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## **Gaming modeling of self-enforcing agreements and free-rider problem**

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# **Gaming modeling of self-enforcing agreements and free-rider problem**

## **Abstract**

The paper justifies the selection of formal conditions under which the rational-minded actors will tend to observe the implicit contract between them.

Self-enforcing agreements are characterized by inappropriateness of arbitration support, primarily due to too high transaction costs of such support.

It is an underdeveloped area of research of self-enforcing agreements does not operate categories of reputation directly. The question is: can there be such conditions for the relationship of agents, in which compliance with the agreement will be beneficial to both of them without them having a priori information?

As the main method for research the problem selected the game theory. Is constructed the game model of subjects' relationships and found the value of the payment functions for which there is Nash equilibrium in pure strategies "to comply with agreement".

It is shown, that above game simulate the relationship of agents, which can lead to a free-rider problem in the theory of collective goods. That is the solution of this game is also a solution to the free-rider problem, that demonstrate the dual tasks of self-enforcing agreements and the free-rider problem in the allocation of collective goods.

The novelty of the study results is to obtain an analytical expression for the automatic compliance with the agreement conditions by rationally acting cognitively perfect agents and formal proof of their adequacy.

The ability to analyze the behavior of economic agents in matters of free-riding by simple formal tools of the game theory makes presented results useful from a practical point of view.

## **Keywords:**

contract theory; self-forced agreement; behavior of economic agents; game model; pure strategies; Nash equilibrium; free-rider problem

## **Introduction**

The paper justifies the selection of formal conditions under which rational-minded actors will tend to compliance with the agreement between them, because the final result is more profitable for them.

The problem in different contexts is covered in theory of self-enforcing agreements that along with theory of implicit (incomplete) contracts is one of the main branches of generally contract theory. At this, different researchers either range self-enforcing contracts assign to class of implicit contracts or equate to them (see e.g., (Hart, 1987)).

## **Literature review**

Over the past few decades, the various concepts of the contract have been stabilized and offered a number of classifications. One of the main is division of all the contracts into three groups: agency contracts (contract type “principal – agent” – a contract of employment, as well as its generalization to the agents’ “horizontal” relations – the contract of sale), incomplete (implicit) and implicit contracts (self-enforcing agreement). At this

- 1) subject of analysis theory of agency contracts is usually the effects of the information’s asymmetric that is inherent in the relations of the parties to agreement (see (Grossman and Hart, 1983), (Holmstrom, 1979), (Milgrom and Roberts, 1982), (Mirrlees, 1976), (Ross, 1973), (Shavell, 1979), (Stiglitz, 1974));
- 2) theory of incomplete contracts research, on the one hand, opportunism ex post, that caused by transaction costs (Williamson, 1971) and is typical for the like agreements, and on the other hand – difficulty of detection of compliance by the parties with their contractual responsibilities (see (Grossman and Hart, 1986), (Hart and Moore, 1988));
- 3) object of theory of implicit (self-enforcing) agreements is conditions and mechanism their automatic compliance, the necessity is caused by the miss (imperfection) of clearly (legal) prescribed a mechanism to enforce the agreement, its control and sanctions for evasion it.

The problem of self-enforcing agreements holds an important place in the new institutional economics. Unlike other self-enforcing agreements does not legal contract. Such agreements arise because of imperfection of the legal system that don't has enough resources for provide the compliance enforcing agreements between economic agents. However the legal details don't subject of proposed article, so the reason of use of self-enforcing agreements can be formulated in more accordance with research area: external, arbitration support of agreements is inexpedient primarily economically – because of the excessive transaction costs of such support.

The activation of researches self-enforcing agreements usually to start from articles Telser (1980), Klein and Leffler (1981), in which behavior of agents was postulate as quite rational – maximizing individual utility. That is, an agent compliance with existing contract only if he obtains economic benefit, otherwise he avoids compliance responsibilities. No other factors are influence his behavior (see (Telser, 1980), (Williamson, 1971)). It is noted that like argumentation, is based on a matching of benefits with costs, used also Coleman (1994) and Frank (1992, 152).

In other informal approach [that it was] proposed by Akerlof, implicit contract regards rather for not allocation of risk, and behavioral norms. Akerlof consider the relationship around employment contract as exchange of gifts, based a certain extent on endogenic behavior's norms (Akerlof, 1982). Among others also could distinguish article by Scott (2003), in which he proposes self-enforcing of implicit agreements.

Applications using of theoretical developments in the field self-enforcing agreements, first of all, relating to the salary theory and the labor markets (see (Carmichael, 1989), (Lazear, 1979; 1981), (Okun, 1981)), and to the research of relationships economic agents at macro level in the cases of easing or full absence of government intervention (e.g. (Kronman, 1985)). In last article Kronman with help of simple model shows implicit contract that support efficiency trade agreements with laissez faire. In a later work Srinivasan and Brush (2006) consider self-enforcing agreements between supplier and buyer in context vertical alliances.

On the ground of above approach Akerlof concludes salary and standards of employer's efforts are interrelated (Akerlof, 1982, 544).

Also trade relations with asymmetric information researched Kreps and Wilson (1982), Milgrom and Roberts (1982).

Usually, in models of self-enforcing agreements consider contractors and referee (e.g. government) are unable to:

- ✓ determine, did be the agreement evasion;
- ✓ enforce the agreements (regarding it is the most often assumed compliance with the agreements is induced by the explicit or implicit threat of them discontinuation) (see (Telser, 1980), (Stiglitz, 1974)).

In addition, it is assumed almost perfect information in relationship – none of parties of the agreement know, is his vis-à-vis fair?

Under such conditions there is important to accumulate information about the history of behavior of potential parties to the agreement in similar situations and to present similar information about own reputation regarding compliance with the agreements. Because of this the theory of self-enforcing agreements is largely based on reputation's concept (e.g. a brand of firm etc.). Klein and Leffler in above said article (Klein and Leffler, 1981) where first accented special attention to role of reputation in consideration of self-enforcing agreements. Shapiro (1983) developed the model of Klein and Leffler by formalizing the concept of reputational equilibrium.

As discussed reputation formed by repeated actions of agents (firstly, this interaction), it is natural to use the theory of repeated game to researching “reputation models”.

This approach presents Bull (1987) (the problem of implicit labour contracts and a role of reputation in increase of completeness of the agreement), Kreps (1990), Bernheim, Peleg and Whinston (1987), who used games with infinite number of repetitions for solving boundary problems. See the example of game-theoretical interpretation of effective salary e.g. (Gibbons, 1992, 107).

At once, it is still an underdeveloped line of research of self-enforcing agreements categories, and reputation does not operate directly. The question is: are there such conditions of relationship of agents, in which compliance with the

agreement is beneficial to them without them having and accumulation a priori additional information (e.g. reputation of vis-à-vis) and don't it by repeated interaction? I.e. are there conditions both rational agents stand by an agreement from first step of relationship.

Actually, this article purpose is to solve this problem.

### **Method's and Models**

The game theory, which is the most appropriate tool for the problems of this type, has been selected as the main method for investigating the problem.

Firstly, the different direct of Contract theory, particular models "principal – agent", use actively game theory. At this in theory "principal – agent" line of research, in which numerically evaluating eventual opportunism (total or partial) of agent (first of all) and of principal is difficult, is not well developed. Sometime careful and exact evaluation of degree of opportunism of agent by principal (and vice versa) is so much costly that it is economically unprofitable. At this obtained in consequence of interaction of principal and agent result is determined quite accurately. This case pretty much corresponds to self-enforcing agreements between principal and agent description.

Second, as noted above, theory of self-enforcing (implicit) agreements use the method repeated games for modelling interaction of contractors to create of own reputation and of study of reputation of vis-à-vis.

As self-enforcing agreements can describe not only "vertical" ("principal – agent") also "horizontal" interaction of equal partners, it is offered game model the same relationship of economic agents. In this case "vertical" relationship is the subcase of "horizontal" ones and do not need a separate modeling.

Following the formal model is constructed for interaction between 2 parties, but the conclusion can extend to any number of contractors.

**Free-rider problem.** Self-enforce of agreement assume lack of control of eventual opportunism of agents, that is income distribution does not depend, the agents compliance with the agreement or they evade one.

Is considered simple case of the interaction 2 agents: both agents are potential investors that agree to invest in some process in defined proportions  $n_1:n_2$  (be definite it is assumed  $n_1 > n_2$ ).

Assume the process is multiplier, that is after  $I$  turnover he multiplies the funds invested by coefficient  $\alpha$  ( $\alpha > I$ ).

Naturally, agents distribute earned revenue in the same proportions  $n:m$ .

Extreme strategies (pure-strategies) of behavior of agents are such: agent can comply the agreement (investing respectively  $n_1$  or  $n_2$  costs' units), and can evade it, investing nothing. Also he can partly evade (and partly comply) the agreement, investing  $0 < v_1 < n_1$  or  $0 < v_2 < n_2$  units. That is in general strategies of agents are as follows:

$$\begin{aligned} 1: & 0 < v_1 < n_1; \\ 2: & 0 < v_2 < n_2. \end{aligned} \tag{1}$$

Nevertheless eventual agreement avoiding by one or both agents, principled lack of control causes the income distribution in proportion  $n:m$  for any real investment amount of agents.

Is set a problem: find value of parameters  $n_1, n_2, \alpha$ : rational agents–maximizers observe the agreement, i.e. they full invest the process.

Considering foregoing conditions, can be described the profits of agents if they use extreme mutual strategies (the markings:  $0$  – total investment evasion;  $I$  – total investment according to the agreement):

✓  $(0; 0)$ : both agents avoid investing. The total investment identically zero, hence the revenue is equals also  $\alpha \cdot 0 = 0$ . Because the agents don't suffered costs (don't invested), their profits are:

$$\begin{aligned} 1: & g(0;0) = \alpha \cdot 0 - 0 = 0; \\ 2: & h(0;0) = \alpha \cdot 0 - 0 = 0; \end{aligned}$$

✓ (0; 1): 1th agent (let it is potentially bigger investor) avoid the investment, 2th – fair invests agreed value  $n_2$ . Obviously total investment both agents is also  $n_2$ , the revenue equals  $\alpha m$ , the revenues of agents distribute as follows:

$$1: \frac{n_1}{n_1 + n_2} \alpha n_2;$$

$$2: \frac{n_2}{n_1 + n_2} \alpha n_2;$$

the profits of agents are equals:

$$1: g(0;1) = \frac{n_1 n_2}{n_1 + n_2} \alpha;$$

$$2: h(0;1) = \frac{n_2}{n_1 + n_2} \alpha n_2 - n_2 = n_2 \left( \frac{n_2}{n_1 + n_2} \alpha - 1 \right);$$

✓ (1; 0): the situation is symmetrical previous one: 1th agent fully invests agreed funds, 2th fully avoid the investment, acting as a free-rider. Obviously the profits of contractors also are the symmetrical in relation to previous case:

$$1: g(1;0) = \frac{n_1}{n_1 + n_2} \alpha n_1 - n_1 = n_1 \left( \frac{n_1}{n_1 + n_2} \alpha - 1 \right);$$

$$2: h(1;0) = \frac{n_1 n_2}{n_1 + n_2} \alpha;$$

✓ (1; 1): both agents fully comply own commitments under the value of funds. Total investment is equal to  $n_1 + n_2$ , total revenue is:  $\alpha(n_1 + n_2)$ , revenues of agents are:  $\frac{n_1}{n_1 + n_2} \alpha(n_1 + n_2)$  and  $\frac{n_2}{n_1 + n_2} \alpha(n_1 + n_2)$  or  $n_1 \alpha$  and  $n_2 \alpha$ , and ones profits are:

$$1: g(1;1) = n_1 \alpha - n_1 = n_1 (\alpha - 1);$$

$$2: h(1;1) = n_2 \alpha - n_2 = n_2 (\alpha - 1).$$

Let's establish the relationship of agents as the game:



$$\Gamma = (S1, S2, (G, H)(S1 \times S2));$$

$$(G, H) = \left( \begin{array}{cc} \{0; 0\} & \left\{ \frac{n_1 n_2}{n_1 + n_2} \alpha; n_2 \left( \frac{n_2}{n_1 + n_2} \alpha - 1 \right) \right\} \\ \left\{ n_1 \left( \frac{n_1}{n_1 + n_2} \alpha - 1 \right); \frac{n_1 n_2}{n_1 + n_2} \alpha \right\} & \{n_1(\alpha - 1); n_2(\alpha - 1)\} \end{array} \right). \quad (2)$$

In this formulation self-enforcing agreement by both agents corresponds to existence unique in the game (2) Nash' equilibrium with the use of mutual strategy  $(I; I)$ , that is compliance with these conditions:

$$\begin{aligned} \frac{n_1 n_2}{n_1 + n_2} \alpha < n_1(\alpha - 1) \quad \wedge \quad \frac{n_1 n_2}{n_1 + n_2} \alpha < n_2(\alpha - 1); \\ n_2 \left( \frac{n_2}{n_1 + n_2} \alpha - 1 \right) > 0 \quad \vee \quad n_1 \left( \frac{n_1}{n_1 + n_2} \alpha - 1 \right) > 0. \end{aligned} \quad (3)$$

Considering  $n_1 > n_2$  and in the first (conjunction) and in the second (disjunction) pair the second inequalities majorize first ones, that is

$$\begin{aligned} \frac{n_1 n_2}{n_1 + n_2} \alpha < n_2(\alpha - 1); \\ n_1 \left( \frac{n_1}{n_1 + n_2} \alpha - 1 \right) > 0. \end{aligned} \quad (3')$$

On rearrangements there is:

$$\begin{aligned} \frac{n_1}{n_1 + n_2} < \frac{\alpha - 1}{\alpha}; \quad \frac{n_1}{n_1 + n_2} < 1 - \frac{1}{\alpha}; \quad \frac{1}{\alpha} < \frac{n_2}{n_1 + n_2}; \quad \frac{n_1}{n_2} < \alpha - 1; \\ n_1 \left( \frac{n_1}{n_1 + n_2} \alpha - 1 \right) > 0; \quad \frac{n_1}{n_1 + n_2} \alpha > 1; \quad \alpha > \frac{n_1 + n_2}{n_1}; \quad \frac{n_2}{n_1} < \alpha - 1. \end{aligned}$$

Because  $\frac{n_2}{n_1} < \frac{n_1}{n_2}$ , the first inequality majorize the second, finally be obtained

the unique Nash' equilibrium condition for game (2):

$$\frac{n_1}{n_2} < \alpha - 1. \quad (4)$$

Inequality (4) shows that to invest in an agreed value is profitable for agents (i.e. it is case that every agent make a maximum possible profit versus other cases), if the ratio them investment does not exceed the profitability of the process. That is even with equal deposits from investors the multiplier should be  $\geq 2$ .

If the multiplier is less than ratio of investment payment, do this means that self-enforce of agreement is inaccessible?

The multiplier coefficient  $\alpha$  define the productivity of process per  $1$  turnover. Clear the multiplier repeated process is equal  $\alpha^\tau$ , where  $\tau$  – amount of repeats of process. If  $\alpha > 1$ , then

$$\exists \tau : \tau = 1, 2, \dots : \alpha^\tau > 1 + \frac{n_1}{n_2}.$$

That is if agents will arrange not income distribution during  $\tau$  turnovers then maximizing strategy for them will be the investments in agreed values.

### **Discussion**

The following explanation are possible, why do not all agreement in real economy realized automatically?

1. Not every agent can and wants to wait during  $\tau$  turnovers. But then it is not clear why in general do such agent agrees to make an agreement?
2. More convincing is argument about the insecurity of result of production process – the multiplier's function will not necessary stably and at each turn will multiply the income by  $\alpha$  (formally is suffices a weaker condition: at every step the incomes could increase as much the formed sequence of quantities of incomes tends as  $\tau \rightarrow +\infty$  to  $N : N > 1 + \frac{n_1}{n_2}$  – sure will be found a timepoint when received required profitability). The desire to insure against a risk can tempt someone of agents to opportunist behavior.
3. The agent has cognitive limitations – he bad knows how to count. Based on “common sense” or certain routines he consider that unpunished opportunism always give more profit than fair compliance with the agreement (especially as

real life has many evidence of this view). Therefore at the first opportunity the agent, that is aware about the lack of control and sanctions for agreement evasion, will try to evade the agreement and not to invest, it without realizing that he chooses an economically ineffective behavior strategy.

Separately, above game that simulates relationship of equal (in general) partners (it completely concerns to special case – relationship “principal – agent”), is also the model of relationship of contractors that could cause free-rider problem in the theory of collective (public, club, common-pool) goods<sup>1</sup>. That is the solution such game is also the solution free-rider problem by indication system parameters in which rational agent it is not profitable to become a free-rider. This conclusion follow from what free-rider problem arise just if it is difficult or even impossible to determinate the amount of contributions of each contractors or if the agreement evasion each contractors it is difficult or of even impossible to prevent – just like conditions are determinant to identify self-enforcing agreements.

Therefore the proposed game model allows uniting the problems of enforcement of self-enforcing agreements and the issue of motivation potential free-riders to non-opportunistic behavior.

### **Conclusion**

Therefore because in modelling of game 2 equitable agents established the contractors essentially don't control eventual mutually opportunism, the game solves the issues formal conditions of self-enforce of agreement between this agents.

The novelty of the study results is to obtain an analytical expression for the self-enforce the agreement conditions by rationally acting cognitively perfect agents and formal proof of their adequacy. There is demonstrated the dual tasks of self-enforcing agreements and the free-rider problem in the allocation of collective goods.

The proof of the tasks duality of seemingly different fields of study (theory of contracts and market failure) suggests the existence of common roots of these areas,

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<sup>1</sup> The term “collective [consumption] good” used, in particular, Samuelson (1954) and Graves (2009).

which in itself is the certain scientific result in these areas of institutional economic theory. At the same time, the ability to analyze the behavior of economic agents, for example, in matters of free-riding by relatively simple but quite formal tools of the game theory makes results presented in the paper useful from a practical point of view.

The main direction of this problem's further development is its generalization for an arbitrary number of interacting entities, i.e. expansion of the self-enforcing agreement to the interaction of  $m$  of contractors: the construction of the model and the proof of the adequacy of the obtained values of the relationship system's parameters. It is also interesting to analyze the possibility to achieve practically conditions for self-enforcing of agreements between agents in terms of specific practical problems of the real economy, e.g. in the areas of environmental-economic management, administration, etc.

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