Firms' Costs, Profits, Entries, and Innovation under Optimal Privatization Policy

Haraguchi, Junichi and Matsumura, Toshihiro

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Junichi Haraguchi†
Faculty of Economics, Gakushuin University
and
Toshihiro Matsumura‡
Institute of Social Science, The University of Tokyo

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Abstract

We investigate how cost conditions of private firms affect optimal privatization policy and private firms' profits. We find that the optimal degree of privatization is decreasing with the costs of private firms unless the public firm is fully privatized in equilibrium. A cost reduction in a private firm increases the degree of privatization and benefits for all private firms. Therefore, each private firm's profit is increasing with its rival private firms' costs, which is in contrast to the result when the degree of privatization is given exogenously. This interesting property yields two important results. The profit of each private firm can increase with the number of private firms, and the positive externality of innovation accelerates private firms' R&D.

JEL classification numbers: D43, H44, L33

Key words: partial privatization, cost-reducing R&D, asymmetric private firms, constant marginal costs

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†Corresponding author. Faculty of Economics, Gakushuin University, 1-5-1, Mejiro, Toshima-ku, Tokyo 171-8588, Japan. Phone: (81)-3-5992-3075. Fax: (81)-3-5992-1007. E-mail: jyunichi.haraguchi@gmail.com

‡Institute of Social Science, the University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Phone: (81)-3-5841-4932. Fax: (81)-3-5841-4905. E-mail: matusmur@iss.u-tokyo.ac.jp

1 Introduction

For more than 50 years, we have observed a worldwide wave of the privatization of state-owned public enterprises. Nevertheless, many public and partially privatized enterprises are still active in planned and market economies in developed, developing, and transitional countries. While some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public and partially privatized enterprises competes with private enterprises in a wide range of industries.[1] Optimal privatization policies in such mixed oligopolies have attracted extensive attention from economics researchers in such fields as industrial organization, international economics, public economics, financial economics, and development economics.[2]

Specifically, the literature on mixed oligopolies has investigated optimal privatization policy in different situations. Matsumura (1998) investigated Cournot mixed duopolies and showed that the optimal degree of privatization is never zero unless full nationalization yields a public monopoly. Lin and Matsumura (2012) and Matsumura and Okamura (2015) found that the optimal degree of privatization increases with the number of private firms and decreases with the foreign ownership share in private firms. In free-entry markets, Matsumura and Kanda (2005) showed that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) found that the optimal degree of privatization is strictly positive when private competitors are foreign and increases with the foreign ownership share in private firms. In addition, Chen (2017) showed that the optimal degree of privatization is positive even in free-entry markets if privatization improves production efficiency. Fujiwara (2007) showed a nonmonotonic (monotonic) relationship between the degree of product differentiation and optimal degree of privatization in a non-free-entry (free-entry) market. Cato and Matsumura (2015) discussed the relationship between optimal trade and privatization policies, showing that a higher tariff rate reduces the optimal degree of privatization in free-entry markets. Lee *et al.* (2017) showed that the optimal degree

[1] Examples include United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication (NTT), Japan Tobacco, Volkswagen, Renault, Électricité de France, Japan Postal Bank, Kampo, Korea Development Bank, and Korea Investment Corporation.

[2] For examples of mixed oligopolies and recent developments in this field, see Heywood and Ye (2009a), Ishida and Matsushima (2009), Chen (2017), and the works cited therein.

of privatization depends on the timing of privatization. Heywood *et al.* (2017) investigated how asymmetric information on demand conditions affects the optimal degree of privatization.

One common assumption in the abovementioned studies is that all private firms share the same cost function. However, in mixed oligopolies, it is often the case that private firms are not symmetric. In the Japanese financial industry, public financial institutions compete with mega banks, such as the Bank of Tokyo-Mitsubishi UFJ, and smaller regional banks, such as Aozora Bank. In the overnight delivery industry, Japan Post competes with Yamato Transport, Nippon Express, Sagawa Express, and Seino Transportation, which are far from symmetric. In the telecommunication industry, NTT group competes with large carriers, such as Softbank and KDDI Corp., as well as many of smaller companies, such as Japan Communication. In the automobile industry, VW and Rouault compete with huge private firms, such as General Motors and Toyota Motor Corp., as well as smaller private firms, such as Hyundai Motor Company, Honda Motor Co. Ltd., and Mazda Motor Corp. Thus, it is realistic to assume that private firms are not always symmetric.

In this study, we allow cost asymmetry among private firms. This enriches the analysis of mixed oligopolies. For example, suppose that a decrease in private firms' costs increases these firms' profits. If we allow asymmetric costs among private firms, we can decompose this cost-reduction effect into the following two effects: the effect of the reduction of a firm's own cost and that of its rival's cost. Then, we can investigate how the rival's cost affects profits and thus, the behavior of other private firms. In this study, we use the model of Pal (1998) with linear demand and constant marginal costs, and we allow cost differences among private firms. We adopt the partial privatization approach of Matsumura (1998) and endogenize the degree of privatization.

We find that under optimal privatization policy, the reduction of a private firm's marginal cost increases the profits of all private firms. Under the optimal privatization policy, a reduction of a private firm's marginal cost increases the degree of privatization, which makes the public firm less aggressive. This is beneficial for all private firms and thus, the reduction of a private firm's marginal cost is beneficial for all private firms. By contrast, if the degree of privatization is given exogenously, the reduction of a private firm's marginal cost increases its own profit but reduces the other private firms' profit.

This basic principle can apply to mixed oligopolies with symmetric private firms, and we derive important implications even in models with these firms. First, we investigate the relationship between the new entry of a private firm and the optimal degree of privatization. We find that the new entry of a private firm increases the degree of privatization, which increases the profits of all private firms. By contrast, if the degree of privatization is given exogenously, the new entry of a private firm decreases the profits of all incumbent private firms.

Next, we investigate an innovation incentive for private firms. We formulate a model in which private firms engage in cost-reducing R&D investments with externality among private firms. We find that private firms more intensively engage in innovation when the degree of privatization is endogenous. In addition, we find that R&D expenditure is increasing with the degree of spillover effect among private firms and the number of private firms when the degree of privatization is endogenous. These findings are because a decrease of one private firm's cost increases the profits of all private firms. These results suggest that the timing of privatization affects the entry decision and innovation activities of private firms.

The rest of this paper is organized as follows. In Section 2, we present the basic model. Section 3 presents an equilibrium analysis and derives the optimal privatization policy. Section 4 discusses the relationship between private firms' profits and their costs. Section 5 investigates the relationship between private firms' profits and new entries. Section 6 endogenizes the costs of private firms by considering cost-reducing R&D. Section 7 concludes.

2 The Model

We consider a mixed oligopoly model in which one public firm (firm 0) competes with n private firms (firms 1, 2,...,n). These firms produce homogeneous products for which the inverse demand function is

$$p(Q) = a - Q,$$

where p denotes price, a is a positive constant, and $Q := \sum_{i=0}^{n} q_i$ is the total output. The marginal costs are constant. Let $c_i \geq 0$ be the firm i's marginal cost. Each firm's profit is given by

$$\pi_i = (p(Q) - c_i)q_i.$$

We assume that $c_i < c_0$ for $i = 1, 2, ..., n$. In other words, we assume that the public firm is less efficient than the private firm.[3]

The social surplus W is given by

$$W = \int_0^Q p(q)dq - pQ + \sum_{i=0}^{n} \pi_i = \int_0^Q p(q)dq - \sum_{i=0}^{n} c_i q_i.$$

Following Matsumura (1998), the public firm's objective Ω is convex-combination of social surplus and their own profit,

$$\Omega = \alpha\pi_0 + (1-\alpha)W,$$

where $\alpha \in [0, 1]$ represents the degree of privatization. In the case of full nationalization (i.e., $\alpha = 0$), firm 0 maximizes social welfare. In the case of full privatization (i.e., $\alpha = 1$), firm 0 maximizes its profit. Each private firm's objective is its profit.

The complete information game runs as follows. In the first stage, the government chooses the degree of privatization α to maximize the social surplus. In the second stage, each firm simultaneously chooses its output to maximize its objective. We solve this game by backward induction and the equilibrium concept is subgame perfect Nash equilibrium. Throughout this study, we assume that a is sufficiently large. It guarantees that the solutions in the second-stage subgames are interior. In other words, the public firm produces a positive output in equilibrium regardless of α.

[3] The assumptions of linear demand and constant marginal costs with cost disadvantage of a public firm over private firms is popular in the literature on mixed oligopolies. See Pal (1998), Capuano and De Feo (2010), and Matsumura and Ogawa (2010). For a discussion on the endogenous cost disadvantage of public firms, see Matsumura and Matsushima (2004).

3 Equilibrium

First, we solve the second stage game given α. The first-order conditions of public and private firms are

$$\begin{aligned}\frac{\partial \Omega}{\partial q_0} &= a-(1+\alpha)q_0-\sum_{i=1}^{n} q_i - c_0 = 0,\\ \frac{\partial \pi_i}{\partial q_i} &= a-2q_i-\sum_{j\neq i} q_j - c_i = 0 \quad (i=1,...n),\end{aligned}$$

respectively. The second-order conditions are satisfied. These first-order conditions yield the following reaction functions of public and private firms

$$\begin{aligned}R_0(q_i) &= \frac{a-\sum_{i=1}^{n} q_i - c_0}{1+\alpha},\\ R_i(q_j) &= \frac{a-\sum_{j\neq i} q_j - c_i}{2} \quad (i=1,2,...,n \quad j\neq i),\end{aligned}$$

respectively. These reaction functions yield the following equilibrium quantities of public and private firms

$$q_0^*(\alpha) = \frac{a-(n+1)c_0+\sum_{i=1}^{n} c_i}{1+(n+1)\alpha}, \tag{1}$$

$$q_i^*(\alpha) = \frac{\alpha(a+\sum_{i=1}^{n} c_i)+c_0-(1+(n+1)\alpha)c_i}{1+(n+1)\alpha} \quad (i=1,2,...,n), \tag{2}$$

respectively. We obtain the following equilibrium total output, price, private firm's profit, and welfare

$$Q^*(\alpha) = \frac{(na-\sum_{i=1}^{n} c_i)\alpha+a-c_0}{1+(n+1)\alpha}, \tag{3}$$

$$p^*(\alpha) = \frac{(a+\sum_{i=1}^{n} c_i)\alpha+c_0}{1+(n+1)\alpha}, \tag{4}$$

$$\pi_i^*(\alpha) = \left(\frac{\alpha(a+\sum_{i=1}^{n} c_i)+c_0-(1+(n+1)\alpha)c_i}{1+(n+1)\alpha}\right)^2 \quad (i=1,2,...,n), \tag{5}$$

$$W^*(\alpha) = \frac{X_1}{2(1+(n+1)\alpha)^2}, \tag{6}$$

respectively, where $X_1 := (a(1+n\alpha)-c_0-\alpha\sum_{i=1}^{n} c_i)^2+2\alpha(a-(n+1)c_0+\sum_{i=1}^{n} c_i)^2+2\sum_{i=1}^{n}(\alpha(a+\sum_{i=1}^{n} c_i)+c_0-(1+(n+1)\alpha)c_i)^2$.

Next, we discuss the government's welfare maximization problem in the first stage. Let α^* be the equilibrium degree of privatization.

Lemma 1 *(i)* $\alpha^* > 0$. *(ii)* $\alpha^* = \min\{\alpha^{**}, 1\}$, *where*

$$\alpha^{**} := \frac{nc_0 - \sum_{i=1}^n c_i}{a - (n+1)^2 c_0 + (n+2)\sum_{i=1}^n c_i}.$$

(iii) α^{**} *is decreasing in* c_i *for* $i = 1, 2, ..., n$ *and increasing in* c_0.

Proof See the Appendix.

Lemma 1(i) was shown by Matsumura (1998) in duopolies and by Matsumura and Kanda (2005) in oligopolies with symmetry among private firms.

Lemma 1(iii) states that as long as the solution is interior (i.e., full privatization is not optimal), the optimal degree of privatization is decreasing with the cost of each private firm. An increase of the degree of privatization makes firm 0 less aggressive, because it is less concerned with consumer surplus. Through the strategic interaction, the less aggressive behavior of firm 0 makes private firms more aggressive. In other words, production substitution from the public firm to the private firms takes place. Because the marginal cost of the public firm is higher than that of each private firm, this production substitution improves welfare (welfare-improving effect).[4] However, because the total output is decreasing in α, an increase of the degree of privatization reduces welfare (welfare-reducing effect). This trade-off determines the optimal degree of privatization. The higher (lower) c_0 (c_i for $i = 1, 2, ..., n$) is, the stronger is the abovementioned welfare improving effect of the production substitution. Therefore, the optimal degree of privatization is increasing in c_0 and decreasing in c_i for $i = 1, 2, ..., n$.

From Lemma 1(ii), we find that the optimal degree of privatization remains unchanged as long as $\sum_{i=1}^n c_i$ remains unchanged. This includes an important policy implication. Given the average productivity among private firms, the distribution of the costs among private firms (or the degree of heterogeneity among private firms) does not affect the optimal privatization policy.

[4] For an excellent discussion on the welfare-improving production substitution, see Lahiri and Ono (1988).

4 Private Firms' Profits

Suppose that the solution of the first stage is interior (i.e., $\alpha^* < 1$). By substituting α^{**} into $\pi_i^*(\alpha)$, we obtain the following equilibrium profit of private firms:

$$\pi_i^* = \left((n+1)c_0 - \sum_{i=1}^{n} c_i - c_i \right)^2 \quad (i = 1, 2, ..., n). \tag{7}$$

We now present our main result.

Proposition 1 *If the optimal privatization policy is not full privatization (i.e., $\alpha^* < 1$), private firm i's profit is decreasing in c_j for $i, j = 1, 2, ..., n$ and increasing in c_0.*

Proof See Appendix.

In order to highlight the implication and intuition of this result, we present a supplementary result as a benchmark.

Proposition 2 *Suppose that the degree of privatization α is given exogenously. (i) Private firm i's profit is decreasing in c_i and increasing in c_0. (ii) Private firm i's profit is nondecreasing in c_j for $i = 1, 2, ..., n$, $j =, 1, ..., n$ and $j \neq i$. (iii) If $\alpha > 0$, private firm i's profit is increasing in c_j for $i = 1, 2, ..., n$, $j =, 1, ..., n$ and $j \neq i$.*

Proof See the Appendix.

In both exogenous and endogenous α cases, private firm i's profit is decreasing with its own cost and this result is intuitive. A reduction of c_i directly increases firm i's profit even when all firms' outputs are given exogenously. In addition, a decrease in c_i improves the competitive advantage of firm i and increases the equilibrium output of firm i. Through the strategic interaction, a reduction of c_i reduces the total output of other firms, which further increases firm i's profit.

Suppose that α is given exogenously. Private firm i's profit is increasing with its rivals' costs, which is an intuitive result. A reduction of c_j ($j \neq i$) increases firm j's output (direct effect) and though strategic interaction, decreases the other firms' outputs, including firm i's output (strategic effect). This reduction of firm i's output reduces its profit. In addition, a reduction of c_j increases the total output, and further reduces firm i's profit.

However, when α is endogenous, an additional effect exists. A reduction of c_j $(j \neq i)$ increases α, and makes the public firm (firm 0) less aggressive. This effect is so strong that a reduction of c_j $(j \neq i)$ increases the profits of all private firms.

This result suggests that as long as the solution is interior (i.e., $\alpha^* < 1$), private firms that compete with a public firm have incentives to reduce the private rivals' costs as well as their own costs. This fact might suggest that a recent open strategy of Tokyo-Mitsubishi UFJ (Nikkei Newspaper, May 3, 2017) to reduce its own cost as well as those of other private firms is reasonable.

From Proposition 1, we guess that the new entry of a private firm might increase the profits of incumbents because it also increases the degree of privatization. In addition, we guess that private firms engage in cost-reducing R&D more intensively when the privatization policy is determined after R&D activity and when there is a stronger spillover effect that reduces the private rivals' costs. We discuss these problems in the following two sections.

5 New Entry

5.1 New entry of a private firm

We consider how the entry of a private firm affects the profits of incumbent private firms. Suppose that a new entrant, firm n+1, enters the market. We assume that $c_{n+1} < c_0$. First, we consider the situation in which α is determined before the entry.

Lemma 2 *Suppose that α is given exogenously. The new entry of a private firm reduces the profits of all incumbent private firms.*

Proof See the Appendix.

Given α, the new entry of a private firm accelerates competition and reduces the market share of each incumbent, and thereby reduces the profits of all firms.

We now consider the situation in which α is determined after the entry.

Proposition 3 *Suppose that $\alpha^* < 1$. The new entry of a private firm increases α^*.*

Proof See the Appendix.

The new entry of a private firm strengthens the welfare-improving effect of privatization (the effect of production substitution from the public firm to the private firm), whereas it reduces the welfare-reducing effect of privatization (the output-reduction effect). Therefore, this new entry increases the optimal degree of privatization.

We now discuss how the new entry of a private firm affects the profits of incumbent private firms.

Proposition 4 *Suppose that α is endogenous. Suppose that $\alpha^* < 1$ even after the entry of firm $n+1$. Then, the entry of firm $n+1$ increases the profits of all incumbent private firms.*

Proof See the Appendix.

A new entry increases the degree of privatization, which makes the public firm less aggressive. This is beneficial for all private firms and thus, a new entry increases the profits of all private firms.[5]

5.2 Free entry

We now discuss a free-entry market. We assume that all potential entrants (private firms) share the same marginal cost c and entry cost F. In the first stage, each firm i ($i = 1, 2, .., n$) chooses whether to enter the market. In the second stage, after observing n, the government chooses α.[6] In the third stage, all firms choose their outputs independently.

Let q be the output of each private firm. In the third stage the first-order conditions of firm 0 and firm i ($i = 1, 2, ...n$) are

$$p + \alpha p' q_0 - c_0 = 0, \tag{8}$$

[5]Matsumura and Sunada (2013) investigated a mixed oligopoly with misleading advertising competition. They showed that the new entry of a private firm might increase the profits of the incumbent private firms, because it increases (decreases) advertising of the public firm (private firms). Some studies on private oligopolies have showed that a new entry could increase the profits of incumbents. Mukherjee and Zhao (2009) considered an asymmetric Stackelberg setting in which there are two incumbent firms (leader) with different marginal costs and a potential entrant (follower) with a higher marginal cost. The authors then show that the existence of an inefficient follower can increase the profit of the more efficient leader. Ishida *et al.* (2011) considered a model in which a dominant firm competes with minor firms and showed that an increase of the number of minor firms accelerates the R&D and profit of the dominant firm. Chen and Riordan (2007) showed that in a differentiated market, an increase of variety by a new entry might soften competition and increase the profits of incumbent firms. The driving force of our study is different from that in these studies.

[6]For examples supporting this timeline (entry then privatization), see Lee *et al.* (2017).

$$p + p'q - c = 0, \tag{9}$$

respectively.

In the second stage, the government chooses α^* such that

$$\alpha^* = \min\left\{\frac{n(c_0 - c)}{a - (n+1)^2 c_0 + n(n+2)c}, 1\right\}. \tag{10}$$

In the first stage, each firm i ($i = 1, 2, ...n$) enters the market as long as the profit is nonnegative. Therefore, as long as $n > 0$,

$$(p - c)q - F = 0. \tag{11}$$

By definition

$$q_0 + nq = Q. \tag{12}$$

These five equations determine the equilibrium values of q_0, q, Q, n, and α. The free-entry equilibrium is locally stable if the profit of each private firm is decreasing in n. As we showed in the previous subsection, the profit of each firm is increasing in n as long as $\alpha^* < 1$. Therefore, as long as $\alpha^* < 1$, the equilibrium must be locally unstable. Under these conditions, α^* must be one (full privatization) at the locally stable equilibrium. These discussions lead to the following proposition.

Proposition 5 *If $n > 0$, at the locally stable equilibrium $\alpha^* = 1$.*

If F is large enough, no private firm enters the market. In this case, $\alpha^* = 0$. Therefore, in the constant marginal cost model, the free-entry equilibrium yields either a public monopoly or a private oligopoly in which the government fully privatizes firm 0.[7]

6 R&D

In this section, we endogenize the marginal costs of private firms. We consider cost-reducing R&D activities by private firms.[8] We assume that all private firms are symmetric at the beginning of

[7]Most studies on mixed oligopolies in free-entry homogeneous product markets such as Matsumura and Kanda (2005), Cato and Matsumura (2012, 2015) and Lee *et al.* (2017), have assumed increasing marginal costs because the constant marginal cost model yields this technical problem.

[8]For discussions on R&D in mixed oligopolies, see Nishimori and Ogawa (2002), Matsumura and Matsushima (2004), Ishibashi and Matsumura (2006), Zikos (2010), Gil-Molto*et al.* (2011), and Lee and Tomaru (2017). How-

the game.

The marginal cost of firm i is given by

$$c_i(x_i; x_j) = C - (x_i + \beta \sum_{j \neq i} x_j) \quad (i, j = 1, 2, ..., n, i \neq j)$$

where C denotes the private firm's marginal cost before R&D, x_i is firm i's R&D level, and $\beta \in [0, 1]$ is the degree of spillover from other private firms' R&D.[9] We assume that $C < c_0$. This assumption guarantees that the public firm is less efficient, which is also assumed in the previous sections.

Each private firm i's profit and social welfare are

$$\pi_i = \big(p(Q) - c_i(x_i; x_j)\big)q_i - \gamma x_i^2,$$

$$W = \int_0^Q p(q)dq - pQ + \sum_{i=0}^{n} \pi_i = \int_0^Q p(q)dq - \sum_{i=0}^{n} c_i q_i - \gamma \sum_{i=1}^{n} x_i,$$

respectively, where γ is a positive constant. We assume that γ is sufficiently large. This assumption guarantees the second-order condition and interior solutions in the quantity competition stage.

The game runs as follows. In the first stage, each private firm i chooses x_i independently. In the second stage, the government chooses α. In the third stage, all firms choose their outputs independently. We restrict our attention to the symmetric equilibrium in which all private firms choose the same x.

We have already solved the second and third subgames. We now solve the first stage game. The first-order condition of each private firm i is[10],

$$\frac{\partial \pi_i}{\partial x_i} = 2((n+1)c_0 - \sum_{i=1}^{n} c_i(x_i; x_j) - c_i(x_i; x_j))((n-1)\beta + 2) - 2\gamma x_i = 0. \quad (i = 1, 2, ..., n).$$

This leads to the following equilibrium R&D level.

$$x^* = \frac{(n+1)(\beta n - \beta + 2)(c_0 - C)}{\gamma - (n+1)(\beta n - \beta + 2)(\beta n - \beta + 1)}. \tag{13}$$

ever, none of these studies discuss optimal privatization policy. Heywood and Ye (2009c) is the exception. They investigated the optimal degree of privatization but assumed that privatization occurs before R&D. Therefore, the authors discussed only our benchmark timeline. Moreover, they discussed a duopoly (only one private firm) and thus, did not discuss how private rivals' costs affected the profit of the private firm.

[9]This type of cost-reducing R&D is intensively discussed in the literature on private oligopolies. See Spence (1984), d'Aspremont and Jacquemin (1988), and Suzumura (1992).

[10]The second-order condition is satisfied if $\gamma > \frac{1}{2((n-1)\beta+2)^2}$

Then, we obtain the following equilibrium marginal cost c^*

$$c^* \quad = \quad \frac{\gamma C - (n+1)(\beta n - \beta + 2)(\beta n - \beta + 1)c_0}{\gamma - (n+1)(\beta n - \beta + 2)(\beta n - \beta + 1)}. \tag{14}$$

By substituting (14) into α^* in Lemma 1(ii), we obtain

$$\alpha^* \quad = \quad \frac{n\gamma(c_0 - C)}{(a - (n+1)^2 c_0 + n(n+2)C)\gamma - (n+1)(\beta n - \beta + 2)(\beta n - \beta + 1)(a - c_0)} \tag{15}$$

as long as $\alpha^* < 1$.

We now discuss the case with exogenous α as a benchmark. Suppose that α is given exogenously. The first-order conditions of each private firm i ($i = 1, 2, ..., n$) is[11],

$$\frac{\partial \pi_i}{\partial x_i} = 2\Big(\frac{\alpha(a + \sum_{j \neq i} c_j(x_j; x_i) - c_0) + c_0 - (1 + n\alpha)c_i(x_i; x_j)}{1 + (n+1)\alpha}\Big)\Big(\frac{(n - \beta n + \beta)\alpha + 1}{1 + (n+1)\alpha}\Big) - 2\gamma x_i = 0.$$

This leads to the following equilibrium R&D level.

$$x^*(\alpha) \quad = \quad \frac{(1 + \alpha - (n-1)\alpha\beta)(\alpha(a - C) + c_0 - C)}{(1 + (n+1)\alpha)^2\gamma - (1+\alpha)(1 + (n-1)\beta)(1 + n\alpha - (n-1)\alpha\beta)}. \tag{16}$$

We now present our result on R&D.

Proposition 6 *Suppose that $\alpha^* < 1$. (i) $x^*(\alpha^*) < x^*$. (ii) x^* is increasing in β, whereas $x^*(\alpha)$ is decreasing in β. (iii) x^* is increasing in n, whereas $x^*(\alpha)$ is decreasing in n.*

Proof See the Appendix.

Proposition 6 states that the timing of privatization is important for innovation. Expecting future privatization, private firms competing with a public firm engage in R&D more intensively to increase the degree of privatization.[12]

Because the cost reduction of private rivals is beneficial for all private firms when α is endogenous, the existence of spillover effect among private firms stimulates R&D. By contrast, when the degree of privatization is given exogenously, each private firm might less intensively engage in

[11] The second-order condition is satisfied if $\gamma > (\frac{1+n\alpha-(n-1)\alpha\beta}{1+(n+1)\alpha})^2$

[12] However, it might not be welfare-improving. Suppose that the government, not each private firm, chooses x. Let x^S be this second-best R&D level. We obtain that $x^S < x^*$ as long as $\alpha^* < 1$ (i.e., the equilibrium R&D investment level is excessive for social welfare).

R&D, because the reduction of private rivals' cost reduces the profits of other private firms in this case. Similarly, expecting future privatization, the new entry of a private firm accelerates private firms' R&D, because it affects the optimal privatization policy.

An increase in β (or n) increases x^* as long as $\alpha^* < 1$. A further increase in β (or n) increases x^* and α^* will eventually reach one. Thereafter, the equilibrium x is $x^*(1)$. Therefore, the equilibrium investment level is nonmonotone with respect to β and n (inverted U-shape).

Finally, we discuss the limitation of our results. We assume that the public firm's cost is given exogenously. If the spillover effect reduces the public firm's cost, Proposition 6 does not hold. A decrease in c_0 reduces the profits of all private firms, and thus, each private firm has a weaker incentive for innovation when the spillover effect is stronger.

7 Concluding remarks

In this study, we introduce cost differences among private firms and investigate how a private firm's cost affects the optimal degree of privatization and profits of private firms. We find that a cost reduction of a private firm reduces the optimal degree of privatization, which is beneficial for all private firms. Therefore, a cost reduction of a private firm increases the profits of all private firms, which never holds when the degree of privatization is given exogenously. In addition, we find that the new entry of a private firm is beneficial for all incumbent private firms because it increases the degree of privatization. Finally, we discuss innovation of private firms and find that expecting future privatization accelerates innovation activities of private firms.

In this study, we assume that private firms are domestic. In the literature on mixed oligopolies, it is known that the nationality of private firms often affects the behavior of a public firm and the optimal privatization policy. To extend our analysis in this direction is difficult work and remains for future research.[13]

[13]Whether the private firm is domestic or foreign often yields contrasting results in the literature on mixed oligopolies. See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), Bárcena-Ruiz and Garzón (2005a, 2005b), and Heywood and Ye (2009b). The optimal degree of privatization is decreasing with the foreign ownership rate in private firms when the number of private firms is given exogenously (Lin and Matsumura, 2012), while it is increasing with the foreign ownership rate in private firms in free-entry markets (Cato and Matsumura, 2012).

Appendix

Proof of Lemma 1

From (6), we obtain

$$\frac{\partial W^*}{\partial \alpha} = -\frac{\big(a-(n+1)c_0+\sum_{i=1}^n c_i\big)\big(\alpha(a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i)-nc_0+\sum_{i=1}^n c_i\big)}{\big(1+(n+1)\alpha\big)^3}.$$

By substituting $\alpha = 0$ into this, we obtain

$$\frac{\partial W^*}{\partial \alpha}\Big|_{\alpha=0} = \big(a-(n+1)c_0+\sum_{i=1}^n c_i\big)\big(nc_0-\sum_{i=1}^n c_i\big) > 0.$$

This implies Lemma 1(i).

By solving $\partial W^*/\partial\alpha = 0$ with respect to α, we obtain

$$\alpha^{**} = \frac{nc_0-\sum_{i=1}^n c_i}{a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i}. \tag{17}$$

The second-order condition

$$-\frac{(a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i)^4}{(a-(n+1)c_0+\sum_{i=1}^n c_i)^2} < 0$$

is satisfied. Therefore, the optimal degree of privatization is α^{**} as long as $\alpha^{**} < 1$. This implies Lemma 1(ii).

From (17), we obtain

$$\begin{aligned}
\frac{\partial \alpha^{**}}{\partial c_i} &= -\frac{(a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i)+(n+2)(nc_0-\sum_{i=1}^n c_i)}{(a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i)^2} < 0 \quad (i=1,2,...,n),\\
\frac{\partial \alpha^{**}}{\partial c_0} &= \frac{n(a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i)+(n+1)^2(nc_0-\sum_{i=1}^n c_i)}{(a-(n+1)^2c_0+(n+2)\sum_{i=1}^n c_i)^2} > 0.
\end{aligned}$$

These imply Lemma 1(iii). ■

Proof of Proposition 1 From (7), we obtain

$$\begin{aligned}
\frac{\partial \pi_i^*}{\partial c_i} &= -4\Big((n+1)c_0-\sum_{i=1}^n c_i - c_i\Big) < 0 \quad (i=1,2,...,n),\\
\frac{\partial \pi_i^*}{\partial c_j} &= -2\Big((n+1)c_0-\sum_{i=1}^n c_i - c_i\Big) < 0 \quad (i,j=1,2,...,n \;\; j\neq i),\\
\frac{\partial \pi_i^*}{\partial c_0} &= 2(n+1)\Big((n+1)c_0-\sum_{i=1}^n c_i - c_i\Big) > 0 \quad (i=1,2,...,n).
\end{aligned}$$

These results imply Proposition 1. ■

Proof of Proposition 2

Because we assume interior solutions in the quantity competition stage, from (2) we obtain $\alpha(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1)\alpha)c_i) > 0$. From (5), we obtain

$$\frac{\partial \pi_i^*(\alpha)}{\partial c_0} = \frac{2(\alpha(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1)\alpha)c_i)}{(1 + (n+1)\alpha)^2} > 0 \quad (i = 1, 2, ..., n), \tag{18}$$

$$\frac{\partial \pi_i^*(\alpha)}{\partial c_i} = -\frac{2(1 + n\alpha)(\alpha(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1)\alpha)c_i)}{(1 + (n+1)\alpha)^2} < 0 \quad (i = 1, 2, ..., n), \tag{19}$$

$$\frac{\partial \pi_i^*(\alpha)}{\partial c_j} = \frac{2\alpha(\alpha\big(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1)\alpha)c_i\big)}{(1 + (n+1)\alpha)^2} \geq 0 \quad (i, j = 1, 2, ..., n \quad j \neq i). \tag{20}$$

The strict inequality in (20) holds if and only if $\alpha > 0$. These imply Proposition 2. ■

Proof of Lemma 2

Let $\pi_i^{(n+1)*}(\alpha)$ and $\pi_i^{n*}(\alpha)$ be the equilibrium profit of firm i $(i = 1, 2, ..., n)$ with and without the entry of firm $n + 1$, respectively, given α. We obtain

$$\pi_i^{(n+1)*}(\alpha) = \left(\frac{\alpha(a + \sum_{i=1}^{n+1} c_i) + c_0 - (1 + (n+2)\alpha)c_i}{1 + (n+2)\alpha}\right)^2 \quad (i = 1, 2, ..., n+1). \tag{21}$$

Let $q_i^{(n+1)*}(\alpha)$ be the equilibrium output of firm i $(i = 1, 2, ..., n+1)$ with the entry of firm $n+1$, given α. We obtain

$$q_i^{(n+1)*}(\alpha) = \frac{\alpha(a + \sum_{i=1}^{n+1} c_i) + c_0 - (1 + (n+2)\alpha)c_i}{1 + (n+2)\alpha} \quad (i = 1, 2, ..., n+1). \tag{22}$$

Because we assume interior solutions in the quantity competition stage, from (2) and (22), we obtain $\alpha(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1))\alpha)c_i > 0$ and $\alpha(a + \sum_{i=1}^{n+1} c_i) + c_0 - (1 + (n+2)\alpha)c_i) > 0$. From (21) and (5), we obtain

$$\pi_i^{(n+1)*}(\alpha) - \pi_i^{n*}(\alpha) = -\frac{\alpha X_2}{(1 + (n+2)\alpha)(1 + (n+1)\alpha)} \leq 0 \quad (i = 1, 2, ..., n) \tag{23}$$

where $X_2 := (1 + (n+1)\alpha)(\alpha(a + \sum_{i=1}^{n+1} c_i) + c_0 - (1 + (n+2)\alpha)c_i) + (1 + (n+2)\alpha)(\alpha(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1)\alpha)c_i)))(\alpha(a + \sum_{i=1}^{n} c_i) + c_0 - (1 + (n+1)\alpha)c_{n+1}) > 0$. The strict inequality in (23) holds if and only if $\alpha > 0$. These imply Lemma 2. ■

Proof of Proposition 3

Let $\alpha^{(n+1)*}$ and α^{n*} be the equilibrium degree of privatization with and without the entry of firm $n+1$, respectively. We obtain

$$\alpha^{(n+1)*} = \frac{(n+1)c_0 - \sum_{i=1}^{n+1} c_i}{a - (n+2)^2 c_0 + (n+2)\sum_{i=1}^{n+1} c_i} \tag{24}$$

as long as $\alpha^{(n+1)*} < 1$. From (17) and (24), we obtain

$$\alpha^{(n+1)*} - \alpha^{n*} = \frac{X_3}{(a - (n+2)^2 + (n+1)\sum_{i=1}^{n+1} c_i)(a - (n+1)^2 c_0 + (n+2)\sum_{i=1}^{n} c_i)} > 0,$$

where $X_3 := (a - (n+1)^2 c_0 + (n+2)\sum_{i=1}^{n} c_i)(c_0 - c_{n+1}) + ((n+2)(c_0 - c_{n+1}) + (n+1)c_0)(nc_0 - \sum_{i=1}^{n} c_i) > 0$. This implies Proposition 3. ■

Proof of Proposition 4

Let $\pi_i^{(n+1)*}$ and π_i^{n*} be the equilibrium profit of firm i $(i = 1, 2, ..., n)$ with and without the entry of firm $n+1$, respectively.

By substituting $\alpha^{(n+1)*}$ into $\pi_i^{(n+1)*}(\alpha)$, we obtain

$$\pi_i^{(n+1)*} = \left((n+2)c_0 - \sum_{i=1}^{n+1} c_i - c_i\right)^2 \quad (i = 1, 2, ..., n+1). \tag{25}$$

From (7) and (25), we obtain

$$\pi_i^{(n+1)*} - \pi_i^{n*} = 2(c_0 - c_{n+1})((n+1)c_0 - \sum_{i=1}^{n} c_i - c_i) + (c_0 - c_{n+1})^2 > 0.$$

This implies Proposition 4. ■

Proof of Proposition 6

By substituting (15) into (16), we obtain

$$x^*(\alpha^*) = \frac{(c_0 - C)(n+1)((a - c_0 - n(c_0 - C))\gamma - (a - c_0)(\beta n - \beta + 1)(\beta n - \beta + 2))X_4}{(\gamma - (n+1)(\beta n - \beta + 1)(\beta n - \beta + 2))X_5}, \tag{26}$$

where $X_4 := (a - c_0 - n(c_0 - C)(\beta n - \beta + 2))\gamma - (n+1)(a - c_0)(\beta n - \beta + 1)(\beta n - \beta + 2)$, and $X_5 := (a - c_0 - n(c_0 - C))^2\gamma^2 - (a - c_0)(\beta n - \beta + 1)((a - c_0)(\beta n^2 - \beta + 2n + 3) - n(c_0 - C)(n +$

$\beta n+3-\beta))\gamma+(a-c_0)^2(n+1)(\beta n-\beta+1)(\beta n-\beta+2)$, and both X_4 and X_5 are positive because we assume that a and γ are sufficiently large. From (13) and (26), we obtain

$$x^* - x^*(\alpha^*) = \frac{(a-c_0)(c_0-C)(n+1)(\beta n-\beta+1)\gamma X_6}{(\gamma-(n+1)(\beta n-\beta+1)(\beta n-\beta+2))X_5} > 0,$$

where $X_6 := (a-c_0-n(c_0-C))\gamma-(a-c_0)(n+1)(\beta n-\beta+1)(\beta n-\beta+2)$, and this is positive because we assume that a and γ are sufficiently large. These imply Proposition 6(i).

From (13), we obtain

$$\frac{\partial x^*}{\partial \beta} = \frac{(n+1)(n-1)(c_0-C)(\gamma+(n+1)(\beta n-\beta+2)^2)}{(\gamma-(n+1)(\beta n-\beta+2)(\beta n-\beta+1))^2} > 0.$$

This implies the former part of Proposition 6(ii).

From (16), we obtain

$$\frac{\partial x^*(\alpha)}{\partial \beta} = -\frac{((a-C)\alpha+c_0-C)X_7}{((1+(n+1)\alpha)^2\gamma-(1+\alpha)\{1+(n-1)\beta)(1+n\alpha-(n-1)\alpha\beta))^2} < 0$$

where $X_7 := \alpha(1+(n+1))^2\gamma-(1+\alpha)(1+n\alpha-(n-1)\alpha\beta)^2$, and this is positive because we assume that γ is sufficiently large. This implies the latter part of Proposition 6(ii).

From (13), we obtain

$$\frac{\partial x^*}{\partial n} = \frac{(c_0-C)(2(1+n\beta)\gamma+\beta(n+1)^2(\beta n-\beta+2)^2)}{(\gamma-(n+1)(\beta n-\beta+2)(\beta n-\beta+1))^2} > 0.$$

This implies the former part of Proposition 6(iii).

From (16), we obtain

$$\frac{\partial x^*(\alpha)}{\partial n} = -\frac{((a-C)\alpha+c_0-C)X_8}{((1+(n+1)\alpha)^2\gamma-(1+\alpha)\{1+(n-1)\beta)(1+n\alpha-(n-1)\alpha\beta))^2} < 0$$

where $X_8 := \alpha(\alpha n+\alpha+1)(2(1+\alpha)+\beta+3\alpha\beta-n\alpha\beta)\gamma+\alpha^2(1+\alpha)(1-\beta)\beta^2n^2-2\alpha\beta(1+\alpha)(1-\beta)(1+\alpha+\alpha\beta)n-\alpha\beta^2(1+\alpha)(2+\alpha+\alpha\beta)-(1+\alpha)^2(\alpha+\beta-\alpha\beta)$, and this is positive for sufficiently large γ. This implies the latter part of Proposition 6(iii). ■

References

Bárcena-Ruiz, C. J., Garzón, B. M. (2005a). Economic integration and privatization under diseconomies of scale. European Journal of Political Economy 21(1):247–267.

Bárcena-Ruiz, C. J., Garzón, B. M. (2005b). International trade and strategic privatization. Review of Development Economics 9(4):502–513.

Capuano, C., De Feo, G. (2010). Privatization in oligopoly: the impact of the shadow cost of public funds. Rivista Italiana Degli Economisti 15(2):175–208.

Cato, S., Matsumura, T. (2012). Long-run effects of foreign penetration on privatization policies. Journal of Institutional and Theoretical Economics 168(3):444–454.

Cato, S., Matsumura, T. (2015). Optimal privatization and trade policies with endogenous market structure. Economic Record 91(294):309–323.

Chen, T. L. (2017). Privatization and efficiency: a mixed oligopoly approach. Journal of Economics 120(3):251–268.

Chen, Y., Riordan, M. H. (2007). Price and variety in the spokes model. Economic Journal, 117(7):897–921.

Corneo, G., Jeanne, O. (1994). Oligopole mixte dans un marche commun. Annales d'Economie et de Statistique 33:73–90.

d'Aspremont, C., Jacquemin, A. (1988). Cooperative and noncooperative R&D in duopoly with spillovers. American Economic Review 78(5):1133–1137.

Fjell, K., Pal, D. (1996). A mixed oligopoly in the presence of foreign private firms. Canadian Journal of Economics 29(3):737–743.

Fujiwara, K. (2007). Partial privatization in a differentiated mixed oligopoly. Journal of Economics 92(1):51–65.

Gil-Molto, M. J., Poyago-Theotoky, J., Zikos, V. (2011). R&D subsidies, spillovers, and privatization in mixed markets. Southern Economic Journal 78(1):233–255.

Heywood, J. S., Ye, G. (2009a). Mixed oligopoly, sequential entry, and spatial price discrimination. Economic Inquiry 47(3):589–597.

Heywood, J. S., Ye, G. (2009b). Mixed oligopoly and spatial price discrimination with foreign firms. Regional Science and Urban Economics 39(5):592–601

Heywood, J. S., Ye, G. (2009c). Partial privatization in a mixed duopoly with an R&D rivalry. Bulletin of Economic Research 61(2):165–178.

Heywood, J. S., Hu, X., Ye, G. (2017) Optimal partial privatization with asymmetric demand information. Journal of Institutional and Theoretical Economics 173(2):347–375.

Ishibashi, I., Matsumura, T. (2006). R&D competition between public and private sectors. European Economic Review 50(6):1347–1366.

Ishida, J., Matsushima, N. (2009). Should civil servants be restricted in wage bargaining? A mixed-duopoly approach. Journal of Public Economics 93(3–4):634–646.

Ishida, J., Matsumura T., Matsushima, N. (2011). Market competition, R&D and firm profits in asymmetric oligopoly. Journal of Industrial Economics 59(3):484–505.

Lahiri, S., Ono, Y. (1988). Helping minor firms reduces welfare. Economic Journal 98:1199–1202.

Lee, S.-H., Matsumura, T., Sato, S. (2017). An analysis of entry-then-privatization model: welfare and policy implications. forthcoming in Journal of Economics.

Lee, S.-H., Tomaru, Y. (2017). Output and R&D subsidies in a mixed oligopoly. Operations Research Letters 45(3):238–241.

Lin, M. H., Matsumura, T. (2012). Presence of foreign investors in privatized firms and privatization policy. Journal of Economics 107(1):71–80.

Matsumura, T. (1998). Partial privatization in mixed duopoly. Journal of Public Economics 70(3):473–483.

Matsumura, T., Kanda, O. (2005). Mixed oligopoly at free entry markets. Journal of Economics 84(1):27–48.

Matsumura, T., Matsushima, N. (2004). Endogenous cost differentials between public and private enterprises: a mixed duopoly approach. Economica 71(284):671–688.

Matsumura, T., Ogawa, A. (2010). On the robustness of private leadership in mixed duopoly. Australian Economic Papers 49(2):149–160.

Matsumura, T., Okamura, M. (2015). Competition and privatization policies revisited: The payoff interdependence approach. Journal of Economics 116(2):137–150.

Matsumura, T., Sunada, T. (2013). Advertising competition in a mixed oligopoly. Economics Letters 119(2):183–185.

Mukherjee, A., Zhao, L. (2009). Profit raising entry. Journal of Industrial Economics 57(4):870–870.

Nishimori, A., Ogawa, H. (2002), Public monopoly, mixed oligopoly and productive efficiency, Australian Economic Papers 41(2):185–190.

Pal, D. (1998). Endogenous timing in a mixed oligopoly. Economics Letters 61(2):181–185.

Pal, D., White, M. D. (1998). Mixed oligopoly, privatization, and strategic trade policy. Southern Economic Journal 65(2):264–281.

Spence, M. (1984). Cost reduction, competition, and industry performance. Econometrica 52(1):101–21.

Suzumura, K. (1992). Cooperative and non-cooperative R&D in an oligopoly with spillovers. American Economic Review 82(5):1307–1320.

Zikos, V. (2010). R&D collaboration networks in mixed oligopoly. Southern Economic Journal 78(1):233–255.