The PPP Puzzle: An Update

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Abstract

We show that the Purchasing Power Parity (PPP) puzzle, whereby the half-life of the shock to the real exchange rate is long and unjustifiable by monetary and financial shocks, is a result of specification and estimation issues. We provide an alternative specification for PPP and show that the half-life of the shock could be as short as 6.8 months and as long as 2 years, which is considerably shorter than what have been reported in the literature.

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1. Introduction

The idea of the absolute version of PPP is that the *equilibrium* value of the exchange rate between two currencies should be equal to the ratios of the countries price level.[2] Taylor and Taylor (2004) quoted Cassel (1922, p.138) who wrote, "our willingness to pay a certain price for a foreign money must ultimately and essentially depend on the fact that this money has a purchasing power as against *commodities and services* in the foreign country…"[3]

Cassel (1961, 1918, 1921, 1928), Keynes (1923, p. 95 and p.97) and Samuelson (1964, p.147) saw the PPP as an equilibrium concept, where short-run deviations occur, and that parity could be restored in the long run.[4] The length of the deviations from equilibrium depends on the nature and the permanency of the shocks. They agreed that deviations from equilibrium are more persistent if the dominant shocks are not monetary or financial shocks or they are *real.*

Following that logic, Obstfeld and Rogoff (2001) characterized the PPP as a puzzle. It has been identified as one of the six major puzzles in international economics. The puzzle is pertinent to the lengthy deviations from equilibrium. They used monthly data from 1973 to 1995 for Canada, France, Germany, Japan and the United States and constructed values for the real exchange rates, $q = sp^* / p$, where s is the exchange rate and, p^* and p are the CPI of the foreign country and at home respectively. They estimated values for the AR(1) coefficient λ in the equation $\log q_t = \alpha + \eta t + \lambda \log q_{t-1} + \varepsilon_t$, to be ranging from 0.99 to 0.97, which imply half-lives of the shock h (i.e., $\lambda^h = 0.5$) 69 months in the U.S. – Canada to 21 months in Germany – Japan, and the mean half-life across the real exchange rates was 39 months or 3¼ year. They argued that these half-lives are long and inconsistent with the fact that monetary and financial shocks play a major role in explaining the high volatility observed in the exchange rate. They say that it is

[2] In the *relative version* of PPP, the rate of change of the exchange rate is equal to the ratio of the change in the price level at home and abroad.

[3] The same paragraph was quoted as Cassel (1921, p. 36) in Katseli-Papaefstratiou (1979, p. 13).

[4] Cassel (1918) recognized large deviations from PPP. His paper was titled "Abnormal Deviations in International Exchanges."

"hard to imagine what source of nominal rigidity could be so persistent as to explain the prolongation of real exchange rate deviations."

The progress in econometrics over the past thirty years added hundreds of papers to the PPP's empirical literature. The general idea is that the nominal exchange rate and the CPI's at home and abroad have unit roots (i.e., highly persistent), but could be cointegrated in a long span of data. Cointegration means that deviations from PPP (the real exchange) are I (0), hence the real exchange rate is mean reverting in the long run. Therefore, the half-life requires that λ is different from unity (because as λ approaches unity, the half-life of the shock approaches infinity).

However, the puzzle is more about the length of the half-life of the deviations from PPP, which is a function of the estimated λ only. There has been many attempts to explain the puzzle. Commonly used tests for unit root have low powers against stationary alternatives, and they may not be able to distinguish between small variations in the size of λ , which can produce a significant difference in the length of the half-life. Here are some examples of the literature. Frankel (1986) and Kim (1990) among many others provided evidence that the real exchange rate is mean reverting. Lothian and M. Taylor (1996) used two centuries of data to show that the half-life is 3 to 6 years. A. Taylor (2002) used 100-year data for 20 countries and reported support for the long-run PPP. However, Engel (2000) and Chen and Engel (2005) for example showed that unit root tests produce biased results. Cheung and Lai (1993) and Cheung and Lai (2000) examined the dynamic adjustments of the real exchange rate using impulse response functions instead of a point estimate to shed more light on the puzzle, and Balli *et al.* (2013) found that structural breaks in time series could explain the high estimated half-life of the shock to the real exchange rate.[5]

Imbs *et al.* (2005) showed that there is an aggregation bias in panel data relative prices and that accounting for price heterogeneity solves the PPP puzzle. We use time series data. The objective

[5] Most of the linear tests for unit roots including the ADF which is used the most in these analysis confuse breaks in the time series with unit root simply because they are linear tests. They fit a straight line through the data. Accounting for the breaks in the times series changes the test results completely.

of this paper is to show that the estimates of the half-life of the shocks in Obstfeld and Rogoff (2001) are misleading because the estimates of λ are subject to a number of specification and estimation problems, which affect the conclusion about the PPP puzzle. Specification issues include: (1) the specific regression equation used to estimate the AR (1) coefficient (λ) of the deviations from PPP (the real exchange rate) has distributional implications that are not innocuous; (2) specifying the PPP in terms of the CPI has been debated throughout the history of this literature. Estimation issues include, most importantly, the issue of causality and whether the use of OLS to estimate the PPP equation is appropriate or not. We show that addressing these specification and estimation issues leads to a shorter half-life of the shocks. We show that the half-lives of the shocks to the real exchange could be as short as 6.8 months.

Next, we discuss the specification and estimation issues, then examine the Canadian, Australian, and the New Zealand – U.S. dollar PPP with different specifications and estimations, provide estimates for the half-life of the shocks to deviations from PPP, and compare with those reported earlier in this literature. Section 3 is a short conclusion.

2. Specification and estimation issues

The first specification issue concerns the equation of the real exchange rate used by Obstfeld and Rogoff (2001). They measured the real exchange rate $q_t = s_t p_t^* / p_t$. Then the half-life of the shock is a function of λ in the equation $\log q_t = \alpha + \eta t + \lambda \log q_{t-1} + \varepsilon_t$. The use of time trend in this equation is also unclear since there is no trend in the PPP equation. The value of λ , which determines the half-life estimate, depends on the specification of the q . Under the null hypothesis of a unit root, the specification $\log q_t = \alpha + \eta t + \lambda \log q_{t-1} + \varepsilon_t$ is *distributionally different* from the same equation without trend (e.g., $\log q_t = \alpha + \lambda \log q_{t-1} + v_t$), and the same equation without a constant and a trend (e.g., $\log q_t = \lambda \log q_{t-1} + u_t$).[6] Assuming that these specifications are the same and do not affect the value of λ is misleading. We will demonstrate

[6] Commonly used tests for unit root tabulate different critical values for the parameter estimates of these specifications, which are non-standard.

that these specifications produce different estimates of λ , hence different estimates of the half-life of the shock.

The second specification issue concerns the measurement of prices. There is a very old debate about the use of the price indices in testing the PPP. In the spatial-arbitrage interpretation of the PPP, the PPP coincides with the “law of one price”. Perfect arbitrage ensures that the price of *each traded* commodity is equalized across countries. The absolute version of the PPP holds under perfect information in the commodity markets, the absence of (or negligible amount of): transportation cost, trade barriers, government interventions in the markets, and price discrimination behavior. For example, articles about PPP such as Dornbusch and Krugman (1976), Wihlborg (1978), Kravis and Lipsey (1971, 1974, and 1978), and Dornbusch (1980) are consistent with the spatial arbitrage condition. Haberler (1975, p.24) argued that *tradable* good prices is the foundation of PPP and that PPP holds when *all commodities* have the same price in both countries.

The question is that since the law of one price holds for a single commodity price, would it also hold for any equally weighted price index? Katseli-Papaefstration (1979) cites Samuelson (1964, p.147) who argued that if transport cost is zero, no trade barriers, and the accounting methods are the same across countries then “every ruling exchange rate would turn out to be the PPP equilibrium.” Obviously these assumptions are questionable. Engel and Rogers (1996) provided empirical evidence that these assumption matter.

But to justify testing the spatial-arbitrage PPP using aggregate price indices such as the CPI, WPI, and GNP deflator, it has been assumed that traded and non-traded good price are highly positively correlated. Essentially, it is assumed that there is a high degree of substitutability between these good. Balassa (1964), however, pointed out that high productivity growth in the non-tradable sector in developed countries relative to that in the developing countries requires an increase in the ratio of traded to non-traded goods in the developed countries. Thus, high substitutability between traded and non-traded goods is not a sufficient condition for the use of general price indices to test PPP.[7]

The case for using the CPI in the PPP specification is weak. The CPI contains non-traded goods and services prices, e.g., haircut price, and prices of inputs that are non-traded goods, which have nothing to do with the exchange rate. So instead of the CPI we argue for using commodity prices, which are more consistent with the theory to test the puzzle. Specifically, a country's trade is usually dominated by a single or a few commodities. In the case of Canada for example, crude oil is the major export. It is about 25 percent of total exports and 10 percent of GDP. Thus, the ratio of the Canadian price of oil relative to the international price of oil would be fluctuating around 1, which is consistent with PPP. The Canadian dollar is mostly affected by the price of oil. The same is true for Australia, where coal and iron ore dominate trade, dairy is in the case of New Zealand, and copper is in Chile…etc. Even commodity price indices are more suitable to test the PPP than general price indices because, presumably, the weight on the dominant traded commodity is high.

Commodity prices are also driven by macroeconomic shocks (Joets et. al, 2015). Dwyer *et al.* (2011) show that the volatility of commodity prices are determined by financial investments in commodity derivative markets. Frankel (2006) investigates the relationship with the interest rate and the desire to carry commodity inventories. Gruber and Vigfusson (2012) show that commodity price volatility attributable to transitory shocks declines with interest rates. At least in our sample of countries, we will show that commodity price ratios have higher variance than the CPI ratios, and that matches the high variance of the exchange rate better.[8]

There are also some estimation issues if one decides to estimate the PPP equation first, by regressing the exchange rate on an intercept and the price ratio, then use the residuals as a measure of the deviation from PPP. In this case the estimated coefficients of the constant term

[7] Officer (1976, p.22) says that the same is true for the relative version of the PPP if there is a symmetric change over time in the developed countries' productivity. Essentially what they say is that structural differences across countries affect relative prices, hence these indices complicate the testing of the PPP.

[8] The volatility changes across different sample periods because the volatility of the shocks that determine the volatility of commodity prices could be different.

and the slope may differ from zero and one. The PPP does not hold if the slope is different from one. In this case the deviation from PPP is not an appropriate measure of the real exchange rate. We don't know what Obstfeld and Rogoff (2001) result would look like had they estimated the PPP rather than imposing a value of zero on the intercept and one on the slope to obtain q . The intercept could be picking up the effects of some shifting variables. If some macro variables, monetary policy shocks, real productivity shocks etc. cause the price ratio, and since they are not present in the PPP equation, we have an omitted variable problem. A single-equation OLS regression would produce a biased and inconsistent parameter estimates.

A second estimation issue is whether we estimate λ using a parametric regression method or a non-parametric partial autocorrelation function. That seems to account for big differences in the estimates of the half-life.

Finally, there is the issue of causality. This is also an old issue in the PPP literature. In the early studies, for example, Cassel (1916) and Keynes (1923), which were reviewed by Officer (1976), causality was implicit, i.e., the price ratio causes the exchange rate. In this case one could justify running an OLS regression of the exchange rate on prices. Then came the Quantity Theory of Money in open economy and models of the Monetary Approach to the Balance of payments, where monetary disturbances affect the price level in the short run, and the changes are fully offset by changes in the nominal exchange rate in the long run, e.g., Frenkel (1976). Keynes (1923, p. 95), also Samuelson (1964), and Officer (1976), agreed that monetary shocks could disturb the equilibrium in the short run, and that the equilibrium could be reestablished later. Keynes (1923, p. 97) also argued that if the shocks were not monetary, such as changes in capital or the relative efficiency of labor, then the deviations may become permanents. Samuelson (1964) argued that if monetary shock do not dominate and if other real shocks dominate, PPP may not hold.

In another interpretation of the PPP within the Asset-Market view (e.g., Dornbusch 1976), whereby the exchange rate and prices are endogenously and simultaneously determined, expectations about the future exchange rate could cause significant departure from PPP.

These models and theories imply that the PPP is treated as a reduced-form relationship. MacDonald and Marsh (1997), for example, used a simultaneous equation model and showed that it could significantly outperform the random walk forecasts. But generally speaking, OLS estimates of the single-equation PPP will produce a biased and inconsistent parameter estimates, i.e., a single-equation problem. One other useful way to resolve the issue is to use an instrumental variable estimator such as GMM for example.

Recent development about the same issue of causality is found in Chen, Rogoff, and Rossi (2010), who argue, contrary to all of the theories above, that the exchange rate predicts commodity prices and not the reverse. They argue that causality runs (in Granger sense) from the exchange rate to commodity prices because the exchange rate is viewed as an asset price, which bears expectations about the value of its future fundamentals such as commodity prices. They also argued that financial markets for commodities tend to be far less developed and much more regulated than for the exchange rate. Therefore, they conclude that commodity prices are less accurate indicators of future conditions than the exchange rate. Finally, they suggest that it is extremely difficult to find an exogenous measure of terms-of-trade.

Frenkel (1978, p. 183) says that causality, in the economic sense, could not be tested empirically by running an OLS regression, "there is no statistical method that is capable of determining causality…" We don't believe that Clive Granger intended to test for causality *per se*; but rather test whether, or not, some past information of a particular variable have predictive power of another. Coining his method as a test for "causality" is also misleading.

We will examine the reverse causality suggested by Chen *et al* by estimating the PPP equation using GMM. We run two specifications. First, we estimate the PPP using commodity prices. The instruments are the changes in temperatures. Changes in temperature are more correlated with commodity prices than with the exchange rate. Then we reverse the regression, i.e. by having the exchange rate as a regressor, which is not correlated with the instruments. We show that the differences in the slope estimates in these two regressions indicate that causality probably runs from commodity prices to the exchange rate.

We begin with Canada. We fit two specifications. One is with the CPI's and the other is with the commodity price index.[9] Similarly, we also examine the Australian PPP and New Zealand PPP.

The data appendix reports the mean and standard deviation and plots the data. The sample is Jan 1999 to August 2016. It is very important to note that the standard deviation of the exchange rate is almost the same as that of the commodity price ratio in Canada, Australia, and New Zealand. The standard deviations of the commodity price ratios are significantly higher than that of the CPI ratio in Canada, and closer to that of the exchange rate. This is important because it has been argued before (Flood and Rose, 1995) that the standard deviation of the fundamentals are significantly smaller that standard deviation of the exchange rate, hence the fundamentals' failure to explain the variation in the exchange rate. Flood and Rose do not include commodity prices in their set of fundamentals. The standard deviation of the exchange rate is 0.12, for the CPI's ratio is 0.01, and for the commodity price ratio is about 0.10. Therefore, commodity prices have larger variance than the CPI. The plots of the Confidence Ellipse show stronger correlation between the exchange rate and the commodity price ratio than with the CPI's ratio. The same is true for Australia, whether we define commodity prices in terms of coal or oil prices. For New Zealand, the log-differenced data fit the PPP very well.

Finally, the data exhibit trend. Whether the trend is stochastic or deterministic is a relevant specification issue. Nevertheless, economists seem to agree that the nominal exchange rate and prices have unit root regardless of the low power of the tests.

Canada

We define the PPP as the average Canadian crude price to international crude prices. Canadian average crude price time series data are not readily available; a good proxy would be the Canadian commodity price index. Clearly this index has a large weight on the crude oil price as we explained earlier. The world crude prices such as the West Texas Intermediate, Brent, and

[9] Obstfeld and Rogoff (2001) also examined the Japanese, the German, and the French currencies from 1973 to 1995. The Japanese – USD exchange rate is subject to significant joint governments' interventions so we do not use it. Germany and France switched to the Euro.

Dubai move very closely together. We use West Texas Intermediate price of oil (WTI) as a proxy. Figure (1) plots the Canadian commodity price index and WTI price of oil. This price ratio is denoted p_t^c / p_t^o, where the superscript c denotes commodity price and the superscript o denotes the price of oil.

We examine the PPP by regressing: (1) the exchange rate s_t defined as the U.S. dollar price of the home currency on the CPI price ratio p_t / p_t^* and an intercept, where p_t is the CPI price level. Asterisk denotes the foreign price; and (2) on p_t^c / p_t^o as defined above. The deviations from PPP equations above represent $\hat{q}_t$, i.e., the estimated residuals.

Table (1) reports four regressions for Canada (column 2 to column 5). Two of the regressions are OLS regressions and the other two are GMM regression. The first regression is the OLS regression of exchange rate on the CPI's ratio. The second regression is the OLS regression of the exchange rate on the commodity price ratio as defined earlier (the Canadian commodity price index / the price of oil index).

In the CPI-based PPP, the slope coefficient is significantly different from one, while it is insignificantly different from one in the commodity price-based PPP. The p-values to test the null hypothesis that the coefficient is different from 1 are in squared brackets. The intercepts are significant in both regressions. Non-zero intercept indicates that the intercept might be picking up the effects of some omitted variables that explain the fluctuations in the exchange rate that are not explained by the price ratios; those could be policy interventions, changes in capital flows, transport cots and other shifters.

GMM regressions are in columns 4 and 5. We use 12 lags of the average changes of world temperature and 12 lags of the average changes in the Northern Hemisphere temperature as instruments to deal with the simultaneity issue. The last regression is the reverse of the third, where the dependent variable is the commodity price ratio and the independent variable is the exchange rate. These regressions test Chen *et al.* (2010) proposition that causality running from the exchange rate to commodity prices. Temperature is strictly exogenous, negatively correlated with commodity prices (oil) and not with the error term. For example, as the temperature rises, the demand for oil falls, and so does the price. We plot the temperature data in the data appendix.[10]

The slope coefficient in the reversed regression is much smaller than one, and statistically significant. The p values of the J statistics are very large; we cannot reject the over-identifying restrictions. These results suggest that the commodity price-based PPP specification estimated by GMM explains nearly 80 percent of the variation in the exchange rate, that the slope coefficient is one, and that causality runs from commodity prices to the exchange rate as suggested by the theory.[11] Thus, imposing absolute PPP with a zero intercept on the data as in Obstfeld and Rogoff (2001) is questionable. It is also unjustifiable to have a time trend in their specification of the real exchange rate.

Figures (2) plots the actual, fitted, and the residuals of the OLS regressions. Figure (3) plots the same data for the GMM regressions. Visually, the residuals, i.e., the estimated deviations from PPP are trendless.

Next we estimate the half-life of the shock to the real exchange rate.

Half-life

We formally test the deviation from PPP for unit root. We use the ADF as originally recommended by Engle and Granger (1987).[12] The ADF test does not reject the null hypothesis often. But we hope to reject the null hypothesis that the residuals have a unit root because we observe no trend in the plot. That said, the power of the test is irrelevant if it rejects the null hypothesis.

[10] The cross-correlations between the lags and leads of the world and the Northern Hemisphere average changes in temperature die off quickly, even though the contemporaneous correlation is high.

[11] The regressions specified in logs give similar results, but the magnitudes of the slope coefficient are smaller, -0.70 and 0.68 in second OLS and the third GMM regressions respectively. The hypothesis that it is equal one is rejected.

[12] Johansen (1988) maximum likelihood test for the case of two variables and one cointegrating vector like this case is the square of the ADF test.

Table (2) reports the ADF test for unit root for different specifications of the real exchange rate. We begin with the residuals of equation (1). We test the residuals from the OLS and the GMM regressions. Then we construct and test additional number of specifications of the real exchange rate, which impose the PPP on the data just as in Obstfeld and Rogoff (2001):

$$q = (sp^* / p) \qquad (1)$$

$$q = (sp^o / p^c) \qquad (2)$$

$$\ln(q) = \ln(sp^* / p) \text{ and} \qquad (3)$$

$$\ln(q) = \ln(sp^o / p^c), \qquad (4)$$

where p_t is the CPI, p_t^c is the commodity price, p_t^o is the price of oil, and asterisk denotes the U.S. CPI. We report the estimated λ (the coefficient of q_{t-1} in the ADF regression) and the estimated half-life of the shock to the real exchange rate. The table has seven columns. Each column has a heading, which indicates which specification of q_t is being tested. The first column lists the specification of the ADF regression. The ADF regression is an OLS regression. The second column, for example, tests the residuals of the OLS regression of PPP, which we reported in table (1). We test for unit root using the ADF with intercept, with an intercept and a trend, and without an intercept and a trend. We then compute the half-life of the shock. In the last row, we also estimate the partial autocorrelation function to estimate the value of the autocorrelation coefficient λ and use that to compute the half-life. The rest of the columns follow the same pattern. Note that when the value of λ approaches 1, the half-life approaches infinity, and we have NA in that space.

For the ADF we use a variety of Information Criteria to determine the lag structure. In addition to the ADF, we used a number of commonly used unit root tests, but we do not report the results because they are similar and take more space. The unit root hypothesis for the GMM residuals is rejected.[13] The half-life of the shock estimates are very sensitive to the values of the estimates of

[13] A stronger test for cointegration is to examine the significance of the error-correction coefficient. For cointegration, this coefficient should be highly significant with a large t-value. Unfortunately, it is not. Therefore, cointegration is hard to confirm. This fact notwithstanding, the estimated autocorrelation coefficient of the real

λ, which vary from one specification to another. Figure (4) plots the distribution of the half-life estimates for Canada.

The results in tables (1) and (2) suggest that: (1) A commodity price-based PPP specification fits the data better than a CPI-based PPP specification for a commodity-exporting country such as Canada. The variance of the price ratio matches the variance of the exchange rate when prices are defined as commodity prices rather than the CPI. (2) The GMM estimator suggests that causality runs from commodity prices to the exchange rate, not the reversed. (3) The deviations from the commodity price-based PPP are trendless and the estimated AR (1) coefficients are significantly different from one. (4) The half-life estimates of the real exchange rate could be as low as 6.8 months. These results cast doubts about the PPP puzzle.

Next, we examine PPP in the Australian and New Zealand data because both countries are also major, albeit different, commodity exporters.

Australia

We showed that the CPI-based PPP does not fit the Canadian data so we do not repeat this regression for Australia. Similar to Canada, we plot the Australian commodity price index, which has a large weight on Australia's major exports of coal, and the average international price of coal index in figure (5). Again, the ratio clearly fluctuates around one, which is consistent with PPP.

Table (3) reports the estimates for Australia. These estimates are not different from those of Canada. The commodity prices of Australia relative to the price of coal are used because Australia is largely a coal exporter. However, the results do not change significantly when we use the price of oil instead of coal. The intercept is also significant and the slope coefficient is insignificantly different from one. Figure (6) plots the actual, fitted values, and the residuals. Again, the deviation from this PPP is trendless. Table (4) reports the half-life analysis for

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exchange rate measured by the deviations from PPP is smaller than any other estimate, hence produces a significantly shorter half-life.

Australia.[14] Figure (7) plots these estimates. The results indicate that the commodity price-based PPP is associated with small half-life estimates of 8 months compared with the specification preferred by Obstfeld and Rogoff (2001).

New Zealand

Finally, we fit the New Zealand data. For New Zealand is a primary commodity exporter, we use the ANZ commodity price index for New Zealand relative to the world commodity price index.[15] Figure (8) plots the data, and again they fluctuate around 1, which is what PPP predicts. However, for New Zealand, the log-differenced variables seem to fit the model best.[16] We report the results in table (5). In the level regression, the intercept is significant just like in the previous cases, but the slope coefficient is significantly different from one, which is different from the Canadian and the Australian PPP. In the first-differenced regression, however, the intercept is zero as we would expect, the slope is one, and the joint hypothesis that the constant term is zero and the slope is one could not be rejected. Figure (9 a) and (9 b) plot the actual, fitted values, and the residuals. Table (6) reports the estimates of λ for the real exchange rate, and the half-life of the shocks.[17] In general, the results reveal a relatively significant persistence of shocks in New Zealand. Most plausibly, the persistence is present in the data because New Zealand is a primary commodity exporter. Figure (10) plots the half-life estimates, which correspond to the deviations from the GMM commodity price-based PPP, the GMM commodity price-based PPP in log-differenced specification, and the $\ln(q) = \ln(sp^* / p)$. Just like the Canadian and Australian cases, the latter specification is associated with the longest half-life. The half-life of the New Zealand real exchange rate shocks could be 19 to 22 months.

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[14] In addition to the ADF tests, the error correction model has very significant lagged residuals. The t-stat is very large, which is a strong test for cointegration (not reported).

[15] https://www.anz.co.nz/about-us/economic-markets-research/commodity-price-index/

[16] That might indicate *relative* PPP is the true underlying model of the data.

[17] The p values for the unit root test of the residuals are significant at the 90 percent level, which reject the unit root in the real exchange rate. Furthermore, the error correction model results in very significant lagged residuals. The t-stat is very large, which is a strong test for cointegration (not reported).

Figure (11) plots the half-life estimates based on deviations from the GMM commodity price-based PPP for the three countries. The Canadian's deviation from the commodity price-based PPP have the shortest half-life, followed by Australia, and New Zealand. Although New Zealand has the longest half-life estimates, they are still half the magnitude of the average half-life reported in Obstfeld and Rogoff (2001).

3. Conclusions

The PPP puzzle is about the long length of the half-life of the real exchange rate measured by the deviations from PPP, which is unjustified by the monetary and financial shocks. The half-life is a function of the estimated autocorrelation coefficient of these deviations. So it matters how these deviations are measured, i.e., whether the PPP is imposed on the data like in Obsfeld and Rogoff (2001) or they are the residuals of a particular specification of PPP. The estimated half-life also depends on the specification of the real exchange rate's equation used to estimate the autocorrelation coefficient matters more. Under the null hypothesis of a unit root, the specifications of the ADF equation whether it has a constant term and a linear trend, no trend, or no trend and no intercept, are distributionally different. They lead to different estimates of the autocorrelation coefficient. The implicit assumption that these specifications are the same is misleading.

We showed that PPP specified in terms of relative commodity prices fit the data of three commodity-exporting countries, Canada, Australia and New Zealand, better than relative CPIs. To resolve the potential simultaneity between the exchange rate and commodity prices, we use the GMM estimator with temperature as instrument. Temperature is exogenous and (negatively) correlated with commodity price, especially oil and gas, and uncorrelated with the error terms. These regressions produce trendless residuals, which pass the unit root tests regardless of their lack of powers against stationary alternatives. The estimated autocorrelation coefficients of the residuals result in a significantly shorter half-life estimates than those reported in Obstfeld and Rogoff (2001). The results cast doubts about the validity of the Obstfeld and Rogoff (2001) characterization of the PPP as a puzzle.

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Table (1)
Canadian PPP – Sample Jan 1999 – August 2016

				Reversed Regression
Dependent Variable ⇒	s_t	s_t	s_t	p_t^c / p_t^o
Regressors ⇓	OLS	OLS	GMM	GMM
Constant	6.0 (0.0000)	1.2 (0.0000)	1.3 (0.0000)	1.1 (0.0000)
p_t / p_t^*	-9.6 [0.0000]	-	-	-
p_t^c / p_t^o	-	-0.91 [0.5890]	-1.05 [0.7195]	-
s_t	-	-	-	-0.81 [0.0573]
$\overline{R}^2$	0.69	0.67	0.77	0.72
DW	0.09	0.15	0.22	0.30
J	-	-	(0.9977)	(0.9930)

- s_t is the USD price of one Canadian dollar.
- p_t is the Canadian CPI; p_t^* is the US CPI; p_t^c is Canada's commodity price index; and p_t^o is oil price.
- The standard errors are estimated using the Newey-West method.
- P values are in parentheses.
- P values of the Wald statistic to test the slope coefficient is one (in absolute value) are in squared brackets.
- The instruments are 1 to 12 lags of the average changes in the world temperature and 1 to 12 lags of the average changes in the Northern Hemisphere temperature.
- The last column is the reverse of the regression in the fourth column.

Table (2)
Canada - ADF test for the residuals, autocorrelation coefficients and half-life estimates

ADF	OLS Residuals	GMM Residuals	$q=(sp^*/p)$	$q=(sp^o/p^c)$	$\ln(q)=\ln(sp^*/p)$	$\ln(q)=\ln(sp^o/p^c)$
With intercept						
λ	0.938542	0.914868	0.984807	0.97464	0.986064	0.979884
	(0.1441)#	(0.0498)*	(0.3913)	(0.2805)	(0.4028)	(0.5037)
Half-Life (month)	11.93	6.79	45.27	26.98	49.39	34.1
Intercept & Trend						
λ	0.939448	0.915089	0.988947	0.965216	0.986962	0.986773
	(0.4028)	(0.1948)#	(0.9453)	(0.6623)	(0.9164)	(0.97787)
Half-Life (month)	11.09	7.81	62.36	19.57	52.81	52.05
None						
λ	0.938342	0.914895	0.9993	0.999057	0.99928	0.998393
	(0.0151)*	(0.0041)*	(0.6975)	(0.5954)	(0.5963)	(0.5870)
Half-Life (month)	10.89	7.79	NA	NA	NA	NA
ACF	0.947	0.905	0.988	0.979	0.989	0.890
Half-Life (month)	12.72	6.94	57.41	32.66	62.66	34.30

- ADF MacKinnon p-values are in parentheses. Asterisk denotes significant at the 95 percent level and # denotes significant at the 90 percent level
- Significant means reject the null hypothesis of a unit root.
- Half-life is $\lambda^h = 0.5$.
- p^* is the US CPI, and p is Canada's CPI.
- p^c is Canada's commodity price index.
- p^o is the price of oil index.
- ACF denotes the autocorrelation function.

Table (3)
Australia - GMM estimates of the commodity price-based PPP
Sample Jan 1999-Aug 2016
Dependent Variable s_t

Constant	1.46
	(0.0000)
p_t^c / p_t^*	-0.82
	[0.2186]
$\overline{R}^2$	0.48
DW	0.14
J	(0.9805)

- p_t^c is the country's commodity price index.
- p_t^* is the price of coal.
- The exchange rate s_t is the USD price of one Australian dollar.
- The standard errors are estimated using the Newey-West method.
- p values are in parentheses.
- p values to test the hypothesis that the coefficient is not different from unity are in square brackets.

-The instruments are 1 to 12 lags of the average change in temperature in the Southern Hemisphere.

Table (4)
Australia ADF test for the residuals, autocorrelation coefficients and half-life estimates

	GMM Residuals	$q = sp^* / p$	$\ln(q) = \ln(sp^* / p)$
With intercept			
λ	0.92076	0.957935	0.97296
	(0.0373)*	(0.2483)	(0.2530)
Half-life (month)	8.39	16.12	25.28
Intercept and trend			
λ	0.920881	0.935513	0.955776
	(0.1408)#	(0.3648)	(0.2856)
Half-life (month)	8.40	10.39	15.32
None			
λ	0.921339	0.963	0.972796
	(0.0030)*	(0.4816)	(0.0343)*
Half-life (month)	8.46	18.38	25.13
ACF	0.925	0.963	0.978
Half-life (month)	8.89	18.38	31.15

- ADF MacKinnon p-values are in parentheses. Asterisk denotes significant at the 95 percent level and # denotes significant at the 90 percent level
- Significant means reject the null hypothesis of a unit root.
- Half-life is $\lambda^h = 0.5$.
- p^* is the price of coal index (or oil).
- p is Australia's commodity price index.
- ACF denotes the autocorrelation function.

Table (5)
New Zealand - GMM estimates of the commodity price-based PPP
Sample Jan 1999-Aug 2016

Dependent Variable⇒	s_t	$\Delta \ln s_t$
Constant	1.50 (0.0000)	0.0002 (0.7437)
p_t^c / p_t^*	-0.74 [0.0000]	- -
$\Delta \ln(p_t^c / p_t^*)$	-	1.02 [0.8053] {0.9171}
$\overline{R}^2$	0.95	0.85
DW	0.14	1.71
J	(0.9793)	(0.3991)

- p_t^c is the country's commodity price index.
- p_t^* is the world commodity price index.
- The exchange rate is the USD price of one New Zealand dollar.
- The standard errors are estimated using the Newey-West method.
- p values are in parentheses.
- p values to test the hypothesis that the coefficient is not different from unity are in square brackets. Curly brackets are the p-value of the joint hypothesis that the constant term is zero and the slope=1 in absolute value.

-The instruments are 1 to 12 leads of the average changes in temperature in the Southern Hemisphere.

Table (6)
New Zealand ADF test for the residuals, autocorrelation coefficients and half-life estimates

	GMM (level)	GMM (log-difference)	$\ln(q) = \ln(sp^* / p)$
With intercept			
λ	0.96469	0.969817	0.985436
	(0.4326)	(0.4245)	(0.5464)
Half-life (month)	19.2	21.61	47.24
Intercept and trend			
λ	0.96420	0.967786	0.960385
	(0.7405)	(0.7445)	(0.3668)
Half-life (month)	19.0	21.16	17.15
None			
λ	0.96459	0.969876	0.992878
	(0.0840)#	(0.0824)#	(0.1899)#
Half-life (month)	19.22	22.66	96.97
Autocorrelation function	0.944	0.968	0.986
Half-life (month)	12.03	21.31	49.16

- ADF MacKinnon p-values are in parentheses. # denotes significant at the 90 percent level
- Significant means reject the null hypothesis of a unit root.
- Half-life is $\lambda^h = 0.5$.
- p^* is the world commodity price index.
- p is New Zealand commodity price index.

Figure (1)
Canada, commodity price index and West Texas Intermediate Crude Price

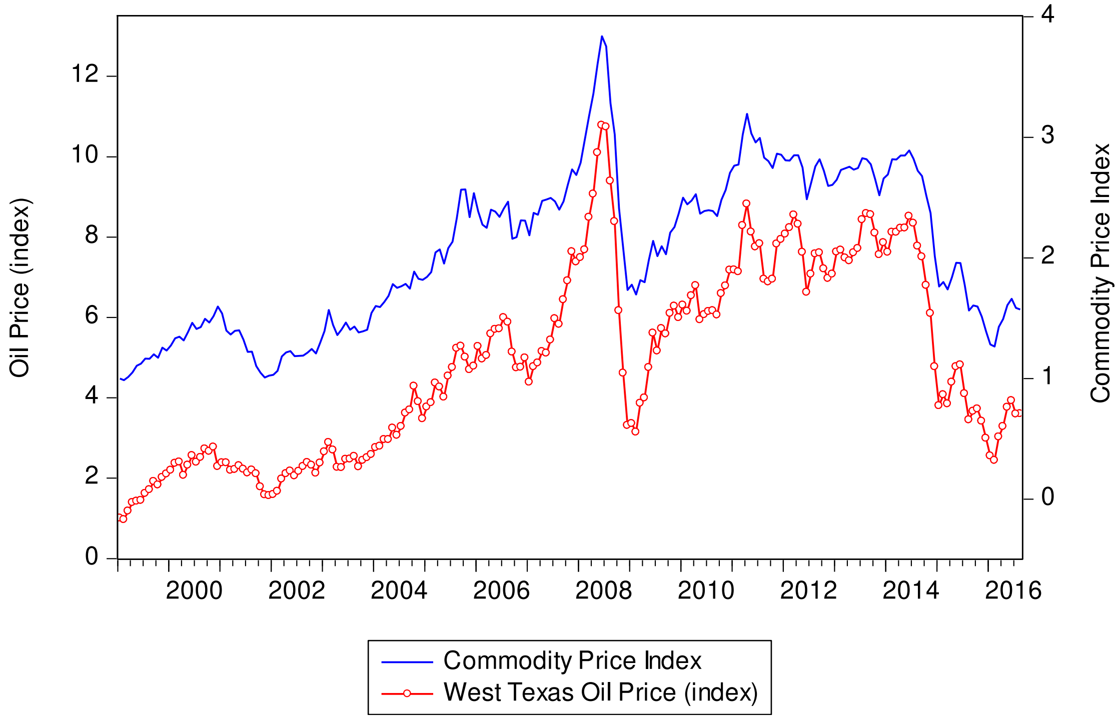


Figure (2)
Canada, actual, fitted values, and the residuals of the PPP specification
Using the CPI's

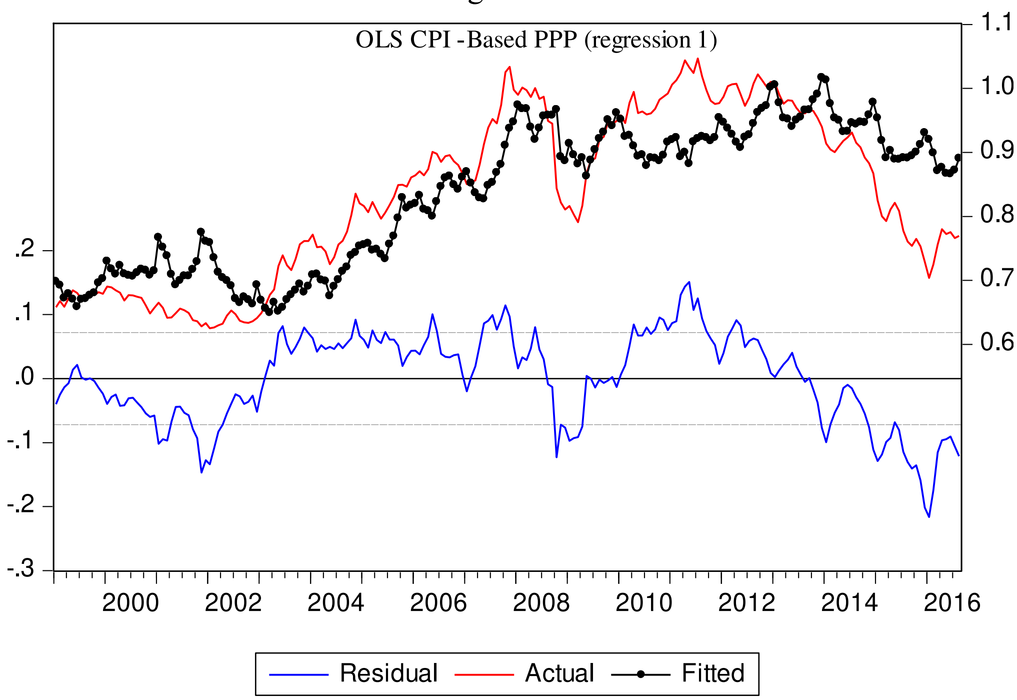


Figure (3)
Canada, actual, fitted values, and the residuals of the PPP specification
Using commodity prices

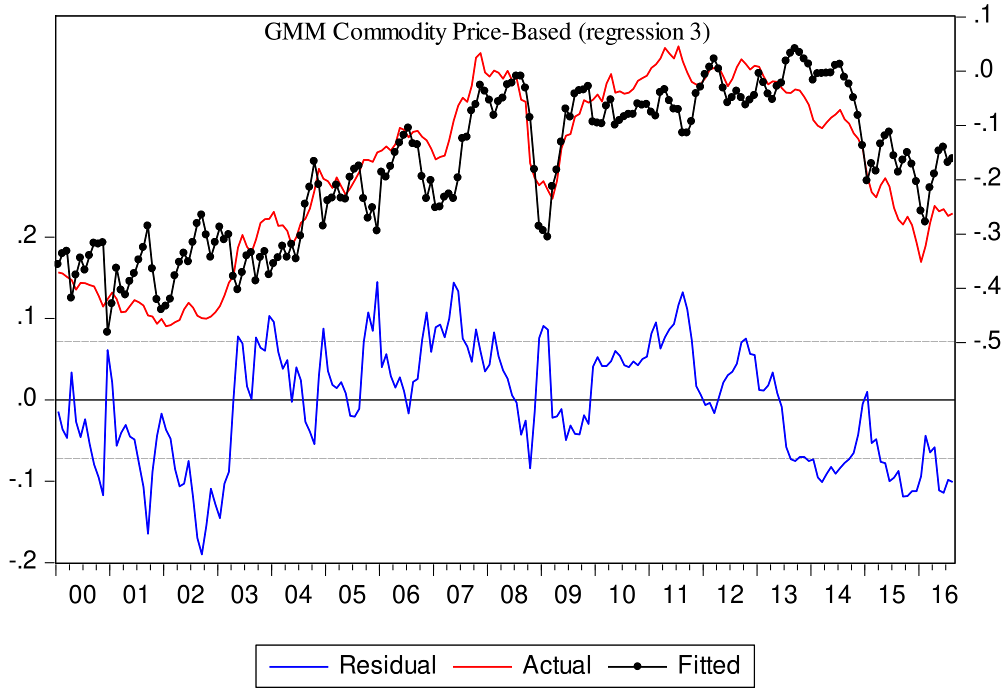
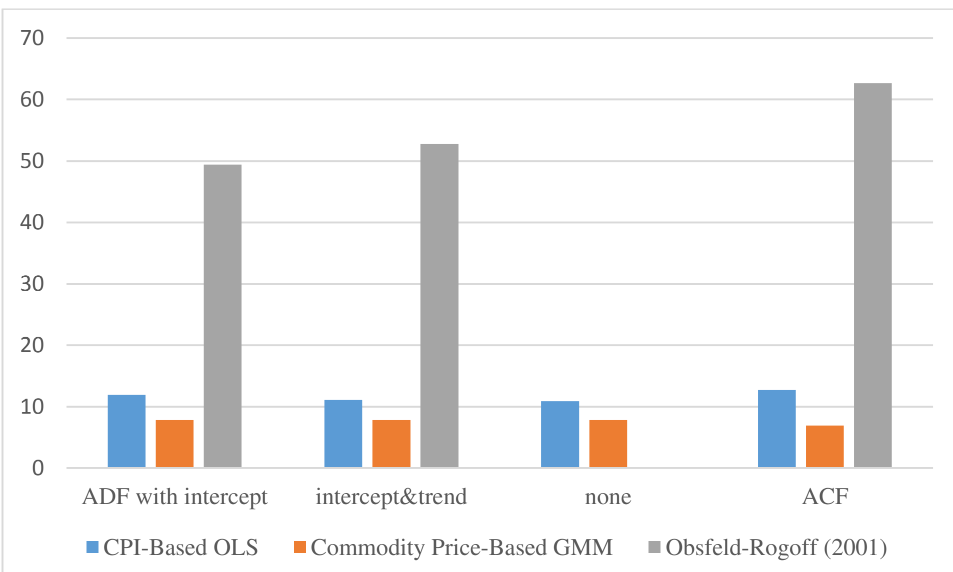


Figure (4)
Estimates of half-life deviations from PPP
Canada



The estimated half-life is based on estimated autocorrelation coefficients using the ADF test and the autocorrelation function

Figure (5)

Australia's commodity price index and the price of coal (index)

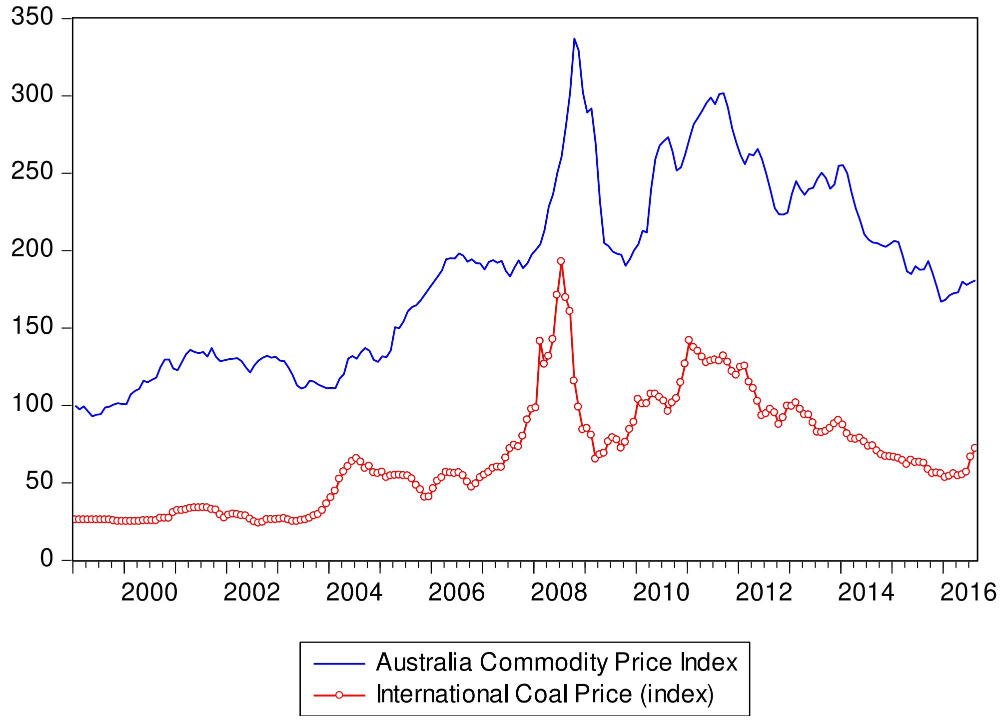


Figure (6)
Australia: GMM regression, actual, fitted values, and the residuals of the PPP using commodity prices

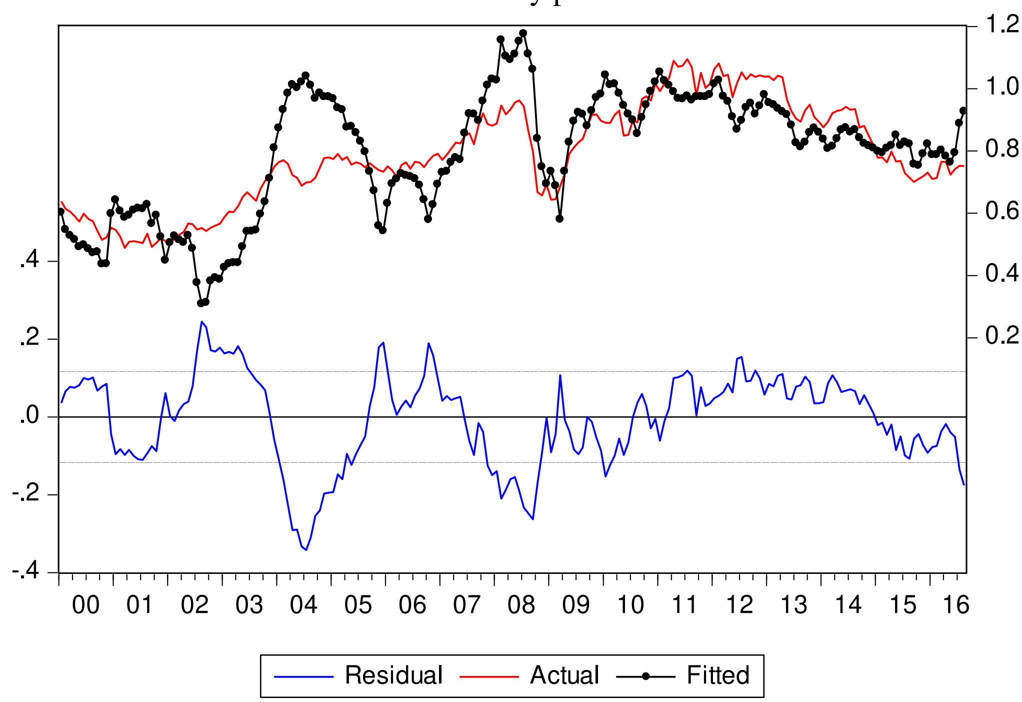
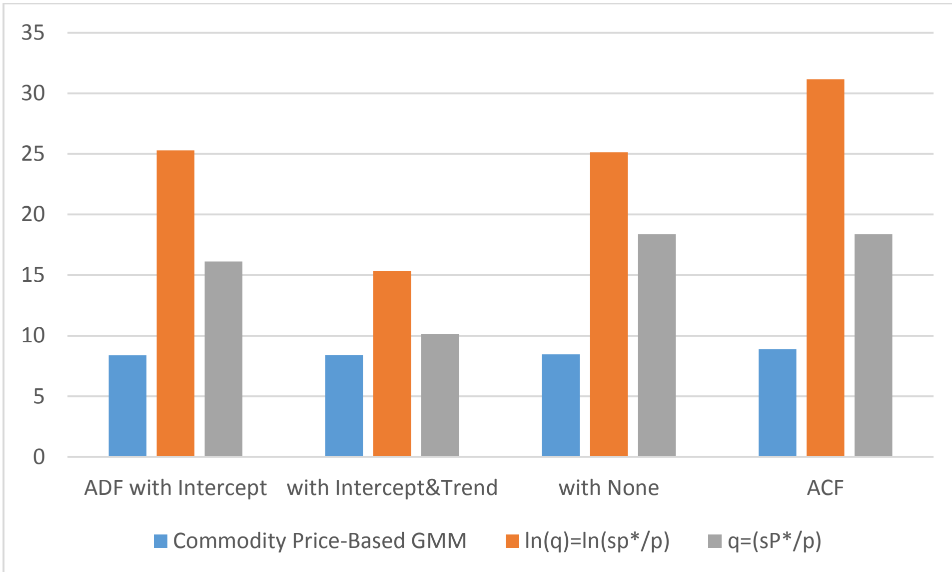


Figure (7)
Estimates of half-life deviations from PPP
Australia



The estimated half-life is based on estimated autocorrelation coefficients using the ADF test and the autocorrelation function

Figure (8)
New Zealand commodity price index and the world commodity price index

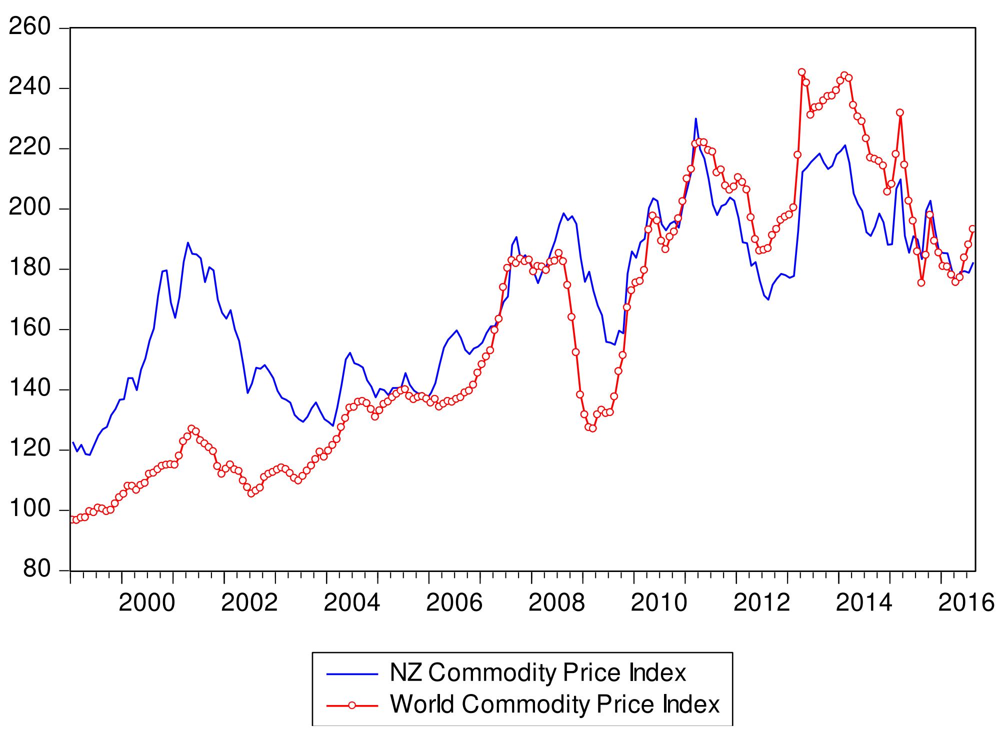


Figure (9-a)
New Zealand: GMM regression, actual, fitted values, and the residuals of the PPP using commodity prices in levels

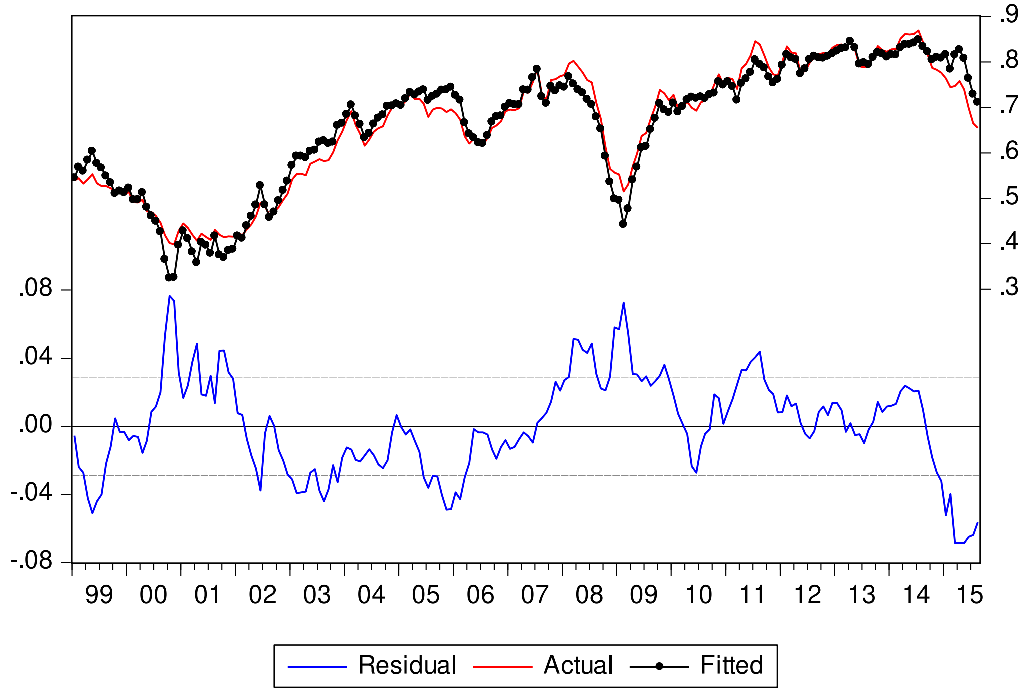


Figure (9-b)
New Zealand: GMM regression, actual, fitted values, and the residuals of the PPP specification $\Delta \ln(s_t) = a + b\Delta \ln(p_t^c / p_t^*)$

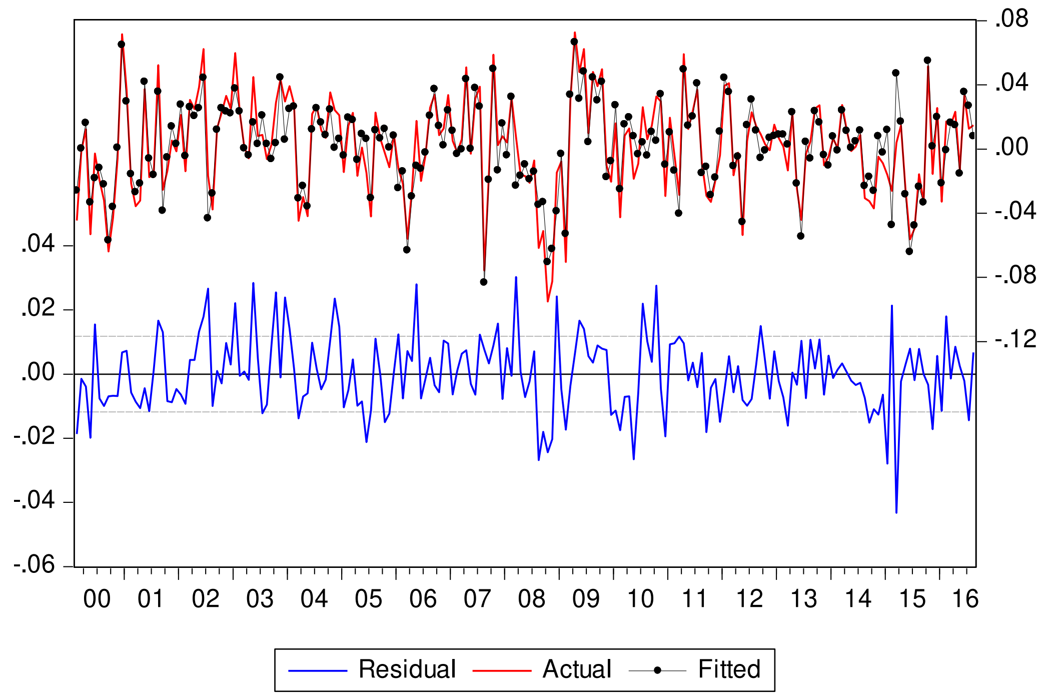
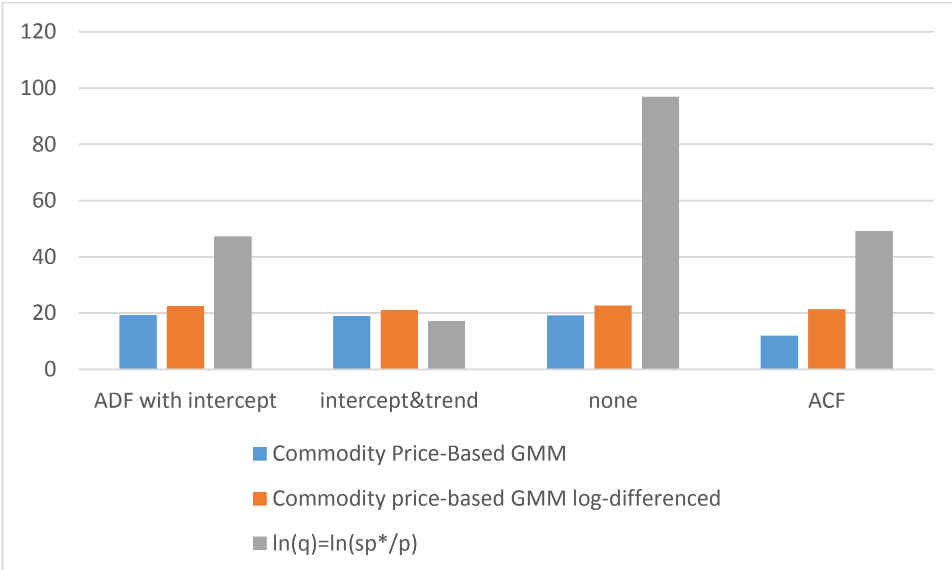
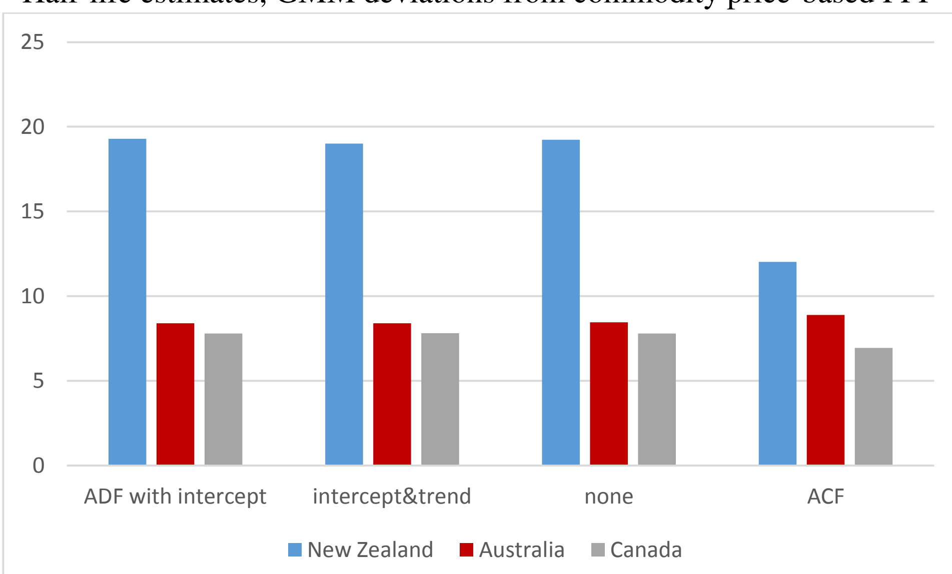


Figure (10)
Estimates of half-life deviations from PPP
New Zealand



The estimated half-life is based on estimated autocorrelation coefficients using the ADF test and the autocorrelation function

Figure (11)
Half-life estimates, GMM deviations from commodity price-based PPP



The estimated half-life is based on estimated autocorrelation coefficients using the ADF test and the autocorrelation function

Data Appendix

Descriptive Statics – Jan 1999 to Aug 2016

Variable	Canada		Australia		New Zealand		Unit Root
	$\bar{x}$	σ	$\bar{x}$	σ	$\bar{x}$	σ	
s_t	0.84	0.12	0.78	0.16	0.66	0.13	✓
p_t^c / p_t^o	0.45	0.09	-	-	-	-	✓
p_t^{cpi} / p_t^{cpi*}	0.54	0.01	-	-	-	-	✓
p_t^c / p_t^*	-	-	0.82	0.23	1.01	0.18	✓
$\Delta \ln(s_t)$	0.0006	0.020	0.00084	0.036	0.00139	0.29	
$\Delta \ln(p_t^c / p_t^o)$	-0.0019	0.052	-	-	-	-	
$\Delta \ln(p_t^{cpi} / p_t^{cpi*})$	-0.0001	0.003	-	-			
$\Delta \ln(p_t^c / p_t^*)$	-	-	-0.002	0.072	-.00140	0.25	

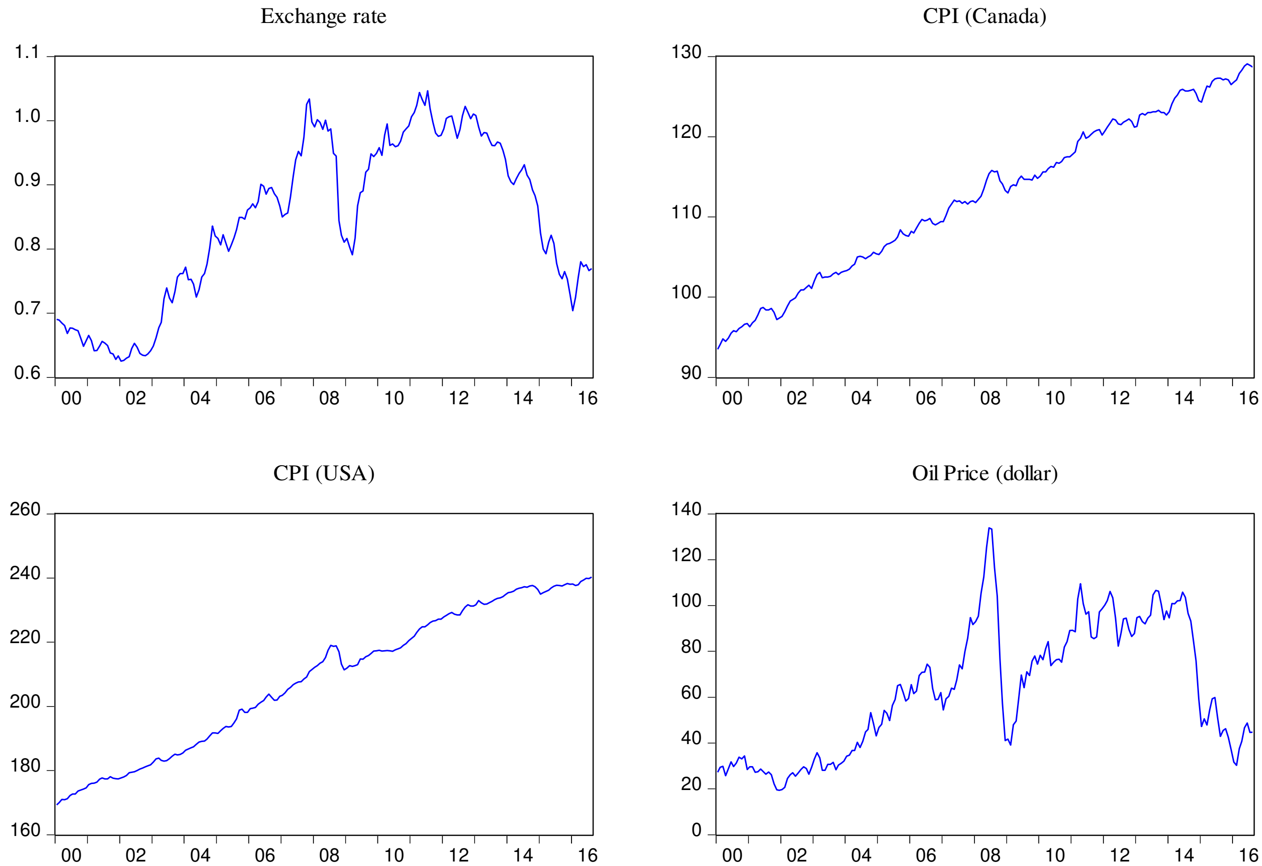
- s_t is the nominal exchange rate defined as the home currency price of one US dollar.

- p_t^c is the country's commodity price index (Jan 1999=100).

- p_t^o is the price of oil measured by the West Texas Intermediate price (index, Jan 1999=100).

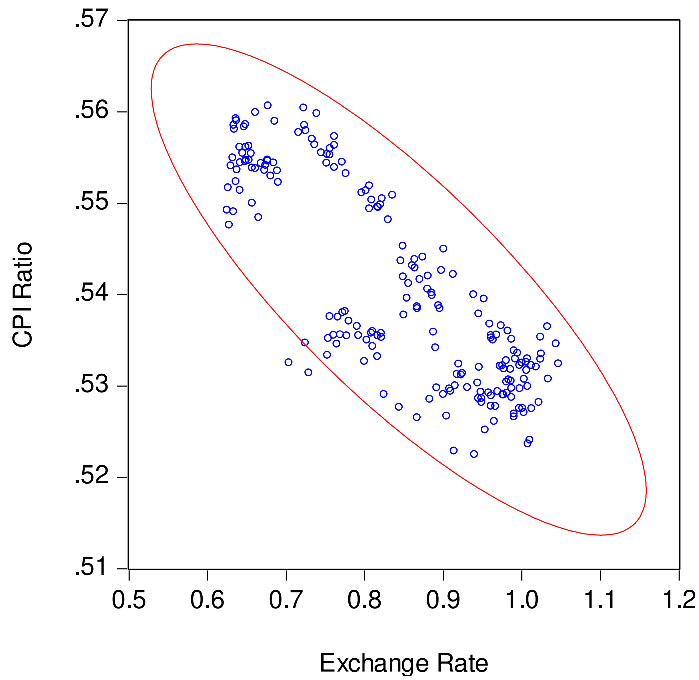
- p_t^* is the price of coal in the case of Australia and the world commodity price index (Jan 1999=100) in the case of New Zealand.
- $\bar{x}$ is the mean and σ is the standard deviation.
- A variety of commonly used unit root tests with different specifications, and different lag structures chosen by a variety of commonly used Information Criteria could not reject the hypothesis of a unit root in the level variables. All the tests have low power against stationary alternatives.

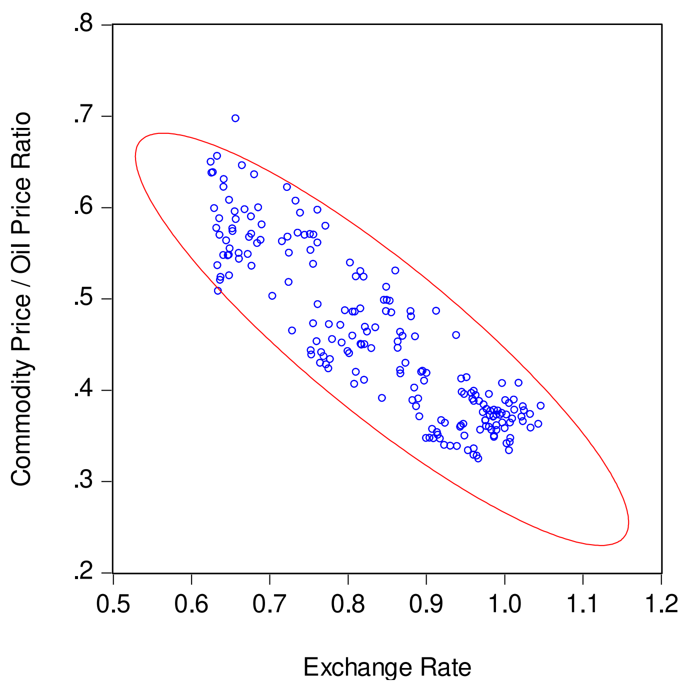
Canadian Data



Sources: The Exchange rate is from the Federal Reserve Bank of St Louis FRED. The Canadian CPI is from CANSIM (statistics Canada). The US CPI is from FRED and is the West Texas Intermediate Price of oil.

Canadian PPP – Confidence Ellipse is distributed $\chi^2_{0.95}$

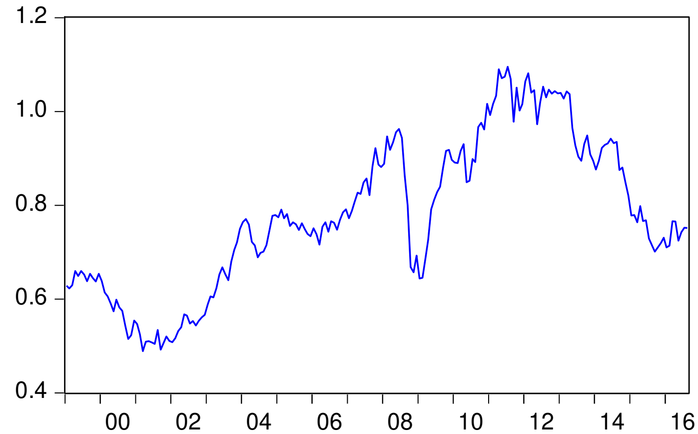




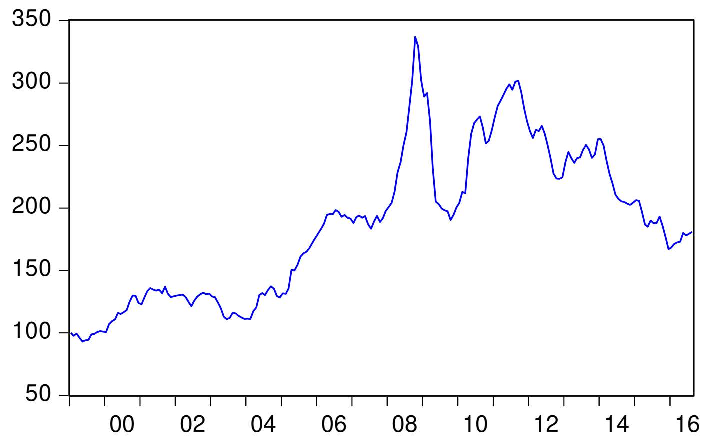
The commodity price-based PPP has a tighter Confidence Ellipse, i.e., stronger correlation. Source of the commodity price index: CANSIM.

Australian Data

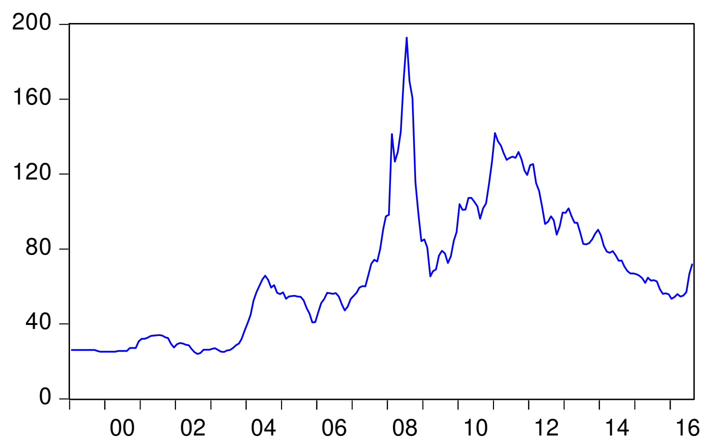
Exchange Rate



Commodity Price Index

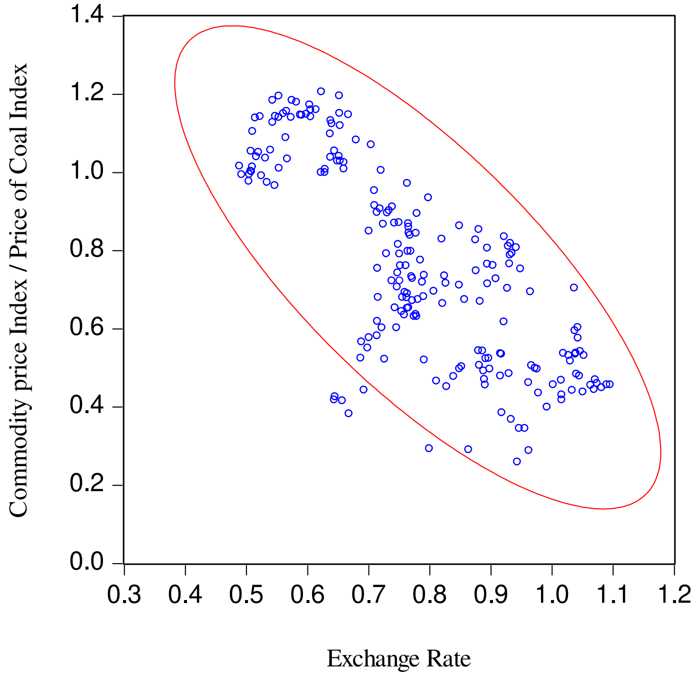


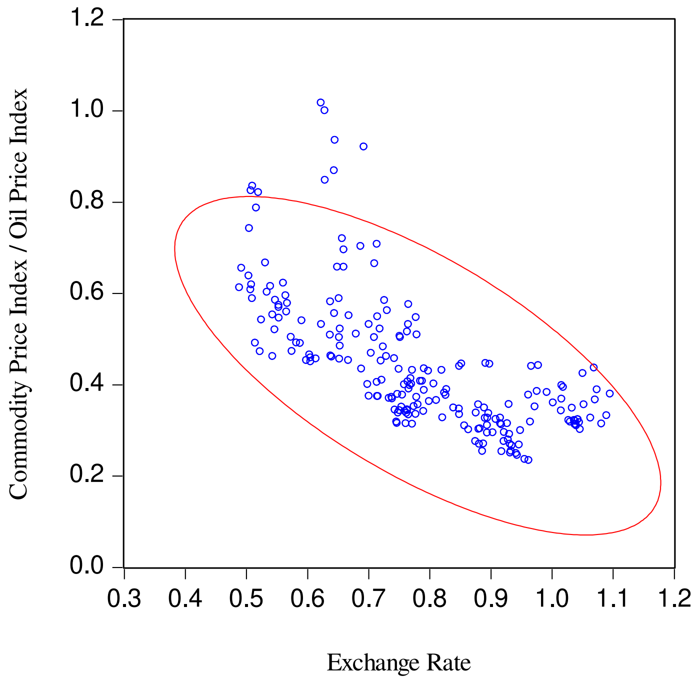
Price of Coal (Index)



Sources: Prices are from Australian Bureau of Statistics and the exchange rate is from the Reserve Bank of Australia

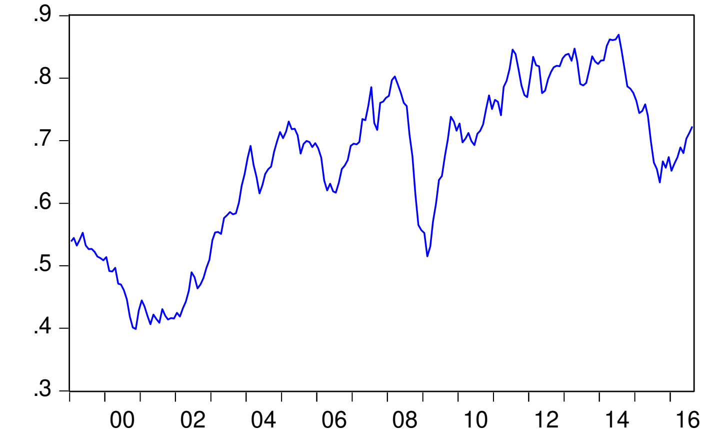
Australian PPP – Confidence Ellipse is distributed $\chi^2_{0.95}$



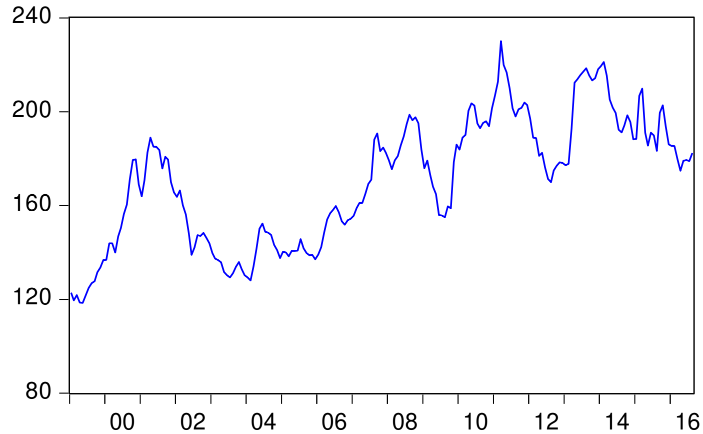


New Zealand Data

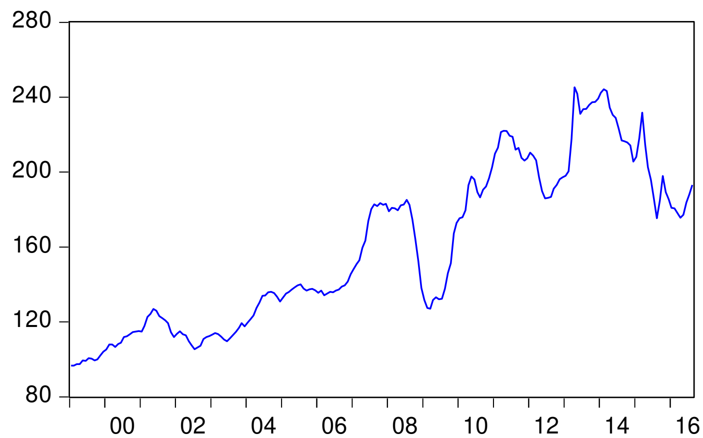
Exchange Rate



Commodity Price Index

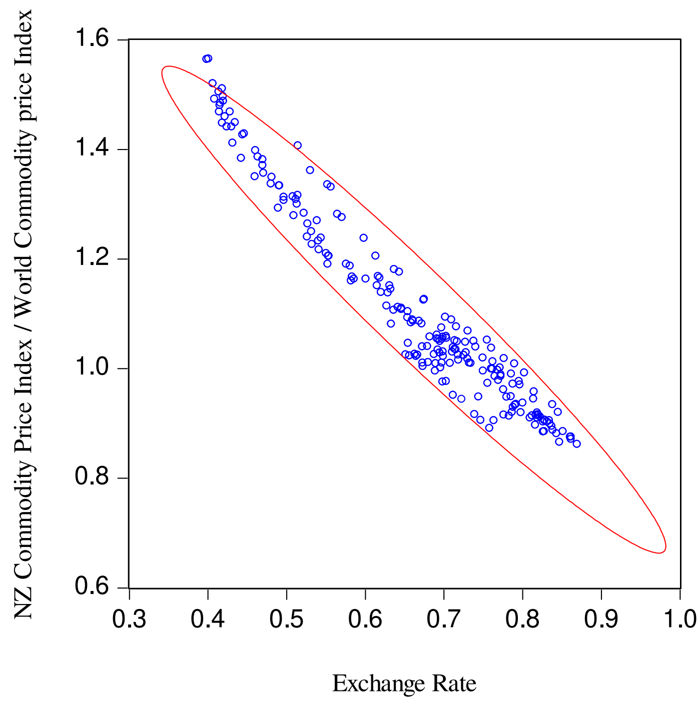


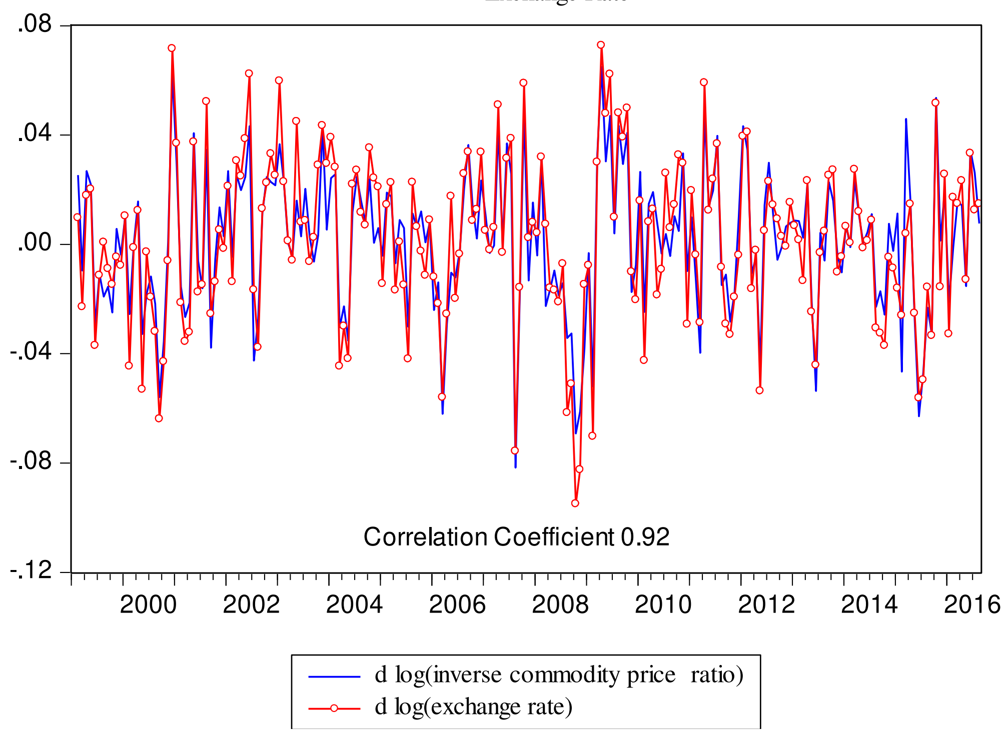
World Commodity Price Index



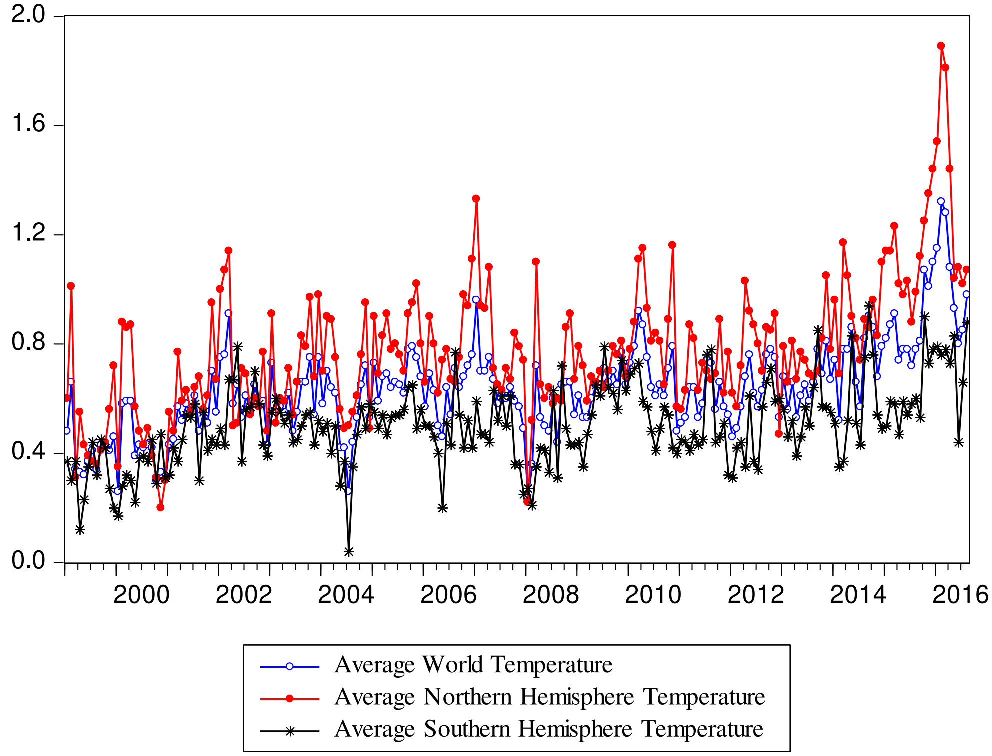
Sources: The exchange rate is from the Reserve Bank of New Zealand, and commodity prices indices are from ANZ Bank.

New Zealand PPP – Confidence Ellipse is distributed $\chi^2_{0.95}$





Average Changes in Temperature



Source: NASA (Hansen et. al, 2010).