New alternative measuring financial stability

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Abstract
If the z-score index is widely used as a measure of the stability in conventional banks, it would be more reliable to find an appropriate measure of the stability for all type of banks. Knowing that the cooperative or Islamic banks follow different contracts forms of investments such the PLS system and are closer to real economic, by considering the illiquid assets, we expect that the new measure labelled g-score, associated to tangible economic growth, reflects multiple risks and allows to track the banking stability.

Keywords
Stability, Liquid assets, Illiquid assets, z-score, g-score, probability of default.

1. Introduction
Intrinsically, the bank investment processes involve committing funds to expected profit earning assets, but the outcomes of short-term compared to long-term processes of investments are particularly affected by multiple type of risks driven by economic and financial conditions (He and Kondor 2016). If a good investment opportunities are expected to support bank soundness, it would lead to more solvency of the bank, while a bad investment may conduct to illiquidity problems; together lead to a situation of complete insolvency, where liabilities exceed assets. This state is called insolvency i.e. there is less value of assets that the bank controls with a certain amount of equity. Consequently, the bank can’t be able to payback its money. According to Demirgüç-Kant et al. (2004) the liquidity variable, considered as exogenous, constructed from cash and some securities, can be used to measure bank risk. As financial behavior of the banks, the liquidity dynamic cannot be addressed as exogenous. In fact, the illiquidity would be more related to bank risk issue, where the share of illiquid assets in total assets exposes bank to insolvency risk even if the bank assets predominate liabilities. Recently, the Basel Committee (2010) established two official regulatory measures of liquidity risk, namely the liquidity coverage ratio (LCR) for measuring short-term liquidity risk, and the net stable funding ratio (NSFR) for determining long-term liquidity risk by considering both side of the balance sheet. A high value of each measure corresponds to high bank liquidity. There is a need of a quantitative analytical index to capture the banking stability through liquidity or illiquidity variables which affect largely the probability of bank default.
Several well-known indices measure the financial stability mostly in banking sector. Among these indices, Value at Risk (Holton 2003, Manganelli and Engle 2001), Stress Test (Aragonés et al. 2001, BIS 2000) and z-score (Altman 1983). The probabilistic version of this latter makes it the most widely used (Lepetit and Strobel 2015), in addition to its advantage in predicting the financial distress of bank and focuses on bank’s risk of insolvency. The other indices signal and find out if the bank may face a liquidity problem. These measures could be affected by the nature of the bank’s activities; for example, the z-score can’t be applied directly to cooperative banks (owned by their customers) or Islamic banks which use specific accounting methods and work with the Profit-Loss Sharing (PLS) model which can expand the bank equity. The banks with capital risk are increasingly willing to consider transacting their assets in counter party arrangements whereby the credit-risk exposure is shared with the reduction in total risk in comparison to the lender system (Altman, Caouette and Narayan 1998). The insolvency problem is more perilous than the liquidity one, as it means that the bank’s liabilities exceed the value of its assets. A bank may become illiquid even when it is solvent, if its assets held are illiquid assets (either long-term financial assets or real assets) that can only be liquidated at high cost, and associated with greater probability of loss. This point is well-documented by Allen and Gale (2004). But, the economic value of a bank’s assets can be sufficient to cover its claims. The bank may be forced to sell part of such assets at lower than normal values. A measure of stable decomposition between liquid and illiquid assets could be more important and serving to manage early the risk of insolvency.

2. Z-score and probability of default

The time varying z-score using banks’ accounting quarterly data, with specific distinction in the balance sheet of each bank, is calculated using the following definitions:

\[ z_{t,q} = \frac{k_{t,q} + \mu_t}{\sigma}, \quad k_{t,q} = \frac{E_{t,q}}{A_{t,q}}, \quad \mu_t = \frac{\bar{R}_t}{A_{t,q}}, \quad \sigma^2 = V \left( \frac{R_{t,q}}{A_{t,q}} \right) \]

where \( k \) is the ratio of equity capital plus total reserves to assets (CAR); \( \mu \) represents the ratio of average returns to assets (ROA) i.e. profitability, where average returns are calculated on base of four observations per year. The variable \( \bar{R}_t \) is the mean of returns per quarter for the year \( t \). The parameter \( \sigma \) stands for the standard deviation of returns to assets, and measures the volatility of returns on assets; it is calculated over the full sample with infra-annual data. The original z-score (1) uses the standard deviation estimate of the return on assets, calculated over the full sample, and combine it with the current values of capital to assets ratio and a specific time-varying mean of returns on assets ratio based one the average per year of the quarterly net profit to avoid the seasonal volatility. In the related literature, there are many versions of z-score with yearly observations (as Boyd, De Nicolo and Jalal 2006, Hesse and Cihak 2007, Yeyati and Micco 2007). Lepetit and Strobel (2013) well-document various measure of z-score with interesting empirical discussion. As supported by Laeven and Levine (2009, page 262) and Houston, Chen, Ping, and Yue (2010), in empirical implementation the logged z-score is used to reduce the skewness of the
simple z-score (for more details see Strobel, 2014); but the logarithm could only smooth the distribution shape.

The z-score reflects the probability of insolvency or bank liabilities \( L \) exceed assets \( A \). From the accounting equation: \( A = L + E \); and hence if \( L > A \), then \( E < 0 \). Assuming normality distribution of bank returns \( R \), and defining insolvency as a state where losses exceed equity or capital i.e. \(-R \geq E \iff R \leq -E \iff R/A \leq -E/A \), we can write in term of probability that \( p(-R \geq E) \iff p(R \leq -E) \iff p(R/A \leq -E/A) \), then the probability of default is

\[
p\left( \frac{R}{A} \leq -\frac{E}{A} \right) \equiv p(\mu \leq -k) = \int_{-\infty}^{-k} \text{N}(0,1)d\mu
\]

\[
p\left( \frac{R}{A} \leq -\frac{E}{A} \right) = p\left( \frac{\frac{R}{A} - \frac{\mu_R}{A}}{\frac{\sigma_R}{A}} \leq - \frac{\frac{E}{A} + \frac{\mu_R}{A}}{\frac{\sigma_R}{A}} \right) = p(Z \leq -z) = \Phi(-z) = 1 - \Phi(z)
\]  \hspace{1cm} (2a)

where \( \Phi \) is the standard normal cumulative distribution function, corresponding to left tail-distribution. The significant lower z-score for a bank indicates that this bank is closer to insolvency than another bank. The z-score measures the number of standard deviations that returns realization have to fall to deplete equity (Čihak 2007). The greater z-score indicates the lower likelihood of bank insolvency risk; meaning that the probability that liabilities exceed assets is small i.e. there is no asset deficiency. The index will take high value when capitalization, measured in terms of risk error, is large. In many applications, the argument \( z \) appears higher than 4, which means that the probability of insolvency is near to zero. There are some biases in z-score that underestimate largely the probability of default.

Empirically, if the stochastic returns \( R \) are not normally distributed, we can determine another probabilistic version of the z-score by using one-sided Chebyshev inequality (Boyd, Graham and Hewitt 1993, and others). By using the Markov inequality, we can show that for any random variable as \( R/A \) with its mean \( \mu \) and variance \( \sigma^2 \) for any shape of distribution, and any positive number \( a \) (which is satisfied because \( a = k + \mu > 0 \), the following one-side Chebyshev inequality holds. It is easy to prove this proposition: Considering \( b > 0 \), we obtain that

\[
p\left( \mu - \frac{R}{A} \geq a \right) = p\left( \mu - \frac{R}{A} + b \geq a + b \right) \leq p\left( \left( \mu - \frac{R}{A} + b \right)^2 \geq (a + b)^2 \right)
\]

Upon applying Markov inequality on the preceding, we get that:

\[
p\left( \frac{R}{A} \leq \mu - a \right) \leq \frac{\text{E}\left( \left( \mu - \frac{R}{A} + b \right)^2 \right)}{(a + b)^2} = \frac{\sigma^2 + b^2}{(a + b)^2}
\]

By letting \( b = \frac{\sigma^2}{a} \) and knowing that \( z\sigma = k + \mu = a \), we obtain the following result:

\[
p\left( \frac{R}{A} \leq \mu - a \right) = p\left( \mu - \frac{R}{A} \geq a \right) \leq \frac{\sigma^2}{\sigma^2 + a^2} = \frac{1}{1 + z^2} < 1
\]  \hspace{1cm} (3)
this gives the upper bound of the probability of insolvency in bank.\(^1\) This result is obtained in Lepetit and Strobel (2015, Proposition 1, page 216). It can be used across time as the probabilistic interpretation of z-score. But in practice, the use of this maximum level makes the default probability more pertinent for large sample. Nevertheless, as indicated by Lapteacru (2016), the empirical distribution of the returns \(R\) have skewness and kurtosis statistical features. To correct the skewness of normal function, he utilizes Owen’s \(T\) function

\[
T(-z, \lambda) = (1/2\pi) \int_0^\lambda \exp{-0.5z^2(1+t^2)}/(1+t^2) \, dt
\]

which gives a probability of independent events in terms of an integral over an area of the standardized bivariate normal distribution.\(^2\) Using Monte Carlo simulations, Lapteacru show for non-normality of the returns, that the z-score with or without logarithmic transformation provide inconsistent results. The skewness parameter \(\lambda\) allows to avoid the under-estimation of the z-score. However, Lapteacru (2016) considers the stable distribution allowing more flexibility in the moments of the empirical distribution; it generalizes the central limit theorem to random variables without first, or second order moments.\(^3\) The cumulative stable distribution function \(\Phi_{\text{stable}}\) has no analytical expression, it is defined only for some parameter values of stability \(\beta\) (from 0 to 2), skewness \(\lambda\) (between –1 and +1), positive scale \(c\) and location \(\mu\) (real number) parameters. Considering \(\Phi_{\text{stable}}\) and \((\Phi)^{-1}\) the inverse of the cumulative distribution; Lapteacru determines the \(x\)-score is from \(-(\Phi)^{-1}[\Phi_{\text{stable}}(-k, \beta, \lambda, c, \mu)]\) where \(k\) stands for equity-assets ratio. He advocates that the stable distribution, via an arbitrary choice of its sharpness parameters, better fits the tails of the empirical distribution of returns in comparison to skew normal distribution and provides too small \(x\)-score with meaningful risk measure.

The z-score could be not appropriate to measure the risk of cooperative and Islamic banks, because the returns on assets depend on the nature of their activities. There is a need to construct an appropriate measure of risk considering the financing modes. We can focus on risks involved through the investment of banks in both liquid and illiquid assets. It would be particularly suitable for banks adopting investment strategies that prefer high risk assets given a high rate of return, or low risk assets even at low rate of returns, which guarantees the objectivity of the probability of default (Čihašek and Hesse 2010). The long-term investments of banks with illiquid assets could lead to higher levels of economic output; in contrast, their short-term investments with liquid assets may conduct to speculative financing and causes economic and financial crises. The link between bank liquidity and economic growth is well-documented in Berger and Sedunov (2017). They find

\(^1\) By applying Cantelli’s inequality (which is a generalization of Chebyshev inequality), for any negative number \(a\) satisfying \(a = k + \mu < 0, \ P\left(\frac{\mu}{\lambda} \leq \mu - a\right) \geq 1 - \frac{a^2}{\sigma^2 + a^2}\). This case corresponds to a huge loss.

\(^2\) The skew normal distribution family \(SN(\mu, \sigma, \lambda)\) is dedicated for modeling by adding a shape parameter \(\lambda\) that reduces the skewness effect. The cumulative distribution function \((\Phi)\) of the skew normal distribution is defined as \(\Phi(-z, \lambda) = \Phi(z) - 2T(-z, \lambda)\), and the corrected z-score “sz-score” by removing the skewness can be determined from \(-((\Phi)^{-1}[\Phi(-z, \lambda)]]\).

\(^3\) In probabilistic terms, a distribution is stable if a linear combination of two independent random variables (returns and \(x\)-score) with this distribution has the same distribution, up to location and scale parameters.
at aggregated per capita level a positive correlation between the output and the liquidity creation of banks. For many banks, there is a continuous trade-off between illiquid and liquid assets, each one influences the financial stability of the banks, but the contribution of illiquid assets to economic growth is more important for the long-run banking stability.

Regarding the mobilization of both side of the balance-sheet i.e. assets and liabilities, the z-score index is not also compatible with the nature of cooperative banking and Islamic banking, because this latter relies mainly on the PLS system combining the liabilities with the assets in the investment processes. The PLS, expanding the bank equity and the investment opportunities, leads to a common risk of the investor and bank via Shariah compliant contracts like “Mudarabah” and “Musharakah”; and in consequence, reduce the financial fragility of the bank. It is probable that the capital value and reserves do not reflect the financial strength of Islamic banks, because the investor shall bear a part of the risk according to Shariah-terms of the PLS contracts, and thus reduce fairly the risk of Islamic Banks. These banks may seek for adjustments processes in risk-taking rates by the investors through appropriate contracts of PLS system and new methods of capital investment. As mentioned by Čihak and Hesse (2010), the conventional banks also seek for adjustment processes of interest rates on deposits and loans to avoid insolvency.

3. G-score and illiquidity level

Wagner (2007) stated, that the liquid asset matters in banking stability. In the context of derivative instruments, he proved that an increase in liquidity rises retained risk and in crisis times reduces banking stability (Wagner 2007, Proposition 2, page 130). Also, He and Kondor (2016), by using stochastic dynamic model of liquidity management, show that the incentive of non-financial firms of the economy to shift from liquid resources to illiquid capital is stronger during booms than during recessions. Considering the financial firms as the banks, such shifting is inherent in their investment behavior.

It is recognized from the bank balance sheet that the bank is in a financially healthy position when its assets are worth more than its liabilities. When the bank liquid assets are depleted, the bank will be unable to meet the demand for withdrawals. It will need cash or reserves from other banks or the central bank to face any withdrawals. But, if the bank cannot borrow or attract additional cash or reserves, it will sell-off at low prices a part of its illiquid assets i.e. loans for commercial banks or sharing for cooperative or Islamic banks. If the shareholder equity is less than the difference between book value and sell-off value, the bank becomes insolvent. The equation (2) describes the first case of commercial banks insolvency.

The second case of the cooperative or Islamic banks, which work with the scheme of capital risk, is different. From the accounting equation, \( L = (1-\theta)D + \theta D \) where \( D \) stands for deposits and \( \theta \) is the invested share of the deposits as a contribution in the bank capital and \( (1-\theta) \) is simply the remaining fraction of deposits as demand deposits; we have: \( A = (1-\theta)D + E(\theta) \)

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4 The bank assets contain liquid (as cash, central bank reserves, bonds) and illiquid (loans or financing) assets. The bank liabilities include customer deposits and shareholder equity.
where \( E(\theta) = \theta D + E \). Hence, if \( L > A \), then \( E < 0 \). Assuming normality distribution of bank returns \( R \), and by the insolvency definition, we have \(-R \geq E(\theta) \geq E \iff R \leq -E(\theta) \leq -E \iff R/A \leq -E(\theta)/A \leq -E/A \), we can write in term of probability that \( p(-R \geq E(\theta)) \iff p(R \leq -E(\theta)) \Rightarrow p(R \leq -E(\theta)) \Rightarrow p(R/A \leq -E(\theta)/A) \), then the probability of default is

\[
p \left( \frac{R}{A} \leq - \frac{E(\theta)}{A} \right) = p \left( \frac{R}{A} - \frac{\mu_R}{\sigma_R} \right) > \Phi \left( - \frac{\mu_E}{\sigma_E} \right) = p \left( Z \leq -z_\theta \right) = \Phi(-z_\theta) \quad (2b)
\]

We observe that the equation (2a) is a special case of a more generalized equation (2b) when \( \theta = 0 \). More importantly, we deduce \( z_\theta > z \) and \( \Phi(-z_\theta) < \Phi(-z) \), meaning that the probability of insolvency is less in banks working with capital risk as PLS system than the probability of insolvent in banks working with loans system.

In the line of the contribution of this paper and in the spirit of the NSFR of Basel III (considering both assets and liabilities), we suggest to use, as indicators of bank risks related to liquidity and leverage having randomness feature, the following ratios: \( r_1 = LA/A \) and \( r_2 = A/E \) where \( r_1 \) is the liquid assets (LA) to assets (A) ratio labelled LAR, its complement ratio is the liquid assets ratio \( IA/A = 1 - r_1 \); and \( r_2 \) is the ratio of assets to equity (E) i.e. capital of bank and total reserves. To exhibit the impact of illiquid assets as long-run investments on bank stability, it would be more accurate to consider the following formulation (4) with stock-based g-score definition:

\[
g = \frac{E(\theta)}{\sigma(A)} + \left( \frac{1A}{A} \right) = \frac{r_2^{-1} + (1 - r_1)}{\sigma(1 - r_1)} \quad (4)
\]

and its operational definition (5)\(^5\)

\[
g_{t,q} = \frac{k_{t,q} + \mu_t}{\sigma}, \quad k_{t,q} = \frac{E(\theta)_{t,q}}{A_{t,q}}, \quad \mu_t = \frac{\overline{T_A}}{A_{t,q}}, \quad \sigma^2 = V \left( \frac{1A_{t,q}}{A_{t,q}} \right) \quad (5)
\]

In the line of z-score measure and as an alternative, we can consider a flow-based z-score, defined in (6) as follows:

\[
z_{t,q} = \frac{k_{t,q} + \mu_t}{\sigma}, \quad k_{t,q} = \frac{E(\theta)_{t,q}}{A_{t,q}}, \quad \mu_t = \frac{\overline{R_A}}{A_{t,q}}, \quad \sigma^2 = V \left( \frac{1A_{t,q}}{A_{t,q}} \right) \quad (6)
\]

where \( \mu \) and \( \sigma \) correspond to the average and standard deviation, respectively. The advantage of this measure is that the liquidity source could be detected in both side of the balance sheet i.e. from assets and liabilities. The measure g-score is based on an indicator of bank insolvency focusing on bank’s risk of illiquidity. It would be more reliable to cooperative or Islamic banks which work in principles with the PLS system, requiring direct participation of banks to the investment processes. The leverage variable \( r_2 \) is related mostly to debt financing, since it is influenced by the share of

\[\text{By applying the skew normal distribution to obtain the corrected g-score “sg-score” by removing the skewness, which will be determined from } -(S\Phi)^{-1}[S\Phi(-g, \lambda)] \text{ where } S\Phi(-g, \lambda) = \Phi(g) - 2T(-g, \lambda).\]
illiquid assets to total assets. In conventional banks, the increasing leverage through financing a part of their assets by debts, may be a sign of apprising problems such the probability of insolvency. The debt finance does not apply to Islamic banks, they use their own sources as equity, retained profits, and provisions, in addition to outside sources as the Islamic Sukuk. It is important to show how the leverage is affected by the illiquid assets of investments. These latter are generally financed by long-term financing Shariah-compliant contracts and equity, but the liquid assets are financed by short-term liabilities. Each financing process could be related to a financial instability issue of banks.

In contrast to short-term liabilities financing liquid assets, the failure in illiquid assets financing long-term debts and equity would be more related to the financial bank instability issue. Dissimilarly to short-term highly liquid assets that can be liquidated easily by sales, the long-term highly illiquid assets can be liquidated only at a high cost i.e. with loss. In consequence, the share of bank illiquid assets in total assets is directly related to bank financial instability, and should be characterized explicitly in indices measuring bank stability. The g-score seems to be more appropriate to describe this important component of bank capitalization.

As in the z-score, the decrease of \( \sigma(I/A/A) \) increases the g-score indicating less risk of illiquidity, whereas the increase of \( \sigma(I/A/A) \) decreases the g-score leading to more illiquidity risk. This latter can generate insolvency, and lead to financial failure of bank when liabilities exceed assets. The greater g-score indicates lower probability of bank insolvency risk; this index could be relatively stable when illiquid capitalization is large. Furthermore, the positive economic growth or at least the stability of the bank returns from illiquid assets can attract more deposits, hence increasing the current liquidity and working capital, and would prepare for more investment in illiquid assets.

The ratio \( I/A/A \) would reflect a bank’s financing risk through long-term investments. If this ratio has a great value since \( 1/2 \leq 1 - r_1 < 1 \), the greater this ratio is, the more likely withdrawals by depositors will expose bank to insolvency risk. In the double inequality, the lower bound is obtained by assuming that at least \( LA = IA \), or in fact the bank tends to keep a small or reasonable fraction as liquid assets, in comparison to illiquid assets, to face any shocks of depositor withdraws through demand deposits. The upper bound is determined by supposing that the illiquid assets are predominant in the total assets. This hypothesis is more realistic since in the banking system there is a regulatory reserve defined by the monetary authority. Also, Wagner (2007) considers that if liquid asset increases, the capital requirement as stability instrument becomes a less effective. Lucas and McDonald (1992) model the optimal level of liquid assets and state that small banks hold more fraction of liquid assets than large and medium banks. Alger and Alger (1999) find that banks relying heavily on the deposits would face larger liquidity shocks. The insolvency risk could be managed by increasing the leverage of bank, which increase the credit risk of conventional bank or Sukuk risk of Islamic bank, and may reduce the ratio \( I/A/A \). Also, a low ratio \( E/A \) may indicate that bank might encounter difficulties in obtaining financing through Sukuk selling, except if the bank expects to earn more from their illiquid assets.
4. Conclusion

The illiquidity could be a simple symptom of bank insolvency, it is important to consider a measure of stability using illiquidity assets to take care about the optimal allocation between liquid and illiquid assets of financial firms involved directly in economic growth. Instead of measuring the returns on all assets, we restrict to returns on illiquid i.e. long-term assets which depend on the real economic growth. The suggested stock-based g-score index can reflect the long-run dynamic of real investments of the banks. This index could be associated to the habitual flow-based z-score by focusing on the returns on illiquid assets. The complementarity of the two indices could be a road map of the banking stability.

References


Lapteacru I., (2016). On the Consistency of the Z-score to Measure the Bank Risk. CR16-EFI05. Hal Id: hal-01301846. https://hal.archives-ouvertes.fr/hal-01301846


