mathbf{z}_t; \mathbf{0}, \mathbf{I}, v) \tag{1}
$$
$$
\boldsymbol{\varepsilon}_t \mid I_{t-1} \sim f[0, \mathbf{H}_t],
$$
$$
vech(\mathbf{H}_t) = vech(\tilde{\mathbf{A}}_0) + vech(\tilde{\mathbf{A}}_1) \circ vech(\boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1}) + vech(\tilde{\Gamma}_1) \circ vech(\tilde{\boldsymbol{\varepsilon}}_{t-1} \tilde{\boldsymbol{\varepsilon}}'_{t-1}) + vech(\tilde{\mathbf{B}}_1) \circ vech(\mathbf{H}_{t-1}),
$$

where the innovation process, $\boldsymbol{\varepsilon}_t$, has an $(n \times n)$ conditional variance covariance matrix $\mathbf{H}_t$, and $\mathbf{z}_t$ is an $(n \times 1)$ vector process such that $E(\mathbf{z}_t) = \mathbf{0}$ and $E(\mathbf{z}_t \mathbf{z}'_t) = \mathbf{I}$, whereas $f(\mathbf{z}_t; \mathbf{0}, \mathbf{I})$ is the multivariate density function. The $vech(.)$ operator stacks the columns of an $(n \times n)$ square matrix from the diagonal downwards in an $(n(n+1)/2) \times 1$ vector[4].

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[4] The Diag-VECH model is estimated as (see Xekalaki and Degiannakis, 2010) $vech(\mathbf{H}_t) = vech(\tilde{\mathbf{A}}_0) + \tilde{\mathbf{A}}_1 vech(\boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1}) + \tilde{\Gamma}_1 vech(\tilde{\boldsymbol{\varepsilon}}_{t-1} \tilde{\boldsymbol{\varepsilon}}'_{t-1}) + \tilde{\mathbf{B}}_1 vech(\mathbf{H}_{t-1})$, however, the parameter matrices are defined in a vech form $vech(\mathbf{H}_t) = vech(\tilde{\mathbf{A}}_0) + vech(\tilde{\mathbf{A}}_1) \circ vech(\boldsymbol{\varepsilon}_{t-1} \boldsymbol{\varepsilon}'_{t-1}) + vech(\tilde{\Gamma}_1) \circ vech(\tilde{\boldsymbol{\varepsilon}}_{t-1} \tilde{\boldsymbol{\varepsilon}}'_{t-1}) + vech(\tilde{\mathbf{B}}_1) \circ vech(\mathbf{H}_{t-1})$.

As mentioned above, the model is estimated for two density functions: for the $N(\mathbf{z}_t;\mathbf{0},\mathbf{I})$ standard normal density function, as well as for the standardized Student-$t$ density function, $t(\mathbf{z}_t;\mathbf{0},\mathbf{I},v)=\dfrac{\Gamma\big((v+n)/2\big)}{\Gamma(v/2)\big(\pi(v-2)\big)^{n/2}}\left(1+\dfrac{\mathbf{z}'_t\mathbf{z}_t}{v-2}\right)^{-\frac{v+n}{2}}$ , where $\Gamma(.)$ is the gamma function and $v$ is the degree of freedom to be estimated, for $v>2$. Student-$t$ distribution allows modelling the excess leptokurtosis which is not captured by the ARCH process. The symbol $\circ$ denotes the Hadamard product. The model demands the estimation of 420 parameters, or $2(n+1)n$ , for $n=14$ , for the computation of the conditional variance-covariance matrix:

$$
vech(\mathbf{H}_t)=\begin{pmatrix}\sigma^2_{1,t}\\ \sigma_{1,2,t}\\ \sigma_{1,3,t}\\ \sigma_{1,4,t}\\ .\\ .\\ .\\ \sigma_{1,n,t}\\ \sigma^2_{2,t}\\ \sigma_{2,3,t}\\ .\\ .\\ .\\ \sigma_{n-1,n,t}\end{pmatrix}=\begin{pmatrix}a_{1,1}+\tilde{a}_{1,1}\varepsilon^2_{1,t-1}+\gamma_{1,1}\varepsilon^2_{1,t-1}d_{1,t-1}+\tilde{b}_{1,1}\sigma^2_{1,t-1}\\ a_{1,2}+\tilde{a}_{1,2}\varepsilon_{1,t-1}\varepsilon_{2,t-1}+\gamma_{1,2}\varepsilon_{1,t-1}d_{1,t-1}\varepsilon_{2,t-1}d_{2,t-1}+\tilde{b}_{1,2}\sigma_{1,2,t-1}\\ a_{1,3}+\tilde{a}_{1,3}\varepsilon_{1,t-1}\varepsilon_{3,t-1}+\gamma_{1,3}\varepsilon_{1,t-1}d_{1,t-1}\varepsilon_{3,t-1}d_{3,t-1}+\tilde{b}_{1,3}\sigma_{1,3,t-1}\\ a_{1,4}+\tilde{a}_{1,4}\varepsilon_{1,t-1}\varepsilon_{4,t-1}+\gamma_{1,4}\varepsilon_{1,t-1}d_{1,t-1}\varepsilon_{4,t-1}d_{4,t-1}+\tilde{b}_{1,4}\sigma_{1,4,t-1}\\ .\\ .\\ .\\ a_{1,n}+\tilde{a}_{1,n}\varepsilon_{1,t-1}\varepsilon_{n,t-1}+\gamma_{1,n}\varepsilon_{1,t-1}d_{1,t-1}\varepsilon_{n,t-1}d_{n,t-1}+\tilde{b}_{1,n}\sigma_{1,n,t-1}\\ a_{2,2}+\tilde{a}_{2,2}\varepsilon^2_{2,t-1}+\gamma_{2,2}\varepsilon^2_{2,t-1}d_{2,t-1}+\tilde{b}_{2,2}\sigma^2_{2,t-1}\\ a_{2,3}+\tilde{a}_{2,3}\varepsilon_{2,t-1}\varepsilon_{3,t-1}+\gamma_{2,3}\varepsilon_{2,t-1}d_{2,t-1}\varepsilon_{3,t-1}d_{3,t-1}+\tilde{b}_{2,3}\sigma_{2,3,t-1}\\ .\\ .\\ .\\ a_{n-1,n}+\tilde{a}_{n-1,n}\varepsilon_{n-1,t-1}\varepsilon_{n,t-1}+\gamma_{n-1,n}\varepsilon_{n-1,t-1}d_{n-1,t-1}\varepsilon_{n,t-1}d_{n,t-1}+\tilde{b}_{n-1,n}\sigma^2_{n-1,n,t-1}\end{pmatrix} \tag{2}
$$

where $d_{i,t-1}$ denotes the indicator function, i.e. $d_{i,t-1}=1$ if $\varepsilon_{i,t-1}<0$ , and $d_{i,t-1}=0$ otherwise. The time-varying correlations between $i^{th}$ and $j^{th}$ indices are estimated as:

$$
\rho_{i,j,t}=\frac{\sigma_{i,j,t}}{\sqrt{\sigma^2_{i,t}\sigma^2_{j,t}}}=\frac{a_{i,j}+\tilde{a}_{i,j}\varepsilon_{i,t-1}\varepsilon_{j,t-1}+\gamma_{i,j}\varepsilon_{i,t-1}d_{i,t-1}\varepsilon_{j,t-1}d_{j,t-1}+\tilde{b}_{i,j}\sigma^2_{i,j,t-1}}{\sqrt{\big(a_{i,i}+\tilde{a}_{i,i}\varepsilon^2_{i,t-1}+\gamma_{i,i}\varepsilon^2_{i,t-1}d_{i,t-1}+\tilde{b}_{i,i}\sigma^2_{i,t-1}\big)\big(a_{j,j}+\tilde{a}_{j,j}\varepsilon^2_{j,t-1}+\gamma_{j,j}\varepsilon^2_{j,t-1}d_{j,t-1}+\tilde{b}_{j,j}\sigma^2_{j,t-1}\big)}}. \tag{3}
$$

## 4.2. Evaluate Forecasting Ability

We evaluate forecasting VaR over a period of two weeks in calendar time, or 10 trading days, and at 99% confidence level based on the Basle Committee on Banking Supervision requirements. The 10-day 99%VaR for the $i^{th}$ index is computed as

$$VaR_{i,t+10|t}^{(99\%)} = f_{1\%}(z_t;0,1,\hat{v})\sqrt{\sigma_{i,t+10|t}^2},\qquad(4)$$

where $f_{1\%}(z_t;0,1,\hat{v})$ denotes the 1%-percentile of the distribution of $z_t$, i.e. either the standard normal ($\hat{v}\to\infty$) or the standardized Student-$t$ density function ($\hat{v}>2$), and $\sigma_{i,t+10|t}^2$ is the 10-step-ahead conditional variance. The 10-step-ahead conditional variance is estimated recursively as

$$\sigma_{i,t+\tau|t}^2 = a_{i,i} + \left(\tilde{a}_{i,i}+0.5\gamma_{i,i}+\tilde{b}_{i,i}\right)\sigma_{i,t-1+\tau|t}^2\qquad(5)$$

for $\tau=2,...,10$, and

$$\sigma_{i,t+1|t}^2 = a_{i,i} + \left(\tilde{a}_{i,i}+\gamma_{i,i}d_{i,t}\right)\varepsilon_{i,t|t}^2 + \tilde{b}_{i,i}\sigma_{i,t-1|t}^2\qquad(6)$$

for $\tau=1$.[5]

A violation occurs if the VaR estimate is not able to cover the realized loss (i.e. when $y_{i,t+10} < VaR_{i,t+10|t}^{(99\%)}$). In the next section we present the 99%VaR failure rates (percentage of VaR violations), the Kupiec's (1995) test for testing whether the observed failure rate, $N/T$,[6] is statistically equal to the expected violation rate, $p=0.01$, as well as the Christoffersen's (1998) test [7] for examining if the 99%VaR failures are independently distributed over time.

The Kupiec's test is based on the $LR_k = 2\log\left((1-N/T)^{T-N}(N/T)^N\right) - 2\log\left((1-p)^{T-N}p^N\right)$, which is asymptotically $\chi^2$

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[5]The analytical calculation of the 10-day ahead VaR is not very widespread due to its innate practical difficulties. Thus, most practitioners apply the square root of time scaling rule. However, Engle (2004) and Danielsson and Zigrand (2006) noted that the square of time rule leads to inadequate VaR forecasts. Recently, Degiannakis et al. (2013) provide evidence that even if the square-root rule (under an appropriate modelling framework i.e. fractionally integrated volatility modelling) is able to provide proper multi-period risk forecasts, there is significant difference of the percentage of observed violations between the square-root rule and an analytical calculation approach. The comparison is in favour of the analytical calculation of the 10-day ahead VaR.

[6] $N$ is the number of days on which a violation occurred across the total number of VaR estimates and $T=5,640/10$.

[7]Christoffersen (1998) test examines the null hypothesis that the observed violation rate is statistically equal to the expected violation rate as well as that the VaR failures are independently distributed over time.

distributed with one degree of freedom. The Christoffersen's test examines the likelihood ratio statistic $LR_c = -2\log\!\left((1-p)^{T-N}p^N\right) + 2\log\!\left((1-\pi_{0,1})^{n_{00}}\pi_{0,1}^{n_{01}}(1-\pi_{1,1})^{n_{10}}\pi_{1,1}^{n_{11}}\right)$, which is $LR_c \sim \chi_2^2$. Note that $\pi_{j,j'} = n_{j,j'}\left(\sum_{j'} n_{j,j'}\right)^{-1}$ indicates the probability that $j'(=0,1)$ occurs at trading day $t$, given that $j(=0,1)$ occurred at trading day $t-10$. $n_{j,j'}$ is the number of observations with value $j$ followed by $j'$, i.e., $j, j' = 1$ indicates a violation, whereas $j, j' = 0$ indicates the opposite.

## 5. Results Analysis

Table 2 provides the average 10-day-ahead 99%VaR results of the Diag-VECH model under the Normal density function as well as the results of the backtesting tests. For the four real estate indices (UK, German, Japan and Singapore) and two stock indices (FTSE100 and NIKKEI225) the backtesting results indicate that either the VaR is over-estimated or the VaR violations tend to be clustered.

Concerning the Diag-VECH model under the Student-*t* density function, the 10-day-ahead 99%VaR is accurately estimated for all the real estate and the stock indices (see Table 3). Kupiec's and Christoffersen's backtesting tests provide p-values which do not reject the hypotheses i) that the observed violation rate is statistically equal to the expected violation rate, as well as ii) that the VaR failures are independently distributed over time. Hence, the Diag-VECH model under the Student-*t* density function is adequate and efficient for multi-period 99%VaR forecasts. Pearson's goodness-of-fit test and Engle and Manganelli's (2004) dynamic quantile test also provide qualitatively similar results.

Moreover, based on the framework of Degiannakis (2008), we investigate whether the model under the Student-*t* distribution provides statistically superior VaR estimates compared to the model under the normal distribution and confirm the superiority of the model framework under the Student-*t* density function. Diebold and Mariano (1995) and Hansen (2005) define methods for testing whether a model statistically provides superior forecasts of the variable into consideration (i.e. the 10-day-ahead 99% VaR forecast), in terms of a predefined loss function (which measures the distance between actual and predicted values). For the purposes of our study, Diebold and

Mariano's and Hansen's tests are applied for the quadratic loss between $VaR_{i,t+10|t}^{(99\%)}$ and

$y_{i,t+10}$. For all the indices, the model under the Student-$t$ distribution provides statistically superior forecasts (the null hypothesis of equivalent predictive ability of the models is rejected in any case with a p-value less than 0.05).

[Insert Table   2   about here]

[Insert Table   3   about here]

Figure 1 presents the 10-day-ahead 99%VaR from the aforementioned model under the Student-$t$ density function. It is obvious from Figure 1 that VaR thresholds evaluate the realized daily log-returns efficiently for all the 14 indices. According to Figure 1 and the 1$^{st}$ column of Table 3, the real estate portfolios are riskier and need higher (in absolute values) VaR estimations than their relative stock portfolios of each country. In the sample period the most risky indices tend to be the Japan, Hong Kong and Singapore real estate indices, with the highest average 10-day-ahead VaR. This phenomenon possibly derives from, firstly, the Asian Flu (1997-1998) where the real estate market plummeted and, secondly, the recent global financial crisis. Concerning Europe, Germany gives larger VaR estimations (in absolute values) for real estate and stock market indices than UK's. Australia and Singapore stock portfolios were found to be the less risky markets.

Another remarkable point derived from Figure 1 is that the estimates of VaR thresholds become sharply higher in the period of global financial crisis (2008-2010) for all the indices under study than the previous period, whereas from 2011 and onwards the VaR estimations go back to a more steady level.

[Insert Figure   1   about here]

Figure 2 depicts the seven local time-varying correlations between stock and real estate markets. The results suggest that time varying correlation fluctuates remarkably for USA, UK, Germany, Singapore and Australia at mostly positive levels – although Germany gives some periods with slight negative correlation. However, during the last three years, there is a trend for highly positive and less volatile correlation concerning the above countries maybe due to the recent global financial crisis. Regarding the rest Asian markets, the results are slightly different. Even though time varying correlation between

the two markets in Japan is positive and varies significantly, it does not present a significant trend during the last three years (of the recent financial crisis). Hong Kong real estate and stock markets are highly positive correlated throughout the whole period under study.

[Insert Figure     2     about here]

Having analyzed the results obtained from the VaR forecasts and the dynamic correlation between the real estate and stock markets across the world, an interesting aspect would be the identification of a possible lead-lag relationship between the 10-day-ahead 99%VaR and the local time-varying correlation. From visual inspection of Figures 1 and 2, we may infer that the high magnitude of the volatility is related to high levels of time varying correlation. However, in order to provide feasible evidence, we investigate whether the lagged value of the local correlation between stock and real estate indices provides statistically significant explanatory power in estimating the 99%VaR. In the regression model which follows, we test the impact of statistical significance of the lagged VaR estimations as well as the lagged value of the local correlation, $\hat{\rho}_{i,j,t-1}$ on the 10-day- ahead VaR estimations $VaR_{i,t|t-10}^{(99\%)}$,

$$VaR_{i,t|t-10}^{(99\%)} = \delta_0 + \delta_1 VaR_{i,t-10|t-20}^{(99\%)} + \delta_2 \hat{\rho}_{i,j,t-1} + \upsilon_t, \tag{7}$$

for normally distributed $\upsilon_t$. In Table 4, we present the estimated values and the relative *t*-statistics of the $\delta_1$, $\delta_2$ coefficients. The statistical significance of coefficient $\delta_2$ denotes that the lagged value of the local correlation, $\hat{\rho}_{i,j,t-1}$, provides explanatory power supplementary to the lagged estimate of $VaR_{i,t|t-10}^{(99\%)}$ ( $\delta_1$ is expected to be statistically significant due to the high autocorrelation of volatility). Interestingly, for 13 of the 14 indices, the lagged value of the local correlation between stock and real estate indices provides statistically significant explanatory power in estimating the 10-day-ahead 99%VaR. Only in the case of the USA real estate index, the VaR estimations are not influenced by the local correlation between stock and real estate indices.

[Insert Table     4     about here]

## 5. Concluding Remarks

The paper contributes to the existing literature as it bridges the gap between VaR theory and practice for the securitized real estate and stock markets across the world. A fourteen-dimensional Diag-VECH model is estimated for equity and their relative real estate indices from USA, Europe and Asia-Pacific. We use daily data from a period of over 20 years. The proposed multivariate modeling framework provides accurate multi-period risk estimates for leptokurtic and asymmetrically distributed real-estate and stock portfolio returns. Following the Basel Committee requirements for financial regulation, we proceed to the efficient estimation of 10-day-ahead 99%VaR for real estate markets together with stock markets of seven countries across the world.

Concerning the time-varying correlations between the local real estate and equity markets we find out mainly high positive dynamic correlations between the real estate and stock price indices for all the countries under study. Future research could examine if there is a structural break on the specific dynamic correlation for the above mentioned markets, which may be caused by the recent global financial crisis.

Finally, we find out the significant explanatory power of the local correlation for the majority (13 out of 14) of stock and real estate indices in estimating the 10-day-ahead 99%VaR.

The accuracy of the VaR model obtained in this paper could have implications for the accuracy of the VaR models used by banks for capital constitutions purposes as well as for international investors and risk managers who should assess market risk accurately. Large violations from an internal bank VaR model could lead to a bank failure. Accurate VaR estimations can be used also from a risk manager to execute portfolio hedging strategies against the amount measured by VaR. Another implication of this paper is that it provides a realistic view of securitized real estate market risk across the world with the help of VaR instrument since there is scarce literature on this topic. Also, the focus of this paper is on the 10-day-ahead VaR and not on daily VaR estimations, since that banks and investors could have an extra information tool for their capital constitution especially at times of liquidity crises. This paper could be helpful for an investor or a risk manager since it shows that between real estate and stock markets, portfolio diversification opportunities are diminished during the recent global financial crisis.

Future research could investigate the benefits of the usage and implications of local correlation's predictive power in estimating the 10-day-ahead 99%VaR in financial applications i.e. portfolio management, market risk minimization, VaR and expected shortfall forecasting.

## References

**Andersen, T., Bollerslev, T., Christoffersen, P. and Diebold, F.X. (2006).** Volatility and Correlation Forecasting.In (eds.) Elliott, G. Granger, C.W.J. and Timmermann, A *Handbook of Economic Forecasting*, North Holland Press, Amsterdam.

**Basel Committee on Banking Supervision (2006).** International Convergence of Capital Measurement and Capital Standards – A Revised Framework. Basel.

**Basel Committee on Banking Supervision (2009).** Revisions to the Basel II Market Risk Framework. Basel.

**Bauwens, L., Laurent, S. and Rombouts, J. (2006).** Multivariate GARCH models: A survey, *Journal of Applied Econometrics*, 21(1), 79-109.

**Bollerslev, T., Engle, R.F. and Wooldridge, J.M. (1988).** A Capital Asset Pricing Model with Time-Varying Covariances, *Journal of Political Economy*, 96,116-131.

**Brooks, C. and Persand, G. (2003).** Volatility Forecasting for Risk Management, *Journal of Forecasting*, 22(1), 1-22.

**Cheong, C.W. (2011).** Univariate and Multivariate Value-at-Risk: Application and Implication in Energy Markets, *Communications in Statistics - Simulation and Computation*, 40(7), 957-977.

**Choudhry, M. (2006).** *An Introduction to Value-at-Risk*, 4th Edition, John Wiley and Sons, New York, NY.

**Christoffersen, P. (1998).** Evaluating Interval Forecasts, *International Economic Review*, 39, 841-862.

**Christoffersen, P. (2009).** Value-at-Risk Models. In T. Andersen, R. Davis, J.-P. Kreiss, and T. Mikosch (Eds.), *Handbook of Financial Time Series*, Springer Verlag.

**Clayton, J. and MacKinnon, G. (2003).** The Relative Importance of Stock, Bond and Real Estate Factors in Explaining REIT Returns, *Journal of Real Estate Finance and Economics*, 27, 39-60.

**Daníelsson, J. and J.P. Zigrand.(2006).** On Time-Scaling of Risk and the Square Root-of-Time-Rule., *Journal of Banking and Finance*, 30(10), 2701–2713.

**Degiannakis, S. (2008).** ARFIMAX and ARFIMAX- TARCH Realized Volatility Modeling, *Journal of Applied Statistics*, 35(10), 1169-1180.

**Degiannakis, S., Floros, C. and Dent, P. (2012).** Forecasting Value-at-Risk and Expected Shortfall using Fractionally Integrated Models of Conditional Volatility: International Evidence, *International Review of Financial Analysis*, forthcoming.

**Degiannakis, S., Floros, C. and Dent, P. (2013).** A Monte Carlo Simulation Approach to Forecasting Multi-period Value-at-Risk and Expected Shortfall using the FIGARCH-skT Specification, *The Manchester School*, forthcoming.

**Diebold, F.X. and Mariano, R. (1995).** Comparing Predictive Accuracy, *Journal of Business and Economic Statistics*, 13(3), 253-263.

**Engle, R.F. (2004).** Risk and Volatility: Econometric Models and Financial Practice. *The American Economic Review*, 94(3), 405-420.

**Engle, R.F. and Manganelli, S. (2004).** CAViaR: Conditional Autoregressive Value at Risk by Regression Quantiles, *Journal of Business and Economic Statistics*, 22(4), 367-381.

**Ftse Epra/Nareit Developed Europe Reits and Non-Reits Indices. (2011).** Definition of a REIT, available in www.ftse.com/Indices at EPRA_NAREIT_ Developed_Europe_REITs_Factsheet.pdf

**Gregoriou, G. (2009).** *The VAR Implementation Handbook*, McGraw-Hill Education, USA.

**Hansen, P.R. (2005).** A Test for Superior Predictive Ability, *Journal of Business and Economic Statistics*, 23, 365-380.

**He, K., Xie, C. and Lai, K.K. (2008).** Estimating Real Estate Value-at-Risk using Wavelet Denoising and Time Series Model, *Computational Science-ICCS*, 5102, 494-503.

**HoesliM. and Reka, K. (2011).** Volatility Spillovers, Comovements and Contagion in Securitized Real Estate Markets, *Swiss Finance Institute*, Research Paper Series, 10, 40.

**Jin, C. and Ziobrowski, A.J. (2011).** Using Value-at-Risk to Estimate Downside Residential Market Risk, *Journal of Real Estate Research*, 33(3), 389-414.

**Jorion, P. (2006).** *Value at Risk: The New Benchmark for Managing Financial Risk*, 3rd Edition, McGraw-Hill Education, USA.

**Kuester, K., Mittnik, S. and Paolella, M. (2006).** Value-at-risk Prediction: A Comparison of Alter-native Strategies, *Journal of Financial Econometrics*, 4(1), 53-89.

**Kupiec, P.H. (1995).** Techniques for Verifying the Accuracy of Risk Measurement Models, *Journal of Derivatives*, 3, 73-84.

**Liow, K.H. and Ibrahim, M.F. (2010).** Volatility Decomposition and Correlation in International Securitized Real Estate Markets, *Journal of Real Estate Finance and Economics*, 40(2), 221-243.

**McAleer, M. and da Veiga, B. (2008a).** Forecasting Value-at-Risk with a Parsimonious Portfolio Spillover GARCH (PS-GARCH) model, *Journal of Forecasting*, 27(1), 1-19.

**McAleer, M. and da Veiga, B. (2008b).** Single-index and Portfolio Models for Forecasting Value at Risk Thresholds, *Journal of Forecasting*, 27(3), 217-235.

**Michayluk, D., Wilson, P.J. and Zurbruegg, R. (2006).** Asymmetric Volatility, Correlation and Returns Dynamics between the U.S. and U.K. Securitized Real Estate Markets, *Real Estate Economics*, 34, 109-131.

**Rossignolo, A.F., Fethi, M.D. and Shaban, M. (2012).** Value-at-Risk models and Basel capital charges: Evidence from Emerging and Frontier stock markets, *Journal of Financial Stability*, 8(4), 303-319.

**Santos, A.P., Nogales, F. and Ruiz, E. (2009).** Comparing Univariate and Multivariate Models to Forecast Portfolio Value-at-Risk, Statistics and Econometrics Series, Universidad Carlos III de Madrid, Madrid, working paper, no.09-72.

**Xekalaki, E. and Degiannakis, S. (2010).** *ARCH Models for Financial Applications*, John Wiley and Sons, New York, NY.

**Yang, J., Zhou, Y. and Leung, W.K. (2012).** Asymmetric Correlation and Volatility Dynamics among Stock, Bond and Securitized Real Estate Markets, *Journal of Real Estate Finance and Economics*, 45(2), 491-521.

**Figures**

Figure 1. The daily log-returns against the 10-day-ahead 99%VaR metrics, from the Diag-VECH model under the Student-$t$ density function.
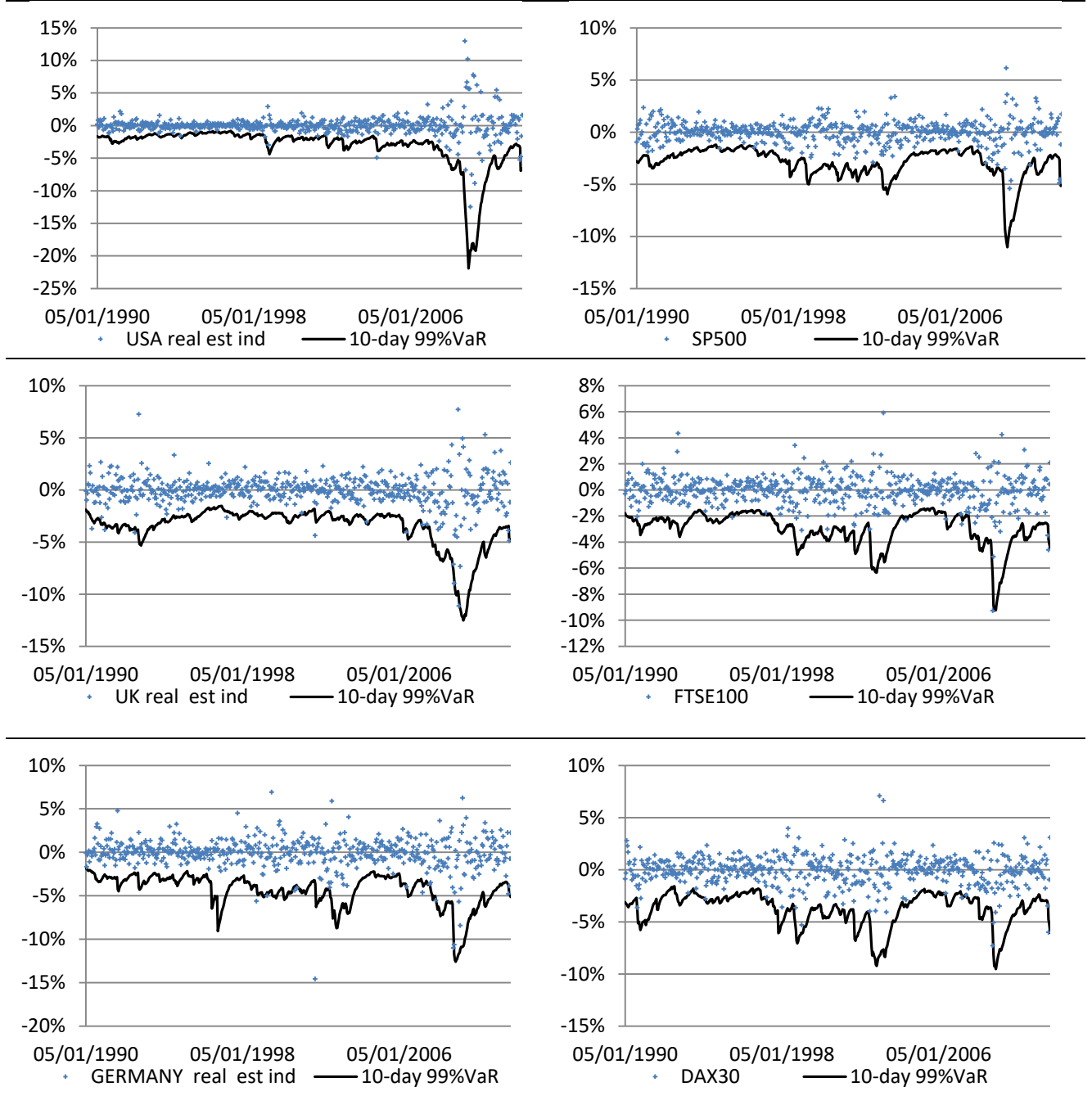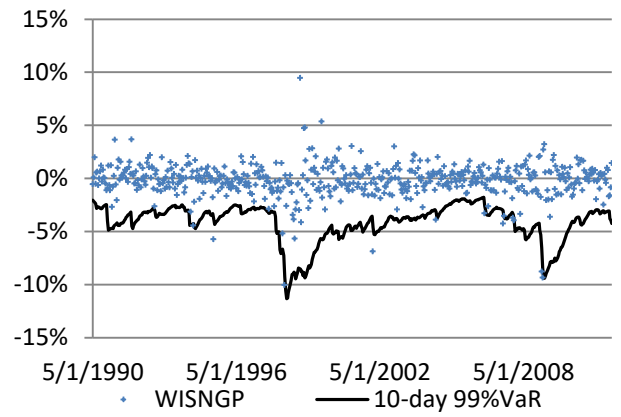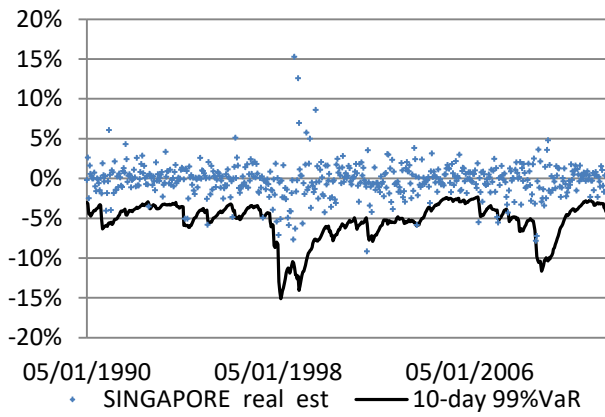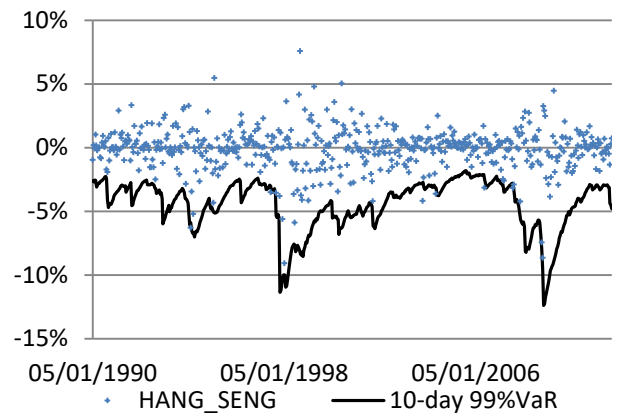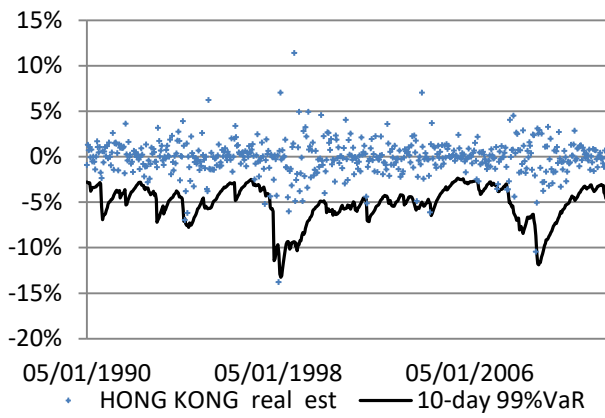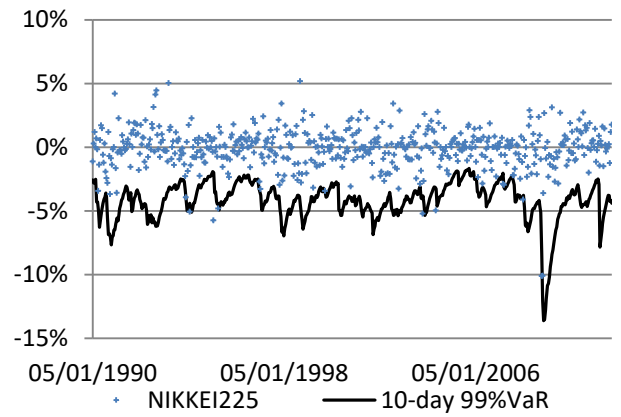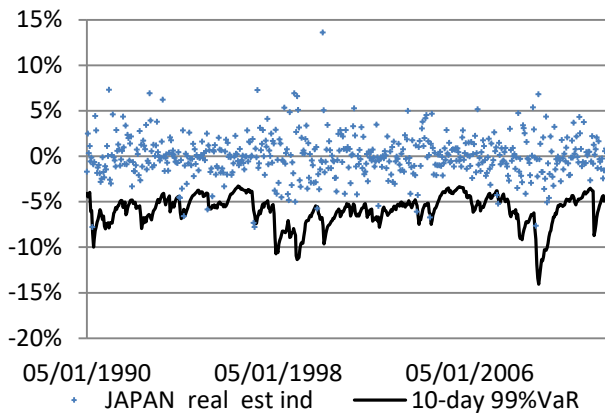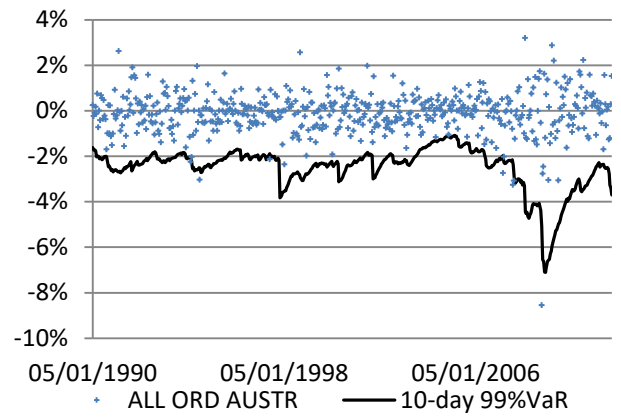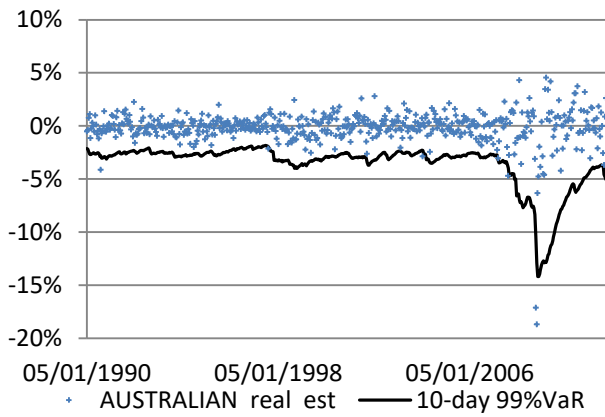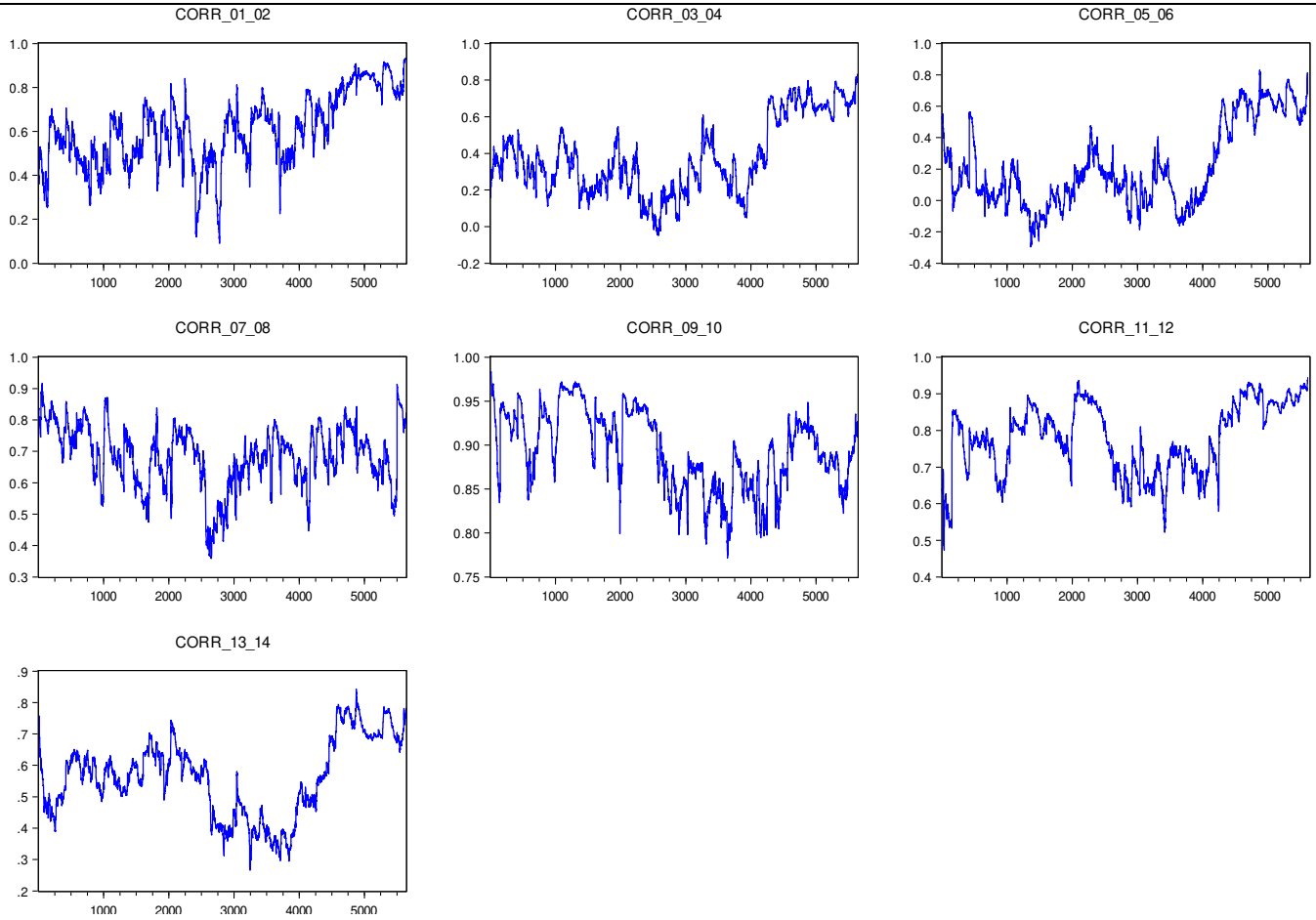
Figure 2. The daily time-varying correlations between the 7 stock and real estate indices.



corr_01_02:(USA real est ind,SP500), corr_03_04:(UK real est ind,FTSE100), corr_05_06:(GERMANY real est ind,DAX30), corr_07_08:(JAPAN real est ind,NIKKEI225), corr_09_10:(HONG KONG real est,HANG_SENG),corr_11_12:(SINGAPORE real est,WISNGP), corr_13_14:(AUSTRALIAN real est,ALL ORD AUSTR).

**Tables**

Table 1. Descriptive statistics of seven countries securitized real estate and stock market returns

| Index | Mean | Median | Maximum | Minimum | SD | Skewness | Kurtosis |
|---|---|---|---|---|---|---|---|
| USA real est | 0.00021 | 0.00019 | 0.16849 | -0.21689 | 0.01615 | -0.29126 | 31.27625 |
| SP500 | 0.00021 | 0.00026 | 0.10957 | -0.09470 | 0.01162 | -0.24209 | 12.02405 |
| UK real est | -0.00005 | 0.00002 | 0.11793 | -0.11135 | 0.01438 | -0.30475 | 11.47116 |
| FTSE100 | 0.00014 | 0.00004 | 0.09384 | -0.09266 | 0.01139 | -0.13066 | 9.38795 |
| GERMANY real est | 0.00001 | 0.00017 | 0.14111 | -0.21700 | 0.01713 | -0.49848 | 15.82820 |
| DAX30 | 0.00020 | 0.00042 | 0.10798 | -0.08875 | 0.01454 | -0.12486 | 7.87059 |
| JAPAN real est | -0.00007 | -0.00059 | 0.19391 | -0.13270 | 0.02194 | 0.38155 | 7.41940 |
| NIKKEI225 | -0.00027 | 0.00000 | 0.13235 | -0.12111 | 0.01528 | -0.09068 | 8.94077 |
| HONG KONG real est | 0.00025 | 0.00007 | 0.19675 | -0.13815 | 0.01896 | 0.11480 | 10.23287 |
| HANG_SENG | 0.00033 | 0.00000 | 0.17247 | -0.14735 | 0.01653 | -0.00180 | 12.68444 |
| SINGAPORE real est | 0.00005 | -0.00003 | 0.23101 | -0.18479 | 0.02043 | 0.54561 | 13.14961 |
| WISNGP | 0.00016 | 0.00033 | 0.18351 | -0.12669 | 0.01518 | 0.19803 | 12.67160 |
| AUSTRALIAN real est | 0.00004 | 0.00044 | 0.10443 | -0.18683 | 0.01450 | -1.24794 | 20.12661 |
| ALL ORD AUSTR | 0.00015 | 0.00016 | 0.06069 | -0.08554 | 0.00927 | -0.54278 | 9.71165 |

Table 2. Diag-VECH model under the normal density function. 10-day-ahead 99%VaR violations, Kupiec's and Christoffersen's tests.

| Index | Average 10-day-ahead 99%VaR | Observed exception rate | Kupiec's p-value | Christoffersen's p-value |
|---|---|---|---|---|
| USA real est | -2.85% | 0.53% | 0.221 | 0.466 |
| SP500 | -2.62% | 1.06% | 0.877 | 0.926 |
| UK real est | -3.09% | 2.66% | 0.001** | 0.001** |
| FTSE100 | -2.58% | 1.95% | 0.044* | 0.007** |
| GERMANY real est | -4.00% | 1.95% | 0.044* | 0.106 |
| DAX30 | -3.30% | 1.60% | 0.189 | 0.131 |
| JAPAN real est | -5.35% | 1.77% | 0.095 | 0.008** |
| NIKKEI225 | -3.62% | 1.95% | 0.044* | 0.106 |
| HONG KONG real est | -4.61% | 1.60% | 0.189 | 0.365 |
| HANG_SENG | -3.95% | 1.06% | 0.877 | 0.926 |
| SINGAPORE real est | -4.68% | 1.95% | 0.044* | 0.059 |
| WISNGP | -2.25% | 0.71% | 0.467 | 0.745 |
| AUSTRALIAN real est | -3.12% | 1.42% | 0.345 | 0.159 |
| ALL ORD AUSTR | -2.19% | 1.24% | 0.576 | 0.163 |
| *denotes significant at 5%, ** denotes significant at 1%. | | | | |

Table 3. Diag-VECH model under the Student-*t* density function. 10-day-ahead 99%VaR violations, Kupiec's and Christoffersen's tests.

| Index | Average 10-day-ahead 99%VaR | Observed exception rate | Kupiec's p-value | Christoffersen's p-value |
|---|---|---|---|---|
| USA real est | -3.19% | 0.35% | 0.076 | 0.206 |
| SP500 | -2.93% | 0.53% | 0.221 | 0.466 |
| UK real est | -3.54% | 1.77% | 0.095 | 0.094 |
| FTSE100 | -2.96% | 1.24% | 0.576 | 0.163 |
| GERMANY real est | -4.33% | 1.42% | 0.345 | 0.571 |
| DAX30 | -3.78% | 0.89% | 0.786 | 0.091 |
| JAPAN real est | -5.92% | 1.60% | 0.189 | 0.059 |
| NIKKEI225 | -4.18% | 1.42% | 0.345 | 0.571 |
| HONG KONG real est | -5.16% | 1.24% | 0.576 | 0.783 |
| HANG_SENG | -4.42% | 1.06% | 0.877 | 0.926 |
| SINGAPORE real est | -5.30% | 1.60% | 0.189 | 0.131 |
| WISNGP | -2.52% | 0.53% | 0.221 | 0.466 |
| AUSTRALIAN real est | -3.60% | 1.06% | 0.877 | 0.138 |
| ALL ORD AUSTR | -2.50% | 1.24% | 0.576 | 0.163 |

Table 4. Estimated values and the relative *t*-statistics of $\delta_1$, $\delta_2$ coefficients from $VaR_{i,t|t-10}^{(99\%)} = \delta_0 + \delta_1 VaR_{i,t-10|t-20}^{(99\%)} + \delta_2 \hat{\rho}_{i,j,t-1} + \upsilon_t$.

| Index | $\delta_1$ | *t*-statistic | $\delta_2$ | *t*-statistic |
|---|---|---|---|---|
| USA real est | 0.988335 | 160.8940* | -0.001715 | -1.500741 |
| SP500 | 0.975682 | 117.9498* | -0.001988 | -2.877431* |
| UK real est | 0.984064 | 171.4080* | -0.001494 | -2.762552* |
| FTSE100 | 0.974601 | 121.3835* | -0.001714 | -3.399565* |
| GERMANY real est | 0.958691 | 93.00739* | -0.002410 | -3.276160* |
| DAX30 | 0.978607 | 114.5437* | -0.000892 | -1.701320** |
| JAPAN real est | 0.930825 | 66.38348* | -0.008783 | -3.585037* |
| NIKKEI225 | 0.923957 | 64.45283* | -0.009319 | -4.146429* |
| HONG KONG real est | 0.966763 | 98.68409* | -0.010540 | -2.316597* |
| HANG_SENG | 0.967722 | 100.4932* | -0.010304 | -2.403878* |
| SINGAPORE real est | 0.977483 | 128.9932* | -0.005636 | -2.918124* |
| WISNGP | 0.975402 | 132.7690* | -0.004637 | -3.362409* |
| AUSTRALIAN real est | 0.986491 | 201.5087* | -0.002889 | -3.270393* |
| ALL ORD AUSTR | 0.962570 | 112.0806* | -0.003159 | -4.649645* |

*denotes significant at 5%, **denotes significant at 10%.