

# The postwar growth slowdown and the path of economic development

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Online at https://mpra.ub.uni-muenchen.de/80988/ MPRA Paper No. 80988, posted 28 Aug 2017 10:59 UTC The Postwar Growth Slowdown and the Path of Economic Development

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Although the persistent slowdown in the growth of per capita output has been observed in virtually all industrialized countries since the early 1970s, no persuasive theoretical explanation for this phenomenon has been given. This paper constructs a modified endogenous growth model that indicates the slowdown is part of the natural process of economic development. Specifically, the model predicts that each economy develops along a path characterized by Malthusian stagnation, economic take-off, demographic transition, growth slowdown, and steady-state. The persistent slowdown in growth indicates that even the most developed countries are not in their steady-state yet, and their future growth could be slower. (JEL E27 O40)

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The slowdown in the growth of per capita output has been observed in virtually all industrialized countries since the early 1970s, and it continues to be a significant source of concern for economists and policy makers (Fischer 1988, Gordon 2013, Antolin-Diaz, Drechsel, and Petrella 2017). Although the causes of the slowdown have been extensively analyzed, it continues to remain somewhat of a puzzle: a wide variety of explanations have been offered, with little consensus as to the clear-cut culprit. Previous explanations include the oil price shock of 1973 (Jorgenson 1988), measurement problems (Baily and Gordon 1988), changes in the quality of the labor force (Bishop 1989), and exhaustion of important innovations (Gordon 2016). Most of the existing investigations have been conducted outside the context of explicit models of economic growth, and none of them are persuasive enough to prevent searching for other explanations.<sup>1</sup>

The current paper hypothesizes that the widespread slowdown of growth, which has lasted for more than 40 years since the early 1970s (see Section IV), is because of the fundamental mechanism of long-run economic growth, not any transitory exogenous shocks. Identifying the fundamental cause of the ongoing long-run growth slowdown has tremendous value both for growth theory and policy. Providing a persuasive theoretical explanation for the growth slowdown is difficult because if it is to be believed the slowdown reflects the mechanism of long-run economic growth, then it must be explained together with other early stylized facts of economic growth: a growth theory that explains the slowdown but that is inconsistent with other stylized facts cannot be persuasive.

Figure 1 presents the stylized facts that need to be explained together with the postwar growth slowdown. Over most of history, human society was locked into Malthusian stagnation with minimal growth in the population and negligible growth in the standard of living. But both "took off" in the first half of the nineteenth century as a spectacular growth in the population matched a rapid growth in per capita income. After long-run increases, the growth rate of the population peaked at the

<sup>&</sup>lt;sup>1</sup> See the arguments against each of the explanations of 1973's oil price shock, measurement problems, changes in the quality of the labor force, and exhaustion of important innovations from Jorgenson (1988), Byrne, Fernald, and Reinsdorf (2016), Maddison (1987), and Baily and Montalbano (2016).

beginning of the twentieth century and declined since then, and the growth rate of per capita income peaked in the second half of the twentieth century and declined since then. In summary, the stylized facts of economic development include Malthusian stagnation, economic take-off, demographic transition, and growth slowdown.

An analyzation of the existing growth theory reveals no candidate that can be directly applied to reconcile the growth slowdown with other stylized facts as presented in Figure 1. Malthus' (1798) classical growth theory does indicate a growth slowdown because it assumes income growth will cause a population explosion that in turn will reduce income to the subsistence level, but it is inconsistent with the simultaneous growth of the population and income seen over the last two centuries. Neoclassical growth models have the potential to explain short-term slowdowns with diminishing returns,<sup>2</sup> but they do not provide a sufficient explanation for the increasing growth over the long run, as noted by Romer (1986). Although the endogenous growth models, such as Romer's (1990) and Grossman and Helpman's (1991), are successful in explaining the increasing growth observed during the vast majority of modern history, the postwar slowdown has been viewed as something of a stumbling block for them. These models contain "scale effects" in the sense that, other things being equal, the growth rate of per capita income is proportional to the *level* of resources devoted to R&D. Given the continual increase in the amount of resources devoted to R&D activities throughout the postwar period (Jones 1995b), these models actually predict increasing postwar rates of growth.<sup>3</sup>

Jones's (1995a) semi-endogenous growth model does suggest the possibility of accounting for

 $<sup>^2</sup>$  It is possible to partly explain the postwar growth slowdown with the transitional dynamics of the neoclassical model: the war induced a decline in physical capital stock, leading to high postwar growth, and the growth rate declined with capital accumulation. However, as Shigehara (1992) points out, the neoclassical growth model cannot account for the bulk growth slowdown since the 1970s. Most computational evidence indicates that the pace of convergence in the neoclassical model is very rapid. For example, King and Rebelo (1993) finds that, for appropriate parameters, the half-life of convergence is only 6 years. Therefore, it is difficult to explain the 40-year postwar growth slowdown using the transitional dynamics of the neoclassical model.

<sup>&</sup>lt;sup>3</sup> Similar criticism applies to the "AK"-style or "broad capital" growth models of Romer (1986), Lucas Jr (1988), and Rebelo (1991). See Jones (1995b) for details.

the postwar growth slowdown together with other early stylized facts. It eliminates the counterfactual scale effect prediction of endogenous growth models and predicts that, other things being equal, the growth rate of per capita income is proportional to the *growth rate* but not the *level* of resources devoted to R&D. It implies that, holding equal the percentage of people engaged in R&D and holding the research productivity of each people, the growth rate of income is proportional to the growth rate of the population. On the other hand, holding the number of people engaged in R&D at the same level, the growth rate of income is proportional to the growth rate of research productivity. Therefore, the first increasing and then decreasing growth of the population has the potential to explain the first increasing and then decreasing growth of income. In addition, growing research productivity, as indicated by the increasing years of schooling, could explain why the slowdown in income growth occurred much later than the slowdown in population growth (see Figure 1).

However, because population and human capital are exogenous in the semi-endogenous growth model, the above explanation is only an unverified hypothesis based on historical observation. It is also possible that these contemporary observations are just a coincidence or that the dynamics of income growth is the cause, but not the result, of the dynamics of population and human capital growth. To explore the possibility of explaining the growth slowdown and other stylized facts using the dynamics of population and human capital, it is imperative to develop a growth model with endogenous population and human capital.

This paper develops a modified endogenous growth model with an endogenous population, human capital, and technology to explain the growth slowdown together with other early stylized facts. The model follows Jones (1995a) in its elimination of the counterfactual scale effect prediction of the early endogenous growth models and follows Becker, Murphy, and Tamura (1990) to model the endogenous growth of the population and human capital. In the model, utility-maximizing parents choose the number of children and the human capital investment in each child to maximize a dynastic utility function. Parents' utility maximizing behavior determines the growth of population

and human capital, and the growth of population and human capital determines the growth of technology and income, which in turn alter parents' decisions regarding the quantity and quality of their children. The endogenous interactions among population, human capital, technology, and income generate a path of development characterized by Malthusian stagnation, economic take-off, demographic transition, and growth slowdown.

The rest of this paper is organized as follows: Section I sets out the basic assumptions of our analysis and derives its main implications in an informal way. Section II outlines the model. Section III details the model's prediction regarding the path of economic development and illustrates that the growth slowdown is a natural part of the process of economic development. Section IV empirically supports the model's predictions, especially the growth slowdown, using long-run time series data from 18 advanced OECD countries. Section IV contains concluding remarks.

#### I. Basic properties of the model

The model is built on a natural rule of the creation of ideas: given the percentage of people engaged in R&D, the number of new ideas discovered is proportional to the size of the population and the level of human capital of each person. The literature has long recognized that given the chance of inventing something by each person, in a larger population, there will be proportionally more people lucky or smart enough to come up with new ideas (Kuznets 1960, Simon 1977). On the other hand, a person with a higher level of human capital is more likely to advance the technological frontier (Phelps 1966, Easterlin 1981). This rule of ideas creation implies that the growth rate of technology is an increasing function of the growth rate of the population and the growth rate of human capital.

As Romer (1986) points out, ideas are non-rivalrous in the sense that the use of an idea by one person does not preclude, at the technological level, the simultaneous use of the idea by another person. Increasing return in the production function for aggregate output introduced by the non-rivalry ideas implies that a growth in ideas has the potential to lead to a growth in per capita

output (Phelps 1966, Romer 1990).

Nevertheless, the current paper stresses that the positive effect of the growth of ideas on income growth depends on human capital accumulation. Human capital is ideas embodied in physical labor through education or training (Lucas Jr 1988, Becker, Murphy, and Tamura 1990). Although ideas are non-rivalrous, individuals still cannot use ideas discovered by others to enhance productivity before they learn how to use these ideas through human capital accumulation. For example, it is unlikely to significantly improve the productivity of a primitive tribe by just providing them with books documenting the latest ideas of production of modern societies. At the very least, this tribe would have to be taught how to read these books and then encouraged to learn the ideas documented within; this learning process is human capital accumulation. Therefore, a lack of human capital investment will limit the positive effect of the growth of ideas on income growth by limiting technology diffusion.

The model also assumes that the rate of return to human capital investment increases with the speed of technological progress. At least since Schultz (1964), economists have recognized that returns to formal schooling will be high when rapid technological progress limits the time available for the informal learning by observation to function. This assumption has been empirically supported by Foster and Rosenzweig (1996) and theoretically modeled by Nelson and Phelps (1966) and Galor and Weil (2000).

Following Becker, Murphy, and Tamura (1990), the current paper assumes an overlapping-generations economy in which identical parents choose the number of children and the human capital investment in each child to maximize a dynastic utility function. The utility is derived from both the consumption and the utility of each child. Combining this assumption with the above assumptions indicates endogenous interactions among population, human capital, technology, and per capita output. First, parents' utility-maximizing behavior determines the growth of the population and human capital; second, the rule of ideas creation suggests that the growth of population and

human capital determines the growth of technology and income; and third, technological progress and income growth alter parents' utility-maximizing behavior by raising the rate of return to human capital investment and by raising the opportunity costs of time spent on the production and rearing of children.

It is this endogenous interaction that generates the path of economic development depicted in Figure 1. Specifically, at the early stage of economic development, parents can only afford to have a small number of children because of the low labor productivity, and hence, the growth rate of the population is small.<sup>4</sup> Since the growth rate of technology is an increasing function of the growth rate of population, a small population growth rate implies a small technology growth rate. Because the rate of return to human capital investment increases with the speed of technological progress, parents do not invest in the human capital of their children when the growth rate of the population, and hence the growth rate of technology, is too small. The absence of human capital investment limits the diffusion of technology.<sup>5</sup> Both the small growth rate of technology and the inefficiency of technology diffusion determine the thousands of years of Malthusian stagnation that came with negligible growth of population, technology, and per capita output.

Nevertheless, as long as new technologies are created and diffused, labor productivity and the growth rate of the population will increase.<sup>6</sup> A higher population growth accelerates technological progress and leads to higher rates of return to human capital investment that eventually induces parents to invest in the human capital of their children. Positive human capital investment not only accelerates the growth of technology, but also enhances the efficiency of technology diffusion. Economic take-off occurs as a result and the growth rates of population, human capital, technology,

<sup>&</sup>lt;sup>4</sup> It is worth pointing out that the fertility rate was actually quite high during Malthusian stagnation, but because of the high mortality rate at the same time, the number of children who survived was very small.

<sup>&</sup>lt;sup>5</sup> See Section III.A for a detailed discussion of the inefficiency of technological diffusion when there is no human capital investment.

<sup>&</sup>lt;sup>6</sup> See Section III.A for empirical evidence supporting that the population growth rate was increasing, although at an extremely small rate, during Malthusian stagnation.

and per capita output increase simultaneously over time after that.

However, because the production of children and human capital investment are time-intensive, the growth rates of the population and human capital cannot increase forever. Continually rising rates of return to human capital investment motivate parents to have fewer children and to invest more in the human capital of each child, and this substitution effect ultimately leads to the decline of the population's growth rate. Declining population growth imposes a negative effect on the growth rate of technology. In addition, the growth rate of human capital also tends to decline when the level of human capital is very high and when the time invested in human capital is bounded. The declining growth of both the population and human capital leads to the slowing of technological growth and hence to the slowing of per capita output growth. The economy eventually converges into a steady-state with a constant population growth rate and a constant human capital investment.

The mechanism of economic development described here can be summarized by the two virtuous circles in Figure 2. Circle 1 is the virtuous circle of population growth: population growth spurs technological progress, and the growth of technology leads to the growth of per capita output, which in turn enhances population growth through the income effect. Circle 2 is the virtuous circle of human capital accumulation: human capital accumulation enhances technology progress, which in turn induces more human capital investments by raising the rate of return to those investments. Initially, Circle 2 is not functioning, and Circle 1 is very weak because technology diffusion is inefficient. But the weak Circle 1 eventually triggers Circle 2, which in turn strengthens Circle 1 by promoting the diffusion of technology, and economic take-off occurs. However, further economic development induces parents to make a trade-off between these two virtuous circles, and it eventually leads to the declining population growth and the declining technology and income growth.

The model depends heavily on the insights of previous theoretical studies. As will be shown in the next section, we have modified the ideas-based growth model of Jones (1995a), which is modified from Romer (1990), to allow for the endogenous growth of the population and human capital following the method proposed by Becker, Murphy, and Tamura (1990). In addition, this model adopts several critical assumptions from the unified growth model of Galor and Weil (2000). They developed the first unified growth model that can explain wholly Malthusian stagnation, economic take-off, and demographic transition. Complementary to Galor and Weil (2000), the current paper extends the unified growth model to include the more recent, ongoing growth slowdown.<sup>7</sup>

#### II. A simple ideas-based growth model

Consider an overlapping-generations economy in which identical agents live for two periods: childhood and adulthood. An adult chooses the number of children  $n_r$  at the beginning of his or her adulthood. The production and rearing of children are expensive and time-intensive. We assume each child consumes fixed hours e of his or her parent's working time and consumes fixed units f of goods. Each adult is endowed with T hours of working time that can be spent on producing consumer goods, rearing children, and investing in the human capital of children. Children spend all their time on human capital accumulation.

A single consumption good  $Y_t$  is produced using technology  $A_t$ , labor  $L_t$ , and physical capital  $K_t$ . Physical capital is accumulated consumer goods that do not wear out. The consumer goods are produced according to a Cobb-Douglas production function in which technology is labor-augmenting, as follows:

$$Y_t = A_t^\beta L_t^\beta K_t^{1-\beta} = C_t + \Delta K_t + N_t n_t f \quad , \tag{1}$$

 $<sup>^{7}</sup>$  There are several other important differences between these two models. For example, the current model explains Malthusian stagnation by the inefficiency of technological diffusion and the small growth of the population, but their model explains it by the existence of a fixed production capital; the current model predicts that economic take-off is triggered by the endogenous rising of the *growth rate* of the population, but their model predicts it is triggered by the endogenous rising of the *growth rate* of the population, but their model predicts the growth rate of technology first increases and then declines after demographic transition, but their model predicts it is monotonically increasing once an economy emerges from Malthusian stagnation.

where  $\beta \in (0,1)$  is a constant,  $C_t$  is the total consumption of generation t,  $\Delta K_t$  is the net investment in physical capital, and  $N_t$  is the number of adults. The production function can be written in per capita terms by dividing both sides by the number of adults:

$$Y_t / N_t = y_t = c_t + \Delta k_t + n_t f = A_t^{\beta} l_t^{\beta} k_t^{1-\beta} , \qquad (1)$$

in which  $y_t$  is the per capita output,  $l_t$  is the per capita time spent on production, and  $k_t$  is the per capita physical capital.

The creation of new ideas is the driving force of long-run per capita income growth. We modify the research functions of Jones (1995a) and Jones (2002) to obtain a research function in which the number of new ideas discovered each period  $\Delta A_t$  is proportional to the total amount of human capital spent on searching for new ideas  $H_t$ :<sup>8</sup>

$$\Delta A_t = \delta H_t^{\mu} A_t^{\phi} = \delta \left( h_t N_t s T \right)^{\mu} A_t^{\phi} \quad , \quad A_0 > 0 \text{ given} , \tag{1}$$

that  $\phi < 1$ ,  $\mu > 0$ ,  $\delta > 0$ , and 0 < s < 1. The research equation assumes that each identical adult spends a constant share *s* of working time on searching for new ideas.<sup>9</sup> As shown on the far-right side of equation (1),  $H_t$  is proportional to the human capital of each adult ( $h_t$ ), the number of adults ( $N_t$ ), and the time each adult spends on searching for new ideas (*sT*). Underlying research equation (1) is the natural rule of ideas creation, stating that the amount of new ideas discovered is proportional to the size of the population and the level of human capital.

Note that research equation (1) accommodates a wide range of beliefs regarding the determinants of the research output. Specifically, it allows each person's research productivity to decrease with

<sup>&</sup>lt;sup>8</sup> To present the long-run dynamics of economic development in the clearest fashion, this model abstracts from many of the important insights of the previous ideas-based growth models, such as the decentralized model specification with intermediate sectors, vertical and horizontal product differentiation, and the uncertainty of innovation.

<sup>&</sup>lt;sup>9</sup> A similar assumption is that a constant fraction of adults is working in the R&D sectors. It is worth pointing out that an important observation of the modern economy is the increasing fraction of workers employed in R&D sectors (Jones 2002). However, allowing for an endogenous fraction of R&D employment would complicate the model to the point of intractability, but it does not facilitate the explanation of the stylized facts of economic development.

 $(0 < \mu < 1)$ , increase with  $(\mu > 1)$ , or be independent of  $(\mu = 1)$  the population; it also allows the rate of innovation to decrease with  $(\phi < 0)$ , increase with  $(0 < \phi < 1)$ , or be independent of  $(\phi = 0)$  existing technology.

New ideas will be discovered ( $\Delta A_t > 0$ ) as long as the total amount of human capital spent on searching for new ideas is positive ( $H_t > 0$ ). However, as argued by Jones (1995a), the growth rate of ideas asymptotically approaches zero in the long run if there is no growth in the population and human capital. To see why, divide both sides of equation (1) by  $A_t$  to get the growth rate of ideas:

$$g_{At} = \frac{\Delta A_t}{A_t} = \delta \frac{\left(h_t N_t s T\right)^{\mu}}{A_t^{1-\phi}} \quad . \tag{1}$$

Because  $1-\phi > 0$ , the denominator  $A_t^{1-\phi}$  increases over time when  $\Delta A_t > 0$ . If the numerator  $(h_t N_t s T)^{\mu}$  is constant, the growth rate of ideas  $g_{At}$  asymptotically approaches zero.

The necessary condition for the persistent growth of ideas is the persistent growth of population and human capital. Taking the logarithm of equation (1) and differentiating it with respect to time, the growth rate of ideas becomes an increasing function of the growth rates of the population  $(g_{Nt})$ and human capital  $(g_{ht})$  and becomes a decreasing function of the proportional changes in the growth rate of ideas ( $\Delta g_{At}/g_{At}$ ):

$$g_{At} = \kappa \left( g_{Nt} + g_{ht} \right) - \kappa \mu^{-1} \frac{\Delta g_{At}}{g_{At}} \quad , \tag{1}$$

with  $\kappa = \mu/(1-\phi)$ . Because the growth rate of per capita output is approximately equal to the growth rate of ideas  $(g_{yt} \approx g_{At})$ ,<sup>10</sup> we obtain that the growth rate of per capita output is an increasing

<sup>&</sup>lt;sup>10</sup> Because there is no adjustment cost in this model, the economy will instantaneously adjust the initial amounts of per capita physical capital  $k_t$  so that the ratio  $A_t l_t / k_t$  is equal to  $\beta / (1 - \beta)$ . Relative to endogenously growing ideas  $A_t$ 

function of the growth rate of population and the growth rate of human capital.

From equation (1) we can also obtain that the level of technology is proportional to the size of the population and the level of human capital per capita:

$$A_{t} = b \left( h_{t} N_{t} \right)^{\kappa} g_{A_{t}}^{-\kappa \mu^{-1}} , \qquad (1)$$

Combining equation (1) and equation (1) implies that per capita output is proportional to the size of the population and the level of human capital of each person.

We follow Lucas Jr (1988) and Becker, Murphy, and Tamura (1990) in seeing human capital as ideas embodied in physical labor through education or training and assume that the human capital of children is accumulated according to a learning technique with the positive externality of the parental level of human capital:

$$h_{t+1} = v h_t^{\gamma} z_t + h_0 \quad , \tag{1}$$

with constants  $0 < \gamma \le 1$ ,  $\upsilon > 0$ , and  $h_0 \ge 0$ . The human capital of a child  $h_{t+1}$  depends on the parental human capital  $h_t$ , the time that a parent invests in the human capital of each child  $z_t$ , and the endowed human capital at birth  $h_0$ .<sup>11</sup>

We follow Becker, Murphy, and Tamura (1990) to assume that altruistic parents choose the number of children  $(n_t)$  and the human capital investment in each child  $(z_t)$  to maximize a dynastic utility function:

and physical capital  $k_t$ , per capita labor input  $l_t$  can be approximately seen as constant, so we have  $g_{At} \approx g_{kt}$  and  $g_{yt} = \beta g_{At} + (1-\beta) g_{kt} \approx g_{At}$ .

<sup>&</sup>lt;sup>11</sup> Human capital  $(h_t)$  does not directly enter into the production function (1) but enters through  $A_t$  instead, as can be seen by transforming equation (1) to get  $A_t = b(h_t N_t)^{\kappa} g_{\lambda t}^{-\kappa \mu^{-1}}$ . This model specification helps avoid the problem of "double-counting" human capital and keeps the model simple. We can also assume a separate human capital term in the Cobb-Douglas production function in the form of an "effective workforce," such as  $Y_t = A_t^{\beta} (h_t L_t)^{\beta} K_t^{1-\beta}$ . To do so will not change the implication of the model because the separate human capital term can be combined with the human capital term that already exists in  $A_t$  to get  $A_t' = bh_t^{\kappa+1} N_t^{\kappa} g_{\lambda t}^{-\kappa \mu^{-1}}$ , and the production function becomes  $Y_t' = A_t'^{\beta} L_t^{\beta} K_t^{1-\beta}$ .

$$V_{t} = u(c_{t}) + a(n_{t})n_{t}V_{t+1} \quad .$$
(1)

The dynastic utility of a parent  $V_t$  depends on his or her consumption  $c_t$ , the degree of altruism per child  $a(n_t)$ , the number of children  $n_t$ , and the utility of each child  $V_{t+1}$ . The dynastic utility function is simplified with the following:

$$u(c_t) = \frac{c_t^{\sigma}}{\sigma}, \quad a(n_t) = \alpha n_t^{-\varepsilon} \quad , \tag{1}$$

where  $0 < \sigma < 1, 0 \le \varepsilon < 1$ , and  $\alpha > 0$ . Parents maximize the utility function subject to the following time and budget constraints:

$$(1-s)T = l_t + n_t \left(e + z_t\right) \quad , \tag{1}$$

$$c_t = A_t^{\beta} l_t^{\beta} k_t^{1-\beta} - \Delta k_t - n_t f \quad . \tag{1}$$

The arbitrage condition between per capita consumption in periods t and t+1 is

$$\frac{u'(c_t)}{au'(c_{t+1})} = \alpha^{-1} n_t^{\varepsilon} \left(\frac{c_{t+1}}{c_t}\right)^{1-\sigma} \ge R_{zt} = 1 + r_{zt}$$

$$\tag{2}$$

where  $r_{zt}$  is the rate of return on investment in human capital, and equality holds when investments are positive. To calculate the rate of return, we rewrite the Bellman equation using the learning technology (1), the time constraint (1), and the budget constraint (1) to yield the following:

$$V_{t}(h_{t}) = \max \begin{cases} \sigma^{-1} \left[ (bh_{t}N_{t})^{\beta} \left[ (1-s)T - n_{t}e - n_{t} (vh_{t})^{-1} h_{t+1} - n_{t} (vh_{t})^{-1} h_{0} \right]^{\beta} k_{t}^{1-\beta} - \Delta k_{t} \end{bmatrix}^{\sigma} \\ + \alpha n_{t}^{1-\varepsilon} V_{t+1}(h_{t+1}) \end{cases}$$
(2)

Here, we apply the simplification assumption of  $\kappa = \gamma = 1$ . Differentiating (2) with respect to  $h_{r+1}$  and using the envelope theorem, we get that the rate of return is determined from the following:

$$R_{zt} = \upsilon n_t \left( l_{t+1} + n_{t+1} z_{t+1} \right) \quad . \tag{2}$$

Because the rate of return measures the effect on  $c_{t+1}$  of increasing  $h_{t+1}$ , it depends on the productivity of greater  $h_{t+1}$ , which depends on  $n_t$ ,  $l_{t+1}$ ,  $z_{t+1}$ , and  $n_{t+1}$  according to the production

functions of the ideas, consumer goods, and human capital.

By differentiating the utility function with respect to  $n_t$ , we get the first-order condition for maximizing the utility with respect to the number of children:

$$(1-\varepsilon)\alpha n_t^{-\varepsilon} V_{t+1}(h_{t+1}) = u'(c_t) \left[\beta A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta} \left(e+z_t\right) + f\right]$$
(2)

The marginal utility from an additional child is given on the left-hand side of equation (2) while the right-hand side gives the total costs of producing and rearing a child. Costs depend on the productivity of labor  $(\beta A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta})$ , the fixed time (*e*) and goods (*f*) inputs, and the endogenous time spent investing in each child (*z<sub>t</sub>*).

For the non-corner solution with a positive human capital investment, the first-order condition with respect to the investment is obtained by differentiating the utility function with respect to  $z_i$ :

$$\alpha n_{t}^{-\varepsilon} \upsilon h_{t} \frac{dV_{t+1}}{dh_{t+1}} = \beta u'(c_{t}) A_{t}^{\beta} l_{t}^{\beta-1} k_{t}^{1-\beta} \quad .$$
<sup>(2)</sup>

The marginal utility of an additional unit of time spent investing in children's human capital is given on the left-hand side of equation (2) while the right-hand side gives the marginal costs of time.

#### III. The path of economic development

This section shows how the model explains the growth slowdown and reconciles it with other early stylized facts of economic development. Specifically, the model predicts that the growth rate of the population  $(g_{Nt})$ , the level of human capital investment  $(z_t)$ , and the growth rate of per capita output  $(g_{yt})$  evolve along the paths depicted in Figure 3. The period before time  $t_1$  represents the thousands of years of Malthusian stagnation during which  $g_{yt}$ ,  $g_{Nt}$ , and  $z_t$  are extremely small. These three variables take off from  $t_1$  and experience long-run increases after that. The growth rate of per capita output peaked at  $t_2$  and declines after that. The growth rate of per capita output peaked at  $t_3$ 

and declines after that. The level of human capital investment increases over time since  $t_1$ . These three variables eventually level off when the economy settles down to its steady-state.

Subsections A and B detail the model's predictions about the path of economic development and explain the endogenous transition from one stage to the next along the path. Subsection C extends the model's predictions regarding the path of economic development to countries sharing technology. Subsection D briefly compares the model's predictions with the literature.

#### A. Malthusian stagnation and economic take-off

During the early stages of economic development, labor productivity as measured by the stock of technology  $(A_i)$  is quite low. Parents can only afford to have a small number of children because the production and rearing of children are expensive and time-intensive. A sufficiently small number of children (n) lead to the strict inequality of the arbitrage condition (2):<sup>12</sup>

$$\alpha^{-1} n^{-(1-\varepsilon)} > \upsilon l \quad , \tag{2}$$

which means the rate of return to human capital investment is too low, and the economy is located in a corner solution in which parents do not invest in the human capital of their children ( $z_t = 0$ ).

Underlying this implication is the model's assumption that the rate of return to human capital investment increases with the rate of technological progress. According to equation (1), the growth rate of technology is an increasing function of the growth rate of the population. A small number of children means a small growth rate of the population ( $g_{Nt} = n_t - 1$ ),<sup>13</sup> which implies a small growth

<sup>&</sup>lt;sup>12</sup> Empirical evidence indicates that the ratio  $c_{t+1}/c_t$  is approximately equal to one during Malthusian stagnation. According to Lee (1980) and Maddison (1982), the standard of living was roughly constant during the thousands of years of Malthusian stagnation. Therefore, the combination of the model's coefficients should be in the way that when the number of children is small, the marginal rate of return to children is high enough so that the small growth in labor productivity will mainly be reflected in the growth of the number of children but not in the growth of per capita consumption.

<sup>&</sup>lt;sup>13</sup> In the model, population growth is reflected only in the increasing number of children because we assume longevity as a constant and that there is no migration. Increasing longevity, which increases the available working time, is likely to lead to higher economic growth rates and facilitate economic take-off, but it should not reverse the process of

rate of technology and hence a low rate of return to human capital investment. Parents do not invest in children's human capital when the rate of return is too low.

In the corner solution with no human capital investment, the level of human capital equals the constant endowment at birth  $(h_t = h_0)$ , and new technologies will still be created each period according to the following:

$$\Delta A_t = \delta \left( h_0 N_t s T \right)^{\mu} A_t^{\phi} \quad . \tag{2}$$

However, no human capital investment implies that ideas created by one person cannot be efficiently learned and adopted by others. This is because human capital investment such as school education represents an efficient way for disseminating ideas that have been collected and documented between individuals who would have no intellectual contact otherwise. This view is in line with the understanding that education speeds up technology diffusion (Nelson and Phelps 1966). Without human capital investment, new ideas only spread slowly through informal observation but not through the more efficient formal learning. Therefore, although ideas are non-rivalrous, a growth in ideas does not lead to a corresponding growth in per capita output.

To show this in detail, we assume  $\Delta \tilde{A}_i$  is the amount of new ideas learned by the average agent and  $\pi = \Delta \tilde{A}_i / \Delta A_i$  is the share of new ideas learned, which measures the efficiency of technology diffusion. If the diffusion of technology is perfect, we have  $\pi = 1$  and  $\Delta \tilde{A}_i = \Delta A_i$ . In this case, the growth rate of per capita output is approximately equal to the growth rate of ideas. However, if no human capital investment is present and each agent can only learn by observing the production behavior of adults who have intellectual contact with them (mainly their parents), the amount of new ideas learned by the average agent is the following:

development that is explained in this paper. See Cervellati and Sunde (2005) for a model in which rising longevity explains the transition from Malthusian stagnation to a regime of significant growth.

$$\Delta \tilde{A}_{t} = \delta \left( h_{0} \tilde{N}_{t} s T \right)^{\mu} A_{t}^{\phi}, \qquad (2)$$

where  $\tilde{N}_t$  is the number of adults who have intellectual contact with the average agent. During Malthusian stagnation when there was no human capital investment,  $\tilde{N}_t$  could be negligible relative to  $N_t$ ; hence, the efficiency of technology diffusion was extremely low:

$$\pi = \Delta \tilde{A}_t / \Delta A_t = \frac{\delta \left( h_0 \tilde{N}_t s T \right)^{\mu} A_t^{\phi}}{\delta \left( h_0 N_t s T \right)^{\mu} A_t^{\phi}} = \left( \frac{\tilde{N}_t}{N_t} \right)^{\mu}.$$
(2)

The growth rate of per capita output is approximately equal to  $\pi$  times the growth rate of ideas:

$$g_{yt} \approx \frac{\Delta \tilde{A}_t}{A_t} \approx \frac{\pi \Delta A_t}{A_t} \approx \pi g_{At} \,. \tag{2}$$

Therefore, the economy was locked in Malthusian stagnation for thousands of years not only because of the extremely small growth rate of technology when the population growth rate was small, but also because of the extreme inefficiency of technology diffusion when there was no human capital investment. The virtuous circle of human capital accumulation was not functioning while the virtuous circle of population growth was limited by the inefficiency of technology diffusion. The growth of population and per capita output was extremely small (see evidence from Tables 1 and 4) and the growth of living standards was even smaller because the growth of per capita output was mainly reflected in the growth of the number of children.

Nevertheless, as long as individuals have intellectual contact with others (i.e.,  $\pi > 0$ ), the weak virtuous circle of population growth still gradually enhances the growth rate of the population, even during Malthusian stagnation. Specifically, although extremely small, a growth in ideas leads to a growth in per capita output, and a higher output allows parents to have more children, hence accelerating population growth, which in turn accelerates the growth of ideas. The prediction of accelerating population growth during Malthusian stagnation is consistent with long-run historical data. As presented in Table 1, although with significant fluctuations, an obvious increasing trend of

the growth rate of the world's population has been observed from 1,000,000 B.C. to 1750.<sup>14</sup>

Increasing population growth eventually triggers economic take-off. Inequation (2) is reversed when the growth rate of the population is high enough and when parents start to invest in the human capital of their children. Positive human capital investment activates the virtuous circle of human capital accumulation: human capital investment enhances the growth of technology, which in turn induces more human capital investment by raising the rate of return to investment. More importantly, it also strengthens the virtuous circle of population growth by improving the efficiency of technology diffusion. These two virtuous circles reinforce each other and generate the dramatic increasing growth of population, human capital, technology, and per capital output.

The historical data presented in Figure 1 support the model's predictions that a) increasing population growth triggers economic take-off, b) economic take-off is accompanied by the rising of human capital investment, and c) the per capita growth rate enters a long-run increasing trend after economic take-off. Specifically, the data show that a) significant accelerating growth of per capita real GDP occurred at the beginning of the nineteenth century, but the annual growth rate of the population had increased from 0.095 percent to 0.454 percent during the eighteenth century; b) years of total schooling by birth cohort had increased from 1.8 in 1800 to 6.0 in 1900 and to 11.3 in 1970; and c) the annual growth rate of per capita real GDP increased from 0.11 percent in 1800 to 1.55 percent in 1900 and to 4.68 percent in 1970.

#### B. Demographic transition, growth slowdown, and steady-state

Because the production and rearing of children are time-intensive, the growth rate of the population cannot increase forever. Utility-maximizing parents choose the optimal time allocation among investments in the quantity and quality of children and the production of consumer goods subject to time constraints (1). Increasing labor productivity generates an income effect that eases parents'

<sup>&</sup>lt;sup>14</sup> See various tests supporting the increasing trend of population growth rate from Kremer (1993).

budget constraints, allowing them to spend more time to have more children. On the other hand, increasing labor productivity raises the opportunity cost of time and generates a substitution effect that induces parents to shift time from investing in the quantity of children to the production of consumer goods. In addition, accelerating technological progress enhances the rate of return to human capital, hence generating a substitution effect that induces parents to shift time from investing in the quantity to the quality of their children. These two substitution effects grow over time and eventually overcome the income effect, leading to a declining population growth.

To see this formally, first assume human capital investment as a constant and focus only on the substitution effect of increasing labor productivity. At the beginning, the opportunity cost of time spent on children production (e) is relatively small, and the main cost is the fixed goods cost (f), such as expenses on food, clothing, and housing. The income effect dominates the substitution effect when time cost is low and the goods cost is high, as determined from

$$\frac{f}{e\beta A_{\iota}^{\beta}l_{\iota}^{\beta-1}k_{\iota}^{1-\beta}+f} > 1-\sigma \quad , \tag{2}$$

where  $e\beta A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta}$  measures the time cost. During this period, higher income allows parents to allocate more resources toward producing a larger number of children, so population growth accelerates. On the other hand, higher population growth accelerates the growth of technology and the growth of labor productivity. Continually rising labor productivity ( $\beta A_t^{\beta} l_t^{\beta-1} k_t^{1-\beta}$ ) eventually reverses the inequality in (2). As a result, parents choose to invest less in the quantity of children, so the growth rate of the population declines.

Therefore, even without the substitution effect of human capital investment, the endogenously growing opportunity cost of time will eventually lower the growth of the population. Nevertheless, the substitution effect of human capital investment still plays a significant role in shaping the path of economic development. The offsetting movement of the growth rates of population and human

capital caused by the substitution effect implies the growth rate of per capita output will continue to rise at least for a while after the decline of the population growth rate, as in Figure 1. To see this, we take the first-order derivative of equation (1) with respect to  $g_{At}$  and obtain:

$$\frac{dg_{At}}{dt} = \kappa \frac{dg_{ht}}{dt} + \kappa \frac{dg_{Nt}}{dt} - \kappa \mu^{-1} \frac{d\left(\Delta g_{At}/g_{At}\right)}{dt} \quad .$$
<sup>(2)</sup>

At  $t = t_2$ , which is the turning point of population growth rate (see Figure 3), we have  $dg_{ht_2}/dt_2 > 0$ and  $dg_{Nt_2}/dt_2 = 0$ . If  $g_{At}$  is not strictly convex at  $t_2$ , we have  $d(\Delta g_{At_2}/g_{At_2})/dt_2 \le 0$ . Therefore, we have  $dg_{At}/dt > 0$  at  $t_2$ . The inequality  $dg_{At}/dt > 0$  must hold for at least some small increases in time from  $t_2$ . Therefore, the growth rate of per capita output  $g_{yt}$ , which is approximately equal to  $g_{At}$ , will increase at least for a while after  $t_2$ .

However, the increasing trend of the income growth rate will eventually be reversed when the negative growth effect of the declining population growth overcomes the positive growth effect of increasing human capital growth. In addition, time invested in human capital will eventually level off considering the time constraint of individuals, and this will lead to a constant or declining growth of human capital. To see this, we write the growth rate of human capital as the following

$$g_{ht} = \begin{cases} \upsilon \frac{z_t}{h_t^{1-\gamma}} + \frac{h_0}{h_t} - 1 & \text{if } 0 < \gamma < 1 \\ \upsilon z_t + \frac{h_0}{h_t} - 1 & \text{if } \gamma = 1 \end{cases}$$
(2)

If  $0 < \gamma < 1$ , a constant investment  $(z_t = z)$  corresponds to a declining growth of human capital because  $h_t^{1-\gamma}$  is increasing. In this case, the slowdown of income growth is because of the declining growth of both the population and human capital. If choosing the knife-edge assumption  $\gamma = 1$ , a constant investment corresponds to a constant growth rate of human capital. In this case, the slowdown of income growth occurs when the negative growth effect of the declining population growth overcomes the positive growth effect of the increasing human capital growth.

The economy eventually converges to a steady-state growth path with a constant time invested in each child's human capital  $(z^*)$ , a constant number of children per parent  $(n^*)$ , and a constant growth rate of per capita output  $(g_y^*)$ . The steady-state values  $z^*$  and  $n^*$  are determined from the first-order conditions, as shown in equations (2) and (2). In the steady-state equilibrium, the time spent investing in each child's human capital is the following:

$$z^* = \frac{\beta \sigma e}{1 - \varepsilon - \beta \sigma} \tag{2}$$

The equilibrium education level of a child rises with the labor share of total output ( $\beta$ ), the elasticity of consumption ( $\sigma$ ), the fixed time cost of children rearing (e), and the elasticity of altruism per child ( $\varepsilon$ ). The steady-state number of children is found by substituting into equations (2) and (2):

$$\alpha^{-1} n^{*_{\varepsilon-1}} (1+g^*)^{1-\delta} = \upsilon [(1-s)T - en^*] \quad .$$
<sup>(2)</sup>

The steady-state growth rate of per capita output is equal to the growth rates of consumption, physical capital, and technology, and is proportional to the sum of the growth rates of the population and human capital:

$$g_{y}^{*} = g_{c}^{*} = g_{k}^{*} = g_{A}^{*} = \kappa \left( g_{h}^{*} + g_{N}^{*} \right) \quad .$$
<sup>(2)</sup>

The growth rate of the population is  $g_N^* = n^* - 1$ , and the growth rate of human capital is the following

$$g_{h}^{*} = \begin{cases} 0 & \text{for } 0 < \gamma < 1 \\ \upsilon z^{*} - 1 & \text{for } \gamma = 1 \end{cases}$$
(2)

For  $0 < \gamma < 1$ , the steady-state growth rate of human capital is zero, so the growth rate of per capita output depends only on the growth rate of the population. This prediction is in line with previous studies such as Judd (1985), Jones (1995a), and Segerstrom (1998). For  $\gamma = 1$ , the steady-state

growth rate of per capita output depends both on the growth rate of the population and the level of human capital investment.

The steady-state technological level is proportional to the size of the population and the level of human capital:

$$A_{t}^{*} = b \left( h_{t}^{*} N_{t}^{*} \right)^{\kappa} g_{A}^{*-\kappa \mu^{-1}} , \qquad (2)$$

and the steady-state per capita output is proportional to the level of technology:

$$y_t^* = A_t^{*\beta} l_t^{*\beta} k_t^{*(1-\beta)} = \left(\beta^{-1} - 1\right)^{1-\beta} l_t^{*\beta(1-\beta)} A_t^{*\beta(1-\beta)}$$
(2)

Therefore, the steady-state per capita output is proportional to the level of human capital and the size of the population.

# C. The path of economic development for countries sharing technology

To analyze the mechanism of economic development in the clearest fashion, the model of this paper assumes an economy that has no intellectual contact with the outside, so all technologies are invented domestically. In the real world, however, countries share technology. This subsection extends the model's prediction regarding the path of economic development to countries sharing technology. It shows that although international technology diffusion has significant effects on economic development, the model's prediction regarding the path of economic development can be extended to countries sharing technology.

For an economy that can import technology from outside, economic take-off no longer must be triggered by increasing domestic population growth. The model predicts that increasing population growth triggers economic take-off by accelerating technological progress. In this sense, events such as opening up to international trade and wars of conquest also have the potential to trigger economic take-off. This is because these events enable a country to access the technologies invented by a larger

population; this has similar effects on technological progress as increasing domestic population growth does.

Nevertheless, economic take-off is still accompanied by significant increases in human capital investment because it is a necessary condition for the learning and use of the imported technologies. The increasing population growth and the subsequent demographic transition will still be observed because of the income effect of economic growth and the substitution effect of human capital investment. However, because there is a large stock of pre-existing technology available for the less-developed countries to import, the population growth rates in these countries could be much higher because of the income effect generated by the imported technology. In addition, the imported technology also raises the rate of return to human capital investment and generates a higher substitution effect that may reduce the period of increasing population growth.

The growth rate of per capita output still increases first, then declines, and eventually converges to a steady-state. Historically, economic take-off first occurred in Western European countries and their offshoots at the beginning of nineteenth century. These countries shared technology and developed at a generally similar pace (see Figure 4), so they can be taken as a whole when analyzing the path of economic development. In addition, these countries were mainly the exporters of technology when interacting with the rest of the world after their economic take-off. Therefore, the mechanism of the model (the rule of ideas creation and the two virtuous circles) applies to these countries as a whole, and each shares the same path of per capita growth rate as predicted by the model.

For the second group of countries that "catch up" later, such as South Korea and Singapore, the path of per capita growth rate is deeply affected by the large stock of pre-existing technology invented by the first group of countries. The pre-existing technology enables these countries to grow faster and catch up. Because these countries take up a relatively small share of the world economy compared to the first group, they are mainly the importers of technology after their economic take-off. Consequently, the slowdown of technological progress in the first group inevitably leads to the slowdown of growth in the second group. Therefore, per capita growth in these countries still increases first and then declines before converging to the steady-state.

The last group of countries, including China, India, and other developing countries, have mainly been the importers of technology over the last two centuries. Similar to countries in the second group, they grew much faster than the first group because of the large stock of pre-existing technology. The difference is that these countries together will have a large economic size after they catch up, so they have the potential to reverse the trend of the global technological growth rate in the future. If this happens, the growth rates in countries of the first and second groups will rebound before declining again. Nevertheless, the growth rates in these countries will still decline and eventually converge to a steady-state as determined by the mechanism proposed in this paper.

To sum up, the model's prediction regarding the path of economic development can be extended to countries sharing technology. Specifically, each country generally develops along the path of Malthusian stagnation, economic take-off, demographic transition, growth slowdown, and steady-state. The only possible exception is that if the currently developing countries become leaders in technological progress in the future, the declining per capita growth rates in currently developed countries will rebound before declining again.

# D. Discussion

This subsection briefly compares the model's predictions of Malthusian stagnation, economic take-off, and steady-state with the literature. First, the current paper explains Malthusian stagnation by the extremely slow growth of technology and the inefficiency of technology diffusion, but previous studies usually follow Thomas Malthus (1798) to explain Malthusian stagnation by the existence of a fixed factor of production that reduces labor productivity when the population grows. The current model assumes no fixed physical capital because it is designed to explain both

Malthusian stagnation and the modern growth regimes and physical capital is unlikely fixed in modern production. Nevertheless, extending the model to include fixed production capital does not substantially alter our explanation of Malthusian stagnation and economic take-off. To see this, assume that the physical capital in production function (1) is in fixed supply ( $K_t = K$ ), so per capita output becomes the following:

$$Y_{t} / N_{t} = y_{t} = A_{t}^{\beta} l_{t}^{\beta} \left(\frac{K}{N_{t}}\right)^{1-\beta} = \frac{A_{t}^{\beta}}{N_{t}^{1-\beta}} l_{t}^{\beta} K^{1-\beta} = B_{t}^{\beta} l_{t}^{\beta} K^{1-\beta} , \qquad (2)$$

where  $B_t^{\beta} = A_t^{\beta} / N_t^{1-\beta}$ . Compared to equation (1), the main difference is that the growth effect of technological progress is partly offset by the dilution effect of population growth:

$$g_{Bt} = \beta g_{At} - (1 - \beta) g_{Nt} \quad , \tag{2}$$

in which  $\beta_{g_{At}}$  measures the positive growth effect of technological progress while  $-(1-\beta)g_{Nt}$ measures the negative dilution effect of population growth. Therefore, the existence of fixed production capital further stabilizes Malthusian stagnation but does not change the explanation of economic take-off as long as the virtuous circle of population growth is still functioning (i.e.,  $\beta\kappa > 1-\beta$ ).

Second, the current paper indicates that economic take-off is triggered by the endogenous rising of the growth rate of the population while Galor and Weil (2000) and others believe that it is triggered by the rising level of population. This difference arises directly from the understanding of the effect of population growth on technological progress. The current paper assumes, other things being equal, the number of new ideas discovered is proportional to the size of the population, but Galor and Weil (2000) assumes the growth rate of ideas is proportional to the size of the population. This difference is clearly reflected in choosing the value of  $\phi$  in equation (1). We assume  $\phi < 1$ ; therefore, the number of new ideas discovered is proportional to the size of the population. By shifting to the knife-edge assumption that the new ideas discovered are strictly linear to the stock of ideas ( $\phi = 1$ ), we obtain that the growth rate of ideas is proportional to the size of the population:

$$g_{At} = \frac{\Delta A_t}{A_t} = \delta \left( h_t N_t s T \right)^{\mu} \quad . \tag{2}$$

The current paper follows Jones (1995a) to assume  $\phi < 1$ , which allows the rate of innovation to decrease with ( $\phi < 0$ ), increase with ( $0 < \phi < 1$ ), or be independent of ( $\phi = 0$ ) existing technology. This is mainly because empirical evidence from developed countries over the last half century strongly rejects the assumption that the growth rate of technology is proportional to the size of the population (Jones 1995b). Interestingly, the unified growth model of Galor and Weil (2000) can be extended to include the growth slowdown if we relax the knife-edge assumption that the new ideas discovered are strictly linear to the stock of ideas.

Finally, the current paper indicates the modern history of economic growth reflects the transition dynamics rather than the steady-state, but previous theoretical models are nearly always constructed so as to generate a steady-state growth path. The current paper predicts that each economy develops along the path characterized by Malthusian stagnation, economic take-off, demographic transition, and growth slowdown, and eventually converges to the steady-state when the population growth rate and human capital investment are constant. Empirical data from each of the 18 advanced OECD countries (see Section IV.A) show that the population growth rate is still declining and that human capital investment is still increasing. Therefore, consistent with the declining trend of per capita output growth rate observed currently (see Table 3), the current paper indicates that even the most developed countries are not in their steady-state yet and their future growth could be slower.<sup>15</sup>

<sup>&</sup>lt;sup>15</sup> A conventional view of the U.S. economy is that it is close to its long-run steady-state balanced growth path. However, the evidence presented in Section IV together with other recent studies show that after filtering out fluctuations caused by business cycles and other shocks, there is a significant increasing trend of per capita growth rate before the 1970s and a significant declining trend after (See, for example, Gordon 2012, Antolin-Diaz, Drechsel, and Petrella 2017).

#### **IV. Empirical evidence**

This section extends the empirical evidence presented in Figure 1 to each of the following 18 advanced OECD countries: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the United Kingdom, and the United States.<sup>16</sup> Subsection A visually presents the country-level dynamics of population growth rate, income growth rate, and human capital investment from 1800–2015. Subsection B econometrically identifies the ongoing postwar growth slowdown and the early stylized facts of Malthusian stagnation and accelerating per capita growth.

#### A. An overview of long-run economic development

As presented in Figure 4, although there are noticeable fluctuations during certain periods such as World War II, the growth of per capita real GDP took off from an extremely low level in the first half of the nineteenth century and experienced long-term increases until a significant growth slowdown occurred in the second half of the twentieth century. These observations support the model's prediction that each economy develops along the path characterized by Malthusian stagnation, economic take-off, accelerating growth, and growth slowdown.

As presented in Figure 5, historical data generally support the prediction that population growth rates first increase and then decline in each country. Contrary to income growth rates that peaked around the same time for each country, population growth rates peaked at several different time periods across the countries. This could be explained by the fact that countries share technology: income growth rates are determined by the overall growth rate of technology of the countries sharing technology, but population growth rates are also influenced by other country-specific factors, such as the relative cost of the production and rearing of children.

<sup>&</sup>lt;sup>16</sup> The empirical examination focuses only on the 18 advanced OECD countries in which data for population and income are available at least since 1800 and that had a population larger than 1 million in 1800. Countries with a small population are excluded from the analysis because their paths of development are more vulnerable to exogenous shocks.

The evidence presented in Figure 6 is consistent with the model's prediction that human capital investment is increasing over time after economic take-off. We measure the level of human capital investment by the average years of total schooling for the population aged 25 and over. The average years of total schooling have experienced dramatic increases from 1870–2010 in each of the 18 OECD countries. For example, it increased from 0.4 to 12.5 in Japan and from 3.7 to 13.6 in the United States from 1870–2010.

# B. Malthusian stagnation, accelerating growth, and growth slowdown

To confirm the slowdown of per capita output growth observed in Figure 4, this subsection adopts a Sup Wald test (Andrews 1993) to determine if and when a statistically significant structural break occurred in each country after World War II.<sup>17</sup> We then use various methods to test that per capita growth rates indeed declined significantly after the break. Besides testing the growth slowdown, this subsection also uses income data from 1–2015 A.D. to confirm the stylized facts of Malthusian stagnation and accelerating growth that characterize the vast majority of human history before the slowdown.

Testing the unit root hypothesis is necessary before implementing the Sup Wald test of Andrews (1993), which does not allow for unit roots. The unit root hypothesis is tested by the method developed in Perron (1994), which extends the augmented Dickey-Fuller (ADF) procedure to reduce the potential bias caused by the misspecification of the deterministic trend.<sup>18</sup> Briefly, Perron (1994) extends the ADF test by allowing for unknown break dates and assuming different intercepts and slopes before and after each possible break date. The maximum ADF *t*-statistic is obtained from a range of possible break dates. The null hypothesis of a unit root is rejected if the

<sup>&</sup>lt;sup>17</sup> This test focuses on the postwar periods (1946–2015) because its purpose is to confirm the postwar slowdown as presented in Figure 4. Including data before 1945 may lead us to identify multiple breaks reflecting the significant increases in growth rates.

<sup>&</sup>lt;sup>18</sup> It is well-known that non-rejection of the unit root hypothesis can be caused by a misspecification of the deterministic trend, which is quite possible in our case considering the presence of structural breaks with unknown break dates.

maximum ADF *t*-statistic is greater than the appropriate critical value. According to Perron (1994), the critical value for rejecting the unit root hypothesis at 1 percent level is 4.32. The maximum *t*-statistic estimated for each country is reported in column (1) of Table 2. We find that the unit root hypothesis can be rejected at the 1 percent significance level for each country.

We now proceed to the Sup Wald break test. Columns (2) and (3) of Table 2 report the break years and Sup Wald statistics estimated by the Sup Wald test. Each Sup Wald statistic is the maximum value of the test statistic obtained from a series of Wald tests over a range of possible break dates in the sample. Let *b* denote a possible break date in the range  $[b_1, b_2]$  for a sample size *T*. The Sup Wald statistic for testing the null hypothesis of no structural change in the time trend is given by the following:

supremum 
$$S_T = \sup_{b_1 \le b \le b_2} S_t(b)$$
, (2)

in which  $S_t(b)$  is the Wald test statistic testing  $H_0: \rho_1 = \rho_2$  in

$$g_{t} = \begin{cases} \eta + \rho_{1}t + \vartheta_{t}, \ t = b_{1}, b_{1} + 1, \dots, b \\ \eta + \rho_{2}t + \vartheta_{t}, \ t = b + 1, \dots, b_{2} - 1, b_{2} \end{cases}$$
(2)

The *p*-value indicating the significance level for each Sup Wald test is computed using the method in Hansen (1997). Statistically significant breaks are identified in 17 of the 18 OECD countries, and the breaks are mainly during 1970–1976, and only the break identified in the United Kingdom is statistically insignificant.<sup>19</sup>

Examining the time trend is necessary to confirm that the growth rate is declining after the break. Column (4) reports the ordinary least square (OLS) estimate of the time trend after the break.<sup>20</sup> A

<sup>&</sup>lt;sup>19</sup> Identifying breaks using the modified Sup Wald test developed by Vogelsang (1997), which permits unit root errors, also finds a significant postwar break for each country, except for the United Kingdom, but with a slightly different break year. Here, we report only the results estimated by the method of Andrews (1993) considering the rejection of the unit root hypothesis and the popularity of the method.

<sup>&</sup>lt;sup>20</sup> This paper tests only the slowdown in the growth rate of per capita real GDP because of the availability of long-run historical data. It is worth pointing out that, as predicted by this model, empirical evidence shows that the declining per capita output growth is mainly because of the declining productivity growth (Shigehara 1992, Baily and Montalbano

statistically significant declining trend is found in each country. As a robustness check, column (5) provides the generalized least squares (GLS) estimate of the time trend, allowing for serial correlation, calculated using the Cochrane-Orcutt procedure (Cochrane and Orcutt 1949). The results are generally robust, and only the declining trend in the United Kingdom becomes statistically insignificant. Column (6) provides another robustness check to deal with the potential bias caused by outliers in the data using the robust regression developed in Berk (1990). Briefly, the robust regression drops the most influential data points and down-weights the data points with large absolute residuals. The estimates reported in column (6) are quite similar to those in column (4).

Although a statistically significant declining trend is found in each of the 18 OECD countries, the actually decline in growth rates could be economically negligible. If this is true, we cannot say the growth slowdown is economically important. To address this concern, column (7) provides the changes in growth rates for each country by comparing the 2006–2015 average with the 1981–1990 average. Quantitatively significant changes in growth rates are found in each country. For example, 12 countries experienced more than 2 percent declines in growth rates, and only two countries experienced less than 1 percent declines. In addition, the changes in growth rates are statistically significant in each country.

Table 2 only supports that a statistically and economically significant postwar growth slowdown has occurred in each country. To fully support the model's prediction, we still need to show that the slowdown after the break is persistent. To do so, we divide the post-break period into four 10-year subperiods (1976–1985, 1986–1995, 1996–2005, and 2006–2015) and estimate the time trend in each subperiod. As reported in Table 3, statistically significant declining trends are found in most countries during each subperiod, except for four countries from 1976–1985, three countries from 1996–2005, and two countries from 2006–2015. We also check the robustness of these estimates to

<sup>2016).</sup> Previous studies find similar evidence of slowing growth by examining the growth rate of real GDP per hour (Duernecker, Herrendorf, and Valentinyi 2016), aggregate total factor productivity (Byrne, Fernald, and Reinsdorf 2016), and labor productivity (Baily and Montalbano 2016).

serial correlation and outliers and find similar results (not reported here). Therefore, as predicted by the model, the growth slowdown after the break is persistent.

We now proceed to empirically examine the model's prediction about Malthusian stagnation and accelerating growth that characterized the vast majority of human history before the growth slowdown. As shown in Figure 4, the growth rate of per capita real GDP significantly rose from an extremely low level in the first half of the nineteenth century and then experienced long-term increases before the slowdown occurred. We confirm these observations by estimating time trends of the growth rates from 1–1800, 1801–1900, and 1991–break.<sup>21</sup>

Column (1) of Table 4 reports the time trend from 1–1800, which is a period before significant economic take-off occurred and hence belongs to Malthusian stagnation. Consistent with the model's prediction, the growth rate of per capita real GDP was increasing at an extremely small pace during Malthusian stagnation. Although the positive time trend is statistically significant for each country from 1–1800, the actual increases in growth rates were generally less than 0.0002 percent per year. In other words, the growth rate had only increased less than 0.2 percent when 1,000 years elapsed. Per capital income was stagnant during this period considering the extremely small growth rate and the extremely small changes in growth rate.

The data also support the prediction of accelerating growth after economic take-off and before the growth slowdown. As shown in column (2) of Table 4, the time trend of growth rate from 1801– 1900 had generally increased to more than 0.01 percent per year, which was 50 times of that from 1– 1800. The growth accelerated further after 1800. As presented in column (3), the trend continued to rise from 1901–break in each country. For example, the time trend in Austria from 1901–break was 0.079 percent per year, which was more than six times of that from 1801–1900. The accelerating growth results in the dramatic increases in the growth rates just over less than two centuries. For

<sup>&</sup>lt;sup>21</sup> The data prior to the nineteenth century are only available for the year 1, 1000, 1500, 1600, and 1700, so we did not estimate time trends for the subperiods of 1-1800.

example, the yearly growth rate in Austria raised from 0.166 percent in 1800 to 1.821 percent in 1900 and to 4.868 percent in 1970.

# **V. Concluding Remarks**

The slowdown in per capita output growth has been observed in virtually all industrialized countries since the early 1970s, but no persuasive theoretical explanation for this phenomenon has been given. The current paper argues that the postwar growth slowdown, which has lasted for more than 40 years, is because of the fundamental mechanism of long-run economic growth, not any transitory exogenous shocks. Therefore, for a persuasive theoretical explanation, the postwar growth slowdown should be reconciled with other early stylized facts that reflect the same mechanism of long-run economic growth.

Depending heavily on the insights of Romer (1990), Becker, Murphy, and Tamura (1990), Jones (1995a), and Galor and Weil (2000), the current paper explains the postwar growth slowdown using the endogenous interactions among the growth of the population, human capital, technology, and income. These endogenous interactions are guided by the natural rule of ideas creation that, given the percentage of people engaged in R&D, the number of new ideas discovered is proportional to the size of the population and the level of human capital of each person. The rule implies that the growth rate of per capita output is an increasing function of the growth rates of the population and human capital. The endogenously determined slowdown of population growth leads to the slowdown of per capita output growth.

To explain the postwar growth slowdown, the current paper develops a modified endogenous growth model that wholly explains Malthusian stagnation, economic take-off, demographic transition, and growth slowdown. Specifically, the model predicts that each economy develops along a path characterized by Malthusian stagnation, economic take-off, demographic transition, slowdown of growth, and steady-state. Therefore, the slowdown of per capita output growth is part of the natural

process of economic development.

The model implies that even the currently most developed countries are not in their steady-states of long-run growth yet. Growth models are nearly always constructed to generate a steady-state growth path. However, the model of the current paper and the empirical evidence over the last two centuries indicates that the modern history of economic growth reflects the transition dynamics rather than the steady-state. The model predicts that an economy eventually converges to a steady-state with a constant growth rate of the population and a constant level of human capital investment. The declining growth of population, increasing human capital investment, and declining growth of per capita output in the 18 OECD countries indicate that none of these countries is in their steady-state yet, and their future growth could be slower.

# Reference

Andrews, Donald WK. 1993. "Tests for parameter instability and structural change with unknown change point." *Econometrica: Journal of the Econometric Society*:821-856.

Antolin-Diaz, Juan, Thomas Drechsel, and Ivan Petrella. 2017. "Tracking the Slowdown in Long-Run GDP Growth." *The Review of Economics and Statistics* 99 (2):343-356.

- Baily, Martin Neil, and Robert Gordon. 1988. "The Productivity Slowdown, Measurement Issues, and the Explosion of Computer Power." *Brookings Papers on Economic Activity* 19 (2):347-432.
- Baily, Martin Neil, and Nicholas Montalbano. 2016. "Why is US productivity growth so slow? Possible explanations and policy responses." *Hutchins Center Working Papers*.
- Barro, Robert J, and Jong Wha Lee. 2013. "A new data set of educational attainment in the world, 1950–2010." *Journal of development economics* 104:184-198.
- Becker, Gary S, Kevin M Murphy, and Robert Tamura. 1990. "Human Capital, Fertility, and Economie Growth." *Journal of Political Economy* 98 (5):S12-S37.
- Berk, Richard A. 1990. A Primer on Robust Regression. S. 292–324 in: John Fox und J. Scott Long (Hg.): Modern Methods of Data Analysis. Newbury Park, CA: Sage.
- Bishop, John H. 1989. "Is the test score decline responsible for the productivity growth decline?" *The American Economic Review*:178-197.
- Byrne, David M, John G Fernald, and Marshall B Reinsdorf. 2016. "Does the United States have a productivity slowdown or a measurement problem?" *Brookings Papers on Economic Activity* 2016 (1):109-182.
- Cervellati, Matteo, and Uwe Sunde. 2005. "Human capital formation, life expectancy, and the process of development." *American Economic Review*:1653-1672.
- Cochrane, Donald, and Guy H Orcutt. 1949. "Application of least squares regression to relationships containing auto-correlated error terms." *Journal of the American statistical association* 44 (245):32-61.
- Duernecker, Georg, Berthold Herrendorf, and Akos Valentinyi. 2016. "Unbalanced Growth Slowdown." Manuscript, University of Mannheim, Arizona State University, and Cardiff Business School.
- Easterlin, Richard A. 1981. "Why isn't the whole world developed?" *The Journal of Economic History* 41 (01):1-17.
- Fischer, Stanley. 1988. "Symposium on the Slowdown in Productivity Growth." *Journal of Economic Perspectives* 2 (4):3-7.
- Foster, Andrew D, and Mark R Rosenzweig. 1996. "Technical change and human-capital returns and investments: evidence from the green revolution." *The American economic review*:931-953.
- Galor, Oded, and David N Weil. 2000. "Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond." *American economic review*:806-828.
- Gordon, Robert J. 2012. "Is US economic growth over? Faltering innovation confronts the six headwinds." *NBER Working Paper No. 18315.*
- Gordon, Robert J. 2013. "US productivity Growth: The Slowdown has returned after a temporary revival." International Productivity Monitor (25):13.
- Gordon, Robert J. 2016. *The rise and fall of American growth: The US standard of living since the civil war:* Princeton University Press.
- Grossman, Gene M, and Elhanan Helpman. 1991. "Quality ladders in the theory of growth." *The Review of Economic Studies* 58 (1):43-61.
- Hansen, Bruce E. 1997. "Approximate asymptotic p values for structuras-change tests." *Journal of Business* & *Economic Statistics* 15 (1):60-67.
- Jones, Charles I. 1995a. "R & D-based models of economic growth." *Journal of political Economy*:759-784.
- Jones, Charles I. 1995b. "Time series tests of endogenous growth models." *The Quarterly Journal of Economics*:495-525.
- Jones, Charles I. 2002. "Sources of US economic growth in a world of ideas." *American Economic Review*:220-239.

- Jorgenson, Dale W. 1988. "Productivity and Postwar U.S. Economic Growth." Journal of Economic Perspectives 2 (4):23-41.
- Judd, Kenneth L. 1985. "On the performance of patents." *Econometrica: Journal of the Econometric Society*:567-585.

King, Robert G, and Sergio T Rebelo. 1993. "Transitional Dynamics and Economic Growth in the Neoclassical Model." *The American Economic Review*:908-931.

- Kremer, Michael. 1993. "Population growth and technological change: one million BC to 1990." *The Quarterly Journal of Economics*:681-716.
- Kuznets, Simon. 1960. "Population change and aggregate output." In *Demographic and economic change in developed countries*, 324-351. Columbia University Press.
- Lee, Jong-Wha, and Hanol Lee. 2016. "Human capital in the long run." *Journal of Development Economics* 122:147-169.
- Lee, Ronald. 1980. "A historical perspective on economic aspects of the population explosion: The case of preindustrial England." In *Population and economic change in developing countries*, 517-566. University of Chicago Press.
- Lucas Jr, Robert E. 1988. "On the mechanics of economic development." *Journal of Monetary Economics* 22 (1):3-42.
- Maddison, Angus. 1982. Phases of capitalist development: Oxford University Press, USA.
- Maddison, Angus. 1987. "Growth and slowdown in advanced capitalist economies: techniques of quantitative assessment." *Journal of economic literature* 25 (2):649-698.
- Maddison, Angus. 2007. Historical Statistics for the World Economy: 1-2003 AD. edited by Angus Maddison. <u>http://www.historicalstatistics.org/:</u> Historicalstatistics.org.
- Malthus, Thomas Robert. 1798. An essay on the principle of population. Cambridge: W. Pickering, 1986.
- Matthews, Robert Charles Oliver, Charles Hilliard Feinstein, and John Odling-Smee. 1982. British economic growth 1856-1973: the post-war period in historical perspective: Oxford University Press.
- Nelson, Richard R, and Edmund S Phelps. 1966. "Investment in humans, technological diffusion, and economic growth." *The American economic review* 56 (1/2):69-75.
- Perron, Pierre. 1994. "Trend, unit root and structural change in macroeconomic time series." In *Cointegration*, 113-146. Springer.
- Phelps, Edmund S. 1966. "Models of technical progress and the golden rule of research." *The Review of Economic Studies*:133-145.
- Rebelo, Sergio. 1991. "Long-Run Policy Analysis and Long-Run Growth." *The Journal of Political Economy* 99 (3):500-521.
- Romer, Paul M. 1986. "Increasing returns and long-run growth." *The journal of political economy*:1002-1037.
- Romer, Paul M. 1990. "Endogenous technological change." *Journal of political Economy*:S71-S102.
- Schultz, Theodore William. 1964. *Transforming traditional agriculture*. New Haven: Yale University Press.

Segerstrom, Paul S. 1998. "Endogenous growth without scale effects." American Economic Review:1290-1310.

- Shigehara, Kumiharu. 1992. "Causes of declining growth in industrialized countries." *Policies for Long-Run Economic Growth*:15-39.
- Simon, Julian Lincoln. 1977. *The economics of population growth*. Princeton: NJ: Princeton University Press.
- Vogelsang, Timothy J. 1997. "Wald-type tests for detecting breaks in the trend function of a dynamic time series." *Econometric Theory* 13 (06):818-848.

Williams, Chris. 2006. A Companion to 19th-Century Britain: John Wiley & Sons.



Figure I

#### The Growth Rates of Population and per Capita Real GDP and the Years of Total Schooling by Birth

# Cohort for 12 Western European Countries as a Whole (1700–2015)

*Note*: The 12 Western European countries are Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Norway, Sweden, Switzerland, and the United Kingdom. The growth rates of population and per capita real GDP are derived from Maddison (2007) (before 1950) and the Conference Board Total Economy Database (2015) (after 1950). Growth rates are calculated as 30-year simple moving average for the 12 countries as a whole. Using overall measures from these 12 geographically connected countries helps reduce measurement errors that arise from migration and border changes. Years of schooling by birth cohort after 1880 are the average of the 12 countries and are calculated by the Education Attainment by Age Group, provided by Barro and Lee (2013). Years of schooling by birth cohort before 1880 are only for the United Kingdom and derived from Table E.1 of Matthews, Feinstein, and Odling-Smee (1982) and Table 17.3 of Williams (2006).



Figure II

Two Virtuous Circles That Generate the Fascinating History of Economic Development



Figure III

Predicted Long-run Movements of the Population Growth Rate, the Human Capital Investment, and the per Capita Output Growth Rate

Year	Population (million)	Growth rate	Comments
-1,000,000	0.125	0.00000297	
-300,000	1	0.00000439	
-25,000	3.34	0.000031	
-10,000	4	0.000045	
-5000	5	0.000336	
-4000	7	0.000693	
-3000	14	0.000657	
-2000	27	0.000616	
-1000	50	0.001386	
-500	100	0.001352	
-200	150	0.000623	
1	170	0.000559	
200	190	0.0	
400	190	0.000256	
600	200	0.000477	
800	220	0.000931	
1000	265	0.001886	
1100	320	0.001178	
1200	360	0.0	Mongol Invasions
1300	360	-0.0002817	Black Death
1400	350	0.001942	
1500	425	0.002487	
1600	545	0.0	30 years war, Ming Collapse
1650	545	0.002253	·
1700	610	0.003316	
1750	720	—	

Table IWorld Population Growth: 1,000,000 B.C. to 1700

Note: The growth rate listed for period t is the average growth rate from t to t+1. The data are derived from Table I of Kremer (1993).





#### The Growth Rates of per Capita Real GDP for the 18 Advanced OECD Countries (1800–2015)

*Note*: The annual data are derived from Maddison (2007) (before 1950) and Conference Board Total Economy Database (2016) (after 1951). Data before 1870 are available only for some years, and the continuous yearly measures are generated by linear interpolation. Growth rates are calculated as 30-year simple moving averages to reduce fluctuations associated with business cycles, wars, and other disturbances.





# The Growth Rates in Population for the 18 High-income OECD Countries (1800–2015)

*Note:* The annual data are derived from Maddison (2007) (before 1950) and Conference Board Total Economy Database (2016) (after 1951). Data before 1870 are available only for some years, and the continuous yearly measures are generated by linear interpolation. Growth rates are calculated as 30-year simple moving averages.



Figure VI

Average Years of Total Schooling for the Population Aged 25 and Over (1870–2010)

*Note:* Average years of total schooling for the population aged 25 and over are used to measure the level of human capital investment. The data with 5-year intervals come from the dataset of Lee and Lee (2016).

Break test		Time	Time trend after break year (break–2015)				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Country	Unit root	Break	$\sup S_{\tau}$	OLS	GLS	Robust	Difference
	test	Year	1 1			Regression	in means
Austria	5.82***	1975	17.8***	-0.26***	-0.38***	-0.13***	-4.4***
				(0.03)	(0.11)	(0.00)	(0.44)
Belgium	6.11***	1970	32.2***	-0.11***	-0.10***	-0.11***	-2.6***
C				(0.00)	(0.01)	(0.00)	(0.11)
Canada	5.18***	1961	14.8**	-0.06***	-0.06***	-0.07***	-1.85***
				(0.00)	(0.01)	(0.00)	(0.10)
Denmark	5.74***	1970	32.2***	-0.09***	-0.10***	-0.09***	-2.23***
				(0.00)	(0.01)	(0.00)	(0.15)
Finland	6.37***	1969	23.4***	-0.12***	-0.11***	-0.12***	-3.34***
				(0.01)	(0.01)	(0.01)	(0.24)
France	5.78***	1975	54.5***	-0.16***	-0.17***	-0.15***	-3.57***
				(0.01)	(0.03)	(0.01)	(0.18)
Germany	6.97***	1975	20.8***	-0.24***	-0.32***	-0.13***	-4.35***
				(0.02)	(0.07)	(0.01)	(0.53)
Greece	5.82***	1976	123.8***	-0.31***	-0.30***	-0.31***	-7.45***
				(0.01)	(0.04)	(0.02)	(0.59)
Italy	4.34***	1976	82.0***	-0.23***	-0.24***	-0.23***	-5.66***
				(0.01)	(0.02)	(0.01)	(0.37)
Japan	5.62***	1975	44.3***	-0.58***	-0.56***	-0.58***	-13.58***
				(0.03)	(0.08)	(0.03)	(0.67)
Netherlands	5.8***	1976	52.3***	-0.08***	-0.10***	-0.07***	-1.71***
				(0.00)	(0.02)	(0.00)	(0.16)
Norway	4.96***	1970	29.9***	-0.09***	-0.08***	-0.09***	-2.82***
				(0.01)	(0.02)	(0.01)	(0.20)
Portugal	4.45***	1970	21.2***	-0.19***	-0.15***	-0.19***	-6.01***
				(0.01)	(0.03)	(0.01)	(0.28)
Spain	5.36***	1966	36.9***	-0.15***	-0.09*	-0.22***	-6.02***
				(0.02)	(0.05)	(0.01)	(0.30)
Sweden	4.54***	1960	22.1***	-0.08***	-0.06***	-0.08***	-1.67***
				(0.00)	(0.01)	(0.00)	(0.06)
Switzerland	5.19***	1975	35.8***	-0.06***	-0.06***	-0.06***	-1.48***
				(0.01)	(0.02)	(0.01)	(0.13)
United	5.94***	1975	9.6	-0.01**	-0.01	-0.01*	-0.26**
Kingdom				(0.00)	(0.01)	(0.00)	(0.12)
	5.07***	1960	29.1***	-0.03***	-0.03*	-0.04***	-0.79***
United States				(0.01)	(0.02)	(0.00)	(0.15)

Table II The Slowdown of per Capita Growth

Note: 1. Column (1) reports the maximum ADF *t*-statistics estimated by the method of Perron (1994). The critical value for rejecting the unit root hypothesis at 1 percent is 4.32. 2. In columns (2) and (3), the break years and Sup Wald statistics are estimated using the method of Andrews (1993), and the *p*-value indicating the significant level for each test is computed using the method in Hansen (1997). 3. Column (4) reports the ordinary least square estimate of the time trend  $\rho$  from the regression  $g_t = \eta + \rho t + \vartheta_t$ . Column (5) reports the generalized least square estimate of  $\rho$  that is robust to serial correlation, and column (6) reports the estimate of  $\rho$  that is robust to outliers and estimated by the robust regression developed by Berk (1990). 4. In column (7), the difference in means is calculated as the 2006–2015 average minus the 1980–1989 average. 5. Standard errors are reported in parentheses. Significance levels are \*\*\* p<0.01, \*\* p<0.05, and \* p<0.10.

	Subperiod time trend after the break year			
-	(1) 1976–1985	(2) 1986–1995	(3) 1996–2005	(4) 2006–2015
Austria	-1.14***	-0.17***	-0.17***	-0.08***
	(0.18)	(0.01)	(0.02)	(0.02)
Belgium	-0.16***	-0.12**	-0.15***	-0.05***
	(0.02)	(0.04)	(0.01)	(0.01)
Canada	-0.03	-0.12***	-0.05 **	-0.03
	(0.03)	(0.03)	(0.02)	(0.02)
Denmark	-0.08**	-0.20***	-0.06***	-0.12***
	(0.03)	(0.02)	(0.02)	(0.01)
Finland	-0. 09***	-0.35***	-0.10***	-0.21***
	(0.02)	(0.06)	(0.03)	(0.03)
France	-0.36***	-0.18***	-0.12***	-0.07***
	(0.04)	(0.03)	(0.01)	(0.01)
Germany	-0.97***	-0.21***	-0.13***	-0.03
	(0.07)	(0.01)	(0.01)	(0.02)
Greece	-0.64***	-0. 43***	-0.15***	-0.28***
	(0.10)	(0.04)	(0.04)	(0.02)
Italy	-0. 53***	-0.32***	-0.14***	-0.18***
	(0.04)	(0.03)	(0.02)	(0.02)
Japan	-1.00***	-0.81***	-0.45***	-0.14***
	(0.06)	(0.06)	(0.07)	(0.02)
Netherlands	-0.33***	-0.07**	-0.09***	-0.05***
	(0.04)	(0.02)	(0.01)	(0.01)
Norway	-0. 08**	-0.10***	-0.12***	-0.20***
	(0.03)	(0.01)	(0.01)	(0.02)
Portugal	-0.04	-0.22**	-0.34***	-0.19***
	(0.07)	(0.08)	(0.04)	(0.02)
Spain	-0.23***	-0.30**	-0.24***	-0.15***
	(0.06)	(0.12)	(0.02)	(0.01)
Sweden	-0.10***	-0.24***	0.00	-0.03*
	(0.02)	(0.02)	(0.00)	(0.02)
Switzerland	-0. 08***	-0.13***	-0.04**	-0.03**
	(0.02)	(0.02)	(0.02)	(0.01)
United Kingdom	-0.02	-0.06**	0.03**	-0.07***
	(0.01)	(0.02)	(0.01)	(0.02)
United States	-0.04	-0.06**	0.02	-0.08***
	(0.04)	(0.02)	(0.02)	(0.02)

Table III	
The Ongoing Slowdown in per Capita Output G	rowth

Note: This table reports the ordinary least square estimates of the time trend  $\rho$  from the regression  $g_t = \eta + \rho t + \vartheta_t$  for each country and each time period. Standard errors are reported in parentheses, and significance levels are \*\*\* p<0.01, \*\* p<0.05, and \* p<0.10.

	Time trend in per capita output growth rate before the growth slowdown				
Country	(1)	(2)	(3)		
	1-1800	1801–1900	1901–Break		
Austria	0.000117***	0.0132***	0.0793***		
	(1.54e-06)	(0.000711)	(0.0143)		
Belgium	0.000117***	0.0245***	0.0446***		
	(2.12e-06)	(0.00179)	(0.00666)		
Canada	0.000127***	0.0162***	0.0130*		
	(6.58e-06)	(0.000764)	(0.00704)		
Denmark	0.000125***	0.0167***	0.0298***		
	(1.71e-06)	(0.000652)	(0.00485)		
Finland	8.99e-05***	0.0158***	0.0707***		
	(1.94e-06)	(0.000925)	(0.00438)		
France	0.000111***	0.0141***	0.0763***		
	(1.71e-06)	(0.00116)	(0.0122)		
Germany	0.000103***	0.0178***	0.0351***		
	(1.38e-06)	(0.000690)	(0.00860)		
Greece	8.75e-05***	0.0132***	0.0821***		
	(1.53e-06)	(0.000642)	(0.0203)		
Italy	0.000107***	0.00551***	0.0708***		
	(4.25e-06)	(0.000682)	(0.0116)		
Japan	4.92e-05***	0.0136***	0.0989***		
	(1.00e-06)	(0.00119)	(0.0183)		
Netherlands	0.000188***	0.0142***	0.0647***		
	(6.88e-06)	(0.000744)	(0.0105)		
Norway	7.02e-05***	0.0213***	0.0613***		
	(1.07e-06)	(0.00112)	(0.00358)		
Portugal	9.21e-05***	0.00589***	0.0862***		
	(1.54e-06)	(0.000822)	(0.00772)		
Spain	9.59e-05***	0.0162***	0.0682***		
	(2.33e-06)	(0.00103)	(0.0109)		
Sweden	0.000120***	0.0206***	0.0655***		
	(1.60e-06)	(0.00128)	(0.00389)		
Switzerland	0.000115***	0.0293***	0.0482***		
	(1.53e-06)	(0.00118)	(0.00509)		
United Kingdom	0.000172***	0.0192***	0.0184***		
	(2.65e-06)	(0.00160)	(0.00221)		
United States	0.000186***	0.0196***	0.0162***		
	(8.10e-06)	(0.000827)	(0.00578)		

Table IV Malthusian Stagnation and Accelerating Growth

Note: This table reports the ordinary least square estimates of the time trend  $\rho$  from the regression  $g_t = \eta + \rho t + \vartheta_t$  for each country and each time period. Standard errors are reported in parentheses, and significance levels are \*\*\* p<0.01, \*\* p<0.05, and \* p<0.10.