# Fiscal and Monetary Policy in a New Keynesian Model with Tobin's Q Investment Theory Features

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Abstract

The purpose of this article is to carefully lay out the internal monetary and fiscal transmission mechanisms in the context of a New Keynesian model, with a particular focus on the role of capital - the most vital ingredient in the transition from the basic framework to the medium - scale DSGE models. The key concept of this paper is the form of the monetary policy: we assume a two-channel monetary policy, i.e. it is conducted through a rule for money supply and a Taylor-type rule for interest rates, in order to keep up with the ECB and Fed's policies. We also adopt a simple fiscal policy rule for public consumption to examine the interactions between fiscal and monetary policy. Finally, in order to capture the crisis effects we introduce exogenous shocks to both monetary and fiscal policy rules.

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**Keywords:** Transmission Mechanisms; New Keynesian Model; Tobin's Q; Two-Channel Monetary Policy

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# I. Introduction

Dynamic Stochastic General Equilibrium (DSGE) models with nominal rigidities ("New Keynesian" models) became very popular in the last decades. In this paper we present a medium-size New Keynesian Dynamic Stochastic General Equilibrium Model (DSGE) with both fiscal and monetary policies analysis. The analysis is distinguished from the conventional New Keynesian studies in three ways:

First, we focus on the role of capital - the key ingredient in the transition from the basic framework to the medium - scale DSGE models. More specifically, we analyze the accumulation of capital process in a New Keynesian context under indeterminacy.

Second, we assume a two-channel monetary policy, which is conducted through a rule for money supply and a Taylor-type rule for interest rates, in order to keep up with the ECB and Fed's policies. Both central banks, in order to deal with the negative consequences of the 2008 crisis, initially proceeded to lower interest rates and then to an increase of the money supply (with the form of the Quantitative Easing -QE). Also, in order to capture the crisis effects we introduce exogenous shocks to both rules.

Last but not least, in order to examine the efficiency of the fiscal policy and its interactions with the monetary one, we adopt a simple rule for public consumption imported in the literature by Heer and Maußner (2014). Again, in order to capture the crisis effects we introduce an exogenous spending shock.

The paper proceeds as follows. In the next section a New Keynesian DSGE model with capital accumulation is derived. In section III, we present the analytic solution of the model. In Section IV, the model is calibrated. In sections V, the model is simulated and its dynamic properties are analyzed. Section VI concludes.

## II. The Model

We analyze the effects of capital accumulation in the context of a commonly used general equilibrium model with Calvo-type price stickiness. More specifically we consider a canonical set-up model in which labor markets are competitive and the goods markets are monopolistically competitive. The key concept of our analysis is that we discern two kinds of firms: capital producing and final good firms. Capital firms convert consumption goods into capital through investment, and rent this capital to goods producing firms for a rental rate. Final Good firms uses this capital parallel with labor for production. But let us have a non-formal overview of the model before we lay out the particular assumptions explicitly:

The model economy features three sectors, a consumption sector, a productive sector, and the government. Note that time is discrete and the planning horizon is infinite and that the number of the households is equal to the number of the firms.

<u>Consumption sector</u>

Households purchase consumption goods, save via bonds and capital, and supply labor services and capital to the productive sector. They derive utility from labor, money and consumption and are assumed to be representable by one stand-in agent who maximizes his recursive lifetime utility.

<u>Productive sector</u>

The economy's output is produced through labor and capital inputs. We discern two kinds of firms: capital producing and final good firms. Capital firms convert consumption goods into capital through investment, and rent this capital to goods producing firms for a rental rate. Moreover, capital accumulation is subject to real adjustment costs which generate a time varying real price of capital, Tobin's q. Final Good firms uses this capital parallel with labor for production. Monopolistic competition in the good's market gives rise to price setting power which is again constraint by Calvo-type stickiness.

Public sector

The government conducts the fiscal policy and an independent monetary authority, the Central Bank, conducts the monetary one. To be more precise, the government exogenously purchases public consumption financed through taxes and government bonds. The government spending follows a very simple autoregressive process. The monetary policy has two parts: i) the determination of the money supply through a simple exogenous money creation process, and ii) the determination of the bonds' nominal interest rate through a Taylor-type feedback rule. Via the household's Euler equation for these bond's real interest, this rule impacts the real economy due to the presence of the above outlined distortions.

## 1. Households

We suppose ex ante symmetry, so we will analyze the behavior of the representative household. Its utility function is given by:

$$E_t \sum_{t=o}^{\infty} \left(\frac{1}{1+\rho}\right)^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\beta}{1-b}\left(\frac{M_t}{P_t}\right)^{1-b} - \frac{L_t^{1+\lambda}}{1+\lambda}\right] \quad (1)$$

where $C_t$ is the consumption, (M/P) is the real money balances and $L_t$ is the labor supply.

The consumption is consisted by many goods, indexed by j, $j \epsilon [0,1]$. The aggregate consumption across the individual goods is defined in the following CES form,

$$C_t = \left[\int_{j=0}^{\infty} c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \, dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (2)$$

where ε is the demand elasticity of substitution for the individual goods and ε>1.

The representative household has to deal with two problems:

### 1.1) Allocation of spending across goods

The household in order to determine this optimal allocation has to minimize the cost of buying $C_t$,

$$min_{c_{tj}} \int_0^1 p_{jt} \; c_{jt} dj \quad (3)$$

$$s.t \;\; C_t = [\int_{j=0}^{\infty} c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \; dj]^{\frac{\varepsilon}{\varepsilon-1}}$$

From the Lagrangian,

$$\mathcal{L}_t = \int_0^1 p_{jt} \;\; c_{jt} dj + \psi_t \{C_t - [\int_{j=0}^{\infty} c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} \; dj]^{\frac{\varepsilon-1}{\varepsilon}} \} \quad (4)$$

$$\text{F.O.C. :} \;\; \frac{\partial L_t}{\partial C_{jt}} = p_{jt} - \psi_t \left( \left[ \int_0^1 c_{jt}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{1}{\varepsilon-1}} c_{jt}^{\frac{-1}{\varepsilon}} \right) = 0 \quad =>$$

$$=>^{(2)} \quad p_{jt} - \psi_t C_t^{\varepsilon} c_{jt}^{-1/\varepsilon} = 0 =>$$

$$=> \;\; c_{jt} = (\frac{p_{jt}}{\psi_t})^{-\varepsilon} C_t \quad (5)$$

Substituting this to the definition of aggregate consumption across the individual goods (2), it follows:

$$C_t = [\int_{j=0}^{\infty} [(\frac{p_{jt}}{\psi_t})^{-\varepsilon} C_t]^{\frac{\varepsilon-1}{\varepsilon}} \; dj]^{\frac{\varepsilon}{\varepsilon-1}} = \;(\frac{1}{\psi_t})^{-\varepsilon} [\int_0^1 p_{jt}^{1-\varepsilon} dj]^{\varepsilon/\varepsilon-1} C_t =>$$

$$=> \;\; \psi_t = [\int_0^1 p_{tj}^{1-\varepsilon} dj]^{\frac{1}{1-\varepsilon}} = P_t \quad (6)$$

The Lagrange multiplier can be considered to be the price index appropriate for the consumption bundle.

And by substituting equation (6) back to the first order condition (5), yields,

$$c_{jt} \;\; = (\frac{p_{jt}}{P_t})^{-\varepsilon} \, C_t \quad (7)$$

As $\varepsilon \rightarrow \infty$ we move towards perfect competition and firms enjoy less market power. This equation is effectively the demand curve facing the firm j for this product.

Additionally, when the household follows this optimal policy it holds that:

$$\int_{j=0}^{1} p_{jt} \, c_{jt} dj = P_t C_t \qquad (8)$$

### 1.2) Allocation of spending across time

The Budget Constraint of the representative household is defined as,

$$\int_{j=0}^{1} p_{jt}\, c_{jt} dj + q_t I_t + M_t + \frac{1}{1+i_t} B_t \leq R_t K_{t-1}^{H} + B_{t-1} + M_{t-1} + W_t L_t^{H} - T_t + \Pi_t \qquad (9)$$

where $p_{jt}$ is the price of good j, W is the nominal wage, i is the nominal interest rate (which also is the nominal gross bond return), B is an one-period bond ,T an external transfer from the government to the household ( e.g. taxes), R is the nominal rental rate on capital, K is the household's capital savings (i.e. the part of the capital stock that is owned by the household), I the investments, q is the price of capital and finally $\Pi_t$ denotes profits received from firms owned by households.

We assume the following law of motion for the capital stock,

$$I_t = K_t - (1-\delta)K_{t-1} \qquad (10)$$

According the relations (9) and (10) the Budget constraint becomes,

$$P_t C_t + q_t[K_t^H - (1-\delta)K_{t-1}^H] + \mathrm{M_t} + \frac{1}{1+i_t} B_t \leq R_t K_{t-1}^H + B_{t-1} + M_{t-1} + W_t L_t^H - T_t + \Pi_t \quad (11)$$

Therefore, forming the Lagrangian from the problem,

$$\mathcal{L}_t = E_t \sum_{i=0}^{\infty}\{(\frac{1}{1+\rho})^i \left[\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{\beta}{1-b}\left(\frac{M_t}{P_t}\right)^{1-b} - \frac{L_t^{1+\lambda}}{1+\lambda}\right] - \mu_{t+i}(C_{t+i} + q_{t+i}[\frac{K_{t+i}^H}{P_{t+i}} - (1-\delta)\frac{K_{t+i-1}^H}{P_{t+i}}] + \frac{M_{t+i}}{P_{t+i}} + \frac{1}{1+i_t}\frac{B_{t+i}}{P_{t+i}} - \frac{W_{t+i}}{P_{t+i}} L_{t+i}^H - \frac{R_{t+i}}{P_{t+i}} K_{t+i-1}^H - \frac{M_{t+i-1}}{P_{t+i}} - (1+i_{t+i})\frac{B_{t+i-1}}{P_{t+i}} + \frac{T_{t+i}}{P_{t+i}} - \frac{\Pi_{t+i}}{P_{t+i}})\} \qquad (12)$$

From the first order conditions we get:

$C_t^{-\sigma} = \frac{1}{1+\rho} E_t\{{C_{t+1}}^{-\sigma}\left[\frac{(1-\delta)q_{t+1}+R_{t+1}}{q_t}\right]\frac{P_t}{P_{t+1}}\}$ (13) -> Euler equation for consumption

$$L_t^{\lambda} = \frac{W_t}{P_t} C_t^{-\sigma} \quad (14) \rightarrow \text{Euler for Labor}$$

$$\beta\left(\frac{M_t}{P_t}\right)^{-b} - \frac{C_t^{-\sigma}}{P_t} + \frac{1}{1+\rho}\frac{C_{t+1}{}^{-\sigma}}{P_{t+1}} = 0 \quad (15) \rightarrow \text{Euler equation for money}$$

$$E_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{\sigma}\frac{P_{t+1}}{P_t}\right\} = \frac{1}{1+\rho}(1+i_t)(16) \rightarrow \text{Euler for bonds}$$

Combining equations (15) and (16) we get the money demand function:

$$\frac{i_t}{1+i_t} C_t^{-\sigma} = \beta\left(\frac{M_t}{P_t}\right)^{-b} \quad (17) \quad \rightarrow \text{Money Demand}$$

## 2. Government

### 2.1) Fiscal Policy

In some period, t, the government collects real taxes $T_t$, consumes a quantity $G_t$= $g_y Y_t$ which is a percentage of the total income of the economy, and issues bonds of nominal volume $B_{t+1}$ which pay the predetermined nominal interest $i_t$. It thereby has to restrict its activity to policies that satisfy its budget constraint, conditional on not defaulting

Government's BC: Revenues = Expenses

$$B_t - B_{t-1} + T_t = G_t + i_t B_{t-1} + M_t - M_{t-1} \qquad (18)$$

As in Heer and Maußner (2014), government spending is exogenous. In particular, we assume the following autoregressive process of first order

$$G_t = (1-\rho_G)G_{ss} + \rho_G G_{t-1} + \varepsilon_{t+1}^G$$

where:
- $\rho_G \in [0,1]$
- $G_{ss}$ = steady state level
- $\varepsilon_t^G \sim iidN(0,1)$

Public consumption and taxes determines the stances of the government's fiscal policy. To be more precise, the three main stances of fiscal policy are:

- **Neutral fiscal policy** is usually undertaken when an economy is in equilibrium. Government spending is fully funded by tax revenue and overall the budget outcome has a neutral effect on the level of economic activity.
- **Expansionary fiscal policy** involves government spending exceeding tax revenue, and is usually undertaken during recessions. It is also known as reflationary fiscal policy.
- **Contractionary fiscal policy** occurs when government spending is lower than tax revenue, and is usually undertaken to pay down government debt.

Furthermore, the government is not allowed to apply a Ponzi scheme to intertemporally finance its expenditures such that the debt growth rate $B_{t+1}/B_t$ is capped.

**2.2) Monetary Policy**

Monetary policy is conducted by an independent Central Bank and it has two pillars: the money supply which is determined through an exogenous process and the interest rate which is determined through a Taylor-type rule.

***2.2.1) <u>Money Supply</u>***

We assume that the monetary authorities control money supply through the following simple money creation process:

$$\frac{M_t}{M_{t-1}} = 1 + \mu_t \quad (19)$$

where:

$$\mu_t = \rho_m \mu_{t-1} + \varepsilon_t^m,\ \varepsilon_t^m \sim iidN(0,1)$$

The sign of the parameter $\mu_t$ partly implies the stances of monetary policy. More specifically,

- If $\mu_t < 0$, i.e. the government reduces the size of the money supply, the monetary policy is "***contractionary"***.
- If $\mu_t > 0$, i.e. the government increases the size of the money supply, the monetary policy is **expansionary**.
- If $\mu_t = 0$, i.e. the government keeps the size of the money supply constant, the monetary policy is **neutral**.

*2.2.2) An Interest Rate Rule*

We assume that the central bank follows a Taylor (1993) rule of the form,

$$i_t = \rho_i + \varphi_y y_t + \varphi_\pi \pi_t + \nu_t \quad (20)$$

where $\varphi_y$ and $\varphi_\pi$ are positive coefficients, and $v$ is an exogenous stochastic disturbance in the nominal interest rate. It is worth noting that because the constant in this rule is equal to $\rho$, this rule is consistent with zero steady state inflation[1]. This rule implies a countercyclical monetary policy. When inflation is positive, the central bank increases nominal interest rates in order to reduce it. When employment is low, i.e. when output is lower than its "natural" level, the central bank reduces nominal interest rates in order to increase employment and nudge output towards its "natural" level. In addition, this feedback interest rate rule does not result in inflation and price level indeterminacy if the Taylor principle is satisfied, i.e. if the reaction of nominal interest rates to inflation is sufficiently strong.

## 3. Firms

We separate firms into goods producing and capital producing firms in order to simplify the derivation of the price setting equation on the one hand, and the investment/Q equation on the other hand. More specifically, Capital firms convert consumption goods into capital

[1] See Woodford (2003), for a more extensive and complete analysis.

through investment, and rent this capital to goods producing firms for a rental rate. Final Good firms uses this capital parallel with labor for production.

In our model, we are going to assume Calvo type price fixities. Hence, before we continue with the analysis of firms' profit maximization problem, it would be helpful to write a few words about Calvo contracts.

### 3.1) Calvo type price fixities

In the Calvo staggered contracts model (1983), there is a constant probability 1-γ that the firm can set a new price. Thus, a proportion 1- γ of firms can reset their prices in any period, whilst the remaining proportion γ keep their prices constant. This approach has very significant consequences for the monetary policy and business cycles in the basic "new Keynesian" model we are analyzing.

According to the above analysis the expected pricing duration will be equal to

$$(1-\gamma)\sum_{s=0}^{\infty} s\gamma^{s} = {}^{\gamma}/_{1-\gamma}.$$

Since all firms set the same prices in period t, it follows that,

$$\mathrm{P_t} = \left(\gamma(P_{t-1})^{1-\varepsilon} + (1-\gamma)(P_t^{*})^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \quad (21)$$

so the dynamic adjustment of the price level is given by,

$$\left(\frac{\mathrm{P_t}}{\mathrm{P_{t-1}}}\right)^{1-\varepsilon} = \gamma + (1-\gamma)\left(\frac{P_t^{*}}{\mathrm{P_{t-1}}}\right)^{1-\varepsilon} \quad (22)$$

In the steady state with zero inflation it holds that, $P_t = P_{t-1} = P_t^{*} = P$.

We can write equation (22) as,

$$f(P_t, P_{t-1}, P_t^{*}) = \left(\frac{\mathrm{P_t}}{\mathrm{P_{t-1}}}\right)^{1-\varepsilon} - \gamma - (1-\gamma)\left(\frac{P_t^{*}}{\mathrm{P_{t-1}}}\right)^{1-\varepsilon} = 0 \;=>$$

$$f(P_t, P_{t-1}, \bar{P}_t) = e^{(1-\varepsilon)[lnP_t - lnP_{t-1}]} - \gamma - (1-\gamma)e^{(1-\varepsilon)[lnP_t^{*} - lnP_{t-1}]} = 0$$

By first order Taylor log-linear approximation around the long run equilibrium with zero inflation we get,

$$f(P_t, P_{t-1}, P_t^*) \cong f(P_t, P_{t-1}, P_t^*)_{LR} + \frac{\partial f}{\partial lnP_t}^{LR} (lnP_t - lnP_t^{LR})$$

$$+ \frac{\partial f}{\partial lnP_{t-1}}^{LR} (lnP_{t-1} - lnP_{t-1}^{LR}) + \frac{\partial f}{\partial lnP_t^*}^{LR} \left(lnP_t^* - lnP_t^{*LR}\right) =$$

$$\mathrm{e}^{(1-\varepsilon)\ln\mathrm{P}} \frac{1-\varepsilon}{\mathrm{P}} \left( \underbrace{\ln\mathrm{P_t} - \ln\mathrm{P}}_{\widehat{\mathrm{P}}_\mathrm{t}} \right)$$

$$+ \left[ -\mathrm{e}^{-(1-\varepsilon)\ln\mathrm{P}} \frac{1-\varepsilon}{\mathrm{P}} + (1-\gamma)\mathrm{e}^{-(1-\varepsilon)\ln\mathrm{P}} \frac{1-\varepsilon}{\mathrm{P}} \right] \left( \underbrace{\ln\mathrm{P_{t-1}} - \ln\mathrm{P}}_{\widehat{\mathrm{P}}_\mathrm{t-1}} \right) + (1$$

$$- \gamma)\mathrm{e}^{(1-\varepsilon)\ln\mathrm{P}} \frac{1-\varepsilon}{\mathrm{P}} \left( \underbrace{\ln P_t^* - \ln\mathrm{P}}_{\widehat{P}_t^*} \right)$$

$$= \widehat{\mathrm{P}}_\mathrm{t} - \widehat{\mathrm{P}}_\mathrm{t-1} - (1-\gamma)(\widehat{P}_t^* - \widehat{\mathrm{P}}_\mathrm{t-1}) = 0$$

=> $$\widehat{\mathrm{P}}_\mathrm{t} - \widehat{\mathrm{P}}_\mathrm{t-1} = (1-\gamma)(\widehat{P}_t^* - \widehat{\mathrm{P}}_\mathrm{t-1}) \quad (17)$$

or $$\widehat{P}_t^* = \frac{1}{1-\gamma}\widehat{P}_t - \frac{\gamma}{1-\gamma}\widehat{\mathrm{P}}_\mathrm{t-1} \qquad (23)$$

The fact that firms set higher prices than the previous period prices causes the inflation.

**3.2) Capital producing firms**

Capital firms convert consumption goods into capital through investment, and rent this capital to goods producing firms for a rental rate R. The capital stock evolves according to

$$I_t = K_t - (1-\delta)K_{t-1} \qquad (9)$$

and period profits for these firms are given by

$$D_t = R_t K_{t-1} - I_t - \frac{\varphi_k}{2} K_{t-1} \left(\frac{I_t}{K_{t-1}} - \delta\right)^2 \qquad (24)$$

where the last term captures convex adjustment costs to physical capital.

The firm wants to maximize its real value, i.e.

$$\max_{\{I_t,K_t\}_{s=t}^{\infty}} V_t = \sum_{s=t}^{\infty} (\frac{1}{1+\rho})^{s-t} \{R_s K_{s-1} - I_s - \frac{\varphi_k}{2} K_{s-1} (\frac{I_s}{K_{s-1}} - \delta)^2\}$$

s.t. $$I_s = K_s - (1-\delta) K_{s-1}$$

Forming the Lagrangian equation, we get:

$$\mathcal{L} = \sum_{s=t}^{\infty} (\frac{1}{1+\rho})^{s-t} \left\{ R_s K_{s-1} - I_s - \frac{\varphi_k}{2} K_{s-1} \left( \frac{I_s}{K_{s-1}} - \delta \right)^2 - q_s (K_s - (1-\delta) K_{s-1} - I_s) \right\}$$

FOCs: $(I_t)$: $$q_t = 1 + \varphi_k \left( \frac{I_t}{K_{t-1}} - \delta \right) \qquad (25)$$

$(K_t)$: $$q_t = \frac{1}{1+\rho} \{R_{t+1} - \frac{\varphi_k}{2} \left( \frac{I_{t+1}}{K_t} - \delta \right)^2 + \varphi_k \left( \left( \frac{I_{t+1}}{K_t} \right)^2 - \delta \frac{I_{t+1}}{K_t} \right) + (1-\delta) q_{t+1} \}$$

(26)

The Lagrange multiplier $q$ plays a central role (this is Tobin's $q$). As any other Lagrange multiplier, it is a shadow price. In this case, $q_t$ is the shadow price of capital in place at the end of period $t$. Under the optimal plan, the firm invests such that the marginal cost of an additional unit of capital (which equals 1 plus the adjustment cost) must equal the shadow price of capital. We can also write this as the investment equation that Tobin (1969) posited:

$$I_t = (\frac{q_t - 1}{\varphi_k} + \delta) K_{t-1} \qquad (27)$$

So investment is only positive when $q_t > 1$, i.e. when the shadow price of capital exceeds the price of new capital (before adjustments costs).

Equation (26) plays the role of an investment Euler condition. The shadow price of capital today must equal the discounted value of:

- the return of capital next period,
- what you save in adjustment costs next period,
- the future shadow price (since capital can be sold next period).

By using the no-bubble condition $\lim_{T\to\infty} \frac{q_{t+T}}{(1+r)^T} = 0$ (27) and using iterative substitution we can rewrite equation (21) as follows,

$$q_t = \sum_{s=t+1}^{\infty} (\frac{1}{1+\rho})^{s-t} \ \{R_{s+1} + \frac{\varphi_k}{2} [(\frac{I_{s+1}}{K_s})^2 - \delta^2]\} \quad (28)$$

so $q_t$ reflects the NPV of all future marginal return and reduced adjustment cost that you get from purchasing one unit of capital.

**3.3) Final Good firms**

We assume that all firms have access to the same technology. Firms face three constraints in order to maximize their profits. Firstly, they have to work with a given production technology given by,

$$Y_t = A_t {K_t}^{\alpha} {L_t}^{1-\alpha} \ (29)$$

which is a Cobb-Douglas production function with two inputs, labor and capital, and an aggregate disturbance $A_t$ (In fact, it is the Total Factor Productivity - TFP). The Final Good firms rent the capital stock from the Capital Producing firms.

Secondly, firms face the downward sloping demand curve given by,

$$c_{jt} = (\frac{p_{jt}}{P_t})^{-\varepsilon} C_t \quad => y_{jt} = (\frac{p_{jt}}{P_t})^{-\varepsilon} Y_t \ \ (8')$$

where $Y_t$ denotes aggregate demand.

Thirdly, we are going to assume Calvo contracts. According to them in any period a random proportion (1-γ) of firms is able to change their price. Thus, in order to set prices today, firms ought to take into consideration the existing future economic conditions.

Hence, all firms which are able to change their prices in period t, solve the following profit maximization problem,

max $\sum_{i=0}^{\infty} \gamma^s E_t(\prod_{s=0}^{i} \left(\frac{1}{1+i_{t+s}}\right) (\frac{p_{jt}}{P_{t+i}} Y_{t+i}^t - \frac{W_{t+i}}{P_{t+i}} L_{t+i}^t - \frac{R_{t+i}}{P_{t+i}} K_{t+i}^t))$ (30)

s.t. $Y_{t+i}^t = \left(\frac{p_{jt}}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i}$ (8')

where $L_{t+i}^t, K_{t+i}^t$ *and* $Y_{t+i}^t$ are the labor and output level in period t+i of the firm which determined its price in period t.

Labor Market

We can determine the real marginal cost of production by solving the following cost-minimization problem for firm j:

$$\min_{L_{jt}} TC = \frac{W_t}{P_t} L_{jt} - \frac{R_t}{P_t} K_{jt}$$

$$s.t.\ Y_{jt} = c_{jt} = A_t {K_{jt}}^{\alpha} {L_{jt}}^{1-\alpha} = \bar{Y} \qquad (29)$$

or by forming the Lagrangian equation,

$$\mathcal{L} = \frac{W_t}{P_t} L_{jt} - \frac{R_t}{P_t} K_{jt} - \lambda\, (A_t {K_{jt}}^{\alpha} {L_{jt}}^{1-\alpha} - \bar{Y})$$

F.O.C.: $\frac{\partial \mathcal{L}}{\partial L_{jt}} = 0 => \frac{W_t}{P_t} = \lambda(1-\alpha) A_t (\frac{K_{jt}}{L_{jt}})^{\alpha}$ (31)

$\frac{\partial \mathcal{L}}{\partial K_{jt}} = 0 => \frac{R_t}{P_t} = \lambda \alpha A_t (\frac{K_{jt}}{L_{jt}})^{\alpha-1}$ (32)

Dividing the above equations, we get,

$$\frac{W_t}{R_t} = \frac{1-\alpha}{\alpha}\left(\frac{K_{jt}}{L_{jt}}\right) \qquad (33)$$

Now, substituting (32) to (30) or (31) we get the Lagrange multiplier:

$$\lambda = (\frac{\mathrm{R_t}}{\alpha})^{\alpha}(\frac{\mathrm{W_t}}{1-\alpha})^{1-\alpha}$$

Of course, the Lagrange multiplier (i.e. the shadow price which shows the change of total cost for a marginal increase of production) is the marginal cost ,

$$\lambda = MC = (\frac{R_t}{\alpha})^{\alpha}(\frac{W_t}{1-\alpha})^{1-\alpha} \qquad (34)$$

Therefore, the profit-maximization problem (30) becomes:

$$\max_{p_{jt}} E_t \textstyle\sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ \left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - \underbrace{\lambda(1-\alpha)A_t(\frac{K_{t+i}}{L_{t+i}})^{\alpha}}_{\frac{W_{t+i}}{P_{t+i}}} L_{t+i} + \underbrace{\lambda\alpha A_t(\frac{K_{t+i}}{L_{t+i}})^{\alpha-1}}_{\frac{R_{t+i}}{P_{t+i}}} K_{t+i} \right]$$

$$=> \max_{p_{jt}} E_t \textstyle\sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ \left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - \lambda\{(1-\alpha)A_t \frac{K_{t+i}^{\alpha}}{L_{t+i}^{\alpha-1}} + \alpha A_t \frac{K_{t+i}^{\alpha}}{L_{t+i}^{\alpha-1}}\} \right]$$

$$=> \max_{p_{jt}} E_t \textstyle\sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ \left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - MC_t \underbrace{A_t {K_t}^{\alpha} {L_t}^{1-\alpha}}_{Y_{t+i}^{t}} \right]$$

$$=> \max_{p_{jt}} E_t \sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ \left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - MC_t Y_{t+i}^{t} \right]$$

$$=> \max_{p_{jt}} E_t \sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ \left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\varepsilon} Y_{t+i} - MC_t \underbrace{\left(\frac{p_{jt}}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i}}_{Y_{t+i}^{t}} \right]$$

$$=> \max_{p_{jt}} E_t \sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ \left(\frac{p_{jt}}{P_{t+i}}\right)^{1-\varepsilon} - MC_t \left(\frac{p_{jt}}{P_{t+i}}\right)^{-\varepsilon} \right] Y_{t+i} \quad (30')$$

For the optimal price p* the first order condition is given by,

$$\boxed{E_t \sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left[ (1-\varepsilon) \left(\frac{p_{jt}}{P_{t+i}}\right)^{-\varepsilon} - \varepsilon MC_{t+i} \right] \left(\frac{1}{p_t^*}\right) \left(\frac{p_t^*}{P_{t+i}}\right)^{-\varepsilon} Y_{t+i} = 0} \quad (35)$$

The above equation describes the optimal pricing policy of a firm j.

**3.4) The New Keynesian Phillips Curve**

By the first order condition for the optimal price p* (35), inflation can be determined as,

$$\left(\frac{p_t^*}{P_t}\right) = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} MC_{t+i} \left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon}}{E_t \sum_{i=0}^{\infty} \gamma^i \Delta_{i,t+1} \left(\frac{P_{t+i}}{P_t}\right)^{\varepsilon - 1}} \quad (36)$$

By log-linearizing the above relation it follows,

$$\widehat{p_t^*} = E_t [\sum_{i=0}^{\infty} \gamma^i \beta^i \left(\widehat{MC}_{t+i} + \hat{P}_{t+i}\right) \quad (36')$$

This can be quasi-differenced to yield a forward-looking difference equation in the optimal reset price,

$$\hat{p}_t^* = \gamma \beta E_t p_{t+1}^* + \widehat{MC}_t + \hat{P}_t \qquad (37)$$

We remind that the log-linear price evolution index function (20) is,

$$\widehat{P}_t - \widehat{P}_{t-1} = (1-\gamma)(\hat{P}_t^* - \widehat{P}_{t-1}) \quad (23)$$

By equations (23) and (37) it follows,

$$\frac{\hat{P}_t}{1-\gamma} - \frac{\gamma}{1-\gamma}\hat{P}_{t-1} = \gamma\beta\left(\frac{\hat{P}_{t+1}}{1-\gamma}\right) - \gamma\beta\frac{\gamma}{1-\gamma}\hat{P}_t + \widehat{MC}_t + \hat{P}_t \quad (38)$$

and solving for inflation yields,

$$\boxed{\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\widehat{MC}_t} \quad (39)$$

where

$$\tilde{\kappa} = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma} \quad (40)$$

Equation (39) is the **new Keynesian Phillips curve** embedded in a General Equilibrium Model. This equation is nothing else but an “expectations augmented Phillips curve”, which states that inflation rises when the real marginal costs rise. Also, it is nothing else but an aggregate supply curve for the whole economy.

## 4. General Equilibrium

In a dynamic general equilibrium, all markets in the economy have to be cleared simultaneously with all agents acting mutually optimal at all time.

Firstly, we assume ex post symmetry:

- $P_{jt} = P_t, \quad \forall$ firm j
- $Y_{jt} = Y_t, \quad \forall$ firm j
- $L_{jt} = L_t, \quad \forall$ firm j
- $K_{jt} = K_t, \quad \forall$ firm j.

Now, we are able to demonstrate each market’s clearing condition.

i) Labor Market: $L_t^H = L_t^f$

ii) Capital Market: $K_t^H = K_t^f$

iii) Bonds Market: $B_t = 0$

iv) Money Market: $\mathrm{M_t} = \overline{\mathrm{M}} = \mathrm{const}$, given by the government.

v) Dividends: $\Pi_t^H = \Pi_t^f = 0$

vi) Goods Market: Adding Households' and government's budget constraints and using the clearing conditions of the other markets we get the desired clearing condition which is nothing else but the resource constraint of the economy. Indeed:

$$C_t + I_t + \mathrm{M_t} + \frac{1}{1+i_t} B_t = R_t K_{t-1}^H + B_{t-1} + M_{t-1} + W_t L_t^H + \Pi_t^H - T_t \qquad (11)$$

$$B_t - B_{t-1} + T_t = G_t + i_t B_{t-1} + M_t - M_{t-1} \qquad (18)$$

$$\Pi_t^H = \Pi_t^f = Y_t - W_t L_t - R_t K_t$$

We conclude:

$$\mathrm{Y_t} = \mathrm{C_t} + \mathrm{I_t} + \mathrm{G_t} \qquad (41) \rightarrow \text{Resource Constraint}$$

An implicit assumption of this constraint is that the elasticity of substitution between individual consumption goods, σ, is the same as the elasticity of substitution between individual investment goods.

Combining the above conditions, imposing symmetry between firms and households the equilibrium of the economy is described by the following equations:

$$\mathrm{Y_t} = \mathrm{C_t} + \mathrm{I_t} + \mathrm{G_t} \qquad (41) \rightarrow \text{Resource Constraint}$$

$$\mathrm{I_t} = \mathrm{K_t} - (1-\delta)\mathrm{K_{t-1}} \ (9) \rightarrow \text{Law Mation of Capital}$$

A) Demand Side

$$C_t^{-\sigma} = \frac{1}{1+\rho} E_t \left\{ {C_{t+1}}^{-\sigma} Z_{t+1} \frac{P_t}{P_{t+1}} \right\} \text{(13)} \rightarrow \text{Euler for consumption}$$

where $Z_{t+1} = \frac{(1-\delta)q_{t+1}+R_{t+1}}{q_t}$

$$L_t^{\lambda} = \frac{W_t}{P_t} C_t^{-\sigma} \quad \text{(14)} \rightarrow \text{Euler for Labor}$$

$$\frac{i_t}{1+i_t} C_t^{-\sigma} = \beta \left(\frac{M_t}{P_t}\right)^{-b} \quad \text{(17)} \quad \rightarrow \text{Money Demand}$$

$$\frac{M_t}{M_{t-1}} = 1 + \mu_t \quad \text{(19)} \rightarrow \textit{Money Creation Process}$$

$$i_t = \rho_i + \varphi_y y_t + \varphi_\pi \pi_t + \nu_t \quad \text{(20)} \rightarrow \text{Taylor Rule}$$

$$q_t = 1 + \varphi_k \left(\frac{I_t}{K_{t-1}} - \delta\right) = 1 + \varphi_k x_t \text{ (25)} \rightarrow \text{Tobin's q}$$

where $x_t = \frac{K_t - K_{t-1}}{K_{t-1}}$ is the net investment rate.

$$q_t = \frac{1}{1+r} \{R_{t+1} + \frac{\varphi_k}{2} [(\frac{I_{t+1}}{K_t})^2 - \delta^2] + (1-\delta)q_{t+1}\} \text{ (26)} \rightarrow \text{Investment Euler}$$

$$Y_t = A_t {K_t}^{\alpha} {L_t}^{1-\alpha} \quad \text{(29)} \rightarrow \text{Aggregate Production Function}$$

$$\frac{W_t}{P_t} = MC_t(1-\alpha)A_t(\frac{K_t}{L_t})^{\alpha} \text{ (31)} \rightarrow \text{Demand of Labor}$$

$$\frac{R_t}{P_t} = MC_t \alpha A_t (\frac{K_t}{L_t})^{\alpha-1} \quad \text{(32)} \rightarrow \text{Demand of Capital}$$

Or combining them,

$$\frac{W_t}{R_t} = \frac{1-\alpha}{\alpha}\left(\frac{K_t}{L_t}\right) \quad \text{(33)}$$

$$MC_t = (\frac{R_t}{\alpha})^{\alpha} (\frac{W_t}{1-\alpha})^{1-\alpha} \text{ (34)} \rightarrow \text{Marginal Cost}$$

$(1+i_t) = (1+R_t)(1+\pi_t)$ (42) → Fisher Equation[2]

B) Supply Side

$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa}\widehat{MC}_t$ (39) → NKPC

Again, the NKPC plays the role of the economy's aggregate supply. Moreover, AS and NKPC are the two sides of the same coin.

Steady State

In Steady State (SS) our variables are unchanging in time. Hence,

(40) => $Y_{ss} = C_{ss} + I_{ss} + G_{ss}$

(9) => $I_{ss} = \delta K_{ss}$

(13) => $Z_{ss} = 1 + \rho$ and $R_{ss} = \delta + \rho$

[2] The well-known **Fisher equation** provides the link between nominal and real interest rates. Here $(1 + \pi)$ is one plus the inflation rate, i is the nominal interest rate and R is the real interest rate. The inflation rate $\pi_{t+1}$ is defined—as usual—as the percentage change in the price level from period *t* to period *t* + 1.

$$\pi_{t+1} = (P_{t+1} - P_t)/P_t.$$

If a period is one year, then the price level next year is equal to the price this year multiplied by $(1 + \pi)$:

$$P_{t+1} = (1 + \pi_t) \times P_t.$$

The Fisher equation says that these two contracts should be equivalent:

$$(1 + i) = (1 + R) \times (1 + \pi).$$

As an approximation, this equation implies: $i \approx R + \pi$.

$$(14) => L_{ss}^{\lambda} = \frac{W_{ss}}{R_{ss}} C_{ss}^{-\sigma}$$

$$(17) => \frac{i_{ss}}{1+i_{ss}} C_{ss}^{-\sigma} = \beta \left(\frac{\mathrm{M_{ss}}}{\mathrm{P_{ss}}}\right)^{-\mathrm{b}}$$

$$(19) => \mu_t = 0$$

$$(24) => \mathrm{q_{ss}} = 1 \quad \text{and} \quad \mathrm{x_{ss}} = 1$$

$$(25) => \mathrm{R_{ss}} = \delta + \rho$$

$$(29) => Y_{\mathrm{ss}} = \mathrm{A_{ss}} \mathrm{K_{ss}}^{\alpha} \mathrm{L_{ss}}^{1-\alpha}$$

$$(31) => \frac{\mathrm{W_{ss}}}{\mathrm{P_{ss}}} = \mathrm{MC_{ss}}(1-\alpha)\mathrm{A_{ss}}(\frac{\mathrm{K_{ss}}}{\mathrm{L_{ss}}})^{\alpha}$$

$$(32) => \frac{\mathrm{R_{ss}}}{\mathrm{P_{ss}}} = \mathrm{MC_{ss}}\alpha\mathrm{A_{ss}}(\frac{\mathrm{K_{ss}}}{\mathrm{L_{ss}}})^{\alpha-1}$$

$$(33) => \frac{W_{ss}}{R_{ss}} = \frac{1-\alpha}{\alpha}\left(\frac{K_{ss}}{L_{ss}}\right) => W_{ss} = = \frac{1-\alpha}{\alpha}\left(\frac{K_{ss}}{L_{ss}}\right)(\delta + \rho)$$

$$(34) => MC_{\mathrm{ss}} = (\frac{\mathrm{R_{ss}}}{\alpha})^{\alpha}(\frac{\mathrm{W_{ss}}}{1-\alpha})^{1-\alpha}$$

$$(41) => i_{ss} = R_{ss} = \rho + \delta$$

$$(39) => MC_{ss} = \frac{(1-\beta)\pi}{\tilde{\kappa}}$$

<u>Log-Linearization</u>

We log-linearize the above equations around steady state (all variables are expressed as percentage point deviations from steady state):

$$\frac{\mathrm{C_{ss}}}{\mathrm{Y_{ss}}} \hat{\mathrm{c}}_{\mathrm{t}} + \frac{\mathrm{K_{ss}}}{\mathrm{Y_{ss}}} \hat{\mathrm{k}}_{\mathrm{t}} - \frac{\mathrm{K_{ss}}}{\mathrm{Y_{ss}}}(1-\delta)\hat{\mathrm{k}}_{\mathrm{t-1}} + \frac{\mathrm{G_{ss}}}{\mathrm{Y_{ss}}} \hat{\mathrm{g}}_{\mathrm{t}} = \hat{\mathrm{y}}_{\mathrm{t}} \qquad (40')$$

$$\hat{I}_t = \frac{1}{\delta}[\hat{k}_t - (1-\delta)\hat{k}_{t-1}]\,(9')$$

$$\hat{c}_{t+1} = \hat{c}_t + \frac{1}{\sigma}\left(\hat{z}_{t+1} + (\underbrace{\hat{p}_{t+1} - \hat{p}_t}_{E_t\pi_{t+1}})\right) \quad (13')$$

where: $\hat{z}_{t+1} = -\hat{q}_t + \alpha_1\hat{q}_{t+1} + \alpha_2\hat{r}_{t+1}$, $\alpha_1 = -\frac{1-\delta}{1+\rho}$, $\alpha_2 = -\frac{\rho-\delta}{1+\rho}$

$$\hat{w}_t - \hat{p}_t = \sigma\hat{c}_t + \lambda\hat{l}_t \quad (14')$$

$$\hat{m}_t - \hat{p}_t = \frac{1}{b}(\sigma\hat{c}_t - \hat{\imath}_t) \quad (17')$$

$$\hat{m}_t - \hat{m}_{t-1} = g_m \quad (19')$$

$$\hat{q}_t = -\varphi_\kappa(\hat{k}_t - \hat{k}_{t-1}) \quad (23')$$

$$(1+\rho)\hat{q}_t + (1-\delta)\hat{q}_{t+1} + (\rho+\delta)\hat{r}_{t+1} + \underbrace{\delta\varphi_\kappa(\hat{k}_{t+1} - \hat{k}_t)}_{-\delta\hat{q}_{t+1}} = 0 \quad (24')$$

$$\hat{y}_t = \hat{a}_t + a\hat{k}_t + (1-a)\hat{l}_t \quad (27')$$

where: $AR(1)$: $\hat{\alpha}_t = \nu\hat{\alpha}_{t-1} + \varepsilon_t$ , $\varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$

$$\hat{w}_t - \hat{p}_t = \widehat{mc}_t + \hat{a}_t + (1-a)(\hat{k}_t - \hat{l}_t) \quad (30')$$

$$\hat{r}_t - \hat{p}_t = \widehat{mc}_t + \hat{a}_t + a(\hat{k}_t - \hat{l}_t) \quad (31')$$

$$\hat{w}_t - \hat{r}_t = \hat{k}_t - \hat{l}_t \,(32')$$

$$\widehat{mc}_t = a\hat{r}_t + (1-a)\hat{w}_t \quad (33')$$

$$\hat{\imath}_t = \hat{r}_t + \pi_t \quad (41')$$

$$\pi_t = \beta E_t\pi_{t+1} + \tilde{\kappa} mc_t \quad (38)$$

## III. Analytical Solution of the Model

By simplifying the conditions of general equilibrium we get the following 16 x16 system of equilibrium first-order difference equations,

$$\hat{c}_t + \underbrace{\hat{k}_t - (1-\delta)\hat{k}_{t-1}}_{\hat{i}_t} + \hat{g}_t = \hat{y}_t \quad (40') \rightarrow \text{Recource Constraint}$$

where: $AR(1): \hat{g}_t = \rho_g \hat{g}_{t-1} + \varepsilon_t^g$

$$\hat{c}_{t+1} = \hat{c}_t + \frac{1}{\sigma}(\hat{z}_{t+1} + E_t \pi_{t+1}) \ (13') \rightarrow \mathit{Euler\ for\ consumption}$$

$$\hat{z}_{t+1} = -\hat{q}_t - \frac{1-\delta}{1+\rho}\hat{q}_{t+1} - \frac{\rho-\delta}{1+\rho}\hat{r}_{t+1}$$

$$\hat{w}_t - \hat{p}_t = \sigma\hat{c}_t + \lambda\hat{l}_t \ \ (14') \rightarrow \mathit{Euler\ for\ Labor}$$

$$\hat{m}_t - \hat{p}_t = \frac{1}{b}(\sigma\hat{c}_t - i_t) \quad (17') \rightarrow \mathit{Money\ Demand}$$

$$\hat{m}_t - \hat{m}_{t-1} = \mu_t \quad (19') \quad \rightarrow \mathit{Money\ creation\ process}$$

where: $AR(1): \mu_t = \rho_m \mu_{t-1} + \varepsilon_t^m$

$$(1+\rho)\hat{q}_t + (1-\delta)\hat{q}_{t+1} + (\rho+\delta)\hat{r}_{t+1} - \delta\hat{q}_{t+1} = 0$$

$$\hat{y}_t = \hat{a}_t + a\hat{k}_t + (1-a)\hat{l}_t \quad (27') \rightarrow \ \mathit{Production\ Function}$$

where: $AR(1):\ \hat{\alpha}_t = \rho_\alpha \hat{\alpha}_{t-1} + \varepsilon_t^\alpha, \quad \varepsilon_t \sim WN(0, \sigma_\varepsilon^2)$

$$\hat{w}_t - \hat{r}_t = \hat{k}_t - \hat{l}_t \ (32')$$

$$\widehat{mc}_t = \alpha\hat{r}_t + (1-\alpha)\hat{w}_t \ \ (33') \ \rightarrow \ \text{Marginal Cost}$$

$$\hat{i}_t = \hat{r}_t + \pi_t \ \ (41') \ \rightarrow \text{Fisher Equation}$$

$$\pi_t = \beta E_t \pi_{t+1} + \tilde{\kappa} mc_t \ \ (38) \ \rightarrow \ \text{NKPC}$$

$$i_t = \rho_i + \varphi_y y_t + \varphi_\pi \pi_t + \nu_t \quad (20) \ \rightarrow \text{Taylor Rule}$$

We discrete our parameters into:

i) State/pre-determined: {k, i, a, g, μ}

ii) Control/jump: {y, c, L, Invest., m, r, w, π, mc, z, q}

In addition, we can categorize the model's parameters as follows:

- RBC Parameters: { α, ρ, δ, σ, λ, $g_y$ }

- New Keynesian Parameters: {β, ε, $\rho_i$, $\varphi_y$, $\varphi_\pi$}

- Shock Parameters: $\{\rho_g, \rho_m, \rho_\alpha\}$

By manipulating the above functions, we conclude to the following final 13 x 13 system of equilibrium first-order difference equations,

$$\hat{k}_{t+1=}\hat{y}_t - \hat{c}_t - \hat{g}_t + (1-\delta)\hat{k}_t$$

$$\hat{g}_{t+1} = \rho_g\hat{g}_t + \varepsilon^g_{t+1}$$

$$\hat{c}_{t+1} - \frac{1}{\sigma}\hat{z}_{t+1} - \frac{1}{\sigma}E_t\pi_{t+1} = \hat{c}_t$$

$$\hat{z}_{t+1} + \frac{\delta}{1+\rho}\hat{q}_{t+1} - \frac{2\delta}{1+\rho}\hat{r}_{t+1} = 0$$

$$\hat{m}_t + \sigma\frac{b-1}{b}\hat{c}_t + \lambda\hat{l}_t - \hat{w}_t + \frac{1}{b}\hat{r}_t + \frac{1}{b}\hat{\pi}_t = 0$$

$$\hat{m}_{t+1} - \mu_{t+1} = \hat{m}_t$$

$$\mu_{t+1} = \rho_m\mu_t + \varepsilon^m_{t+1}$$

$$\hat{q}_{t+1} + \varphi_\kappa\hat{k}_{t+1} = \varphi_\kappa\hat{k}_t \quad (23')$$

$$\hat{y}_t - \hat{a}_t - a\hat{k}_t - (1-a)\hat{l}_t = 0$$

$$\hat{\alpha}_{t+1} = \rho_\alpha\hat{\alpha}_t + \varepsilon^\alpha_{t+1}$$

$\widehat{w}_t - \hat{r}_t - \hat{k}_t + \hat{l}_t = 0$

$\beta E_t \pi_{t+1} = \pi_t - \tilde{\kappa}\alpha\hat{k}_t - \tilde{\kappa}(1-a)\hat{l}_t$

$\hat{r}_t + \pi_t - \rho_i - \varphi_y y_t - \varphi_\pi \pi_t - \nu_t = 0$

Now, we have the following 13 variables,

State/pre-determined: $s_t =$ [k , a, g, μ],

Control/jump: $x_t =$ [m, y, c, l, r, z, q, w, π].

Now we can give the state-space form of our system,

$$A\begin{bmatrix} s_{t+1} \\ x_{t+1} \end{bmatrix} = B\begin{bmatrix} s_t \\ x_t \end{bmatrix} + Ce_t$$

where:

$$A=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1/\sigma & 0 & 0 & -1/\sigma \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2\delta/1+\rho & 1 & \delta/1+\rho & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\varphi_\kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$

$$
B=\begin{bmatrix}
1-\delta & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_g & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & \frac{\sigma(1-b)}{b} & \lambda & \frac{1}{b} & 0 & 0 & -1 & \frac{1}{b} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \rho_m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\varphi_\kappa & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-a & -1 & 0 & 0 & 0 & 1 & 0 & -(1-a) & 0 & 0 & 0 & 0 & 0 \\
0 & \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 \\
-\widetilde{\kappa}\alpha & 0 & 0 & 0 & 0 & 0 & 0 & \widetilde{\kappa}(1-\alpha) & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & -\varphi_y & 0 & 0 & 1 & 0 & 0 & 0 & 1-\varphi_\pi
\end{bmatrix}
$$

$$
C=\begin{bmatrix}
0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix} \quad \text{and} \quad e_t=\begin{bmatrix}
\varepsilon_t^g \\
\varepsilon_t^m \\
\varepsilon_t^\alpha \\
\rho_i+\nu_t
\end{bmatrix}
$$

Since matrix A is non-singular (det(A)=0) the Blanchard-Kahn's Method cannot be applied to solve the model. Hence, we adopt the QZ – decomposition form.

## IV. Calibration

Table 1 contains the calibrated parameters. The choice of parameters is one of the main features of the analysis as it must represent economic features and to ensure the stability of the system. The parameters are separated into RBC and New Keynesian parameters. For the latter, we follow the standard literature. To be more precise, new Keynesian parameters are mostly chosen as in Galí (2008) and the recent work by Poutineau, Sobczak and Vermandel (2015). Regarding the Taylor rule, the monetary authorities should respond more than proportionally to inflation developments (namely, $\varphi_{\pi} > 1$) according to the Taylor principle[3]. In this case a rise in inflation leads to a more than proportional rise in nominal interest causing an increase in real interest rates that affects agents' economic decisions and thus the real macroeconomic equilibrium of the model. In addition, the intra-temporal elasticity between intermediate goods is set at 6 which implies a steady state mark-up of 20 % in the goods' market corresponding to what is observed in main developed

| Parameters | Explanation | Value |
| --- | --- | --- |
| $\alpha$ | share of capital in output | 0.36 |
| $\rho$ | discount factor | 0.2 |
| $\delta$ | depreciation of capital | 0.025 |
| $\sigma$ | risk aversion for consumption | 1 |
| $\lambda$ | labor disutility | 1 |
| b | risk aversion for cash | 1 |
| $\beta$ | NKPC, forward term | 0.75 |
| $\varepsilon$ | elasticity/ mark-up on prices | 6 |
| $\gamma$ | portion of firms that cannot change their prices in t | 0.75 |
| $\varphi_{\kappa}$ | capital adjustment cost parameter | 3 |
| $\varphi_{y}$ | monetary policy GDP growth market | 0.125 |
| $\varphi_{\pi}$ | monetary policy inflation growth target | 1.5 |
| $\rho_{i}$ | monetary Policy smoothing parameter | 0.4 |
| $\rho_{g}$ | government sending's shock smoothing parameter | 0.87 |
| $\rho_{m}$ | money supply's shock smoothing parameter | 0.5 |
| $\rho_{a}$ | productivity's shock smoothing parameter | 0.90 |

**Table 1:** Calibrated Parameters

[3] In particular, the Taylor rule stipulates that for each one-percent increase in inflation, the central bank should raise the nominal interest rate by more than one percentage point. This aspect of the rule is often called the **Taylor principle**.

economies. For the RBC parameters we also follow the standard bibliography and more specifically we use the values from Cooley and Prescott (1995). Regarding the policy shock's smoothing parameters, for the government shock's smoothing parameter we follow the work by Heer and Maußner (2014) and the money supply's smoothing parameter is chosen by Sim's (2015) paper. Finally, in order to examine the effect of the capital to the fiscal and monetary policy we allow the adjustment cost parameter to take three possible values: 0, 1.5, and 3. The case of $\varphi_{\kappa} = 0$ corresponds with the case of the standard RBC model without adjustment costs.

## V. Impulse Responses

We next explore the internal mechanics of the model by plotting some impulse response functions. Each impulse response reports the effect of a one standard deviation shock on the variables of the model, expressed in percent deviation from their steady state level.

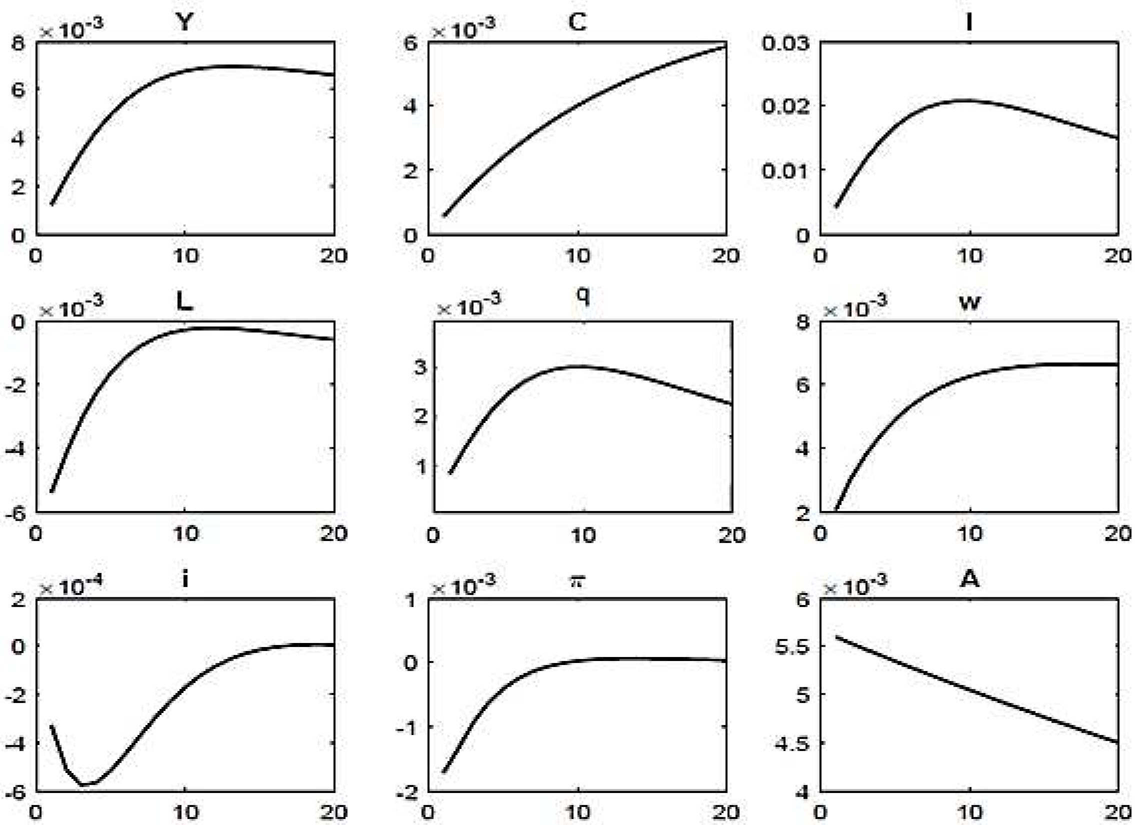


**Figure 1:** Productivity Shock

Let's start our analysis with impulse responses to a one percent positive productivity shock for output, consumption, investment, labor, Tobin’s q, real wages, nominal interest rate, inflation and technology, which are plotted in Figure 1. Output, consumption, and investment all increase on impact. Hours worked decline. This decline is driven by real frictions (in particular the investment adjustment cost). The path of the real wage is similar to output. In addition, the path of capital’s shadow price, i.e. Tobin’s q, follows an analogous path to investment. Finally, inflation falls.

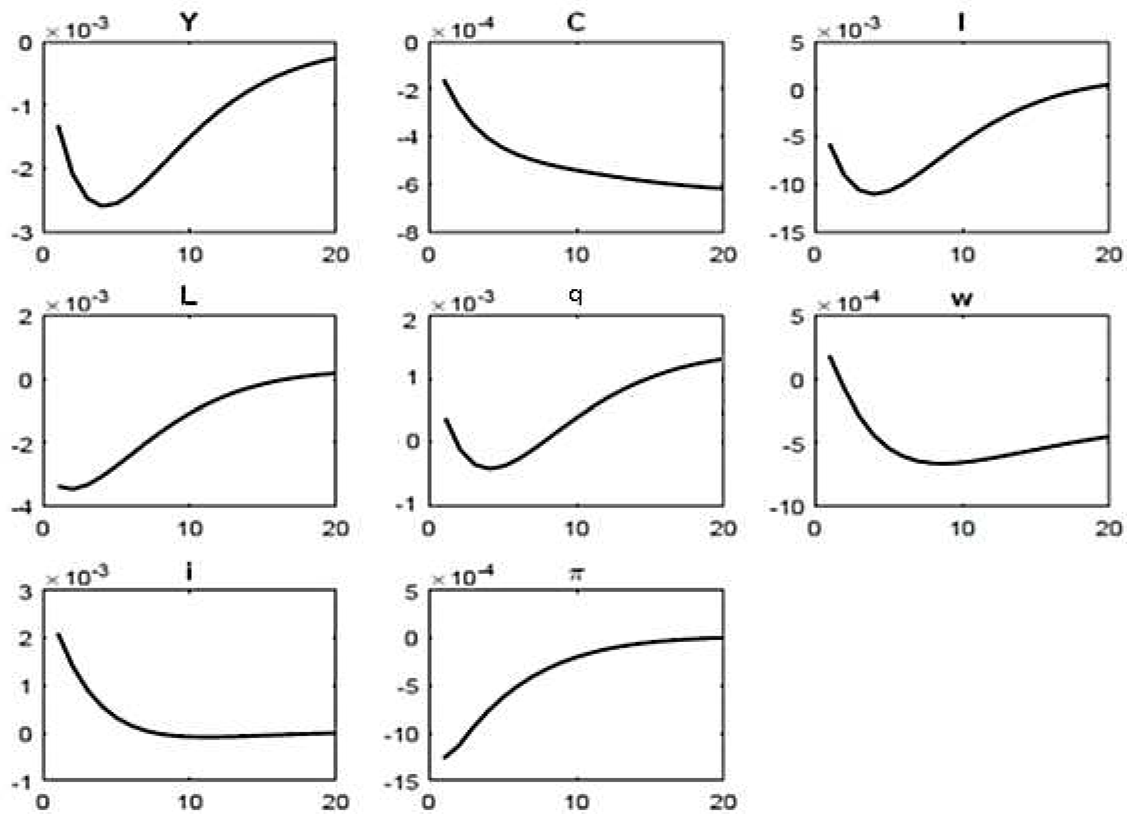


**Figure 2:** Interest Rate Policy Shock

Now, let's focus our analysis on the impulse responses to a one percent interest rate policy shock for output, consumption, investment, labor, Tobin’s q, real wages, nominal interest rate and inflation. We plot the effects of such a shock in Figure 2. Since this is a positive shock to the interest rate rule, it implies contractionary monetary. Output falls on impact and follows a hump-shape before reverting back to trend. Consumption and investment both fall. Hours worked again decline due to the investment adjustment cost. In addition, the path of capital's shadow price, i.e. Tobin's q, follows an analogous path to investment. Interestingly, real wage rises on impact at the beginning but after some time it falls. Inflation falls until it returns to the zero level.

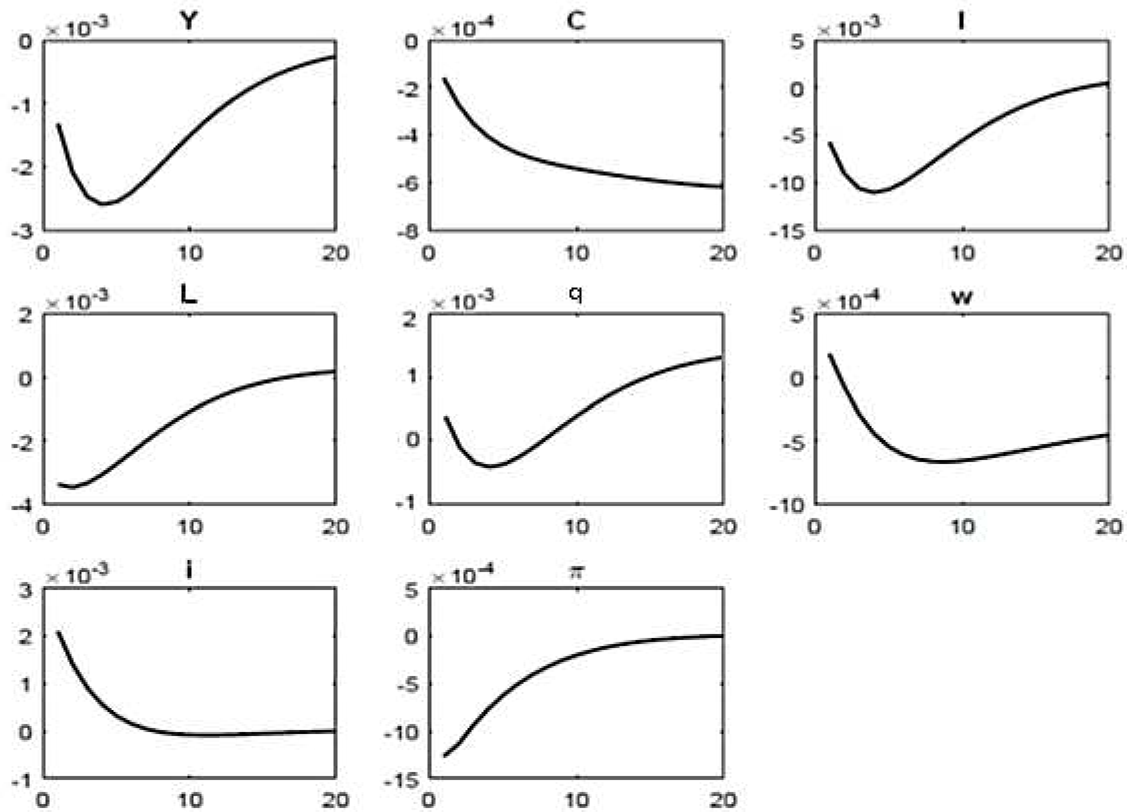


**Figure 3:** Money Supply Shock

Now, let's focus our analysis on the impulse responses to a one percent negative money supply shock (which implies contractionary monetary policy) for output, consumption, investment, labor, Tobin's q, real wages, nominal interest rate and inflation. We plot the effects of such a shock in Figure 3. As we can see the impulse responses of a negative money supply shock are identical with those of a positive interest rate policy rule. This implies that the two tools of monetary policy (namely, interest rate and money supply) have very similar impact on the real economy.

Both FED and ECB followed an expansionary monetary policy in order to deal with the negative consequences of the 2008 crisis. More specifically, they initially proceeded to lower interest rates and then to an increase of the money supply (with the form of the Quantitative Easing -QE). Of course, the adoption of such a policy re-sparkled a great deal of controversy between Keynesian and neoclassical economists around the issue of the liquidity trap. The formers claim that further injections of cash into the private banking system by a central bank will fail to decrease interest rates and hence make monetary policy ineffective[4]. On the other hand, the neoclassical economists asserted that, even in a liquidity trap, expansive monetary policy could still stimulate the economy via the direct effects of increased money stocks on aggregate demand. This essentially was the hope of the central banks of the United States and Europe in 2008–2009, with their foray into quantitative easing. These policy initiatives tried to stimulate the economy through methods other than the reduction of short-term interest rates.

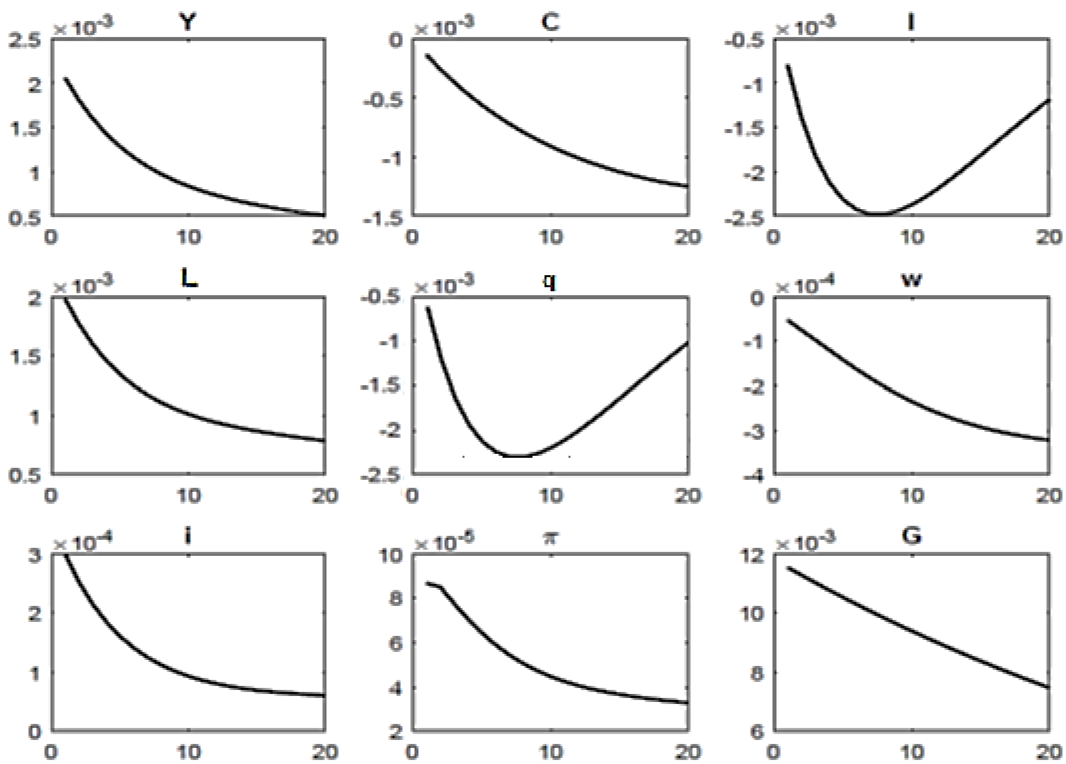


**Figure 4:** Government Spending Shock

[4] See for example Krugman Paul's article "How much of the world is in a liquidity trap?" in *The New York Times* (17 March 2010).

Finally, we examine the impulse responses to a one percent government spending shock for output, consumption, investment, labor, Tobin's q, real wages, nominal interest rate, inflation and governmental expenditures in Figure 4. This also raises output and inflation. Labor goes up, while the real wage falls. Consumption and investment both fall. Again, the path of capital's shadow price, i.e. Tobin's q, follows an analogous path to investment.

## VI. Conclusions

The purpose of this article is to carefully lay out the internal monetary and fiscal transmission mechanisms in the context of a New Keynesian model. More specifically, this paper presents fiscal and monetary policies analysis in a context of a medium-size New Keynesian Dynamic Stochastic General Equilibrium Model (DSGE) with Calvo type price stickiness and capital accumulation.

The analysis is distinguished from the conventional New Keynesian studies in three ways. First, we focus on the role of capital - the key ingredient in the transition from the basic framework to the medium - scale DSGE models. Second, we assume a two-channel monetary policy, i.e. it is conducted through a rule for money supply and a Taylor-type rule for interest rates, in order to keep up with the ECB and Fed's policies. Both central banks, in order to deal with the negative consequences of the 2008 crisis, initially proceeded to lower interest rates and then to an increase of the money supply (with the form of the Quantitative Easing -QE). Third, in order to examine the efficiency of the fiscal policy and its interactions with the monetary one we adopt a simple rule for public consumption imported in the literature by Heer and Maußner (2014). Finally, in order to capture the dynamic crisis effects we introduce exogenous shocks to both monetary and fiscal policy rules.

Our paper is a further step to the effort for the invigoration of the link between economic reality and theory. More specifically, both ECB and FED conducted a two stage expansionary monetary policy after the burst of the 2008 global financial crisis: they initially proceeded to lower interest rates and then to an increase of the money supply (through the Quantitative Easing -QE). In this paper we adopted a two channel monetary policy in order to simulate the monetary policy of the above central banks: it is conducted through a rule for money supply and a Taylor-type rule for interest rates. We found that the two tools of monetary policy (interest rate and money supply) have very similar impact on the economy. More specifically, we found that a contractionary monetary policy (which can be achieved either by a higher interest rate for government bonds or a lower money supply) leads to a significant decrease of output, consumption, employment and investment. It also drives to a temporary deflation. Of course, these changes are not persistent over time. The growth of the economy will return to its trend and the inflation to its zero level. Hence, an expansionary monetary policy is expected to temporary tone up the economy (by increasing the output, the investments and the consumption) and mitigate the negative effects of the crisis without creating a persistent inflation. Thus, our model is compatible with the recipe of the expansionary monetary for the tackling of the severe financial crisis which was adopted by the aforementioned central banks.

Finally, regarding the government's fiscal policy we found that an expansionary spending policy raises output and inflation. In addition, labor goes up but the real wages fall. Also, investments are decreased a fact that implies that such a policy crowds out the private sector's spending. Hence, our model is compatible with the standard implications of economic theory.

## References


Calvo, G., 1983, “Staggered prices in a utility-maximizing framework”, *Journal of Monetary Economics*, Vol. 12 (3), pp. 383-398.

Christiano, L.J., Eichenbaum, M., and Evans, C.L., 2005, “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy”, *Journal of Political Economy*, Vol. 113 (1), pp. 1–45.

Cooley, T., and Prescott, E., 1995, “Frontiers of Business Cycle Research”, chapter 1, Princeton University Press.

Del Negro, M., and Sims, C. A., 2015. "When does a central bank's balance sheet require fiscal support?," *Journal of Monetary Economics*, Elsevier, Vol. 73(C), pp. 1-19.

Gali, J., 2008, “Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework and Its Applications”, Princeton University Press.

Gali, J., Smets, F., and Wouters, R., 2011, “Unemployment in an Estimated New Keynesian Model.” NBER Working Paper, no. 17084.

Heer, B., and Maußner, A., 2014, “Q-Targeting in New Keynesian Models.” Mimeo. University of Augsburg.

Lucas, R., 1987, “Models of business cycles”, New York: Basil Blackwell editions.

Poutineau, J. C., Sobczak, K., and Vermandel, G., 2015, “The analytics of the New Keynesian 3-equation Model”, *Economics and Business Review*, Vol. 1 (15), pp. 110–129.

Prescott, E.C., 1986, “Theory Ahead of Business Cycle Measurement.” Federal Reserve Bank of Minneapolis Quarterly Review, No. 10, pp. 9–22.

Smets, F., and Wouters, R., 2003, “An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area”, *Journal of the European Economic Association,* Vol. 1 (5), pp. 1123–1175.

Taylor, J., 1993, “Discretion versus Policy Rules in practice", Carnegie-Rochester Conference Series on Public Policy, Vol. 39, pp. 195-214.

Tobin, J., 1958, “Estimation of Relationships for Limited Dependent Variables”, *Econometrica*, Vol. 26 (1), pp. 24-36.