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Abstract

We consider an infinite-horizon general equilibrium model with heterogeneous agents and financial market imperfections. We investigate the role of dividend taxation on economic growth and asset price. The optimal dividend taxation is also studied.

Keywords: Intertemporal equilibrium, recession, growth, R&D, dividend taxation, asset price bubbles.
JEL Classifications: C62, D31, D91, G10, E44.

1 Introduction

The interplay between financial market and production sector is an important issue to understand the real effects of the financial sector. On the one hand, as mentioned in Kiyotaki and Moore (1997), the financial friction may amplify the macroeconomic impact of the exogenous changes. On the other hand, in some situations, the financial market may be beneficial to the production sector by providing financial support for the purchase of the physical capital (Le Van and Pham, 2016). Few papers study financial taxation and its role on the the interaction between financial and real sectors, in spite of a large literature on capital and labor income taxations (Atkinson and Sandmo, 1980; Chamley, 1986; Judd, 1985; Kocherlakota, 2010). The current paper aims to fill this gap. More precisely, we investigate the role of asset dividend taxation in an economy with the presence of financial market imperfection. Several questions will be addressed: How do we use dividend taxation to avoid recession and promote economic growth and welfare? What is its impact on asset price and asset bubble? To this purpose, we construct an infinite-horizon general equilibrium model with heterogeneous consumers, a firm and a government. In this economy, a long-lived asset is traded and a single good is consumed or/and used to produce. An agent buys the long-lived

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asset today and may resell it tomorrow after receiving exogenous dividends (in term of consumption good). This asset can be interpreted as land or Lucas tree (Lucas, 1978) or security (Santos and Woodford, 1997) or stock (Kocherlakota, 1992). In the following, it will be referred to as financial asset. Consumers may invest in physical capital or in financial asset, and borrow by selling a financial asset within the limit of a borrowing constraint: the repayment of each consumer cannot exceed a fraction of her (physical) capital income. The representative firm maximizes its profit by computing its capital demand. The government taxes the dividends on the asset. The government spends these taxes to finance research and development (henceforth, R&D) activities that improve in turn the firm's productivity.[1] This kind of endogenous growth is in the spirit of Barro (1990).

After proving the existence of equilibrium, we wonder whether recessions arise and how to avoid them with a positive growth. A recession in the productive sector is said to appear if the capital used for production falls below some critical level, say $\bar{k}$. We show that recessions appear at infinitely many dates if the firm's productivity is too low. The novelty of our work is that taxation on asset dividends allows us to avoid recessions and possibly promote economic growth according to the following mechanism: the government levies taxes on consumers' asset dividends and spends these taxes to finance the R&D. The R&D then increases the Total Factor Productivity (henceforth, TFP) and hence rules out the recessions, promoting economic growth in the end. Given a low initial productivity, recession will be prevented and the economy may grow without bounds if (1) the R&D process is efficient or/and (2) the dividends are high or/and (3) the tax on dividends is high. By contrast, when these three conditions are violated, the economy cannot escape from recession. We also compare the above mechanism with other subsidy policies such as consumption and investment subsidies. We prove that the only way to get unbounded growth is to invest in R&D which improves the productivity. The current paper contributes to the endogenous growth theory. The added-value is that our results are obtained in a model with heterogeneous agents and borrowing constraints, which raises technical difficulties that methods in the standard optimal growth theory (Le Van and Dana, 2003; Acemoglu, 2009) are no longer applied. It should be noticed that our results hold for any equilibrium including recursive ones. Although some authors (Acemoglu and Jensen, 2015; Datta et al., 2017) study comparative statics of recursive equilibria, intertemporal equilibria in our paper may be not recursive and therefore their methods cannot be directly applied in our framework.

When the government increases the tax rate (τ) on dividends, the net dividends decrease but the production level increases. Hence, the total amount of good may decrease or increase. It is natural to study the optimal dividend taxation to grasp this trade-off. In this respect, we assume that the government maximizes the aggregate consumption of the economy at the steady state by choosing the tax rate. If the TFP or the efficiency of R&D or the asset dividends are high, the government should choose the highest feasible tax rate on dividends. By contrast, if these factors are low, the government has to apply the lowest tax rate. In the intermediate case for TFP, R&D and dividends, the optimal level of dividend taxation depends on these

[1]The reader is refereed to Alstadsaeter et al. (2015) for the role of dividend taxes on corporate investment.

three factors as follows. It is increasing in the R&D efficiency and the firm's TFP, but decreasing in the dividend. Our analysis contributes to the optimal taxation theory. The main difference is that the standard literature (Chamley, 1986; Judd, 1985; Kocherlakota, 2010) studies capital and labor income taxations while we focus on dividend taxation. Moreover we consider a model with heterogeneous consumers and financial frictions while Kocherlakota (2010) studies representative agent models without financial friction.[2]

The last avenue of our contribution focuses on the impact of dividend taxation on asset price and bubbles. Following Santos and Woodford (1997), we say that an asset bubble arises if at equilibrium the fundamental value (i.e., the sum of discounted values) of asset dividends (after tax) exceeds the asset's equilibrium price. Although there is a large literature on the non-existence of rational bubble in general equilibrium models,[3] few examples of bubbles of assets having positive dividends have been provided. We present an example, inspired by Le Van and Pham (2016), where there may be continuum of equilibria with bubble. This is when endowments of agents fluctuate over time. Indeed, with such a fluctuation, at any date there is at least one agent who needs to buy asset (even the asset price exceeds the fundamental value) because this agent has to transfer her wealth from this date to the next date (this is the only way she can smooth consumption because she is prevented from borrowing). Differently from Le Van and Pham (2016), the asset fundamental value in our example is not monotonic in dividends. This is from the fact that the real returns and discount factors in our example depend on dividends through R&D investment. More interestingly, we show that asset bubbles are more likely to arise when dividend taxes increase. The intuition is that if such taxes increase, then the after-tax dividends decrease, which makes the fundamental value of asset decrease and may be lower than the asset price.

The paper is organized as follows. Section 2 presents the model and provides some basic equilibrium properties. Section 3 investigates the role of dividend taxation on recessions and economic growth. Section 4 studies the optimal dividend taxation. Section 5 considers the role of dividend taxation on asset bubbles. Section 6 concludes. Formal proofs are gathered in Appendix A.

2 Framework

Our model is based on Santos and Woodford (1997), Le Van and Pham (2016). We consider a deterministic infinite-horizon general equilibrium model à la Ramsey. Time is discrete: $t = 0, \dots, \infty$. However, we introduce two additional ingredients: a government and an externality on the production function. So, there are three types of agents: a representative firm without market power, m heterogeneous households and the government.

[2]The representative agent in Kocherlakota (2010) faces a unique intertemporal constraint. We refer to Aiyagari (1995) for optimal capital income taxation and Bhandari et al. (2013) for optimal labor income taxation in models with incomplete markets.

[3]See Tirole (1982), Santos and Woodford (1997) or more recently Le Van and Pham (2016) and references therein.

Households

Each household invests in physical or financial asset, and consumes.

Consumption good: there is a single good which can be consumed or used to produce. p_t is its price at period t and $c_{i,t}$ the amount of good consumed by agent i.

Physical capital: $\delta \in (0,1)$ denotes the capital depreciation rate, while r_t the return of capital. If agent i buys $k_{i,t} \geq 0$ units of physical capital at date $t-1$, then she will receive in the following period $(1-\delta)k_{i,t}$ units of physical capital (after depreciation) and returns $r_t k_{i,t}$.

Financial asset: if agent i buys $a_{i,t}$ units of financial asset at a price q_t at date t, she will receive in the following period ξ_{t+1} units of consumption good as dividend. Then, she will resell $a_{i,t}$ units of financial asset at a price q_{t+1}. This asset takes on different meanings: land,[4] security (Santos and Woodford, 1997) or stock (Kocherlakota, 1992).

Differently from the existing literature, we introduce a government taxing the revenue from asset dividends. For each unit of dividend, any consumer must pay τ units of consumption good.

Each household i takes the sequence of prices $(p,q,r) := (p_t, q_t, r_t)_{t=0}^{\infty}$ as given, and solves the following program:

$$(P_i(p,q,r)) : \max_{(c_{i,t},k_{i,t+1},a_{i,t})_{t=0}^{\infty}} \left[\sum_{t=0}^{\infty} \beta_i^t u_i(c_{i,t}) \right] \tag{1}$$

$$\text{subject to :} \quad k_{i,t+1} \geq 0 \tag{2}$$

$$\begin{aligned} & p_t(c_{i,t} + k_{i,t+1} - (1-\delta)k_{i,t}) + q_t a_{i,t} \\ & \leq r_t k_{i,t} + q_t a_{i,t-1} + p_t \xi_t (1-\tau) a_{i,t-1} + \theta_t^i \pi_t \end{aligned} \tag{3}$$

$$(q_{t+1} + (1-\tau)p_{t+1}\xi_{t+1})a_{i,t} \geq -f_i \Big[p_{t+1}(1-\delta) + r_{t+1} \Big] k_{i,t+1}. \tag{4}$$

where $k_{i,0}0$ and $a_{i,-1}$ are given. The exogenous parameter $f_i \in [0,1]$ which is set by law, represents the borrowing limit of agent i. This parameter can be viewed as an index of financial development. At date t, π_t is the firm's profit, $(\theta_t^i)_{i=1}^m$ is the exogenous share of profit with $\theta_t^i \geq 0$ for any i and t, and $\sum_{i=1}^{m} \theta_t^i = 1$ for any t.

In our model, consumers can borrow by using the financial asset but they face borrowing constraints. Agent i can borrow an amount if the repayment of this amount does not exceed a fraction of the market value of her (physical) capital income (including returns and depreciation). In other terms, the physical capital plays the role of collateral. The fraction f_i is less than 1 to ensure that the market value of collateral of each agent is greater than her debt. At equilibrium, as we will see (after Lemma 1), the borrowing constraint (4) becomes equivalent to $q_t a_{i,t} \geq -f_i p_t k_{i,t+1}$.

The government

In our model, the government levies tax on dividends and uses it to invest in research and development (R&D). The government fixes the tax rate τ on dividends. The aggregate tax is denoted by T_t (in terms of consumption good). By construction, we

[4] This is the case where $f_i = 0$ for any i. See constraint (4).

have

$$T_t = \sum_{i=1}^{m} \tau \xi_t a_{i,t-1}.$$

Let us denote by G_t the government spending at date t. In the spirit of Barro (1990), we assume that the government spending in R&D will affect the productivity of the firm at the next date. More precisely, the production function at date t is given by $F_g(G_{t-1}, \cdot)$ with $F_g(G, K) = f(G)F(K)$ where f is an increasing function and $f(0) = 1$. F is the original production function without government spending in R&D. If $G = 0$, then $F_g(G, K) = F(K)$ and we recover Le Van and Pham (2016).

Firm

At date t, the representative firm takes prices (p_t, r_t) and government spending G_{t-1} as given and maximizes its profit by choosing the physical capital amount K_t.

$$(P(p_t, r_t, G_{t-1})): \quad \pi_t := \max_{K_t \geq 0} \Big[p_t F_g(G_{t-1}, K_t) - r_t K_t\Big]. \tag{5}$$

The production function at date t is $F_g(G_{t-1}, \cdot)$ which is non-stationary and depends on the government's spending at date $t - 1$.

2.1 Equilibrium

We denote an infinite-horizon sequence of prices and quantities by

$$(p, q, r, (c_i, k_i, a_i)_{i=1}^m, K, G, T)$$

with $(x) := (x_t)_{t\geq 0}$ for $x \in \{p, q, r, c_i, a_i, K, G, T\}$ and $(k_i) := (k_{i,t+1})_{t\geq 0}$ for any i.

The economy is denoted by $\mathcal{E}$ and is characterized by a list

$$\mathcal{E} := \Big((u_i, \beta_i, k_{i,0}, a_{i,-1}, f_i, \theta^i)_{i=1}^m, F, f, (\xi_t)_{t=0}^{\infty}, \delta, \tau\Big).$$

Definition 1. *A list* $\Big(\bar{p}_t, \bar{q}_t, \bar{r}_t, (\bar{c}_{i,t}, \bar{k}_{i,t+1}, \bar{a}_{i,t})_{i=1}^m, \bar{K}_t, \bar{G}_t, \bar{T}_t\Big)_{t=0}^{\infty}$ *is an equilibrium of the economy* $\mathcal{E}$ *if the following conditions are met.*

(i) Price positivity: $\bar{p}_t, \bar{q}_t, \bar{r}_t > 0$ *for* $t \geq 0$.

(ii) Market clearing conditions: for any $t \geq 0$,

$$\textit{good: } \sum_{i=1}^{m}(\bar{c}_{i,t} + \bar{k}_{i,t+1} - (1-\delta)\bar{k}_{i,t}) = f(\bar{G}_{t-1})F(\bar{K}_t) + (1-\tau)\xi_t,$$

$$\textit{capital: } \bar{K}_t = \sum_{i=1}^{m} \bar{k}_{i,t},$$

$$\textit{financial asset: } \sum_{i=1}^{m} \bar{a}_{i,t} = 1,$$

(iii) Optimal consumption plans: for any i, $(\bar{c}_{i,t}, \bar{k}_{i,t+1}, \bar{a}_{i,t})_{t=0}^{\infty}$ *is a solution of the problem* $(P_i(\bar{p}, \bar{q}, \bar{r}))$.

(iv) Optimal production plan: for any $t \geq 0$, $\bar{K}_t$ is a solution of the problem $(P(\bar{p}_t, \bar{r}_t, \bar{G}_{t-1}))$.

(v) Government: $\bar{G}_t = \bar{T}_t$ where $\bar{T}_t = \sum_{i=1}^{m} \tau \xi_t \bar{a}_{i,t-1}$.

At equilibrium, we have $G_t = T_t = \tau \xi_t$. Therefore, the consumption market clearing condition writes

$$C_t + K_{t+1} + G_t = f(G_{t-1})F(K_t) + (1-\delta)K_t + \xi_t, \tag{6}$$

where $C_t := \sum_{i=1}^m c_{i,t}$, $K_t := \sum_{i=1}^m k_{i,t}$. The output of the economy is $f(G_{t-1})F(K_t) + (1-\delta)K_t + \xi_t$ and decomposes into three parts: private consumption C_t, private investment K_{t+1} and public investment G_t.

In the rest of this paper, if we do not explicitly mention, the following standard assumptions are required.

Assumption (H1). u_i is C^1, strictly increasing and concave with $u_i(0) = 0$ and $u_i'(0) = \infty$.

Assumption (H2). The function $F(\cdot)$ is C^1, strictly increasing, concave with $F(0) \geq 0$, $F(\infty) = \infty$. The function $f(\cdot)$ is increasing and $f(0) = 1$.

Assumption (H3). For every $t \geq 0$ and $0 < \xi_t < \infty$.

Assumption (H4). $k_{i,0}, a_{i,-1} \geq 0$, and $(k_{i,0}, a_{i,-1}) \neq (0,0)$ for $i = 1, \dots, m$. Moreover, $\sum_{i=1}^{m} a_{i,-1} = 1$ and $K_0 := \sum_{i=1}^{m} k_{i,0} > 0$.

Assumption (H5). $\sum_{t=0}^{\infty} \beta_i^t u_i(D_t) < \infty$ where

$$\begin{aligned} D_0 :=& \quad F_g(\xi_0, K_0) + (1-\delta)K_0 + \xi_0, \\ D_t :=& \quad F_g(\xi_{t-1}, D_{t-1}) + (1-\delta)D_{t-1} + \xi_t \quad \forall t \geq 0. \end{aligned}$$

Before presenting equilibrium analysis, we prove the existence of equilibrium.

Proposition 1. *Under assumptions (H1, H2, H3, H4, H5), there exists an equilibrium.*

Proof. See Appendix A.1. □

It should be noticed that the equilibrium in the current paper is with externalities on the productivity. The detailed proof of Proposition 1 is presented in Appendix A.

Price normalization: Since $p_t > 0 \ \forall t$ at equilibrium, in the rest of the paper, we will normalize by setting $p_t = 1 \ \forall t$. In this case, we also call $\Big(q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i=1}^m, K_t, G_t, T_t\Big)_t$ an equilibrium.

2.2 Basis properties

Let $\Big(q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i=1}^m, K_t, G_t, T_t\Big)_t$ be an equilibrium. Denote by $\mu_{i,t}$ and $\nu_{i,t+1}$ the multipliers associated to the budget and the borrowing constraint of the agent i at date t. Denote $\lambda_{i,t+1}$ the multiplier associated with constraint $k_{i,t+1} \geq 0$. We obtain

$$\beta_i^t u_i'(c_{i,t}) = \mu_{i,t} \tag{7}$$
$$\mu_{i,t} = (r_{t+1} + 1 - \delta)(\mu_{i,t+1} + f_i \nu_{i,t+1}) + \lambda_{i,t+1} \tag{8}$$
$$q_t \mu_{i,t} = (q_{t+1} + (1-\tau)\xi_{t+1})(\mu_{i,t+1} + \nu_{i,t+1}). \tag{9}$$

Notice that $k_{i,t+1}\lambda_{i,t+1} = 0$ and

$$\nu_{i,t+1}\Big[\Big(q_{t+1} + (1-\tau)\xi_{t+1}\Big)a_{i,t} + f_i\Big(1 - \delta + r_{t+1}\Big)k_{i,t+1}\Big] = 0.$$

The following lemma sums up the FOCs.

Lemma 1 (non-arbitrage condition)**.**

$$\frac{q_{t+1} + (1-\tau)\xi_{t+1}}{q_t} = \frac{1}{\max\limits_i \left\{\dfrac{\mu_{i,t+1}}{\mu_{i,t}}\right\}} \geq r_{t+1} + 1 - \delta \tag{10}$$

for any t. Moreover, the inequality holds with equality if $K_{t+1} > 0$.

According to Lemma 1, we have that

$$f_i(1 - \delta + r_{t+1})k_{i,t+1} = f_i \frac{q_{t+1} + (1-\tau)\xi_{t+1}}{q_t} k_{i,t+1}. \tag{11}$$

Therefore, borrowing constraint (4) is equivalent to $q_t a_{i,t} \geq -f_i k_{i,t+1}$.

It should be noticed that in our model with borrowing constraints, we only have the following Euler inequality, instead of Euler equation as in the representative consumer model without financial frictions,

$$1 \geq (r_{t+1} + 1 - \delta) \max_i \left\{\frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})}\right\}. \tag{12}$$

3 The role of dividend taxation

3.1 How to use dividend taxation to avoid recession?

We consider the specific definition of recession introduced by Le Van and Pham (2016). In Section 3.2, a more general case will be treated.

Definition 2 (recession)**.** *The productive sector experiences a recession at date t if no one invests in this sector, that is the aggregate capital equals zero ($K_t = 0$).*

Consumers diversify their portfolio by investing in capital and the financial asset. The real return on physical capital is $r_{t+1} + 1 - \delta$, and the physical capital's maximum return is $F'(0) + 1 - \delta$. The real return on the financial asset is $\frac{q_{t+1}+(1-\tau)\xi_{t+1}}{q_t}$. By comparing these two returns, Le Van and Pham (2016) obtain the following result.

Proposition 2. *Consider the case without government (i.e., $\tau = 0$). Assume that $F'(0) \leq \delta$ and there exists $\xi > 0$ such that $\xi_t \geq \xi$ for every $t \geq 0$. Then, there is an infinite sequence $(t_n)_{n=0}^{\infty}$ such that $K_{t_n} = 0$ for every $n \geq 0$.*

Proposition 2 shows that if the original productivity is low (in the sense that $F'(0) < \delta$) and there is no R&D investment, recessions will appear at infinitely many dates. Since the bound ξ does not depend on technology, the cause of economic recession is no longer the financial market, but the low productivity. Proposition 2 suggests that we should invest in R&D to improve the productivity and avoid recessions. In what follows, we will focus on the role of R&D. For simplicity, we consider a simple case where $\xi_t = \xi > 0$ for any t and $f(x) = (1 + bx)^{\alpha_1}$ with $\alpha_1 > 0$, and the positive parameter b represents the efficiency of the R&D process.

We denote by $\rho_i \equiv 1/\beta_i - 1$ which may be interpreted as the exogenous subjective interest rate of agent i. We have the following result showing how recession can be avoided.

Proposition 3. *Assume that $\xi_t = \xi > 0$ for any t and $f(x) = (1 + bx)^{\alpha_1}$ with $\alpha_1 > 0$. Then, $K_t > 0$ if*

$$(1 + b\tau\xi)^{\alpha_1} F'(0) > \delta + \max_{i=1,\dots,m} \rho_i. \tag{13}$$

Proof. See Appendix A.2. □

Condition (13) means that the return from the productive sector is higher than the investment cost. In this case, someone is willing to invest in the productive sector and recession is avoided. It should be noticed that condition (13) is satisfied if productivity $F'(0)$ and/or R&D efficiency b and/or dividend ξ are high.

Proposition 3 has an interesting implication. Consider a "bad" technology F (in the sense that $F'(0) < \delta$). In this case, without taxation on dividends, there is no R&D investment and the recession will arise at infinitely many dates (according to Proposition 2). When the government levies tax on asset dividends to finance efficient R&D (in the sense of condition (13)), the economy never falls in recession.

However, we would like also to point out that, given a low initial productivity, recession becomes unavoidable if the R&D is inefficient and dividends are low. Formally, we have.

Proposition 4. *Assume that $\bar{\xi} := \sup_t \xi_t < \infty$ and $\xi := \inf_t \xi_t > 0$ with $f(x) = (1 + bx)^{\alpha_1}$ and $(1 + b\tau\bar{\xi})^{\alpha_1} F'(0) \leq \delta$. Then, there exists a sequence $(t_n)_{n=0}^{\infty}$ such that $K_{t_n} = 0$ for every $n \geq 0$.*

Proof. See Appendix A.3 □

We now provide some implications of Propositions 3 and 4.

1. **Human capital**. Let us introduce the human capital in the production function: $F(K)L^{\alpha_1}$. Our model can be also interpreted as an economy with exogenous labor supply $L_0 = 1$. With a government spending in human capital, the effective labor becomes $(1 + bG_t)L_0$ and the marginal productivity (with respect to capital) $F'(K)(1 + bG_t)^{\alpha_1}$. In this case, all the above results still hold and we would say that recession in the productive sector may be prevented if the government uses the tax on dividends to invest in human capital.

2. **Taxes on land dividends**.

   If $f_i = 0$ for any i, we recover the asset structure of land: an agent buys land today to receive fruits (i.e., consumption good) tomorrow as land dividends and resell land thereafter. Proposition 3 shows that a good government is able to prevent recessions when land dividends are high enough. This interpretation leads to another interesting remark. Focus on a two-sector economy: agriculture (represented by land) and industry (represented by a firm). In this case, if the productivity $F'(0)$ of the industrial sector is low, the government may collect taxes on land dividends to finance R&D activities and, therefore, to improve the industrial productivity and shelter this sector from recessions. In some cases, this strategy not only avoids recession but also creates more consumption good. In Section 4, the issue of optimal tax level will be addressed.

3.2 Dividend taxation and economic growth

Consider now a more general concept of recession than Definition 2.

Definition 3. *There is a $\bar{k}$-recession in the productive sector at date t if $K_t \leq \bar{k}$.*

We have the following result which generalizes Proposition 4.

Proposition 5. *Assume that $\bar{\xi} := \sup_t \xi_t < \infty$ and $\xi := \inf_t \xi_t > 0$ with $f(x) = (1+bx)^{\alpha_1}$ and $(1+b\tau\bar{\xi})^{\alpha_1} F'(\bar{k}) \leq \delta$. Then, there exists a sequence $(t_n)_{n=0}^{\infty}$ such that $K_{t_n} \leq \bar{k}$ for every $n \geq 0$.*

The proof of Proposition 5 is similar to that of Proposition 4. According to Proposition 5, $\bar{k}$-recessions will appear at infinitely many dates if b, ξ_t and productivity are low. However, we will prove that $\bar{k}$-recessions can be prevented when dividends are high enough. First, we require an additional assumption on utility functions.

Assumption 1. *For each function u_i, there exists the function $y_i(\cdot)$: $\mathbb{R}^+ \to \mathbb{R}^+$ such that*

(1) $y_i(x) > 0$ and $y_i'(x) > 0$ for any $x > 0$. Moreover, $\lim_{x\to\infty} y_i(x) = \infty$.

(2) Given $x > 0$, we have $(u_i')^{-1}\Big(\frac{u_i'(a)}{x}\Big) \geq y_i(x)a$ for any $a > 0$, where $(u_i')^{-1}$ is the inverse function of u_i'.

Notice that Assumption 1 is satisfied with standard utility functions. For example, if $u_i'(c) = c^{-\sigma}$ with $\sigma \in (0,1]$, then $y_i(x) = x^{\frac{1}{\sigma}}$.

According to point 2 of this assumption, condition $u_i'(a) \geq xu'(b)$ implies that $b \geq y_i(x)a$. Combining this with the following Euler inequality

$$u_i'(c_{i,t-1}) \geq \Big(f(\tau\xi)F'(K_t) + 1 - \delta\Big)\beta_i u_i'(c_{i,t}).$$

we can show that consumption at date t is higher an endogenous proportion of consumption at the next date. This is the key argument to obtain our main results in this section. First, we show that a $\bar{k}-$recession can be avoided if dividends are high enough.

Proposition 6. *Let Assumption 1 be satisfied. We assume that (1) $\xi_t = \xi > 0$ for any t, (2) $f(x) = (1 + bx)^{\alpha_1}$.*

Given $\bar{k} > 0$, there exists $\bar{\xi}$ such that $K_t > \bar{k}$ for any $\xi > \bar{\xi}$ and for any $t \geq 1$.

Proof. See Appendix A.4. □

We may wonder whether the dividend taxation can be growth-enhancing. The next result shows the important role of dividend taxation and efficient R&D in economic growth.

Proposition 7. *Let Assumption 1 be satisfied. We assume that (1) $\xi_t = \xi > 0$ for any t, (2) $f(x) = (1 + bx)^{\alpha_1}$, and (3) $F'(K) \geq A > 0$ for any K.*

Then, we have $\lim_{t\to\infty} K_t = \infty$ at equilibrium if

$$x := \min_i \left\{ y_i\Big(\beta_i\Big(f(\tau\xi)A + 1 - \delta\Big)\Big) \right\} > 1 \tag{14}$$

$$\text{and} \quad \frac{xf(\tau\xi)}{f(\tau\xi) + \frac{1-\delta+x}{A}} > 1. \tag{15}$$

where the function $y_i(\cdot)$ is defined in Assumption 1.

Proof. See Appendix A.5. □

Different from Proposition 6, in Proposition 7 we require condition $F'(K) \geq A > 0$ $\forall K$, which is essential to obtain economic growth. Indeed, if $F'(\infty) = 0$, we can prove, by using the following condition

$$C_t + K_{t+1} \leq (1 - \delta)K_t + f(\xi)F(K_t) + \xi \tag{16}$$

that, given ξ and the function f, the sequence (K_t) is uniformly bounded from above.

Comparative statics. By definition, x increases in $\tau\xi$ and A, and hence conditions (14, 15) are more likely satisfied if $\tau\xi$, b, A are high. It means that dividend taxation and efficiency of R&D process play the key role on growth.

Proposition 7 has an interesting implication. To see the point, let us consider a simple case with linear technology $F(K) = AK$ and the productivity is low in the sense that $A < \delta$.

1. If there is no dividend ($\xi_t = 0$ for any t), then, according to (6), we have $K_{t+1} \leq (A + 1 - \delta)K_t$ for any t, which implies that $\lim_{t\to\infty} K_t = 0$: the economy collapses.

2. In the case with constant positive dividend ($\xi_t = \xi > 0$ for any t), Proposition 7 suggests that, if the government levies taxes on asset dividends and invests in R&D or human capital (in the sense of condition (14), (15)), growth will be unbounded.

Our result is related to the literature on optimal growth with increasing returns (Jones and Manuelli, 1990; Kamihigashi and Roy, 2007; Bruno et al., 2009). Our added-value is twofold. First, we point out the role of dividend taxation which can provide investment in R&D, and thanks to this, the host country may grow. Second, we consider a decentralized economy while these authors study centralized economies. Working in a general equilibrium framework is more difficult than in optimal growth context. The reason is that, in general equilibrium context, there does not exist a representative agent who chooses the level of aggregate capital K_t to maximize her intertemporal utility. So, it is not easy to prove some nice properties such at monotonicity of capital stock (K_t) as in the optimal growth theory (see Le Van and Dana (2003); Acemoglu (2009) among others).

Acemoglu and Jensen (2015), Datta et al. (2017) study comparative statics of recursive equilibria. However, intertemporal equilibria in our paper may not be recursive and therefore their methods cannot be directly applied here. It should also be noticed that equilibrium indeterminacy may arise in our model (see Proposition 10 in Section 5.1).

3.3 R&D versus other subsidy policies

So far, we have analyzed the impact of dividend taxation and R&D policy. This section aims to compare this policy with others subsidy policies. Let us consider two alternative policies: consumption and investment subsidies.

1. Consumption subsidy. In this case, the government uses taxes to produce public good which increases the utility function of all households. The utility function of agent i at date t now depends on both her consumption $c_{i,t}$ and public investment in public good $u(c_{i,t}, G_t)$. The production function in this case is the original production function: F(K). The good market clearing condition (6) becomes

$$C_t + K_{t+1} + G_t = F(K_t) + (1-\delta)K_t + \xi_t.$$

and hence

$$K_{t+1} \leq F(K_t) + (1-\delta)(K_t + G_{t-1}) + \xi_t. \tag{17}$$

2. Investment subsidy. In this case, the production function has the following form: $F_g(G, K) = F(K + G)$. The good market clearing condition (6) becomes

$$C_t + K_{t+1} + G_t = F(K_t + G_{t-1}) + (1-\delta)K_t + \xi_t.$$

and hence

$$K_{t+1} + G_t \leq F(K_t + G_{t-1}) + (1-\delta)(K_t + G_{t-1}) + \xi_t. \tag{18}$$

If $\bar{\xi} := \sup_{t\geq 0} \xi_t < \infty$ and $F'(\infty) < \delta$, we can prove, by using (17) or (18) that $K_{t+1} + G_t$ is uniformly bounded from above. Hence, consumption and investment subsidy policies cannot help us to have unbounded growth. In our framework, the only way to get unbounded growth is to invest in R&D which improves the productivity.

4 Optimal dividend taxation

When the government raises the tax rate τ, the net dividend $(1-\tau)\xi_t$ drops but the production level increases. It is worthy to deepen this trade-off by considering the optimal taxation on dividends. To this purpose, we assume that the government chooses $\tau \in [\underline{\tau}, \bar{\tau}] \subset [0,1]$, where $\underline{\tau}$ and $\bar{\tau}$ are exogenous parameters,[5] in order to maximize the aggregate consumption at the steady state. Let us start by defining the steady state formally.

Definition 4. *Assume that $\xi_t = \xi > 0$ and $\tau_t = \tau \in [0,1]$ for any t. A steady state is an equilibrium $\Big(q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i=1}^m, K_t, G_t, T_t\Big)_t$ such that $q_t = q, r_t = r$, $c_{i,t} = c_i, k_{i,t} = k_i$ and $a_{i,t} = a_i$ for any i and t, and $K_t = K$, $G_t = G$ and $T_t = T$ for any t.*

We provide now sufficient conditions for steady state uniqueness.

Lemma 2. *Let $\beta_1 > \beta_i$ for any $i \geq 2$ and $f_i < 1$ for any i. Assume also that $\xi_t = \xi, \tau_t = \tau \in [0,1]$ for any t and that F is strictly concave and linear with $F'(0) = \infty$. Then, there is a unique steady state:*

$$1 = \beta_1\Big(f(\tau\xi)F'(K) + 1 - \delta\Big) \tag{19}$$

$$r = f(\tau\xi)F'(K) \text{ and } q = \frac{(1-\tau)\xi\beta_1}{1-\beta_1} \tag{20}$$

$$k_1 = K,\ a_i = 1 \text{ and } c_1 = (r-\delta)K + \theta_1\pi + (1-\tau)\xi \tag{21}$$

$$a_i = k_i = 0 \text{ and } c_i = \theta_i\pi \text{ for } i = 2, \dots, m. \tag{22}$$

Proof. See Appendix A.6. □

Since $\beta_1 > \beta_i$ for any $i = 2, \dots, m$, the borrowing constraints of any consumer $i = 2, \dots, m$ are binding. Moreover, the condition $f_i < 1$ for any i implies that no agent $i = 2, \dots, m$ will invest in physical capital.[6] Hence, the income of any agent $i = 2, \dots, m$ equals their profit share.[7]

Since the aggregate capital level K is determined by (19) and F is strictly concave, we see that K is uniquely determined. Moreover, we also see that K is increasing in β_1, τ and ξ, and decreasing in δ. For simplicity, in what follows, we write β instead of β_1.

The aggregate consumption is $C = (1-\tau)\xi + f(\tau\xi)F(K) - \delta K$. For the sake of simplicity, we consider a Cobb-Douglas production function $F(K) = AK^\alpha$ with

[5] The exogenous parameters $\underline{\tau}$ and $\bar{\tau}$ represent political or institutional constraints that we do not microfound here.

[6] If $f_i = 1$ for any i, there may be an equilibrium indeterminacy (in term of assets held by agents).

[7] Notice that, when there are at least 2 agents, say 1 and 2, whose rates of time preference are $\beta_1 = \beta_2 > \beta_i$ for any $i = 3, \dots, m$, the aggregate capital stock K remains unique and still determined by (19) but their income distribution depends on their initial distribution of capital.

$\alpha \in (0,1)$. In this case, we have

$$K = \left(\frac{\alpha A f(\tau\xi)}{\frac{1}{\beta} + \delta - 1}\right)^{\frac{1}{1-\alpha}} \tag{23}$$

$$C = f(\tau\xi)AK^{\alpha} - \delta K + (1-\tau)\xi = B_1\Big(Af(\tau\xi)\Big)^{\frac{1}{1-\alpha}} + (1-\tau)\xi \tag{24}$$

where $B_1 := \alpha^{\frac{\alpha}{1-\alpha}} \dfrac{\frac{1}{\beta} - 1 + \delta(1-\alpha)}{\left(\frac{1}{\beta} - 1 + \delta\right)^{\frac{1}{1-\alpha}}}$.

If $f(\tau\xi) = (1 + b\xi\tau)^{\alpha_1}$, the government's problem writes

$$\max_{\tau \in [\underline{\tau}, \bar{\tau}]} \Big[B_1 A^{\frac{1}{1-\alpha}}(1 + b\xi\tau)^{\sigma} - \xi\tau\Big] \tag{25}$$

where $\sigma := \frac{\alpha_1}{1-\alpha}$. If $\alpha_1 < 1 - \alpha$, then, $\sigma < 1$, which implies in turn that the objective function in (25) is strictly concave.[8] By consequence, we obtain the following result.

Proposition 8. *Let $F(K) = AK^{\alpha}$ and $f(x) = (1+bx)^{\alpha_1}$ with $\alpha + \alpha_1 < 1$. There are three possibilities.*

1. *If $\sigma b B_1 A^{\frac{1}{1-\alpha}} \geq (1 + b\bar{\tau}\xi)^{1-\sigma}$, then $\tau^* = \bar{\tau}$.*
2. *If $\sigma b B_1 A^{\frac{1}{1-\alpha}} \leq (1 + b\underline{\tau}\xi)^{1-\sigma}$, then $\tau^* = \underline{\tau}$.*
3. *If $(1+b\underline{\tau}\xi)^{1-\sigma} < \sigma b B_1 A^{\frac{1}{1-\alpha}} < (1+b\bar{\tau}\xi)^{1-\sigma}$, then τ^* is the solution of the following equation*
$$\sigma b B_1 A^{\frac{1}{1-\alpha}} = (1 + b\tau\xi)^{1-\sigma}.$$

Comparative statics

Consider the role of parameters b and A that represent R&D efficiency and the original TFP. Proposition 8 shows that when R&D efficiency b and TFP A are very high (in the sense of the first point in Proposition 8), the optimal tax rate equals $\bar{\tau}$, the highest affordable tax rate. But, when b and A are low (enough), the optimal tax rate equals $\underline{\tau}$ and the government implements the lowest taxation.

The following result is immediate.

Corollary 1. *In the third case of Proposition 8, the optimal level τ^* is increasing in β, A and b, but decreasing in ξ.*

Remark 1. *When the government objective is a measure of welfare such as the aggregation of agents' intertemporal utilities, it is difficult to find closed solutions. Indeed, because of the financial market imperfections, it may become impossible to provide a closed form for equilibrium prices: given a tax rate τ, the equilibrium may fail to be unique (see Proposition 10). Even in the case of uniqueness, equilibrium allocations and prices may fail to be smooth in τ and the government's maximization problem becomes a hopeless challenge.*[9]

[8] If $\alpha_1 \geq 1 - \alpha$, the objective function is convex and the solution becomes either $\underline{\tau}$ or $\bar{\tau}$.
[9] This is different from Chamley (1986) and Judd (1985).

5 Dividend taxation and asset price bubbles

This section investigates the impact of the dividend tax on asset price and bubbles. We allow for non-stationary tax (τ_t) and non-stationary dividends (ξ_t). Before starting, a definition of asset bubble is needed. Since Lemma 1 still holds with non-stationary tax rates τ_t, we have the following asset-pricing equation:

$$q_t = \gamma_{t+1}(q_{t+1} + (1-\tau_{t+1})\xi_{t+1})$$

where $\gamma_{t+1} := \max\limits_i \dfrac{\beta_i u_i(c_{i,t+1})}{u_i(c_{i,t})}$ is the discount factor of the economy from date t to date $t+1$. Then, we can decompose the asset price q_0/p_0 (in term of consumption good at the initial date) into two parts:

$$q_0 = \sum_{t=1}^{\infty} Q_t(1-\tau_t)\xi_t + \lim_{T\to\infty} Q_T q_T$$

where $Q_t := \prod_{s=1}^{t} \gamma_t$ is the discount factor of the economy from the initial date to date t. Following Kocherlakota (1992), Santos and Woodford (1997), we define the fundamental value and bubble of asset.

Definition 5. *$\sum_{t=1}^{\infty} Q_t(1-\tau_t)\xi_t$ is the asset fundamental value. Bubbles exist at equilibrium if the asset price exceeds the fundamental value: $q_0 > \sum_{t=1}^{\infty} Q_t(1-\tau_t)\xi_t$.*

Apply the same argument by Montrucchio (2004) and Le Van et Pham (2014) to characterize the existence of bubbles.

Proposition 9. *Bubbles exist (i.e., $\lim\limits_{t\to\infty} Q_t q_t > 0$) if and only if $\sum\limits_{t\geq 1} \frac{(1-\tau_t)\xi_t}{q_t} < \infty$.*

The following result provides conditions (based in exogenous parameters) under which bubbles are ruled out.

Corollary 2. *Assume that $\xi_t = \xi > 0$ for any t. If $f(\xi)F'(\infty) < \delta$ and $\limsup_{t\geq 0} \tau_t < 1$ for any t, then bubbles are ruled out.*

Proof. See Appendix A.7. □

Corollary 2 implies that bubbles are ruled out in a stationary economy and tax rates are bounded below from 1. So, the effect of dividend taxation on the existence of bubbles appears only in non-stationary economy or/and when $\lim_{t\to\infty} \tau_t = 1$.

Proposition 9 and Corollary 2 suggest that bubbles are more likely to exist if (τ_t) are high. In the next section, we will study this effect through some examples.

5.1 Examples of bubbles: the role of dividend taxation

Although there is a large literature on the non-existence of rational bubble in general equilibrium models,[10] few examples of bubbles of assets having positive dividends have

[10] See Tirole (1982), Santos and Woodford (1997) or more recently Le Van and Pham (2016).

been found. In this section, we provide some examples of asset bubbles and look at the role of dividend taxation. Our examples are inspired by Section 6.1 in Le Van and Pham (2016).

Fundamentals. Assume that there are 2 consumers H and F. Let $u_i(c) = ln(c)$, $\beta_i = \beta \in (0,1)$ and $f_i = 0$ for $i = \{H, F\}$ with $\delta \in (0,1)$. Agents' initial endowments are given by $k_{H,0} = 0$, $a_{H,-1} = 0$, $k_{F,0} > 0$ and $a_{F,-1} = 1$, while their profit shares by:

$$\left(\theta^H_{2t}, \theta^H_{2t+1}\right) = (1,0), \quad \left(\theta^F_{2t}, \theta^F_{2t+1}\right) = (0,1) \quad \forall t \geq 0.$$

Focus on a linear production function: $F(K) = AK + B$, where $A, B > 0$ and $\beta(1 - \delta + f(\bar{\xi})A) \leq 1$ where $\bar{\xi} \equiv \sup_t \xi_t$.[11] This production function can be viewed as a particular case of the function $F(K, L) = AK + BL$ with inelastic labor supply (equal to one). Notice that $F_g(G_{t-1}, K_t) = f(\tau_{t-1}\xi_{t-1})(AK_t + B)$ and $\pi_t = f(\tau_{t-1}\xi_{t-1})B$ for any t.

Equilibrium. Let us now construct an equilibrium. The allocations of consumer H are given by

$$k_{H,2t} = 0, a_{H,2t-1} = 0 \tag{26}$$
$$c_{H,2t-1} = (1 - \delta + r_{2t-1})K_{2t-1} + q_{2t-1} + (1 - \tau_{2t-1})\xi_{2t-1} \tag{27}$$
$$k_{H,2t+1} = K_{2t+1}, a_{H,2t} = 1 \tag{28}$$
$$c_{H,2t} = \pi_{2t} - K_{2t+1} - q_{2t} \tag{29}$$

while the allocations of consumer F by

$$k_{F,2t} = K_{2t}, a_{F,2t} = 1 \tag{30}$$
$$c_{F,2t-1} = \pi_{2t-1} - K_{2t} - q_{2t-1} \tag{31}$$
$$k_{F,2t+1} = 0, a_{F,2t} = 0 \tag{32}$$
$$c_{F,2t} = (1 - \delta + r_{2t})K_{2t} + q_{2t} + (1 - \tau_{2t})\xi_{2t}. \tag{33}$$

Prices and the aggregate capital solve the following system: for any t,

$$K_{t+1} + q_t = \frac{\beta}{1+\beta}(F_t(K_t) - r_tK_t) = B_t \tag{34}$$
$$q_{t+1} + (1 - \tau_{t+1})\xi_{t+1} = q_t(r_{t+1} + 1 - \delta) \tag{35}$$
$$q_t > 0, \quad K_t > 0 \tag{36}$$

with $p_t = 1$ and $r_t = f(\tau_{t-1}\xi_{t-1})A$, where $B_t := \frac{\beta f(\tau_{t-1}\xi_{t-1})B}{1+\beta}$.

By using Lemma 3 in Appendix A.8, we can prove that any sequence of allocations and prices satisfying the above conditions is an equilibrium.

The asset fundamental value is equal to $FV := \sum_{s=1}^{\infty}(1 - \tau_s)\xi_s Q_s$ where

$$Q_s := \frac{1}{(1 - \delta + f(\tau_0\xi_0)A) \cdots (1 - \delta + f(\tau_{s-1}\xi_{s-1})A)}$$

[11]Condition $\beta(1 - \delta + f(\bar{\xi})A) \leq 1$ ensures that FOCs are satisfied. This and condition (13) are not mutually exclusive since (13) implies $K_t > 0$. However, in some cases, we do not need (13) to have $K_t > 0$.

is the discount factor of the economy.

It is easy to see that FV is decreasing in τ_t for any t. However, FV is not monotonic in dividend ξ_t while the fundamental value of the asset in Section 6.1 in Le Van and Pham (2016) is increasing in dividends. This difference is from the fact that the interest rates and discount factors in our example depend on dividends through R&D investment.[12] Notice that if $\tau_t = 0\ \forall t$, we recover Le Van and Pham (2016).

To find an equilibrium, we have to find a sequence $(K_{t+1}, q_t)_{t\geq 0}$ satisfying the system (34, 35, 36). To do so, we choose $q_0 \geq FV$ and $(q_t)_{t\geq 1}$ such that

$$q_0 = \sum_{s=1}^{t}(1-\tau_s)\xi_s Q_s + q_t Q_t \tag{37}$$

$$q_t < \frac{\beta f(\tau_{t-1}\xi_{t-1})B}{1+\beta}. \tag{38}$$

Condition (38) ensures that $K_{t+1} > 0$. Condition $q_0 \geq FV$ implies that $q_t > 0$ for any t. Hence, such a sequence $(q_t)_{t\geq 0}$ is a sequence of equilibrium prices because it satisfies the system (34, 35, 36). In this case, a bubble exists when $q_0 > FV$. Summing up, we obtain the following result.

Proposition 10 (continuum of equilibria)**.** *Any sequence (q_t) with $q_0 \in [FV, B_0)$ and $(q_t)_{t\geq 1}$ satisfying (37, 38) is a sequence of equilibrium price.*

If $q_0 = FV$, then the equilibrium is bubbleless.

If If $q_0 > FV$, then the equilibrium is bubbly.

Our result is also related to Tirole (1985) where he shows that there may be continuum of bubbly equilibria. The difference is that Tirole (1985) works in an overlapping generations model without financial frictions while we consider an infinite-horizon general equilibrium model with borrowing constraints.

Let us provide some implications of Proposition 10.

- **Asset bubble and dividend taxes**. Since $q_0 \geq FV$, Proposition 10 indicates that FV is the minimum level above which q_0 is an equilibrium price with bubbles. It is easy to see that FV is decreasing in each τ_t. Thus, we concludes that bubbles are more likely to appear when sequence of tax τ_t increases. The intuition is that, when the tax rates τ_t increases, the after-tax dividend $(1-\tau_t)\xi_t$ decreases and the financial asset fundamental value may turn out to be lower than its price. In this case, an asset bubble arises.

- **Asset price and dividend taxes**. In Proposition 10, let $q_0 = FV + \bar{d}$ with $\bar{d} \in [0, B_0 - FV)$, and then bubbles arise. According to (37), we can compute

[12] Indeed, given ξ_t, we write

$$FV = \Big(\sum_{s=1}^{t-1}(1-\tau_s)\xi_s Q_s\Big) + (1-\tau_t)\xi_t Q_t + \frac{\sum\limits_{s=t+1}^{\infty}(1-\tau_s)\xi_s \frac{Q_s}{Q_{t+1}}}{(1-\delta+f(\tau_0\xi_0)A)\cdots(1-\delta+f(\tau_t\xi_t)A)}$$

The first term do not depend on ξ_t. The second term increases in ξ_t but the last term decreases in ξ_t.

the asset price at date t as follows

$$q_t = \Big((1-\delta+f(\tau_0\xi_0)A\Big)\cdots\Big(1-\delta+f(\tau_{t-1}\xi_{t-1})A\Big)\bar{d} + \sum_{s=t+1}^{\infty}\frac{(1-\tau_s)\xi_s}{(1-\delta+f(\tau_t\xi_t)A)\cdots(1-\delta+f(\tau_{s-1}\xi_{t-1})A)} \quad (39)$$

It is easy to see that q_t is increasing τ_s for any $s \leq t-1$ but decreasing in τ_s for any $s \geq t$.

6 Conclusion

We have proved that a low productivity entails recessions at infinitely many dates. However, when the government taxes the consumers' dividends and spend this fiscal revenue to invest in R&D activities, the productivity of firms is enhanced and recession may be avoided. This happens if: (1) the R&D process is efficient or (2) dividends are high. The economy may grow without bounds when the R&D process becomes very efficient.

Some steady state analyses have been studied. For example, given the objective function is the aggregate consumption, the optimal level of dividend taxation increases in the R&D efficiency the TFP, but decreases in the level of dividends. Moreover, we have also shown that equilibrium indeterminacy may arise. In this case, asset bubbles are more likely to appear if dividend taxes increases.

A Appendix: Formal proofs

A.1 Proof of Proposition 1

The existence of equilibrium. We consider the intermediate economy $\tilde{\mathcal{E}}$ as the economy $\mathcal{E}$ but the government is not taken into account. Denote $\tilde{\xi}_t := (1-\tau)\xi_t$ and the function $\tilde{F}_t$ defined by $\tilde{F}_t(K) := F_g(\tau\xi_{t-1}, K)$. According to Le Van and Pham (2016), there exists an equilibrium $\Big(\tilde{p}_t, \tilde{q}_t, \tilde{r}_t, (\tilde{c}_{i,t}, \tilde{k}_{i,t+1}, \tilde{a}_{i,t})_{i=1}^m, \tilde{K}_t\Big)_{t=0}^{\infty}$ of the economy $\tilde{\mathcal{E}}$, i.e., the following conditions hold:

1. $\tilde{p}_t, \tilde{q}_t, \tilde{r}_t > 0$ for $t \geq 0$.

2. For any $t \geq 0$,

$$\sum_{i=1}^{m}(\tilde{c}_{i,t} + \tilde{k}_{i,t+1} - (1-\delta)\tilde{k}_{i,t}) = \tilde{F}_t(\tilde{K}_t) + (1-\tau)\xi_t \quad (A.1)$$

$$\tilde{K}_t = \sum_{i=1}^{m}\tilde{k}_{i,t} \quad (A.2)$$

$$\sum_{i=1}^{m}\bar{a}_{i,t} = 1. \quad (A.3)$$

3. Optimal consumption plans: for any i, $(\tilde{c}_{i,t}, \tilde{k}_{i,t+1}, \tilde{a}_{i,t})_{t=0}^{\infty}$ is a solution of the problem $(P_i(\tilde{p}, \tilde{q}, \tilde{r}))$.

4. Optimal production plan: for any $t \geq 0$, $\bar{K}_t$ is a solution of the following problem

$$\max_{K_t \geq 0} \Big[\tilde{p}_t \tilde{F}_t(\tilde{K}_t) - \tilde{r}_t \tilde{K}_t\Big]. \tag{A.4}$$

It is easy to see that $\Big(\tilde{p}_t, \tilde{q}_t, \tilde{r}_t, (\tilde{c}_{i,t}, \tilde{k}_{i,t+1}, \tilde{a}_{i,t})_{i=1}^{m}, \tilde{K}_t, G_t, T_t\Big)_{t=0}^{\infty}$, where $G_t = \tau\xi_t, T_t = \tau\xi_t$, is an equilibrium the economy $\mathcal{E}$.

A.2 Proof of Proposition 3

If $K_{t+1} = 0$, we have

$$\begin{aligned}\sum_{i=1}^{m} c_{i,t} &= F(K_t) + (1-\delta)K_t + (1-\tau)\xi, \\ \sum_{i=1}^{m} c_{i,t+1} + K_{t+2} &= F(0) + (1-\tau)\xi.\end{aligned}$$

Therefore, we have

$$\sum_{i=1}^{m} c_{i,t} \geq F(0) + (1-\tau)\xi \geq \sum_{i=1}^{m} c_{i,t+1}. \tag{A.5}$$

Consequently, there exists $i \in \{1, \cdots, m\}$ such that $c_{i,t} \geq c_{i,t+1}$, hence $u_i'(c_{i,t+1}) \geq u_i'(c_{i,t})$. Thus, we have that

$$\frac{1}{(1+b\tau\xi)^{\alpha_1}F'(0) + 1 - \delta} \geq \max_j \frac{\beta_j u_j'(c_{j,t+1})}{u_j'(c_{j,t})} \geq \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} \geq \beta_i$$

So $1 \geq \Big((1+b\tau\xi)^{\alpha_1}F'(0) + 1 - \delta\Big)\beta_i$, contradiction!

A.3 Proof of Proposition 4

We claim that there exists an infinite increasing sequence $(t_n)_{n=0}^{\infty}$ such that $q_{t_n} + (1-\tau)\xi_{t_n} > q_{t_n - 1}$ for every $n \geq 0$.
Indeed, if not, there exists t_0 such that $q_{t+1} + (1-\tau)\xi_{t+1} \leq q_t$ for every $t \geq t_0$. Combining with $\xi_t \geq \xi$ for every $t \geq 0$ and by using induction argument, we can easily prove that

$$q_{t_0} \geq q_{t+t_0} + t(1-\tau)\xi$$

for every $t \geq 0$. Let $t \to \infty$, we have $q_{t_0} = \infty$, contradiction![13]

[13]Our result is still valid if the condition "$\xi_t \geq \xi > 0$ for every $t \geq 0$" is replaced by "$\sum_{t=0}^{\infty} \xi_t = \infty$".

Therefore, there exists a sequence (t_n) such that for every $n \geq 0$, $q_{t_n} + (1-\tau)\xi_{t_n} > q_{t_n - 1}$. Therefore, by assumptions in Proposition 4, we have

$$\frac{q_{t_n} + (1-\tau)\xi_{t_n}}{q_{t_n-1}} > 1 \geq (1 + b\tau\bar{\xi})^{\alpha_1} F'(0) + 1 - \delta.$$

Assume that $K_{t_n} > 0$. According to Lemma 1, we see that

$$\frac{q_{t_n} + (1-\tau)\xi_{t_n}}{q_{t_n-1}} = (1 + b\tau\xi_{t_n})^{\alpha_1} F'(K_{t_n}) + 1 - \delta < (1 + b\tau\bar{\xi})^{\alpha_1} F'(0) + 1 - \delta.$$

This is a contradiction. Therefore, $K_{t_n} = 0$ for any n.

A.4 Proof of Proposition 6

We see that

$$\frac{\beta_i u_i'(c_{i,t})}{u_i'(c_{i,t-1})} \leq \max_j \frac{\beta_j u_j'(c_{j,t})}{u_j'(c_{j,t-1})} \leq \frac{1}{(1 + b\tau\xi)^{\alpha_1} F'(K_t) + 1 - \delta}.$$

Let us denote $B_i(\tau\xi, K_t) := \Big(f(\tau\xi)F'(K_t) + 1 - \delta\Big)\beta_i$. The above inequality implies that $c_{i,t} \geq y_i(B_i(\tau\xi, K_t))c_{i,t-1}$ for any i, where the function $y_i(\cdot)$ is defined in Assumption 1.

Denote

$$x(\tau\xi, K_t) := \min_i y_i(B_i) = \min_i \Big\{ y_i\Big(\beta_i\Big(f(\tau\xi)F'(K_t) + 1 - \delta\Big)\Big)\Big\}.$$

Notice that x_t increases in $b\xi$ (because $f(\tau\xi) = (1 + b\xi\tau)^{\alpha_1}$) but decreases in K_t.

Since $c_{i,t} \geq y_i(B_i(\tau\xi, K_t))c_{i,t-1}$ for any i, we have $C_t \geq x(\tau\xi, K_t)C_{t-1}$. By market clearing conditions, we have

$$C_{t-1} + K_t = f(G_{t-2})F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi \tag{A.6}$$
$$C_t + K_{t+1} = f(G_{t-1})F(K_t) + (1-\delta)K_t + (1-\tau)\xi. \tag{A.7}$$

By consequence, condition $C_t \geq x_t C_{t-1}$ implies that

$$\begin{aligned}
&f(\tau\xi)F(K_t) + (1-\delta)K_t + (1-\tau)\xi \\
&\geq C_t \geq x(\tau\xi, K_t)C_{t-1} = x_t\Big(f(\tau\xi)F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi - K_t\Big).
\end{aligned}$$

From this, we have

$$\begin{aligned}
\frac{f(\tau\xi)F(K_t) + (1-\delta)K_t + (1-\tau)\xi}{x(\tau\xi, K_t)} + K_t &\geq f(\tau\xi)F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi \\
&\geq (1-\tau)\xi.
\end{aligned}$$

Thus, we get

$$\frac{f(\tau\xi)F(K_t)}{x(\tau\xi, K_t)} + \frac{(1-\delta)K_t}{x(\tau\xi, K_t)} + K_t \geq (1-\tau)\xi \frac{x(\tau\xi, K_t) - 1}{x(\tau\xi, K_t)}. \tag{A.8}$$

By definition, $x(\tau\xi, K_t)$ decreases in K_t, and so does $\dfrac{x(\tau\xi, K_t) - 1}{x(\tau\xi, K_t)}$. Hence, we have: for each $\xi > 0$, there exists a unique $K(\xi)$ such that

$$\frac{f(\tau\xi)F(K(\xi))}{x(\tau\xi, K(\xi))} + \frac{(1-\delta)K(\xi)}{x(\tau\xi, K(\xi))} + K(\xi) = (1-\tau)\xi\frac{x(\tau\xi, K(\xi)) - 1}{x(\tau\xi, K(\xi))}. \tag{A.9}$$

Since $\sigma_i < 1$ for any i, we see that $\dfrac{f(\tau\xi)}{x(\tau\xi, K(\xi))}$ is decreasing in ξ. Combining with the fact that $x(\tau\xi, K(\xi))$ is increasing in ξ, we get that $K(\xi)$ is increasing in ξ.

Moreover, by using Assumption 1 and definition of $K(\xi)$, we see that $\lim_{\xi\to\infty} K(\xi) = \infty$. Consequently, there exists $\bar{\xi} > 0$ such that $K(\bar{\xi}) > \bar{k}$ for any t. Therefore, $K_t > \bar{k}$ for any t.

A.5 Proof of Proposition 7

We see that

$$\frac{\beta_i u_i'(c_{i,t})}{u_i'(c_{i,t-1})} \leq \max_j \frac{\beta_j u_j'(c_{j,t})}{u_j'(c_{j,t-1})} \leq \frac{1}{(1+b\tau\xi)^{\alpha_1}F'(K_t) + 1 - \delta} \leq \frac{1}{(1+b\tau\xi)^{\alpha_1}A + 1 - \delta}$$

where the last inequality comes from the fact that $F'(x) \geq A$ for any x.

Let us denote $B_i := \Big(f(\tau\xi)A + 1 - \delta\Big)\beta_i$. The above inequality implies that $B_i u_i'(c_{i,t}) \leq u_i'(c_{i,t-1})$. According to Assumption 1, we get that $c_{i,t} \geq x_i c_{i,t-1}$ for any i, where $x_i \equiv y_i(B_i)$. Denote $x := \min_i y_i(B_i)$. Using the same argument in the proof of Proposition 6, we have

$$\begin{aligned}
&\frac{f(\tau\xi)F(K_t) + (1-\delta)K_t + (1-\tau)\xi}{x} + K_t \geq f(\tau\xi)F(K_{t-1}) + (1-\delta)K_{t-1} + (1-\tau)\xi \\
&\quad\Rightarrow f(\tau\xi)F(K_t) + (1-\delta)K_t + xK_t \geq xf(\tau\xi)F(K_{t-1}) + x(1-\delta)K_{t-1} + (1-\tau)\xi(x-1) \\
&\quad\qquad \geq xf(\tau\xi)F(K_{t-1})
\end{aligned}$$

where the last inequality is based on condition $x > 1$.

Since $F(K) \geq F'(K)K \geq AK$, we have that

$$f(\tau\xi)F(K_t) + (1-\delta)K_t + xK_t = F(K_t)\Big(f(\tau\xi) + \frac{(1-\delta+x)K_t}{F(K_t)}\Big) \tag{A.10}$$

$$\leq F(K_t)\Big(f(\tau\xi) + \frac{1-\delta+x}{A}\Big). \tag{A.11}$$

Thus, we get that

$$\frac{F(K_t)}{F(K_{t-1})} \geq \frac{xf(\tau\xi)}{f(\tau\xi) + \frac{1-\delta+x}{A}} > 1. \tag{A.12}$$

By consequence, we obtain that $\lim_{t\to\infty} F(K_t) = \infty$, and hence $\lim_{t\to\infty} K_t = \infty$.

A.6 Proof of Lemma 2

Let $\Big(p, q, r, (c_i, k_i, a_i)_{i=1}^m, K, G, T\Big)$ be a steady state equilibrium. By FOCs, there exists $x_i \geq 0$, and $y_i \geq 0$ such that

$$1 = (r + 1 - \delta)(\beta_i + f_i x_i) + y_i \tag{A.13}$$
$$q = (q + (1 - \tau)\xi)(\beta_i + x_i) \tag{A.14}$$
$$k_i y_i = 0, \quad x_i\Big((q + (1 - \tau)\xi)a_i + f_i(1 - \delta + r)k_i\Big) = 0. \tag{A.15}$$

According to (A.14) and $\beta_1 > \beta_i$ for any $i \geq 2$, we have $x_1 = 0$ and $x_i > 0$ for any $i \geq 2$ which implies that $(q + (1 - \tau)\xi)a_i + f_i(1 - \delta + r)k_i = 0$ for any $i \geq 2$.

Since $F'(0) = \infty$, we have $r + 1 - \delta = \frac{q+(1-\tau\xi)}{q} = \frac{1}{\beta_i + x_i}$. According to (A.13), we obtain that, for any i,

$$1 = \frac{\beta_i + f_i x_i}{\beta_i + x_i} + y_i \tag{A.16}$$

For each $i \geq 2$, since $x_i > 0$, and $f_i < 1$, we obtain that $y_i > 0$. Therefore, we get that $k_i = 0$, and hence $a_i = 0$ for each $i \geq 2$. So, we can compute $c_i = \theta_i \pi$ for each $i \geq 2$.

Since $F'(0) = \infty$ we have $K > 0$, so $k_1 = K > 0$. According to (A.13), we see that K is determined by

$$1 = \Big(f(\xi\tau)F'(K) + 1 - \delta\Big)\beta_1. \tag{A.17}$$

It is now easy to obtain that $a_i = 1$ and $c_1 = (r - \delta)K + \theta_1\pi + (1 - \tau)\xi$.

A.7 Proof of Corollary 2

According to (6), we have

$$C_t + K_{t+1} + G_t = f(G_{t-1})F(K_t) + (1 - \delta)K_t + \xi_t \tag{A.18}$$
$$= f((1 - \tau_{t-1})\xi_{t-1})F(K_t) + (1 - \delta)K_t + \xi_t \tag{A.19}$$
$$\leq f(\xi)F(K_t) + (1 - \delta)K_t + \xi. \tag{A.20}$$

Therefore, $K_{t+1} < f(\xi)F(K_t) + (1-\delta)K_t + \xi$. Since $f(\xi)F'(\infty) < \delta$, it is easy to prove that the capital stock (K_t) is uniformly bounded from above.

By using $\sum_{t=1}^{\infty} Q_t(1 - \tau_t)\xi < \infty$ and $\limsup_t \tau_t < 1$, we get that $\sum_{t=1}^{\infty} Q_t < \infty$ and hence $\lim_{t\to\infty} Q_t = 0$.

Since (K_t) is uniformly bounded from above we have $\lim_{T\to\infty} Q_T k_{i,T+1} = 0$ for any i, and

$$\sum_{t=1}^{\infty} f(\tau_t\xi)F(K_t)Q_t \leq \sum_{t=1}^{\infty} f(\xi)F(K_t)Q_t < \infty.$$

We can prove that there is no financial asset bubble by using the argument in the proof of Proposition 8 in Le Van and Pham (2016).

A.8 A sufficient condition for the equilibrium

Let us denote $I := \{1, 2, \dots, m\}$. We give sufficient conditions for a sequence

$$\Big(p_t, q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i\in I}, K_t, G_t, T_t\Big)_t$$

to be an equilibrium. This result is used in our examples in Section 5.1. Notice that the utility may satisfy $u_i(0) = -\infty$.

Lemma 3. *Let $f_i = 0$ for any i. A sequence $\Big(p_t, q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t})_{i\in I}, K_t, G_t, T_t\Big)_t$ is an equilibrium, if the sequence $\Big(p_t, q_t, r_t, (c_{i,t}, k_{i,t+1}, a_{i,t}, \zeta_{i,t}, \epsilon_{i,t})_{i\in I}, K_t, G_t, T_t\Big)_t$ satisfies the following conditions.*

(i) *For any i and t, $c_{i,t} > 0$, $k_{i,t+1} \geqslant 0$, $a_{i,t} \geqslant 0$, $\zeta_{i,t} \geqslant 0$ and $\epsilon_{i,t} \geqslant 0$.*
For any t, $p_t = 1$, $q_t > 0$ and $r_t > 0$.

(ii) *The first-order conditions hold*

$$\frac{1}{r_{t+1} + 1 - \delta} = \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} + \zeta_{i,t}$$
$$\frac{q_t}{q_{t+1} + (1 - \tau_{t+1})\xi_{t+1}} = \frac{\beta_i u_i'(c_{i,t+1})}{u_i'(c_{i,t})} + \epsilon_{i,t}$$

with $\zeta_{i,t} k_{i,t+1} = 0$ and $\epsilon_{i,t} a_{i,t} = 0$.

(iii) *The transversality conditions are satisfied:*

$$\lim_{t\to\infty} \beta_i^t u_i'(c_{i,t}) k_{i,t+1} = \lim_{t\to\infty} \beta_i^t u_i'(c_{i,t}) q_t a_{i,t} = 0.$$

(iv) *For any t, $F_g(G_{t-1}, K_t) - r_t K_t = \max_{K:K\geq 0}\{F_g(G_{t-1}, K) - r_t K\}$.*

(v) *For any t, $c_{i,t} + k_{i,t+1} - (1-\delta)k_{i,t} + q_t a_{i,t} = r_t k_{i,t} + (q_t + (1-\tau_t)\xi_t)a_{i,t-1} + \theta_t^i \pi_t$ where $\pi_t = F_g(G_{t-1}, K_t) - r_t K_t$.*

(vi) *For any t, $K_t = \sum_{i\in I} k_{i,t}$.*

(vii) *For any t, $\sum_{i\in I} a_{i,t} = 1$.*

(viii) *For any t, $G_t = T_t = (1-\tau_t)\xi_t$.*

Proof. The proof is left to the reader. □

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