Rational Inattention and the Dynamics of Consumption and Wealth in General Equilibrium

Luo, Yulei and Nie, Jun and Wang, Gaowang and Young, Eric

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Yulei Luo†
University of Hong Kong

Jun Nie‡
Federal Reserve Bank of Kansas City

Gaowang Wang§
Shandong University

Eric R. Young¶
University of Virginia

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Abstract

We use a recursive utility version of a basic Huggett (1993) model to study the cross-sectional dispersion of consumption and wealth (relative to income). The basic model implies too little dispersion compared to the data, whereas a one-parameter extension to include rational inattention (limited information processing) delivers a better fit to both facts in general equilibrium. In particular, intertemporal substitution plays an important role in determining the two key dispersion moments via affecting the degree of optimal attention in equilibrium. Alternative models that rely on habit formation, incomplete information about current income, or borrowing constraints are not consistent with the facts we document.

Keywords: Rational Inattention; General Equilibrium; Consumption and Wealth Volatility.

JEL Classification Numbers: C61; D83; E21.

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†Faculty of Business and Economics, The University of Hong Kong, Hong Kong. E-mail: `yulei.luo@gmail.com`.

‡Research Department, Federal Reserve Bank of Kansas City. E-mail: `jun.nie@kc.frb.org`.

§Center for Economic Research, Shandong University, Jinan, China. E-mail: `wanggaowang@gmail.com`.

¶Department of Economics, University of Virginia, Charlottesville, VA 22904. E-mail: `ey2d@virginia.edu`.

1. Introduction

Our interest in this paper is to evaluate the general equilibrium implications of limited information-processing capacity (rational inattention or RI) for the joint cross-sectional dispersions of consumption, wealth, and income. The evolution of the consumption, income, and wealth inequality based on optimal consumption-saving choice is a central topic in modern macroeconomics. In intertemporal consumption-savings problems, *prudent* households save today for three reasons: (i) they anticipate future declines in income (saving for a rainy day), (ii) they face uninsurable risks (precautionary savings), and (iii) they are patient relative to the interest rate. For example, the "permanent income hypothesis" (PIH) of Friedman (1957) emphasizes the motive (i) in which consumption is solely determined by permanent income (the annuity value of total wealth) and follows a random walk (see Hall 1978).[1]

A growing recent literature inspired by Sims (2003) shows that RI plays an important role in influencing the consumption and saving dynamics and has also gained some empirical support. Specifically, Sims (2003), Luo (2008), and Luo and Young (2010) introduce RI into the basic partial equilibrium PIH environment; RI implies that agents process signals slowly and therefore appear to respond sluggishly to innovations in permanent income. This sluggish response appears to deliver changes in consumption in response to anticipated income changes, and as a result also delivers smaller responses to permanent income changes; that is, the model delivers both excess sensitivity and excess smoothness in the consumption behavior we observed in the US data.[2] Some empirical studies found that incomplete information about the state plays an important role in affecting individual agents' optimal decisions. For example, Coibion and Gorodnichenko (2015) and Andrade and Le Bihan (2013) find pervasive evidence consistent with Sims (2003)'s rational inattention theory using the U.S. and European surveys of professional forecasters and other agents, respectively.[3] Crucial for our purposes, the RI model delivers only one new free parameter, which maps directly into the speed of learning (one can think of this parameter as a filter gain).

However, the above RI-PIH models are partial equilibrium, taking as given a constant exogenous risk-free rate. By construction, the models shut off the feedback loop from the equilibrium

[1]This statement holds, for example, if households have quadratic utility and have access to a single risk-free bond with a constant return. If utility is not quadratic, the random walk nature of consumption is only approximately true, but the PIH still holds.

[2]The tests of the PIH, while robust to the possibility that agents have more information than the econometrician (see Campbell and Deaton 1989), may not be robust to the RI implication that the econometrician may actually have more information than the agents (at least at the time of decisions). The reason is that agents with more information will still set consumption changes to be orthogonal to everything the econometrician observes; it is clear that the converse would not necessarily hold.

[3]Hong, Torous, and Valkanov (2007) find evidence for rational inattention in the financial markets. Specifically, they find that investors in the stock market react gradually to information contained in industry returns about their fundamentals and that information diffuses only gradually across markets.

interest rate to the dynamics of consumption and wealth. Furthermore, these RI models do not separate risk aversion from intertemporal substitution in determining consumption dynamics, while many papers on recursive utility (RU) preferences highlight the importance of the separation between risk aversion and intertemporal substitution in affecting optimal consumption-portfolio rule and asset pricing. (See, for example, Epstein and Zin 1989, Campbell 2003, Vissing-Jï¿œrgensen and Attanasio 2003, Guvenen 2006.) This paper therefore fills the gap by providing a general theoretical RU framework to explore the general equilibrium implications of rational inattention for the joint cross-sectional dispersion of consumption, wealth and income. We investigate both the theoretical mechanism (how RI influences the equilibrium interest rate) and the empirical performance (how RI could lead the model to fit the data better compared to alternative hypotheses).

Specifically, we propose an equilibrium precautionary saving model in which inattentive consumers have recursive utility with exponential risk and intertemporal attitudes and face uninsurable labor income. We disentangle two distinct aspects of preferences: the agent's elasticity of intertemporal substitution (EIS; attitudes towards variation in consumption across time) with the coefficient of absolute risk aversion (CARA; attitudes towards variation in consumption across states).[4] We solve our model in closed-form and analytically inspect how risk aversion and intertemporal substitution interact with rational inattention and then affect the equilibrium interest rate as well as the cross-sectional dispersion of consumption and wealth (relative to income). We find that if attention is elastic (i.e., consumers can adjust the degree of attention in response to changes in the real world), the EIS plays an important role in affecting the equilibrium properties of the model economy.[5] In particular, we show that an increase in EIS affects the equilibrium interest rate through two channels: (i) it increases the relative importance of the impatience-induced dissaving effect (*the direct channel*) and (ii) it reduces the optimal attention level and thus increases the inattention-induced precautionary saving amount (*the indirect channel*). In addition, the optimal level of attention in our RU model is mainly determined by the EIS, and is independent of the degree of risk aversion. We also show that the general equilibrium exists and is unique in such an RI-RU economy.

We then bring the theory to the data as well as compare with alternative hypotheses to ex-

[4]Constant-relative-risk-aversion (CRRA) utility functions are more common in macroeconomics, mainly due to balanced-growth requirements. CRRA utility would greatly complicate our analysis because the intertemporal consumption model with CRRA utility and stochastic labor income has no explicit solution and leads to non-linear consumption rules. Introducing RI would then be substantially more difficult and involve approximations of unknown quality.

[5]Coibion and Gorodnichenko (2015) find that information rigidities were falling from the late 1960s to the early 1980s as the volatility of macroeconomic variables was rising, while these rigidities had been consistently increasing since the start of the Great Moderation ($1983-1984$). They then argue that one should be careful when treating information rigidities at the macro level as a structural parameter because these rigidities vary over time in response to changes in macroeconomic conditions.

amine whether our RI model can better explain the data. We use the RI model to interpret the dispersion of consumption and wealth (relative to income) that we observe in the data.[6] We find that RI substantially improves the predictions of the model for these relative dispersions. The full-information model, for the income process we estimate from micro data, implies that these dispersions are much lower than in the data; the separation of risk aversion and intertemporal substitution made feasible by recursive utility does not help increase these dispersions. For plausible value of the CARA and EIS, the dispersion of consumption and wealth relative to income is still well below their empirical counterparts. With RI, as households become less attentive, relative dispersions in consumption and wealth both rise. Interestingly, we find that the same value of the RI parameter roughly matches both moments, providing an over-identifying test of the model. General equilibrium effects turn out to be less significant when consumers are less information-constrained, acting to slightly reduce the dispersions in the model relative to a partial equilibrium exercise with a fixed interest rate.

Next, we ask whether our model provides a story for the observed changes in the cross-sectional distributions of consumption and income. The significant increase in household income inequality or dispersion for the U.S. in the 1980s and 1990s is a well-documented fact. Many studies have found that the dispersions of U.S. household earnings and incomes have a strong upward trend.[7] In addition, the literature has documented that the recent increase in income inequality in the U.S. has not been accompanied by a corresponding rise in consumption inequality over the 1980s and 1990s. In other words, over the sampling period income and consumption inequality diverged and the relative volatility of consumption to income has decreased, as discussed Krueger and Perri (2006) and Blundell, Pistaferri, and Preston (2008). With elastic attention we find that our model naturally predicts a decline in the relative volatility of consumption to income, in response to a rise in income volatility, and the magnitude is close to that found in our data; this experiment provides additional evidence for our model.

To further justify the validation of RI in explaining the data, we compare our RI model with several alternative models that are commonly used to study consumption dynamics in the literature. We first compare our RI model with models of sluggish movements in consumption, namely habit formation and incomplete information about current income. As the habit parameter increases, consumption changes become sluggish as households try to smooth changes in consump-

[6] As the PSID does not have complete consumption information, we construct a panel with consumption and income based on the approach from Guvenen and Smith (2014) to impute total consumption via a demand system estimated on the CEX (see also Blundell, Pistaferri, and Preston 2008). We also use the PSID to construct measures of wealth, and then we compare our measures to those predicted by the model.

[7] See Katz and Autor (1999) for a survey of these empirical findings. Table 1 also reports the increases in the variance of income growth over the 1980s and 1990s.

tion rather than levels. Thus, there is a sense in which the two model frameworks look similar; in fact, Luo (2008) shows that the two models are observationally equivalent at the aggregate level (in terms of consumption growth dynamics), but not at the individual level: RI delivers high consumption volatility from the noise shocks. We show that this similarity does not extend to the equilibrium cross-sectional dispersion moments we examine here, even though the noise shocks aggregate out; unlike RI, habit formation moves the relative dispersion of consumption to income away from its empirical counterpart, and the model's prediction on the wealth dispersion is well below its empirical counterpart.[8] In addition, we find that stronger habit formation leads to higher, not lower, interest rates.

With incomplete information, households cannot distinguish between permanent and transitory innovations in current income; although similar to rational inattention on its face, there is a key difference – under RI, agents cannot observe the value of their assets in addition to their income, and therefore need to extract endogenous as well as exogenous components. It turns out that incomplete information economies behave very much like habit models – the larger the MA coefficient on the forecast of future income (which captures the speed of learning), the higher the equilibrium interest rate. Consequently, the consumption dispersion measure gets too small, and the wealth dispersion measure remains far too low.

Finally, we consider whether introducing borrowing constraints can deliver the observed dispersion measures. We argue that they cannot, based on an additional observation – we find that our relative dispersion measure for consumption does not vary across the income and wealth distributions. In models that rely mainly on borrowing constraints, households near the constraint have high marginal propensities to consume, which leads them to respond very strongly to income changes; as a result, the relative variation in consumption for "poor" households is much higher than that for "rich" ones. Given that this difference is decisively rejected by the data we conclude that borrowing constraints are not a critical ingredient in a model of cross-sectional consumption, income, and wealth dynamics.

This paper is organized as follows. Section 2 constructs a precautionary saving model with a continuum of inattentive consumers who have the recursive utility and face uninsurable labor income. Section 3 solves optimal consumption-saving rules under rational inattention and characterizes the unique general equilibrium of this economy. Section 4 examines how RI affects the interest rate and the joint dynamics of consumption, income, and wealth quantitatively. Section 5 compares the rational inattention model to the models with habit formation, incomplete infor-

[8]Relative wealth dispersion is not strictly monotone in the habit parameter, but the change occurs at very high habit levels that cannot be reconciled with observed individual consumption volatility unless substantial measurement error is assumed.

mation about current income, and borrowing constraints. In the appendices we provide the key proofs and derivations.

2. A Caballero-Huggett-Wang Economy with Rational Inattention

2.1. A Full-information Rational Expectations Model with Recursive Utility and Precautionary Savings

In this section, we first consider a full-information rational expectations (FI-RE) recursive utility model with labor income and precautionary savings. Although the expected utility model has many attractive features, it implies that the agent's elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. However, risk aversion (attitudes towards atemporal risks) and intertemporal substitution (attitudes towards shifts in consumption over time) capture two distinct aspects of preferences, and should not necessarily be linked so tightly.[9] In this paper, we assume that agents in our model economy have recursive preferences of the Kreps-Porteus/Epstein-Zin type, and can disentangle the degree of risk aversion from the elasticity of intertemporal substitution. Specifically, for every stochastic consumption stream, $\{c_t\}_{t=0}^{\infty}$, the utility stream, $\{f(U_t)\}_{t=0}^{\infty}$, is recursively defined as follows:[10]

$$f(U_t) = f(c_t) + \frac{1}{1+\rho} f(\mathcal{CE}_t[U_{t+1}]) \tag{1}$$

where $\rho > 0$ is the agent's subjective discount rate, $f(x) = -\psi \exp(-x/\psi)$,

$$\mathcal{CE}_t[U_{t+1}] = g^{-1}(E_t[g(U_{t+1})]), \tag{2}$$

is the certainty equivalent of U_{t+1} conditional on the period t information, and $g(U_{t+1}) = -\exp(-\alpha U_{t+1})/\alpha$. In (1), $\psi > 0$ governs the elasticity of intertemporal substitution (EIS), while $\alpha > 0$ governs the coefficient of absolute risk aversion (CARA). If $\psi = 1/\alpha$, the functions f and g are the same and the recursive utility reduces to the standard time-separable expected utility function used in Caballero (1990) and Wang (2003). In addition, $\psi = 1/\alpha$ also implies that the consumer is indifferent about the time at which uncertainty is resolved.[11]

[9] Risk aversion describes the agent's reluctance to permit consumption to vary across different states of the world and is meaningful even in a static setting. In contrast, intertemporal substitution describes the agent's willingness to substitute consumption over time and is meaningful even in a deterministic setting. Furthermore, many estimates in the literature reject the special case of expected utility.

[10] Angeletos and Calvet (2006) adopt similar recursive utility preferences to study the effects of idiosyncratic production risks on business cycles and growth.

[11] Consumers prefer early resolution of uncertainty if $\alpha > 1/\psi$ and late resolution if $\alpha < 1/\psi$. Luo and Young (2016) show how the distaste for late resolution of uncertainty and rational inattention deliver a high equity premium and a low portfolio weight of risky assets.

Following Caballero (1990) and Wang (2003), the flow budget constraint is written as

$$a_{t+1} = (1+r)\, a_t + y_t - c_t, \tag{3}$$

where r is a constant rate of interest and labor income, y_t, follows a stationary AR(1) process with Gaussian innovations

$$y_t = \phi_0 + \phi_1 y_{t-1} + w_t,\ t \geq 1,\ \ |\phi_1| < 1, \tag{4}$$

where $w_t \sim N\left(0, \sigma^2\right)$, $\phi_0 = (1-\phi_1)\, \overline{y}$, $\overline{y}$ is the mean of y_t, and the initial levels of labor income y_0 and asset a_0 are given.[12]

In order to facilitate the introduction of rational inattention, we follow Luo (2008) and Luo and Young (2010) and reduce the multivariate model to a univariate model with iid innovations to total wealth. Let total wealth, $s_t = a_t + h_t$, be defined as a new state variable. Here

$$h_t \equiv \frac{1}{1+r} E_t \left[\sum_{j=0}^{\infty} \left(\frac{1}{1+r} \right)^j y_{t+j} \right], \tag{5}$$

is human wealth (defined as the discounted expected present value of current and future labor income); evaluating the sum yields

$$h_t = \phi \left(y_t + \frac{\phi_0}{r} \right),$$

where $\phi = 1/\left(1+r-\phi_1\right)$.[13] Using (3) and (4), it is straightforward to show that the evolution of the new state variable can be written as

$$s_{t+1} = (1+r)\, s_t - c_t + \zeta_{t+1}, \tag{6}$$

where the time $(t+1)$ innovation to total wealth can be written as

$$\zeta_{t+1} \equiv \frac{1}{1+r} \sum_{j=t+1}^{\infty} \left(\frac{1}{1+r} \right)^{j-(t+1)} (E_{t+1} - E_t) \left[y_j \right], \tag{7}$$

which can be reduced to $\zeta_{t+1} = \phi w_{t+1}$ if we use income process (4).[14]

[12] Assuming that the individual income shock includes two components, one permanent and the other transitory, does not change the main results in this paper. Here we follow Wang (2003) and adopt specification (4), in order to simplify the algebra. A detailed derivation of the model with the two-income shock specification is available from the corresponding author by request. For the empirical studies on the income specification, see Attanasio and Pavoni (2011).

[13] See Appendix 7.1 for the derivation.

[14] See Appendix 7.1 for the derivation.

2.2. Incorporating Rational Inattention

In this section, we follow Sims (2003) and incorporate rational inattention (RI) due to finite information-processing capacity into the above permanent income model with the CARA-Gaussian specification. Under RI, consumers have only finite Shannon channel capacity available to observe the state of the world. Specifically, we use the concept of entropy from information theory to characterize the uncertainty about a random variable; the reduction in entropy is thus a natural measure of information flow. With finite capacity $\kappa \in (0, \infty)$, a random variable $\{s_t\}$ following a continuous distribution cannot be observed without error and thus the information set at time $t+1$, denoted $\mathcal{I}_{t+1}$, is generated by the entire history of noisy signals $\left\{s_j^*\right\}_{j=0}^{t+1}$. Following the literature, we first assume the noisy signal takes the additive form:[15]

$$s_{t+1}^* = s_{t+1} + e_{t+1},$$

where e_{t+1} is the endogenous noise caused by finite capacity.[16] We further assume that e_{t+1} is an iid idiosyncratic shock and is independent of the fundamental shocks hitting the economy. The reason that the RI-induced noise is idiosyncratic is that the endogenous noise arises from the consumer's own internal information-processing constraint. Agents with finite capacity will choose a new signal $s_{t+1}^* \in \mathcal{I}_{t+1} = \left\{s_1^*, s_2^*, \cdots, s_{t+1}^*\right\}$ that reduces the uncertainty about the variable s_{t+1} as much as possible. Formally, this idea can be described by the information constraint

$$\mathcal{H}\left(s_{t+1} | \mathcal{I}_t\right) - \mathcal{H}\left(s_{t+1} | \mathcal{I}_{t+1}\right) \leq \kappa, \tag{8}$$

where κ is the investor's information channel capacity, $\mathcal{H}\left(s_{t+1} | \mathcal{I}_t\right)$ denotes the entropy of the state prior to observing the new signal at $t+1$, and $\mathcal{H}\left(s_{t+1} | \mathcal{I}_{t+1}\right)$ is the entropy after observing the new signal. κ imposes an upper bound on the amount of information flow – that is, the change in the entropy – that can be transmitted in any given period. Finally, following the literature, we suppose that the prior distribution of s_{t+1} is Gaussian.

In a linear-quadratic-Gaussian (LQG) setting, Sims (2003) and Shafieepoorfard and Raginsky (2013) show that *ex post* Gaussian distributions, $s_t | \mathcal{I}_t \sim N\left(E\left[s_t | \mathcal{I}_t\right], \Sigma_t\right)$ with $\Sigma_t = E_t\left[\left(s_t - \widehat{s}_t\right)^2\right]$,

[15]Mackowiak and Wiederholt (2009) show that this additive noisy signal form is optimal if the state being tracked is a stationary Gaussian AR(1) process. The proof for the optimal form of the noisy signal in their paper can be extended to our model in which the state being tracked is a random walk if the channel capacity, κ, is greater than a lower bound so that all conditional moments appearing in the proof are finite and well-defined. Please see the proof in Online Appendix A for the details.

[16]For other types of imperfect information about state variables, see Pischke (1995) and Wang (2004). Pischke (1995) assumes that consumers ignore the aggregate income component, and Wang (2004) assumes that consumers cannot distinguish two individual components in the income process.

are optimal. Here we first assume *ex post* Gaussian distributions of the true state and Gaussian noise but adopt negative exponential recursive utility preferences. Because both the optimality of *ex post* Gaussianity and the standard Kalman filter are based on the linear-quadratic-Gaussian (LQG) specification, the applications of these results in the RI models with negative exponential preferences are only approximately valid.[17] In the next section we verify that the loss function due to RI is approximately quadratic and consequently the optimality of the *ex post* Gaussianity of the state approximately holds in the recursive utility model.

Since both *ex ante* and *ex post* distributions of the state are Gaussian, (8) reduces to

$$\ln\left(|\Psi_t|\right) - \ln\left(|\Sigma_{t+1}|\right) \leq 2\kappa, \tag{9}$$

where $\Sigma_{t+1} = \mathrm{var}_{t+1}\left(s_{t+1}\right)$ and $\Psi_t = \mathrm{var}_t\left(s_{t+1}\right) = (1+r)^2\,\Sigma_t + \mathrm{var}_t\left(\zeta_{t+1}\right)$ are the posterior and prior variances of the state variable, s_{t+1}, respectively. In our univariate model, (9) fully determines the value of the steady state conditional variance Σ:

$$\Sigma = \frac{\mathrm{var}_t\left(\zeta_{t+1}\right)}{\exp\left(2\kappa\right) - (1+r)^2}, \tag{10}$$

which means that Σ is entirely determined by the variance of the exogenous shock $\left(\mathrm{var}_t\left(\zeta_{t+1}\right)\right)$ and finite capacity (κ). To guarantee that the state is stabilizable and the unconditional variance converges, we need to make the following assumption on the amount of channel capacity:

Assumption 1

$$\kappa > \ln\left(1+r\right). \tag{11}$$

It is worth noting that this restriction is very weak if r is small; in general equilibrium r will be smaller than ρ, so for short time periods this condition is not restrictive at all.

Following the steps outlined in Luo and Young (2014), we can compute the Kalman gain in the steady state θ as

$$\theta = 1 - \frac{1}{\exp\left(2\kappa\right)}; \tag{12}$$

θ measures the fraction of uncertainty removed by a new signal in each period, and is the only new parameter introduced by the rational inattention framework.

The evolution of the estimated state $\widehat{s}_t$ is governed by the Kalman filtering equation

$$\widehat{s}_{t+1} = (1-\theta)\left((1+r)\,\widehat{s}_t - c_t\right) + \theta s^*_{t+1}, \tag{13}$$

[17] See Peng (2004), Mondria (2010), and Van Nieuwerburgh and Veldkamp (2009, 2010) for applications of CARA preferences in RI models.

where $\widehat{s}_t = E_t[s_t]$ is the conditional mean of the state, s_t. Combining (6) with (13) yields

$$\widehat{s}_{t+1} = (1+r)\,\widehat{s}_t - c_t + \widehat{\zeta}_{t+1}, \tag{14}$$

where

$$\widehat{\zeta}_{t+1} = \theta\,(1+r)\,(s_t - \widehat{s}_t) + \theta\,(\zeta_{t+1} + e_{t+1}) \tag{15}$$

is the innovation to $\widehat{s}_{t+1}$ and is independent of all the other terms on the RHS of (14). $\widehat{\zeta}_{t+1}$ is an MA(∞) process with $E_t\left[\widehat{\zeta}_{t+1}\right] = 0$ and

$$\operatorname{var}\left(\widehat{\zeta}_{t+1}\right) = \Gamma(\theta, r)\,\omega_{\zeta}^2, \tag{16}$$

where $\omega_{\zeta}^2 = \operatorname{var}_t(\zeta_{t+1})$,

$$\Gamma(\theta, r) = \frac{\theta}{1-(1-\theta)(1+r)^2} > 1 \tag{17}$$

for $\theta < 1$, and

$$s_t - \widehat{s}_t = \frac{(1-\theta)\,\zeta_t}{1-(1-\theta)(1+r)\cdot L} - \frac{\theta e_t}{1-(1-\theta)(1+r)\cdot L} \tag{18}$$

is the estimation error with $E_t[s_t - \widehat{s}_t] = 0$ and $\operatorname{var}(s_t - \widehat{s}_t) = \frac{1-\theta}{1-(1-\theta)(1+r)^2}\omega_{\zeta}^2$; L is the standard lag operator. To guarantee that the sum of these infinite series converges, we impose the restriction $\kappa > 0.5\ln(1+r)$, which is weaker than (11). From (17), it is clear that $\frac{\partial\Gamma}{\partial r} > 0$ and $\frac{\partial\Gamma}{\partial\theta} < 0$.

3. General Equilibrium under RI

3.1. Optimal Consumption and Savings Functions

The consumption function and the value function under RU and RI can be obtained by solving the Bellman equation:

$$f(J(\widehat{s}_t)) = \max_{c_t}\left\{f(c_t) + \frac{1}{1+\rho} f\left(\mathcal{CE}_t\left[J(\widehat{s}_{t+1})\right]\right)\right\}, \tag{19}$$

subject to (14)-(18). The following proposition summarizes the main results from the above precautionary-savings model with RI.

Proposition 1. *For a given Kalman gain, θ, the value function is*

$$\widehat{v}(\widehat{s}_t) = -\frac{\psi}{r}\exp\left(-\frac{1}{\psi}\left\{r\widehat{s}_t - \psi\ln(1+r) + \left[\frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\omega_{\widehat{\zeta}}^2\right]\right\}\right), \tag{20}$$

the consumption function is

$$c_t^* = r\widehat{s}_t + \frac{\psi}{r}\Psi(r) - \Pi(\theta, r), \tag{21}$$

and the saving function is

$$d_t^* = \widetilde{f}_t + r\left(s_t - \widehat{s}_t\right) + \Pi\left(\theta, r\right) - \frac{\psi}{r}\Psi\left(r\right), \tag{22}$$

where $\widetilde{f}_t \equiv (1-\phi_1)\,\phi\,(y_t - \overline{y})$ captures the consumer's demand for savings "for a rainy day", $\widehat{s}_t$ *is governed by (14),* $s_t - \widehat{s}_t$ *is an* $MA(\infty)$ *estimation error process given in (18),* $\Pi(\theta, r) \equiv \alpha r \omega_{\widehat{\zeta}}^2/2 = r\alpha\Gamma(\theta, r)\,\omega_{\zeta}^2/2$ is the precautionary saving demand, *and* $\Psi(r) \equiv \ln\left(\frac{1+\rho}{1+r}\right)$ *captures the patience-induced saving.*

Proof. See Appendix 7.1 for the derivations. ■

Given the value function (20), we can show that the loss function due to RI is approximately quadratic and the optimality of the ex post Gaussianity of the state still approximately holds in the RU model, which justifies the Kalman filtering equation, (14). See Appendix 7.2 for the detailed proof.

As κ converges to ∞ (or θ converges to 1), $\widehat{s}_t$ and $\omega_{\widehat{\zeta}}^2$ reduce to $\widehat{s}_t$ and ω_{ζ}^2, respectively; consequently, our RI model reduces to the full-information rational expectations (FI-RE) model.[18] To explore the effects of limited attention on precautionary savings within the RU framework, we first define the precautionary saving premium due to limited attention as

$$P_{ri} \equiv \frac{1}{2}\left(\Gamma\left(\theta, r\right) - 1\right)\alpha r \omega_{\zeta}^2, \tag{23}$$

where $\Gamma(\theta, r)$ is given in (17). It is clear that this premium is decreasing with the degree of attention θ, and is increasing with the coefficient of absolute risk aversion (α) and the persistence and volatility of the income shock (ϕ_1 and σ) for any given θ. Thus, the incomplete information that RI forces upon the households leads to an increase in saving.

To further explore the precautionary savings premium in (23), we isolate the effects of RI on individual consumption and saving by rewriting (21) as

$$c_t^* = r\widehat{s}_t + \left\{\frac{\psi}{r}\Psi\left(r\right) - \frac{1}{r\alpha}\left[\begin{array}{c} \ln\left(E_t\left[\exp\left(-r\alpha\theta\left(1+r\right)\left(s_t - \widehat{s}_t\right)\right)\right]\right) + \frac{1}{2}\left(r\alpha\theta\omega_{\zeta}\right)^2 + \\ \frac{1}{2}\left(1-\theta\right)\Gamma\left(\theta, r\right)\left(r\alpha\omega_{\zeta}\right)^2 \end{array}\right]\right\}, \tag{24}$$

where $\Psi(r) = \ln\left(\frac{1+\rho}{1+r}\right)$ measures the relative importance of patience to the interest rate in determining optimal consumption (it is greater than 0 if $\rho > r$),

$$\frac{1}{\alpha r}\ln\left(E_t\left[\exp\left(-\alpha r\theta\left(1+r\right)\left(s_t - \widehat{s}_t\right)\right)\right]\right) = \frac{1}{2}r\alpha\theta\left(1-\theta\right)\Gamma\left(\theta, r\right)\left(1+r\right)^2\omega_{\zeta}^2$$

[18] If $\alpha = 1/\psi$ and $\theta = 1$, the consumption function with RI, (21), reduces to that of the Wang (2003) model.

is the precautionary savings premium due to the time t estimation error, $\left(r\alpha\theta\omega_{\zeta}\right)^2/2$ is the precautionary savings premium driven by the exogenous fundamental income shocks $\{w_t\}$, and $(1-\theta)\,\Gamma(\theta)\left(r\alpha\omega_{\zeta}\right)^2/2$ captures the precautionary savings premium driven by the endogenous noise shocks, $\{e_t\}$.[19] From (21), for finite capacity ($\kappa < \infty$ or $\theta \in (0,1)$), the precautionary saving premium due to fundamental shocks is smaller than that in the full-information case, $\left(r\alpha\theta\omega_{\zeta}\right)^2/2 < \left(r\alpha\omega_{\zeta}\right)^2/2$, because of the incomplete adjustment of consumption to the fundamental shock; however, we have two new positive terms that increase the total savings more than the absolute value of the reduced savings: (i) the premium due to the estimation error and (ii) the premium due to the RI-induced endogenous noise.

Given the time t available information and the fact that $E_t\left[s_t - \widehat{s}_t\right] = 0$, the conditional mean of (22) can be written as

$$\widetilde{d}_t = \widetilde{f}_t + \Pi(\theta, r) - \frac{\psi}{r}\Psi(r). \tag{25}$$

3.2. Existence and Uniqueness of General Equilibrium

As in Wang (2003), we assume that the economy is populated by a continuum of *ex ante* identical, but *ex post* heterogeneous agents, of total mass normalized to one, with each agent solving the optimal consumption and savings problem with RI specified in (19). Similar to Huggett (1993), we also make the following assumption:

Assumption 2 *The risk-free asset in our model is a pure-consumption loan and is in zero net supply. The initial cross-sectional distribution of permanent income is a stationary distribution* $\Phi(\cdot)$*.*

By the law of large numbers in Sun (2006), provided that the spaces of agents and the probability space are constructed appropriately, aggregate permanent income and the cross-sectional distribution of permanent income $\Phi(\cdot)$ are constant over time.

Proposition 2. *The total savings demand "for a rainy day" in the precautionary savings model with RI equals zero for any positive interest rate. That is,* $F_t(r) = \int_{y_t} \widetilde{f}_t(r)\, d\Phi(y_t) = 0$*, for* $r > 0$*.*

Proof. The proof uses the LLN and is the same as that in Wang (2003). ∎

Proposition 2 states that the total savings "for a rainy day" is zero, at any positive interest rate. Therefore, from (22), for $r > 0$, the expression for total savings under RI in the economy at time t is

$$D(\theta, r) \equiv \Pi(\theta, r) - \frac{\psi}{r}\Psi(r), \tag{26}$$

[19] This result is derived by using Equation (18) and the iid property of the processes $\left\{\widehat{\zeta}_t\right\}$, $\{\zeta_t\}$, and $\{e_t\}$.

where $\Pi(\theta, r) = r\alpha\Gamma(\theta, r)\,\omega_\zeta^2/2$ measures the amount of precautionary savings, and $\Psi(r)$ captures the patience-induced saving effect. Given (26), an equilibrium under RI is defined by an interest rate r^* satisfying

$$D(\theta, r^*) = 0. \tag{27}$$

The following proposition shows the existence of the equilibrium and the PIH holds in the RI general equilibrium.

Proposition 3. *There exists a unique equilibrium with an interest rate $r^* \in (0, \rho)$ in the precautionary-savings model with RI. In equilibrium, each agent's consumption is described by the PIH, in that*

$$c_t^* = r^* \widehat{s}_t, \tag{28}$$

where $\widehat{s}_t = E[s_t|\mathcal{I}_t]$ is the perceived value of permanent income. The evolution equations of wealth and consumption are

$$\Delta c_{t+1}^* = r^* \widehat{\zeta}_{t+1}, \tag{29}$$

$$\Delta a_{t+1}^* = \frac{1-\phi_1}{1+r^*-\phi_1}(y_t - \overline{y}) + r^*(s_t - \widehat{s}_t), \tag{30}$$

respectively, where $\widehat{\zeta}_{t+1}$ is specified in (15) with $E_t\left[\widehat{\zeta}_{t+1}\right] = 0$, $\mathrm{var}\left(\widehat{\zeta}_{t+1}\right) = \Gamma(\theta, r^*)\,\omega_\zeta^2$, and $\Gamma(\theta, r^*) = \frac{\theta}{1-(1-\theta)(1+r^*)^2}$. In the general equilibrium, the value function under RI can be written as

$$\widehat{v}(\widehat{s}_t) = -\frac{\psi(1+r)}{r}\exp\left(-\frac{r}{\psi}\widehat{s}_t\right), \tag{31}$$

Proof. If $r > \rho$, the two terms, $\Pi(\theta, r)$ and $-\psi\Psi(r)/r$, in the expression for total savings $D(\theta, r^*)$, are positive, which contradicts the equilibrium condition, $D(\theta, r^*) = 0$. Since $\Pi(\theta, r) - \psi\Psi(r)/r < 0$ (> 0) when $r = 0$ ($r = \rho$), the continuity of the expression for total savings implies that there exists at least one interest rate $r^* \in (0, \rho)$ such that $D(\theta, r^*) = 0$. From (21), we can obtain the individual's optimal consumption rule under RI in general equilibrium as $c_t^* = r^* \widehat{s}_t$. Substituting (14) and (28), we can obtain (29). (28) into (3) yields (30). The proof of uniqueness is longer and relegated to Appendix 7.3. ■

The intuition behind Proposition 3 is similar to that in Wang (2003). With an individual's constant total precautionary savings demand $\Pi(\theta, r)$, for any $r > 0$, the equilibrium interest rate r^* must be such that each individual's dissavings demand due to impatience is exactly balanced by their total precautionary-savings demand, $\Pi(\theta, r^*) = \psi\Psi(r)/r$. Figure 1 plots aggregate saving as a function of θ, given the parameters $\alpha = 2$, $\psi = 0.54$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.[20] It is clear from the figure that the equilibrium interest rate increases with θ.

Regarding uniqueness, Toda (2017) demonstrates that the FI-RE model used here can have multiple stationary equilibria, provided the income process is sufficiently rich; the AR(1) process we use here does not satisfy the requirements for multiple equilibria, though. Our results suggest that RI does not deliver any new insights into the nature of multiple equilibria, so we do not investigate this issue further.

The magnitude of the EIS (ψ) is a key issue in macroeconomics and asset pricing. For example, Parker (2002) and Vissing-Jorgensen and Attanasio (2003) estimate the IES to be well in excess of one. Hall (1988) and Campbell (2003), on the other hand, estimate its value to be well below one. Here we choose $\psi = 0.5$ for illustrative purposes and will examine how EIS affects the general equilibrium under RI in Section 4 when we do the quantitative analysis.[21] From the equilibrium condition (27), it is clear that a high value of ψ would amplify the relative importance of the dissaving effect $\Psi(r)$ for the equilibrium interest rate. The intuition behind this result is simple. When ψ is higher, consumption growth responds less to changes in the interest rate. In order to clear the market, the consumer must be offered a higher equilibrium risk free rate in order to be induced to save more and making his consumption tomorrow even more in excess of what it is today (less smoothing).

Given (21) and (27), it is clear that even though the individual increases their total precautionary savings in response to information frictions for a given r, the level of aggregate savings equals zero. That is, RI does not affect the aggregate wealth in the economy, because the equilibrium interest rate is pushed down to counteract this precautionary savings increase.[22] With lower Shannon channel capacity, the equilibrium interest rate is lower.

From the equilibrium condition,

$$\frac{1}{2} r^* \alpha \Gamma(\theta, r^*) \omega_\zeta^2 - \frac{\psi}{r^*} \ln\left(\frac{1+\rho}{1+r^*}\right) = 0, \tag{32}$$

[20]In Section 4.1, we will provide more details about how we estimate the income process using the U.S. data. The main result here is robust to the choices of these parameter values.

[21]Guvenen (2006) finds that stockholders have a higher EIS (around 1.0) than non-stockholders (around 0.1). Crump, Eusepi, Tambalotti, and Topa (2015) find that the EIS is precisely and robustly estimated to be around 0.8 in the general population using the newly released FRBNY Survey of Consumer Expectations (SCE).

[22]If we introduced an asset with elastic supply, such as the capital stock in Aiyagari (1994), the same effects would be present but the stock of capital would rise (and the change in the interest rate would be smaller as a result). How much smaller depends on the elasticity of output with respect to capital (the share parameter on capital in a Cobb-Douglas production function).

it is straightforward to show that

$$\frac{dr^*}{d\theta} = \frac{r^{*3}\left(2+r^*\right)}{\left[1-\left(1-\theta\right)\left(1+r^*\right)^2\right]^2}\left\{\frac{2r^*\theta\left[1-\left(1-\theta\right)\left(1+r^*\right)\right]}{\left[1-\left(1-\theta\right)\left(1+r^*\right)^2\right]^2}+\frac{2\psi}{\alpha\left(1+r^*\right)\omega_\zeta^2}\right\}^{-1}. \tag{33}$$

where $1-\left(1-\theta\right)\left(1+r^*\right)^2 > 0$. It is clear from this expression that r^* is decreasing in the degree of inattention $1-\theta$. The first row of Table 2 reports the general equilibrium interest rates for different values of θ.[23] We can see from the table that r^* decreases as the degree of inattention increases. For example, if θ is reduced from 1 to 0.1, r^* is reduced from 3.41 percent to 2.89 percent. In addition, it is clear that

$$\frac{dr^*}{d\alpha} < 0 \text{ and } \frac{dr^*}{d\psi} > 0.$$

That is, the equilibrium interest rate decreases with the degree of risk aversion and increases with the degree of intertemporal substitution. Here it is worth noting that although both the CARA model and the LQ model lead to the PIH in general equilibrium, both risk aversion and intertemporal substitution play roles in affecting the dynamics of consumption and wealth in the CARA model via the equilibrium interest rate channel.

One might ask what a reasonable value of θ is, and if there is any way to calibrate it outside a model. Unfortunately, there is no direct survey evidence on the value of channel capacity of ordinary households in the economics literature, and thus it is not straightforward to answer these questions; estimates of learning capacity exist, but they are not directly useful since we are interested in the capacity that will be devoted to economic activity (specifically, consumption and saving). In lieu of such evidence, we simply note that 0.1 is the value needed to match portfolio holdings in Luo (2010) and is therefore not obviously unreasonable (a caveat can be found in Luo and Young 2016, where a significantly larger number is obtained using recursive utility). Coibion and Gorodnichenko (2015) have the most "model independent" measure of θ, and they find $\theta = 0.5$ provides a good fit for a variety of forecast and survey data, and a variety of other papers obtain a number of different values depending on what facts they bring to bear. We will show below that $\theta = 0.1$ allows us to match some cross-sectional dispersion facts, but are cognizant that this parameter's value is quite uncertain.

3.3. Elastic Attention

Instead of using fixed channel capacity to model finite information-processing ability, one could assume that the marginal cost of information-processing (i.e., the shadow price of information-

[23]Here we also set $\alpha = 3$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

processing capacity) is fixed. That is, the Lagrange multiplier on (9) is constant.[24] In the univariate case, the objective of the agent with finite capacity in the filtering problem is to minimize the discounted expected mean square error (MSE),

$$L_t = E_t \left[v_0 \left(s_t \right) - v \left(x_t \right) \right], \tag{34}$$

where $v_0(\cdot)$ and $v(\cdot)$ are the value functions in the FI-RE model and the RI model, respectively, s_t is the unobservable state variable and x_t is the best estimate of the true state, subject to the information-processing constraint, or

$$\min_{\{\Sigma_t\}} \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \Phi \Sigma_t + \lambda \ln \left(\frac{(1+r)^2 \Sigma_{t-1} + \omega_\zeta^2}{\Sigma_t} \right) \right] \right\},$$

where $\Phi = (r/\psi)^2$, Σ_t is the conditional variance at t, and λ is the Lagrange multiplier corresponding to (9).[25]

Solving this problem yields the optimal steady state conditional variance:

$$\Sigma \left(r, \lambda^* \right) = \frac{(1+r)^2 (1-\beta) \lambda^* - \Phi + \sqrt{\left[(1+r)^2 (1-\beta) \lambda^* - \Phi \right]^2 + 4\lambda^* (1+r)^2 \Phi}}{2\Phi (1+r)^2} \omega_\zeta^2, \tag{35}$$

where $\lambda^* = \lambda / \omega_\zeta^2$ is the normalized shadow price of information-processing capacity. It is straightforward to show that as λ goes to 0, $\Sigma = 0$; and as λ goes to $\pi\infty$, $\Sigma = \infty$. Note that $\frac{\partial \ln \Sigma}{\partial \ln \omega_\zeta^2} < 1$ if we adopt the assumption that λ is fixed, while $\frac{\partial \ln \Sigma}{\partial \ln \omega_\zeta^2} = 1$ in the fixed κ case. Comparing (35) with (10), it is clear that the two RI modeling strategies are observationally equivalent in the sense that they lead to the same conditional variance if the following equality holds:

$$\kappa \left(r, \lambda^* \right) = \ln (1+r) + \frac{1}{2} \ln \left(1 + \frac{2\Phi}{(1+r)^2 (1-\beta) \lambda^* - \Phi + \sqrt{\left[(1+r)^2 (1-\beta) \lambda^* - \Phi \right]^2 + 4\lambda^* (1+r)^2 \Phi}} \right). \tag{36}$$

[24]See Maćkowiak and Wiederholt (2015) and Matejka, Filip and Alisdair McKay (2015) for applications of elastic attention in other macroeconomic settings.

[25]As in the fixed-capacity case, although we adopt the CARA-Gaussian setting, the loss function due to imperfect-state-observation is approximately quadratic. See Appendix 7.2 for derivation of the objective function and discussion of the approximation quality.

In this case, the Kalman gain is

$$\theta\left(r,\lambda^{*}\right)=1-\frac{1}{1+r}\left\{1+\frac{2\Phi}{\left(1+r\right)^{2}\left(1-\beta\right)\lambda^{*}-\Phi+\sqrt{\left[\left(1+r\right)^{2}\left(1-\beta\right)\lambda^{*}-\Phi\right]^{2}+4\lambda^{*}\left(1+r\right)^{2}\Phi}}\right\}^{-1}. \tag{37}$$

It is obvious that κ converges to its lower limit $\underline{\kappa}=\ln\left(1+r\right)\approx r$ as λ goes to ∞; and it converges to ∞ as λ goes to 0. In addition, it is also clear that the higher the income uncertainty, the more capacity is devoted to monitoring the evolution of the state. In other words, using this RI modeling strategy, the consumer is allowed to adjust the optimal level of capacity in such a way that the marginal cost of information-processing for the problem at hand remains constant. Figure 2 clearly shows that the optimal level of attention is decreasing with the value of EIS for different values of the marginal information-processing cost.[26] That is, when consumers are more reluctant in substituting their consumption over time (low EIS), they choose to devote more capacity and attention to monitoring the evolution of the state. The intuition behind this result is that when inattentive consumers really like flat consumption profile, they devote more attention to monitoring the state in order to avoid making the consumption profile steep, which occurs with low attention due to "accidental savings".[27] In summary, we use the following two-stage optimization procedure to solve the joint optimal control-filtering model under RI:

1. Given that κ is constant channel capacity, guess that the ex post Gaussian distribution of the true state and additive iid Gaussian noise due to RI are still optimal when the agent has recursive utility. Given the optimality of ex post Gaussianity and Gaussian noise, we can apply the standard Kalman filter and dynamic programming to solve the RI model explicitly. Using the loss function derived from the value functions under RI and FI-RE, we can verify that our guess about the optimality of ex post Gaussianity and Gaussian noise is correct.

2. Minimizing the same loss function due to the information-processing constraint obtained in Stage 1 and fixed marginal cost leads to optimal conditional variance (Σ^{*}) and endogenous attention (κ), which verifies that the assumption of constant channel capacity we used in Stage 1 is correct.

Although the above two RI modeling strategies, inelastic and elastic capacity, are observationally equivalent in the "static" sense, they have distinct implications for the model's propagation

[26] Here we set $\beta=0.96$ and $r=0.025$.

[27] Given the relationship between λ and θ (or κ), in the following analysis we just use the value of θ to measure the degree of optimal attention.

mechanism if the economy is experiencing regime switching. With inelastic capacity, the propagation mechanism governed by the Kalman gain is fixed regardless of changes in fundamental uncertainty, whereas with elastic capacity the propagation mechanism will change in response to changes in fundamental uncertainty. In a recent study, Coibion and Gorodnichenko (2015) used the SPF forecast survey data to test the degree of information rigidities governed by the Kalman gain and find that the information rigidities were decreasing with the volatility of the macroeconomic conditions. Specifically, they find that information rigidities were falling from the late 1960s to the start of the Great Moderation ($1983-1984$) and have declined since then, and argue that one should be wary of treating the degree of information rigidities as a structural parameter because it responds to changes in macroeconomic conditions. In the following analysis, we show that elastic attention delivers an accurate prediction about the response of consumption dispersion to changes in income volatility.

Since κ and θ in the elastic attention case depend on both the equilibrium interest rate and labor income uncertainty, the equilibrium interest rate is now determined implicitly by the following function:

$$D\left(\theta\left(r^*,\widetilde{\lambda}\right),r^*\right)\equiv\Pi\left(\theta\left(r^*,\widetilde{\lambda}\right),r^*\right)-\frac{\psi}{r}\Psi\left(r^*\right). \tag{38}$$

Figure 3 illustrates how r^* varies with labor income uncertainty, σ, for fixed information-processing cost, λ – the aggregate saving function is increasing with the interest rate and the general equilibrium interest rate is decreasing with labor income uncertainty. We can see from Table 4 that if the economy becomes more volatile (i.e., larger σ), the Kalman gain (θ) increases while the equilibrium interest rate (r^*) decreases. This result is different from that obtained in the fixed capacity case in which θ and r^* move in the same direction. (See Table 2.) The main reason for this result is that income uncertainty affects the equilibrium interest rate via two channels: (i) The direct channel which leads to higher aggregate savings (i.e., the ω_ζ^2 term in (38)) and (ii) the indirect channel which leads to lower aggregate savings (i.e., the $\theta\left(r^*,\widetilde{\lambda}\right)$ term in (38)), and the direct channel dominates.[28] In the next section, after estimating the income process using the U.S. data, we will examine how changes in income uncertainty affects the level of optimal attention and the equilibrium interest rate, and the relative volatility of consumption and wealth to income.

It is worth noting that the analysis in the benchmark model can be extended to an economy with risky financial assets, which consumers can use to partially hedge their idiosyncratic labor income shocks. Under some reasonable assumptions, the general equilibrium results obtained in our benchmark model remain valid when we reinterpret the variance of the labor income as the

[28]We have been unable to prove that r^* is unique under elastic capacity, precisely because of the indirect channel.

variance of the residual risk faced by the consumers after hedging using the risky financial assets.[29]

4. Empirical and Quantitative Results

In this section we assess our GE-RI model's implications for the dynamics of consumption, income and wealth. To construct empirical counterparts that are comparable with the theoretical moments derived in the model, we construct a panel with individual consumption, income and wealth based on the Panel Study of Income Dynamics (PSID) and the Consumer Expenditure Survey (CEX), using the imputation approach from Blundell, Pistaferri, and Preston (2008) as extended by Guvenen and Smith (2014). Then, using the estimated income process, we show the GE-RI model significantly fits the data better than the full information model in terms of the consumption and wealth dynamics. Our closed-form solutions explicitly show the different channels through which RI drives the results.

4.1. Empirical Evidence

In order to measure the relative consumption dispersion in the data, $\mathrm{sd}(\Delta c) / \mathrm{sd}(\Delta y)$, we construct a panel data set which contains both consumption and income at the household level. The PSID does not include enough consumption expenditure data to create full picture of household nondurable consumption. Such detailed expenditures are found, though, in the CEX from the Bureau of Labor Statistics. But households in this study are only interviewed for four consecutive quarters and thus do not form a panel. To create a panel of consumption to match the PSID income measures, we use an estimated demand function for imputing nondurable consumption following Guvenen and Smith (2014). Using an IV regression, they estimate a demand function for nondurable consumption that fits the detailed data in the CEX. The demand function uses demographic information and food consumption which can be found in both the CEX and PSID. Thus, we use this demand function of food consumption and demographic information (including age, family size, inflation measures, race, and education) to estimate nondurable consumption for PSID households, creating a consumption panel.

Following Blundell, Pistaferri, and Preston (2008), we define the household income as total household income (including wage, financial, and transfer income of head, wife, and all others in household) minus financial income (defined as the sum of annual dividend income, interest income, rental income, trust fund income, and income from royalties for the head of the household only) minus the tax liability of non-financial income. This tax liability is defined as the total federal tax liability multiplied by the non-financial share of total income. Tax liabilities after 1992 are

[29] See Online Appendix B for the detailed proof.

not reported in the PSID and so we estimate them using the TAXSIM program from the National Bureau of Economic Research. Our final household income measure can be expressed as:

$$\text{income measure} = (\text{total HH income} - \text{financial income}) - \text{taxes} \times \frac{\text{total HH income} - \text{financial income}}{\text{total HH income}}.$$

Our household sample selection closely follows that of Blundell, Pistaferri, and Preston (2008) as well.[30] We exclude households in the PSID poverty and Latino subsamples. We exclude households in years of family composition change, change in marital status, or female headship, as well as in years where the head or wife is under 30 or over 65. Households in years with missing education, region, income, and imputed consumption responses are also excluded. We also exclude households in years where they report a negative income or a food consumption level in the top or bottom 5 percent of all reported values in that year. Income and consumption values are then deflated by the CPI to constant $1982 - 1984$ dollars. Our final panel contains $7,111$ unique households with $58,034$ yearly income responses and $48,990$ imputed nondurable consumption values.[31]

With this constructed panel of household income and consumption, we next drop households in years where year-over-year food consumption changes are more than 20 percent or less than -20 percent. To exclude extreme outliers, we then follow Floden and Linde (2001) and normalize both income and consumption measures as ratios of the mean of each year, and exclude household in bottom and top 1 percent of the distribution of those ratios. Figure 4 shows the relative dispersion of consumption, defined as the ratio of the standard deviation of the consumption change to the standard deviation of the income change between 1980 and 2000. The basic pattern confirms but extends the findings in Blundell, Pistaferri, and Preston (2008) – relative consumption dispersion declines in the 1980s, but this decline stops around 1990.

In order to calculate the relative volatility of wealth to income ratio,$\mathrm{sd}(\Delta a)/\,\mathrm{sd}(\Delta y)$, we use wealth information included in the PSID data. Notice that the PSID only reports household wealth variables every five years starting in 1983, and then every other year starting in 1998. To be consistent with the model, we construct household wealth in the following way. We use measure of wealth defined as the sum of the net value of liquid assets (checking, savings, money market, etc.), vehicles, home equity, and other assets such as bonds, insurance policies, and trusts. All reported

[30]They create a new panel series of consumption that combines information from PSID and CEX, focusing on the period when some of the largest changes in income inequality occurred. For other explanations for observed consumption and income inequality, see Krueger and Perri (2006) and Attanasio and Pavoni (2011).

[31]There are more household incomes than imputed consumption values because food consumption - the main input variable in Guvenen and Smith's nondurable demand function - is not reported in the PSID for the years 1987 and 1988. Dividing the total income responses by unique households yields an average of 7-8 years of responses per household. These years are not necessarily consecutive as our sample selection procedure allows households to be excluded in certain years but return to the sample if they later meet the criteria once again.

values are again deflated by the CPI to constant $1982 - 1984$ dollars. We normalize each reported wealth and income value to the mean of the year reported, and then exclude outliers of this distribution at the top and bottom 1 percent. We then take the standard deviations of the change in normalized value from the previous report for both wealth and income to calculate our ratio. Our final panel for wealth and income has $23,630$ observations across 6232 households. This panel is somewhat smaller than our panel of consumption and income due to the limited number of years that wealth measures are reported. Figure 5 reports the results, which shows the relative volatility of wealth to income has been relatively stable in the sample period.

When estimating the income process, we focus on the sample period to the years $1980 - 1996$, due to the PSID survey changing to a biennial schedule after 1996. To further restrict the sample to exclude households with dramatic year-over-year income changes, we eliminate household incomes with year-over-year level changes in the top and bottom 5 percent of the distribution in each year. Then, again following Floden and Linde (2001), we normalize household income measures as ratios of the mean for that year and exclude all household values in years in which the income is in the top and bottom 1 percent of the normalized income measure for the year. To eliminate possible heteroskedasticity in the income measures, we follow Floden and Linde (2001) and regress each on a series of demographic variables in a fixed-effect panel regression to remove variation caused by differences in age and education. We next subtract these fitted values from each measure to create a panel of income residuals. We then use this panel to estimate the household income process as specified by equation (4) by running panel regressions on lagged income. As the last row of Table 1 reports, the estimated values of ϕ_0, ϕ_1, and σ are 0.0005, 0.919, and 0.175, respectively.

4.2. Empirical Implications for the Cross-Sectional Dispersion of Consumption, Wealth, and Income

Luo (2008) examines how RI affects consumption volatility in a partial equilibrium version of the PIH model presented above. In general equilibrium, since RI affects the equilibrium interest rate, it will have an additional effect on consumption dynamics. Using (29) and (30), we can obtain the key stochastic properties of the joint cross-sectional dispersion of individual consumption and wealth relative to income. The following proposition summarizes the implications of RI for the relative dispersion of consumption to income as well as the relative dispersion of financial wealth to income. Note that mathematically, the cross-sectional dispersion of consumption and wealth (relative to income) can be measured by the relative volatility of consumption to income and the relative volatility of wealth to income.

Proposition 4. *Under RI, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{r^*}{1+r^*-\phi_1}\sqrt{\frac{\left(1+\phi_1\right)\Gamma\left(\theta,r^*\right)}{2}}, \tag{39}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{1}{\sqrt{2}\left(1+r^*-\phi_1\right)}\sqrt{1-\phi_1+\frac{r^{*2}\left(1-\theta\right)\left(1+\phi_1\right)}{1-\left(1-\theta\right)\left(1+r^*\right)^2}+\frac{2r^*\left(1-\theta\right)\left(1-\phi_1^2\right)}{1-\phi_1(1-\theta)\left(1+r^*\right)}}. \tag{40}$$

Proof. See Appendix 7.1. ■

Expression (39) shows that RI has two opposing effects on the relative consumption dispersion. The first effect is direct through its presence in the expression of $\Gamma\left(\theta,r^*\right)$, whereas the second effect is through the general equilibrium interest rate $\left(r^*\right)$ and is thus indirect. Using the expression of $\Gamma\left(\theta,r^*\right)$, it is straightforward to show that the direct effect of RI is to increase consumption volatility. The intuition is very simple: the presence of the RI-induced noise dominates the slow adjustment of consumption in determining consumption volatility at the individual level. In contrast, the indirect effect of RI will reduce consumption volatility because it reduces the general equilibrium interest rate and $\partial\Gamma\left(\theta,r^*\right)/\partial r^* > 0$. Following the literature of precautionary savings and the estimated income process in the preceding subsection, we set $\rho = 0.04$, $\alpha = 3$, $\sigma = 0.175$, and $\phi_1 = 0.919$. The second to fourth rows of Table 2 reports how the interest rate and the relative volatility of consumption and wealth to income vary with θ in general equilibrium when $\psi = 0.54$. It is clear from the second row of Table 2 that RI can significantly affect the equilibrium interest rate. For example, when θ decreases from 1 to 0.10, r^* decreases from 3.41 percent to 2.89 percent, which is very close to 2.97 percent, the average annual equilibrium real interest rate from 1980 to 1996 estimated in Laubach and Williams (2015) using $1961-2014$ U.S. quarterly data (note that if $\theta = 0.11$, the equilibrium interest rate obtained in our model is exactly the same as its empirical counterpart). Here we focus on the $1980-1996$ period because we use it to estimate the income process and the relative volatility of consumption to income.

From the third row of Table 2, the relative volatility of consumption growth to income growth increases with the degree of inattention. For example, when θ decreases from 1 to 0.1, μ_{cy} increases from 0.290 to 0.375, which is the same as the empirical counterpart. It is clear from these results that the direct effect of inattention via the $\Gamma\left(\theta,r^*\right)$ term in (39) dominates its indirect general equilibrium effect via r^*. We can get the same conclusion by shutting down the general equilibrium (GE) channel, see the corresponding partial equilibrium (PE) results reported in the same table. Comparing the GE and PE results in Table 2, we can see the values of μ_{cy} are lower in the GE case if the interest rate is fixed as θ decreases. In other words, the general equilibrium effect of RI tends to reduce the volatility of individual consumption in this case.[32] Furthermore, from the second panel of Table 2, we can see that the equilibrium relative volatility of consumption to income increases with the EIS. The main reason for the result is that the equilibrium interest rate increases with the EIS. Note that the higher the value of the EIS, the higher the dissaving effect due to impatience, and the higher the equilibrium interest rate.

Another important implication of RI in general equilibrium is that RI leads to more skewed wealth dispersion measured by μ_{ay}, the relative volatility of financial wealth to labor income. From the fourth row of Table 2, we can see that when θ is reduced from from 1 to 0.1, μ_{ay} increases from 1.748 to 2.62 in the $\psi = 0.54$ case, which is much closer to the empirical counterpart. (For example, μ_{ay} is 3.11 in 1993 and is 2.59 in 1998.) From (30), it is clear that the main driving force behind this result is the presence of the estimation error, $s_t - \widehat{s}_t$, because $\partial \operatorname{var}\left(s_t - \widehat{s}_t\right) / \partial\theta < 0$. Note that although $\partial r^* / \partial\theta > 0$, the estimation error channel dominates the general equilibrium channel and increases wealth dispersion. Therefore, RI also increases wealth inequality, which makes the model fit the data better.[33] We can also see from the second panel of Table 2, the equilibrium relative volatility of wealth to income increases with the EIS. The main reason for the result is that the equilibrium interest rate increases with the EIS. We can clearly inspect this mechanism by considering a special case when $\phi_1 = 1$. Specifically, in this case, (30) reduces to

$$\mu_{ay} = \sqrt{\frac{1-\theta}{1-(1-\theta)\left(1+r^*\right)^2}}, \tag{41}$$

which implies that μ_{ay} increases with ψ and r^*.

To briefly summarize the key discussions above, Table 3 compares the performances of the FI-RE model, the general equilibrium rational inattention model (RI-GE), and the partial equilibrium rational inattention model (RI-PE) with the data. Overall, it shows under the estimated income process and at a single value of rational inattention parameter (θ), the GE-RI model can do a significantly better job than the FI-RE model in generating a lower interest rate, a higher consumption volatility, and a higher wealth volatility, bring all of them much closer to the data. In terms of welfare loss, as the last row in Table 3 shows and will be discussed in detail in the next subsection, the

[32] We cannot examine the stochastic properties of aggregate consumption dynamics because all idiosyncratic shocks (income shocks and RI-induced noise shocks) cancel out after aggregating across consumers.

[33] The literature has found that simple models based on standard CRRA preferences and on measured uninsurable shocks to labor income cannot account for the observed U.S. wealth distribution. For example, Aiyagari (1994) finds considerably less wealth concentration in a model with only idiosyncratic labor earnings uncertainty. Given the CARA-Gaussian setting, the model here is not suitable to address the issue like why the top 1 percent or 5 percent richest families hold a large fraction of financial wealth in the U.S. economy.

partial equilibrium model significantly underestimates the welfare loss, though the welfare loss is generally small.

Table 4 reports how elastic Kalman gain, the general equilibrium interest rate, and the relative volatility of consumption and wealth to income vary with different values of income uncertainty measured by σ (and σ_y). We have reached four key findings. First, it is clear from the second row of Table 4 that the Kalman gain increases with income volatility. For example, if $\lambda = 0.38$ (the value calibrated to the data as explained in the next paragraph) and $\psi = 0.54$, θ increases from 0.11 to 0.15 when σ increases from 0.2 to 0.4. This means agents optimally allocate more attention to the state variable when income uncertainty increases. Second, RI has significant effects on the equilibrium interest rate. For example, r^* declines from 2.79 percent to 1.94 percent when σ increases from 0.2 to 0.4. It is worth noting that in the elastic capacity case an increase in income volatility affects the equilibrium interest rate via two channels: (i) the direct channel (the ω_ζ^2 term in (38)) and (ii) the indirect channel (the elastic capacity θ term in (38)). The third panel of Table 4 reports the results when we shut down the indirect channel and assume that $\theta = 1$. Comparing the first and third panels of Table 4, we can see that the indirect channel is more important when σ is relatively low. For example, given that $\sigma = 0.2$, r^* decreases from 3.23 percent to 2.79 percent when we switch from the FI economy to the RI economy, whereas r^* only decreases from 2.14 percent to 1.94 percent when $\sigma = 0.4$. Third, EIS can significantly affect the dispersions of consumption and wealth via affecting the optimal attention level and the equilibrium interest rate. For example, when $\sigma = 0.2$ and ψ increases from 0.54 to 0.8, the optimal attention level is reduced from 11% to 8% and the equilibrium interest rate is reduced from 2.79 percent to 2.73 percent. Note that under RI, EIS affects the equilibrium interest rate via two channels: (i) the optimal attention channel and (ii) the impatience-induced dissaving channel. In this quantitative analysis, we can see that the optimal attention channel dominates the impatience-induced dissaving channel. Consequently, μ_{cy} and μ_{ay} increases from 0.34 to 0.43 and from 2.53 to 2.87, respectively. Fourth, the relative volatility of consumption growth to income growth decreases with the value of σ in general equilibrium. That is, consumption becomes smoother when income becomes more volatile. For example, in the equilibrium RI economy when $\psi = 0.54$, μ_{cy} decreases from 0.34 to 0.21 when σ increases from 0.2 to 0.4.[34]

The last finding highlighted above might provide a potential explanation for the empirical evidence documented in Blundell, Pistaferri, and Preston (2008) that income and consumption

[34]It is not surprising that μ_{cy} is greater in the equilibrium RI economy than in the equilibrium FI economy because the value of θ is less than 1 in the RI case. This result is the same as that we obtained in the fixed capacity case and reported in Table 2.

inequality diverged over the sampling period they study.[35] To explore this issue in our model, we do the following exercise. First, we divide the full sample into two sub-samples ($1980-1986$ and $1987-1996$) and apply the same estimation procedure to re-estimate σ and ϕ_1 (see the first and second rows of Table 1 for the estimation results). Household income is more volatile in late sub-periods than earlier ones. Specifically, the standard deviation of y is 0.386 in the sub-sample ($1980-1986$), while it is 0.427 in the sub-sample ($1987-1996$). The average values of μ_{cy} are 0.46 and 0.30 in the first and second sub-samples, respectively. In the elastic capacity case, using the estimated income processes in the first sub-sample, we first use $\mu_{cy}=0.46$ to calibrate $\lambda=0.38$; the corresponding value of θ is 0.08 in the first sub-sample. Using this calibrated value of λ, we find that μ_{cy} is reduced to 0.38 in the second sub-sample, which is much closer to the empirical counterpart than the value obtained in the fixed capacity case (0.41). Note that here we assume that the marginal information-processing cost is invariant across sub-samples.

4.3. Welfare Losses due to RI in Equilibrium

We now turn to the welfare cost of RI – how much utility does a consumer lose if the actual consumption path he chooses under RI deviates from the first-best FI-RE path in which $\theta=1$? To answer this question, we follow Barro (2007) and Luo and Young (2010) by computing the marginal welfare cost due to RI. The following proposition summarizes the main result.

Proposition 5. *Given the initial value of the state, $\widehat{s}_0$, the marginal welfare cost* (mwc) *due to RI is given by*

$$\mathrm{mwc}(\theta)\equiv\frac{\left(\partial v\left(\widehat{s}_0\right)/\partial\theta\right)\theta}{\left(\partial v\left(\widehat{s}_0\right)/\partial\widehat{s}_0\right)\widehat{s}_0}=\frac{\theta\psi}{r^{*2}}\left[\frac{r^*}{\psi}+\frac{1}{\left(1+r^*\right)\widehat{s}_0}\right]\frac{dr^*}{d\theta},\tag{42}$$

where $dr^*/d\theta$ is given in (33) and $\widehat{v}\left(\widehat{s}_0\right)=-\exp\left(-r^*\alpha\widehat{s}_0+\ln\left(1+r^*\right)\right)/\left(r^*\alpha\right)$. The monthly dollar loss due to deviating from the FI-RE path $(\theta=1)$ can be written as

$$\$\,\mathrm{loss}\,(\theta<1)\equiv\frac{r^*}{12}\mathrm{mwc}\,(1)\,(1-\theta)\,\widehat{s}_0.\tag{43}$$

Proof. See Appendix 7.4. Since we are interested in the deviation from the FI-RE path, $\theta=1$ is considered as the starting point. If we change from 1 to θ, the percentage change is $(\theta-1)$. $\widehat{s}_0$ is initial total wealth. Finally, we need to convert the change in the $\widehat{s}_0$ term to monthly rates by multiplying by $r^*/12$. ■

Expression (42) gives the proportionate reduction in the initial level of the perceived state ($\widehat{s}_0$) that compensates, at the margin, for a percentage decrease in θ (i.e., stronger degree of RI) —

[35] Other mechanisms have been proposed for this decline; see Krueger and Perri (2006) and Athreya, Tam, and Young (2009) for examples.

in the sense of preserving the same effect on welfare for a given $\widehat{s}_0$. To do quantitative welfare analysis we need to know the value of $\widehat{s}_0$. First, we set $\widehat{y}_0 \equiv E[y_t] = 1$, $\phi_1 = 0.919$, and the ratio of the initial level of financial wealth $(\widehat{a}_0)$ to mean income $(\widehat{y}_0)$ equal to 5.[36] Second, given that $\widehat{s}_0 = \widehat{a}_0 + \widehat{y}_0 / (1 + r^* - \phi_1) + \overline{y}/r^*$, we can calculate the values of the monthly dollar loss ($ loss) for different values of θ and the corresponding values of the general equilibrium interest rate. The fifth row of Table 2 reports the welfare losses (measured by the dollar) for different degrees of inattention. For example, when $\theta = 0.1$, the welfare loss due to RI in general equilibrium is about $14.64 per month, or about 0.06 percent of mean income per month, which is relatively small.[37] This welfare loss decreases to $8.51 when $\theta = 0.6$, or 0.04 percent of monthly income. This result is similar to the findings by Pischke (1995), Luo (2008), Luo and Young (2010), and Luo, Nie, and Young (2015), and is robust to changes in the income process and the degrees of patience and risk aversion.[38] Thus, it seems reasonable for agents to devote low channel capacity to observing and processing information because the welfare improvement from increasing capacity would be trivial.

Another implication of the welfare losses due to RI reported in Table 2 is that there is a general equilibrium effect of RI on the welfare loss. For example, when r is set to be 3.41 percent (the general equilibrium interest rate obtained under FI) in the partial equilibrium (PE) case, the monthly dollar loss due to RI is significantly less than that obtained in the GE case.[39] For example, when $\theta = 0.2$, the welfare loss in the GE case is $15.64, while it is close to 0.01 in the PE case. The main intuition behind this result is that the general equilibrium channel governed by $dr^*/d\theta$ is shut down in the PE model.

4.4. Policy Implications

In this section, we discuss the effects of changes in public policy on the size of precautionary savings, the interest rate, and the cross-sectional distribution of consumption, wealth, and income in general equilibrium. Some public policies are important for ordinary consumers because they cover such risks as unemployment, health, and longevity, and may affect the need for the consumers to accumulate financial wealth.

Here for simplicity we consider a public policy that can be used to provide social insurance

[36] This number varies largely for different individuals, from 2 to 20. 5 is the average wealth/income ratio in the Survey of Consumer Finances 2001. We find that changing the value of this ratio only has minor effects on the welfare calculation.

[37] In our estimation, we normalize the household income to the 1982 unit. The average value of real disposable personal income per capita is $24, 146 from 1980 to 1996.

[38] Pischke (1995) found that in most cases the utility losses arising from households having no information about aggregate income shocks are less than $1 per quarter in the LQ permanent income model.

[39] See Appendix 7.4 for the derivations of the welfare loss due to RI in partial equilibrium.

by reducing the income variance. Specifically, we assume that the government now imposes a marginal tax rate on labor income (τ) by increasing it from 0 to 1/3. In this case, the labor income risk measured by $(1-\tau)\,\sigma$ is reduced from σ to $(2/3)\,\sigma$. Using the expression for the precautionary saving demand, (21), it is clear that the change in the tax rate leads to a reduction in precautionary savings, holding other parameter values fixed. From the individual consumption function, it is clear that the presence of rational inattention measured by $\Gamma(\theta, r) > 1$ can amplify the impact of this public policy on the precautionary saving demand. In the general equilibrium, given

$$\frac{1}{2}r^{*}\alpha\Gamma\left(\theta, r^{*}\right)\left(1-\tau\right)^{2}\omega_{\zeta}^{2}-\frac{\psi}{r^{*}}\ln\left(\frac{1+\rho}{1+r^{*}}\right)=0, \tag{44}$$

it is clear that the reduction in the labor income risk has two opposite effects on the equilibrium interest rate: (i) it increases the interest rate by reducing the uncertainty about labor income from ω_{ζ}^{2} to $(1-\tau)^{2}\,\omega_{\zeta}^{2}$ (the direct effect) and (ii) it reduces the interest rate by reducing the optimal attention level, θ, and then increasing $\Gamma(\theta, r^{*})$ (the indirect effect). In the quantitative analysis reported in Tabel 4 in which we set $\sigma = 0.3$, it is clear that the direct channel dominates the indirect channel, which drives up the interest rate in the RI model. It is worth noting that in the FI-RE case, the indirect channel disappears, and this public policy unambiguously increases the equilibrium interest rate. Comparing Columns 2 and 3 of Table 4 in the paper, we can see that under RI, r^{*} increases by 20%, while under FI-RE, it is increased by 23%.

Furthermore, from the expression for the relative volatility of consumption to income (39), this public policy can increase μ_{cy} via two channels: (i) increasing the equilibrium interest rate and (ii) increasing $\Gamma(\theta, r^{*})$ by reducing θ.[40] In the FI-RE case, the second channel disappears and only the interest rate channel matters in determining μ_{cy}. For example, Table 4 shows that μ_{cy} increases by 31% under RI, while it only increases by 17% under FI-RE, which means that under RI, the public policy has larger impact on μ_{cy} than under FI-RE.

However, this policy may have opposite effects on the relative volatility of wealth to income, μ_{ay}, in the RI and FI-RE models. In the FI-RE case, this ratio is only determined by the equilibrium interest rate. As shown in Expression (40), it is clear that the higher the value of the interest rate, the less the ratio is. In contrast, in the RI case, the RI-induced noises term drives up μ_{ay}. In Table 4, we can see that the RI-induced noise channel may dominate the interest rate channel, and the ratio increases with the tax rate in the RI case. In contrast, in the FI-RE case, the RI-induced noise channel disappears and only the interest rate channel remains, which makes μ_{ay} decrease with the tax rate. As shown in Table 4, when the policy is implemented, μ_{ay} increases from 2.39 to 2.53

[40]Note that $\Gamma(\theta, r^{*})$ is also increasing in r^{*} for given θ.

under RI, while it is reduced from 1.88 to 1.78 under FI-RE.

5. Comparison with Alternative Models

5.1. Comparison with Habit Formation

An alternative structure that delivers slow consumption dynamics is the habit formation (HF) model of Constantinides (1990). With HF preferences, households try to smooth consumption growth (roughly speaking), rather than the level of consumption; the result is that consumption tends to respond slowly to changes in permanent income. Luo (2008) shows that RI and HF deliver identical aggregate consumption growth movements, but at the individual level RI models deliver more consumption volatility due to the noise shocks (which are canceled out by aggregation).[41]

In this section, we compare the different implications of HF and RI in the general equilibrium framework. Following Alessie and Lusardi (1997), we introduce HF into the FI-RE model specified in Section 2.1 by assuming that the utility function takes the following form:

$$f\left(U_t\right) = f\left(c_t - \gamma c_{t-1}\right) + \frac{1}{1+\rho} f\left(\mathcal{CE}_t\left[U_{t+1}\right]\right), \tag{45}$$

where $\gamma > 0$ is the habit parameter, $f\left(c_t - \gamma c_{t-1}\right) = \left(-\psi\right)\exp\left(-\left(c_t - \gamma c_{t-1}\right)/\psi\right)$, $f\left(U_t\right) = \left(-\psi\right)\exp\left(-U_t/\psi\right)$, $\mathcal{CE}_t\left[U_{t+1}\right] = g^{-1}\left(E_t\left[g\left(U_{t+1}\right)\right]\right)$, and $g\left(U_{t+1}\right) = -\exp\left(-\alpha U_{t+1}\right)/\alpha$. Using the same solution method used in Section 2.1, we can solve for the consumption function under HF:

$$c_t = \frac{\gamma}{1+r} c_{t-1} + r\left(1 - \frac{\gamma}{1+r}\right) s_t + \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\left(1 - \frac{\gamma}{1+r}\right)^2 \omega_\zeta^2. \tag{46}$$

(See Online Appendix D for the derivation.) The corresponding saving function can thus be written as

$$d_t = \left(1-\phi_1\right)\phi\left(y_t - \overline{y}\right) + \frac{r\gamma}{r+1}\frac{\zeta_t}{1-\gamma\cdot L} - \frac{1}{1-\gamma}\left(\frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\left(1-\frac{\gamma}{1+r}\right)^2 \omega_\zeta^2\right).$$

Following the same definition of general equilibrium in our benchmark model, it is straightforward to show that there exists a unique equilibrium interest rate r^* such that

$$\frac{\psi}{r^*}\ln\left(\frac{1+\rho}{1+r^*}\right) - \frac{1}{2}r^*\alpha\widetilde{\Gamma}\left(\gamma, r^*\right)\omega_\zeta^2 = 0,$$

[41] Otrok (2001) notes that HF models deliver an aversion to high frequency movements in consumption, while RI noise shocks would introduce precisely those kinds of movements. Thus, the spectrum of consumption growth would look very different across the two models. We leave to future work an exploration of which framework can better fit that spectrum.

where $\widetilde{\Gamma}(\gamma, r^*) = \left(1 - \frac{\gamma}{1+r^*}\right)^2 < 1$. In general equilibrium, the consumption and saving functions can be written as

$$c_t^* = r^* \left(1 - \frac{\gamma}{1+r^*}\right) s_t + \frac{\gamma}{1+r^*} c_{t-1}^*,$$
$$d_t^* = (1-\phi_1)\,\phi\,(y_t - \overline{y}) + \frac{r\gamma}{1+r}\frac{\zeta_t}{1-\gamma \cdot L},$$

for $t \geq 0$ and given c_{-1}, a_0, y_0, and s_0, and

$$\frac{dr^*}{d\gamma} > 0.$$

That is, the stronger the habit persistence, the higher the equilibrium interest rate.[42] In summary, we can conclude that although both RI and HF lead to slow adjustments in consumption, they have opposite effects on the equilibrium interest rate.[43] RI reduces the equilibrium interest rate, while HF increases it. The second row of Table 5 reports the general equilibrium interest rates for different values of γ.[44] We can see from the table that r^* increases as the degree of habit formation increases. For example, if γ is raised from 0.4 to 0.9, r^* increases from 3.57 percent to 3.79 percent.

The following proposition summarizes the implications of habit formation for the relative volatility of consumption to income as well as the relative volatility of financial wealth to income:

Proposition 6. *Under habit formation, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{r^*\left(1+r^*-\gamma\right)}{1+r^*}\sqrt{\frac{(1+\phi_1)\left\{\frac{\left[(1-\phi_1)\phi+(r^*\phi-1)\right]^2}{1-\phi_1^2} + \frac{(r^*\gamma\phi)^2}{(1-\gamma^2)(1+r^*)^2} + 2\frac{(r^*\gamma\phi)\left[(1-\phi_1)\phi+(r^*\phi-1)\right]}{(1+r^*)(1-\gamma\phi_1)}\right\}}{2\left[1-\left(\frac{\gamma}{1+r^*}\right)^2\right]}}, \tag{47}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{\phi}{\sqrt{2}}\sqrt{(1+\phi_1)\left[\frac{1-\phi_1}{1+\phi_1} + \left(\frac{r^*\gamma}{1+r^*}\right)^2\frac{1}{1-\gamma^2} + \frac{2\left(1-\phi_1\right)r^*\gamma}{\left(1+r^*\right)\left(1-\gamma\phi_1\right)}\right]}. \tag{48}$$

Proof. See Online Appendix C for the derivation. ■

[42]In a partial equilibrium model, Alessie and Lusardi (1997) show that the stronger the habit, the smaller the effect of income uncertainty on the precautionary saving term. Seï¿œkin (2000) shows that in general one cannot determine the connection between habit and the precautionary savings premium in models with more than two periods.

[43]The mechanisms of RI and HF that generate slow adjustment are distinct. Under RI, slow adjustment is forced upon the agent due to finite information processing capacity (learning is slow). In contrast, slow adjustment is optimal under HF because consumers are assumed to prefer to smooth consumption growth.

[44]Here we also set $\gamma = 3$, $\psi = 0.54$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

Expressions (47) and (48) show that habit formation affects the relative volatility of consumption and wealth to income via two channels. The first channel is direct through the presence of γ, whereas the second channel is indirect and operates through the equilibrium interest rate r^*. Given the complexity of these expressions, we cannot obtain explicit results about how habit formation affects the relative volatility of consumption and wealth to income. We therefore use the same parameter values as used in the preceding subsection to do a quantitative analysis. Table 5 reports the general equilibrium and partial equilibrium interest rates for different values of γ. It is clear from the table that the relative volatility of consumption to income is decreasing with the degree of habit formation. For example, when γ is increased from 0.4 to 0.9, μ_{cy} is reduced from 0.21 to 0.09 in general equilibrium; as with RI, the general equilibrium effects are small and the slow adjustment in consumption channel dominates the general equilibrium channel. Thus, the relative volatility of consumption to income is even lower than that predicted in the FI-RE case, and makes the model fit the data worse in this dimension.

From the fourth row of the table, we can see that the relative volatility of wealth to income (μ_{ay}) increases with the degree of habit formation. For example, if γ is increased from 0.4 to 0.9, μ_{ay} increases from 1. 76 to 2.30 in general equilibrium. The main reason for this result is that habit formation affects the wealth accumulation response to an income shock by slowing the adjustment of consumption to the shock, which creates a temporary gap between consumption and permanent income, which exaggerates wealth accumulation effects and thereby increases the volatility of wealth accumulation that results from income shocks. Even for a high degree of habit ($\gamma = 0.9$), the model's prediction on μ_{ay} is still well below its empirical counterpart (3.28). In addition, values of γ that high cannot be reconciled with some of the large changes in consumption observed at the individual level; in effect, one is forced to suppose that these movements are almost entirely measurement error.[45]

5.2. Comparison with Incomplete Information about Income

In this subsection, we consider an incomplete information (IC) model in which the income process has two components and consumers cannot distinguish the two components. Following Muth (1960) and Pischke (1991), we assume that observed (measured) labor income includes a unit root and the whole income process has two kinds of structural shocks to labor income: One has a per-

[45] Technically, this statement holds only if the utility function does not permit "effective consumption" $c_t - \gamma c_{t-1}$ to be negative, as would the case with CRRA preferences. CARA preferences are defined for negative values. Nevertheless, we find the general principle likely to be true. Multiplicative habits of the form c_t / c_{t-1}^{γ} permit much larger movements in individual consumption; see Gayle and Khorunzina (2016) for estimates using the PSID that control for measurement error.

manent impact on the level of labor income and the other has only transitory impact.[46] Specifically, the income process can be written as:

$$y_{t+1} = y_{t+1}^{p} + y_{t+1}^{i}, \tag{49}$$

$$y_{t+1}^{p} = y_{t}^{p} + \varepsilon_{t+1}, \tag{50}$$

$$y_{t+1}^{i} = \overline{y} + \xi_{t+1}, \tag{51}$$

where y_{t+1}^{p} and y_{t+1}^{i} are permanent and transitory components in measured income, respectively, ε_{t+1} and ξ_{t+1} are orthogonal permanent and transitory iid shocks with mean 0 and variance ω_{ε}^{2} and ω_{ξ}^{2}, respectively. Note that here we can interpret the iid component, ξ_{t+1}, as measurement error which would destroy the identification of the true level of labor income.

Given that the change in income is

$$\Delta y_{t+1} = \varepsilon_{t+1} + \epsilon_{t+1} - \epsilon_{t}, \tag{52}$$

the best forecast is to recognize that Δy_{t+1} is a moving-average process of order one:

$$\Delta y_{t+1} = \nu_{t+1} - \tau \nu_{t}, \tag{53}$$

where the innovation, $\nu_{t} \sim N\left(0, \omega_{\nu}^{2}\right)$, is not a fundamental driving process – it contains information on current and lagged permanent and transitory income shocks. Equating the variances and autocorrelation coefficients of the original and derived processes (52) and (53), we have

$$\omega_{\nu}^{2} = \frac{\mathrm{var}\left(\Delta y_{t+1}\right)}{1+\tau^{2}} = \frac{\omega_{\epsilon}^{2}}{\tau},$$

where $\tau = -\left(1-\sqrt{1-4\varrho^{2}}\right)/\left(2\varrho\right)$ and $\varrho = -\omega_{\xi}^{2}/\left(\omega_{\varepsilon}^{2}+2\omega_{\xi}^{2}\right) \in (-0.5, 0]$. $\tau \in [0,1]$ will be large if the variance of the transitory shock ω_{ξ}^{2} is large relative to the variance of the permanent shock ω_{ε}^{2} and will converge to 0 as ω_{ξ}^{2} approaches 0. In the following analysis, we use τ to measure the relative importance of measurement error and thus the degree of incomplete information.

Following the same procedure in Section 2.1, we define a new state variable, $s_{t} = a_{t} + y_{t}/r -$

[46] Wang (2004) considered a similar incomplete information and signal extraction problem in continuous-time, and assumed that the two individual components in income follow different Ornstein-Uhlenbeck processes. Here for simplicity we just consider the permanent-transitory decomposition in income. The main conclusion about the comparision between the IC model and the RI model still holds if we assume that the two individual components follow AR(1) processes. The proof is available from the corresponding author by request.

$\tau\nu_t / \left(r\left(1+r\right)\right)$, for the IC problem, and rewrite the original budget constraint as follows:

$$s_{t+1} = \left(1+r\right)s_t - c_t + \zeta_{t+1}, \tag{54}$$

where

$$\zeta_{t+1} = \frac{1+r-\tau}{r\left(1+r\right)}\nu_{t+1}. \tag{55}$$

Maximizing the typical consumer's lifetime utility subject to (54) leads to the following consumption and saving functions:

$$c_t^* = rs_t + \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) - \frac{1}{2}\alpha r\omega_\zeta^2,$$

and

$$d_t^* = \tau\nu_t - \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \frac{1}{2}\alpha r\omega_\zeta^2.$$

Since ν_t is an idiosyncratic innovation with mean zero, following the same definition of general equilibrium in our benchmark model, it is straightforward to show that there exists a unique equilibrium interest rate r^* such that

$$\frac{1}{2}\alpha r^*\omega_\zeta^2 - \frac{\psi}{r^*}\ln\left(\frac{1+\rho}{1+r^*}\right) = 0. \tag{56}$$

It is straightforward to show that

$$\frac{dr^*}{d\omega_\zeta^2} < 0.$$

Given that

$$\omega_\zeta^2 = \left[\frac{1+r-\tau}{r\left(1+r\right)}\right]^2 \frac{\mathrm{var}\left(\Delta y_{t+1}\right)}{1+\tau^2}$$

is decreasing with τ, it is clear that the higher the degree of incomplete information, the higher the equilibrium interest rate.

In summary, we can conclude that although both RI and IC lead to slow adjustments in consumption, they have opposite effects on the equilibrium interest rate. RI reduces the equilibrium interest rate, while IC increases it. The second panel of Table 5 reports the general equilibrium interest rates for different values of τ.[47] We can see from the table that r^* increases as the degree of incomplete information increases. For example, when τ is raised from 0.4 to 0.9, r^* increases from 1.30 percent to 4.10 percent.

The following proposition summarizes the implications of incomplete information for the relative volatility of consumption to income as well as the relative volatility of financial wealth to

[47] Here we also set $\gamma = 3$, $\psi = 0.54$, $\phi_1 = 0.92$, $\sigma = 0.175$, and $\rho = 0.04$.

income:

Proposition 7. *Under incomplete information, the relative volatility of individual consumption growth to income growth is*

$$\mu_{cy} \equiv \frac{sd\left(\Delta c_t^*\right)}{sd\left(\Delta y_t\right)} = \left(1 - \frac{\tau}{1+r^*}\right)\sqrt{\frac{1}{1+\tau^2}}, \tag{57}$$

and the relative volatility of financial wealth to income is

$$\mu_{ay} \equiv \frac{sd\left(\Delta a_t^*\right)}{sd\left(\Delta y_t\right)} = \frac{\tau}{1+r^*}\sqrt{\frac{1}{1+\tau^2}}. \tag{58}$$

Proof. Using the expressions for the equilibrium consumption and asset accumulation functions, it is straightforward to show that

$$\Delta c_{t+1}^* = r\zeta_{t+1} = \left(1 - \frac{\tau}{1+r^*}\right)\nu_{t+1} = \left(1 - \frac{\tau}{1+r^*}\right)\frac{\Delta y_{t+1}}{1-\tau\cdot L}, \tag{59}$$

$$\Delta a_{t+1}^* = \frac{\tau}{1+r^*}\nu_t = \frac{\tau}{1+r^*}\frac{\Delta y_t}{1-\tau\cdot L}. \tag{60}$$

Taking unconditional variance on both sides of (59) and (60) yields (57) and (58). ∎

From (57) and (58), we can see that incomplete information about current income affects the relative volatility of consumption and wealth to income via two channels. The first channel is direct through the presence of τ, whereas the second channel is indirect and operates through the equilibrium interest rate r^*. Using the same parameter values as used in the preceding subsection, Table 5 reports the general equilibrium results for different values of τ. It is clear from the table that the relative volatility of consumption to income decreases with the degree of incomplete information. For example, if τ is increased from 0.4 to 0.9, μ_{cy} falls from 0.56 to 0.10 in general equilibrium.

From the second panel of the table, we can see that the relative volatility of wealth to income (μ_{ay}) increases with the degree of incomplete information. For example, if τ is increased from 0.4 to 0.9, μ_{ay} increases from 0.37 to 0.64 in general equilibrium. The main reason for this result is that incomplete information about income affects the wealth accumulation response to an income shock by slowing the adjustment of consumption to the shock, which creates a temporary gap between consumption and permanent income and thereby increases the volatility of wealth accumulation that results from income shocks. Even for a high degree of incomplete information (e.g., $\tau = 0.9$), the model's prediction on μ_{ay} is still well below its empirical counterpart (3.28).

5.3. Comparison with Models with Borrowing Constraints

Our version of Huggett (1993) abstracts from a key feature – borrowing constraints. As noted in many papers, borrowing constraints can deliver consumption dynamics that display excess sensitivity to predictable movements in income, although Ludvigson and Michaelides (2001) and Hryshko (2014) show that the basic model with borrowing constraints does not deliver the observed excess sensitivity in micro or macro data. We argue in this subsection that borrowing constraint models make predictions regarding the dispersion of consumption relative to income that is inconsistent with our data, and therefore that RI models are to be preferred.

Standard models with borrowing constraints (Huggett 1993, Aiyagari 1994) imply that poor households (those close to the borrowing constraint) have a relatively high marginal propensity to consume out of wealth (and income); the result is that for a given dispersion in income, consumption changes are more volatile and wealth changes less volatile among the poor. Indeed, using a simple benchmark version of Aiyagari (1994), we find that μ_{cy} for the poor (those with below median wealth) is roughly twice μ_{cy} for the rich (those with above median wealth); similarly, μ_{ay} is about 80 percent smaller for the poor than the rich. And these relationships are even stronger if we consider percentiles further out in the tails.[48]

This variation across wealth is strongly rejected in our data. Table 6, based on the PSID data, shows that the relative dispersion of consumption for the whole sample is almost the same as that of the top 50 percent by income levels. That is, even if we exclude the bottom 50 percent of households by income the ratio is nearly unchanged, which suggests borrowing constraints do not play a significant role in the dynamics of consumption relative to income. The dispersion ratio varies slightly more if we select households by their wealth levels, although our sample size shrinks significantly due to limited wealth information in our data.

6. Concluding Remarks

In this paper we have studied how rational inattention affects the interest rate and the joint dynamics of consumption and income in a Huggett-type general equilibrium model with recursive utility. We highlight our two main results here in the conclusion. First, RI helps the basic model deliver a good fit to the dispersions of consumption and wealth relative to income; more inattention leads to more dispersion, pushing the models closer to the data, and we find that there is a common value of the inattention parameter that delivers a good fit for both. The effects on the equilibrium

[48] We use a version of Aiyagari (1994) with inelastic labor to make these points; see Online Appendix D for details. Elastic labor strengthens the results – the relative volatility of consumption is three times as large for the poor as the rich. Dividing households by income leads to similar conclusions.

interest rate are modest, and work to slightly offset the added dispersion. These results are robust to the exact way attention is modeled (elastic or inelastic), and the elastic attention version also captures the observed movements in consumption dispersion relative to income dispersion evident in US data. Second, we compare RI to three alternative models – habit formation, incomplete information about income, and borrowing constraints. We find that the RI model delivers better predictions regarding these dispersions than the alternatives.

7. Appendix

7.1. Deriving the Consumption, Saving, and Value Functions in the Cabellero-Huggett Model with RU and RI

Using the income process, the original budget constraint (3) can be rewritten as

$$a_{t+1} + \phi y_{t+1} + \frac{\phi\phi_0}{r} = (1+r)\,a_t + y_t - c_t + \phi\left(\phi_0 + \phi_1 y_t + w_{t+1}\right) + \frac{\phi\phi_0}{r}$$
$$= (1+r)\left(a_t + \phi y_t + \frac{\phi\phi_0}{r}\right) - c_t + \zeta_{t+1},$$

where the $(t+1)$-innovation $\zeta_{t+1} = \phi w_{t+1}$ is Gaussian innovation process with mean zero and variance $\phi^2\sigma^2$. Denote $s_t = a_t + \phi y_t + \phi\phi_0/r$, the new budget constraint can be rewritten as

$$s_{t+1} = (1+r)\,s_t - c_t + \zeta_{t+1}.$$

As shown in Section 2.2, under RI, the typical consumer uses the Kalman filter, (14), to update the perceived state, $\widehat{s}_t$. The objective of the consumer is to solve the following Bellman equation based on the recursive utility defined in Section 2.1:

$$f\left(J\left(\widehat{s}_t\right)\right) = \max_{c_t}\left\{f\left(c_t\right) + \frac{1}{1+\rho} f\left(\mathcal{CE}_t\left[J\left(\widehat{s}_{t+1}\right)\right]\right)\right\}, \tag{61}$$

subject to (14). We first conjecture that $J\left(\widehat{s}_t\right) = A\widehat{s}_t + A_0$, where A and A_0 are undetermined coefficients. Substituting the guessed function into the definition of the certainty equivalent: $\exp\left(-\alpha\mathcal{CE}_t\right) = E_t\left[\exp\left(-\alpha J\left(\widehat{s}_{t+1}\right)\right)\right]$, we have

$$\exp\left(-\alpha\mathcal{CE}_t\right) = \exp\left(-\alpha A E_t\left[\widehat{s}_{t+1}\right] + \frac{1}{2}\alpha^2 A^2 \operatorname{var}_t\left[\widehat{s}_{t+1}\right] - \gamma A_0\right)$$
$$= \exp\left(-\alpha A\left[(1+r)\,\widehat{s}_t - c_t\right] + \frac{1}{2}\alpha^2 A^2 \omega_{\widehat{\zeta}}^2 - \gamma A_0\right),$$

which implies that

$$\mathcal{CE}_t = A\left[(1+r)\,s_t - c_t - \frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right] + A_0.$$

Substituting these expressions into (61) yields:

$$f\left(J\left(\widehat{s}_t\right)\right) = \max_{c_t}\left\{f\left(c_t\right) + \frac{1}{1+\rho}f\left(J\left((1+r)\,\widehat{s}_t - c_t - \frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right)\right)\right\}. \tag{62}$$

The FOC for c_t is thus

$$f'\left(c_t\right) = \frac{A}{1+\rho}f'\left(A\left[(1+r)\,s_t - c_t - \frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right] + A_0\right).$$

The Envelop theorem can be written as

$$f'\left(J\left(\widehat{s}_t\right)\right) = \frac{1+r}{1+\rho}f'\left(A\left[(1+r)\,s_t - c_t - \frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right] + A_0\right).$$

Combining these two conditions yields

$$c_t = A\widehat{s}_t + A_0 - \psi\ln\left(\frac{A}{1+r}\right). \tag{63}$$

Substituting the consumption function into (62) yields:

$$\begin{aligned}\exp\left(-\frac{1}{\psi}\left(A\widehat{s}_t + A_0\right)\right) &= \exp\left(-\frac{1}{\psi}\left(A\widehat{s}_t + B - \psi\ln\left(\frac{A}{1+r}\right)\right)\right)\\ &\quad + \frac{1}{1+\rho}\exp\left(-\frac{1}{\psi}\left[A\left((1+r)\,\widehat{s}_t - \left(A\widehat{s}_t + B - \psi\ln\left(\frac{A}{1+r}\right)\right) - \frac{1}{2}\alpha A\omega_{\widehat{\zeta}}^2\right) + A_0\right]\right).\end{aligned}$$

Matching the $\widehat{s}_t$ terms in the exponential functions, we obtain that

$$A = r.$$

Matching the constant coefficient terms yields:

$$A_0 = \frac{\psi}{r}\ln\left(\frac{1+\rho}{1+r}\right) + \psi\ln\left(\frac{r}{1+r}\right) - \frac{1}{2}\alpha r\omega_{\widehat{\zeta}}^2$$

Substituting the expressions for A and A_0 into (63) yields (21) in the main text. The corresponding value function can be written as

$$\widehat{v}\left(\widehat{s}_t\right) = -\psi\exp\left(-\frac{1}{\psi}J\left(\widehat{s}_t\right)\right) = -\psi\exp\left(-\frac{1}{\psi}\left(A\widehat{s}_t + A_0\right)\right),$$

which is just (20) in the main text.

Using (14), (21), and (63), we can derive the savings function:

$$
\begin{aligned}
d_t^* &= ra_t + y_t - c_t^* \\
&= ra_t + y_t - c_t + (c_t - c_t^*) \\
&= ra_t + y_t - r\left[a_t + \phi y_t + \frac{\phi\phi_0}{r} + \frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right)\right] + \\
&\left\{\begin{array}{c} r\left[a_t + \phi y_t + \phi\frac{\phi_0}{r} + \frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\phi w_{t+1}\right)\right]\right)\right)\right] \\ -r\left[\widehat{s}_t + \frac{1}{r^2\alpha}\left(\ln\left(\frac{1+\rho}{1+r}\right) - \ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right)\right)\right] \end{array}\right\} \\
&= (1-\phi_1)\,\phi\,(y_t - \overline{y}) + r\,(s_t - \widehat{s}_t) + \frac{1}{r\alpha}\left[\ln\left(E_t\left[\exp\left(-r\alpha\widehat{\zeta}_{t+1}\right)\right]\right) - \ln\left(\frac{1+\rho}{1+r}\right)\right].
\end{aligned}
$$

To derive the relative volatility of financial wealth (a_t) and labor income (y_t) in general equilibrium, we first rewrite the above saving equation as follows:

$$
\Delta a_{t+1}^* = d_t^* = (1-\phi_1)\,\phi\,(y_t - \overline{y}) + r^*\,(s_t - \widehat{s}_t)\,.
$$

Taking unconditional variance on both sides yields:

$$
\begin{aligned}
\operatorname{var}\left(d_t^*\right) &= \operatorname{var}\left((1-\phi_1)\,\phi\,(y_t-\overline{y}) + r^*\,(s_t-\widehat{s}_t)\right) \\
&= \operatorname{var}\left((1-\phi_1)\,\phi\,(y_t-\overline{y})\right) + \operatorname{var}\left(r^*\,(s_t-\widehat{s}_t)\right) + 2\operatorname{cov}\left((1-\phi_1)\,\phi\,(y_t-\overline{y}),\, r^*\,(s_t-\widehat{s}_t)\right) \\
&= \left[\frac{1-\phi_1}{1+\phi_1} + \frac{(1-\theta)\,r^{*2}}{1-(1-\theta)\,(1+r^*)^2} + \frac{2r^*\,(1-\phi_1)\,(1-\theta)}{1-\phi_1(1-\theta)\,(1+r^*)}\right]\frac{\omega^2}{(1+r^*-\phi_1)^2},
\end{aligned}
$$

which is just (40) in the main text, where we use the expression for $s_t - \widehat{s}_t$ specified in (18). Furthermore, using (17) and (29), it is straightforward to show that the relative volatility of consumption growth to income growth is (39) in the main text.

7.2. Optimality of Ex Post Gaussianity under RI

Following Sims (2003, 2010), we first define the expected loss function due to limited information-processing capacity as

$$
L_t = E_t\left[v_0\left(s_t\right) - \widehat{v}\left(x_t\right)\right], \tag{64}
$$

where s_t is the unobservable state variable, x_t is the best estimate of the true state, $\widehat{v}\left(x_t\right) = -\frac{\psi(1+r)}{r}\exp\left(-\frac{r}{\psi}x_t\right)$ is the value function under RI and $v_0\left(s_t\right) = -\frac{\psi(1+r)}{r}\exp\left(-\frac{r}{\psi}s_t\right)$ is the corresponding value function when $\kappa = \infty$. It is straightforward to show that:

$$\begin{aligned}
&\min E_t \left[v_0 \left(s_t\right) - \widehat{v} \left(x_t\right)\right] \\
&= \min -\frac{\psi \left(1+r\right)}{r} E_t \left[\exp\left(-\frac{r}{\psi} s_t\right) - \exp\left(-\frac{r}{\psi} x_t\right)\right] \\
&\simeq \min -\frac{\psi \left(1+r\right)}{r} E_t \left[-\frac{r}{\psi} \exp\left(-\frac{r}{\psi} x_t\right) \left(s_t - x_t\right) + \frac{1}{2} \left(\frac{r}{\psi}\right)^2 \exp\left(-\frac{r}{\psi} x_t\right) \left(s_t - x_t\right)^2\right] \\
&\Longleftrightarrow \min -\frac{\psi \left(1+r\right)}{2r} E_t \left[\left(\frac{r}{\psi}\right)^2 \exp\left(-\frac{r}{\psi} x_t\right) \left(s_t - x_t\right)^2\right] \\
&\Longleftrightarrow \min -\frac{r \left(1+r\right)}{2\psi} \exp\left(-\frac{r}{\psi} x_t\right) E_t \left[\left(s_t - x_t\right)^2\right].
\end{aligned}$$

Since x_t is non-stationary process, we normalize this objective function by dividing it by the value function under RI:

$$\min \frac{E_t \left[v_0 \left(s_t\right) - \widehat{v} \left(x_t\right)\right]}{\widehat{v} \left(x_t\right)} \Longleftrightarrow \min \frac{1}{2} \left(\frac{r}{\psi}\right)^2 E_t \left[\left(s_t - x_t\right)^2\right] \Longleftrightarrow \min \frac{1}{2} \left(\frac{r}{\psi}\right)^2 \Sigma_t, \tag{65}$$

where Σ_t is the conditional variance at t. The normalization method is also used in Maenhout (2004) and Liu, Pan, and Wang (2005) in which they used this normalization method to assure the homothecity or scale invariance of the optimal consumption and portfolio decision problem when investors are concerned about model misspecification. Note that the agent's filtering problem, (65), is invariant to the scale of perceived total resources x_t after normalization, which we use so that the optimal choice of attention does not disappear as the value of total wealth increases.

The (approximate) loss function under CARA derived above is essentially the same as that obtained in the LQG RI model proposed in Sims (2003). Since the only difference in these two settings is just in the constant coefficient in the loss function, the CARA specification does not affect the optimality of ex post Gaussianity in Sims' LQG setting after we approximate the value functions we obtained in the CARA-Gaussian setting.

Furthermore, we use the following procedure to justify the quadratic approximation, (65):

1. We first conjecture that the quadratic approximation is an accurate approximation for the original exponential loss function, and the higher-order moments in the expansion of the CARA loss function are trivial.

2. Given that the loss function is quadratic, we can obtain the optimality of the ex post Gaussian variables and Gaussian noise and noisy signal (i.e., both x and s are Gaussian).

3. For Gaussian variables, s_t and x_t, the higher-order moments (the third and fourth moments)

of the loss function can be written as

$$\frac{(r/\psi)^2}{6}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)E_t\left[(s_t-x_t)^3\right]-\frac{(r/\psi)^3}{24}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)E_t\left[(s_t-x_t)^4\right]$$
$$=-\frac{(r/\psi)^3}{24}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)\Sigma^2\times 3,$$

where we use the facts that for Gaussian variables, we have

$$E_t\left[(s_t-x_t)^j\right]=\begin{cases}\sigma^j(j-1)!!, \text{ when } j \text{ is even,}\\ 0, \text{ when } j \text{ is odd.}\end{cases},$$

where $x_t=E_t[s_t]$ and σ is the standard deviation of x.

4. It is straightforward to calculate that the ratio of the sum of the third and fourth moments to the second moment is:

$$ratio=\frac{-\frac{(r/\psi)^3}{24}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)\Sigma^2\times 3}{-\frac{r}{2\psi}\exp\left(\frac{-rb_0-rx_t}{\psi}\right)\Sigma}=\frac{(r\alpha)^2\Sigma}{4}=0.0287^2\times 15.77=0.013,$$

where we use the fact that $\Sigma=15.77$ when $\theta=10\%$. We can then verify that our guess that the quadratic approximation is accurate is correct.

7.3. Proof of Uniqueness of General Equilibrium in the Benchmark Model

To prove uniqueness, consider the derivative of the aggregate saving function, $D(\theta,r)=\Pi(\theta,r)-\psi\Psi(r)/r$, with respect to r: we have

$$\begin{aligned}\frac{dD}{dr}&=\left[\frac{\psi}{r(1+r)}+\frac{\psi}{r^2}\ln\left(\frac{1+\rho}{1+r}\right)\right]\\&+\frac{1}{2}\alpha\sigma^2\left\{\Gamma(\theta,r)\frac{(1+r-\phi_1)^2-2r(1+r-\phi_1)}{(1+r-\phi_1)^4}+\frac{r}{(1+r-\phi_1)^2}\frac{2\theta(1-\theta)(1+r)}{\left[1-(1-\theta)(1+r)^2\right]^2}\right\}\\&=\frac{\psi}{r(1+r)}+\frac{\psi}{r^2}\ln\left(\frac{1+\rho}{1+r}\right)\left[\frac{2(1-\phi_1)}{1+r-\phi_1}+\frac{2r(1-\theta)(1+r)}{1-(1-\theta)(1+r)^2}\right]>0\end{aligned}$$

when θ is fixed, because $\phi_1<1$, where the last line is obtained by using the general equilibrium condition, $\frac{1}{2}r\Gamma(\theta,r)\alpha\left(\frac{\sigma}{1+r-\phi_1}\right)^2+\frac{\psi}{r}\ln\left(\frac{1+r}{1+\rho}\right)=0$. From this expression, we can see that fixed capacity does not change the equilibrium property of the model because $\frac{2r(1-\theta)(1+r)}{1-(1-\theta)(1+r)^2}$ is always positive.[49]

7.4. Computing the Welfare Loss due to RI

Given that the value function under RI in general equilibrium is

$$\widehat{v}\left(\widehat{s}_0\right) = -\frac{\psi\left(1+r^*\right)}{r^*}\exp\left(-\frac{r^*}{\psi}\widehat{s}_t\right),$$

we can compute the following partial derivatives:

$$\frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\theta} = \frac{\exp\left(-r^*\widehat{s}_0/\psi\right)}{r^{*2}/\psi}\left[1+\frac{r^*\left(1+r^*\right)}{\psi}\widehat{s}_0\right]\frac{dr^*}{d\theta} \text{ and } \frac{\partial\widehat{v}(\widehat{s}_0)}{\partial\widehat{s}_0} = \left(1+r^*\right)\exp\left(-r^*\widehat{s}_0/\psi\right).$$

The marginal welfare cost due to RI can thus be written as:

$$\text{mwc} \equiv \frac{\left(\partial v\left(\widehat{s}_0\right)/\partial\theta\right)\theta}{\left(\partial v\left(\widehat{s}_0\right)/\partial\widehat{s}_0\right)\widehat{s}_0} = \frac{\theta\psi}{r^{*2}}\left[\frac{r^*}{\psi}+\frac{1}{\left(1+r^*\right)\widehat{s}_0}\right]\frac{dr^*}{d\theta},$$

where we use the facts that in general equilibrium (i.e., $\ln\left(\frac{1+\rho}{1+r}\right) = \ln\left(E_t\left[\exp\left(-r^*\widehat{\zeta}_{t+1}/\psi\right)\right]\right)$), and $dr^*/d\theta$ is given in (33).

In the partial equilibrium setting in which $r^* = r$ is fixed,

$$\begin{aligned}
\widehat{v}\left(\widehat{s}_t\right) &= -\frac{\psi}{r}\exp\left(-\frac{r}{\psi}\left\{\widehat{s}_t - \frac{\psi}{r}\ln\left(1+r\right)+\frac{\psi}{r^2}\left[\ln\left(\frac{1+\rho}{1+r}\right)-\ln\left(E_t\left[\exp\left(-\frac{r}{\psi}\widehat{\zeta}_{t+1}\right)\right]\right)\right]\right\}\right) \\
&= -\frac{\psi\left(1+r\right)}{r}\exp\left(-\frac{r}{\psi}\widehat{s}_t\right)\exp\left(-\frac{1}{r}\ln\left(\frac{1+\rho}{1+r}\right)+\frac{1}{2}\frac{\theta}{1-\left(1-\theta\right)\left(1+r\right)^2}\frac{r}{\psi^2}\omega_\zeta^2\right).
\end{aligned}$$

Note that in partial equilibrium, $\ln\left(\frac{1+\rho}{1+r}\right)$ and $\ln\left(E_t\left[\exp\left(-r\widehat{\zeta}_{t+1}/\psi\right)\right]\right)$ do not cancel out. The marginal welfare cost due to RI in partial equilibrium can thus be written as:

$$\text{mwc} = \frac{\theta\omega_\zeta^2}{2\psi\widehat{s}_0}\frac{\left(1+r\right)^2-1}{\left[1-\left(1-\theta\right)\left(1+r\right)^2\right]^2}.$$

The month dollar loss can be thus written as:

$$\$\,\text{loss}\left(\theta<1\right) \equiv \frac{1}{12}r\text{mwc}\left(1\right)\left(1-\theta\right)\widehat{s}_0 = \frac{1}{12}r\left(\frac{\omega_\zeta^2}{2\psi}\right)\left(\left(1+r\right)^2-1\right)\left(1-\theta\right).$$

[49] For the elastic RI case, the derivatives are too complicated to sign because θ itself is a function of r. However, given the plausible parameter values used in this paper, the equilibrium is unique.

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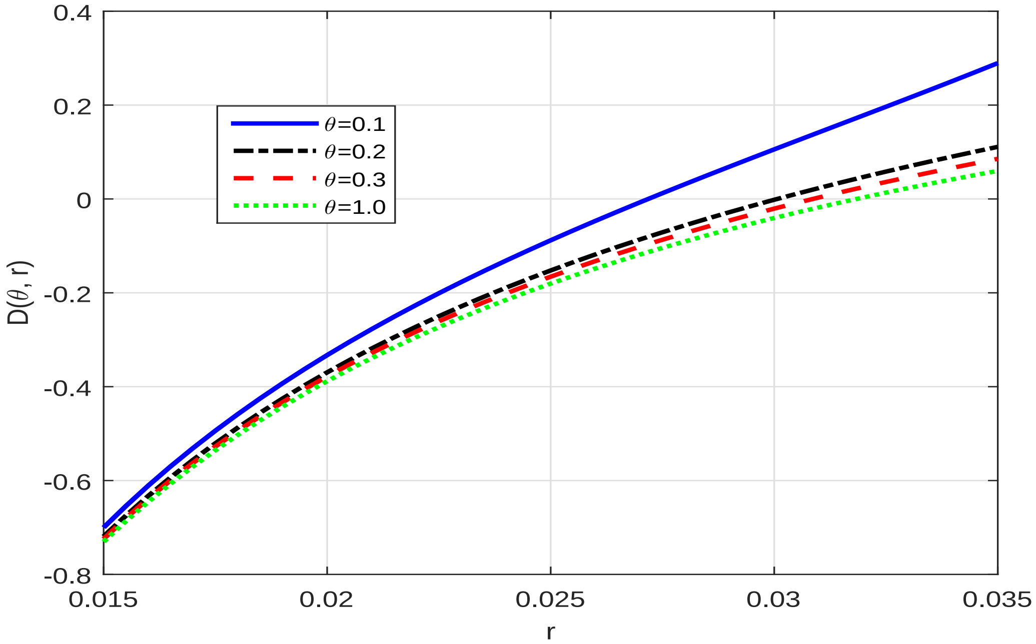

Figure 1. *Effects of RI on Aggregate Saving*

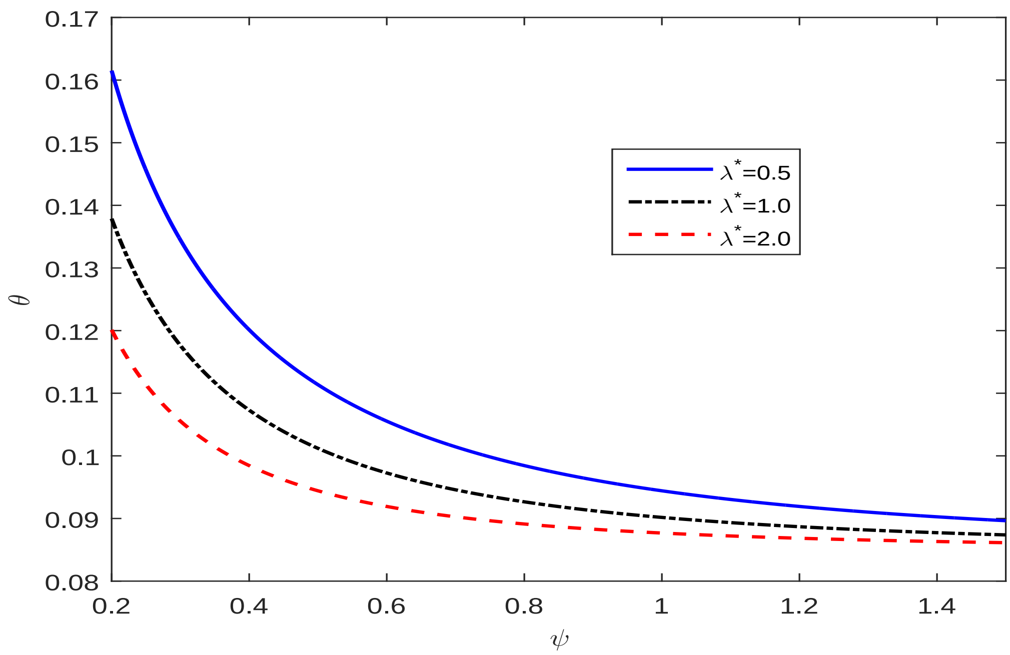

Figure 2. *Effects of ψ on Elastic Attention*

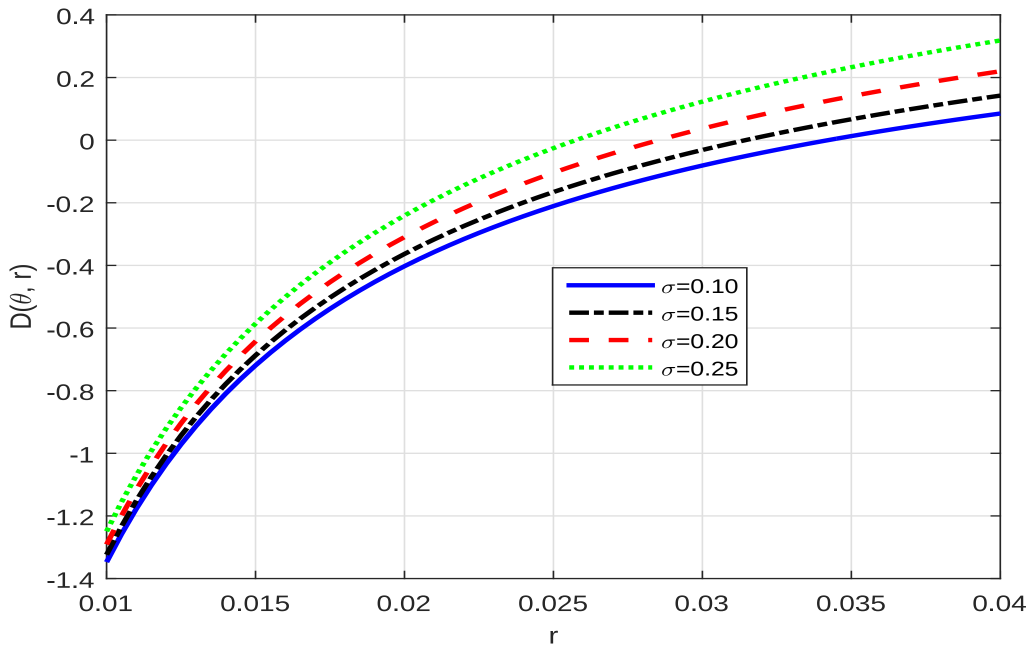

Figure 3. *Effects of Income Volatility on the Interest Rate in GE (Elastic κ)*

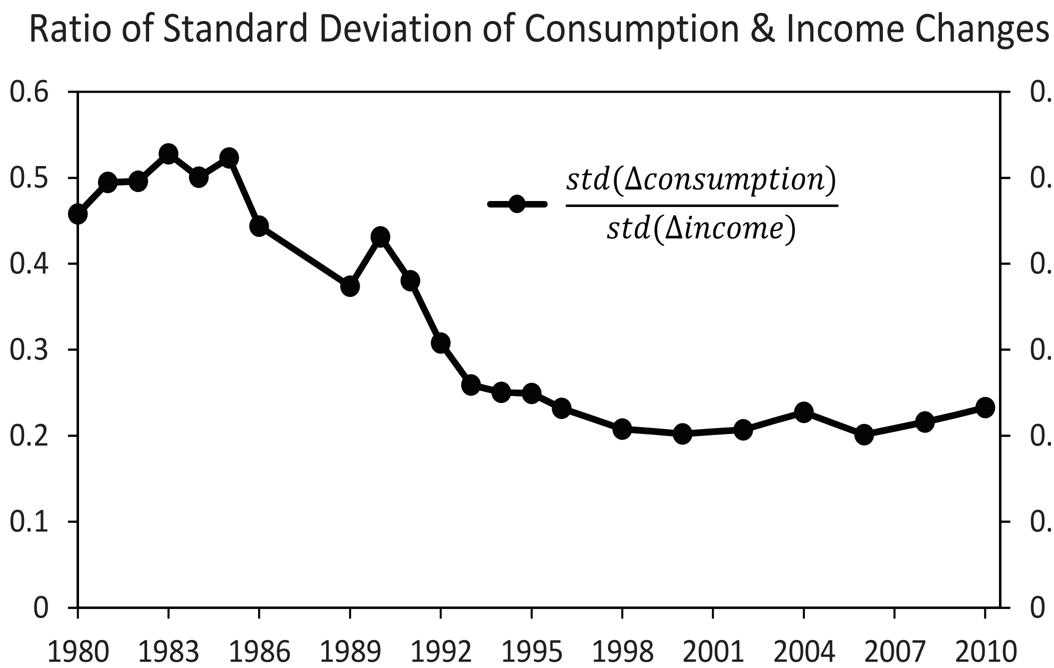

Figure 4. *Relative Consumption Dispersion*

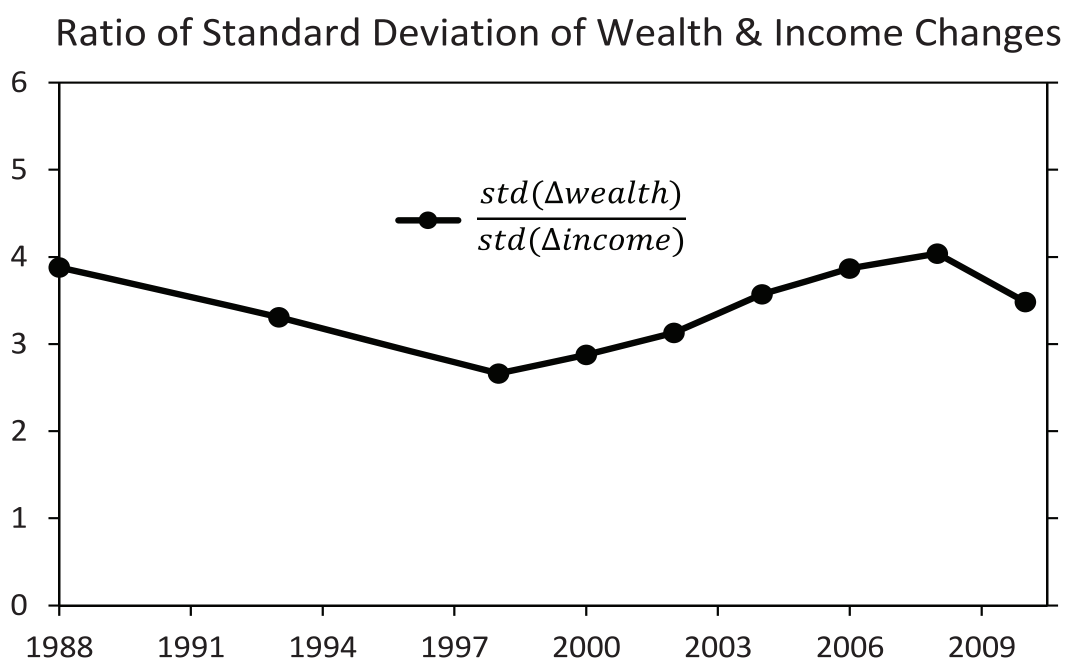

Figure 5. *Relative Dispersion of Wealth to Income*

Table 1. *Estimation of the Income Process*

	std (ϵ_{it})	ϕ_{it}	std (y_{it})
Period 1 (1980 – 1986)	0.154	0.917	0.386
Period 2 (1987 – 1996)	0.175	0.912	0.427
Full Period (1980 – 1996)	0.175	0.919	0.444

Table 2. *Implications of RI for Interest rates, the Relative Volatility of Consumption and Wealth to Income, and Welfare*

	θ	10%	20%	40%	60%	100%
GE ($\psi = 0.54$)	$r^*(\%)$	2.89	3.22	3.35	3.38	3.41
	μ_{cy}	0.375	0.324	0.302	0.296	0.290
	μ_{ay}	2.620	2.208	1.945	1.840	1.748
	\$loss	14.64	15.64	12.53	8.51	0
GE ($\psi = 0.8$)	$r^*(\%)$	3.12	3.46	3.57	3.60	3.63
	μ_{cy}	0.416	0.346	0.318	0.309	0.303
	μ_{ay}	2.671	2.203	1.922	1.812	1.716
	\$loss	15.78	16.58	13.11	8.87	0
PE ($\psi = 0.54$)	μ_{cy}	0.474	0.342	0.307	0.297	0.290
($r = 3.41\%$)	μ_{ay}	2.751	2.204	1.938	1.837	1.748
	\$loss	0.006	0.005	0.004	0.003	0

Table 3. *Model Comparison*

	Data	FI-RE ($\psi = 0.54$)	FI-RE ($\psi = 0.8$)	RI-GE ($\theta = 0.1$, $\psi = 0.54$)	RI-GE ($\theta = 0.1$, $\psi = 0.8$)	RI-PE ($\theta = 0.1$, $\psi = 0.54$)
$r^*(\%)$	2.97	3.41	3.63	2.89	3.12	n.a.
μ_{cy}	0.38	0.29	0.30	0.38	0.42	0.47
μ_{ay}	3.28	1.75	1.72	2.62	2.67	2.75
\$loss	n.a.	0	0	14.64	15.78	0.006

Table 4. *Implications of RI (Elastic κ and θ)*

	$\sigma\left(\sigma_y\right)$	0.2 (0.51)	0.3 (0.77)	0.4 (1.02)
GE-RI	θ	11%	14%	15%
($\lambda = 0.38$, $\psi = 0.54$)	$r^*(\%)$	2.79	2.32	1.94
	μ_{cy}	0.34	0.26	0.21
	μ_{ay}	2.53	2.39	2.35
GE-RI	θ	8%	10%	12%
($\lambda = 0.38$, $\psi = 0.8$)	$r^*(\%)$	2.73	2.46	2.14
	μ_{cy}	0.43	0.31	0.25
	μ_{ay}	2.87	2.56	2.46
GE-FI	r^*	3.23	2.62	2.14
($\theta = 1$, $\psi = 0.54$)	μ_{cy}	0.28	0.24	0.20
	μ_{ay}	1.78	1.88	1.97

Table 5. *Implications of Habit Formation and Incomplete Information*

Habit Formation	γ	0.4	0.6	0.8	0.9
GE ($\psi = 0.54$)	$r^*(\%)$	3.85	4.02	4.14	4.18
	μ_{cy}	0.21	0.17	0.12	0.09
	μ_{ay}	1.76	1.83	2.03	2.30

Incomplete Info	τ	0.4	0.6	0.8	0.9
GE ($\psi = 0.54$)	$r^*(\%)$	1.30	3.01	3.89	4.10
	μ_{cy}	0.56	0.36	0.18	0.10
	μ_{ay}	0.37	0.50	0.60	0.64

Table 6. *Relative Consumption Dispersion*

μ_{cy}	By Income Level	(Obs.)	By Wealth Level (with wealth data)	(Obs.)
Entire sample	0.321	$(22,370)$	0.247	$(9,401)$
Top 75%	0.328	$(16,785)$	0.235	$(7,067)$
Top 50%	0.315	$(11,195)$	0.212	$(4,708)$
Top 25%	0.293	$(5,600)$	0.179	$(2,354)$