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Endogenous Mergers in Markets with Vertically Differentiated Products

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Abstract

This paper studies the incentives of firms selling vertically differentiated products to merge. To this aim, we introduce a three-stage game in which, at the first stage, three independent firms can decide to merge with their competitors via a sequential game of coalition formation and, at the second and third stage, they can optimally revise their qualities and prices, respectively. We study whether such binding agreements (i.e. full or partial mergers) can be sustained as subgame perfect equilibria of the coalition formation game, and analyze their effects on equilibrium qualities, prices and profits. We find that, although profitable, the merger-to-monopoly of all firms is not an outcome of the finite-horizon negotiation, where only partial mergers arise. Moreover, we show that all stable mergers always include the firm initially producing the bottom quality good and reduce the number of variants on sale.

Keywords: *Mergers, Price Collusion, Vertically Differentiated Products, Sequential Game of Coalition Formation.*

JEL Classification: D42, D43, L1, L12, L13, L41.

1 Introduction

This paper studies the incentives of firms selling vertically differentiated products to merge. Empirical investigations show that the firms involved in mergers and acquisitions usually revise their prices and qualities.¹ In some cases, they may also decide to shut down part of their product lines, thereby putting in place a *market pruning* of the variants on sale.² For instance, Steven and Waldfogel (2001) found that the series of mergers following the 1996 Telecommunications Act drastically reduced the number of stations but increased the relative number of varieties of formats available. Sweeting (2010) and George (2007) reported similar evidence for the U.S. radio music industry and Fan (2013) for the U.S. newspaper market.³ For airline industry, Peters (2006) observed a reduction of flight frequency on segments where merging carriers are competing against each other, whereas Mazzeo (2003) showed that carriers deteriorate their on-time performance as result of a less competitive after-merger market structure.

These possibilities are explicitly considered by the U.S. Horizontal Merger Guidelines (2010) when stating that:

“Enhanced market power can also be manifested in non-price terms and conditions that adversely affect customers, including reduced product quality, reduced product variety, reduced service, or diminished innovation.” (U.S. Horizontal Merger Guidelines, 2010).⁴

Anecdotal evidence shows that frequently mergers and acquisitions occur among firms selling products which are fairly differentiated along the quality spectrum. For example, most of mergers that took place after the deregulation of U.S. airline market in 1979 occurred between one big national/international carrier and one low fare local carrier (e.g. the merger between American Airlines and AirCal in 1986 or between Delta and Atlantic Southeast Airlines in 1999)⁵ or, alternatively, among intermediate-quality carriers (as for Southwest Airlines and

¹See, for instance, Steven and Waldfogel (2001) and Sweeting (2010) for an analysis of mergers in the radio music industry, George (2007) and Fan (2013) for the newspapers market, Giraud-Heraud *et al.* (2003) for mergers occurred in the mineral water industry, Peters (2006), Lee (2013) and Mazzeo (2003) for those in the airline industry and Draganska *et al.* (2009) for those in the ice-cream market.

²A top cited case is Apple withdrawing from the market its i-phone 5 when marketing its enhanced smart-phone i-phone 6. See also Johnson and Myatt (2003) for a detailed description of pruning by an incumbent as effect of an entrant in the market.

³In particular, using data on the assignment of reporters to topical areas at 706 newspapers in the US, George (2007) observes that differentiation increases with ownership concentration. Sweeting (2010) finds instead that those firms that buy competing stations tend to emphasize "service differentiation" among themselves.

⁴For an analysis of recent US antitrust trials in which quality issues arise see, for instance, McMillan (2015).

⁵A complete list of U.S. airlines industry mergers is available at: <http://www.airlines.org>.

AirTran Airways in 2010).⁶ The European Airlines industry similarly experienced a high number of mergers among highly differentiated airlines as, for instance, those between Air France and Air-Inter in 1999 or between Lufthansa and Air Dolomiti, started in 1993 and concluded in 2003.⁷ In a similar way, the automotive industry is plenty of examples of premium segment car producers absorbing economy automobile manufacturers, as in the merger between Volkswagen Group and Skoda in 1991 or between Nissan and Renault in 1999.⁸ As a consequence of these consolidation processes, often firms re-position their lower quality brand towards a higher segment of the market as well as, in some other cases, to *un-brand* intermediate quality products to create a *fighting brand* thereby competing more aggressively with the firms positioned at the bottom of the quality spectrum. Under this perspective, the boom of mergers recently observed in pharmaceutical industries, involving top pharmaceutical companies acquiring generics drugs manufacturers (as in the recent case of Teva absorbing Allergan Generics), might be seen as an attempt of branded firms to sell generic versions of their branded products without being objected by antitrust authorities.⁹ However, the latter strategy appears more as a temporary strategy, since a fighting brand presents the high risk to cannibalize the market of the merging firms. This could have been one of the reasons why Lufhtansa decided to sell its share of the low-cost airlines Condor in 2006. Ultimately, a consolidated group can find more advantageous to *re-brand* its economy products rather than *un-brand* some of its intermediate quality outlets. Instead of letting Smart for Two competing in the low segment of the market, Daimler-Benz preferred to transform this city car into a premium car.

To study these issues, in this paper we introduce a simple framework in which three firms selling three vertically differentiated products contemplate merging with all or some of their rival firms. Once merged, firms can optimally reshape their qualities and prices according to the new market structure. Thus, taking into account all alternative price-quality equilibrium configurations, we study whether full or partial mergers can be sustained as subgame perfect equilibria of a coalition formation game played at the first stage. Moreover, we analyze the effects of different coalition structures on equilibrium qualities, prices and profits accruing to

⁶Other mergers between medium and small airlines are also those between Republic Airways and Midwest Airlines in 2009, Republic Airways and Frontier Airlines in 2010 and many others. Such a long series of mergers finally turned the U.S. Airlines industry into a quadriopoly between Delta, United Airlines, Southwest and American Airways which, together, control more than 80% of the passenger capacity.

⁷In some other cases the low-cost carriers have attempted to take over small-medium companies, as in the recent hostile takeover launched by Ryanair to Air Lingus.

⁸Also the purchase in 1964 by Volkswagen of Auto Union (later known as Audi) from Daimler-Benz was mainly due, at that time, to the production of economy cars by Auto Union.

⁹See, for instance, Wieczner (2015), “The real reasons for the pharma merger boom”, *Fortune*, July 28, 2015.

firms.

In the remaining of this section we briefly review some of the existing literature on collusive agreements and mergers under product differentiation and detail the main content of our paper.

1.1 Related Literature

The relationship between collusive agreements and vertical product differentiation was formerly analyzed by Hackner (1994). In his work, the key question is whether price collusion is more likely to arise when products are close substitutes or, rather, highly differentiated. In a duopoly setting, he finds that monopoly pricing is easier to sustain in markets in which products are similar. Further, he proves that the incentive to deviate from a collusive agreement is always stronger for the high-quality firm. The main reason is that when the quality gap between products is significant, the profit of the top-quality firm is already high under no collusion, so that its incentive to collude is weak. As the quality gap decreases and the noncooperative payoff become smaller for the high-quality firm, reaching a collusive agreement becomes more and more attractive. Along the same research line, Ecchia and Lambertini (1997) study how the stability of price collusion in a duopoly setting is affected by the introduction of a minimum quality standard. They observe how the introduction of a welfare-maximizing minimum quality standard makes collusive agreements more difficult to sustain. This is because the existence of a standard decreases product differentiation by providing the bottom quality firm with a stronger incentive to break the agreement.¹⁰

There are two common traits in these works. First, (i) the degree of product differentiation does not change after a coalition has formed, since the collusive behavior is restricted to pricing. The former assumption is a natural entry point in the literature on cartel stability under product differentiation, as it enables to disentangle the effect of quality gap on the stability of a cartel. Further, conceiving collusion in terms of pricing is particularly reasonable in a short-run perspective. Still, it leaves unexplored a companion question, namely the effect of the cartel on product differentiation. This analysis is particularly pregnant in a long-run perspective since one cannot exclude that in a more extended time span a coalition (typically a cartel or a merger) entails structural changes, such as relocations of production facilities, or adjustment in the product range and quality.

Secondly, (ii) the market is populated by two firms so that it turns out to be fully mo-

¹⁰In Hackner, the opposite holds since, due to the cost structure, in his model the asymmetry in profits gives an advantage to the high quality firm.

nopolized by a *grand coalition* in the case of cooperation between firms.¹¹ While considering at the start a duopoly enables to detail the effects of a full cooperation, casual observations show that, there exist circumstances under which firms choose to form a partial merger (i.e. one including a subset of firms in the market) rather than the *grand coalition*. In any partial merger, colluding firms compete against some rivals outside the coalition so that a noncooperative behavior is still preserved. Thus, the effects of a partial merger are not equivalent to those observed when all agents merge and mimic a monopolist.

The aim of our analysis is to consider the incentive of firms to merge when the degree of differentiation may change once a coalition has formed and the market is populated by more than two firms. Accordingly, we assume that the market is initially populated by three firms, each of them producing a variant which is differentiated along a quality ladder.

As far as we know, the only model of vertical differentiation with three independent firms competing in quality and price is provided by Scarpa (1998).¹² Considering the role of a minimum quality standard, Scarpa (1998) highlights how the demand level of a firm in a vertically differentiated market depends on quality and price of adjacent firms in the product space. This property, reminiscent of a spatial competition approach, is rather interesting when considering the rationale adopted by the colluding firms to define the optimal range of variants. Indeed, since only adjacent variants compete against each other, under partial collusion defining the optimal set of products to market requires to put in balance *the cannibalization effect* that a variant produced by the coalition may exert *within* the coalition with the possibility that this variant *steals* consumers from the rival firm (henceforth *stealing effect*).¹³ We nest in this formal setting the possibility that firms merge thereby generating either a grand coalition or a partial merger. Then, the merged entity optimally chooses price and quality against the competitor, if any. Considering that firms in the post-merger scenario can choose price and quality relates our paper to Lambertini (2000). He studies how cartel stability is related to R&D activity in a duopoly with convex costs, and assumes that a collusive quality choice can either occur under price or quantity-setting behaviour.¹⁴ The issue concerning merger

¹¹The grand coalition is the one formed by *all* firms in the market.

¹²Pezzino (2010) analyses quantity competition among three firms in a vertically differentiated market.

¹³These effects resembles the so called *peer effect* and *pecking order effect*. The peer effect takes place when joining organization with high-quality agents increases the payoff of its members. This effects explains why outstanding researchers tend to join top research department. On the other hand, the pecking order effect takes place when the payoff an individual gets depends on his/her relative position in a ranking. Typically, people at the top in the pecking order have a greater chance to obtain further internal promotions.

¹⁴A different strand of literature considers the possible impacts of R&D joint ventures on product market collusion. See on this, Martin (1995) and Lambertini *et al.* (2002).

formation with more than two firms in a vertically differentiated market is, however, unexplored in Lambertini (2000), like so the impact of partial collusion on market equilibrium. The introduction of an intermediate quality firm sheds light on some interesting features of the coalition formation process.

Finally, it can be useful to briefly mention here other related strands of literature dealing with mergers in markets in which products are not vertically differentiated. In particular, our paper is close in the spirit to Lommerud and Sorgard (1997) and, within the literature on horizontally differentiated products, to Gandhi *et al.* (2008) and Brekke *et al.* (2014). Indeed, although all these authors do not consider quality as the main source of product differentiation, their analysis centers around the price-quality post-merger re-positioning which is our main aim.¹⁵ The first paper is inspired by Salant *et al.* (1983) and Deneckere and Davidson (1985) and it is devoted to evaluate the profitability of a merger under both Cournot and Bertrand competition in a symmetric model of product differentiation. The authors assume that the market is initially populated by three firms and, therefore, two firms can merge and decide on the number of brands to market. When the fixed cost of marketing a brand is ‘high’, the merged entity reduces its product range. This increases the profitability of mergers both under Bertrand and Cournot competition due to reduced marketing costs. With a ‘low’ cost of marketing, the effect on the product range depends both on the nature of competition and on the degree of product differentiation. For example, under Cournot or Bertrand competition and sufficiently differentiated products, the non-merging firm finds profitable to introduce a new brand, thereby damaging the merged entity. In Gandhi *et al.* (2008), firms are assumed able to instantaneously and costlessly reposition their products after a merger, thereby choosing both price and location in a Hotelling market. They show that after a merger the products are repositioned away from each other to reduce the resulting cannibalization effect. Consequently, non-merging substitutes are repositioned between the merged products and, after all these location strategies, the merged firm’s incentive to raise prices decreases. In a spatial competition model *à la* Salop with three *ex ante* identical firms, Brekke *et al.* (2014) show that any two-firm merger reduces its product quality whereas the non-merging firm responds increasing its quality. Final prices can either increase or decrease according to the responsiveness of demand functions. Moreover, it is shown that if a merger entails the closure of one of the two merged firms, it always leads to higher qualities and prices for all firms in the market.¹⁶

¹⁵Other recent papers by Mazzeo (2002), Einav (2003) and Seim (2006) focus on the price-quality strategies decided by industry entrants. These models are particularly relevant for the analysis of the strategic behaviour of *ex ante* symmetric firms deciding their price-quality positioning (see on this, the discussion of Section 3).

¹⁶While these works represent important advancements in considering the relationship between merger and

However, as also the literature above highlights, the mechanics of mergers in markets with horizontally differentiated products is quite different from that in markets with vertically differentiated products. As observed in Norman *et al.* (2005), "eliminating product lines is unlikely to be an issue in markets with undifferentiated products or in markets in which the products are best thought as horizontally differentiated. [...] (p.1205). In these markets, "firms have an incentive to maintain their pre-merger product lines as a means to reach as many consumers as possible" (*ibidem*, p.1205). Of course, the cannibalization effect still plays a role in an Hotelling setting and, thus, can weaken the incentive to keep on sale differentiated varieties. Nonetheless, whenever firms produce horizontally differentiated goods, the equilibrium configuration is also (and strongly) determined by transport costs. When transport costs are extremely high, firms behave like local monopolists and the cannibalization is weak. In contrast, as transport costs tends to zero, the market with horizontally differentiated goods tends to mimic market with homogeneous products where firms do not have market power. In this alternative case, the cannibalization effect is likely to be more significant than the incentive to satisfy heterogeneous preferences so that firms can decide to remove some goods from the market in the post-merger scenario.¹⁷

1.2 Our Paper

In the present paper we consider a setting in which three firms initially produce three vertically differentiated products as independent firms. In this environment, we study their incentives to sign full or partial binding agreement among firms, knowing in advance that the resulting mergers will be able to manipulate collusively their quality-price combinations.

More specifically, we introduce a three-stage game where, at the first stage, every firm expresses its willingness to merge with its competitors or, alternatively, to remain independent. A merger can either contains all firms in the market (*grand coalition*) or a subset of them (two firms colluding against the third one playing alone). As in Bloch (1995, 1996) and Ray and Vohra (1999) we assume that the coalition formation game is *sequential*, with an exogenous order of play. Differently from them, we assume that every firm proposes not only an alliance, but also a division of the coalition joint payoff.¹⁸ Each recipient of the proposal can either

competition, they are concerned with ingredients, such as transport costs, which do not matter when goods are differentiated in quality. Still, they identify strategic motives for withdrawing products which shall be relevant in our analysis. We discuss this point in the next section.

¹⁷See also Gandhi *et al.* (2008) for an analysis of the effect of transport cost on the firm's choice to prune goods.

¹⁸Bloch (1995, 1996) and Ray and Vohra (1999) assume identical players and a fixed allocation rule within

accept or reject the offer and, in case of rejection, it becomes its turn to make a proposal. The game is assumed finite-horizon and every firm only possesses one turn of proposal in each period.¹⁹ Once a coalition structure has formed, at the second stage firms decide simultaneously the optimal quality of their products. When considering this issue, we take into account how a full or partial merger among firms may affect their incentives to differentiate products in the market. Choosing the optimal quality after colluding affects, in turn, their incentives to merge. Finally, at the third and final stage, firms set simultaneously prices. When in a coalition, quality and price are set so as to maximize the joint profit of firms which belong to it. Notice also that, when merging, firms can choose at the second stage (resp. third stage) to produce a quality so low (resp. to quote a price so high) that no consumer is willing to buy it. This is equivalent to stop producing the variant, thereby reducing the range of products sold at equilibrium.

We find that, although the full monopoly merger would arise in an infinite-horizon sequential game of coalition formation, under a finite horizon the incentive for firms to enter the full market merger is always dominated by that to form partial coalition structures (partial mergers). Furthermore, we prove that all equilibrium mergers always contain the bottom quality firm which, in all cases, drops its low-quality variant from the market. In particular, whoever is the additional player included in coalition (either the intermediate or the top quality firm), equilibrium prices and qualities always coincide with that observed in the case of a duopoly, with a high-quality firm competing against a low-quality rival, as in Motta (1992). At first sight, this result seems to be counterintuitive. A natural conjecture when considering that players producing different variants merge is that either the range of variants or the quality gap between variants in the market changes with the players involved in the merger. We find on the contrary that only profits accruing to the single players change with the type of partial merger, range of products, quality gap and price being unchanged. Indeed, the cannibalization effect and the stealing effect induce the merger, whatever its members, to withdraw from the market the lowest quality variant between the set which can be produced *a priori*. Interestingly, depending on the intensity of these effects, in some circumstances this variant is withdrawn from the market at the price stage, in some other circumstances at the quality stage. In particular, the merger formed by the intermediate quality and by the low-quality

each coalition. See also Belleflamme (2000) for an extension of the model to the formation of asymmetric alliances, and also Bloch (2002, 2003), Marini (2009) and Currarini and Marini (2015) for extensive surveys on alliance formation models applied to oligopoly.

¹⁹Both Bloch's (1995, 1996) and Ray and Vohra's (1999) models are, instead, infinite-horizon. Our assumptions are meant to describe an environment in which the time of negotiation is quite limited in each period.

firm stops immediately to market the bottom-quality product at the price stage. In contrast, the merger formed by the top and the bottom-quality firm keeps the bottom product (as a fighting brand) at the price stage whereas ultimately drops it at the quality stage. As argued above, keeping a fighting brand in an alliance is mostly a short-run (price) than a medium/long run strategy (quality) and it is, therefore, dropped when the merging group can re-position its product lines.

Finally, we find that, in all equilibrium *partial* mergers, the bottom-quality firm is always present. This appears in line with numerous theoretical and experimental studies on coalition formation in triads of heterogeneous individuals possessing different skills or fighting ability (e.g. Caplow 1956, 1959, 1968, Vinacke and Arkoff 1957, Gamson 1961). A central conclusion of these studies is that “weakness is strength” (see, for instance, Mesterton-Gibbons *et al.* 2011, p.189), with this meaning that less-powered individuals have usually more chances to be part of a coalition. We obtain the same result with the firms competing in a vertically differentiated market. Here the main reason to merge with a lower quality firm is to soften competition.

The structure of the paper is as follows. Section 2 describes in detail the market and the various equilibrium market configurations, the *noncooperative case*,²⁰ the *full merger* case and all different cases of *partial mergers*. Section 3 introduces the coalition formation game and characterizes the mergers which are stable. Section 5 briefly concludes. Most of the proofs are gathered in the Appendix.

2 The Market

As mentioned in the introduction, firms are assumed to play a three-stage game: (i) a *coalition formation game* (stage 1) assumed sequential; (ii) a *market game* including a *quality stage* (stage 2) and a *price stage* (stage 3) both played simultaneously by the firms. Since the game is solved backward, we start below with the description of the market game and, after solving for the equilibria of the second and third stage of the game, we introduce the coalition formation game and all results obtained therein.

For the market, to keep things simple we adopt the well known specification of Mussa and Rosen’s (1978) and Gabszewicz and Thisse’s (1979) models of a vertically differentiated market. In particular, we assume an uncovered market initially populated by three *firms*, $i = H, M, L$

²⁰Part of this analysis is also contained in Scarpa (1998).

selling three vertically differentiated goods, denoted v_H, v_M, v_L with $v_H > v_M > v_L$.²¹ Also, for every i , $v_i \in [0, \bar{v}]$, where $\bar{v} \in \mathcal{R}_+$ is the highest quality level which is technologically feasible.²² There exists a quality specific fixed cost, say $c_i = \frac{1}{2}v_i^2$. Notice that this cost does not depend on quantity, while being strictly increasing and convex in quality. On the one hand, the most appropriate way to model the cost function may depend on the features of the industry in which firms compete. Whenever a firm mainly faces a quality development cost (namely when the cost depends on a technology enabling it to produce a specific quality), then assuming that costs do not depend on quantity could be a reasonable assumption. Most importantly though, it can be shown that the inclusion of quality dependent production costs would not alter the qualitative results of the model as long as firms' marginal and unitary production cost are either constant or increase less than proportionally with quality. For instance, it is easy to prove that, for this range of cost functions and for any number of firms, the merger involving all firms in the market would put on sale only the top quality product among those previously put on sale by the competing firms.²³ Therefore, to simplify calculations, in what follows we will assume zero variable production costs.

Let consumers be indexed by θ , which is uniformly distributed in the interval $[0, 1]$, with density function denoted $f(\theta)$.²⁴ The parameter θ captures consumers' willingness to pay (henceforth WTP) for quality: the higher θ , the higher the corresponding WTP. Each consumer can either buy one variant or not buying at all. Formally, consumers' utility can be written as

$$U(\theta) = \begin{cases} \theta v_i - p_i & \text{if she/he buys variant } i \\ 0 & \text{if she/he refrains from buying.} \end{cases}$$

where p_i is the price charged by a firm for variant i . From the above formulation, the consumer indifferent between buying variant i and not buying is:

$$\theta_L = \frac{p_L}{v_L},$$

while the consumer θ_M indifferent between buying variant L and M (resp. M and H) is:²⁵

²¹Since for our model specification the market is always *endogenously* uncovered in the case of a monopolist, the assumption of *uncovered market*, namely that some of the consumers refrain from buying goods, appears in our model as the most natural one.

²²We share this assumption on the quality interval with Wauthy (1996).

²³See on this, also Gabszewicz *et al.* (1986).

²⁴Considering an interval $[0, m]$ would simply lead to the addition of a parameter on which prices, quantities and quality levels would depend linearly, with no substantial changes in the payoff structure (see, for instance, Scarpa 1998).

²⁵We easily deduce the expression of the indifferent consumer from: $U_L(\theta) = U_M(\theta)$ and $U_M(\theta) = U_L(\theta)$

$$\theta_M = \frac{p_M - p_L}{v_M - v_L} \text{ (resp. } \theta_H = \frac{p_H - p_M}{v_H - v_M}\text{)}.$$

Of course, since qualities are endogenously defined at stage 1, the demand function for firms when producing v_H , v_M , and v_L can be written, respectively, as:

$$\begin{aligned} D_H &= \int_{\theta_H}^1 f(\theta) d\theta = (1 - \theta_H), \\ D_M &= \int_{\theta_M}^{\theta_H} f(\theta) d\theta = (\theta_H - \theta_M), \\ D_L &= \int_{\theta_L}^{\theta_M} f(\theta) d\theta = (\theta_M - \theta_L), \end{aligned}$$

and, the corresponding profit functions are:

$$\Pi_H = \left(1 - \frac{p_H - p_M}{v_H - v_M}\right) p_H - \frac{1}{2} v_H^2 \quad (1)$$

$$\Pi_M = \left(\frac{p_H - p_M}{v_H - v_M} - \frac{p_M - p_L}{v_M - v_L}\right) p_M - \frac{1}{2} v_M^2 \quad (2)$$

$$\Pi_L = \left(\frac{p_M - p_L}{v_M - v_L} - \frac{p_L}{v_L}\right) p_L - \frac{1}{2} v_L^2. \quad (3)$$

2.1 Equilibrium Analysis: Prices and Qualities

Since the whole game is solved backward, we can start characterizing the two final stages of the game. In particular, we firstly present the benchmark case of the analysis, that is the case in which all firms decide *noncooperatively* prices and qualities (*noncooperative equilibrium*); secondly, we turn to the case in which the grand coalition of firms has formed and they can jointly decide prices and qualities (*full collusion*); finally, we look at what happens when firms form intermediate coalitions (*partial mergers*). Since prices are usually more easily adjusted than qualities, it is reasonable to assume that firms define qualities at the second stage (quality stage) and set prices at the third one (price stage).

The game is solved by backward induction. So, we consider first the price stage under the assumption that qualities have been fixed. Then, we move to the quality stage.

2.1.1 Noncooperative equilibrium

In this section, we briefly summarize price and quantity equilibrium obtained when the three firms compete in the market against each other, while referring the interesting reader to Scarpa (1998) for further details. We assume that at the first stage, no collusive agreement has been reached so that firms decide their quality and then their price in a fully noncooperative fashion.

Price stage At the price stage, given that costs are fixed, we can study the noncooperative price behaviour of the three firms by simply characterizing their revenue functions in the quality spectrum: (i) top quality H , (ii) intermediate quality M and (iii) bottom quality L .

Thus differentiating (1), (2) and (3) w.r.t p_H , p_M and p_L , respectively, we can easily derive all firms' best-replies as:²⁶

$$p_H(p_M) = \frac{1}{2}p_M + \frac{1}{2}(v_H - v_M), \quad (4)$$

$$p_M(p_H, p_L) = \frac{1}{2} \frac{p_H(v_M - v_L) + p_L(v_H - v_M)}{v_H - v_L} \quad (5)$$

and

$$p_L(p_M) = \frac{1}{2}p_M \frac{v_L}{v_M}. \quad (6)$$

As stressed by Scarpa (1998), the best-reply function of a firm depends on the quality and price of the firm itself and of its neighboring rivals, while products that are farther away in the product space do not play any role. From the above, equilibrium prices p_i at the price stage are obtained as:

$$p_H^*(v_H, v_M, v_L) = \frac{1}{2} \frac{(v_H - v_M)(4v_Mv_H - v_Lv_H - 3v_Lv_M)}{(4v_Mv_H - v_Lv_H - 2v_Lv_M - v_M^2)} \quad (7)$$

$$p_M^*(v_H, v_M, v_L) = \frac{1}{2} \frac{(v_H - v_M)(v_M - v_L)v_M}{(4v_Mv_H - v_Lv_H - 2v_Lv_M - v_M^2)} \quad (8)$$

$$p_L^*(v_H, v_M, v_L) = \frac{1}{2} \frac{(v_H - v_M)(v_M - v_L)v_L}{(4v_Hv_M - v_Hv_L - 2v_Lv_M - v_M^2)}, \quad (9)$$

with corresponding profits

$$\Pi_H(p^*(v_H, v_M, v_L)) = \frac{1}{4} \frac{(v_H - v_M)(v_Hv_L - 4v_Hv_M + 3v_Lv_M)^2}{(v_M^2 + v_Lv_H - 4v_Mv_H + 2v_Lv_M)^2} - \frac{1}{2}v_H^2 \quad (10)$$

²⁶Firms' profit functions (1)-(3) are strictly concave in their respective prices.

$$\Pi_M(p^*(v_H, v_M, v_L)) = v_M^2 \frac{(v_H - v_M)(v_M - v_L)(v_H - v_L)}{(v_M^2 + v_L v_H - 4v_M v_H + 2v_L v_M)^2} - \frac{1}{2} v_M^2 \quad (11)$$

$$\Pi_L(p^*(v_H, v_M, v_L)) = \frac{1}{4} \frac{v_L (v_H - v_M)^2 (v_M - v_L) v_M}{(v_M^2 + v_L v_H - v_M 4v_H + 2v_L v_M)^2} - \frac{1}{2} v_L^2, \quad (12)$$

where $p^* = (p_H^*, p_M^*, p_L^*)$ denote the Nash equilibrium prices of firms obtained at the price stage (stage 3). Let us now consider the choice of qualities by firms.

Quality stage In order to characterize the Nash equilibrium quality choices occurring at the second stage, it suffices to maximize payoff function (10), (11) and (12) w.r.t. quality v_H, v_M, v_L , respectively, thereby getting:²⁷

$$v_H^* = 0.2526, \quad v_M^* = 0.0497, \quad v_L^* = 0.0095. \quad (13)$$

Moreover, the corresponding subgame perfect Nash equilibrium prices $p^*(v^*)$ and profits $\Pi_i(p^*(v^*))$, for $v^* = (v_H^*, v_M^*, v_L^*)$, are immediately obtained as:

$$p_H^*(v^*) = 0.10601, \quad p_M^*(v^*) = 0.00912, \quad p_L^*(v^*) = 0.0008, \quad (14)$$

and

$$\Pi_H(p^*(v^*)) = 0.02348, \quad \Pi_M(p^*(v^*)) = 0.00124, \quad \Pi_L(p^*(v^*)) = 0.00005. \quad (15)$$

2.1.2 Mergers

By definition a merger can either involve the set of all firms, denoted $N = \{H, M, L\}$ (*grand coalition*) or, alternatively, any other nonempty subset $S \subset N$ of them, with $S \in \mathcal{N}$, where $\mathcal{N} = 2^N \setminus \emptyset$ is the set of all nonempty coalitions of the N firms, in this case simply:

$$\mathcal{N} = (\{H\}, \{M\}, \{L\}, \{H, M\}, \{H, L\}, \{M, L\}, \{H, M, L\}).$$

Thus, while if the firms form the grand coalition they commit irrevocably to jointly set qualities and prices so as to maximize the sum of all firms' profits (full merger), in the second scenario (partial merger), a smaller subset of firms jointly decide qualities and prices, again irrevocably, so as to maximize the sum of their own profits, while competing against a rival(s), if any. In

²⁷It can be easily checked that all firm second-stage profit functions are strictly concave in their own qualities. In what follows, for ease of exposition, we truncate our numerical results to five decimals.

general, we can describe any type of (full or partial) collusive or noncooperative behaviour by simply indicating the *coalition structure* $C = (S_1, S_2, \dots, S_m)$ representing a collection of firms in coalition having null intersection and summing up to N , with $m \leq n$. The set \mathcal{C} of all coalition structures C that can be formed by the three firms is, therefore, simply given by:

$$\mathcal{C} = ((\{H\}, \{M\}, \{L\}), (\{H, M\}, \{L\}), (\{H\}, \{M, L\}), (\{H, L\}, \{M\}), (\{H, M, L\})).$$

The game is solved backward so that we first analyze the price and then the quality stage under the assumption that either the grand coalition or any other intermediate coalition structure have formed at the first stage. After the full characterization of market equilibrium in any of these cases, we wonder which type of merger (if any) can prevail in equilibrium.

2.1.3 Full Merger

Let us assume that, at the first stage, firms have formed the grand coalition. In the following, we consider the price and then the quality decision.

Price stage When the grand coalition $\{N\}$ forms, at the price stage each firm maximizes the sum of all firms' payoffs (1)-(3) for arbitrary levels of the quality chosen at the second stage. Thus, by the price maximization of the joint payoff of the grand coalition, firms' *optimal replies* p_L^c , p_M^c and p_H^c are obtained as

$$p_H^c(p_M) = p_M + \frac{1}{2}(v_H - v_M), \quad (16)$$

$$p_M^c(p_H, p_L) = \frac{p_H(v_M - v_L) + p_L(v_H - v_M)}{v_H - v_L}, \quad (17)$$

and

$$p_L^c(p_M) = p_M \frac{v_L}{v_M}. \quad (18)$$

By solving the system (16)-(18), the optimal price profile $p^{\{N\}}(v)$, for $v = (v_H, v_M, v_L)$, is:

$$p_H^{\{N\}}(v_H) = \frac{1}{2}v_H, \quad p_M^{\{N\}}(v_M) = \frac{1}{2}v_M, \quad p_L^{\{N\}}(v_L) = \frac{1}{2}v_L. \quad (19)$$

Hence, in this case the firms' market shares at the price stage are:

$$D_H(p^{\{N\}}(v)) = \frac{1}{2}, \quad D_M(p^{\{N\}}(v)) = 0, \quad D_L(p^{\{N\}}(v)) = 0. \quad (20)$$

It is immediate to see that, for the prices selected by the grand coalition, consumers are willing to buy only the top quality variant v_H , the demand for the intermediate and bottom variants being nil. Accordingly, the profit accruing to the grand coalition at the price stage is

$$\Pi^{\{N\}}(p^{\{N\}}(v)) = \frac{1}{4}v_H - \frac{1}{2}v_H^2.$$

Quality stage In order to fully characterize the behaviour of the grand coalition, we can easily find its optimal quality, given by $v_H^{\{N\}} = 0.25$ and its final profit is simply given by:

$$\Pi^{\{N\}}\left(p^{\{N\}}(v^{\{N\}})\right) = 0.03125. \quad (21)$$

The logic underlying this finding has been well described by Mussa and Rosen (1978): “Serving customers who place smaller valuations on quality creates negative externalities for the monopolist that limit possibilities for capturing consumer surplus from those who do value quality highly” (p.306).²⁸ Rather interestingly, this finding does not depend on the initial assumption on market coverage. Indeed, even if one would develop the above analysis under the alternative assumption that the market is covered, still at the price-quality equilibrium the grand coalition would offer only the top-quality, while serving half of the market.

Finally, it is worth remarking that, under a full collusive behaviour, the level of prices is, for all firms, always higher than under Nash equilibrium. This can be easily checked by the following simple reasoning: (i) Start with the Nash equilibrium price of firm H and let the remaining firms respond using their optimal collusive replies (16)-(17). (ii) Since comparing (4)-(5) with (16)-(17) it turns out that optimal collusive replies are twice as sloped as the noncooperative best-replies and both upward sloping, as effect of (i) all $N \setminus \{H\}$ firms will increase their prices. (iii) Let now also firm H respond collusively using its optimal reply (18) and, as a result, it will increase its price. (iv) By continuing the adjustment process of all firms along their collusive optimal replies, since these are all contraction mappings (due to the inequality $v_H > v_M > v_L$), a new price profile $p^{\{N\}}$ will be reached with the property that

²⁸Further discussion on this result are provided by Gabszewicz *et al.* (1982) and by Gabszewicz and Wauthy (2002) under the assumption of zero quality costs. Along the same research line, Accharya (1998) shows that when the cost for quality improvement is not too convex, a multiproduct monopolist offers only the top variant among the ones which *a priori* can be sold in the market. Indeed, if the costs are not so significant, offering the top variant only allows firms to escape from the cannibalization effect which would take place if the more than one variant would be saled at equilibrium. Finally, Lambertini (1997) analyses the Mussa-Rosen’s model with quality specific variable costs under the alternative assumptions of both full market and partial market coverage.

$p^{\{N\}} \gg p^*$, where p^* is the corresponding profile of noncooperative Nash equilibrium prices.²⁹

2.1.4 Partial Mergers

In this section we analyze all market configurations arising when partial mergers take place among firms. We characterize three different market scenarios occurring, in turn, under the following coalition structures: (i) $C_{H,ML} = (\{H\}, \{M, L\})$, (ii) $C_{HL,M} = (\{H, L\}, \{M\})$ and (iii) $C_{HM,L} = (\{H, M\}, \{L\})$.

Before computing in detail prices and qualities of firms under partial mergers, note that from (1)-(3) when either the bottom quality firm or the top quality firm collude in prices with their *direct* competitor, i.e. the intermediate quality firm, they just behave as in the fully collusive case, with optimal replies given by (16) and (18), respectively. On the other hand, when bottom and top quality firms form a coalition, due to the structure of the vertical differentiation model, they set prices exactly as in the noncooperative case, with optimal replies given by (4) and (6). Thus, under a partial merger only the price behaviour of the firm producing the intermediate quality variant v_M (henceforth denoted *intermediate firm*) varies according on whether it is allied either with its left (lower quality) or with its right (higher quality) competitor. In particular, when the intermediate firm coordinates its price with its left competitor, its first-order condition implies

$$\frac{\partial \Pi_M}{\partial p_M} + \frac{\partial \Pi_L}{\partial p_M} = \frac{2p_L - 2p_M}{v_M - v_L} + \frac{p_H - 2p_M}{v_H - v_M} = 0,$$

whereas, when it coordinate its price with its right-competitor, it sets p_M such that

$$\frac{\partial \Pi_M}{\partial p_M} + \frac{\partial \Pi_H}{\partial p_M} = \frac{p_L - 2p_M}{v_M - v_L} + \frac{2p_H - 2p_M}{v_H - v_M} = 0.$$

As a result, the optimal reply of the *intermediate firm*, $p_M^{lc}(p_L, p_H)$ in the left-partial (resp. $p_M^{rc}(p_L, p_H)$ in the right-partial) merger writes as

$$p_M^{lc}(p_L, p_H) = \frac{p_L(v_H - v_M) + \frac{1}{2}p_H(v_M - v_L)}{(v_H - v_L)} \quad (22)$$

$$\text{(resp. } p_M^{rc}(p_L, p_H) = \frac{\frac{1}{2}p_L(v_H - v_M) + p_H(v_M - v_L)}{(v_H - v_L)}). \quad (23)$$

²⁹For an extension of this result with any number of firms, see Gabszewicz *et al.* (2017).

2.1.5 Partial merger between the intermediate and the bottom quality firm

We consider initially the scenario where at the first stage a merger has occurred between firm M and firm L , with firm H playing as singleton against them. We assume, as a start, that variants v_M and v_L are produced by the colluding firms M and L , respectively. Firm H , outside the merger, produces the high quality variant v_H . We need to check whether this quality assignment remains optimal at the equilibrium.

Price stage As coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$ forms, prices p_H , p_M and p_L set by firms 1, 2 and 3 at the price stage can be obtained through the maximization of the following objective functions

$$\begin{aligned}\Pi_H &= \left(1 - \frac{p_H - p_M}{v_H - v_M}\right) p_H \\ \Pi_M + \Pi_L &= \left(\frac{p_H - p_M}{v_H - v_M} - \frac{p_M - p_L}{v_M - v_L}\right) p_M + \left(\frac{p_M - p_L}{v_M - v_L} - \frac{p_L}{v_L}\right) p_L.\end{aligned}$$

Using (4), (18), and (22), the following optimal replies are obtained, respectively as,

$$\begin{aligned}p_H^{pc}(p_M) &= \frac{1}{2}(p_M + (v_H - v_M)) \\ p_M^{pc}(p_H, p_H) &= \frac{p_L(v_H - v_M) + \frac{1}{2}p_H(v_M - v_L)}{(v_H - v_L)} \\ p_L^{pc}(p_M) &= \frac{v_L}{v_M}p_M.\end{aligned}$$

Therefore, the following equilibrium prices are set by firms:

$$\begin{aligned}p_H^{(\{H\}, \{M, L\})}(v) &= \frac{2v_H(v_H - v_M)}{4v_H - v_M}, \\ p_M^{(\{H\}, \{M, L\})}(v) &= \frac{v_M(v_H - v_M)}{4v_H - v_M}, \\ p_L^{(\{H\}, \{M, L\})}(v) &= \frac{v_L(v_H - v_M)}{4v_H - v_M},\end{aligned}$$

with corresponding profits:

$$\begin{aligned}
\Pi_H(p^{\{\{H\},\{M,L\}\}}(v)) &= 4 \frac{v_H^2 (v_H - v_M)}{(4v_H - v_M)^2} - \frac{1}{2}v_H^2, \\
\Pi_M(p^{\{\{H\},\{M,L\}\}}(v)) &= \frac{v_H (v_H - v_M) v_M}{(4v_H - v_M)^2} - \frac{1}{2}v_M^2, \\
\Pi_L(p^{\{\{H\},\{M,L\}\}}(v)) &= 0.
\end{aligned}$$

Note that in this case the price of the low quality variant is set so high that no consumer is willing to buy this variant and, therefore, $D_L^{\{M,L\}} = 0$. Thus, firm L ceases to be active in the market: selling the bottom-quality variant would determine a cannibalization effect within the coalition since variant v_L would be in competition only with the adjacent product v_M . Of course, it still plays a role in the coalition as the decision to stop producing benefits the coalition as a whole.³⁰

Quality stage Then, moving to the quality stage and using the best reply functions, it is immediate to see that top variant and intermediate variant are set, respectively, equal to

$$v_H^{\{\{H\},\{M,L\}\}} = 0.25331, \quad v_M^{\{\{H\},\{M,L\}\}} = 0.04823. \quad (24)$$

Given the above, we can easily find the equilibrium prices as

$$p_H^{\{\{H\},\{M,L\}\}} = 0.10766, \quad p_M^{\{\{H\},\{M,L\}\}} = 0.01025, \quad (25)$$

and the corresponding equilibrium profits:

$$\Pi_H^{\{\{H\},\{M,L\}\}} = 0.02443, \quad \Pi_{\{M,L\}}^{\{\{H\},\{M,L\}\}} = 0.00152. \quad (26)$$

It is easy to see that, at equilibrium, firm H continues to produce the top quality while coalition $\{M, L\}$ sells the intermediate quality only. Note also that the above findings coincide with those emerging, for instance, in Motta (1992) where only two firms compete in a traditional duopoly setting. Indeed, coalition $\{M, L\}$ behaves like a multiproduct firm: since it withdraws from the market one of its variant, *it is as if only* two single-product firms would be active in the market, each of them setting noncooperatively their quality and price. We resume these results in the next proposition.

³⁰Its role will be clarified at the coalition formation stage.

Proposition 1 *When the intermediate and bottom quality firm merge against the top quality firm (playing as singleton), namely under coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$, at the price stage colluding firms set a price so high for the low quality variant that no consumer is willing to buy it. Thus, at the price-stage only two variants are marketed and the equilibrium configuration in terms of quality and price coincides with that occurring in a traditional duopoly setting.*

Proof. It directly follows by expressions (24), (25) and by their comparison with the results obtained, for instance, in Motta (1992). ■

Finally, it is worth remarking that this merger benefits both the merging firms and the rival H which plays as a singleton. Indeed, not only the lowest quality variant is dropped out from the market, but also the gap between variants in the market is now larger than the one emerging in the noncooperative setting with three independent firms: under partial collusion, the optimal quality of the intermediate variant is lower (and the top quality higher) than the corresponding levels set noncooperatively. This relaxes price competition between firms, thereby increasing the resulting profits.

2.1.6 Partial merger between the top and the bottom quality firm

Let us move now to the case in which at the first stage firms H and L have merged, whereas firm M plays as singleton. As usual, we have to verify whether this quality assignment holds at the equilibrium.

Price stage To obtain the optimal prices decided by the merging firms H and L and by firm M alone, we need to take into account the fact that colluding firms H and L maximize the sum of their profits $\Pi_H + \Pi_L$, while M is only concerned with its own profit function Π_M . However, since firm H and L are not direct price competitor and are separated by firm M , at the price stage their equilibrium prices and profits coincides with those obtained in the noncooperative case (cfr. Section 2.1.1).

Quality stage We can now move to the quality stage. In order to identify the optimal qualities, notice that the revenue of coalition $\{H, L\}$ is monotonically decreasing in v_L , as

$$\frac{\partial \left(\Pi_H^{\{\{H,L\},\{M\}\}} + \Pi_L^{\{\{H,L\},\{M\}\}} \right)}{\partial v_L} = \frac{1}{4} v_M^2 \frac{(v_H - v_M)^2 (v_M^2 + v_H v_L + 20 v_M v_H - 22 v_M v_L)}{(v_M^2 + v_H v_L - 4 v_H v_M + 2 v_M v_L)^3} < 0.$$

Accordingly, at the quality stage for colluding firms H and L it is profitable to set $v_L = 0$, whatever the quality chosen by the intermediate rival M . The economic intuition underlying this finding is that the low quality variant and the intermediate variant are strategic complements. So, if the merging firm increases v_L , the independent firm producing v_M would increase its quality variant, thereby making tighter the competition with the top quality producer.³¹ Since the profit loss suffered by firm L when decreasing its quality level is lower than the gain obtained by firm H (since the competition between v_M and v_H relaxes), the merging firms will optimally set $v_L = 0$ restricting their production only to the high quality variant v_H .

As a result, from the first-order conditions obtained maximizing, in turn, the profit of coalition $\{H, L\}$ w.r.t to v_H and the profit of rival M w.r.t v_M , namely

$$\begin{aligned}\frac{\partial(\Pi_H + \Pi_L)}{\partial v_H} &= \frac{(v_H v_M^3 - 64v_H^4 + 48v_H^3 v_M + 16v_H^3 - 12v_H^2 v_M^2 + 8v_H v_M^2 - 12v_H^2 v_M)}{(4v_H - v_M)^3} = 0 \\ \frac{\partial \Pi_M}{\partial v_M} &= \frac{(v_M^4 - 12v_H v_M^3 - 64v_H^3 v_M + 4v_H^3 + 48v_H^2 v_M^2 - 7v_H^2 v_M)}{(4v_H - v_M)^3} = 0\end{aligned}$$

given that, at equilibrium $v_L^{\{\{H,L\},\{M\}\}} = 0$, we obtain the following equilibrium qualities, prices and profits under $C_{HL,M} = (\{H, L\}, \{M\})$:

$$v_H^{\{\{H,L\},\{M\}\}} = 0.25331, \quad v_M^{\{\{H,L\},\{M\}\}} = 0.04823, \quad (27)$$

$$p_H^{\{\{H,L\},\{M\}\}} = 0.10766, \quad p_M^{\{\{H,L\},\{M\}\}} = 0.01025, \quad (28)$$

and

$$\Pi_{\{H,L\}}^{\{\{H,L\},\{M\}\}} = 0.02443, \quad \Pi_M^{\{\{H,L\},\{M\}\}} = 0.00152. \quad (29)$$

Proposition 2 *When the top and the bottom quality firm merge whereas the intermediate firm remains singleton, namely under coalition structure $C_{HL,M} = (\{H, L\}, \{M\})$, at the quality stage the low quality variant is set equal to zero. Prices and qualities offered in equilibrium coincide with those observed under $C_{H,ML} = (\{H\}, \{M, L\})$.*

Proof. It directly follows by the above calculations and by comparing expressions (24) and (25) with (27) and (28). ■

³¹See also Scarpa (1998), p. 669 for the same effect in a three-firm noncooperative setting.

It is worth noting that from a market structure viewpoint, the formation of coalition structures $C_{H,ML}$ and $C_{HL,M}$ are equivalent, as both of them entail a duopoly structure with the same quality gap between variants. Still, the *rationale* underlying the equilibrium configuration in coalition $C_{H,ML}$ cannot be extended to $C_{HL,M}$. In the former case, namely when the intermediate and bottom quality firm compete against the top quality one, the colluding firms decide to set a price so high for the bottom variant that no consumer is willing to buy it, whatever its quality. So, this finding would be observed even if firms would be unable to define endogenously the quality of their products. This is the case, for instance, in a collusive agreement between an intermediate and a bottom quality producer, where firms have no reason to maintain a fighting brand. Variant v_L is adjacent only to variant v_M and, if kept in the market, would reap consumers only to the other colluding player without playing any role in the competition against the top quality firm. Rather, in the latter scenario top and bottom quality firms can decide to reduce the bottom quality to such an extent that the corresponding market share for this variant turns out to be nil. When the coalition decides to withdraw variant v_L from the market, it takes into account two different effects. On one hand, since the low quality variant is adjacent to the intermediate variant, *ceteris paribus*, increasing its quality can enable the coalition to gain market share from the competitor producing variant v_M and, thus, to benefit from the higher profits obtained by the bottom quality firm. On the other hand, as these two variants v_M and v_L are strategic complements, the higher quality of the bottom quality variant boosts the quality of the intermediate variant. The latter variant is, in turn, in direct competition with the top variant: since the lower the quality gap, the fiercer price competition between players, the higher the intermediate quality, the lower, *ceteris paribus*, will be the profit accruing to the top quality firm. Since the loss for this player when the low quality is produced is higher than the gain obtained by the bottom producer, coalition $\{H, L\}$ will prefer to stop producing this variant.

2.1.7 Partial merger between the top and the intermediate quality firm

We finally characterize the equilibrium configuration when the top and the intermediate quality firm decide to merge, with the bottom quality rival playing as singleton.

Price stage At the price stage, firms top and intermediate quality firms maximize the sum of their own profits, namely $\Pi_H + \Pi_M$, whereas the bottom quality firm is playing independently. Using (6), (16) and (23), the optimal replies under coalition structure $C_{HM,L} = (\{H, M\}, \{L\})$ are obtained, respectively, as

$$\begin{aligned}
p_H^{pc}(p_M) &= p_M + \frac{1}{2}(v_H - v_M) \\
p_M^{pc}(p_L, p_H) &= \frac{\frac{1}{2}p_L(v_H - v_M) + p_H(v_M - v_L)}{(v_H - v_L)} \\
p_L^{pc}(p_M) &= \frac{1}{2} \frac{v_L}{v_M} p_M.
\end{aligned}$$

Thus, the last stage equilibrium prices can be easily found as:

$$\begin{aligned}
p_H^{((\{H,M\},\{L\}))}(v_H, v_M, v_L) &= \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{2(4v_M - v_L)}, \\
p_M^{((\{H,M\},\{L\}))}(v_H, v_M, v_L) &= \frac{2v_M(v_M - v_L)}{4v_M - v_L}, \\
p_L^{((\{H,M\},\{L\}))}(v_H, v_M, v_L) &= \frac{v_L(v_M - v_L)}{4v_M - v_L},
\end{aligned}$$

with corresponding profits,

$$\begin{aligned}
\Pi_H^{((\{H,M\},\{L\}))} &= \frac{1}{4} \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{(4v_M - v_L)} - \frac{1}{2} v_H^2, \\
\Pi_M^{((\{H,M\},\{L\}))} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_M^2, \\
\Pi_L^{((\{H,M\},\{L\}))} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_L^2.
\end{aligned}$$

Quality stage We saw above that, at the price stage, when the coalition structure $C_{H,M,L} = (\{H, M\}, \{L\})$ forms, no variant is withdrawn from the market. Still, at the quality stage, it can be proved that a case of *quality reversal* occurs. This is done in the next proposition.

Lemma 1 *In order to escape from the cannibalization taking place between adjacent variants, merging top and intermediate quality firms enhance maximal differentiation between their products by putting one of their variants at the bottom of the quality ladder. As a result, the rival L "leapfrogs" this variant thereby selling a product which lies now in the middle of the quality ladder.*

Proof. See the Appendix. ■

Notice that now, profit $\Pi_L^{((\{H,M\},\{L\}))}$ coincides with that obtained by firm M when producing variant v_M in coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$, namely $\Pi_M^{((\{H\},\{M,L\}))}$. Thus,

the variant produced by the merging firms coincide now with those produced under coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$ where intermediate and bottom quality firms were colluding. Moreover, in $C_{HM,L} = ((\{H, M\}, \{L\}))$ the independent firm produces now the variant that in the the previous scenarios was sold by the intermediate quality firm. In line with the analysis performed in the previous case, the optimal variants are immediately obtained here as:

$$v_H^{\{\{H,M\},\{L\}\}} = 0.25331, \quad v_M^{\{\{H,M\},\{L\}\}} = 0 \quad v_L^{\{\{H,M\},\{L\}\}} = 0.04823, \quad (30)$$

while the equilibrium profits write as:

$$\begin{aligned} \Pi_{\{H,M\}}^{\{\{H,M\},\{L\}\}} &= 0.02443, \\ \Pi_L^{\{\{H,M\},\{L\}\}} &= 0.00152. \end{aligned} \quad (31)$$

Thus, one can state the following proposition.

Proposition 3 *When the top and intermediate quality firms merge whereas the bottom quality firm remains as singleton, namely under coalition structure $C_{HM,L} = ((\{H, M\}, \{L\}))$, at the quality stage one of the merged firms leapfrogs the adjacent rival whose variant is no longer on sale in the market. The obtained qualities coincide with those occurring under the alternative coalition structures $C_{H,ML} = ((\{H\}, \{M, L\}))$ and $C_{HL,M} = (\{H, L\}, \{M\})$.*

Proof. This follows directly by Proposition 1 and by the comparison of (27), (24) and (30). ■

For ease of exposition, we summarize in the following table the payoffs accruing to each firm or coalition in each feasible coalition structure.

$(\{H\}, \{M\}, \{L\})$	$\Pi_H^* = 0.02348$	$\Pi_M^* = 0.00124$	$\Pi_L^* = 0.00005$
$\{N\}$	$\Pi_{\{\{H,M,L\}\}}^{\{N\}} = 0.03125$		
$(\{H\}, \{M, L\})$	$\Pi_H^{\{\{H\},\{M,L\}\}} = 0.02443$	$\Pi_{\{M,L\}}^{\{\{H\},\{M,L\}\}} = 0.00152$	
$(\{H, L\}, \{M\})$	$\Pi_{\{H,L\}}^{\{\{H,L\},\{M\}\}} = 0.02443$	$\Pi_M^{\{\{H,L\},\{M\}\}} = 0.00152$	
$(\{H, M\}, \{L\})$	$\Pi_{\{H,M\}}^{\{\{H,M\},\{L\}\}} = 0.02443$	$\Pi_L^{\{\{H,M\},\{L\}\}} = 0.00152$	

Table 1 - Firm payoffs in every coalition structure.

It is worth remarking that the market structure (duopoly) arising in all *partial mergers* does not vary with the coalition structure induced by the firms. Still, the profits accruing to firms depend on the coalitions to (against) which they belong (compete).

3 Equilibrium Analysis: Stable Mergers

In this section we first consider the problem of coalition formation between three firms from a cooperative perspective, in order to see whether and when there are advantages and incentives for firms to merge. For this purpose we first compare the cooperative stability of the grand coalition when firms are *ex ante* identical to the case in which, as in our setting, they are *ex ante* heterogeneous, namely they are initially selling vertically differentiated products as independent firms. After showing that, in term of stability, things go quite differently in these two cases, and that, in principle, heterogeneous firms possess very strong incentives to merge, we introduce our specific finite-horizon game of coalition formation. We show that in equilibrium firms get engaged in *partial* rather than *full* mergers. Hence, it becomes important to evaluate which, among the partial mergers, are more likely to arise in our setting, and which underlying forces determine such results.

3.1 The profitability and cooperative stability of the grand coalition

As initial observation, and as clearly illustrated in Table 1 above, in terms of total payoff the grand coalition is, not surprisingly, the most profitable coalition structure obtainable in the vertical differentiated market. Thus, we may wonder whether the grand coalition is, in general, robust against coalitional deviations. More specifically, we may wonder whether, there are feasible allocations of the monopoly profit belonging to the *core* of the transferable utility game associated to our simple model. An accurate analysis shows that the answer to this question crucially depends on the initial assumptions of the game. In particular, if the players (firms) are assumed *ex ante* identical and (contrarily to our case) there is no pre-assigned level of quality among them when the negotiation starts, the *core* of the corresponding cooperative game (in partition function form), turns out to be *empty*. Even worse, no intermediate coalition structures are stable even against individual deviations.

3.1.1 Empty core with ex ante identical firms

The emptiness of the *core* with *ex ante* identical firms can be easily shown as follows: it is natural to think that three *ex ante* identical firms $i = 1, 2, 3$ would equally divide the monopoly profit, that in our model is obtained by producing only the top quality product, thus obtaining $\Pi_i = \frac{\Pi^{\{N\}}}{3} = \frac{0.03125}{3} = 0.0104$. In this case, at least one of the firms could decide to break the agreement and start producing alone variant v_H , thus obtaining $\Pi_H^{\{\{H\},\{M,L\}\}} = 0.02443$ if the remaining firms jointly produce in response the intermediate quality variant (and a duopoly

forms);³² alternatively, if the remaining firms split up into singletons and a triopoly forms, the firm leaving the merger would obtain $\Pi_H^{\{\{H\},\{M\},\{L\}\}} = 0.02348$.³³ In both cases the deviation is profitable and the efficient equally-split monopoly payoff is not sufficient to prevent that at least one of the firms breaks the cooperative agreement to become the top-quality producer. Analogously, all partial mergers are *unstable*. In fact, inside every partial merger $\{i, j\}$, for every $i, j \in N$, jointly selling either H or M against an independent rival, at least one of the two firms could always try to break the agreement and sell, independently, either H or M . Thus, since an *ex ante* identical firm i in a partial merger receives either $\Pi_{\{i,j\}}^{\{\{i,j\},\{h\}\}}/2 = 0.0122$ when the merger produces H or $\Pi_{\{i,j\}}^{\{\{h\},\{ij\}\}}/2 = 0.00076$ when it produces M , these payoffs are largely dominated, respectively, by $\Pi_H^{\{\{i\},\{j\},\{h\}\}} = 0.02348$ and $\Pi_M^{\{\{i\},\{j\},\{h\}\}} = 0.00124$. The same result would arise in a sequential bargaining protocol, since the first firm along the sequence would always announce its willingness to remain singleton to produce v_H , the second to remain singleton to produce v_M and the third, similarly, would remain alone producing v_L . As a result, in a vertically differentiated market in which firms are *ex ante* identical and free to select their qualities and prices in a two-stage market game, any negotiation procedure would presumably yields a coalition structure in which all firms remain independent. This outcome, by the way, is what we take as starting point of our coalition formation game. Before any merger can take place, firms are characterized by pre-assigned quality levels, due to their previous history: say, they are either Volkswagen or Skoda. However, as in our model, once entered a coalition, they can jointly adjust their quality-price combinations and nothing may prevent that, say the high quality firm turns into the firm producing the lowest quality good in a merger. However, the *ex ante* firms' heterogeneity is somehow crucial for the negotiation outcome, since it provides players with *ex ante* different outside options.

3.1.2 Nonempty core with ex ante heterogeneous firms

If firms are assumed, as in our game, *ex ante* heterogeneous, since at the beginning they produce noncooperatively three different variants, it can be easily proved that the core is *nonempty*. This is because now the monopoly profit can be allocated asymmetrically according to the initial identities (and outside options) of the players, i.e., H , M and L . Formally, we can associate to the vertically differentiated market a *partition function game* $\rho = (N, v(S, C(S)))$,

³²Inside the monopoly at least one of the three firms is producing the top quality good. Therefore, this firm could decide to produce it alone. The core is rather silent on the behaviour of players inside or outside the cooperative agreement.

³³Consistently with our model, we assume here that firms first decide whether enter a coalition or not and then set qualities and price as individual or coalitional "Nash players".

where N is the set of firms and $v(S, C(S)) \in \mathcal{R}$ is the worth associated to every coalition of firms $S \subset N$ embedded in a given coalition structure $C \in \mathcal{C}$ of which S is part. In our model, when an alliance $S \subset N$ forms, its maximal payoff obtains when the remaining firms stick together in the complementary coalition $\{N \setminus S\}$.³⁴ Therefore, if the core of the partition function game ρ exists when every $S \subset N$ is embedded in $C = (\{S\}, \{N \setminus S\})$, it will *a fortiori* exist in any other coalition structure containing S . Let us formally state this result.

Definition 1 *The core of the partition function game $\rho = (N, v(S, C))$ consists of all efficient profit allocations $\Pi \in \mathcal{R}_+^{|N|}$ such that $\sum_{k \in S} \Pi_k \geq v(S, C(S))$ for all $S \subset N$ and for all $C(S)$ in which S can be embedded.*

Thus, we can prove the following result.

Proposition 4 *In the three-firm vertically differentiated market with ex ante heterogeneous firms H, M, L and endogenous qualities and prices, the core of the corresponding partition function game $\rho = (N, v(S, C))$ is nonempty.*

Proof. See the Appendix. ■

The above result simply says that in a vertical differentiated market with firms initially competing noncooperatively in prices and qualities, there would always be room for a *cooperative* agreement between them.³⁵ This is because, starting from their initial outside options, there exist divisions of the monopoly payoff that cannot be improved upon by any coalition of firms, which includes their departure as singletons. The grand coalition of firms would also be the outcome of an infinite-horizon sequential coalition formation game with *ex ante* heterogeneous players, where for a sufficiently high discount rate there would always be room for full cooperation.³⁶ However, as we show in the next section, if the bargaining process is sequential and in each period the firms possess only a *finite* set of possibilities to propose coalitions and divisions of the joint profit to the rivals, i.e. the game possesses a finite-horizon, the grand

³⁴With only three firms, the behaviour outside a coalition S matters only if $S = \{i\}$, i.e. when an individual firm i is competing with the remaining firms in $N \setminus \{i\}$, which, in turn, can either stay together, or play as singletons. Moreover, from Section 3 we know that whenever two firms form a coalition they eliminate one of the variant either at the quality or at the price stage. Therefore, a firm playing as singleton prefers that its competitors merge rather than compete independently in the market: in game-theoretic terms there are *positive coalition externalities* (see, for instance, Yi, 1997 and 2003).

³⁵In a companion paper, Gabszewicz, Marini and Tarola (2016) show that the core-existence result of this section extends to a n -firm oligopoly if firms sell *equispaced* variants of the products.

³⁶In Bloch (1996) it is proved that if a coalition structure is core-stable it can also be sustained as a subgame perfect equilibria of the infinite-horizon sequential game of coalition formation (with a fixed allocation rule).

coalition cannot be enforced in equilibrium.³⁷ In particular, we show that only *intermediate coalition structures* (partial mergers) can be sustained as subgame perfect equilibria of the alliance formation game. The study of *which*, among all feasible partial mergers, are more likely to arise in the three-firm negotiation, it is the purpose of the next section.

3.2 The Coalition Formation Game

Our game of coalition formation occurs at the first stage of the game. Following Bloch (1995, 1996) and Ray and Vohra (1999) we model the process of coalition formation as a *sequential* unanimity game in which, in an exogenous order, firms propose to their rivals an alliance to which they also belong.³⁸ The firm which follows in the given order among those receiving the proposal may, in turn, either accept or reject it. In case of acceptance, the turn passes to the subsequent firm in the proposed alliance according to the exogenous order and, if all proposed firms accept, the alliance is irrevocably formed and its members can decide cooperatively quantities and prices. If, alternatively, one of the firms rejects the offer, it becomes its turn to make a proposal and the game continues with the same logic until a given coalition structure of the firms is obtained. Differently from Bloch (1996) and Ray and Vohra (1999) and following Selten (1981) and Chatterjee *et al.* (1993) we let the allocation rule be part of the bargaining process.³⁹ Specifically, when it is its turn to offer, a firm proposes both a *merger* and a *division* of the merger profit among its members. The reason for this departure from their model is that since in our coalition formation game the players (firms) are *ex ante* heterogeneous, no obvious fixed allocation rule can be assumed (for instance, an equal split division of joint payoff) before the negotiation takes place. A second distinction of our coalition formation game with respect to Bloch's (1996) and Ray and Vohra's (1999) is that our game is finite-horizon, one in which every player is allowed to make *at most* one proposal at each period. This means that when a firm proposes a merger and is rejected, it can enter a coalition only when proposed by another firm (and it accepts), remaining singleton otherwise. As explanation of this model assumption we can simply say that mergers' time horizon is usually finite, rather than infinite. In particular, publicly traded companies commonly issue tender offers to accomplish mergers and acquisitions, thus offering a given price to shareholders who, in turn, have to decide with

³⁷In the real world there is also be an important additional reason for which the grand coalition is unfeasible: anti-trust authorities would always prevent a monopoly to form. This also justifies the great attention we devote to the stability of partial agreements among firms.

³⁸To be formed, an alliance needs the unanimous agreement of all participants (hence, *unanimity* game). As a result, a player can always remain independent by simply declaring the coalition only containing himself/herself.

³⁹The same assumption is also made in Moldovanu's (1992).three-player coalition formation game.

a yes or no by a given time. Our finite-horizon-single-proposal bargaining protocol may be intended to capture an institutionally constrained negotiation process for merging parties. In this setting, the order of play can play a crucial role for the final outcome. We will discuss again below the implications of our assumptions.

In formal terms, our *merger formation game* is a triple $G = (N, \{\Sigma_i, \Pi_i\}_{i \in N})$, with player set $N = \{L, M, H\}$, strategy set Σ_i and payoff $\Pi_i(\sigma) : \Sigma \rightarrow \mathcal{R}$. For every firm (player) $i \in N$, a strategy $\sigma_i \in \Sigma_i$ defines the *actions* $a_i \in A_i$ available at each node (or information set $I_i \in \mathcal{I}_i$) in which it is its turn to play. In our game, an *action* for a firm $i \in N$ can either be an element of the set $\{\text{Yes}, \text{No}\}$ coming in response to another firm's proposal p_j with $j \neq i$ or, in turn, a proposal $p_i = (S, \Pi)$ including an alliance $S \subset N$ to which i belongs to and a division $\Pi \in \mathcal{R}^{|S|}$ of the alliance joint profit Π_S , such that $\sum_{i \in S} \Pi_i = \Pi_S$. Thus, for a firm a *strategy* $\sigma_i \in \Sigma_i$ is a mapping from its information sets to the set of its feasible actions A_i available therein, namely, $f(I_i) : \mathcal{I}_i \rightarrow A_i$, where $A_i \subset \left(\left(2^N \setminus \{\emptyset\}, \mathcal{R}^{|S|} \right) \cup \{\text{Yes}, \text{No}\} \right)$, with the property that, in every period, a proposal $p_i \in \left(2^N \setminus \{\emptyset\}, \mathcal{R}^{|S|} \right)$ can be made by a firm only if, when it is its turn to play, there are no other players' proposals on the floor and the firm itself has not already made a proposal. That is, for every firm $i \in N$ the action available at every information set I_i^t is $a_i(I_i^t) = p_i$ if both $p_j(I_j^t) = \emptyset$ for $j \neq i$ and $p_i(\{I_i^t\}_{\tau < t}) = \emptyset$ for any previous information set, and $a_i(I_i^t) \in \{\text{Yes}, \text{No}\}$ otherwise. Note that every *strategy profile* $\sigma = (\sigma_H, \sigma_M, \sigma_L)$ of G induces an *outcome* $O(\sigma) = (C(\sigma), \Pi(\sigma))$, namely a coalition structure $C \in \mathcal{C}$ and a profile of payoffs $\Pi = (\Pi_H, \Pi_M, \Pi_L)$ assigned to firms in C . The payoff of every firm $\Pi_i(p(v)) \in \Pi$ is obtained by associating to each coalition structure C a price-quality equilibrium profile $p(v)$ as already illustrated in Section 2. As last step, we need an equilibrium concept for the game occurring at the first stage, namely a *subgame perfect Nash equilibrium* (SPE) of the coalition formation game and, accordingly, a notion of *stable coalition structure*.

Definition 2 *A subgame perfect Nash equilibrium (SPE) of the coalition formation game is a strategy profile σ such that, for every firm $i \in N$, every proper subgame $G' \subset G$, and every $\sigma_i \in \Sigma_i$, $\Pi_i(\sigma_i^*, \sigma_{-i}^*) \geq \Pi_i(\sigma_i, \sigma_{-i}^*)$.*

Definition 3 *A coalition structure $C \in \mathcal{C}$ (a partition of the N firms) is stable if and only if it is sustained by a SPE σ^* of the coalition formation game, namely, $C = C(\sigma^*)$.*

Once again, it is important to mention that the outcome of the game would be completely different if the firms were *ex ante* identical, i.e. they would not possess any pre-assigned quality level. In this case no merger would arise and all firms would remain independent producing

three vertically differentiated goods, as at the starting point of our coalition formation game. So, we might imagine our three heterogeneous firms as the result of a (failed) negotiation process previously occurred among three *ex ante* identical firms.⁴⁰

3.3 Stable Mergers

In this section we characterize all equilibria of the sequential game of coalition formation. Since this game is sensitive to the identity of the initial player, we consider, in turn, the outcomes obtained by the game when either firm H , M and L are the initiators of the bargaining process. Let us first consider the case in which the firm producing the top-quality good (firm H) is the initiator of the coalition formation game.

It can be proved the following:

Proposition 5 *When firm H is the initiator of the coalition formation game, the only stable coalition structure is $C_{H,ML} = (\{H\}, \{M, L\})$, where firm H continues to produce variant v_H and the two remaining firms M and L only market variant v_M .*

Proof. See the Appendix. ■

The main mechanism driving this result is that when in the negotiation every player has only one turn of proposal, or in general the negotiation occurs in a finite horizon, the last player can potentially exploit its "last mover advantage", by inducing the other players to accept their outside options. For this reason the game initiator (here firm H) has an incentive to avoid to be exploited inside a coalition and, anticipating this, will prefer to play as singleton, also knowing in advance that the remaining firms will always prefer to merge to play in a duopoly than being independent and competing in a triopoly. Therefore, the result of Proposition 5 strongly hinges on the identity of the initiator of the game, here firm H .

Applying the same *rationale* as above, in the next proposition, we can easily show that, when firm M is the initial player, $C_{HL,M} = (\{H, L\}, \{M\})$ is the *only stable coalition structure*.

⁴⁰ Another possibility would be to assume two *ex ante* identical firms initially competing with one firm selling a different quality. In this case the two identical firms would presumably prune one of their duplicate variant, thus ending in a duopoly with two heterogeneous firms. In this case the game of coalition formation would result of no much interest. We thank one referee for raising this possibility.

Proposition 6 *When firm M is the initiator of the coalition formation game, the only stable coalition structure is $C_{HL,M} = (\{H, L\}, \{M\})$, where firm H and L jointly produce variant v_H and firm M produces variant v_M .*

Proof. See the Appendix. ■

Notice how, in both Proposition 5 and 6, the initiator of the game is never part of a merger in equilibrium. Indeed, as shown in detail in the proofs of both propositions (see the Appendix) the payoff of a firm when remaining singleton (and rationally expecting that the other firms will merge) dominates that of being part of the grand coalition, since in this case the distribution of profits will be unfavourable for the initial proposer. The equilibrium profit accruing to either firm H or M when initiating the game and competing against a merger is, therefore, larger than when they are part of the merger itself. The optimal strategy is, therefore, to induce the remaining firms to merge. About the role played by the (exogenous) order of play (here H , M , L except when a player rejects a proposal becoming the new game's initiator) it should be observed that this does not play a crucial role in shaping the stable mergers but rather the firms' final equilibrium payoffs. For instance, in Proposition 6, if it was L to play after M , the main difference would be that firm H could exploit in full its last mover advantage, granting its merger's ally (firm L) a lower payoff.

Another relevant force underlying the outcomes of the coalition formation game is the inherent difficulty of firms H and M to engage in a merger together. This explains the different type of merger occurring when firm L (the bottom quality one) is the initiator of the negotiation. In fact, in this case, firm L cannot credibly commit to remain independent when the remaining firms (H and M) prefer to play as singletons rather than merge (see Table 1). The reason is that the merger between firm H and M is problematic just because in this circumstance M optimally leapfrogs the firms selling the bottom quality firm in the merger, and ends up sharing the top quality firm duopoly payoff, which is lower than the sum of their profits under triopoly (cf. section 2). Knowing in advance the infeasibility of coalition $\{H, M\}$, firm L strategically prefers to let firm H playing independently, and form an alliance with firm M . This is shown in the next proposition.

Proposition 7 *When firm L is the initiator of the coalition formation game, the only stable coalition structure is $C_{H,ML} = (\{H\}, \{M, L\})$, where firm H produces variant v_H and M and L jointly produce variant v_M .*

Proof. See the Appendix. ■

It is worth noting that if the game initiator would be selected at random, the most likely outcome of the merger formation game would be that in which the coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$ forms, the other possible outcome implying the formation of $C_{HL,M} = (\{H, L\}, \{M\})$. Moreover, although at equilibrium the same coalition structure $C_{H,ML}$ forms both when either firm H or L starts the negotiation, there is a difference in term of rent extraction, in the two cases, for merged firms M and L : when firm H is the one starting the negotiation, firm M in merger $\{M, L\}$ only receives its outside option $\Pi_M = \Pi_M^* = 0.00124$, whereas firm L is able to get a profit $\Pi_L^{(\{H\}, \{M, L\})} = \Pi_{\{M, L\}}^{(\{H\}, \{M, L\})} - \Pi_M^* = 0.00027 > \Pi_L^*$, exploiting its last-mover advantage in the sequential game. When, on the other hand, it is the firm L to start the game, firm M in alliance $\{M, L\}$ receives $\Pi_M^{(\{H\}, \{M, L\})} - \Pi_L^* = 0.00147 > \Pi_M^* = 0.00124$, while firm L only receives its noncooperative payoff $\Pi_L^* = 0.00005$. In both cases, firm H receives its duopoly payoff $\Pi_H = 0.02443$.

Quite surprisingly, in the merger formation game firm M enjoys a first-mover advantage, just because, when it starts the negotiation, it is able to enforce $C_{HL,M} = (\{H, L\}, \{M\})$ extracting a profit of $\Pi_M^{(\{H, L\}, \{M\})} = 0.00152$ higher than in all other cases. Moreover, this comes at expense of firm H , which in coalition structure $C_{HL,M}$ only receives its noncooperative payoff $\Pi_H^* = 0.02348$.

Finally, it can be noticed that, since for any order of play our one-shot coalition formation game always sustains only one equilibrium alliance structure, the subgame perfect equilibrium of the *finite* repeated version of the game will generate the same outcome period after period. We condense this conclusion in the next corollary.

Corollary 1 *If the coalition formation game is repeated for a finite number of periods, the coalition structures which are stable in the one-shot game will continue to be so in the finite-horizon repeated version of the game, sustained by the same SPE strategy profile repeated at every period.*

Therefore, even in a repeated finite-horizon framework, the stability of $C_{H,ML}$ and $C_{HL,M}$ would continue to hold.

The results of our coalition formation game, confirms that the most likely mergers occur between intermediate and bottom-quality producers, with the premium quality brands preferably running alone. This is the case of some top car producers as, for instance, Daimler-Benz, whose only participation is in the production of Smart, initially started as a joint venture with

Nicolas Hayek, the inventor and producer of Swatch. What our model stylized results also indicate, is that mergers between intermediate and bottom quality firms, as those occurred between Volkswagen and Skoda, or between Fiat and Chrysler in the automotive industry, should be the norm. In these cases the intermediate quality product is withdrawn from the market, which can be interpreted saying that of all products sold by the merger have a tendency to converge towards the same level of quality of their premium brand products. The model also underlines how the mergers between top and bottom quality firms are also likely, as for instance those occurring between generics pharmaceutical manufacturers and premium brand pharmaceutical companies. Our model results just suggests that in this case low quality products can be profitably retired from the market, in order to soften the competition among the remaining goods.

4 A brief discussion of our main assumptions

There are some assumptions of the model which deserve to be discussed, also in view of its possible extensions. We have assumed that only three players populate the market. While this assumption represents a natural entry point, one may wonder whether our main findings can be extended to the case of a market populated by N firms, with $N > 3$. In the case of N firms, partial collusion can take place among a group of firms selling variant located at the bottom of the quality ladder, in the middle or at the top. In the first case (resp. the last case), a *bottom cartel* (resp. a *top cartel*) competes against a fringe of firms, each of which sells a higher (resp. lower) quality variant than the ones produced by the cartel. In the case of an intermediate cartel involving firms whose qualities lie in the middle of the quality ladder, it competes against lower and higher quality rivals. Whatever the type of cartel, two drivers affect the choice to keep all variants on sale vs. withdrawing some of them. The former choice (all variants on sale) is driven by the profitability of being where the demand is, namely to meet different types of consumers through a wide set of price/quality mix (*demand driver*). Keeping on sale all variants however determines a cannibalization effect inside the coalition since the variants produced by the colluding firms are perceived by consumers as mutually exclusive alternatives (*cannibalization driver*). Finally, it is worth remarking that the quality gap between variants (both when they are produced by colluding firms and when they are produced by competing firms) directly affects the prices and the corresponding market share of these variants at equilibrium. In a companion paper (Gabszewicz, Marini, Tarola, 2017) we analyze how these drivers interact in a market populated by N vertically differentiated

firms where some of them decide to collude in prices. There, we adopt a short-run perspective thus restricting the analysis to exogenously given qualities. In that setting, we observe that whatever the type of coalition (bottom, top or intermediate cartel), the colluding firms always fix prices in such a way that some variants stop having a positive demand at equilibrium. Accordingly, we conclude that the driver inducing *product pruning* is stronger than the one inducing *product proliferation*.⁴¹ This paper in a way confirms and reinforces that finding since we prove that, even if a variant is not withdrawn at the price stage, it is removed from the market at the quality stage.

The above statement immediately opens the door to a discussion of another key assumption in our model. We assume that initially each player is a single-product firm. In line with this, in the post-merger scenario, *a priori* at most three variants could be observed at equilibrium: two of them would be produced by the merged entity, while the third would be marketed by the competitor. Nonetheless, in our analysis we allow for pruning and, accordingly, at equilibrium we obtain that only two variants are sold and the merged entity always withdraws from the market one of the two available variants.

It is worth noting that, as clearly illustrated by Norman *et al.* (2005), allowing for pruning turns out to be very meaningful when products are vertically rather than horizontally differentiated. Indeed, they argue that while undifferentiated or horizontally differentiated products induce firms "to maintain their pre-merger product lines as a means to reach as many consumers as possible" (Norman *et al.*, p.1205, 2005), the same argument no longer holds when products are differentiated along a quality dimension. In this latter case, "the removal of intermediate quality goods can soften competition between goods remaining in the post-merger market" (Norman *et al.*, p.1206, 2005).

From a theoretical viewpoint the solution to the dilemma between single-product vs. multi-products is mainly depending on the income dispersion and the specification of the cost function. In the seminal papers by Shaked and Sutton (1982, 1983) and Gabszewicz *et al.* (1986), it is shown that, in the case in which a firm acts as a monopolist, offering a single variant can be optimal, in absence of costs, if the income dispersion is sufficiently narrow. So, taking the parameter α as a lower bound of the market, a necessary condition for the single-product to emerge at equilibrium is that $\alpha > 0$. Otherwise, namely when $\alpha = 0$, then the specification of the cost function determines the optimal choice of variants.

Interestingly, income dispersion and costs have been identified as major drivers of the

⁴¹See Johnson and Myatt (2003) on product pruning.

production choice even later and in a different setting of vertical differentiation. Acharyya (1998), for example, proves that without any ex-ante restriction on market coverage, the choice of being single-product can depend on the specification of the cost function. In his analysis, a firm optimally offers a single variant whenever the cost of quality does not increase too fast. Even more relevant, a further driver can affect the solution to this dilemma, namely the distribution of consumers in the market. On this point, when there exists only two different types of consumers and each type is parametrized by a parameter capturing his preferences (or tastes), it may well happen that, regardless of the cost, the monopolist decides to serve only a part of the market and to drive the low-type consumers out of the market. In Acharyya (2005), however, the constraining effect of income can induce the monopolist to discriminate among consumers. In particular, Acharyya (2005) shows that while differences in preferences do not prevent the monopolist to optimally sell a single variant, under income disparity, a proliferation strategy (quality discrimination) is observed at equilibrium. Moreover, quality discriminating can be observed both when the low-income consumers are income-constrained and in the alternative case when also the high-income consumers are income constrained. Still, in this latter case, the profitability to discriminate among consumers is related to the tastes differences compared with income differences so that discrimination can also induce the monopolist to underprovide quality to the high-income consumers. When moving from a monopoly to a imperfectly competitive market, even under an income constraint, it is shown that a firm refrains from selling more than a single variant. For example, in Bonisseau and Lahmandi-Ayed (2006), the incentive to product proliferation for a incumbent is considered when an entrant contemplates to enter a vertically differentiated market and consumers are uniformly distributed but income-constrained. In their analysis, adopting a multi-product strategy to face entry is never profitable since the lower quality variant which the incumbent could sell, under multi-product strategy, would not contribute to increase its equilibrium profit. Accordingly, they conclude, in line with Shaked and Sutton (1982 1983) and Gabszewicz *et al.* (1986) that income dispersion rather than threat of entry could be a significant force driving firms to adopt a multiproduct strategy.

Although cannibalization represents in our model one of the major factors affecting product pruning in the ex-post merger scenario, one may wonder whether this driver could still hold in an alternative specification where the lowest willingness to pay is strictly positive. In this case, it can be shown that when the intermediate and bottom quality firm merge, the low quality variant is withdrawn from the market at the quality stage rather than at the price stage. So, when consumers are income-constrained, it is no longer profitable for firms to use

a price strategy to drop a variant from the market and thus, if quality would be exogenously given, it would be possible to observe a multi-product strategy by the merging firms.

5 Concluding Remarks

We have investigated the endogenous formation of mergers in vertically differentiated markets in which full or partial binding agreements among initially heterogeneous firms can be signed over prices and qualities of the products. We have shown that regardless of the profitability of the full collusive agreement (i.e. the one signed by all firms in the market), such an arrangement is not obtained in a (finite horizon) sequential negotiation process requiring the unanimity of firms. Conversely, we found that the sequential bargaining process enforces only partial collusive agreements, namely those involving subsets of firms. In particular, stable associations of firms always include the firm producing the bottom quality variant, which is, however, never sold by the coalition at equilibrium. Further, whatever the coalition structure arising at the equilibrium, the market moves from a triopoly to a duopoly with only two variants on sale. The rationale underlying this result can be found in the nature of competition among vertically differentiated firms. Indeed, in a partial merger, the optimal set of products to market is defined by balancing *the cannibalization effect* within the coalition and the *stealing effect* occurring between the coalition and the firm outside. When the bottom quality is kept for sale in the market under a collusive agreement, the former effect always dominates the latter. As immediate consequence, this variant is withdrawn from the market and the equilibrium configuration coincides with that observed in the case of a duopoly in terms of price and quality gap between variants. In a complementary perspective, we can state that moving from a triopoly (observed in the noncooperative scenario) to a duopoly under partial collusion, firms can soften price competition in the market and magnify the quality differentiation between the variants kept on sale. This view is in line with the empirical findings, where mergers emphasize "product differentiation" among merging firms and also with respect to the rivals. Partial mergers are, thus, as a means to enhance *dynamic* competition *for* the market, while decreasing *static* competition in the market.

6

Appendix: Omitted Proofs

A.1 Proof of Lemma 1

Lemma 1. *In order to escape from the cannibalization taking place between adjacent variants, merging top and intermediate quality firms enhance maximal differentiation between their products by putting one of their variants at the bottom of the quality ladder. As a result, the rival L "leapfrogs" this variant thereby selling a product which lies now in the middle of the quality ladder.*

Proof. At the quality stage, firms' profits are:

$$\begin{aligned}\pi_H^{\{\{H,M\},\{L\}\}} &= \frac{1}{4} \frac{(4v_H v_M - v_H v_L - 3v_L v_M)}{(4v_M - v_L)} - \frac{1}{2} v_H^2 \\ \pi_M^{\{\{H,M\},\{L\}\}} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_M^2 \\ \pi_L^{\{\{H,M\},\{L\}\}} &= \frac{v_L v_M (v_M - v_L)}{(4v_M - v_L)^2} - \frac{1}{2} v_L^2\end{aligned}$$

It is easy to see that, the joint profit of merger $\{H, M\}$ is monotonically decreasing in v_M , as

$$\frac{\partial(\pi_H^{\{\{H,M\},\{L\}\}} + \pi_M^{\{\{H,M\},\{L\}\}})}{\partial v_M} = \frac{(4v_L^3 v_M + v_L^3 - 48v_L^2 v_M^2 + 20v_L^2 v_M + 192v_L v_M^3 - 256v_M^4)}{4(v_L - 4v_M)^3} < 0.$$

Thus, the merging firms find it profitable to set the quality of one of their variants at the minimum admissible value, say 0. By doing this, they choose to produce a variant which is at the bottom of the quality ladder. If the competitor L would keep its own variant at the same quality level, then it would obtain nil profits. Rather, choosing to produce an intermediate variant $v_M > 0$ would yield positive equilibrium profits equal to

$$\pi_L^{\{\{H,M\},\{L\}\}} = \frac{v_M^2 (v_H - v_L) (v_M - v_L) (v_H - v_M)}{(v_M^2 + v_H v_L - 4v_H v_M + 2v_M v_L)^2} > 0.$$

As this profit is strictly positive for any $v_H > v_M > v_L = 0$, one can conclude that firm L will find profitable to leapfrog the post-merger bottom quality variant put on offer by coalition $\{H, M\}$, namely $v_L = 0$, by producing an intermediate quality variant (denoted above v_M). ■

A.2 Proof of Proposition 4

Proposition 4. *In the three-firm vertically differentiated market with ex ante heterogeneous firms and endogenous qualities and prices, the core of the partition function game $\rho = (N, v(S, C))$*

is nonempty

Proof. Core allocations are individually-rational and group-rational profit division $\Pi = (\Pi_H, \Pi_M, \Pi_L)$ of the efficient monopoly payoff $v(N) = \Pi^{\{N\}} = 0.03125$. Thus, the set of $\Pi \in \text{Core}(\rho)$ must respect the following inequalities:

$$\begin{aligned}
\sum_{i=H,M,L} \Pi_i &= v(N) = \Pi^{\{N\}} = 0.03125, \\
\Pi_H + \Pi_M &\geq v(\{H, M\}, (\{H, M\}, \{L\})) = \Pi_{\{H,M\}}^{\{\{H,M\}, \{L\}\}} = 0.02443 \\
\Pi_H + \Pi_L &\geq v(\{H, L\}, (\{H, L\}, \{M\})) = \Pi_{\{H,M\}}^{\{\{H,L\}, \{M\}\}} = 0.02443 \\
\Pi_M + \Pi_L &\geq v(\{M, L\}, (\{H\}, \{M, L\})) = \Pi_{\{H,M\}}^{(\{H\}, \{M,L\})} = 0.00152 \\
\Pi_H &\geq v(\{H\}, (\{H\}, \{M, L\})) = \Pi_H^{(\{H\}, \{M,L\})} = 0.02443 \\
\Pi_M &\geq v(\{M\}, (\{H, L\}, \{M\})) = \Pi_M^{\{\{H,L\}, \{M\}\}} = 0.00152 \\
\Pi_L &\geq v(\{L\}, (\{H, M\}, \{L\})) = \Pi_L^{\{\{H,M\}, \{L\}\}} = 0.00152
\end{aligned}$$

which surely hold, since:

$$0.02443 + 0.00152 + 0.00152 = 0.02749 < 0.03125.$$

Note that for every i -th firm, $v(\{i\}, (\{i\}, \{N \setminus \{i\}\})) > v(\{i\}, (\{i\}, \{j\}, \{h\}))$ for every $j, h \in N \setminus \{i\}$, implying that each firm gains more when the remaining firms form a coalition than when playing alone. Thus, the last numerical inequality holds *a fortiori* when, after one firm leaves the grand coalition, the remaining firms split-up in singletons. As a result, all efficient payoff allocations $\Pi = (\Pi_H, \Pi_M, \Pi_L)$ rewarding every firm *at least* its maximal deviating payoff and distributing the remaining surplus Z between the three firms, namely,

$$Z = \Pi^{\{N\}} - \Pi_H^{(\{H\}, \{M,L\})} - \Pi_M^{\{\{H,L\}, \{M\}\}} - \Pi_L^{\{\{H,M\}, \{L\}\}} = 0.0085$$

belong to the core, which is, therefore, nonempty. ■

A.3 Proof of Proposition 5

Proposition 5. *When firm H is the initiator of the coalition formation game, the only stable coalition structure is $C_{H,ML} = (\{H\}, \{M, L\})$, where firm H continues to produce variant v_H and the two remaining firms M and L only market variant v_M .*

Proof. The game can be solved backward. Firm's H available actions at the initial node

(information set $I_H^1 \in \mathcal{I}_H$) are the following (proposals):

$$A_H(I_H^1) = [(\{N\}, \Pi), (\{H, M\}, \Pi), (\{H, L\}, \Pi), (\{H\})].$$

Assume first that firm H proposes the grand coalition $\{N\}$ associated to a given division $\Pi \in \Pi^{\{N\}}$ of the efficient monopoly profit between the three firms. By the order of the game, firm M can either accept or reject. If it rejects the offer, it is its turn to make a proposal and can propose one of the following:

$$A_M(I_M^1) = [(\{N\}, \Pi), (\{H, M\}, \Pi), (\{M, L\}, \Pi), (\{M\})].$$

We know (from Table 1) that, for any associated payoff division, the coalition structure $C_{HM,L} = (\{H, M\}, \{L\})$ is dominated by the choice of firm H and M to play as singletons, since

$$\Pi_{\{H,M\}}^{\{H,M\},\{L\}} < \Pi_H^* + \Pi_M^*.$$

Therefore, when made by firm M , the proposal $p_M^{HM} = (\{H, M\}, \Pi)$ will always be rejected by firm H . In this event, firm H has no more proposals to make. Thus, firm L can gain its highest payoff by proposing $\{N\}$, offering the noncooperative profits to H and M and get the difference $\Pi^{\{N\}} - \Pi_H^* - \Pi_M^*$, which is its most profitable outcome. To break the ties, we can initially assume that, when gaining equal payoffs all firms prefer being in a coalition than remaining singletons (although the reasoning can be repeated when the alternative case holds). A similar outcome would be reached if, after a rejection, firm M proposes $p_M^{ML} = (\{M, L\}, \Pi)$ or $p_M^N = (\{N\}, \Pi)$ which, in turn, would be both refused by firm L , aiming to propose (as last proposer) the grand coalition, obtaining $\Pi_L = \Pi^{\{N\}} - \Pi_H^* - \Pi_M^*$. Analogously, if firm M accepts to enter the grand coalition when proposed by firm H , it knows that, when it is its turn to play, firm L will always reject such proposal to propose, in turn, the grand coalition with a payoff allocation which assigns to its rivals their Nash equilibrium payoffs. Reasoning backward, firm H knows that, if it proposes the grand coalition, it would obtain at most its Nash equilibrium payoff. For this reason, firm H can try to make alternative offers. Proposing $p_H^{HM} = (\{H, M\}, \Pi)$ is out of question, since player M would always reject it, and the game would return to the situation described above. Another chance for firm H is to propose $p_H^{HL} = (\{H, L\}, \Pi)$ that, in turn, would be rejected by firm L with the aim to propose again $(\{H, L\}, \Pi)$, offering to firm H its noncooperative outside option. Alternative proposals by firm L (after its rejection of $\{H, L\}$ proposed by firm H) involving firm M , as $p_L^N = \{\{N\}, \Pi\}$

or $p_L^{ML} = \{\{M, L\}, \Pi\}$ would similarly be rejected by firm M to enforce, as last proposer, the grand coalition payoff. Thus, at the initial node the most profitable action for firm H is to propose $p_H^H = \{H\}$, signalling the intention to play irrevocably as singleton. By doing this, it is aware that firm M can either propose $p_M^{ML} = (\{M, L\}, \Pi)$ or $p_M = (\{M\})$. In the first case, firm M knows that firm L will reject to propose, in turn, $p_L^{ML} = (\{M, L\}, \Pi)$, offering Π_M^* to firm M and keeping the difference, since: $\Pi_{\{ML\}}^{\{\{H\}\{M,L\}\}} - \Pi_M^* > \Pi_L^*$. In the second case, namely when firm M proposes $p_M^M = \{M\}$, a triopoly arises and firm M obtains Π_M^* . Since with equal payoffs firms prefer by assumption to be in a coalition rather than remaining as singletons, in this subgame the choice of firm M will be $p_M^{ML} = (\{M, L\}, \Pi)$. Therefore, the coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$ is stable because can be sustained by the following SPE strategy profile along the *equilibrium path*:⁴²

$$\sigma^* = \left(\sigma_H^* = \{H\}, \sigma_M^* = \left(\{M, L\}, \Pi' \right), \sigma_L^* = \left(\text{No}, \{M, L\}, \Pi'' \right) \right),$$

where $\Pi' = (\Pi'_M, \Pi'_L)$, for $\Pi'_M = \Pi_{\{ML\}}^{\{\{H\}\{M,L\}\}} - \Pi_{L,L}^*$, $\Pi'_L = \Pi_L^*$, and $\Pi'' = (\Pi''_M, \Pi''_L)$, for $\Pi''_M = \Pi_M^*$ and $\Pi''_L = \Pi_{ML}^{\{\{H\}\{M,L\}\}} - \Pi_M^*$. If we assume, to break ties, that with equal payoffs firms prefer to be singletons rather than being in coalition, the same coalition structure $C_{H,ML}$ can be enforced by a SPE of the coalition formation game with the difference that, along the equilibrium path, $\Pi'_M = \Pi_{\{ML\}}^{\{\{H\}\{M,L\}\}} - (\Pi_L^* + \epsilon)$, $\Pi'_L = \Pi_L^* + \epsilon$ and $\Pi''_M = \Pi_M^* + \epsilon$, $\Pi''_L = \Pi_{\{ML\}}^{\{\{H\}\{M,L\}\}} - (\Pi_M^* + \epsilon)$, for $\epsilon > 0$. The same occurs in all other proposals implying the presence of a coalition. The reason is that to convince a firm to join an alliance it must receive something *more* (an $\epsilon > 0$) than its noncooperative payoff. Therefore, coalition structure $C_{H,ML}$ remains stable (namely supported by a SPE strategy profile of the sequential coalition formation game) for every adopted rule to break ties. Finally, to see that $C_{H,ML}$ is the *only* stable coalition structure arising when firm H is the initiator of the game, note that any alternative strategy profile cannot be SPE just because firm H possesses an incentive to profitably deviate by proposing $p_H = \{H\}$ with the expectation to compete in a duopoly (namely under $C_{H,ML}$) and gaining a payoff $\Pi_H^{((H),(ML))} = 0.02443$ which dominates its triopoly profit $\Pi_H^* = 0.02348$ (or in turn, $\Pi_H^* + \epsilon$). ■

⁴²We have verbally described all *out of equilibrium path* actions which compose the SPE strategy profile σ^* and, therefore, for ease of simplicity, we do not repeat it here.

A.4 Proof of Proposition 6

Proposition 6. *When firm M is the initiator of the coalition formation game, the only stable coalition structure is $C_{HL,M} = (\{H, L\}, \{M\})$.*

Proof. As above, the game can be solved backward. Firm's M available actions at the initial node (information set $I_M^1 \in \mathcal{I}_M$) are:

$$A_M(I_M^1) = [(\{N\}, \Pi), (\{H, M\}, \Pi), (\{M, L\}, \Pi), (\{M\})].$$

Again, if firm M proposes the grand coalition $\{N\}$, with an associated division of the monopoly profit $\Pi \in \Pi^{\{N\}}$, the next player, firm H , would reject the offer to propose, in turn, one of the following:

$$A_H(I_H^1) = [(\{N\}, \Pi), (\{H, M\}, \Pi), (\{H, L\}, \Pi), (\{H\})].$$

Coalition structure $C_{HM,L} = (\{H, M\}, \{L\})$ is dominated by the choice of firm H and M to play as singletons and proposal $p_H^{HM} = (\{H, M\}, \Pi)$ is, therefore, rejected by firm M . If this occurs, firm M has no more proposals and, hence, firm L can propose $\{N\}$, obtaining $\Pi_L = \Pi^{\{N\}} - \Pi_H^* - \Pi_M^*$, which is its most profitable outcome. Similar outcome would be reached if, after a rejection, firm H offers, in turn, $p_H^{HL} = (\{H, L\}, \Pi)$ or $p_H^N = (\{N\}, \Pi)$, which can either be accepted or refused by firm M , but nevertheless the final payoff would, for firm H and M , be their noncooperative outside options. Thus, reasoning backward, firm M knows that by proposing the grand coalition it would receive at most its noncooperative payoff. Its alternative proposals are $p_M^{HM} = (\{H, M\}, \Pi)$ which would be rejected by firm H (so the game would reach the same outcome described above) or $p_M^{ML} = (\{M, L\}, \Pi)$ which, in turn, would be rejected by firm L with the aim to propose $p_L = (\{M, L\}, \Pi)$, offering firm M its noncooperative outside option, which turns out to be better than any other coalition containing firm H that would, in fact, exploit its last mover advantage. Note that forming alliance $\{M, L\}$ would, for firm L , be better than any other proposal involving firm H , that could exploit its last-mover advantage. Thus, at the initial node, the most profitable action for firm M is to propose $p_M^M = \{M\}$, with the knowledge that firm H prefers to be in coalition than playing as singleton proposing $p_H^{HL} = (\{H, L\}, \Pi)$ rather than $p_H^H = \{H\}$. Hence, the proposal p_H^{HL} by firm H would be rejected by firm L , that can counter-offer, in turn, $p_L^{HL} = (\{H, L\}, \Pi)$, giving Π_H^* to firm H and keeping the difference for itself, since $\Pi_{HL}^{\{H,L\}\{M\}} - \Pi_H^* > \Pi_L^*$. As a result, the coalition structure $C_{HL,M} = (\{H, L\}, \{M\})$ is stable since it can be sustained by

the following SPE strategy profile along the *equilibrium path*.⁴³

$$\sigma^* = \left(\sigma_H^* = \left(\{H, L\}, \Pi' \right), \sigma_M^* = \{M\}, \sigma_L^* = \left(\text{No}, \{H, L\}, \Pi'' \right) \right),$$

with $\Pi' = (\Pi'_H, \Pi'_L)$, where $\Pi'_H = \Pi_{\{HL\}}^{\{\{H,L\}\{M\}\}} - \Pi_L^*$ and $\Pi'_L = \Pi_L^*$ and $\Pi'' = (\Pi''_M, \Pi''_L)$, where $\Pi''_H = \Pi_H^*$ and $\Pi''_L = \Pi_{\{HL\}}^{\{\{H,L\}\{M\}\}} - \Pi_H^*$. As in the proof of Proposition 6, if, under equal payoffs, firms prefer to be singletons than being in a coalition, the coalition structure $C_{HL,M}$ can be enforced as a SPE of the coalition formation game for, $\Pi'_H = \Pi_{\{HL\}}^{\{\{H,L\}\{M\}\}} - (\Pi_L^* + \epsilon)$, $\Pi'_L = \Pi_L^* + \epsilon$, $\Pi''_H = \Pi_H^* + \epsilon$ and $\Pi''_L = \Pi_{\{HL\}}^{\{\{H,L\}\{M\}\}} - (\Pi_H^* + \epsilon)$, for $\epsilon > 0$, and, similarly for all other proposal involving coalitions outside the equilibrium path. Finally, $C_{HL,M}$ is the *only* stable coalition structure when firm M is the initiator of the coalition formation game just because in any alternative strategy profile firm M would always prefer to propose $p_M^M = \{M\}$ and compete in a duopoly with a payoff $\Pi_M^{\{\{H,L\}\{M\}\}} = 0.00152$ rather than getting its triopoly profit $\Pi_M^* = 0.00124$ (or in turn, $\Pi_M^* + \epsilon$), which occurs in all other subgames. ■

A.5 Proof of Proposition 7

Proposition 7. *When firm L is the initiator of the coalition formation game, the only stable coalition structure is $C_{ML,H} = (\{H\}, \{M, L\})$.*

Proof. Again in this proof we reason backward. Note that when firm L is the initiator of the game, the line of reasoning is slightly different than in Proposition 6 and 7. Firm's L available actions at the initial node (information set $I_L^1 \in \mathcal{I}_L$) are:

$$A_L(I_L^1) = [(\{N\}, \Pi), (\{H, L\}, \Pi), (\{M, L\}, \Pi), (\{L\})].$$

To break ties assume initially that, with equal payoffs, firms prefer to be in coalition rather than act as singletons. Note first that if firm L proposes $p_L^L = \{L\}$, the turn passes to player H , who can either propose $p_H^H = \{H\}$, in which case the game ends with $C_{H,M,L} = (\{H\}, \{M\}, \{L\})$ or instead $p_H^{HM} = (\{H, M\}, \Pi)$, which again forces the game to end with $C_{H,M,L} = (\{H\}, \{M\}, \{L\})$, since $C_{H,ML} = (\{H\}, \{M, L\})$ is dominated by $C_{H,M,L}$ for both firm H and M (see Table 1). So differently from above, firm L is unable to enforce the formation of its complementary coalition $N \setminus \{L\} = \{H, M\}$ by signalling its willingness to play alone as singleton. Alternatively, if firm L proposes either $p_L^N = (\{N\}, \Pi)$ or $p_L^{HL} = (\{H, L\}, \Pi)$ it always induces the formation of coalition structure $C_{H,ML}$ with $\Pi_H = \Pi_H^{\{\{H\}\{M,L\}\}}$, $\Pi_M =$

⁴³ Again, for simplicity, we skip the description of all players' out of equilibrium actions.

$\Pi_M^{\{\{H\},\{M,L\}\}} - \Pi_L^*$ and $\Pi_L = \Pi_L^*$. The reason is that, by the order of play, after both proposals the turn passes to firm H whose optimal strategy is to reject the offer and to announce $p_H^H = \{H\}$, thus inducing proposal $p_M^{ML} = (\{M, L\}, \Pi)$ by firm M with $\Pi_M = \Pi_M^{\{\{H\},\{M,L\}\}} - \Pi_L^*$ and $\Pi_L = \Pi_L^*$, which firm L will accept. Finally, if firm L proposes at the beginning of the game $p_L^{ML} = (\{M, L\}, \Pi)$, for any profit distribution Π firm M will reject it to propose, in turn, $p_M^{ML} = (\{M, L\}, \Pi)$, again offering $\Pi_L = \Pi_L^*$ to firm L that, in turn, will accept. Therefore, since by assumption with equal payoffs firms prefer to be in coalition, the game possesses as unique outcome the intermediate coalition structure $C_{H,ML} = (\{H\}, \{M, L\})$, which can be sustained as a SPE strategy profiles. Again, it can be easily checked that the game outcome does not change if, to break ties, we assume that under equal payoffs firms prefer to play as singletons rather than being in coalitions. ■

REFERENCES

- Accharya, R. (1998). "Monopoly and Product Quality: Separating or Pooling Menu?", *Economics Letters*, 61, 187–194.
- Belleflamme, P., (2000). "Stable Coalition Structures with Open Membership and Asymmetric Firms", *Games and Economic Behavior*, 30, 1-21.
- Bloch, F. (1995). "Endogenous Structures of Associations in Oligopolies", *Rand Journal of Economics*, 26, 537-556.
- Bloch, F. (1996). "Sequential Formation of Coalitions with Fixed Payoff Division", *Games and Economic Behaviour*, 14, 90-123.
- Bloch, F. (2002). "Coalition and Networks in Industrial Organization", *The Manchester School*, 70, 36-55.
- Bloch, F. (2003). "Coalition Formation in Games with Spillovers" in: Carraro C. (eds.) *The endogenous formation of economic coalitions*, Fondazione Eni Enrico Mattei Series on Economics and the Environment, Cheltenham, U.K. and Northampton, Mass., Elgar.
- Bonnisseau, J-M, Lahmandi-Ayed, R. (2006). "Vertical Differentiation: Multiproduct Strategy to Face Entry?", *The B.E. Journal of Theoretical Economics*, 6, 1, 1-14
- Brekke, K. R., Siciliani, L., Straume, O. R. (2014). "Horizontal Mergers and Product Quality", *NHH Discussion Paper*, February 2014.
- Caplow, T. (1956). "A Theory of Coalitions in the Triad", *American Sociological Review*, 21, 489-493.

- Caplow, T. (1959). "A Theory of Coalitions in the Triad", *American Journal of Sociology*, 64, 488-493.
- Caplow, T. (1968). *Two Against One: Coalitions in Triads*, Prentice-Hall, Englewood Cliff, NJ.
- Chatterjee K. B., Dutta, B. , Ray, D., Sengupta, K. (1993). "A Noncooperative Theory of Coalitional Bargaining", *Review of Economic Studies*, 60, 463-477.
- Currarini, S., Marini, M. A. (2015). "Coalitional Approaches to Collusive Agreements in Oligopoly Games", *The Manchester School*, 83, 3, 253-287.
- Deneckere, R. and Davidson, C., (1985). "Incentive to form coalitions with Bertrand competition", *Rand Journal of Economics*, 16 , pp.473-486.
- Draganska, M., Mazzeo, M., Seim, K., (2009). "Beyond Plain Vanilla: Modeling Joint Product Assortment and Pricing Decisions", *Quantitative Marketing and Economics*, 7, 105-146.
- Ecchia, G., Lambertini, L. (1997), "Minimum Quality Standards and Collusion," *Journal of Industrial Economics*, 45, 1, 101-13.
- Einav, L. (2010). "Not all Rivals Look Alike: Estimating an equilibrium Model of the Release Date Timing Game", *Economic Inquiry*, 48, 2, 369–390.
- Fan, Y. (2013). "Ownership consolidation and product characteristics: A study of the U.S. daily newspaper market", *American Economic Review*, 103, 1598–1628.
- Gabszewicz, J. J. and J-F. Thisse (1979).. "Price Competition, Quality and Income Disparities", *Journal of Economic Theory*, 20, 340-359.
- Gabszewicz, J. J., Shaked, A., Sutton, J., Thisse, J-F. (1986). "Segmenting the Market: the Monopolist's Optimal Product Mix" *Journal of Economic Theory*, 39, 2, 273-289.
- Gabszewicz, J. J., Wauthy, X.Y.(2002). "Quality Underprovision by a Monopolist when Quality is not Costly" *Economics Letters*, 77, 1, 65-72.
- Gabszewicz, J. J., Marini, M. A., Tarola O. (2016). "Core Existence in Vertically Differentiated Markets ", *Economics Letters*, 149, 28-32.
- Gabszewicz, J. J., Marini, M. A., Tarola O. (2017). "Vertical Differentiation and Collusion: Pruning or Proliferation?", *Research in Economics*, 71,1, 129-139.
- Gamson, W. A. (1961). "A Theory of Coalitions in the Triad", *American Sociological Review*, 21, 489-493.
- Gandhi, A., Froeb, L., Tschanz, S.,Werden, G. (2008). "Post-merger product repositioning", *Journal of Industrial Economics*, 56, 49–67.
- George, L. (2007). "What's fit to print: the effect of ownership concentration on product variety in daily newspapers markets", *Information Economics and Policy*, 19, 285-303.

- Giraud-Heraud, E., H. Hammoudi, Mokrane, M. (2003). "Multiproduct Firm Behaviour in a Differentiated Market", *Canadian Journal of Economics*, 36, 1, 41-61.
- Hackner, J. (1994). "Collusive pricing in markets for vertically differentiated products", *International Journal of Industrial Organization*, 12, 2, 155-177.
- Johnson J. P, Myatt, D. (2003). "Multiproduct Quality Competition: Fighting Brands and Product Line Pruning," *American Economic Review*, 93, 748-774.
- Lambertini, L. (2000). "Technology and Cartel Stability under Vertical Differentiation", *German Economic Review*, 1, 4, 421-444.
- Lambertini, L., S. Poddar, D. Sasaki (2002). "Research joint ventures, product differentiation, and price collusion", *International Journal of Industrial Organization* 20, 829–854.
- Lee, J. (2013) "Endogenous Product Characteristics in Merger Simulation: A Study of the U.S. Airline Industry", *mimeo*.
- Lommerud, K. and Sorgard, L. (1997). "Merger and Product Range Rivalry", *International Journal of Industrial Organization*, 16, 1, 21-42.
- Marini, M. A. (2009). "Games of Coalition & Network Formation: a Survey", in Naizmada A.K. *et al.* (eds.). *Networks, Topology and Dynamics. Lectures Notes in Economics & Mathematical Systems*, 613, 67-93, Berlin Heidelberg, Springer-Verlag.
- Martin, S., (1995). "R&D Joint ventures and tacit product market collusion", *European Journal of Political Economy*, 11, 733–741.
- Mesterton-Gibbons, M., Gavriletz, S., Janko, G., Akcay, E. (2011). "Models of Coalition or Alliance Formation", *Journal of Theoretical Biology*, 274, 187-204.
- Mazzeo, M., (2002). "Product Choice and Oligopoly Market Structure," *Rand Journal of Economics*, 33, 221-242.
- Mazzeo, M., (2003). "Competition and service quality in the U.S. airline industry", *Review of Industrial Organization*, 22, 275–296.
- Mc Millan, M., (2012). "Quality collusion: news, if it ain't broke, why fix it?", *Fordham Urban Law Journal*, 39, 5, 1895-1942.
- Moldovanu, B. (1992). "Coalition-Proof Nash Equilibrium and the Core in Three-Player Games", *Games and Economic Behavior*, 9, 21-34.
- Motta, M., (1992). "Cooperative R&D and vertical product differentiation", *International Journal of Industrial Organization* 10, 643–661.
- Mussa, M. and S. Rosen (1978). "Monopoly and product quality", *Journal of Economic Theory*, 18, 301–317.
- Norman, G., Pepall L., Richards D. "Product Differentiation, Cost-Reducing Mergers, and

- Consumer Welfare", *Canadian Journal of Economics*, 38, 4, 1204-1223.
- Pezzino, M. (2010). "Minimum Quality Standards with More Than Two Firms Under Cournot Competition.", *The IUP Journal of Managerial Economics*, 8, 3, 26-45.
- Peters, C. (2006) "Evaluating the performance of merger simulation: evidence from the U.S. airline industry", *Journal of Law and Economics*, 49, 2, 627-649.
- Ray, D. and R. Vohra (1999). "A Theory of Endogenous Coalition Structures", *Games and Economic Behavior*, 26, 2, 286-336.
- Salant, S.W., Switzer, S. and Reynolds, R., J. (1983). "Losses from Horizontal Merger: the Effects of an Exogenous Change in Industry Structure on Cournot-Nash Equilibrium", *Quarterly Journal of Economics*; 98, 2, 185-99.
- Scarpa, C. (1998). "Minimum quality standards with more than two firms", *International Journal of Industrial Organization*, 16, 5, 665-676.
- Selten, R. (1981). "A Non-cooperative Model of Characteristic Function Bargaining", in *Essays in game Theory and Mathematical Economics in honor of O. Morgenstern*. (V. Bohm and H. Nachcamp, Eds.). Mannheim: Bibliographisches Institute.
- Seim, K. (2006). "An empirical model of firm entry with endogenous product-type choices", *Rand Journal of Economics*, 37, 619-640.
- Steven T. B., Waldfoegel, J. (2001). "Do Mergers Increase Product Variety? Evidence from Radio Broadcasting", *Quarterly Journal of Economics*, 116, 3, 1009-1025.
- Sweeting, A. (2010). "The Effects of Mergers on Product Positioning: Evidence from the Music Radio Industry", *Rand Journal of Economics*, 41, 372-397.
- Vinacke, V. E., Arkoff, A. (1957). "An Experimental Study of Coalitions in the Triad", *American Sociological Review*, 22, 406-414.
- U.S. Department of Justice and the Federal Trade Commission (2010). *Horizontal Merger Guidelines*, August 19, 2010.
- Wauthy, X. (1996). "Quality Choice in Models of Vertical Differentiation", *Journal of Industrial Economics*, 44, 3, 345-53.
- Wieczner, J. (2015). "The real reasons for the pharma merger boom", *Fortune*, July 28, 2015.
- Yi, S.-S. (1997). "Stable Coalition Structure with Externalities", *Games and Economic Behaviour*, 20: 201-237.
- Yi, S. S., (2003). "The Endogenous Formation of Economic Coalitions: The Partition Function Approach" in C. Carraro (ed.) *The endogenous formation of economic coalitions*, Fondazione Eni Enrico Mattei Series on Economics and the Environment, Cheltenham, U.K. and Northampton, Mass.: Elgar.