What do the shadow rates tell us about future inflation?

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Abstract

This paper investigates whether shadow interest rates contain predictive power for U.S. inflation in a data-rich environment. We find that shadow rates are useful leading indicators of inflation. Shadow rates contain substantial in-sample and out-of-sample predictive power for inflation in both the zero lower bound (ZLB) and non-ZLB periods. We find that the shadow rate suggested by Wu and Xia (2016) contains more information about future inflation than the shadow rate suggested by Krippner (2015b).

Keywords: shadow interest rates, zero lower bound, unconventional monetary policy, inflation forecasting, data-rich environment, factor models

JEL codes: C38, C53, E37, E43, E44, E58

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1. Introduction

During the recent global financial crisis central banks around the world, including the Federal Reserve (Fed), have cut interest rates to near zero and introduced unconventional monetary policy measures such as quantitative easing and forward guidance.[1] These measures have been useful in easing the economic conditions as the primary monetary policy instrument, the short-term nominal interest rate, has been constrained by the zero lower bound (ZLB).[2] When policy rates are in the ZLB range for a prolonged period of time, the stance of the Fed's monetary policy cannot be evaluated by the observable variations of the federal funds rate. This has raised the question of how to measure the overall stance of the monetary policy in the ZLB environment.

One possible way to approach this question is to consider longer maturity interest rates. They can be seen as a proxy for the policy rate, but they are not constrained by the ZLB. They are also influenced by current and expected short-term interest rates. Thus, several researchers (e.g., Krippner 2015b; Wu and Xia 2016) have used longer-term interest rates to construct a shadow (short) rate using a term structure model. The shadow rate is a quantitative measure that indicates the overall stance of the monetary policy when the conventional monetary policy instrument (the short-term policy rate) is at the ZLB. It takes into account the effects of unconventional monetary policy actions and can freely take on negative values in the ZLB environment. The shadow rate is the shortest maturity from the estimated shadow yield curve. In the non-ZLB environment the shadow rate is essentially equal to the policy rate.

The previous shadow rate literature has mostly focused on modeling the shadow rates, discussing the sensitivity of the shadow rate estimates, and analyzing the con-

[1]For comprehensive analysis of unconventional monetary policy measures, see, e.g., Bernanke et al. (2004) and their effectiveness, see, e.g., Neely (2015) and the references cited therein.

[2]Because currency is available as an alternative asset, interest rates are bounded below by zero (or a slightly negative value because of the storing and transferring costs of currency). Otherwise negative nominal interest rates would offer a risk-free arbitrage opportunity.

sistency of shadow rate movements with unconventional monetary policy actions. Kim and Singlenton (2012) estimated several two-factor models using Japanese Government Bond yields. They found that shadow rate models capture the variation in bond yields during periods of near-zero short-term rates. Christensen and Rudebusch (2015) derived an option-based approach to estimated one-, two-, and three-factor models and showed the sensitivity of the shadow rate estimates to the model fit and specification. Bauer and Rudebusch (2016) forecasted the expected time to the policy rate lift-off and how long the policy rate will remain near zero with U.S. data.

This paper contributes to the existing literature by examining whether the shadow rates contain predictive power for U.S. inflation. It is well known that monetary policy affects future inflation and thus the shadow rates, as measures of the stance of monetary policy in the ZLB period are potentially useful leading indicators of inflation. Chung et al. (2012) and Mishkin (2017) point out that ZLB periods can be more frequent and long-lasting than the standard macro literature has suggested. The standard macro models have underestimated the incidence and effects of ZLB events, because contractionary shocks may appear more frequently than previously anticipated and lead to ZLB constraint binding more often. That is, the ZLB periods have become more significant to the central banks than was foreseen before the recent financial crisis. Therefore investigating the indicator properties of the shadow rates in the ZLB environment is essential.

We analyze the in-sample and out-of-sample predictive power of the shadow rates in a data-rich environment using factor models. We investigate whether the predictive power remains stable over time and especially whether the shadow rates contain predictive power in the recent ZLB/unconventional monetary policy era. To the best of our knowledge, the extant literature has not analyzed the relationship between shadow rates and future inflation in a systematic way.[3] Thereby, our paper is intended to

[3]Hännikäinen (2017) shows that shadow rates contain out-of-sample predictive power for future inflation but not for real activity. Importantly, he compares only the predictive content of one WX and

bridge this gap.

We consider shadow rates estimated by two alternative term structure models. The first is the shadow rate term structure model by Wu and Xia (2016) (henceforth WX), which is a three-factor model, and the other is the Krippner arbitrage-free Nelson and Siegel (1987) model with two state-variables by Krippner (2015b) (henceforth K-ANSM). Because Bauer and Rudebusch (2016) and Krippner (2015a) show that the estimated shadow rates are very sensitive to the selected values of the lower bound parameter, we carefully analyze the robustness of our forecasting results. To implement this we use four different lower bound parameter values for both the WX and the K-ANSM shadow rates. We also analyze whether the results remain robust when the sample period used in the estimation of the shadow rates is changed.

The main finding from this study is that the shadow rates are useful leading indicators of U.S. inflation. Shadow rates contain substantial predictive power for inflation both in the non-ZLB and ZLB periods irrespective of which model specification or forecast horizon is considered. We find that the shadow rate suggested by Wu and Xia (2016) produces more accurate inflation forecasts than the shadow rate suggested by Krippner (2015b), probably because it fits better to the yield curve data and thus contains more information. Our results are robust regardless of the lower bound parameter or the estimation period considered. We believe that our results are important for forecasters, central banks and other policymakers, because it has been difficult to find good leading indicators for inflation in the post-1985 period (see, e.g., Stock and Watson 2003; 2007).

The remainder of the paper is organized as follows. Section 2 describes the methodology. Section 3 introduces the data used for the estimations. Section 4 presents the results of the in-sample and out-of-sample forecasting exercises. Section 5 concludes. Appendix A at the end of the paper provides a detailed description of the dataset.

K-ANSM shadow rate and thus does not consider the uncertainty related to shadow rate estimates.

2. Methodology

In this section, we describe the econometric methodologies used in this paper. The purpose of this study is to evaluate whether different shadow rates contain predictive power for U.S. inflation. To this end, we conduct both in-sample and out-of-sample forecasting exercises.

We examine the predictive content of shadow rates in a data-rich environment. We use factor models, because they provide a parsimonious way of dealing with a large set of candidate predictors. The key insight of factor models is that predictor variables are often strongly correlated, and thus, the information encoded in a large number of candidate predictors can be summarized by a handful of unobserved factors. These factors can be consistently estimated by principal components (Stock and Watson, 2002a). Factor models have become popular in macroeconomic analysis in recent years, especially when the focus is on forecasting.[4]

To assess the predictive ability of shadow rates, we estimate the following linear h-step-ahead factor model (henceforth, the shadow rate forecasting model):

$$\pi_{t+h}^{h} = \alpha_h + \sum_{j=1}^{m}\sum_{i=1}^{k}\beta_{hij}\hat{F}_{i,t-j+1} + \sum_{j=1}^{p}\gamma_{hj}\pi_{t-j+1} + \phi_h z_t + \varepsilon_{t+h}^{h}, \tag{1}$$

where the dependent variable and the lagged dependent variable are $\pi_{t+h}^{h} = (1200/h)$ $ln(P_{t+h}/P_t)$ and $\pi_t = 400ln(P_t/P_{t-1})$, respectively, P_t is the price index at month t, $\hat{F}_{i,t}$ is the ith principal component constructed from a large set of predictors, z_t is either the WX or K-ANSM shadow rate, and ε_{t+h}^{h} is the forecast error. The constant term is included in the forecasting regression (1), and the superscripts h indicate that the parameters are horizon specific.

Forecasts of consumer price index (CPI) inflation and personal consumption ex-

[4]For a comprehensive survey of factor models and their use in macroeconomic analysis, see Stock and Watson (2016).

penditures (PCE) inflation are generated for horizons of h = 3, 6, 9, and 12 months. The factors are extracted by principal components, and the forecasting regression (1) is estimated by OLS. The lags of factors m, the number of factors k, and the number of autoregressive lags p of each specification are determined by the Bayesian information criterion (BIC), with $1 \leq m \leq 2$, $1 \leq k \leq 4$, and $0 \leq p \leq 6$. The error term in model (1) is autocorrelated, because the sampling interval is smaller than the forecasting horizon. The MA (h-1) structure of the error term induced by overlapping observations is taken into account by computing Newey and West (1987) HAC standard errors with the lag truncation parameter equal to h - 1.

Furthermore, we are interested in whether the beginning of the ZLB period changed the relationship between the shadow rates and future inflation. To address this question, we create a dummy variable that takes a value of one when the federal funds rate is at the ZLB (2009:M1–2015:M12) and zero otherwise. To study the differences between the non-ZLB and ZLB periods, an interaction term that is the product of the ZLB dummy and a shadow rate is included in the estimation, i.e., we consider a predictive model of the form:

$$\pi^h_{t+h} = \alpha_h + \sum_{j=1}^{m}\sum_{i=1}^{k}\beta_{hij}\hat{F}_{i,t-j+1} + \sum_{j=1}^{p}\gamma_{hj}\pi_{t-j+1} + \phi_h z_t + \psi_h(ZLB_t * z_t) + \varepsilon^h_{t+h}, \quad (2)$$

where ZLB_t denotes the ZLB dummy.

Our main interest lies in the interaction coefficient ψ_h that captures the effects of the ZLB/unconventional monetary policy environment on the relationship between the shadow rates and future inflation. If the coefficient turns out to be insignificant, the relation between the shadow rate and future inflation is similar in both periods. On the other hand, a statistically significant coefficient implies that the relationship has changed since the beginning of the ZLB/unconventional monetary policy period.

We also evaluate the forecasting performance of the shadow rates in a pseudo out-of-sample forecasting exercise. The out-of-sample forecasting period runs from October 1996 to December 2015. The model selection and estimation is recursive as the forecasting exercise proceeds through time. That is, we extract the factors and estimate the parameters of the forecasting model (1) using all available prior data at each forecast origin. The lags of factors m, the number of factors k, and the number of autoregressive lags p are determined by the data, with $1 \leq m \leq 2$, $1 \leq k \leq 4$ and $0 \leq p \leq 6$. At each forecast origin we select the model with the lowest BIC.

We quantify out-of-sample performance by comparing the forecasting accuracy of a shadow rate forecasting model relative to that of a benchmark model. In our framework, natural benchmark models are obtained by excluding the shadow rate from the forecasting model (1). By comparing the accuracy of the model augmented with a shadow rate and the benchmark model, we investigate the marginal predictive power of the shadow rate. The results in the previous literature show that factor models typically produce better macroeconomic forecasts than other popular forecasting models, such as autoregressive or vector autoregressive models (see, e.g., Bernanke and Boivin 2003; Clements 2016; Stock and Watson 2002a,b). As a consequence, factor models provide a stiff benchmark against which to compare the shadow rate forecasting model.

To facilitate comparisons between the shadow rate forecasting model and the benchmark model, we report the results in terms of their relative mean squared forecast error (MSFE), which is the ratio of the MSFE from the shadow rate forecasting model over the MSFE from the benchmark. The values of the relative MSFE below (above) unity indicate that the shadow rate forecasting model produces more (less) accurate forecasts than the benchmark model, implying that the shadow rate contains (does not contain) marginal predictive power. To assess the statistical significance of improvements in forecast accuracy relative to the benchmark model, we employ the one-sided Diebold and Mariano (1995) test (DM henceforth) with the small sample modification proposed by Harvey et al. (1997).[5]

In addition, we report the fraction of observations for which the shadow rate forecasting model produces a smaller absolute forecast error than the benchmark model. This exercise allows us to analyze whether the shadow rate forecasting model qualitatively outperforms the benchmark model. This is an important robustness check. As is well known, the MSFE measure gives more weight to large errors. Therefore, the MSFE results might give a misleading picture of the true predictive power in the presence of a few extreme forecast errors. On the other hand, the sign statistic gives equal weight to each observation and is thus less sensitive to outliers. We test whether the fraction of observations for which the shadow rate forecasting model generates a more accurate forecast is statistically significantly above 0.5 using the sign test developed by Diebold and Mariano (1995).

3. Data

Our data consist of monthly observations of macroeconomic variables and shadow rates from November 1985 to December 2015. We use CPI inflation and PCE inflation from the Federal Reserve Bank of St. Louis FRED Economic database and measure inflation by the annualized rate of inflation.[6] We also use macroeconomic data from the FRED-MD database, which includes 133 macroeconomic variables to create the factors in

[5]Our forecasting models are nested in the sense that the benchmark model is a restricted version of the shadow rate forecasting model (1). The DM test is not designed for nested model comparison. We are aware of the limitations of the DM approach. However, our use of the DM test is a deliberate choice. The Monte Carlo results in Clark and McCracken (2013) indicate that the DM test with the small sample correction suggested by Harvey et al. (1997) provide a good sided test of the null hypothesis of equal finite-sample forecast accuracy even when the models are nested. Faust and Wright (2013) and Groen et al. (2013) also use the DM test when comparing inflation forecasts from nested models.

[6]We have also estimated our results for a CPI ex food and energy inflation series. However, that inflation series varies much less than the other two inflation series that we use. Therefore, it is difficult to find leading indicators for the CPI ex food and energy inflation series that could improve the predictive power of the benchmark model. We leave these results outside of the analysis.

forecasting regressions (1) and (2).[7] The FRED-MD database includes variables from several categories, such as inflation, exchange rates, stock prices, and employment.[8] We estimate four factors from the FRED-MD database by principal components. The first factor captures the variation of real activity and employment variables. The second factor catches forward-looking variables, such as interest rate spreads and inventories. The third factor can be interpreted as an inflation factor, because it associates with price variables. The fourth factor captures the variation of a mix of housing and interest rate variables. Typically BIC chooses a model that contains three factors.

We use 14 different shadow rates estimated from two different shadow (short) rate models:

1. Shadow rate term structure model by Wu and Xia (2016) (WX)
2. Krippner arbitrage-free Nelson and Siegel (1987) model with two state-variables (level and slope) by Krippner (2015b) (K-ANSM)

The WX model has already become a widely used model to summarize the overall stance of monetary policy.[9] It uses three factors to estimate the shadow rates, but the problem with three-factor shadow rate estimates is that they do not correlate well with unconventional monetary policy announcements and sometimes produce counter-intuitive positive values during unconventional monetary policy periods. They are also very sensitive to precise model specification, creating very different magnitudes and profiles for minor changes in the lower bound specification. The problems related to the WX model are discussed in more detail in Krippner (2015a). Krippner (2015a) also shows that shadow rate estimates from a two-factor model are more suitable for

[7]The FRED-MD database consists originally of 134 variables, but we have excluded variable number 64 (New Orders for Consumer Goods) from the original database to get the balanced panel that we need for estimating the principal components. We have also standardized and stationarized the variables so that their sample mean is equal to zero and sample variance is equal to one. See Appendix A for a data description and Figure A1 for factors over time.

[8]See McCracken and Ng (2016) for details of the FRED-MD database.

[9]The WX shadow rate is published at the Federal Reserve Bank of Atlanta website: *https://www.frbatlanta.org/cqer/research/shadow_rate.aspx?panel=1*

describing the stance of monetary policy, because they correlate well with the unconventional monetary policy announcements and are relatively robust, producing similar magnitudes and profiles. The advantage of the WX model is that it fits better to the yield curve data used in estimating the shadow rates, especially at the short end, than the K-ANSM model.

To estimate the WX shadow rates we construct monthly forward rates for maturities of 0.25, 0.5, 1, 2, 5, 7, and 10 years using the Gürkaynak et al. (2007) dataset[10] and observations at the end of the month. Our sample period for the forward rate data is from January 1990 to December 2015. For the period from November 1985 to December 1989 we use the federal funds rate as the WX shadow rate. For K-ANSM shadow rates we use the same Gürkaynak et al. (2007) dataset as before for maturities 1, 2, 3, 5, 7, and 10 years and 3-month and 6-month T-bill rates from the FRED database to calculate the continuously compounding T-bill rates for the short end of the yield curve.[11]

For both shadow rate models we use four different lower bound (LB) parameters and four different estimation periods. The WX shadow rates that we use are LB = 0, 14, 19, and 25 bps, and estimation periods end at December 2013 (Dec-13), April 2014 (Apr-14), December 2014 (Dec-14), and December 2015 (Dec-15). For K-ANSM shadow rates we use LB = 0, 14, 16, and 25 bps and the same estimation periods as for the WX shadow rates.[12] These are chosen according to Krippner (2015a) except that we have replaced the Sep-15 shadow rate with the Dec-15 shadow rate to obtain a full sample for 2015.

Because the Federal Open Market Committee (FOMC) lowered the target range for the federal funds rate to 0–25 bps in December 2008, we refer the period from

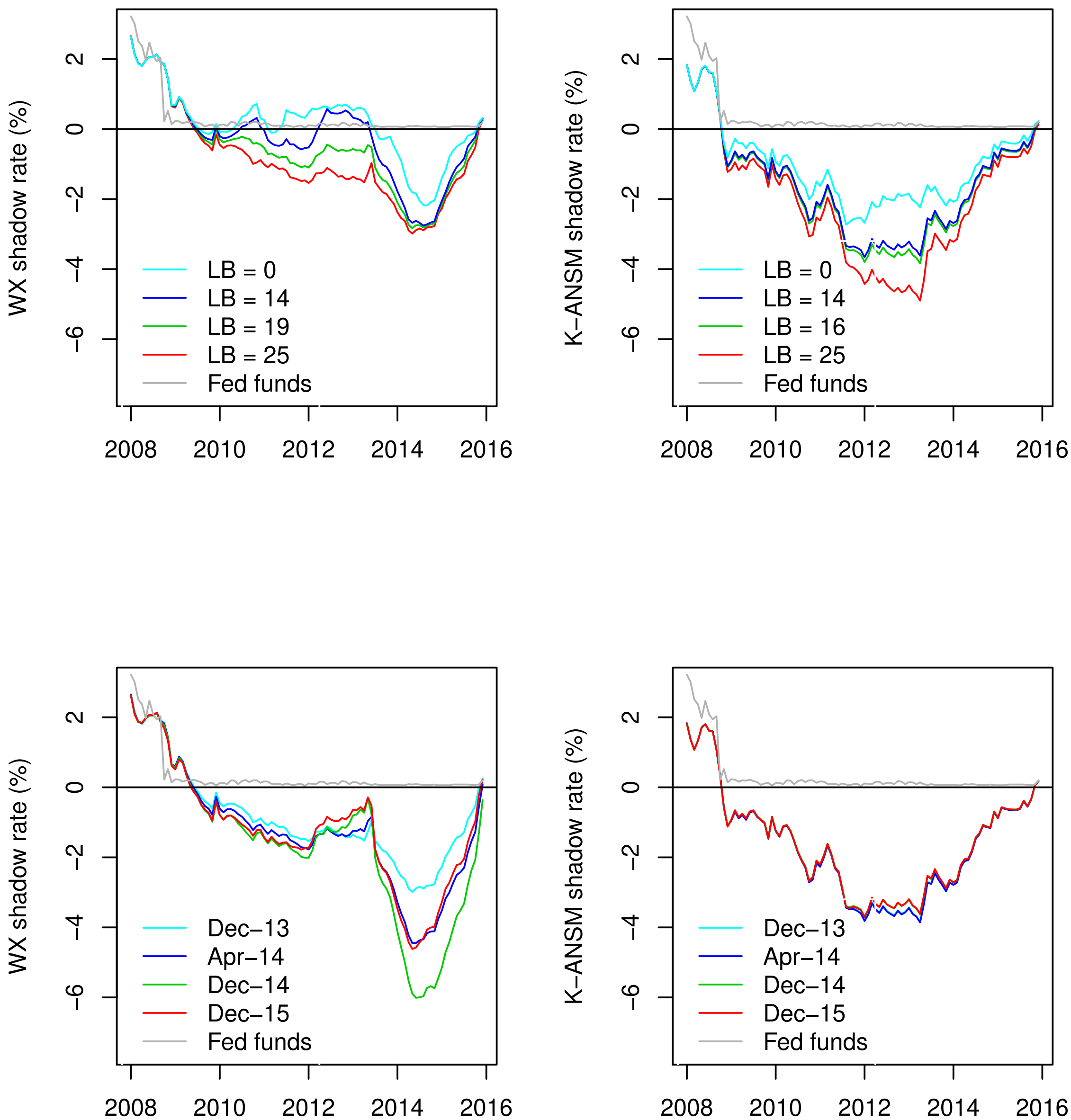
[10] This dataset is available at
*http://www.federalreserve.gov/pubs/feds/*2006/200628/200628*abs.html*

[11] For a similar approach, see Krippner (2015a).

[12] Note that the WX shadow rate with LB = 25 bps is the same as the WX shadow rate with estimation period Dec-13, and the K-ANSM shadow rate LB = 16 bps is the same as the K-ANSM shadow rate with estimation period Dec-13.

January 2009 to the end of our sample as the ZLB period. The shadow rates are highly correlated and receive almost identical values in the non-ZLB period. Therefore Figure 1 plots the shadow rates merely over the 2008:1–2015:M12 period. An inspection of Figure 1 reveals that the shadow rate models are very sensitive to the LB parameters and data used for the estimation. Figure 1 also shows that the WX shadow rates are more sensitive than the K-ANSM shadow rates.

Figure 1: The shadow rates and federal funds rate



Notes: The estimated WX and K-ANSM shadow rates and the federal funds rate from 2008:M1 to 2015:M12 in percentage points.

4. Empirical results

This section presents the results of the in-sample and out-of-sample forecasting exercises.

4.1. In-sample analysis

In this section we examine the in-sample predictive power of shadow rates for U.S. inflation. First, we consider the estimation results for the forecasting regression (1), which are presented in Tables 1 and 2. We present the results for shadow rates with different values of the LB parameter and for different estimation periods.

Tables 1 and 2 show that the relationship between the shadow rates and future inflation is positive for all 14 shadow rates and for all four forecasting horizons.[13] This finding indicates that as the shadow rate decreases the future inflation also decreases. All the shadow rate coefficients are statistically significant at the 1% level of significance. The LB specification or selection of the estimation period does not seem to affect the results much.

The coefficients and adjusted R^2s of the WX shadow rates are typically larger than the coefficients and adjusted R^2s of the K-ANSM shadow rates, especially when the forecasting horizon is 9 or 12 months. These findings suggest that the WX shadow rates perform better than the K-ANSM shadow rates in forecasting future inflation. The LB parameter of 14 bps for the WX shadow rate and LB parameter of 0 bps for the K-ANSM shadow rate give the highest adjusted R^2s, which gives evidence that these LB specifications could be the best predictors of inflation. The coefficient typically increases as the forecasting horizon becomes longer, which supports the argument that monetary policy affects inflation over a longer period of time. The adjusted explanation ratios are higher for CPI inflation than for PCE inflation when the forecasting horizon

[13] Correlation between the shadow rates and future inflation is also positive.

Table 1: In-sample predictive regressions

		$h=3$		$h=6$		$h=9$		$h=12$	
		ϕ_3	AdjR^2	ϕ_6	AdjR^2	ϕ_9	AdjR^2	ϕ_{12}	AdjR^2
CPI inflation									
WX									
	$LB=25$	0.249***	0.160	0.252***	0.228	0.271***	0.322	0.284***	0.403
	$LB=19$	0.257***	0.162	0.261***	0.231	0.281***	0.326	0.293***	0.407
	$LB=14$	0.266***	0.163	0.270***	0.233	0.289***	0.328	0.301***	0.408
	$LB=0$	0.275***	0.162	0.279***	0.231	0.260***	0.306	0.310***	0.410
K-ANSM									
	$LB=25$	0.221***	0.151	0.212***	0.202	0.198***	0.263	0.228***	0.345
	$LB=16$	0.238***	0.153	0.229***	0.206	0.211***	0.268	0.245***	0.352
	$LB=14$	0.242***	0.154	0.232***	0.207	0.214***	0.269	0.248***	0.353
	$LB=0$	0.266***	0.158	0.256***	0.213	0.233***	0.277	0.271***	0.362
PCE inflation									
WX									
	$LB=25$	0.202***	0.189	0.200***	0.228	0.209***	0.288	0.205***	0.327
	$LB=19$	0.209***	0.191	0.207***	0.230	0.216***	0.292	0.212***	0.331
	$LB=14$	0.215***	0.192	0.213***	0.233	0.223***	0.294	0.216***	0.342
	$LB=0$	0.223***	0.191	0.222***	0.231	0.231***	0.291	0.224***	0.340
K-ANSM									
	$LB=25$	0.142***	0.195	0.123***	0.183	0.131***	0.229	0.134***	0.259
	$LB=16$	0.153***	0.197	0.132***	0.186	0.140***	0.233	0.178***	0.273
	$LB=14$	0.156***	0.197	0.134***	0.186	0.141***	0.234	0.180***	0.274
	$LB=0$	0.172***	0.200	0.147***	0.191	0.155***	0.240	0.198***	0.282

Notes: The in-sample forecasting period runs from 1985:M11 to 2015:M12. Asterisks denote statistical significance at the 1% (***), 5% (**), and 10% (*) levels, respectively.

is six months or longer.

The estimation results of the forecasting regression (2) are presented in Tables 3 and 4. Now we have added an interaction term to the forecasting regression to see if the relationship between the shadow rates and future inflation is different between the non-ZLB and ZLB periods. Table 3 shows that the coefficients of the interaction terms for the WX shadow rates are positive and typically statistically significant when the forecasting horizon is six months or longer. This finding indicates that the relationship between the WX shadow rate and future inflation is positive in both periods, but the relationship tends to be stronger in the ZLB period when the Fed conducts unconventional monetary policies. The coefficients of the interaction term for the K-ANSM shadow rates are typically statistically insignificant, indicating that the relationship between the shadow rate and future inflation is similar in both periods.

One possible explanation for our results is the forward-looking nature of monetary policy.[14] The positive relationship between the shadow rates and future inflation arises probably because the Fed anticipates lower future inflation and reacts to this new information by decreasing the policy rate (or by using unconventional monetary policies in the ZLB period). That is, we observe the shadow rate decreasing, because the Fed predicts that inflation will decrease in the future.[15] Monetary policy affects inflation with a long lag. Thus, the impacts of (unconventional) monetary policy actions on inflation probably do not show entirely within a short time period.[16] Due to this, the short-term (3–12 months) inflation can fall even though the Fed pursues an accommodative monetary policy. The reverse causality from inflation to the shadow rate causes a positive correlation between these two variables in the short term.

[14]Rearranging the variables in our forecasting regression gives us an expression that looks like a forward-looking Taylor rule.

[15]Our results refer to the price puzzle (see, e.g., Sims 1992). Wu and Xia (2016) use the FAVAR model and obtain results similar to ours for the relationship between the policy rate and future inflation.

[16]Typically monetary policy is designed to have impact on inflation over a longer time period, such as 1–2 years.

Table 2: In-sample predictive regressions (re-estimated shadow rates)

		$h=3$		$h=6$		$h=9$		$h=12$	
		ϕ_3	AdjR^2	ϕ_6	AdjR^2	ϕ_9	AdjR^2	ϕ_{12}	AdjR^2
CPI inflation									
WX ($LB=25$)									
	Dec-13	0.249***	0.160	0.252***	0.228	0.271***	0.322	0.284***	0.403
	Apr-14	0.240***	0.164	0.244***	0.236	0.261***	0.332	0.273***	0.414
	Dec-14	0.228***	0.167	0.231***	0.240	0.246***	0.336	0.256***	0.419
	Dec-15	0.240***	0.162	0.243***	0.232	0.262***	0.328	0.274***	0.410
K-ANSM ($LB=est.$)									
	Dec-13	0.238***	0.153	0.229***	0.206	0.211***	0.268	0.245***	0.352
	Apr-14	0.238***	0.153	0.228***	0.206	0.210***	0.268	0.244***	0.352
	Dec-14	0.241***	0.154	0.232***	0.207	0.213***	0.269	0.248***	0.353
	Dec-15	0.241***	0.154	0.231***	0.207	0.213***	0.269	0.248***	0.353
PCE inflation									
WX ($LB=25$)									
	Dec-13	0.202***	0.189	0.200***	0.228	0.209***	0.288	0.205***	0.327
	Apr-14	0.194***	0.193	0.192***	0.234	0.201***	0.296	0.196***	0.335
	Dec-14	0.183***	0.195	0.181***	0.237	0.188***	0.299	0.184***	0.338
	Dec-15	0.193***	0.190	0.192***	0.230	0.201***	0.292	0.197***	0.331
K-ANSM ($LB=est.$)									
	Dec-13	0.153***	0.197	0.132***	0.186	0.140***	0.233	0.178***	0.273
	Apr-14	0.153***	0.197	0.131***	0.186	0.139***	0.233	0.177***	0.273
	Dec-14	0.155***	0.197	0.133***	0.186	0.141***	0.234	0.180***	0.274
	Dec-15	0.155***	0.197	0.133***	0.186	0.141***	0.234	0.180***	0.274

Notes: The in-sample forecasting period runs from 1985:M11 to 2015:M12. Asterisks denote statistical significance at the 1% (***), 5% (**), and 10% (*) levels, respectively.

Table 3: In-sample predictive regressions with interaction terms

		$h = 3$			$h = 6$			$h = 9$			$h = 12$		
		ϕ_3	ψ_3	AdjR^2	ϕ_6	ψ_6	AdjR^2	ϕ_9	ψ_9	AdjR^2	ϕ_{12}	ψ_{12}	AdjR^2
CPI inflation													
WX													
	$LB = 25$	0.195**	0.351	0.163	0.196**	0.362	0.236	0.219***	0.343	0.332	0.233***	0.334	0.415
	$LB = 19$	0.193**	0.498	0.168	0.194**	0.522*	0.246	0.217***	0.496**	0.345	0.233***	0.473**	0.429
	$LB = 14$	0.207***	0.541	0.171	0.208***	0.563**	0.251	0.233***	0.511**	0.349	0.251***	0.464**	0.430
	$LB = 0$	0.225***	0.799*	0.174	0.226***	0.842***	0.259	0.235***	0.771***	0.341	0.275***	0.583***	0.433
K-ANSM													
	$LB = 25$	0.326***	-0.375*	0.163	0.311***	-0.354*	0.222	0.296***	-0.342*	0.289	0.304***	-0.270	0.366
	$LB = 16$	0.327***	-0.398	0.163	0.313***	-0.375*	0.222	0.299***	-0.361*	0.290	0.305***	-0.268	0.366
	$LB = 14$	0.328***	-0.405	0.163	0.313***	-0.381*	0.222	0.299***	-0.366*	0.290	0.305***	-0.266	0.366
	$LB = 0$	0.329***	-0.460	0.163	0.315***	-0.430	0.223	0.301***	-0.406	0.291	0.306***	-0.248	0.367
PCE inflation													
WX													
	$LB = 25$	0.173***	0.194	0.190	0.167***	0.211	0.231	0.177***	0.209	0.293	0.174***	0.205	0.333
	$LB = 19$	0.172***	0.287	0.194	0.165***	0.320	0.239	0.175***	0.319*	0.304	0.173***	0.306*	0.344
	$LB = 14$	0.181***	0.311	0.196	0.175***	0.350*	0.244	0.186***	0.337**	0.308	0.188***	0.261*	0.351
	$LB = 0$	0.192***	0.498*	0.200	0.187***	0.559***	0.251	0.199***	0.504***	0.313	0.200***	0.398***	0.356
K-ANSM													
	$LB = 25$	0.211***	-0.236	0.203	0.198***	-0.254*	0.200	0.202***	-0.242	0.250	0.194***	-0.204	0.276
	$LB = 16$	0.212***	-0.249	0.203	0.200***	-0.277	0.200	0.204***	-0.259	0.250	0.226***	-0.214	0.287
	$LB = 14$	0.212***	-0.253	0.203	0.201***	-0.283	0.201	0.204***	-0.263	0.251	0.226***	-0.214	0.287
	$LB = 0$	0.214***	-0.285	0.204	0.204***	-0.336	0.201	0.206***	-0.301	0.252	0.227***	-0.211	0.288

Notes: The in-sample forecasting period runs from 1985:M11 to 2015:M12. Asterisks denote statistical significance at the 1% (***), 5% (**), and 10% (*) levels, respectively.

Table 4: In-sample predictive regressions with interaction terms (re-estimated shadow rates)

		$h = 3$			$h = 6$			$h = 9$			$h = 12$		
		ϕ_3	ψ_3	AdjR^2	ϕ_6	ψ_6	AdjR^2	ϕ_9	ψ_9	AdjR^2	ϕ_{12}	ψ_{12}	AdjR^2
CPI inflation													
WX ($LB = 25$)													
	Dec-13	0.195**	0.351	0.163	0.196**	0.362	0.236	0.219***	0.343	0.332	0.233***	0.334	0.415
	Apr-14	0.192**	0.222	0.166	0.192**	0.235	0.241	0.215***	0.209	0.337	0.230***	0.192	0.421
	Dec-14	0.194**	0.116	0.166	0.195***	0.122	0.241	0.218***	0.094	0.337	0.234***	0.076	0.419
	Dec-15	0.204**	0.167	0.162	0.202***	0.193	0.235	0.224***	0.181	0.332	0.238***	0.168	0.415
K-ANSM ($LB = est.$)													
	Dec-13	0.327***	-0.398	0.163	0.313***	-0.375*	0.222	0.299***	-0.361*	0.290	0.305***	-0.268	0.366
	Apr-14	0.327***	-0.398*	0.163	0.313***	-0.375*	0.222	0.299***	-0.361*	0.290	0.305***	-0.267	0.366
	Dec-14	0.328***	-0.404	0.163	0.313***	-0.380*	0.222	0.299***	-0.366*	0.290	0.305***	-0.266	0.366
	Dec-15	0.327***	-0.404	0.163	0.313***	-0.381*	0.222	0.299***	-0.366*	0.290	0.305***	-0.266	0.366
PCE inflation													
WX ($LB = 25$)													
	Dec-13	0.173***	0.194	0.190	0.167***	0.211	0.231	0.177***	0.209	0.293	0.174***	0.205	0.333
	Apr-14	0.169***	0.113	0.192	0.163***	0.132	0.236	0.173***	0.123	0.298	0.170***	0.117	0.338
	Dec-14	0.170***	0.045	0.193	0.164***	0.058	0.236	0.175***	0.047	0.298	0.172***	0.041	0.337
	Dec-15	0.177***	0.073	0.188	0.170***	0.102	0.230	0.179***	0.102	0.293	0.175***	0.101	0.333
K-ANSM ($LB = est.$)													
	Dec-13	0.212***	-0.249	0.203	0.200***	-0.277	0.200	0.204***	-0.259	0.250	0.226***	-0.214	0.287
	Apr-14	0.212***	-0.249	0.203	0.200***	-0.276	0.200	0.204***	-0.259	0.250	0.226***	-0.214	0.287
	Dec-14	0.212***	-0.252	0.203	0.201***	-0.282	0.200	0.204***	-0.263	0.250	0.226***	-0.213	0.287
	Dec-15	0.212***	-0.252	0.203	0.201***	-0.282	0.200	0.204***	-0.263	0.250	0.226***	-0.213	0.287

Notes: The in-sample forecasting period runs from 1985:M11 to 2015:M12. Asterisks denote statistical significance at the 1% (***), 5% (**), and 10% (*) levels, respectively.

4.2. Out-of-sample forecasting results

Next, we present the results of the out-of-sample forecasting exercise outlined in Section 2. The aim of this exercise is to analyze (i) whether the shadow rates contain predictive power for U.S. inflation in a data-rich environment, (ii) whether the choice of the LB parameter matters for the predictive ability of the shadow rates, and (iii) whether the results remain robust when the sample period used in the estimation of the shadow rates is changed.

We start our analysis by considering the shadow rates estimated with data from January 1990 to December 2013 (i.e., the shadow rates in the upper panel of Figure 1). The MSFE results for the whole 1996:M10–2015:M12 out-of-sample period are summarized in Table 5. This table shows the MSFE value of the shadow rate forecasting model relative to the MSFE value of the benchmark model.[17] Values below (above) unity indicate that the model augmented with a shadow rate has produced more (less) accurate forecasts than the benchmark model, implying that the shadow rate contains (does not contain) marginal predictive power. The statistical significance is evaluated using the one-sided DM (1995) test with the small sample modification proposed by Harvey et al. (1997).

Three main results emerge from Table 5. First, the relative MSFE values are below one, indicating that the models augmented with the shadow rates produce more accurate inflation forecasts than the benchmark models. The improvements in forecast accuracy are typically large (up to 30%) and statistically significant for the WX shadow rates. This result holds irrespective of which shadow rate or forecast horizon is considered. Therefore, both the WX and K-ANSM shadow rates contain predictive power for U.S. CPI and PCE inflation when the predictive information encoded in a

[17] The results in the previous literature suggest that a random walk model performs well in inflation forecasting exercises (see, e.g., Atkeson and Ohanian 2001). As a consequence, the random walk model is seen as a hard-to-beat benchmark model. In unreported results, we find that the random walk model produces less accurate inflation forecasts than the factor model in our out-of-sample period. For this reason, we use the factor model as a benchmark model in the out-of-sample forecasting exercise.

large number of macroeconomic variables is already taken into account. This is an important finding, because the results in the previous literature suggest that it is difficult to find good leading indicators for inflation in the post-1985 period (see, e.g., Stock and Watson 2007). Second, the WX shadow rates produce better out-of-sample forecasts than the K-ANSM shadow rates. The relative MSFE values for the WX shadow rates are lower than those for the K-ANSM shadow rates, sometimes by quite a substantial margin. Indeed, for all dependent variable/forecast horizon combinations, the worst performing WX shadow rate outperforms the most accurate K-ANSM shadow rate. Third, the choice of the LB parameter does not matter much for the out-of-sample forecasting performance. The relative MSFE values for all LB parameters are quite similar. Thus, the best performing LB parameter makes only a very slight improvement over the alternatives. The results suggest that the LB parameter of 14 bps performs the best for the WX shadow rate, whereas the K-ANSM shadow rate with the LB parameter of 0 bps yields the most accurate forecasts (cf. the in-sample results in Section 4.1.).

Table 5 focuses on the average predictive power over the whole out-of-sample period. However, the purpose of this study is to examine whether the shadow rates contain predictive power in the recent ZLB/unconventional monetary policy era. To shed light on this question, we divide the out-of-sample period into two parts. The non-ZLB period runs from 1996:M10 to 2008:M12, and the ZLB period runs from 2009:M1 to 2015:M12. The results for these two subperiods are reported in tables 6 and 7, respectively.

The results in Table 6 show that the WX and K-ANSM shadow rates have predictive power for inflation in the non-ZLB period. The relative MSFE values are below one for both measures of inflation regardless of which forecast horizon is considered. However, the improvements in forecast accuracy are smaller than those reported in Table 5, and

Table 5: Relative out-of-sample MSFE values

		$h=3$	$h=6$	$h=9$	$h=12$
CPI inflation					
WX					
	$LB=25$	0.945	0.865*	0.757*	0.709*
	$LB=19$	0.943	0.861*	0.750*	0.701**
	$LB=14$	0.941*	0.857*	0.744**	0.696**
	$LB=0$	0.943*	0.853*	0.747**	0.707**
K-ANSM					
	$LB=25$	0.982	0.918	0.847	0.880
	$LB=16$	0.978	0.914	0.838	0.862
	$LB=14$	0.977	0.912	0.836	0.859
	$LB=0$	0.970	0.900	0.822	0.838
PCE inflation					
WX					
	$LB=25$	0.926*	0.903*	0.846	0.830
	$LB=19$	0.924*	0.899*	0.842*	0.824
	$LB=14$	0.922*	0.896*	0.837*	0.819
	$LB=0$	0.922**	0.896*	0.835*	0.824*
K-ANSM					
	$LB=25$	0.990	0.971	0.935	0.890
	$LB=16$	0.984	0.968	0.926	0.883
	$LB=14$	0.983	0.966	0.923	0.882
	$LB=0$	0.974	0.962	0.910	0.870

Notes: The out-of-sample forecasting period runs from 1996:M10 to 2015:M12. Each row reports the ratio of the MSFE of the shadow rate forecasting model to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test with the small sample modification by Harvey et al. (1997) at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

Table 6: Relative out-of-sample MSFE values for the non-ZLB period

		$h=3$	$h=6$	$h=9$	$h=12$
CPI inflation					
WX					
	$LB=25$	0.989	0.924	0.865	0.829
	$LB=19$	0.989	0.924	0.865	0.830
	$LB=14$	0.989	0.924	0.865	0.830
	$LB=0$	0.989	0.924	0.865	0.830
K-ANSM					
	$LB=25$	0.994	0.912	0.870	0.888
	$LB=16$	0.994	0.912	0.869	0.887
	$LB=14$	0.994	0.912	0.869	0.887
	$LB=0$	0.993	0.909	0.869	0.886
PCE inflation					
WX					
	$LB=25$	0.968	0.960	0.935	0.925
	$LB=19$	0.968	0.960	0.935	0.925
	$LB=14$	0.968	0.960	0.935	0.925
	$LB=0$	0.968	0.959	0.935	0.924
K-ANSM					
	$LB=25$	0.999	0.976	0.970	0.943
	$LB=16$	0.999	0.980	0.969	0.942
	$LB=14$	0.999	0.980	0.969	0.942
	$LB=0$	0.999	0.979	0.967	0.940

Notes: The out-of-sample forecasting period runs from 1996:M10 to 2008:M12. Each row reports the ratio of the MSFE of the shadow rate forecasting model relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test with the small sample modification by Harvey et al. (1997) at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

Table 7: Relative out-of-sample MSFE values for the ZLB period

		$h=3$	$h=6$	$h=9$	$h=12$
CPI inflation					
WX					
	$LB=25$	0.813*	0.716	0.550	0.436
	$LB=19$	0.805*	0.700	0.527	0.408
	$LB=14$	0.796**	0.686	0.510	0.394
	$LB=0$	0.801**	0.677*	0.522	0.430
K-ANSM					
	$LB=25$	0.949	0.934	0.803	0.861
	$LB=16$	0.932	0.918	0.775	0.804
	$LB=14$	0.928	0.913	0.769	0.794
	$LB=0$	0.900	0.877	0.728	0.728
PCE inflation					
WX					
	$LB=25$	0.796*	0.739	0.641	0.489
	$LB=19$	0.787*	0.724	0.624	0.459
	$LB=14$	0.778*	0.709	0.607	0.437
	$LB=0$	0.779*	0.712	0.607	0.463
K-ANSM					
	$LB=25$	0.961	0.955	0.857	0.735
	$LB=16$	0.935	0.933	0.830	0.709
	$LB=14$	0.931	0.928	0.823	0.703
	$LB=0$	0.896	0.911	0.785	0.662

Notes: The out-of-sample forecasting period runs from 2009:M1 to 2015:M12. Each row reports the ratio of the MSFE of the shadow rate forecasting model relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test with the small sample modification by Harvey et al. (1997) at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

they are never statistically significant.[18] The predictive ability of the shadow rates seem to be similar. There is a simple explanation for this finding. As discussed in Section 3, shadow rates are constructed such that they are strongly correlated and display similar properties in the non-ZLB period. As a consequence, the WX and K-ANSM shadow rates perform almost equally well in the non-ZLB period. For this reason, in what follows we save space and focus exclusively on the ZLB period.

An examination of Table 7 leads us to a number of important observations regarding the predictive ability of shadow rates. The most important finding is that both the WX and K-ANSM shadow rates contain substantial predictive power for CPI and PCE inflation in the ZLB period. The models augmented with the shadow rates produce systematically smaller MSFE values than the benchmark forecasting models in all

[18]One possible explanation for this finding is related to the length of the forecasting period. The non-ZLB period is much shorter than the whole out-of-sample period. Thus, the DM (1995) test has less power to reject the null of equal forecast accuracy in the non-ZLB period.

cases.[19] The improvements in forecast accuracy are especially large (up to 60%) at longer forecast horizons ($h = 9$ and 12). Despite the large differences in the predictive ability, the DM (1995) test rejects the null of equal forecast accuracy at conventional significance levels only for the WX shadow rates at the shortest $h = 3$ horizon.[20] Broadly speaking, these results support the conclusion of Wu and Xia (2016) that shadow rates contain useful information about the state of the economy when the short-term rates are stuck at the ZLB.[21]

Another important finding from Table 7 is that the WX shadow rates produce more accurate inflation forecasts than the K-ANSM shadow rates in the ZLB/unconventional monetary policy environment. This result is a bit surprising. Krippner (2015a) shows that the K-ANSM shadow rates are better suited for monitoring the stance of unconventional monetary policy than the WX shadow rates. Furthermore, he finds that the WX shadow rates are sometimes counterintuitive relative to the evolution of major unconventional monetary policy events.[22] Our results convey that, although the WX shadow rates are not always well correlated with unconventional monetary policy events, they are more informative about future inflation than the K-ANSM shadow rates.

[19] Stock and Watson (2007) point out that the U.S. inflation has been much less volatile in the post-1985 period. Thus, inflation forecasting has become more difficult, because it is harder to improve upon simple benchmark models, such as the AR model, in the post-1985 period. In our data, the inflation variance is substantially smaller in the ZLB period than in the non-ZLB period. Nevertheless, the shadow rate forecasting models outperform the benchmark models in both periods.

[20] The ZLB period is relatively short, and thus the DM (1995) test might have low power against the null of equal forecast accuracy.

[21] As discussed in Rossi (2013), different estimation windows may lead to different out-of-sample results. We check the robustness of our results by estimating the parameters of the forecasting models using a rolling window of 120 observations. The results of this sensitivity analysis by and large confirm our main findings. In particular, all shadow rates contain substantial predictive power at longer horizons ($h = 9$ and 12). The most notable difference between the rolling window and expanding window results is that the shadow rates do not contain predictive power at the two shortest horizons when the rolling window is used. A careful examination of the results reveals that forecast accuracy deteriorates substantially at $h = 3$ and $h = 6$ when the rolling window estimator is used, implying that the rolling window results are less reliable than those for the expanding window estimator. For this reason, we report the results for the expanding window estimator only.

[22] Note that our intention is not to analyze the consistency of the shadow rates with unconventional monetary policy events. Rather, we investigate whether the shadow rates are useful leading indicators for inflation in a data-rich environment.

Table 8: Relative out-of-sample MSFE values for the ZLB period (re-estimated shadow rates)

		$h=3$	$h=6$	$h=9$	$h=12$
CPI inflation					
WX ($LB=25$)					
	Dec-13	0.813*	0.716	0.550	0.436
	Apr-14	0.796*	0.687	0.518	0.386
	Dec-14	0.787*	0.670	0.500	0.367
	Dec-15	0.806*	0.701	0.534	0.412
K-ANSM ($LB=est.$)					
	Dec-13	0.932	0.918	0.775	0.804
	Apr-14	0.932	0.919	0.775	0.805
	Dec-14	0.929	0.914	0.770	0.796
	Dec-15	0.929	0.914	0.770	0.797
PCE inflation					
WX ($LB=25$)					
	Dec-13	0.796*	0.739	0.641	0.489
	Apr-14	0.780*	0.704	0.615	0.444
	Dec-14	0.772*	0.690	0.593	0.421
	Dec-15	0.793*	0.718	0.622	0.489
K-ANSM ($LB=est.$)					
	Dec-13	0.935	0.933	0.830	0.709
	Apr-14	0.937	0.934	0.831	0.710
	Dec-14	0.932	0.929	0.824	0.704
	Dec-15	0.932	0.929	0.824	0.705

Notes: The out-of-sample forecasting period runs from 2009:M1 to 2015:M12. Each row reports the ratio of the MSFE of the shadow rate forecasting model relative to the MSFE of the benchmark model. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test with the small sample modification by Harvey et al. (1997) at the 1% (***), 5% (**), and 10% (*) significance levels, respectively.

The results in Table 7 also show that the choice of the LB parameter does not have a large effect on the forecasting performance of the shadow rates. The relative MSFE values for the WX shadow rate are quite similar for all four LB parameters considered in this study. Interestingly, the LB parameter seems to be somewhat more important for the predictive power of the K-ANSM shadow rate. Consistent with the in-sample results and the results in Table 5, the WX shadow rate with the LB parameter of 14 bps outperforms the alternatives, whereas the K-ANSM shadow rate with the LB parameter of 0 bps generates the best forecasts. These findings are intriguing. The previous literature emphasizes that the LB parameter has a critical influence on the shadow rate estimates. The sensitivity of the shadow rate estimates to the LB parameter has been discussed, inter alia, in Christensen and Rudebusch (2015), Bauer and Rudebusch (2016), and Krippner (2015a,b). This literature has shown that the WX shadow rate estimates are particularly sensitive to the LB parameter. On the other hand, the K-ANSM shadow rates have been found to be relatively robust. We conclude from the evidence in Table 7 that when the purpose is to forecast future inflation, the choice of the LB parameter does not matter much for the forecast accuracy of the shadow rates.

As a sensitivity check, we repeat the above analysis using shadow rates estimated with alternative sample periods. We consider the WX shadow rate with the LB parameter of 25 bps and the K-ANSM shadow rate with the estimated LB parameter ($LB = 16$), given their prevalent use in the extant literature. By comparing the forecasting performance of the shadow rates estimated with different data samples, we are able to study whether the results in Table 7 are dependent on the specific data period used in the estimation of the shadow rates. We think that this is a highly relevant exercise. Krippner (2015a) demonstrates that the WX shadow rates re-estimated with updated samples have different profiles over time. In contrast, he shows that the K-ANSM shadow rate estimates are not sensitive to the choice of data sample used in the estimation (cf. Figure 1).

The results of this sensitivity analysis, reported in Table 8, reveal that the estimation sample does not matter much for the predictive power of the shadow rates. Both the WX and K-ANSM shadow rates contain incremental predictive information in the ZLB period regardless of which data sample is used in the estimation of the shadow rates. In fact, the K-ANSM shadow rates perform almost identically in the forecasting exercise. As one might expect based on the discussion in Krippner (2015a), the choice of the estimation sample is more important for the WX shadow rate. Still, the differences in the predictive ability are relatively small. The WX shadow rates estimated with different data samples are very successful at predicting inflation in the ZLB period, and they all dominate the best K-ANSM shadow rate.

We proceed by analyzing whether the model augmented with a shadow rate qualitatively outperforms the benchmark model. To this end, Table 9 reports the fraction

Table 9: Qualitative differences in predictive ability over the ZLB period

		$h = 3$	$h = 6$	$h = 9$	$h = 12$
CPI inflation					
WX					
	$LB = 25$	0.631***	0.687***	0.725***	0.779***
	$LB = 19$	0.655***	0.687***	0.738***	0.779***
	$LB = 14$	0.667***	0.687***	0.750***	0.779***
	$LB = 0$	0.679***	0.711***	0.750***	0.805***
K-ANSM					
	$LB = 25$	0.607**	0.675***	0.662***	0.597**
	$LB = 16$	0.607**	0.675***	0.675***	0.636***
	$LB = 14$	0.607**	0.675***	0.675***	0.636***
	$LB = 0$	0.607**	0.675***	0.688***	0.649***
PCE inflation					
WX					
	$LB = 25$	0.607**	0.627**	0.700***	0.701***
	$LB = 19$	0.619**	0.639***	0.712***	0.714***
	$LB = 14$	0.643***	0.675***	0.712***	0.727***
	$LB = 0$	0.655***	0.687***	0.738***	0.727***
K-ANSM					
	$LB = 25$	0.607**	0.663***	0.662***	0.688***
	$LB = 16$	0.607**	0.675***	0.675***	0.688***
	$LB = 14$	0.607**	0.675***	0.675***	0.701***
	$LB = 0$	0.607**	0.663***	0.700***	0.701***

Notes: The out-of-sample forecasting period runs from 2009:M1 to 2015:M12. Each row reports the fraction of observations for which the shadow rate forecasting model produces more accurate out-of-sample forecasts than the benchmark model. Asterisks mark rejection of the Diebold and Mariano (1995) sign test at the 1%(***), 5%(**), and 10%(*) significance levels, respectively.

of observations for which the model with a candidate shadow rate generates a smaller absolute forecast error than the benchmark model. We test the statistical significance using the Diebold and Mariano (1995) sign test. The results imply that the forecasts from the models including a shadow rate are qualitatively superior to those from the benchmark models. The shadow rate forecasting models provide more accurate forecasts for more than 50% of the observations for all dependent variable/forecast horizon combinations. The differences in the forecasting accuracy are statistically significant. This finding provides further evidence supporting the view that the WX and K-ANSM shadow rates contain predictive power for U.S. inflation in the ZLB period when the predictive information encoded in a large set of macroeconomic variables is already taken into account. The WX shadow rates typically perform a bit better than the K-ANSM shadow rates when we quantify out-of-sample forecast accuracy with a qualitative measure. Consistent with the results in Table 7, the choice of the LB parameter

Table 10: Out-of-sample performance of the WX versus the K-ANSM shadow rates

		$h=3$	$h=6$	$h=9$	$h=12$
CPI inflation					
Non-ZLB					
	$LB=25$	0.997	0.986	0.959	0.947
	$LB=16$	0.997	0.986	0.959	0.948
	$LB=14$	0.997	0.986	0.959	0.948
	$LB=0$	0.996	0.989	0.960	0.949
ZLB					
	$LB=25$	0.841***	0.768**	0.685**	0.522**
	$LB=16$	0.856***	0.788**	0.710**	0.558**
	$LB=14$	0.860***	0.793**	0.715**	0.565**
	$LB=0$	0.890***	0.825**	0.756**	0.616**
PCE inflation					
Non-ZLB					
	$LB=25$	0.994	0.969	0.960**	0.927*
	$LB=16$	0.994	0.966	0.961**	0.927*
	$LB=14$	0.994	0.966	0.961**	0.927*
	$LB=0$	0.995	0.966	0.963**	0.928*
ZLB					
	$LB=25$	0.849**	0.759**	0.709**	0.513**
	$LB=16$	0.865**	0.777**	0.731**	0.534**
	$LB=14$	0.869**	0.782**	0.735**	0.539**
	$LB=0$	0.891**	0.804**	0.768**	0.572**

Notes: Each row reports the ratio of the MSFE of the shadow rate forecasting model augmented with the WX shadow rate ($LB=25$) relative to the MSFE of the shadow rate forecasting model augmented with the K-ANSM shadow rates. Asterisks mark rejection of the one-sided Diebold and Mariano (1995) test with the small sample modification by Harvey et al. (1997) at the 1% (***), 5% (**), and 10% (*) significance levels, respectively. The forecasting periods are as defined before.

has only a relatively small effect on the predictive power of the shadow rates.[23]

As a final exercise, we formally compare the relative out-of-sample forecasting performance of the WX and K-ANSM shadow rates in Table 10. To save space, we report the ratio of the MSFE of the shadow rate forecasting model augmented with the WX shadow rate ($LB = 25$) relative to the MSFE of a forecasting model augmented with different K-ANSM shadow rates ($LB = 0, 14, 16, 25$). The results for the other WX shadow rate specifications, shadow rates estimated with different data samples, and qualitative differences in out-of-sample performance yield very similar conclusions.

The results in Table 10 show that the WX shadow rate produces better out-of-

[23] As a robustness check, we investigate whether the choice of the estimation sample matters for the qualitative forecasting performance of the shadow rates. The results of this sensitivity analysis corroborate the findings in Tables 8 and 9. Again, both shadow rates contain predictive power for inflation in the ZLB/unconventional monetary policy environment. The WX shadow rates are typically slightly more informative about subsequent inflation than the K-ANSM shadow rates. Most importantly, we find that the choice of the data sample used in the estimation of the shadow rates plays only a minor role for the predictive power of the shadow rates.

sample inflation forecasts than the K-ANSM shadow rates. The WX shadow rate dominates the K-ANSM shadow rates in both periods irrespective of which forecast horizon is considered. As one might expect, the differences in forecast accuracy are modest in the non-ZLB period. However, in the ZLB period, the WX shadow rate produces substantially more accurate forecasts than the K-ANSM shadow rates. The differences in forecast accuracy are statistically significant in the ZLB period. These results provide further evidence supporting the findings in Hännikäinen (2017).

One possible explanation for the relative forecasting performance of the shadow rates is related to the way they are estimated from the yield curve data. The K-ANSM shadow rates are estimated from a two-factor model (level and slope), whereas the WX shadow rates are estimated using a three-factor term structure model (level, slope, and curvature). Although the two-factor model generates more robust shadow rate estimates than the three-factor model, it fits the yield curve data less closely than the three-factor model (Krippner 2015a). It is well known that the yield curve (especially the level factor) contains information about future inflation (see, e.g., Diebold and Rudebusch 2013). Therefore, the fact that the WX shadow rate term structure model fits better to the yield curve data and thus contains more information than the K-ANSM model could, at least in part, explain why the WX shadow rate produces better inflation forecasts than the K-ANSM shadow rate.

5. Conclusions

This paper examines whether the shadow interest rates contain predictive power for U.S. inflation in a data-rich environment. We focus on the shadow rates discussed in Wu and Xia (2016) and Krippner (2015b), given their prevalent use in the literature.

Our empirical analysis leads us to three main conclusions. First, the shadow rates contain substantial in-sample and out-of-sample predictive power for U.S. CPI and PCE inflation when the predictive information encoded in macroeconomic factors extracted from a large set of 133 macroeconomic variables is already taken into account. This finding holds both in the non-ZLB and ZLB periods. The forecasting performance of the shadow rates is particularly good in the ZLB period. Therefore, our results suggest that the shadow rates provide a valuable source of information about future inflation for forecasters, central bankers, and other policymakers when the short-term rates are stuck at the ZLB. Second, the relationship between the shadow rates and future inflation is positive; i.e., if the shadow rate decreases, future inflation tends to decrease. This result stems probably from the forward-looking nature of monetary policy. The relationship between the shadow rate and future inflation tends to be stronger in the ZLB environment for the WX shadow rate; but for the K-ANSM shadow rate the relationship remains similar in both periods. Third, the WX shadow rates are more informative about future inflation than the K-ANSM shadow rates. The WX shadow rates produce better in-sample and out-of-sample inflation forecasts than the K-ANSM shadow rates both in the non-ZLB and ZLB periods.

Interestingly, we find that the choice of the LB parameter or the sample period used in the estimation of the shadow rates do not matter much for the predictive power of the shadow rates. The results reveal that both the WX and K-ANSM shadow rates are useful leading indicators for inflation regardless of which LB parameter or data sample is used in the estimation of the shadow rates.

Our results could be extended in several ways. We have examined the predictive power of the shadow rates for U.S. inflation. Evidence from other countries (e.g., from the eurozone, Japan and the U.K.) may lead to a better understanding of the indicator properties of the shadow rates. Furthermore, we consider only point forecasts in our out-of-sample forecasting exercise. However, density forecasts contain more information than point forecasts, because they summarize the information regarding the uncertainty around point forecasts. For this reason, central banks and policy institutions are interested in density forecasts. It would be interesting to know whether the models augmented with a shadow rate produce better density forecasts than the benchmark models. We leave these issues for future research.

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Appendix A

Table A1: Data description

id	Mnemonic	Trans. code	Description
1	RPI	5	Real Personal Income
2	W875RX1	5	Real Personal Income ex transfer receipts
3	DPCERA3M086SBEA	5	Real Personal Consumption Expenditures
4	CMRMTSPLx	5	Real Manu. and Trade Industries Sales
5	RETAILx	5	Retail and Food Services Sales
6	INDPRO	5	IP Index
7	IPFPNSS	5	IP: Final Products and Nonindustrial Supplies
8	IPFINAL	5	IP: Final Products (Market Group)
9	IPCONGD	5	IP: Consumer Goods
10	IPDCONGD	5	IP: Durable Consumer Goods
11	IPNCONGD	5	IP: Nondurable Consumer Goods
12	IPBUSEQ	5	IP: Business Equipment
13	IPMAT	5	IP: Materials
14	IPDMAT	5	IP: Durable Materials
15	IPNMAT	5	IP: Nondurable Materials
16	IPMANSICS	5	IP: Manufacturing (SIC)
17	IPB51222s	5	IP: Residential Utilities
18	IPFUELS	5	IP: Fuels
19	NAPMPI	1	ISM Manufacturing: Production Index
20	CUMFNS	2	Capacity Utilization: Manufacturing
21	HWI	2	Help-Wanted Index for United States
22	HWIURATIO	2	Ratio of Help Wanted/No. Unemployed
23	CLF16OV	5	Civilian Labor Force
24	CE16OV	5	Civilian Employment
25	UNRATE	2	Civilian Unemployment Rate
26	UEMPMEAN	2	Average Duration of Unemployment (Weeks)
27	UEMPLT5	5	Civilians Unemployed - Less Than 5 Weeks
28	UEMP5TO14	5	Civilians Unemployed for 5-14 Weeks
29	UEMP15OV	5	Civilians Unemployed - 15 Weeks & Over
30	UEMP15T26	5	Civilians Unemployed for 15-26 Weeks
31	UEMP27OV	5	Civilians Unemployed for 27 Weeks and Over
32	CLAIMSx	5	Initial Claims
33	PAYEMS	5	All Employees: Total nonfarm
34	USGOOD	5	All Employees: Goods-Producing Industries
35	CES1021000001	5	All Employees: Mining and Logging: Mining
36	USCONS	5	All Employees: Construction
37	MANEMP	5	All Employees: Manufacturing
38	DMANEMP	5	All Employees: Durable Goods
39	NDMANEMP	5	All Employees: Nondurable Goods
40	SRVPRD	5	All Employees: Service-Providing Industries
41	USTPU	5	All Employees: Trade, Transportation & Utilities
42	USWTRADE	5	All Employees: Wholesale Trade
43	USTRADE	5	All Employees: Retail Trade
44	USFIRE	5	All Employees: Financial Activities
45	USGOVT	5	All Employees: Government
46	CES0600000007	1	Avg Weekly Hours: Goods-Producing
47	AWOTMAN	2	Avg Weekly Overtime Hours: Manufacturing
48	AWHMAN	1	Avg Weekly Hours: Manufacturing
49	NAPMEI	1	ISM Manufacturing: Employment Index
50	HOUST	4	Housing Starts: Total New Privately Owned
51	HOUSTNE	4	Housing Starts, Northeast
52	HOUSTMW	4	Housing Starts, Midwest
53	HOUSTS	4	Housing Starts, South
54	HOUSTW	4	Housing Starts, West
55	PERMIT	4	New Private Housing Permits (SAAR)
56	PERMITNE	4	New Private Housing Premits, Northeast (SAAR)
57	PERMITMW	4	New Private Housing Permits, Midwest (SAAR)
58	PERMITS	4	New Private Housing Permits, South (SAAR)
59	PERMITW	4	New Private Housing Permits, West (SAAR)
60	NAPM	1	ISM: PMI Composite Index

(Continued)

Table A1 – *(Continued)*

id	Mnemonic	Trans. code	Description
61	NAPMNOI	1	ISM: New Orders Index
62	NAPMSDI	1	ISM: Supplier Deliveries Index
63	NAPMII	1	ISM: Inventories Index
65	AMDMNOx	5	New Orders for Durable Goods
66	ANDENOx	5	New Orders for Nondefense Capital Goods
67	AMDMUOx	5	Unfilled Orders for Durable Goods
68	BUSINVx	5	Total Business Inventories
69	ISRATIOx	2	Total Business: Inventories to Sales Ratio
70	M1SL	6	M1 Money Stock
71	M2SL	6	M2 Money Stock
72	M2REAL	5	Real M2 Money Stock
73	AMBSL	6	St. Louis Adjusted Monetary Base
74	TOTRESNS	6	Total Reserves of Depository Institutions
75	NONBORRES	7	Reserves of Depository Institutions, Nonborrowed
76	BUSLOANS	6	Commercial and Industrial Loans, All Commercial Banks
77	REALLN	6	Real Estate Loans at All Commercial Banks
78	NONREVSL	6	Total Nonrevolving Credit Owner and Securitized Outstanding
79	CONSPI	2	Nonrevolving Consumer Credit to Personal Income
80	S & P 500	5	S&P's Common Stock Price Index: Composite
81	S & P: indust	5	S&P's Common Stock Price Index: Industrials
82	S & P div yield	2	S&P's Composite Common Stock: Dividend Yield
83	S & P PE ratio	5	S&P's Composite Common Stock: Price-Earnings Ratio
84	FEDFUNDS	2	Effective Federal Funds Rate
85	CP3Mx	2	3-Month AA Financial Commercial Paper Rate
86	TB3MS	2	3-Month Treasury Bill
87	TB6MS	2	6-Month Treasury Bill
88	GS1	2	1-Year Treasury Rate
89	GS5	2	5-Year Treasury Rate
90	GS10	2	10-Year Treasury Rate
91	AAA	2	Moody's Seasoned Aaa Corporate Bond Yield
92	BAA	2	Moody's Seasoned Baa Corporate Bond Yield
93	COMPAPFFx	1	3-Month Commercial Paper Minus FEDFUNDS
94	TB3SMFFM	1	3-Month Treasury C Minus FEDFUNDS
95	TB6SMFFM	1	6-Month Treasury C Minus FEDFUNDS
96	T1YFFM	1	1-Year Treasury C Minus FEDFUNDS
97	T5YFFM	1	5-Year Treasury C Minus FEDFUNDS
98	T10YFFM	1	10-Year Treasury C Minus FEDFUNDS
99	AAAFFM	1	Moody's Aaa Corporate Bond Minus FEDFUNDS
100	BAAFFM	1	Moody's Baa Corporate Bond Minus FEDFUNDS
101	TWEXMMTH	5	Trade Weighted U.S. Dollar Index: Major Currencies
102	EXSZUSx	5	Switzerland / U.S. Foreign Exchange Rate
103	EXJPUSx	5	Japan / U.S. Foreign Exchange Rate
104	EXUSUKx	5	U.S. / U.K. Foreign Exchange Rate
105	EXCAUSx	5	Canada / U.S. Foreign Exchange Rate
106	PPIFGS	6	PPI: Finished Goods
107	PPIFCG	6	PPI: Finished Consumer Goods
108	PPIITM	6	PPI: Intermediate Materials
109	PPICRM	6	PPI: Crude Materials
110	OILPRICEx	6	Crude Oil, spliced WTI and Cushing
111	PPICMM	6	PPI: Metals and Metal Products
112	NAPMPRI	1	ISM Manufacturing: Prices Index
113	CPIAUCSL	6	CPI: All Items
114	CPIAPPSL	6	CPI: Apparel
115	CPITRNSL	6	CPI: Transportation
116	CPIMEDSL	6	CPI: Medical Care
117	CUSR0000SAC	6	CPI: Commodities
118	CUUR0000SAD	6	CPI: Durables
119	CUSR0000SAS	6	CPI: Services
120	CPIULFSL	6	CPI: All Items Less Food
121	CUUR0000SA0L2	6	CPI: All Items Less Shelter
122	CUSR0000SA0L5	6	CPI: All Items Less Medical Care
123	PCEPI	6	Personal Cons. Expend.: Chain Price Index
124	DDURRG3M086SBEA	6	Personal Cons. Expend.: Durable Goods
125	DNDGRG3M086SBEA	6	Personal Cons. Expend.: Nondurable Goods
126	DSERRG3M086SBEA	6	Personal Cons. Expend.: Services

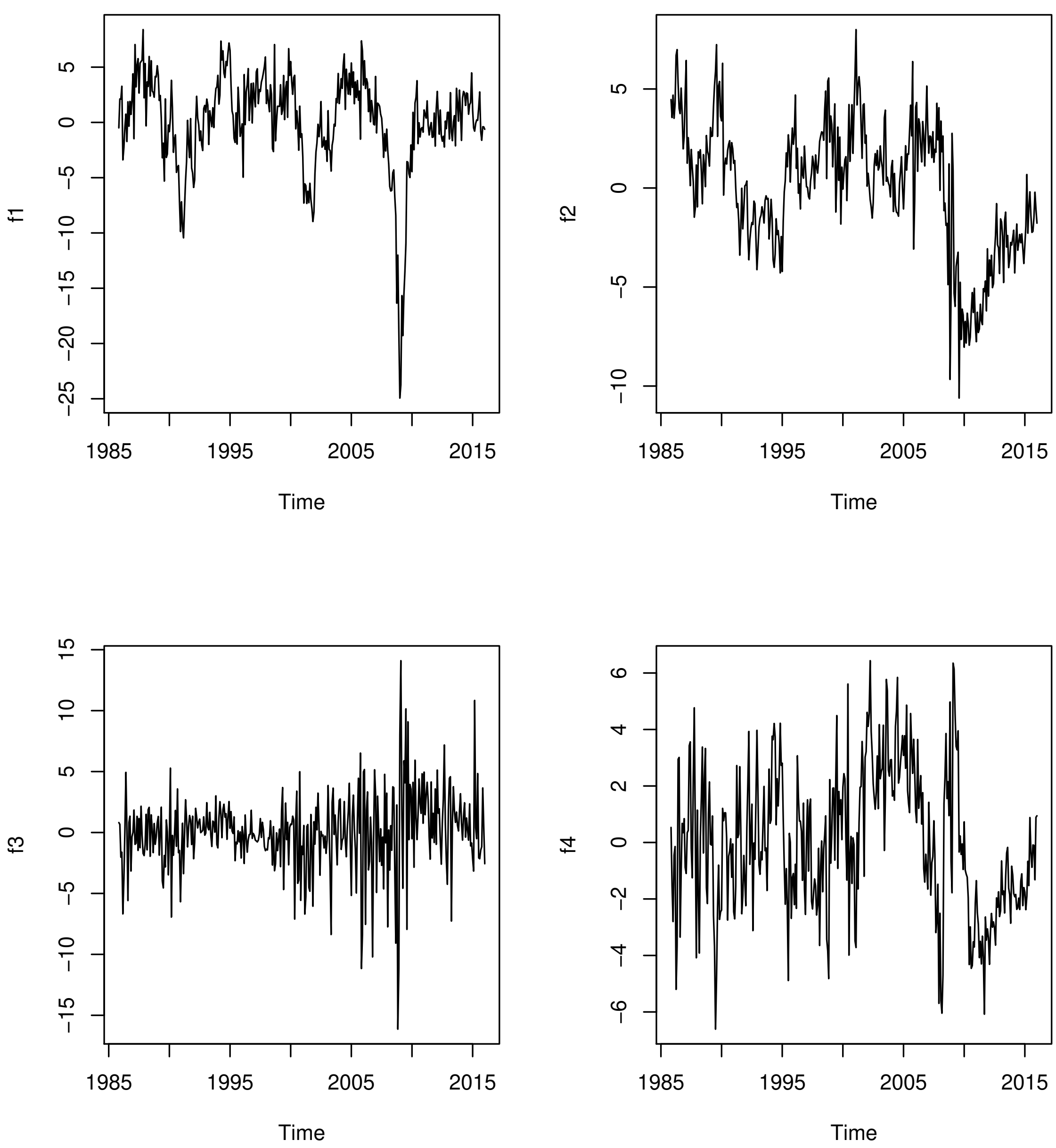
(Continued)

Table A1 – *(Continued)*

id	Mnemonic	Trans. code	Description
127	CES0600000008	6	Avg Hourly Earnings: Goods-Producing
128	CES2000000008	6	Avg Hourly Earnings: Construction
129	CES3000000008	6	Avg Hourly Earnings: Manufacturing
130	UMCSENTx	2	Consumer Sentiment Index
131	MZMSL	6	MZM Money Stock
132	DTCOLNVHFNM	6	Consumer Motor Vehicle Loans Outstanding
133	DTCTHFNM	6	Total Consumer Loans and Leases Outstanding
134	INVEST	6	Securities in Bank Credit at All Commercial Banks

Notes: The transformation code (column 3) denotes the transformation applied to the variable before the principal components are calculated. The transformation codes are 1 = no transformation, 2 = first difference, 3 = second difference, 4 = natural logarithm, 5 = first difference of logarithms, and 6 = second difference of logarithms. The data sample is 1985:M11–2015:M12. The data source is the FRED-MD database.

Figure A1: Factors over time



Notes: The sample period is from 1985:M11 to 2015:M12.