

Resident Bid Preference, Affiliation, and Procurement Competition: Evidence from New Mexico

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Abstract

In public procurement auctions, governments routinely offer preferences to qualified firms in the form of bid discounts. Previous studies on bid discounts do not account for affiliation – a form of cost dependence between bidders that is likely to occur in a public procurement setting. Utilizing data from the New Mexico Department of Transportation's Resident Preference Program, this paper uses an empirical model of firm bidding and entry behavior to investigate the effect of affiliation on auctions with bid discounting. I find evidence that firms have affiliated project-completion costs and show how this type of affiliation changes preference auction outcomes.

1 Introduction

Procurement auctions are widely used by governments as a means of securing goods and services for the lowest possible price. Internationally, government procurement accounts for anywhere from 10 to 25 percent of GDP, and in the Unites States alone, government spending on goods and services accounted for 15.2 percent of GDP in 2013, totaling \$2.55 trillion.¹ In these procurement auctions, governments routinely offer preferential treatment to a certain subset of bidders. This treatment often takes the form of bid discounting – a policy where the government will lower the bids of preferred bidders for comparison purposes and pay the full asking price upon winning. These preferential policies can affect auction outcomes and have been studied extensively in the literature.²

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¹These numbers are taken from the World Bank national accounts data and OECD National Accounts data files.

²See Krasnokutskaya and Seim (2011), Marion (2007), and Hubbard and Paarsch (2009) for papers discussing bid discounting.

In many cases, the purpose of offering these preference programs is to encourage the participation of a particular type of bidder. For example, California offers a bid discount to small businesses to encourage these business to bid on larger projects, and the Inter-American Development Bank offers a bid discount to domestic firms to encourage domestic development. The total effect of these programs, however, has been shown to be ambiguous. Although offering bid discounts can encourage preferred bidders to bid less aggressively, which means they bid further from their costs, bid discounts also encourage non-preferred bidders to bid more aggressively, or closer to their costs, and can increase competition and discourage non-preferred participation. This type of trade-off is highlighted in McAfee and McMillan (1989) where the authors show that the government can minimize procurement costs by choosing an optimal discount level when participation is fixed and in Corns and Schotter (1999) where the authors use experiments to show that preferences can lead to increases in both cost effectiveness and the representation of preferred bidders.³ Krasnokutskaya and Seim (2011) show that the magnitudes of these effects are altered when participation is endogenous.

Another potential factor in evaluating these programs is the possibility of affiliation, or dependence, between a firm's cost of completing a project, which I will now call its project cost, and the project costs of its competitors. These costs are private information, and the literature has typically taken them to be independent, which implies that a firm that learns its own project cost has no additional information on the project costs of other bidders. There are a number of reasons why this independence assumption may not hold. For instance, firms may use the same subcontractors when submitting a bid, so firms sharing subcontractors should have some form of dependence in their project costs. Firms may also buy raw materials from the same suppliers, which again can generate dependence in project costs.

The existence of affiliation can potentially change a number of preference auction outcomes. For a given number of participants, affiliation makes firms more "similar" in that they are more likely to have similar project costs relative to independence. Firms will therefore adjust how they bid, which can change both procurement costs and firm profits conditional on entry. If a firm's incentive to participate is influenced by the expected profitability of a project, then affiliation can also affect the number of favored entrants and auction efficiency. Consequently, the total effectiveness of these preference programs can hinge on presence of affiliation.

This paper contributes to the bid preference literature by allowing firms to have affiliated private project

³Additional studies that show the theoretical implications of granting preference to certain groups of bidders include Vagstad (1995) who extends the analysis of McAfee and McMillan (1989) to incentive contracts and Naegelen and Mougeot (1998) who extend the analysis of McAfee and McMillan (1989) to include objectives concerning the distribution of contracts over preferred and non-preferred bidders.

costs in procurement auctions with bid discounting and endogenous entry.⁴ Affiliation is a stronger notion of positive correlation, and it captures the idea that firm project costs may be related to each other. Using copula methods developed by Hubbard, Li, and Paarsch (2012) and extended by Li and Zhang (2015), I evaluate a bid preference program favoring resident bidders in New Mexico and show the bias that can arise from assuming independence.

I collect the data from New Mexico Department of Transportation (NMDOT) highway construction contracts. New Mexico is one of a few states that offer qualified resident firms a 5 percent bid discount on state-funded projects. Affiliation is plausible in this setting; firms located close to each other are more likely to buy from the same suppliers and use similar subcontractors, potentially generating dependence in project costs. In fact, 30 percent of items⁵ on construction projects qualifying for bid preferences had at least two firms bid the same amount in the data. This statistic suggests that firms may have similar costs of completing some portions of a project.

To then determine the extent to which affiliation is present in NMDOT highway construction contracts, compare outcomes under affiliation and independence, and investigate alternative discount levels, I develop and estimate an empirical model of bidding with endogenous entry, where I allow for affiliation in firm project costs. The parameter that captures the degree of affiliation is positive and statistically significant, which indicates that firms do have affiliated project costs. Counterfactual auctions using alternative discount levels show that New Mexico's preference program accounts for a 1.2 percent increase in procurement costs. At New Mexico's current discount level, procurement costs are 2.9 percent higher than would be predicted if project costs were distributed independently. Furthermore, I find that the proportion of preferred winners is more responsive to the discount level with affiliation and that affiliation can lead to substantial differences in efficiency at particular discount levels relative to independence. These results highlight the relevance of affiliation in the evaluation of public procurement auctions with bid discounting.

The remainder of the paper proceeds as follows. Section 2 gives the details of the New Mexico procurement process and describes the data. Section 3 presents the theoretical framework by which I analyze the effect of affiliation on bidding and entry behavior, and section 4 shows how I represent affiliated distributions using copulas. Section 5 shows the different ways in which affiliation can affect bidding, and section 6 shows how I estimate the theoretical model. Section 7 presents the empirical findings, while section 8 contains the counterfactual policy analysis. Section 9 concludes.

⁴This paper also complements the existing literature on auctions with endogenous entry. These papers include Athey et al. (2011), Li (2005), Sweeting and Bhattacharya (2015) and Bajari and Hortacsu (2003).

⁵Items are portions of a construction project. The final bid is calculated as the sum of the bids on each item.

2 New Mexico's Highway Procurement Market and Data

This section describes the process by which the NMDOT awards their highway construction contracts and the data collected for the empirical portion of this paper. The sample contains 376 highway construction contracts awarded by the NMDOT between 2010 and 2014 for the maintenance and construction of transportation systems. New Mexico applies preferences to resident firms on state-funded projects. Over the sample period, there are a total 23 of these state-funded contracts while the remaining 353 projects are federally-assisted projects. An immediate limitation of the New Mexico data is that there are a small number of preference projects relative to the number of non-preference projects.⁶ In response to this limitation, much of the analysis relies on the empirical model of entry and bidding. The empirical model allows me to use information in both the preference and non-preference auctions in identifying the model primitives while accounting for strategic behavior due to bid discounting.

2.1 Letting

Four weeks prior to the date of bid opening, the NMDOT advertises construction projects estimated to cost more than \$60,000. The Contracts Unit is responsible for gathering the necessary contract documents used during this advertisement phase. Each document is unique to the work required on each project and contains details such as the location of the project, the nature of the work, the number of working days to complete the project, and the length of the project. The NMDOT summarizes these details in an "Invitation for Bids" document, and I use this document to form the set of observable project characteristics.

Another feature of advertising is providing a rough approximation of firms who could potentially bid for a contract. To advertise potential competitors, the NMDOT publishes a list of "planholders" ten days prior to bid opening. Status as a planholder requires that the firms provide some documented evidence that they have the contract documents either directly through the NMDOT or through written communication.⁷ Moreover, failure to seek planholder status results in the bid becoming unresponsive and subsequently rejected. Given that the list of planholders is known prior to bidding and planholder status is required to submit a valid bid, I use the firms who are registered as planholders as a measure of the set of potential bidders.⁸

⁶Based on my conversations with NMDOT employees, one reason why there are so few state-funded projects is because a project must be entirely funded by state funds in order to be listed as a state project. Some projects use a mix of state and federal funds, but if any part of the project uses federal funds, then that project is listed as a federally-assisted project. Every once in a while, the state will receive "capital outlay" funds for NMDOT projects or use state maintenance funds or state severance tax funds to fund entire projects, but these sources of funding are not prevalent in financing these types of auctions.

⁷For more information the planholder requirement, see the NMDOT website.

⁸This measure is not perfect. Some firms seek planholder status after the list is published, resulting in a larger set of potential bidders than what is listed in the planholder document. To account for this difference, I include any actual bidders that do not appear in the planholder document in the set of potential entrants. Moreover, the set of planholders may contain firms that do

In awarding these construction projects, the NMDOT uses a competitive first-price sealed-bid procurement auction format. Potential firms who decide to bid on a project submit bids in a sealed envelope or secure online submission website to the NMDOT. The firm with the lowest bid (usually) wins the contract, and the state pays the winner their bid. The NMDOT tabulates and publishes submitted bids as well as an engineer's estimate for the cost of the project in an Apparent Low Bids document directly after bid opening. I use the bids and estimates in these documents as the bids and estimates received by the NMDOT for each project.

2.2 Resident Preference Program

New Mexico offers bid preferences to qualified resident firms on construction projects funded exclusively by the state. New Mexico implements its preference through a 5 percent discount on bids, which lowers resident bids by 5 percent in evaluation and pays the full asking price conditional on winning. For example, suppose that a resident firm and a non-resident firm are the only two firms bidding for a contract. Furthermore, suppose that the resident firm bids \$1,000,000 and the non-resident firm bids \$975,000. After applying the five percent discount to the resident firm, its bid is lowered to \$950,000, it wins the contract, and the state pays it \$1,000,000.

To qualify for resident preference, firms must meet a certain list of conditions. In particular, firms must have paid property taxes on real property owned in the state of New Mexico for at least five years prior to approval and employ at least 80 percent of their workforce from the state of New Mexico. There are also a number of penalties in place to prevent firms from exploiting residency status. Providing false information to the state of New Mexico in order to qualify as a resident results in automatic removal of any preferences, ineligibility to apply for any more preference for at least five years, and administrative fines of up to \$50,000 for each violation. I obtain a list of qualified resident firms through the New Mexico Inspection of Public Records Act, which allows anyone to view public documents.

In general, non-resident firms tend to be local despite their status, and resident firms tend to be more prevalent in the data. Most non-resident firms have offices within the state (60 percent of bidders and 64 percent of planholders), while only a small number of non-resident firms have offices outside of states bordering New Mexico (15 percent of bidders and 12 percent of planholders). Out of the 110 different firms observed in the data, 66 firms are residents while the remaining 44 firms are non-residents. Resident firms account for 80 percent of planholders and 72 percent of submitted bids, and resident firms win 76 percent of

not have the means to bid as a prime contractor. In order to get a more accurate representation of the set of firms who could potentially bid, I do not include firms who are unsuccessful in submitting a valid bid during the sample period in the set of planholders.

federally-assisted projects and 78 percent of state-funded projects.

3 Theoretical Model

This section provides the theoretical foundation by which I analyze the market for NMDOT construction contracts. In order to preserve the main institutional features, I model New Mexico's market for highway construction contracts as a first-price sealed-bid procurement auction with asymmetric bidders, affiliated private values, and endogenous entry. The model proceeds in two stages as in Levin and Smith (1994), Krasnokutskaya and Seim (2011), and Li and Zhang (2015). In the first stage, potential resident and non-resident bidders decide whether to pay the entry cost and participate in the auction. Bidders will enter if their expected profits from participation exceed their costs of entry. In the New Mexico setting, the entry cost represents the effort required to gather information about the project and the opportunity cost of time, which is analogous to reading the invitation for bids and requesting project information. In the second stage, bidders learn the identity and number of actual competitors, draw their project costs from an affiliated distribution, and submit a bid for the project.

3.1 Affiliation

I model the possibility of project cost dependence across firms through affiliation. First introduced into auctions by Milgrom and Weber (1982), affiliation can arise as a result of shared subcontractors and suppliers. Theoretically, affiliation describes the relationship between two or more random variables; if two or more random variables are affiliated, then they exhibit some form of positive dependence. de Castro (2010) shows that affiliation is a sufficient condition for positive correlation, so affiliation can roughly be interpreted as a stronger form of positive correlation.⁹ Formally, affiliation is defined as follows:

Definition. The density function
$$f: [\underline{c}, \overline{c}]^n \to \mathbb{R}_+$$
 is affiliated if $f(c) f(c') \leq f(c \wedge c') f(c \vee c')$, where $c \wedge c' = (\min\{c_1, c'_1\}, ..., \min\{c_n, c'_n\})$ and $c \vee c' = (\max\{c_1, c'_1\}, ..., \max\{c_n, c'_n\})$.

In a procurement setting, affiliation in project costs means that when a firm draws a high project cost, it is more likely that competing firms also have drawn high project costs. Note that affiliation essentially gives bidders extra information on the opponent's project costs, which is plausible if bidders are located close to each other and share similar subcontractors.

⁹See de Castro (2010) for a detailed discussion on the relationship between affiliation and other notions of positive dependence.

Affiliation is also the key modeling assumption that explains the correlations across bids observed in the data. Other studies such as Krasnokutskaya and Seim (2011), Athey et al. (2011), and Athey et al. (2013) explain these correlations under the independent private value paradigm with unobserved auction heterogeneity. While similar in explaining the observed bidding patterns, these two approaches have distinct implications on how firms bid and therefore on how bid preferences affect auctions; a firm's own cost realization impacts their belief about other firms' costs under affiliation but not under independence. In the data, each project has an engineer's estimate, which contains a detailed break down of each project's tasks. Since the engineer's estimate explains a large part of the variation in observed bids, I treat affiliation as the prime explanation for correlations across bids.¹⁰

3.2 Environment

Turning to the bidding environment, N_R potential resident bidders and N_{NR} potential non-resident bidders compete in a first-price sealed-bid procurement auction for the completion of one indivisible construction project. Resident and non-resident bidders are risk neutral and draw entry costs, k_i , independently from the distribution $G_k^m(\cdot)$, where $m \in \{R, NR\}$ denotes firm i's group affiliation. Firms draw their project costs, c_i , from the joint distribution $F_c(\cdot, \ldots, \cdot)$ with support $[\underline{c}, \overline{c}]^n$, where n is the total number of actual bidders. The marginal distribution for a bidder of group m is $F_c^m(\cdot)$, which allows for heterogeneity in the group-specific marginal distributions. Joint project cost distributions can be affiliated, but I assume that project costs are independent of entry costs. These distributions are common knowledge to every potential bidder.

Additionally, resident firms in auctions funded exclusively by the state of New Mexico receive a discount of δ on their submitted bid. In terms of the model, the auctioneer will lower every resident bid by a factor of $(1 - \delta)$ when comparing it against a non-resident bid in a preference auction, so a resident firm will win if its bid is less than the lowest competing resident bid and the lowest competing non-resident bid scaled by a factor of $\frac{1}{1-\delta}$. The value of the discount is 5 percent for New Mexico residents.

¹⁰In other environments where unobserved auction heterogeneity may dominate affiliation, econometric methods developed in Krasnokutskaya (2011) and empirical methods found in Hong and Shum (2002) and Haile et al. (2006) would be more suitable. Balat (2016) discusses identification in environments with both affiliation and unobserved project heterogeneity.

¹¹This assumption implies that bidders do not base entry decisions on their realized project costs. Samuleson (1985) discusses the opposite case where bidders are completely informed of their project costs prior to entry, and Roberts and Sweeting (2010) discuss the intermediate case where bidders are partially informed. Sweeting and Bhattacharya (2015) study various auction designs when entry is endogenous and selective in the sense that bidders with higher valuations are more likely to enter. Within a procurement setting, Li and Zheng (2009) provide evidence that supports a model in which bidders are initially uniformed prior to entry.

3.3 Bidding

After bidders learn their project costs and the number of actual entrants, bidders submit their bids to complete the construction contract. Heterogeneity in residency status along with bid discounting leads to group-symmetric equilibria as in Krasnokutskaya and Seim (2011), where bidders of each group m follow potentially different monotone and differentiable bid functions $\beta_m(\cdot): [\underline{c}, \overline{c}] \to \mathbb{R}_+$. In particular, a bidder of group m solves the following optimization problem to determine the equilibrium bids:

$$\pi(c_i; n_{NR}, n_R) = \max_{b_i} (b_i - c_i) \Pr\left((1 - \delta)^{D_R} b_i < B_j \, \forall j \in NR, (1 - \delta)^{-D_{NR}} b_i < B_l \, \forall l \in R \mid c_i \right),$$

where $\pi(c_i; n_{NR}, n_R)$ is the value function, b_i is the bid choice of bidder i, B_j and B_l are the competing bids, D_m is an indicator variable that takes on a value of one if firm i is associated with group m and zero otherwise, and $\delta = 0$ if the auction is not a preference auction. The objective function illustrates how firms view preference when submitting a bid. For positive δ , preference increases the probability of a resident beating a non-resident bidder without requiring the resident bidder to submit a lower bid. Residents therefore have a higher probability of winning a preference auction with the same choice of b_i relative to a non-preference auction yet face the same payment if they win.¹²

terizes the optimal bid is then given by

$$0 = (b_{i} - c_{i})$$

$$\times \left[\sum_{j=1}^{n_{NR} - D_{NR}} \bar{F}_{c_{-i},j} \left(\beta_{NR}^{-1} \left((1 - \delta)^{D_{R}} b_{i} \right), \beta_{R}^{-1} \left((1 - \delta)^{-D_{NR}} b_{i} \right) \mid c_{i} \right) \right]$$

$$\times \beta_{NR,1}^{-1} \left((1 - \delta)^{D_{R}} b_{i} \right) (1 - \delta)^{D_{R}}$$

$$+ \sum_{j=n_{NR} - D_{NR} + 1}^{n-1} \bar{F}_{c_{-i},j} \left(\beta_{NR}^{-1} \left((1 - \delta)^{D_{R}} b_{i} \right), \beta_{R}^{-1} \left((1 - \delta)^{-D_{NR}} b_{i} \right) \mid c_{i} \right)$$

$$\times \beta_{R,1}^{-1} \left((1 - \delta)^{-D_{NR}} b_{i} \right) (1 - \delta)^{-D_{NR}}$$

$$+ \bar{F}_{c_{-i}} \left(\beta_{NR}^{-1} \left((1 - \delta)^{D_{R}} b_{i} \right), \beta_{R}^{-1} \left((1 - \delta)^{-D_{NR}} b_{i} \right) \mid c_{i} \right),$$

where $\bar{F}_{c_{-i},j}(\cdot,\ldots,\cdot \mid c_i)$ is the partial derivative of the conditional survival function with respect to the j'th coordinate, $\beta_{NR,1}^{-1}(\cdot)$ is the partial derivative of a non-resident's inverse bid function with respect to its first coordinate, and $\beta_{R,1}^{-1}(\cdot)$ is the partial derivative of a resident's inverse bid function with respect to its first coordinate. These first-order conditions form a system of differential equations that characterize the equilibrium bids.

A complete characterization of the bidding equilibrium requires one to specify boundary conditions. Following Hubbard and Paarsch (2009) and Krasnokutskaya and Seim (2011), I set four group-specific boundary conditions.

The left boundary condition requires that bidders who draw the lowest project cost submit the same bid while accounting for the level of the bid discount. Let \underline{b} be the common low bid. The left boundary conditions for both groups of bidders is as follows:

1. Resident left boundary:

$$\beta_R^{-1}\left(\frac{\underline{b}}{(1-\delta)}\right) = \underline{c}.$$

2. Non-resident left boundary:

$$\beta_{NR}^{-1}\left(\underline{b}\right) = \underline{c}.$$

The right boundary condition restricts bidding behavior at the highest possible project cost draw. This

condition can loosely be interpreted as bidders who draw the highest project cost bid their project costs while making any necessary adjustments for the group affiliation of the competing bidders. The right boundary condition for both groups of bidders is as follows:

3. Resident right boundary:

$$\beta_R^{-1}\left(\overline{b}\right) = \overline{c},$$

where $\bar{b} = \bar{c}$ if $n_R > 1$ and $\bar{b} = \arg\max_b \left[(b - \bar{c}) \Pr\left((1 - \delta) \, b < b_j \, \forall j \in NR \, | \, \bar{c} \right) \right]$ if $n_R = 1$. That is to say, if there is only one resident firm bidding on a project, it will choose a bid that maximizes its expected profits, since the discount may lower its bid enough to be competitive with the non-resident firms.

4. Non-resident right boundary:

$$\beta_{NR}^{-1}\left(\overline{c}\right) = \overline{c}.$$

Observe that bid preference introduces another equilibrium feature mentioned by Hubbard and Paarsch (2009) and Krasnokutskaya and Seim (2011). In particular, if a non-resident firm draws a project cost $c \in [(1-\delta)\bar{b},\bar{c}]$, then it also bids its project cost. Note that, as long as there is at least one competing resident bidder, a project cost draw in this region for a non-resident will never win the auction, yielding a payoff of zero as long as the non-resident firm does not bid below its cost. Since bidders are indifferent between not winning an auction and winning an auction with a bid equal to their cost, this assumption can be made without changing the equilibrium payoffs.

Existence and uniqueness of a bidding equilibrium is key in empirically implementing these types of auctions. Existence establishes that there is, in fact, a solution to the auction, while uniqueness establishes that the bidders are playing one equilibrium as opposed to potentially multiple different equilibria. Reny and Zamir (2004) show that a monotone pure strategy equilibrium exists in a more general setting than this type of auction. Uniqueness follows from Theorem 1 in Lebrun (2006) provided that the conditional survival function is log concave.

3.4 Entry

In the entry stage, firms make participation decisions based on their knowledge of the number of potential entrants of each group, their knowledge of their own entry cost, and their knowledge of the distributions of project costs and entry costs. Firms calculate ex-ante expected profits as

$$\Pi_{m}\left(N_{m}, N_{-m}\right) = \sum_{n_{m}-1 \subseteq N_{m}, n_{-m} \subseteq N_{-m}} \int_{\underline{c}}^{\overline{c}} \pi\left(c_{i}; n_{m}, n_{-m}\right) dF_{c}^{m}\left(c_{i}\right) \Pr\left(n_{m}-1, n_{-m} \mid N_{m}, N_{-m}\right),$$

where the -m subscript indicates the bidders not affiliated with the group of bidder i and $F_c^m(\cdot)$ is the marginal project cost distribution of group m.¹³ These profits are only a function of the observed number of potential bidders, since the only payoff relevant information available to a given firm before entry is the number of potential bidders and its entry cost. Also note that the subscript is group specific, since members of the same group face the same ex-ante expected profits. The entry cost distribution determines the group-specific equilibrium entry probabilities, which I denote as p_m . That is,

$$p_m = \Pr\left(k_i < \Pi_m\right) = G_k^m \left(\Pi_m\right),\,$$

where $G_k^m(\cdot)$ is the marginal distribution of entry costs for a bidder in group m. The above equality follows from the fact that a firm's beliefs about its competitors' entry probabilities must be consistent with their actual entry probabilities in equilibrium. An application of Brouwer's fixed point theorem demonstrates the existence of the threshold probabilities p_m .¹⁴ Note here that the existence and uniqueness results from the bidding equilibrium still hold after entry, since bidders behave as if entry was exogenous upon entering.

4 The Copula Representation

One difficulty in implementing auction models with affiliation is dealing with the joint cost distribution. To overcome this difficulty, I rely on copula methods developed by Hubbard, Li, and Paarsch (2012). Copulas are an expression of the joint distribution of random variables as a function of the marginals. Formally, if c_1, c_2, \ldots, c_n are n possibly correlated random variables with marginal distributions $F_c^1(c_1), F_c^2(c_2), \ldots, F_c^n(c_n)$

¹³When computing these profits, there is a case where no competing bidders enter the auction. This case is problematic since the NMDOT does not explicitly post a reserve price. The NMDOT does, however, reserve the right to reject all bids if the lowest price is excessively high. To capture this power to reject bids, I follow Krasnokutskaya and Seim (2011) in assuming that firms compete against the government (which is modeled as a resident bidder) when faced with no other competition.

¹⁴Uniqueness, however, is not guaranteed and must be verified through simulation.

respectively, then the joint distribution can be written as a function of the marginal distributions as

$$F_{c}(c_{1}, c_{2}, \ldots, c_{n}) = C \left[F_{c}^{1}(c_{1}), F_{c}^{2}(c_{2}), \ldots, F_{c}^{n}(c_{n}) \right],$$

where $C[\cdot, \ldots, \cdot]$ is the copula function.

The particular type of copula I use to model the joint cost distribution of resident and non-resident bidders is a Clayton copula. This type of copula has the following closed-form representation:

$$C\left[F_{c}^{1}\left(c_{1}\right), F_{c}^{2}\left(c_{2}\right), \dots, F_{c}^{n}\left(c_{n}\right)\right] = \left(\sum_{i=1}^{n} F_{c}^{i}\left(c_{i}\right)^{-\theta} - n + 1\right)^{-\frac{1}{\theta}},$$

where $\theta \in [-1, \infty) \setminus \{0\}$ is the dependence parameter. Besides having a tractable representation, Clayton copulas are useful in the sense that affiliation only requires θ to be greater than zero.¹⁵ Moreover, θ has the nice interpretation that a higher value of θ implies a higher degree of affiliation between random variables, so θ contains all of the relevant information on cost dependence.¹⁶

Since I study procurement auctions in this paper, I must model the conditional survival function. For this reason, I use two results from Hubbard, Li, and Paarsch (2012) to construct an expression for the conditional survival function using copulas:

Result 1:

The survival function, $\bar{F}_{\mathbf{c}}(c_1, c_2, \dots, c_n)$, can be written as

$$\bar{F}_{\mathbf{c}}(c_1, c_2, \dots, c_n) = \Pr(C_1 > c_1, C_2 > c_2 \dots, C_n > c_n)
= 1 - \sum_{i=1}^n \Pr(C_i < c_i) + \sum_{1 \le i < j \le n} \Pr(C_i < c_i, C_j < c_j)
- \dots + (-1)^n \Pr(C_1 < c_1, C_2 < c_2 \dots, C_n < c_n).$$

This result provides an expression of the survival function in terms of the cumulative density function (CDF), which has a copula representation. Let $S\left[1-F_c^1\left(c_1\right),1-F_c^2\left(c_2\right),\ldots,1-F_c^n\left(c_n\right)\right]$ denote the survival copula evaluated at the survival marginals. The first result shows that the survival copula can be

¹⁵For a formal proof of this statement, see Müller and Scarsini (2005).

¹⁶A limitation of the Clayton copula, however, is that there is only one parameter governing the affiliation between both groups of bidders. If residents and non-residents have different degrees of affiliation between them, then this setup may not capture those differences. To assess whether this is the case for resident and non-resident bidders in New Mexico, I calculate and compare the intraclass correlations between bids for residents and non-residents, where the classes are the separate auctions. I find that the correlations across bids for the two groups of bidders does not differ substantially from each other or the entire sample, suggesting that a single parameter governing all affiliation is reasonable.

expressed as

$$S\left[1 - F_{c}^{1}(c_{1}), 1 - F_{c}^{2}(c_{2}), \dots, 1 - F_{c}^{n}(c_{n})\right] = 1 - \sum_{i=1}^{n} C\left[F_{c}^{i}(c_{i})\right] + \sum_{1 \leq i < j \leq n} C\left[F_{c}^{i}(c_{i}), F_{c}^{j}(c_{j})\right] - \dots + (-1)^{n} C\left[F_{c}^{1}(c_{1}), \dots, F_{c}^{n}(c_{n})\right].$$

Result 2:

 $\Pr(C_2 > c_2 \dots, C_n > c_n \mid c_1) = \mathbf{S}_1 \left[1 - F_c^1(c_1), 1 - F_c^2(c_2), \dots, 1 - F_c^n(c_n) \right]$, where $\mathbf{S}_1 \left[\cdot, \dots, \cdot \right]$ is the partial derivative of the survival copula with respect to the first coordinate.

Result 2 shows that the conditional survival copula is equivalent to the partial derivative of the full survival copula with respect to the conditioning argument.

Given these two results, the second stage profits of bidder 1 can be rewritten using copulas as

$$\pi(c_{1}; n_{NR}, n_{R}) = \max_{b_{1}} (b_{1} - c_{1}) \times \mathbf{S}_{1} \left[1 - F_{c}^{m_{1}}(c_{1}), 1 - F_{c}^{NR}(\beta_{NR}^{-1}), \dots, 1 - F_{c}^{NR}(\beta_{NR}^{-1}), 1 - F_{c}^{R}(\beta_{R}^{-1}), \dots, 1 - F_{c}^{R}(\beta_{R}^{-1}) \right]$$

where m_1 is the group affiliation of bidder 1, F_c^m is the marginal distribution of a bidder in group m, $\beta_{NR}^{-1} = \beta_{NR}^{-1} \left((1-\delta)^{D_R} b_1 \right)$, and $\beta_R^{-1} = \beta_R^{-1} \left((1-\delta)^{-D_{NR}} b_1 \right)$. The first-order conditions are now given by

$$S_{1}\left[1-F_{c}^{m_{1}}\left(c_{1}\right),1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),\ldots,1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),1-F_{c}^{R}\left(\beta_{R}^{-1}\right),\ldots,1-F_{c}^{R}\left(\beta_{R}^{-1}\right)\right]$$

$$= \left(b_{1}-c_{1}\right)\left[\left(n_{NR}-D_{NR}\right)\beta_{NR,1}^{-1}\left(1-\delta\right)^{D_{R}}f_{c}^{NR}\left(\beta_{NR}^{-1}\right)\right]$$

$$\times S_{12}\left[1-F_{c}^{m_{1}}\left(c_{1}\right),1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),\ldots,1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),1-F_{c}^{R}\left(\beta_{R}^{-1}\right),\ldots,1-F_{c}^{R}\left(\beta_{R}^{-1}\right)\right]$$

$$+\left(n_{R}-D_{R}\right)\beta_{R,1}^{-1}\left(1-\delta\right)^{-D_{NR}}f_{c}^{R}\left(\beta_{R}^{-1}\right)$$

$$\times S_{1n}\left[1-F_{c}^{m_{1}}\left(c_{1}\right),1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),\ldots,1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),1-F_{c}^{R}\left(\beta_{R}^{-1}\right),\ldots,1-F_{c}^{R}\left(\beta_{R}^{-1}\right)\right]\right],$$

where $f_{c}^{m}\left(\cdot\right)$ is the marginal probability density function (PDF) associated with the marginal CDF $F_{c}^{m}\left(\cdot\right)$.

5 A Simulation Study

Before moving into the estimation methodology and to illustrate the possible effects affiliation can have on bid preference auctions at the bidding stage, I conduct simulations over a range of different affiliated distributions with a fixed number of entrants. This section presents the results from those simulation studies. Here, I parameterize the group-specific marginal project cost distributions as beta distributions in order to remain flexible with their shape; I set the copula joining these marginal distributions to a Clayton copula. Figure 1 shows the full set of marginal project cost distribution CDFs used in this analysis.

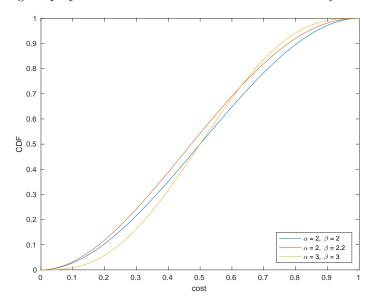


Figure 1: Beta (Marginal Cost) Distribution CDFs

I calculate bid functions in a variety of different environments. Except in a few special cases, the solution to the system of equations in (1) together with the boundary conditions does not have a closed-form solution. As a result, I approximate and invert each group's inverse bid functions with a modified version of the third algorithm found in Bajari (2001), which essentially approximates inverse bid functions using polynomials.¹⁷

I set the remaining simulation parameters to mirror a common New Mexico preference auction. I set the number of actual bidders to a commonly observed configuration of 1 non-resident bidder and 3 resident bidders, and I set the preference level to New Mexico's current discount of 5 percent. For each marginal project cost distribution, I approximate bid functions under independence and affiliation, where affiliation is calculated by setting the affiliation parameter to 1. I denote independence by an affiliation parameter of 0.

5.1 Equal Strength Bidders

As a start, I study a case where both group of bidders are of equal strength. Let α_R and β_R be the parameters characterizing the resident beta distribution, and let α_{NR} and β_{NR} be the parameters characterizing the non-resident beta distribution. I construct the equal strength case by setting each group's beta distribution

 $^{^{17}\}mathrm{See}$ the appendix for a detailed explanation of how I estimate the bid functions.

parameters to α_R , $\alpha_{NR} = 2$ and β_R , $\beta_{NR} = 2$ so that project costs are symmetric. Observe that in this case, the preference is the sole driver of any asymmetry between bidders. Figure 2 displays the equilibrium bid functions corresponding to these marginal project cost distributions.

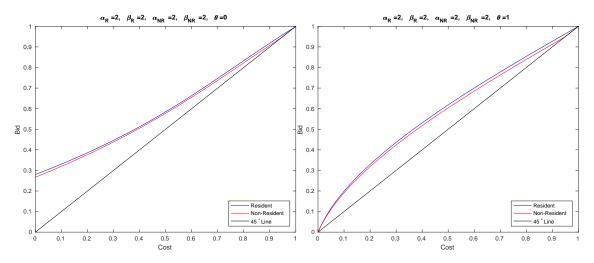


Figure 2: Bid Functions with Equal Strength Bidders $(n_R = 3, n_{NR} = 1)$

Several patterns emerge from these simulations:

- 1. With affiliation, a bidder with a low project cost bids more aggressively relative to independence. Intuitively, competing bidders are more likely to have similar project costs when the joint distribution is affiliated. A bidder with a low project cost draw is then more likely to face competitors with low project costs and will therefore bid more aggressively relative to independence.
- 2. For higher project cost draws, bidders tend to bid less aggressively relative to independence. Note that a bidder who draws a high project cost will believe that other bidders also have high project costs when these costs are affiliated, but her beliefs will not change when these costs are independent. This difference in beliefs will affect equilibrium bids because a bidder bids less aggressively when she believes her competition has higher project costs.
- 3. Affiliation can affect the separation in resident and non-resident bid functions caused by bid preferences. Indeed, the simulations show that the common low bid for both groups of bidders decreases when project costs become affiliated. The left boundary condition then implies that the common low bids are closer together. For higher project cost draws, there is more separation under affiliation. This separation comes from both groups of bidders bidding less aggressively and resident bidders receiving a preference (which makes them bid even less aggressively).

5.2 A Weak Group and a Strong Group of Bidders

Next I turn to a case where both groups of bidders differ in strength. For this case, the "weak" bidders are the resident bidders, and I set their beta distribution parameters to the previous configuration of $\alpha_R = 2$ and $\beta_R = 2$. The "strong" bidders here are the non-resident bidders, and I set their beta distribution parameters to $\alpha_{NR} = 2$ and $\beta_{NR} = 2.2$. Note that this arrangement of distribution parameters generates a situation where the resident project cost distribution first-order stochastically dominates the non-resident project cost distribution, which means that residents are more likely to draw higher project costs. Figure 3 shows the results from this case.

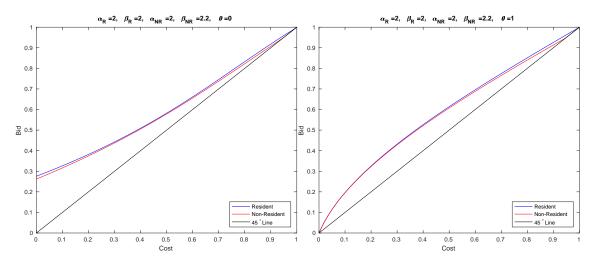


Figure 3: Bid Functions with a Weak and Strong Group of Bidders $(n_R = 3, n_{NR} = 1)$

Many of the same patterns observed in the equal strength bidder case appear in this case as well. Bidders still bid more aggressively for low cost realizations and less aggressively for high cost realizations when their joint project cost distribution is affiliated. The main difference here, especially when there is affiliation, is the amount of separation due to bid preferences, and that difference is generated by the asymmetry in resident and non-resident project costs. The idea here is that the resident marginal project cost distribution now first-order stochastically dominates the non-resident marginal project cost distribution. Whether costs are independent or affiliated, this asymmetry causes non-residents to bid less aggressively relative to the equal strength case, since non-residents are more likely to have lower project costs compared to residents. Affiliation intensifies this effect in that affiliation causes residents and non-residents to draw from similar quantiles on their respective distributions, so a low cost draw for a resident is likely to be even lower for a non-resident relative to independence.¹⁸ As a result, non-residents bid closer to residents under affiliation.

 $^{^{18}}$ To illustrate this point with an example, suppose that a resident bidder draws a cost in the 10^{th} percentile of her marginal

5.3 A High Variance Group and a Low Variance Group of Bidders

The final case I consider in this section is a case where each group of bidders have different levels of dispersion in their project cost distributions. I construct this case by holding the resident beta distribution parameters at their previous levels of $\alpha_R = 2$ and $\beta_R = 2$ while setting the non-resident beta distribution parameters to $\alpha_{NR} = 3$ and $\beta_{NR} = 3$. Observe that this composition of distribution parameters implies that residents and non-residents have the same mean project cost, but residents have more variance in their project costs relative to non-residents. Figure 4 presents the bid functions corresponding to these distributions.

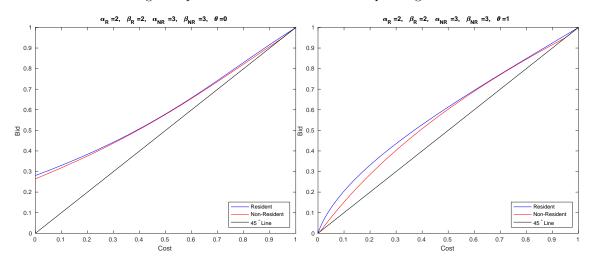


Figure 4: Bid Functions with a High and Low Variance Group of Bidders $(n_R = 3, n_{NR} = 1)$

There are a few differences between this asymmetry's effect on equilibrium bidding relative to the previous cases. Perhaps the most visible difference is that non-resident bidders bid less aggressively for high project cost draws and more aggressively for low project cost draws with affiliation. Intuitively, residents and non-residents become more likely to draw from the same quantiles with affiliation. When residents have more variable project costs than non-residents, this property of affiliation means that high draws for non-residents likely lead to even higher draws for residents, while low draws for non-residents likely lead to even lower draws for residents. As a result, non-residents bid less aggressively for high project cost draws and more aggressively for low project cost draws.

These results, although conditional on a fixed number of entrants, have implications for entry decisions. Bidders who face more (less) aggressive bidding conditional on entry due to affiliation are less (more) likely to project cost distribution. Under affiliation, this draw means that competing bidders are more likely to draw their project costs from the 10th percentile of their marginal distributions. Since the resident marginal distribution first-order stochastically dominates the non-resident marginal distribution, the project cost corresponding to the 10th percentile of the resident marginal distribution, so that resident bidder would believe competing non-resident bidders have even lower project costs relative to the equal strength case.

enter because their expected profits are lower (higher). Affiliation can therefore alter entry decisions within and across groups of bidders, which can change the procurement costs and the composition of actual bidders.

6 Empirical Model and Estimation

While the theoretical model provides a foundation for understanding the market for NMDOT procurement contracts, it does not lend itself to estimation without further distributional assumptions. This section outlines the distributional assumptions needed to produce an empirical model that can be estimated from the data. First, I discuss the distributional assumptions; then, I lay out the estimation routine. I end this section with a discussion of how the parameters are parametrically identified through the estimation procedure.

6.1 Parametric Specifications

The size of the data requires that I take a parametric approach in estimating the theoretical model. For this purpose, I assume that an auction, indexed by w, is characterized by the vector of observables $(\boldsymbol{x}_w, \boldsymbol{z}_w, n_{Rw}, n_{NRw}, N_{Rw}, N_{NRw})$, where \boldsymbol{x}_w is a vector of auction-level observables that affect project costs, \boldsymbol{z}_w is a vector of auction-level observables that affect entry costs, n_{Rw} and n_{NRw} are the observed number of resident and non-resident entrants respectively, and N_{Rw} and N_{NRw} are the advertised number of potential resident entrants and non-resident entrants respectively. The group-specific marginal distributions of project costs conditional on \boldsymbol{x}_w are given by $F_c^m(\cdot \mid \boldsymbol{x}_w)$, and the group-specific marginal distribution of entry costs conditional on \boldsymbol{z}_w are given by $G_k^m(\cdot \mid \boldsymbol{z}_w)$.

To address entry, I require parametric assumptions on the probability firms assign to the entry of competing firms. To this end, I model entry probabilities, $p_{mw}(x_w, z_w, N_{Rw}, N_{NRw})$, as a binomial distributions:

$$\Pr\left(n_{Rw}, n_{NRw} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{Rw}, N_{NRw}\right) = \Pr\left(n_{Rw} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{Rw}, N_{NRw}\right) \times \Pr\left(n_{NRw} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{Rw}, N_{NRw}\right),$$

where

$$\Pr\left(n_{mw} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right) = \begin{pmatrix} N_{mw} \\ n_{mw} \end{pmatrix} \left(p_{mw}\right)^{n_{mw}} \left(1 - p_{mw}\right)^{N_{mw} - n_{mw}},$$

and

$$p_{mw} = G_k^m \left(\Pi_{mw} \left(\boldsymbol{x}_w, N_{mw}, N_{-mw} \right) \mid \boldsymbol{z}_w \right). \tag{2}$$

This assumption on entry probabilities means that each firm calculates the probability that firms in their group and firms in their competing group enter the auction given their knowledge of the project and entry cost distributions. Observe that equation (2) comes from the equilibrium condition that beliefs are consistent.

A complication that arises in empirically implementing the theoretical model is the presence of the inverse bid function in the first-order conditions of the second-stage bidding problem. This complication would require that the inverse bid functions be approximated for every set of second-stage parameter guesses. Instead, this paper relies on approximations based on indirect methods introduced by Guerre, Perrigne, and Vuong (2000, henceforth abbreviated GPV) further extended by Krasnokutskaya (2011) for the case of unobserved auction heterogeneity and Hubbard, Li, and Paarsch (2012) for the case of affiliation using copulas. In particular, I infer a firm's cost from the observed bid distribution by noting that $F_b^m(b) = F_c^m\left(\beta_m^{-1}(b)\right)$ and $f_b^m(b) = f_c^m\left(\beta_m^{-1}(b)\right)\beta_{m,1}^{-1}(b)$. Making these substitutions in the first-order conditions of the second stage bidding problem obviates the need for estimating the inverse bid function when determining project costs. As a result, the empirical model will now focus on the marginal distribution of bids, $F_b^m(\cdot \mid \boldsymbol{x}_w)$, instead of the marginal distribution of project costs, $F_c^m(\cdot \mid \boldsymbol{x}_w)$.

I place the final set of distributional assumptions on the distribution of bids and entry costs. In order to have positive bids, allow for affiliation, and allow for heterogeneity across resident and non-resident bidders, I model the log of the submitted bids as follows:

$$\log(b_{iw}) = \boldsymbol{x}'_{iw}\boldsymbol{\beta} + \epsilon^{m_i}_{iw},$$

where

$$\begin{aligned} \epsilon_{iw}^{m_i} \mid \boldsymbol{x}_{iw} \sim \mathcal{N}\left(0, \exp\left(\boldsymbol{y}_{iw}'\sigma\right)^2\right), \\ \left(\epsilon_{1w}^{NR}, \dots, \epsilon_{n_{NR}w}^{NR}, \epsilon_{n_{NR+1}w}^{R}, \dots, \epsilon_{n_{NR}+n_{R}w}^{R} \mid \boldsymbol{x}_{iw}\right) \equiv \boldsymbol{\epsilon}_w \sim F_{\boldsymbol{\epsilon}_w}, \\ F_{\boldsymbol{\epsilon}_w} = \boldsymbol{C}\left[F_{\epsilon_{1w}^{NR}}, \dots, F_{\epsilon_{n_{NR}w}^{NR}}, F_{\epsilon_{n_{NR}+1}}, \dots, F_{\epsilon_{n_{NR}+n_{R}}}\right], \end{aligned}$$

 x_{iw} is the set of auction-level observables with an indicator variable for bidder i's residency status, and y_{iw} is a subset of the x_{iw} covariates also containing the resident indicator. Likewise, I assume that the entry

 $^{^{19}}$ For a complete description on how to approximate the inverse bid functions using GPV (2000) in this setting, see the appendix.

costs take the following form:

$$\log\left(k_{iw}\right) = \boldsymbol{z}'_{iw}\gamma + u_{iw}^{m_i},$$

where

$$u_{iw}^{m} \mid \boldsymbol{z}_{iw} \sim \mathcal{N}\left(0, \exp\left(\boldsymbol{v}_{iw}^{\prime}\alpha\right)^{2}\right),$$

 z_{iw} is the set of auction-level observables with an indicator for residency status, and v_{iw} is a subset of the z_{iw} covariates that also includes the resident indicator.

6.2 Estimation

I estimate the parameters of the empirical model using generalized method of moments (GMM). In using GMM, I match the theoretical predictions of the empirical model to the data by selecting the parameter values that minimize the weighted distance between model moments and data moments. This subsection gives a general overview of how I construct and use the moment conditions in estimation. For a more detailed explanation on how to derive the moments from the empirical model, see the appendix.

I use the first set of moment conditions to identify the parameters of the bid distribution. These moment conditions are

$$E\left[\boldsymbol{x}_{iw}\left(\log\left(b_{iw}\right) - \boldsymbol{x}_{iw}'\boldsymbol{\beta}\right)\right] = 0 \tag{3}$$

and

$$E\left[\boldsymbol{y}_{iw}\left(\log\left(b_{iw}\right) - \boldsymbol{x}'_{iw}\beta\right)\left(\log\left(b_{iw}\right) - \boldsymbol{x}'_{iw}\beta\right)\right] = E\left[\boldsymbol{y}_{iw}\exp\left(\boldsymbol{y}'_{iw}\sigma\right)^{2}\right].$$
(4)

Observe that equation (4) yields the standard deviation parameter, σ , and equations (3) and (4) yield the mean parameter, β .

In addition to estimating the parameters of the marginal distributions, the affiliation parameter, θ , must also be estimated through the moment conditions of the model. I estimate this parameter by relying on methods developed by Oh and Patton (2013) to estimate copulas using method of moments. In particular, one can summarize the degree of dependence between two random variables by a statistic called Kendall's tau. This statistic's equation for Clayton copulas together with its closed-form solution motivate the following moment condition:

$$\frac{\theta}{\theta + 2} = 4E \left[C \left[\Phi \left(\frac{\log(b_{iw}) - \mathbf{x}'_{iw}\beta}{\exp(\mathbf{y}'_{iw}\sigma)} \right), \Phi \left(\frac{\log(b_{jw}) - \mathbf{x}'_{jw}\beta}{\exp(\mathbf{y}'_{jw}\sigma)} \right) \right] \right] - 1 \quad i \neq j,$$
 (5)

where $\Phi(\cdot)$ is the standard normal CDF.

I use the last set of moment conditions to identify the parameters of the unobserved entry cost distribution.

These moment conditions are

$$E[n_{mw}] = \int N_{mw} p_{mw} dF(\boldsymbol{x}_w, \boldsymbol{z}_w, N_{mw}, N_{-mw}), \qquad (6)$$

$$E[n_{mw}^{2}] = \int N_{mw} p_{mw} (1 - p_{mw}) + N_{mw}^{2} p_{mw}^{2} dF(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}), \qquad (7)$$

$$E\left[n_{mw}^{3}\right] = \int N_{mw} p_{mw} \left(1 - 3p_{mw} + 3N_{mw} p_{mw} + 2p_{mw}^{2} - 3N_{mw} p_{mw}^{2} + N_{mw}^{2} p_{mw}^{2}\right) dF\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right),$$
(8)

and

$$E\left[n_{mw}^{4}\right] = \int N_{mw}p_{mw}\left(1 - 7p_{mw} + 7N_{mw}p_{mw} + 12p_{mw}^{2} - 18N_{mw}p_{mw}^{2} + 6N_{mw}^{2}p_{mw}^{2}\right) - 6p_{mw}^{3} + 11N_{mw}p_{mw}^{3} - 6N_{mw}^{2}p_{mw}^{3} + N_{mw}^{3}p_{mw}^{3}\right)dF\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right),$$

$$(9)$$

where

$$p_{mw} = G_k^m \left(\Pi \left(\boldsymbol{x}_w, N_{mw}, N_{-mw} \right) \mid \boldsymbol{z}_w \right)$$

is the group-specific entry probability. I derive these moment conditions from the assumption that entry is dictated by a joint binomial distribution, where the probabilities bidders assign to entry is consistent with the actual entry probabilities.

6.3 Parametric Identification

This section concludes with a brief discussion on what parts of the data I use to identify the model's parameters. The parameters of the model are the mean and standard deviation parameters of the bid distribution, β and σ , the mean and standard deviation of the entry cost distribution, γ and α , and the affiliation parameter, θ . I rely on the distribution of bids in the data to identify the bid distribution parameters. Given those parameters, I identify the affiliation parameter through the dependence of bids as measured by Kendall's tau in equation (5); if the observed bids tend to be positively dependent conditional on the observables, then the model attributes that dependence to the affiliation parameter.²⁰ I then identify the entry cost distribution parameters through the entry probabilities in the data.

7 Empirical Results

This section presents the empirical findings from the NMDOT highway procurement data. I first show descriptive summary statistics to illustrate some of the main components of the data relevant to residency status and firm bidding and entry behavior. Next, I display and interpret the structural parameter estimates from the empirical model and the corresponding cost distributions. These estimates suggest affiliation among bidder project costs and higher entry costs for resident firms relative to non-resident firms.

7.1 Descriptive Statistics

Table 1 contains the summary statistics for all highway procurement contracts in the sample tabulated by the source of funding. For each auction, I observe the following project characteristics: an engineer's estimated cost, the number of projected working days, the nature and location of the work, the number of licenses required, the length in miles, and the number of bidders and planholders. Additionally, I observe the number of subprojects²¹ as well as any Disadvantaged Business Enterprise (DBE) participation goals. I observe residency status and entry decisions at the firm level.

The top panel of table 1 summarizes the average estimated cost, bid, number of potential entrants, and number of actual entrants. Relative to federal-aid projects, state-funded projects are slightly larger and more expensive on average. The average estimated cost across state-funded projects exceeds that of federal-aid

²⁰Note here that a limitation of using bid dependence to identify affiliation is that any unobserved heterogeneity would also be attributed to the affiliation parameter. As a result, the estimates from this paper should be viewed as an upper bound on the affiliation parameter.

²¹A subproject is a smaller portion of the main project. For example, if a roadway rehabilitation project requires the installation of a fence, the fence installation would be a subproject of the main roadway rehabilitation project. For an example of project and subproject descriptions in the data, see the appendix.

Table 1: Summary Statistics for New Mexico Highway Construction Projects

	Federal-Aid Projects	State Projects	All Projects
Number of Contracts	353.00	23.00	376.00
Number of Bidders	1469.00	92.00	1561.00
Number of Planholders	4195.00	261.00	4456.00
Average Bid (in 1000s)	4068.05	5469.58	4156.93
Average Engineer's Estimate (in 1000s)	3679.79	4628.75	3737.84
Average Resident Planholders	9.50	9.91	9.52
Average Resident Bidders	2.97	3.39	3.00
Average Non-Resident Planholders	2.34	2.22	2.33
Average Non-Resident Bidders	1.17	0.91	1.15
Fraction of Projects by Type of Road:			
Federal Highway	0.59	0.52	0.59
Other Road	0.41	0.48	0.41
Fraction of Projects by Type of Work:			
Road Work	0.61	0.52	0.60
Bridge Work	0.20	0.09	0.19
Other Work	0.20	0.39	0.21
Average Contract Observables:			
Length (in miles)	5.02	3.79	4.94
Working Days	123.76	121.87	123.65
Number of Licenses Required	1.50	1.48	1.50
DBE Goal (%)	2.06	0.00	1.93
Number of Subprojects	8.14	7.65	8.11

projects by about \$949,000, while the bids received on state-funded projects are about \$1,401,000 higher than the bids received on federal-aid projects. Across the potential and actual entrant dimensions, federal-aid and state-funded projects are similar, attracting around the same average number of resident and non-resident planholders and bidders. These set of descriptive statistics also indicate substantial differences in how bidders of both groups enter auctions. On average, only about 3 of the possible 10 resident planholders become actual bidders, while about 1 out of every 2 non-resident planholders becomes an actual bidder.

The next two panels of table 1 separates state and federal aid projects by the type of road and the nature of the work requested. I separate the nature of work into three mutually exclusive categories: road work, bridge work, and other work. State and federal-aid projects are similar in terms of their location; roughly 50 to 60 percent of work is conducted on federal highways. State and federal-aid projects differ, however, in the nature of the work requested. Relative to federal-aid projects, state-funded projects require less road and bridge work, while work falling into neither of these categories is relatively higher.

The bottom panel of table 1 lists the summary statistics on the remaining project-level observables. State and federally funded contracts are, on average, similar across these observable dimensions with the exception being the level of the DBE participation goal. New Mexico does not specify DBE participation goals on its state-funded projects, which explains the lack of DBE participation goals observed on state projects in the data.

7.2 Structural Estimates

I use the estimated empirical model to disentangle strategic participation and bidding decisions. I use both preference and non-preference auctions in estimation, but I drop projects with 20 or more planholders for computational reasons – amounting to 1 state-funded project and 10 federally funded projects. In order to mitigate the effect of unobserved project heterogeneity on submitted bids, I include the number of potential entrants in each group in the set of control variables. The idea behind these controls is that unobservable project characteristics may attract more potential entrants in the form of planholders, since the NMDOT advertises projects before they publish the list of planholders. I use a rich set of project controls so that the correlation in submitted bids is primarily generated through affiliation in costs as opposed to unobserved project characteristics that are common knowledge to the bidders. I include a group-specific indicator for residency status in the set of control variables to allow for heterogeneity between resident and non-resident bidders.

Table 2: Estimated Parameters for the Log-Bid Distribution

	Coefficient	Standard Error
Constant	0.849	0.175
Resident	-0.011	0.011
New Mexico project	-0.034	0.069
log(Engineer's Estimate)	0.913	0.020
log(Length+1) (in miles)	0.038	0.015
log(Working Days)	0.070	0.023
Resident Planholders	0.001	0.004
Non-Resident Planholders	-0.005	0.007
Bridge Work	-0.021	0.033
Road Work	-0.0001	0.034
Number of Licenses Required	0.013	0.019
Federal Highway	-0.004	0.021
Urban	-0.044	0.018
DBE Goal(%)	-0.008	0.004
log(Subprojects)	0.077	0.025
Standard Deviation Parameters		
Constant	0.697	0.325
Resident	0.263	0.707
log(Engineer's Estimate)	-0.180	0.030
Affiliation Parameter		
Theta	0.831	0.189

Note: Standard deviation of the bid distribution is estimated as $\sigma = \exp(b_0 + b_1 resident + b_2 engineer)$, where resident is an indicator for being a resident bidder and engineer is the log of the enginner's estimate.

Table 2 contains the parameter estimates for the bid distribution. The coefficients indicate that the submitted bids vary according to a project's size and observable characteristics. The coefficients also show small and statistically insignificant differences in how the two groups of bidders bid. Residents bid only 1 percent less than non-residents across procurement projects, which need not be attributed to similarities in

resident and non-resident costs.

Conversely, the affiliation parameter estimate is positive and statistically significant, which indicates the presence of affiliation in firm project costs. This estimate can be interpreted using Kendall's tau as a measure of concordance²². In particular, the value of Kendall's tau for the Clayton copula is $\tau = \frac{\theta}{\theta+2}$. Applying that formula to the estimated affiliation parameter of $\theta = 0.831$ results in a Kendall's tau of 0.294, which means that a given pair of cost draws are 29.4 percent more likely to be concordant than discordant.

This tau estimate can be compared to other studies using a similar affiliated private value framework. On one hand, the Kendall's tau of 0.294 estimated here is higher than the tau of 0.06 estimated by Li and Zhang (2015) for the case of timber sales auctions in Oregon, implying that the costs for firms competing for NMDOT construction contracts are more concordant than the values of firms competing for Oregon timer sales auctions. On the other hand, Hubbard, Li, and Paarsch (2012) estimate a tau of 0.655 using Michigan Department of Transportation data under the assumption that costs are drawn from a Clayton copula. The difference between the Michigan and New Mexico tau estimates suggests that affiliation can vary in prevalence across states for similar types of auctions.

In order to evaluate differences in the marginal resident and non-resident project costs, I use methods of bid inversion developed by GPV (2000) on the estimated bid distributions. These methods use the equilibrium bid distributions in conjunction with the first-order conditions on optimal bidding to back out the cost associated with an observed bid. Heterogeneity in project characteristics will result in different marginal cost distributions for each separate project in the data. To keep the analysis concise, I calculate resident and non-resident marginal cost distributions for two types of projects: one project with the average characteristics of a preference project and one project with the average characteristics of a non-preference project. For each of these projects, I simulate and invert bids from the estimated marginal bid distributions to obtain costs using the average number of resident and non-resident bidders as the number of participants and taking into account the estimated affiliation parameter. I estimate the marginal project cost distribution using a kernel density estimator with a normal kernel and optimal bandwidth, yielding a marginal cost CDF for both types of bidders.

Figure 5 displays the different marginal project cost CDFs for the average preference and non-preference project. As evidenced by the shape of the CDFs and consistent with the observed marginal bid distributions, residents have a more disperse cost distribution than non-residents across projects. Also, no one cost

²²Concordance is similar to affiliation in that more concordant random variables exhibit a higher degree of positive dependence. Formally, if $(x_1, y_1) \dots (x_n, y_n)$ are n observations from random variables X and Y such that all values of x_i and y_i , $i = 1 \dots n$, are unique, then a pair of observations (x_i, y_i) and (x_j, y_j) , $i \neq j$, are concordant if $x_i > x_j$ and $y_i > y_j$.

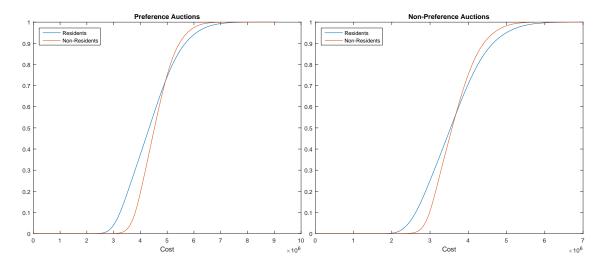


Figure 5: Kernel Density Estimates of the Marginal Cost CDFs for the Average Preference and Non-Preference Auctions

distribution first-order stochastically dominates the other in any of the average projects, which can lead to ambiguity in the ranking of resident and non-resident firms in terms of cost efficiency.

Table 3: Estimated Parameters for the Log-Entry Cost Distribution

	Coefficient	Standard Error
Constant	-0.121	0.800
log(Engineer's Estimate)	0.565	0.757
Resident	2.256	0.410
Resident Planholders	0.228	0.994
Non-Resident Planholders	0.109	0.244
Standard Deviation Parameters		
Constant	-0.589	0.190
Resident	1.854	0.301

Note: Standard deviation of the entry distribution is estimated as $\alpha = \exp(b_0 + b_1 resident)$, where resident is an indicator for being a resident bidder.

Turning to firm entry costs, table 3 presents the estimated parameters for the log-normal entry cost distribution. The entry parameters have the expected signs and magnitudes, although some of the parameters are statistically insignificant due to high standard errors relative to the bid distribution parameters. The entry parameters suggest noticeable differences among resident and non-resident costs of entry. Residents have higher average entry costs compared to non-residents and more variation in these entry costs.²³ A plausible explanation for these differences is that there may be a separate entry process into planholder

 $^{^{23}}$ Recall that these parameter estimates are the mean and variance of the natural logarithm of the entry costs. Let μ be the mean of the natural logarithm of the entry costs, and let σ be the standard deviation of the natural logarithm of the entry costs. The mean of the actual distribution of entry costs is then calculated as $\exp\left(\mu + \frac{\sigma^2}{2}\right)$, while the variance is calculated as $\left(\exp\left(\sigma^2\right) - 1\right)\exp\left(2\mu + \sigma^2\right)$.

status that selects non-resident firms who have innately lower entry costs, which is outside the scope of the data and model. The parameter estimates are nonetheless consistent with the lower conversion rate of potential resident bidders into actual bidders observed in the data.

8 Counterfactual Analysis

This section contains counterfactual policy experiments using the structural parameter estimates from section 7.2. Given the computational burden associated with calculating equilibrium bid functions, I focus on a representative construction project qualifying for preference in the data.²⁴ I first describe how I simulate the counterfactuals and then explore how affiliation and bid preferences affect bidding under fixed participation. As a final point, I compare bidder responses to different discount levels under the estimated level of affiliation and independence, allowing for endogenous entry decisions.

8.1 Simulation Method

I take a number of steps to simulate counterfactual bidding and entry behavior. First, I obtain a kernel density estimate of the underlying marginal project cost distributions, F_c^R and F_c^{NR} , by inverting a large number of bids drawn from the bid distributions implied by the empirical model using GPV (2000).²⁵ These group-specific cost distributions are primitives of the model and are fixed across all counterfactual policies and affiliation levels. Next, I approximate and invert the group-specific inverse bid functions using the modified third algorithm of Bajari (2001). Different discount levels will result in different equilibrium bid functions, so I recalculate the bid functions every time the preference level changes. I use the estimated bid functions and project cost distributions to simulate group-specific ex-ante profits, and, when entry is endogenous, I simulate entry decisions by comparing draws from the estimated entry cost distribution and the simulated ex-ante profits. For entrants, I draw project costs from an affiliated cost distribution using methods described in Marshall and Olkin (1988), and I apply the bid functions to the costs to determine the counterfactual bids. The average number of resident and non-resident planholders are similar for preference auctions and non-preference auctions in the data, suggesting that the number of potential entrants may not be sensitive to

²⁴To construct this project, I take the average of all numerical observables on projects qualifying for preference as the representative project characteristics. For categorical variables, I use the most common category as the representative category. ²⁵Note that the marginal project cost distribution will depend on the number of bidders and must be truncated to be consistent with the theory. Following Athey et al. (2013), I use a common configuration of three resident entrants and one non-resident entrant to determine the marginal project cost distribution. To deal with truncation, I truncate the support of the nonparametric project cost distribution to an interval of 0.5 to 1.6 times the engineer's estimate, corresponding to an interval with a lower bound of \$2,314,400 and an upper bound of \$7,406,000. This particular interval is tight enough to avoid extended regions of the project cost distribution with no density, which adversely affects bid function estimation, yet large enough to contain the vast majority (about 99.9%) of inverted project cost draws.

the preference level. For this reason, I set the number of potential entrants to the average preference auction level of 10 resident and 2 non-resident bidders for the auction simulations across discount levels, but the simulated number of entrants can vary given draws of the entry costs. I simulate a total of 10,000 auctions for each grid point in a grid of discount levels to generate the auction outcomes.

8.2 Affiliation, Bid Preferences, and Optimal Bidding

As a first step in understanding the interplay between affiliation and bid preferences in New Mexico's auctions, I use the numerical methods to approximate bid functions under fixed participation and varying degrees of preference and cost dependence. The bid functions use the cost distributions and average number of participants associated with the representative preference project, comparing bids under the estimated affiliation parameter with counterfactual bids under independence. To investigate the impact of bid preferences, I compare bid functions across auctions with the 5 percent preference policy and auctions without any preference. Figure 6 presents the equilibrium bid functions.²⁶

In general, the bid functions from New Mexico resemble the bid functions simulated with a high and low variance group of bidders, so many of the observations from those simulations apply to firms bidding on NMDOT construction contracts. In particular, affiliation, which can be seen by comparing the left two panels and the right two panels of figure 6, causes firms to bid more aggressively for lower project costs and less aggressively for higher project costs independent of the level of preference, since competing firms are more likely to have similar project costs. Another feature of affiliation is that it changes the relative aggression of resident and non-resident bidders. Comparing the top-left and top-right panels of figure 6, residents and non-residents behave almost as if the auction is symmetric when project costs are independent, but when project costs become affiliated, bid functions become more distinct, with residents bidding less aggressively than non-residents for lower project costs and non-residents bidding less aggressively than residents for higher project costs. This change comes from the higher variance in the resident bid distribution; since affiliation makes it more likely for groups of firms to draw project costs from the same quantiles of their marginal distributions, low draws for a non-resident are likely to be even lower for a resident, while high draws for a non-resident are likely to be even lower for a resident, while high draws for a non-resident are likely to be even higher for a resident. Residents will therefore bid less aggressively relative to non-residents for lower project costs and more aggressively for higher project costs.

Moving on to preference auctions, affiliation also affects how residents and non-residents adjust their bids when there is bid discounting. Bid preferences drive a wedge between preferred and non-preferred bidders,

 $^{^{26}}$ For an analysis of the error associated with these simulated bid functions, see the appendix.

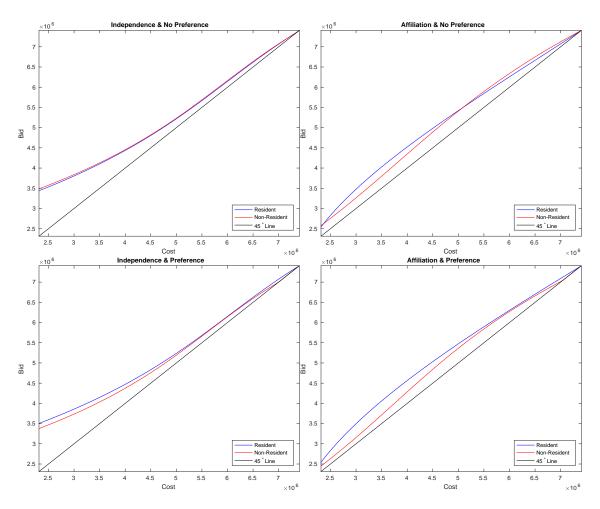


Figure 6: Bid Functions under Fixed Participation ($n_R=3,\,n_{NR}=1$)

meaning that non-preferred bidders lower their bids and preferred bidders increase their bids relative to the no preference case to account for discounting. The size of this wedge, which can be seen by comparing the top two panels with the bottom two panels of figure 6, depends on how aggressively firms bid and is therefore tied to affiliation. Observe that when preferences are offered in the independence case, the wedge between resident and non-resident bidders is large for lower project costs and decreases for higher project costs. When there is affiliation, the wedge is smaller than independence for lower project costs (since firms are bidding closer to their project costs) but becomes large enough to decrease the separation in the two bid functions for higher project cost draws. These differences suggest that the degree of affiliation can lead to substantial changes in how firms adjust bids with discounting.

8.3 Alternative Discount Rates, Efficiency, and the Role of Affiliation

Although New Mexico offers a 5 percent discount for its resident bidders, the discount level for preferred bidders can vary across states and the type of good being procured. Different discount levels will have different implications for the participation and bidding behavior of firms, and I investigate these changes in behavior for the representative construction project using the structural parameter estimates in conjunction with the project cost and entry cost distribution estimates. In order to assess the role of affiliation in these auctions, I contrast bidding and participation behavior under the estimated affiliation level against auctions where costs are assumed independent.

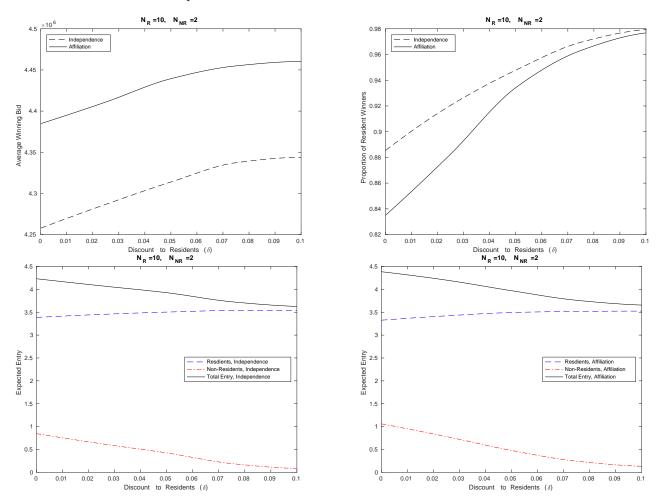


Figure 7: Average Winning Bid, Proportion of Resident Winners, and Entry under Alternative Discount Rates

Figure 7 plots the how the procurement cost, the proportion of preferred winners, and the expected participation changes across affiliation and preference levels. Increasing the discount level increases the

average procurement cost in these preference auctions, since there is less overall participation when the discount level increases. Relative to independence, affiliation leads to higher average procurement costs for all counterfactual discount levels because there is a wider range of project cost values where firms bid further from their costs under affiliation as evidenced by figure 6. The expected participation rate under affiliation is similar to the expected participation rate under independence, but the drop-off in non-resident bidders is more pronounced under affiliation. Despite the similarities in expected participation, affiliation tends to result in a lower proportion of resident winners relative to independence, and that difference decreases with higher discount levels. This behavior comes from how aggressively non-residents bid for lower project costs to account for affiliation, and that difference becomes smaller with higher discount levels because there are less non-resident participants.

In addition to changing bidding and participation, changes in the preference level can also alter economic efficiency. In the auction literature, an efficient auction is one that allocates an object to the firm with the lowest cost. Although auctions with symmetric bidders will always be efficient, auctions with asymmetric bidders, such as the ones considered in this paper, may not allocate objects efficiently. To gauge how efficiency changes over preference levels, I calculate the average efficiency loss, which is the average difference in cost between the lowest cost bidder and the winning bidder over auction simulations, and the proportion of inefficient auctions for a number of counterfactual preference levels. Project cost dependence may affect economic efficiency, so I calculate efficiency for auctions with the estimated level of affiliation and for auctions that assume independence.

Table 4: Counterfactual Preference Simulations

	Winning Bid (\$ 1000s)			Efficiency Loss (\$)			Prop. Inefficient	
Discount (%)	Aff.	Ind.	Diff. (%)	Aff.	Ind.	Diff. (\$)	Aff.	Ind.
0.0	4384.73	4257.80	2.98	4384.33	73.22	4311.10	0.038	0.004
2.5	4411.00	4286.74	2.90	1949.35	893.13	1056.23	0.021	0.014
5.0	4439.36	4313.84	2.91	1106.25	1299.34	-193.09	0.012	0.013
7.5	4454.92	4337.30	2.71	686.37	1328.59	-642.22	0.007	0.009
10.0	4460.31	4343.78	2.68	1298.39	830.01	468.38	0.008	0.005

This table shows the average winning bid, the average efficiency loss, and the proportion of inefficient auctions under independent and affiliated project-completion costs for 10,000 simulated preference auctions. Each potential entrant is given a draw from their group's respective entry cost distribution, and the number of entrants is determined endogenously by comparing their entry cost to their expected profit. Upon entry, each participating firm draws their project cost from their group's marginal project cost distribution to determine bids. Under affiliation, there will be dependence in the project cost draws.

Table 4 breaks down the average procurement cost and efficiency loss over the counterfactual affiliation and preference levels. New Mexico's current policy is responsible for a small change in procurement costs. An increase in the discount rate from 0 percent to its current level of 5 percent under affiliation increases

the average procurement cost of the representative construction project by \$54,631, which is a 1.2 percent cost increase. This increase is relatively smaller than the bias associated with the independence assumption. At the established 5 percent discount level, procurement costs are 2.9 percent higher than they would be if costs were assumed independent.

Table 4 also illustrates the role of affiliation in the evaluation of economic efficiency. At the 5 percent discount level, the average efficiency loss under affiliated project costs is \$1,106.25 (0.025 percent of the average winning bid) and generally decreases with the discount level; the average efficiency loss under independence is \$1,299.34 (0.030 percent of the average winning bid) and generally increases with the discount level. These patterns reverse themselves at the 10 percent discount level. The proportion of inefficient auctions under affiliation decreases with the discount level, but the proportion of inefficient auctions under independence first increases and then decreases with the discount level.

These patterns are generated by differences in bidding under affiliation and independence. Intuitively, efficiency is driven by the separation in the bid functions, which depends on both the level of affiliation and the composition of bidders. As bid functions become more distinct, the likelihood of an inefficient auction increases, and more separation is likely to increase the average efficiency loss.

With that in mind, the proportion of inefficient auctions first increases under independence because firms are virtually symmetric when there is no discount, which can be seen in figure 6. As the discount level increases, the separation in the bid functions also increases, leading to more inefficient auctions. The decrease in the proportion of inefficient auctions comes from the change in the composition of bidders. A higher discount level deters non-residents from entering, so auctions are more likely to be efficient since they only have resident bidders. The efficiency loss follows a similar pattern.

With affiliation, there is generally more separation in the bid functions with no preference, which explains why the proportion of inefficient auctions and the efficiency loss is higher than independence. Although increasing the preference leads to more separation in the bid functions for lower project cost draws, the bid functions are generally closer together with higher project cost draws under affiliation. That and the lower participation of non-resident bidders leads to a decrease in the proportion of inefficient auctions with higher discount levels. The general decrease in the efficiency loss under affiliation comes from the proportion of inefficient auctions together with the discount change. As the discount level increases, both the number of inefficient auctions and the average number of non-resident entrants decreases. The combination of these two forces leads to a general decrease in the efficiency loss. At the 10 percent discount level, the increased separation in the bid function is sufficiently large to increase the average efficiency loss despite the decreased

proportion of inefficient auctions from the 7.5 percent discount level.

Taken together, these simulations suggest that the discount rate can be used as a mechanism to increase the proportion of contracts won by resident bidders and alter the proportion of inefficient auctions at the expense of higher procurement costs. Relative to the independence case, affiliation leads to a higher expected procurement cost, a lower proportion of resident winners, and a lower average efficiency loss under New Mexico's current policy. These results depend on the discount level, which illustrates the significance of accounting for affiliation in public procurement with bid preferences.

9 Conclusion

In this paper, I empirically examine the presence of affiliation and its effect on procurement auctions in an environment where preferred bidders have their bids discounted. My analysis is based on NMDOT construction contracts – a unique environment where resident bidders receive a 5 percent discount over non-resident bidders in construction contracts using state funds. For the purpose of measuring affiliation and its effect on procurement, I develop a two-stage theoretical model, where firms with potentially affiliated private project costs first decide entry and then decide how much to bid. I implement the theoretical model through the use of copulas, capturing affiliation through a tractable parametric assumption on the project cost distribution. I estimate the model via GMM by using moments from firm bidding and entry decisions.

My structural analysis establishes the presence of affiliation and demonstrates the importance of affiliation in assessing procurement auctions with bid discounting. I find that the parameter measuring affiliation is positive and significant, indicating that firms have affiliated project costs. My counterfactual policy simulations reveal that affiliation can lead to differences in the proportion of preferred winners, the proportion of inefficient auctions, and the efficiency loss generated from auctions with asymmetric bidders, and these differences are contingent on the discount level. In fact, I find that although New Mexico's current policy is responsible for a 1.2 percent increase in procurement costs, affiliation results in a 2.9 percent increase in procurement costs relative to independence under New Mexico's policy.

There are a couple of areas open to future research. In line with how the NMDOT awards preferences in its procurement auctions, I focus on how affiliation can affect a particular type of preference policy where preferred bidders have their bids discounted. An interesting research direction for the future would be to explore how affiliation acts in settings where governments use other types of preference policies, such as group-specific entry subsidies and reserve prices. Also, I have one parameter governing the affiliation between all bidders. In other settings where the two groups of bidders are more distinct, a richer copula structure may

be a promising modeling possibility.

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A Applying GPV to Auctions with Bid Preferences and Affiliation

The first-order conditions in equation 1 can be rewritten as follows:

$$c_{1} = b_{1} - \frac{S_{1} \left[1 - F_{c}^{m_{1}} \left(c_{1} \right), 1 - F_{c}^{NR} \left(\beta_{NR}^{-1} \right), \dots, 1 - F_{c}^{NR} \left(\beta_{NR}^{-1} \right), 1 - F_{c}^{R} \left(\beta_{R}^{-1} \right), \dots, 1 - F_{c}^{R} \left(\beta_{R}^{-1} \right) \right]}{\frac{\partial S_{1} \left[1 - F_{c}^{m_{1}} \left(c_{1} \right), 1 - F_{c}^{NR} \left(\beta_{NR}^{-1} \right), \dots, 1 - F_{c}^{R} \left(\beta_{RR}^{-1} \right), \dots, 1 - F_{c}^{R} \left(\beta_{R}^{-1} \right) \right]}{\frac{\partial b_{1}}{\partial b_{1}}}, \quad (10)$$

where

$$\frac{\partial S_{1}\left[1-F_{c}^{m_{1}}(c_{1}),1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),...,1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),1-F_{c}^{R}\left(\beta_{R}^{-1}\right),...,1-F_{c}^{R}\left(\beta_{R}^{-1}\right)\right]}{\partial b_{1}}$$

$$= (n_{NR}-D_{NR})\beta_{NR,1}^{-1}(1-\delta)^{D_{R}}f_{c}^{NR}\left(\beta_{NR}^{-1}\right)$$

$$\times S_{12}\left[1-F_{c}^{m_{1}}\left(c_{1}\right),1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),...,1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),1-F_{c}^{R}\left(\beta_{R}^{-1}\right),...,1-F_{c}^{R}\left(\beta_{R}^{-1}\right)\right]$$

$$+(n_{R}-D_{R})\beta_{R,1}^{-1}(1-\delta)^{-D_{NR}}f_{c}^{R}\left(\beta_{R}^{-1}\right)$$

$$\times S_{1n}\left[1-F_{c}^{m_{1}}\left(c_{1}\right),1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),...,1-F_{c}^{NR}\left(\beta_{NR}^{-1}\right),1-F_{c}^{R}\left(\beta_{R}^{-1}\right),...,1-F_{c}^{R}\left(\beta_{R}^{-1}\right)\right].$$

Define $\tilde{b} = (1 - \delta)^{D_R} b$ as the adjusted resident bid and $\hat{b} = (1 - \delta)^{-D_{NR}} b$ as the adjusted non-resident bid. These adjusted bids come from the opposing group of bidders calculating their optimal bid. Following the methodology outlined in GPV (2000), the marginal CDF and PDF of costs can be expressed solely as functions of the bids by noting that

$$\begin{array}{lcl} F_b^{NR} \left(\tilde{b} \right) & = & F_c^{NR} \left(\beta_{NR}^{-1} \left(\tilde{b} \right) \right) \\ F_b^{R} \left(\hat{b} \right) & = & F_c^{R} \left(\beta_{R}^{-1} \left(\hat{b} \right) \right) \end{array}$$

and

$$\begin{split} f_b^{NR}\left(\tilde{b}\right) &= f_c^{NR}\left(\beta_{NR}^{-1}\left(\tilde{b}\right)\right)\beta_{NR,1}^{-1}\left(\tilde{b}\right) \\ f_b^{R}\left(\hat{b}\right) &= f_c^{NR}\left(\beta_R^{-1}\left(\hat{b}\right)\right)\beta_{R,1}^{-1}\left(\hat{b}\right). \end{split}$$

Equation 10 can now be written as

$$c_{1} = b_{1} - \frac{S_{1}\left[1 - F_{b}^{m_{1}}\left(b_{1}\right), 1 - F_{b}^{NR}\left(\tilde{b}_{1}\right), \dots, 1 - F_{b}^{NR}\left(\tilde{b}_{1}\right), 1 - F_{b}^{R}\left(\hat{b}_{1}\right), \dots, 1 - F_{b}^{R}\left(\hat{b}_{1}\right)\right]}{\frac{\partial S_{1}\left[1 - F_{b}^{m_{1}}\left(b_{1}\right), 1 - F_{b}^{NR}\left(\tilde{b}_{1}\right), \dots, 1 - F_{b}^{R}\left(\hat{b}_{1}\right), \dots, 1 - F_{b}^{R}\left(\hat{b}_{1}\right), \dots, 1 - F_{b}^{R}\left(\hat{b}_{1}\right)\right]}{\frac{\partial S_{1}\left[1 - F_{b}^{m_{1}}\left(b_{1}\right), 1 - F_{b}^{NR}\left(\tilde{b}_{1}\right), \dots, 1 - F_{b}^{R}\left(\hat{b}_{1}\right), \dots, 1 - F_{b}^{R}\left(\hat{b}_{1}\right)\right]}{\partial b_{1}}, \dots, 1 - \frac{1}{2}$$

which expresses costs as the sum of the bid and a strategic markdown.

B Solving for the Inverse Bid Functions

In order to solve for the inverse bid functions, I implement a modified version of the third algorithm found in Bajari (2001). In particular, I assume that the equilibrium inverse bid functions for bidders in group $m \in \{R, NR\}$ take on the following flexible functional form:

$$\hat{\beta}_{m}^{-1}(b) = \underline{b} + \sum_{k=0}^{K} \alpha_{m,k} (b - \underline{b})^{k},$$

where \underline{b} is the unknown common low bid and $\{\alpha_{m,k}\}$, $k=0,\ldots,K$ are polynomial coefficients for bidders in group m. The first-order conditions can now be expressed in terms of the polynomial approximations. Let α be a vector that collects the polynomial coefficients of all groups of bidders, $\hat{\beta}_{NR}^{-1} = \hat{\beta}_{NR}^{-1} \left((1-\delta)^{D_R} b \right)$, $\hat{\beta}_R^{-1} = \hat{\beta}_R^{-1} \left((1-\delta)^{-D_{NR}} b \right)$, and define $G_m(b; \underline{b}, \alpha)$ as the first-order conditions with the approximated inverse bid functions set equal to 0 at b:

$$\begin{split} G_{m}\left(b;\underline{b},\alpha\right) &= \\ S_{1}\left[1-F_{c}^{m}\left(\hat{\beta}_{m}^{-1}\right),1-F_{c}^{NR}\left(\hat{\beta}_{NR}^{-1}\right),\dots,1-F_{c}^{NR}\left(\hat{\beta}_{NR}^{-1}\right),1-F_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right),\dots,1-F_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right)\right] \\ &- \left(b-\hat{\beta}_{m}^{-1}\right)\left[\left(n_{NR}-D_{NR}\right)\hat{\beta}_{NR,1}^{-1}\left(1-\delta\right)^{D_{R}}f_{c}^{NR}\left(\hat{\beta}_{NR}^{-1}\right)\right. \\ &\times S_{12}\left[1-F_{c}^{m}\left(\hat{\beta}_{m}^{-1}\right),1-F_{c}^{NR}\left(\hat{\beta}_{NR}^{-1}\right),\dots,1-F_{c}^{NR}\left(\hat{\beta}_{NR}^{-1}\right),1-F_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right),\dots,1-F_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right)\right] \\ &+ \left(n_{R}-D_{R}\right)\hat{\beta}_{R,1}^{-1}\left(1-\delta\right)^{-D_{NR}}f_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right) \\ &\times S_{1n}\left[1-F_{c}^{m}\left(\hat{\beta}_{m}^{-1}\right),1-F_{c}^{NR}\left(\hat{\beta}_{NR}^{-1}\right),\dots,1-F_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right),\dots,1-F_{c}^{R}\left(\hat{\beta}_{R}^{-1}\right)\right]\right]. \end{split}$$

I evaluate these first-order conditions at T evenly spaced grid points within the intervals $b \in \left[\frac{\underline{b}}{(1-\delta)}, \overline{b}\right]$ for residents and $b \in \left[\underline{b}, (1-\delta)\,\overline{b}\right]$ for non-residents. I determine \overline{b} by the number of resident bidders: $\overline{b} = \overline{c}$ if $n_R > 1$ and $\overline{b} = \arg\max_b \left[(b-\overline{c})\Pr\left((1-\delta)\,b < b_j\,\forall j \in NR \mid \overline{c}\right)\right]$ if $n_R = 1$. In order to capture the flat spot in the inverse bid functions, I assume non-residents who have costs $c \in \left[(1-\delta)\,\overline{b},\overline{c}\right]$ bid their cost. Taken together, the modified boundary conditions are

$$\begin{array}{lll} 0 & = & \hat{\beta}_R^{-1} \left(\frac{\underline{b}}{(1-\delta)} \right) - \underline{c} \\ \\ 0 & = & \hat{\beta}_{NR}^{-1} \left(\underline{b} \right) - \underline{c} \\ \\ 0 & = & \hat{\beta}_R^{-1} \left(\overline{b} \right) - \overline{c} \\ \\ 0 & = & \hat{\beta}_{NR}^{-1} \left((1-\delta) \, \overline{b} \right) - (1-\delta) \, \overline{c} \end{array}$$

Define $H(b; \boldsymbol{\alpha})$ as

$$H(\underline{b}; \boldsymbol{\alpha}) = \sum_{m} \sum_{t=1}^{T} G_{m}(b_{t}; \underline{b}, \boldsymbol{\alpha}) + w(T) \left(\hat{\beta}_{R}^{-1} \left(\frac{\underline{b}}{(1-\delta)} \right) - \underline{c} \right) + w(T) \left(\hat{\beta}_{NR}^{-1} (\underline{b}) - \underline{c} \right)$$

$$+ w(T) \left(\hat{\beta}_{R}^{-1} (\overline{b}) - \overline{c} \right) + w(T) \left(\hat{\beta}_{NR}^{-1} ((1-\delta)\overline{b}) - (1-\delta)\overline{c} \right),$$

where I use the w(T) terms as positive weights to get the boundary conditions to hold. Approximating the inverse bid functions is equivalent to finding a vector of polynomial coefficients $\hat{\boldsymbol{\alpha}}$ to minimize $H(\underline{b}; \boldsymbol{\alpha})$.

In practice, I set the simulation parameters as follows. I use a cubic polynomial to approximate each group's inverse bid function (K = 3), and I set the number of grid points to 50 (T = 50). After performing an extensive set of simulation studies, I find that this particular arrangement of grid points and polynomials produces the most numerically stable results for the range of actual entrants possible during the counterfactual simulations. I set the weighting function for the boundary conditions to w(T) = 4T under affiliation and w(T) = 15T when project costs are independent, and I determine these weights by simulating the bid functions and choosing the lowest coefficient on T sufficient for the boundary conditions to hold during the simulations.

C Inverse Bid Function Accuracy

In order to evaluate the accuracy of the approximated inverse bid functions, I assess the first-order conditions of the resident and non-resident bidding problem on a grid of 100 bid points for the bid functions displayed in figure 6. Here, accuracy is determined by how close the first-order conditions are to reaching zero. Figure 8 shows the results. To my knowledge, the literature has not yet established a benchmark accuracy for the approximation of inverse bid functions with asymmetric bidders, but the results from this paper's approximations appear to be reasonable.

D Estimation Method

I estimate the parameters of the model with GMM, which essentially matches the predictions of the empirical model to the moments of the data. This matching process requires assumptions on the bid distribution and entry cost distribution, which were outlined in section 6.1. For completeness, I list these assumptions below:

$$\log(b_{iw}) = \boldsymbol{x}'_{iw}\beta + \epsilon^{m_i}_{iw}$$

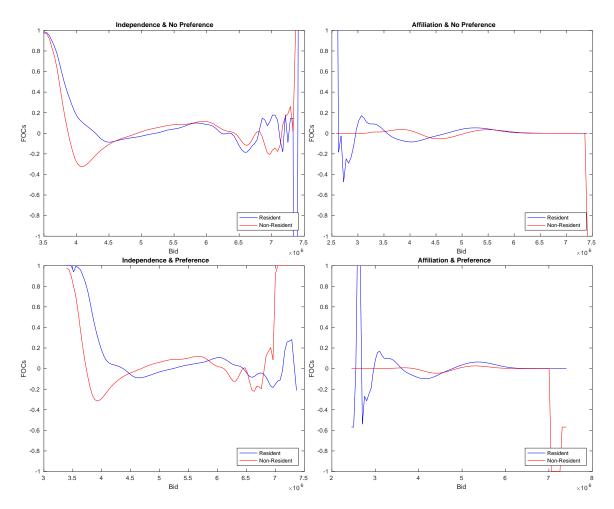


Figure 8: Errors for Approximated Bid Functions

This figure plots the first-order conditions associated with the bid functions approximated in figure 6. I evaluate the first-order conditions on a grid of potential bids, with accuracy determined by how close the first-order conditions are to zero.

$$\epsilon_{iw}^{m_i} \mid \boldsymbol{x}_{iw} \sim \mathcal{N}\left(0, \exp\left(\boldsymbol{y}_{iw}'\sigma\right)^2\right)$$

$$\left(\epsilon_{1w}^{NR}, \dots, \epsilon_{n_{NR}w}^{NR}, \epsilon_{n_{NR+1}w}^{R}, \dots, \epsilon_{n_{NR}+n_{R}w}^{R} \mid \boldsymbol{x}_{iw}\right) \equiv \boldsymbol{\epsilon}_{w} \sim F_{\boldsymbol{\epsilon}_{w}}$$

$$F_{\boldsymbol{\epsilon}_{w}} = \boldsymbol{C}\left[F_{\epsilon_{1w}^{NR}}, \dots, F_{\epsilon_{n_{NR}w}^{NR}}, F_{\epsilon_{n_{NR}+1}}, \dots, F_{\epsilon_{n_{NR}+n_{R}}^{R}}\right]$$

$$\log\left(k_{iw}\right) = \boldsymbol{z}_{iw}^{\prime}\gamma + u_{iw}^{m_i}$$

$$u_{iw}^{m} \mid \boldsymbol{z}_{iw} \sim \mathcal{N}\left(0, \exp\left(\boldsymbol{v}_{iw}^{\prime}\alpha\right)^{2}\right).$$

I derive the first and second moment conditions from the first and second moments of the bidding distribution:

$$E\left[\boldsymbol{x}_{iw}\left(\log\left(b_{iw}\right)-\boldsymbol{x}_{iw}'\boldsymbol{\beta}\right)\right]=E\left[E\left[\boldsymbol{x}_{iw}\left(\log\left(b_{iw}\right)-\boldsymbol{x}_{iw}'\boldsymbol{\beta}\right)\mid\boldsymbol{x}_{iw}\right]\right]$$

$$= E\left[\mathbf{x}_{iw}E\left[\left(\log\left(b_{iw}\right) - \mathbf{x}'_{iw}\beta\right) \mid \mathbf{x}_{iw}\right]\right] = E\left[\mathbf{x}_{iw}E\left[\epsilon_{iw} \mid \mathbf{x}_{iw}\right]\right] = 0$$

and

$$E\left[\boldsymbol{y}_{iw}\left(\log\left(b_{iw}\right) - \boldsymbol{x}_{iw}'\beta\right)\left(\log\left(b_{iw}\right) - \boldsymbol{x}_{iw}'\beta\right)\right] =$$

$$E\left[\boldsymbol{y}_{iw}E\left[\left(\log\left(b_{iw}\right) - \boldsymbol{x}_{iw}'\beta\right)\left(\log\left(b_{iw}\right) - \boldsymbol{x}_{iw}'\beta\right) \mid \boldsymbol{x}_{iw}\right]\right] =$$

$$E\left[\boldsymbol{y}_{iw}E\left[\epsilon_{iw}^{2} \mid \boldsymbol{x}_{iw}\right]\right] = E\left[\boldsymbol{y}_{iw}\exp\left(\boldsymbol{y}_{iw}'\sigma\right)^{2}\right].$$

The corresponding empirical moments are

$$\frac{1}{W} \sum_{w=1}^{W} \frac{1}{n_{Rw} + n_{NRw}} \sum_{i=1}^{n_{Rw} + n_{NRw}} \left[\boldsymbol{x}_{iw} \left(\log \left(b_{iw} \right) - \boldsymbol{x}'_{iw} \beta \right) \right]$$

for the first moment and

$$\frac{1}{W} \sum_{w=1}^{W} \frac{1}{n_{Rw} + n_{NRw}} \sum_{i=1}^{n_{Rw} + n_{NRw}} \left[y_{iw} \left(\log (b_{iw})^2 - (x'_{iw}\beta)^2 - \exp (y'_{iw}\sigma)^2 \right) \right]$$

for the second moment.

I derive the next moment condition from the equation for Kendall's tau for Clayton copulas. In particular, when the dependence between random variables is modeled as a copula, Kendall's tau takes the following form:

$$\tau_{ij} = 4E\left[\boldsymbol{C}\left[F_u^i\left(u_i\right), F_u^j\left(u_j\right)\right]\right] - 1,\tag{11}$$

where τ_{ij} is Kendall's tau, and u_i and u_j are random variables that are related through the copula $C[\cdot, \cdot]$ with marginal distributions F_u^i and F_u^j respectively. Given the assumption that the copula is a Clayton

copula, the equation for Kendall's tau takes the following form:

$$\tau_{ij} = \frac{\theta}{\theta + 2}.\tag{12}$$

Combining equations 11 and 12 gives the next moment condition, which can be expressed as

$$\frac{\theta}{\theta + 2} = 4E \left[C \left[\Phi \left(\frac{\log(b_{iw}) - \mathbf{x}'_{iw}\beta}{\exp(\mathbf{y}'_{iw}\sigma)} \right), \Phi \left(\frac{\log(b_{jw}) - \mathbf{x}'_{jw}\beta}{\exp(\mathbf{y}'_{jw}\sigma)} \right) \right] - 1 \quad i \neq j.$$

The empirical counterpart for the above moment condition is

$$\frac{4}{W} \sum_{w=1}^{W} \frac{1}{\left(n_{Rw} + n_{NRw}\right)} \sum_{1 \leq i < j \leq n_{Rw} + n_{NRw}} C \left[\Phi\left(\frac{\log(b_{iw}) - \boldsymbol{x}'_{iw}\beta}{\exp(\boldsymbol{y}'_{iw}\sigma)}\right), \Phi\left(\frac{\log(b_{jw}) - \boldsymbol{x}'_{jw}\beta}{\exp(\boldsymbol{y}'_{jw}\sigma)}\right)\right]$$

$$-1 - \frac{\theta}{\theta + 2}.$$

There is one subtlety in the above equation. The equation for τ_{ij} (equation 11) is given for copulas with two random variables, yet many auctions require that I draw bids from copulas with three or more random variables. In response to this requirement, I first take averages over all combinations of pairs of bids in an auction and then average over all auctions in order to use all of the information in the sample. In other words, I find the average Kendall's tau for each possible pair of bids in each auction and I use that average when computing the empirical moment condition.

I derive the final set of moment conditions from the moments of the entry distribution. Given that I assume entry follows a binomial distribution, the first, second, third and fourth moments of the entry distribution given the number of potential entrants and project characteristics are

$$E[n_{mw} | \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}] = N_{mw}p_{mw},$$

$$E\left[n_{mw}^{2} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right] = N_{mw}p_{mw}\left(1 - p_{mw}\right) + N_{mw}^{2}p_{mw}^{2},$$

$$E\left[n_{mw}^{3} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right] = N_{mw}p_{mw}\left(1 - 3p_{mw} + 3N_{mw}p_{mw} + 2p_{mw}^{2} - 3N_{mw}p_{mw}^{2} + N_{mw}^{2}p_{mw}^{2}\right),$$
 and

$$E\left[n_{mw}^{4} \mid \boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right] = N_{mw}p_{mw}\left(1 - 7p_{mw} + 7N_{mw}p_{mw} + 12p_{mw}^{2} - 18N_{nw}p_{mw}^{2} + 6N_{mw}^{2}p_{mw}^{2}\right)$$
$$- 6p_{mw}^{3} + 11N_{mw}p_{mw}^{3} - 6N_{mw}^{2}p_{mw}^{3} + N_{mw}^{3}p_{mw}^{3}\right)$$

respectively. Taking unconditional expectations over the number of potential entrants and the project characteristics yields the moment conditions described in section 6.2. These moment conditions are

$$E\left[n_{mw}\right] = \int N_{mw} p\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right) dF\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right),$$

$$E\left[n_{mw}^{2}\right] = \int N_{mw}p\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right)\left(1 - p\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right)\right)$$

$$+ N_{mw}^{2}p\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right)^{2}dF\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right),$$

$$E[n_{mw}^{3}] = \int N_{mw} p_{mw} \left(1 - 3p_{mw} + 3N_{mw} p_{mw} + 2p_{mw}^{2} - 3N_{mw} p_{mw}^{2} + N_{mw}^{2} p_{mw}^{2}\right) dF(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}),$$

and

$$E\left[n_{mw}^{4}\right] = \int N_{mw}p_{mw}\left(1 - 7p_{mw} + 7N_{mw}p_{mw} + 12p_{mw}^{2} - 18N_{nw}p_{mw}^{2} + 6N_{mw}^{2}p_{mw}^{2}\right)$$
$$- 6p_{mw}^{3} + 11N_{mw}p_{mw}^{3} - 6N_{mw}^{2}p_{mw}^{3} + N_{mw}^{3}p_{mw}^{3}\right)dF\left(\boldsymbol{x}_{w}, \boldsymbol{z}_{w}, N_{mw}, N_{-mw}\right)$$

The corresponding empirical moments are then given by

$$\frac{1}{W} \sum_{w=1}^{W} \left[n_{mw} - N_{mw} p_{mw} \right],$$

$$\frac{1}{W} \sum_{w=1}^{W} \left[n_{mw}^2 - N_{mw} p_{mw} \left(1 - p_{mw} \right) - N_{mw}^2 p_{mw}^2 \right],$$

$$\frac{1}{W} \sum_{w=1}^{W} \left[n_{mw}^3 - N_{mw} p_{mw} \left(1 - 3 p_{mw} + 3 N_{mw} p_{mw} + 2 p_{mw}^2 - 3 N_{mw} p_{mw}^2 + N_{mw}^2 p_{mw}^2 \right) \right],$$

and

$$\frac{1}{W} \sum_{w=1}^{W} \left[n_{mw}^{4} - N_{mw} p_{mw} \left(1 - 7 p_{mw} + 7 N_{mw} p_{mw} + 12 p_{mw}^{2} - 18 N_{nw} p_{mw}^{2} + 6 N_{mw}^{2} p_{mw}^{2} \right) - 6 p_{mw}^{3} + 11 N_{mw} p_{mw}^{3} - 6 N_{mw}^{2} p_{mw}^{3} + N_{mw}^{3} p_{mw}^{3} \right]$$

E Project and Subproject Examples

This section contains two example project descriptions in the data: one state project (left) and one federal-aid project (right). The main project is written in capital letters under the "Construction Consists Of:" line, and the subprojects are listed afterwards.

NEW MEXICO PROJECT

FEDERAL AID PROJECT

A300013

3100340

CN A300013

CN 3100340

Construction Consists Of:

ROADWAY REHABILITATION, Cold Milling w/Inlay (Flexible), In-Place Recycling and Stabilization (Flexible), Curb & Gutter w/Sidewalk, Traffic Control (Phasing), Permanent Signing and Miscellaneous Construction.

Construction Consists Of:

BRIDGE REPLACEMENT (Replace Existing Bridge w/3-Span Prestressed Girders, Approach Slabs, Concrete Barrier Railing), Roadway Reconstruction, Pavement Sections (Flexible), Earthwork (Borrow, Subexcavation), Curb & Gutter w/Sidewalk, Concrete Wall Barrier, Structures (Culverts, Drop Inlets), Erosion Control Measures, Traffic Control (Phasing), Permanent Signing, Lighting and Miscellaneous Construction.