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**Abstract**

The assumption of multiplicative non-separable (Cobb-Douglas) consumer preferences is a key assumption for analyzing the interdependence of consumption and leisure choices. In this paper we solve the consumer utility maximization problem under these preferences and derive a simultaneous system of two equations corresponding to a static and an inter-temporal equation of consumption and leisure choice. The system is estimated with GMM to obtain consistent estimates of the consumer's preference parameters, of which the relative weight of consumption in the utility function is found to be much higher than that commonly assumed in DSGE model calibration exercises.

*JEL classification*: C36, C61, D12, E44

*Keywords:* Cobb-Douglas consumer preferences; consumption and leisure choices; GMM estimation; weight of consumption in utility

## 1. Introduction

An important task for economists is to study consumer preferences as revealed by his intra-temporal or inter-temporal choices and estimate a broad range of preference parameters that have an essential role in determining how the consumer behaves, i.e. how he decides about the level of consumption and leisure. An interesting aspect of this behavior is whether consumption and leisure choices are interdependent or not. The literature has generally paid little attention to this issue. In representative agent models, when preferences are assumed to be separable (either additive or multiplicative) or additive non-separable, interdependence is not a feature of the model. The only case in which consumption and leisure decisions are cross-dependent is when preferences are multiplicative non-separable (Cobb-Douglas preferences). The advantage of adopting this form of non-separable utility function is not so much that it is an important ingredient in explaining the co-movements in consumption and leisure but that it represents a better choice for the analysis of consumer behavior since it does not require, as other forms of the utility function do, any a priori constraint on the preference parameters.

Unfortunately, there have been very few empirical studies to date that have attempted to endogenize the link between consumption and leisure choices (Eichenbaum et al., 1988; Domeij and Flodén, 2006; Lopez-Salido and Rabanal, 2006). These studies, by solving the consumer maximization problem, obtained an aggregate labor supply equation and a consumption Euler equation. Eichenbaum et al. (1988) applied GMM estimation to the consumption equation, while they considered the labor supply equation as an exact relation among current wage, consumption, and leisure. They reported evidence against the over-identifying restrictions in the Euler equation and a non-sensible estimated value of the discount factor. Domeij and Flodén (2006) again estimated only the consumption equation by using synthetic micro-data or panel data. They did not test the validity of the instruments used and obtained a non-sensible value for the weight of consumption in the utility function. Their model was estimated by setting exogenously values for the inter-temporal elasticity of substitution and the discount factor. Finally, Lopez–Salido and Rabanal (2006) used Bayesian methods to estimate a DSGE model, but for the household sector of that model all parameters were fixed instead of being estimated.

The purpose of this paper is to extend previous work, in particular that of Eichenbaum et al. (1988), in a number of ways. Unlike previous studies, we estimate, using aggregate quarterly data for the last twenty years, the simultaneous system of both the labor supply equation and the inter-temporal consumption equation and test the cross-equation restriction regarding the weight of consumption in the utility function. A number of specification tests are applied to establish the robustness of the results and the soundness of the specification and estimation procedures; they include an autocorrelation test for the residuals, the J-test for instrument exogeneity, the test for the normality of the residuals and finally a Wald-test for parameter stability. The empirical results presented in Section 2 indicate that all preference parameters are significantly estimated, have the correct sign and take plausible values. A notable result is that the estimated value of the weight of consumption in the utility function is much higher than both the value of this parameter estimated by Eichenbaum et al. (1988) and the values used in model calibrations by other researchers (e.g. Domeij and Flodén, 2006; Heathcote et al., 2008; Collard and Dellas, 2012).

The paper is organized as follows. Section 2 develops the theoretical model of household decisions regarding consumption and leisure and presents the estimation methodology and empirical results, and Section 3 concludes.

## 2. Model and estimation results

In this section we develop the consumption-leisure framework in which a representative consumer derives utility from consuming goods and leisure time. We assume that this agent is liquidity constrained and obtains loans to support consumption smoothing.

The consumer maximizes a lifetime utility function given by:

$$U_t = E_t \sum_{j=0}^{\infty} (\beta)^j \, u(C_{t+j}, l_{t+j}) \qquad (1)$$

where $\beta$ is the discount factor, and $u$ denotes utility which is related to real consumption ($C_{t+j}$) and leisure ($l_{t+j}$, expressed as the ratio of leisure time to total available time per period). The utility function is assumed to be twice differentiable with respect to consumption and leisure, the marginal utilities of which are positive and non-increasing.

Some problems of specification arise in the choice of the appropriate form of the utility function. Thus the assumption of additive separable preferences between consumption and leisure appears quite restrictive (see e.g., Bennet and Farmer, 2000; Domeij and Flodén, 2006), while that of multiplicative separable and additive non-separable preferences implies the existence of non-trivial constraints on the preference parameters that are necessary to ensure positive non-increasing marginal utilities. For these reasons, it seems that the most appropriate form of the utility function without any a priori constraint is the Cobb-Douglas function, which incorporates multiplicative non-separable preferences as below:

$$U_t = \frac{\left(C_t^{\gamma}(l_t)^{1-\gamma}\right)^{1-\sigma} - 1}{1-\sigma} \tag{2}$$

where $1/\sigma$ is the intertemporal elasticitiy of the consumption-leisure composite good, and $\gamma$ is the weight of consumption relative to leisure.

The consumer is also assumed to be subject to a sequence of budget constraints. The constraint for period *t* (in real terms) is:

$$C_t + (1 + i_{t-1})\frac{1}{P_t}L_{t-1} = w_t(1 - l_t) + \frac{1}{P_t}L_t \tag{3}$$

where $i_t$ is the interest rate, $P_t$ is the consumer price level, $L_t$ is consumer loans and $w_t$ is the real wage rate.

Next, we set up the Lagrangian for the consumer maximization problem:

$$\mathscr{L} = \sum_{j=0}^{\infty} \beta^j \left\{ \begin{array}{c} \dfrac{\left[(C_{t+j})^{\gamma}(l_{t+j})^{1-\gamma}\right]^{1-\sigma} - 1}{1-\sigma} + \\ \lambda_{t+j}\left[\begin{array}{c} w_{t+j}(1 - l_{t+j}) + \dfrac{1}{P_{t+j}}L_{t+j} - \\ C_{t+j} - (1 + i_{t-1+j})\dfrac{1}{P_{t+j}}L_{t-1+j} \end{array}\right] \end{array} \right\} \tag{4}$$

where $\lambda_{t+j}$ is the Lagrange multiplier.

By taking derivates with respect to consumption, leisure and loans, the following FOC are obtained:

$$\lambda_t = \gamma (C_t)^{\gamma(1-\sigma)-1} (l_t)^{(1-\gamma)(1-\sigma)} \quad (5)$$

$$\lambda_t = (1-\gamma)(C_t)^{\gamma(1-\sigma)} (l_t)^{(1-\gamma)(1-\sigma)-1} \frac{1}{w_t} \quad (6)$$

$$\lambda_t = \beta \lambda_{t+1} (1+i_t) \frac{P_t}{P_{t+1}} \quad (7)$$

By combining eqs. (5) and (6), we derive the static labor supply equation, which corresponds to the optimal intra-temporal choice for consumption and leisure:

$$lnl_t = l\,n\left(\frac{1-\gamma}{\gamma}\right) + lnC_t - lnw_t \quad (8)$$

Also, by combining eqs. (5) and (7), we take the following Euler equation describing the optimal consumption-leisure inter-temporal choice of the representative household:

$$lnC_t = lnC_{t+1} + \frac{1}{\gamma(1-\sigma)-1} \begin{bmatrix} ln(1+i_t) - ln\frac{P_{t+1}}{P_t} + ln\beta + \\ (1-\gamma)(1-\sigma)ln\frac{l_{t+1}}{l_t} \end{bmatrix} \quad (9)$$

The system of eqs. (8) and (9) suggests that consumption and leisure decisions are indeed interdependent. The reason for this originates from the fact that the labor supply plans of households have both an intra- and an inter-temporal dimension. We estimate these equations by using aggregate quarterly U.S. data for 1999Q1 - 2015Q4. The data are seasonally adjusted (except for the interest rate). Sources of the data are the Federal Reserve Economic Data (FRED) and the Organization for Economic Co-operation and Development (OECD) databases. The interest rate is the average of the commercial bank interest rate on credit card plans and the finance rate on personal loans. Inflation is defined in terms of the implicit price deflator of personal consumption expenditure. Consumption refers to non-durable goods and services consumption expenditure expressed in billions of chained 2009 US dollars. The wage variable measures average real weekly earnings before taxes and other deductions, of both private and public sector employees but not of self-employed persons.

In Table I, we report summary statistics of the variables used in the empirical analysis.

**Table I**

**Descriptive statistics**

| Variable | Mean | Std. dev. | Min. | Max. |
|---|---|---|---|---|
| Real consumption expenditure (bn of US$) | 8,576.4 | 742.2 | 7,025.6 | 9,828.2 |
| Interest rate (percent %) | 12.41 | 1.13 | 10.7 | 15.05 |
| Inflation rate (percent %) | 1.89 | 0.97 | -0.94 | 3.99 |
| Real wage rate (US$) | 4,370.7 | 55.13 | 4,212.0 | 4,485.0 |
| Leisure time (ratio) | 0.79 | 0.00 | 0.79 | 0.80 |

The parameters of the system of eqs. (8) and (9) are estimated consistently using single equation GMM subject to the theory restriction as regards the relative weight of consumption in the utility function. The instruments of choice for the two equations are shown in Table II below. Estimation biases that are likely to be due to measurement errors and unobserved heterogeneity across households, which usually afflict the estimated values, are accounted for by specifying a parametric process for the errors (cf. Arellano, 2002). Since our data do indicate the presence of autocorrelation, we assume that the errors follow a first-order autoregressive process with parameter $\rho$.

The estimation results under the cross-equation restriction that permits to identify the parameters of the simultaneous system of the two equations are reported in Table II. All estimated coefficients have the anticipated sign, are statistically significant and take plausible values. The results of the SK- and Q-tests show that the hypothesis that the residuals are further autocorrelated can be rejected while that of residual normality can be marginally accepted. The J-test indicates that all instruments are exogenous. We further apply a Wald-test for the validity of the cross-equation restriction, the p-value of which is equal to 0.40. Thus the hypothesis that the parameter $\gamma$ takes the same value across the two equations cannot be rejected.

**Table II**

**GMM estimation of the system's equations under a cross-equation restriction**

| *Equation* | *1/ σ* | *γ* | *β* | *ρ* | *J-test* | *SK-test* | *Q-test* |
|---|---|---|---|---|---|---|---|
| Eq. (8) | - | 0.74<br>(58.14) | - | 0.965<br>(67.71) | 0.43 | 0.07 | 0.27 |
| Eq. (9) | 0.26<br>(2.07) | 0.74<br>(58.14) | 0.905<br>(177.78) | 0.676<br>(10.50) | 0.43 | 0.04 | 0.58 |

**Notes:**

Instruments for eq. (8): $l_{t-2}, l_{t-3}, l_{t-4}, \Delta l_{t-4}, c_{t-4}, w_{t-1}, \Delta w_{t-1}$

Instruments for eq. (9): $c_{t-2}, c_{t-3}, c_{t-4}, \Delta l_{t-4}, l_{t-4}, r_{t-1}, \Delta r_{t-1}$

Columns 2, 3, 4 and 5 of the Table present coefficient estimates with their t-values in parenthesis. Columns 6,7 and 8 show the p-value of the J-test for instrument exogeneity, the skewness and kurtosis test for normality of the residuals and the Box-Pierce test for higher order autocorrelation of the residuals, respectively. Finally, $r_t$ refers to the real interest rate.

The value of the inter-temporal elasticity of the consumption-leisure composite good is estimated to be 0.26 which lies in the range 0.15 to 0.31 that Eichenbaum et al. (1988) obtained. Further, the discount factor is highly significant and its value is 0.905, which is lower compared to that of the majority of calibrated models, which set this parameter at values not smaller than 0.94 for liquidity constrained households.

The most notable finding in Table 2 is that the weight of consumption is estimated at 0.74. This value is more than four times the estimated values in Eichenbaum et al. (1988) which range from 0.12 to 0.18, while it is about twice as large as the values used in model calibrations (e.g. Domeij and Flodén, 2006; Heathcote et al., 2008; Collard and Dellas, 2012) which range from 0.33 to 0.39. The prior choice of the parameter values draws mainly on Kydland and Prescott (1982), who have set this parameter equal to 1/3 on the grounds that “households’ allocation of time to nonmarket activities is about twice as large as the allocation to market activities”.

## 3. Conclusion

In this paper we have examined the links between consumption and leisure by solving the consumer utility maximization problem under multiplicative non-separable (Cobb-Douglas) preferences. Our strategy involved estimating a static and an inter-temporal equation of consumption and leisure choice and testing the restriction inherent in these equations, which concerns the relative weight of consumption in the utility function. Our empirical results provide strong support for the above non-seperability of preferences and suggest that consumers derive about three fourths of their satisfaction from current consumption and only the remaining one fourth from their current leisure time. In this respect, the choice in many DSGE models to rely, among other parameters, on a "standard value" for the share of consumption in utility would seem unwarranted in view of the estimates presented in this paper.

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