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Dynamic Status Effects, Savings, and Income Inequality*

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Abstract

This paper advances the hypothesis that the intensity of status preferences depends negatively on the average wealth of society (endogenous dynamic status effect), in accordance with empirical evidence. Our theory replicates the contradictory historical facts of an increasing saving rate along with declining returns to capital over time. By affecting the dynamics of the saving rate, the dynamic status effect raises inequality, thereby providing a behavioural mechanism for the observed diverse dynamics of income inequality across countries. In countries in which the dynamic status effect is strong (weak) inequality rises (declines) over time in response to a positive productivity shock.

JEL classification: D11, D31, O11.

Keywords: Status preferences, saving rate, growth, inequality

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1 Introduction

It is well documented that individuals are concerned with social comparisons and status, particularly as it pertains to consumption. This paper advances the hypothesis that the degree to which individuals in a society are concerned with status is determined by that society’s stage of development, which we proxy by its average level of wealth. Social comparisons in terms of consumption seem to be more important during the early stages of development rather than in later stages, due to, among other factors, the evolution of institutions (education), culture, and social norms that are opposed to, or at least discourage, conspicuous consumption activities. To address this process, we endogenize the degree of status concern, by relating it to average national wealth, and demonstrate that over time, as a country develops, this degree (with respect to consumption) declines. We refer to this mechanism as “endogenous dynamic status preferences.”

The idea that individuals are often motivated in their behavior by a quest for social status is not new. It goes back to the earliest writings known to humanity and has been a recurring theme in a diverse range of endeavors long before the birth of economics.\footnote{Dubey and Geanakoplos (2017) provide a comprehensive introduction on the importance of status motivating and shaping individual behavior.} While economic theory has focused on the implication of status preferences on economic outcomes and policy, little work has been done on the bi-directional interaction of status preferences and economic development.\footnote{Examples include early ‘modern’ models and applications like Pigouvian taxation, Buchanan and Stubblebine’s (1962) treatment of externalities, Becker’s (1971) analysis of discrimination, Becker’s (1974) theory of social interaction, and Frank’s (1985) model of positional goods.}

Introducing endogenous dynamic status preferences enables us to address and explain important phenomena that cannot be satisfactorily explained by the standard neoclassical growth model. These include: (i) the historical and contemporary evolution of the saving rate, together with the evolution of the real return on capital; (ii) the historical dynamics and contemporary dynamics of wealth and income inequality.

In this paper we consider the following stylized facts pertaining to the transitional dynamics of the saving rate and income inequality that the standard growth model augmented by endogenous dynamic status preferences can readily replicate. (i) Historical data show that from the dawn of the modern world the saving rate increases (along with declining...
returns to capital), a fact that cannot be reproduced by the standard neoclassical growth model, for reasonable calibrations (Fact 1, poor countries save less, Dynan et al. 2004). 

(ii) World income inequality decreases from the 1900s until the 1970s (Fact 2, illustrated in Figs. A-1 and A-2). (iii) After the 1970s, for one group of countries income inequality remains approximately constant, yielding an L-shaped pattern (Fact 3, illustrated in Fig. A-1). (iv) In contrast, for another group of countries inequality increases sharply after the 1970s reaching the level of income inequality of 1900s, yielding a U-shaped pattern (Fact 4, illustrated in Fig. A-2). In addition, Saez and Zucman (2016) confirm a U-shaped behavior of wealth inequality using historical data for the US from 1913 to 2013.

Our key mechanism enabling reconciliation with Facts 1 to 4 – endogenous dynamic status preferences – operates through the transitional dynamics of the saving rate, which in turn affects the development of income inequality, as discussed below. This mechanism relies on behavioral changes that occur during the development process. As already noted, it is well documented that people derive utility not only from their own consumption but also from their relative social position (Easterlin, 2001). As long as consumption is visible (Heffetz, 2011, 2012), the social position of individuals can largely be inferred from their own consumption relative to the average consumption of others.³ Thus, by consuming more, people increase their own relative position, and in turn, their utility. However, the pursuit of one’s own status initiates a race with others, which results in excessively high equilibrium consumption that strains savings and intertemporal utility. We argue that during the development process, increases in average wealth lead to the formation of educational institutions, cultures, and social norms that discourage such conspicuous consumption. As a result, the increase in average wealth induces behavioral changes that lead to a lower degree of status concern, which tends to reduce the initial level of the saving rate followed by a rising saving rate along subsequent transitional paths. We show that this latter effect dominates over long periods during (the stages of) development, so that the saving rate is observed to increase over an extended period of time.

The hypothesis of a declining degree of status concern during development is supported by a number of empirical studies. Clark and Senik (2010), using a large European survey, demonstrate that comparisons are mostly in an upward direction. In this respect, there is

³A commodity is visible if, in the cultural context in which it is consumed, society has direct means to correctly assess the expenditures involved (Heffetz, 2011).
much more scope for upward comparisons for the poor (for poor countries) than exists for
the rich (for rich countries). Moreover, the poor tend to care more about status with respect
to relative consumption.\(^4\) In line with our hypothesis, Fig. 1 demonstrates that citizens of
rich European countries find it less important to compare their income with that of others
(Clark and Senik, 2010). In the figure, the mean importance of income comparisons is
monotonically increasing, while the trend in income per capita is uniformly decreasing.

[Figure 1 about here]

Heffetz (2011) estimates income elasticities for the consumption of “status” goods and
confirms the negative relationship between the degree of status concern and income. Looking
across countries, Moav and Neeman (2012) provide examples where the consumption basket
of the poor countries includes many goods that do not appear to alleviate poverty. In their
theoretical model, unobservable income is correlated with observable human capital. As a
result, they conclude that in rich countries people signal status rather through professional
titles and degrees and have less motivation to signal it through conspicuous consumption.

Our explanation of the long-run development of income inequality is based on the inter-
play between the dynamics of the saving rate, on the one hand, and the dynamics of the
return to capital, on the other, during the development process. While there is an extensive
literature that examines the effect of capital returns on income inequality (among many
others, Piketty 2014), we highlight how their interaction with the savings rate is impacted
by the evolution of the dynamic status preferences. In a standard neoclassical world, as
the capital stock increases, the rate of return to capital declines. This “return-to-capital
effect” benefits the poor, who hold less capital than do the rich.\(^5\) In contrast, the additional
mechanism being emphasized here – the endogenous dynamic status effect – impacts both
the level and the rate of change of the saving rate. This effect initially reduces the level
of the saving rate, while during the development process, as the economy’s capital stock
increases, people tend to increase their saving rate due to a reduction of the degree of status

\(^4\)Importantly, literature in psychology states that individuals seem to care about their ranking and the
esteem of others, even if they derive no clear economic benefits, and are willing to pay respect to others
and to modify their behavior accordingly, without receiving any direct benefit (cf. Heffetz and Frank 2011).
Importantly, literature in psychology states that individuals seem to care about their ranking and the esteem
of others, even if they derive no clear economic benefits, and are willing to pay respect to others and to modify their behavior accordingly, without receiving any direct benefit (cf. Heffetz and Frank 2011).

\(^5\)During transition, a decline in the return on capital lowers the (capital) income of the rich by more than
that of the poor.
concerns. The lower level of the saving rate implies a lower rate of capital accumulation. That is, the rate of interest declines at a slower pace. This latter effect benefits the wealthy households relative to the poor households. Hence, in a society with a heterogeneous wealth distribution, the dynamic status effect contributes to a more unequal wealth- or income distribution. Overall then, the strength of the endogenous dynamic status effect relative to that of the standard return-to-capital effect governs the evolution of income inequality.

We characterize analytically and simulate numerically the effects of a positive technology shock on savings and income inequality. Our results show how the interplay between the return-to-capital and the endogenous dynamic status effects can play an important role in reconciling the implications of the augmented neoclassical growth model with the empirical evidence illustrated in Figs. A-1 and A-2. Starting in 1900, when all economies were relatively undeveloped, the return-to-capital was strong and clearly dominated the status-effect; accordingly inequality declined in response to a positive productivity shock. Over the period 1900-1970 as economies developed, the strength of the dynamic status effect increased relative to the return-to-capital effect, and the rate of decline in income inequality decreased. After around 1970, with the different rates of development characterizing different economies, for the slower developing countries cultural developments occur slowly, so that the two effects are roughly in balance and inequality remains roughly constant, yielding the L-shaped curve as in Fig. A-1. For other economies where the dynamic status effect is stronger and continues to increase, it begins to dominate the return-to-capital effect. Income inequality starts to increase, eventually yielding the U-shaped curve illustrated in Fig. A-2.

The remainder of the paper is structured as follows. Section 2 relates our contribution to the relevant prior literature. Section 3 sets out the model and provides further empirical evidence for status concerns to decline in average wealth over time. Section 4 solves the optimization problem of households and firms, and studies the impact of the dynamic status effect on the transitional dynamics of the saving rate. Section 5 analyzes the dynamics on wealth inequality. Finally, Section 6 concludes the paper and discusses further research directions. Technical details and a number of figures are relegated to the Appendix.
2 Related Literature and Contribution

To our knowledge, this is the first paper that theoretically formalizes and analyzes the implications of the hypothesis asserting that the subjective evaluation of status preferences declines as a country develops. In this regard, it contributes to three bodies of literature. These include: (i) implications of positional goods in utility; (ii) studies of the dynamics of the saving rate; (iii) the dynamics of wealth and income inequality.

2.1 Degree of positionality

The proposition that people derive utility not only from their own consumption but also from their relative consumption level can be traced back to Smith (1759) and Veblen (1889). Veblen’s observation has been empirically justified by Easterlin’s (1995) “paradox”, who found that increases in income of all individuals had a negligible effect on their happiness. This finding was confirmed in empirical studies by Clark and Oswald (1996) and Frank (1997). The consequences of positional preferences have been extensively investigated in a number of areas. These include their effects on capital accumulation and growth (Brekke and Howarth 2002, Carroll et al., 1997, Alvarez-Cuadrado et al., 2004, Liu and Turnovsky, 2005, Wendner, 2010), on asset pricing (Abel 1999, Campbell and Cochrane 1999, Dupor and Liu 2003), on optimal tax policy over the business cycle (Ljungqvist and Uhlig 2000) and on public good provision (Micheletto 2011, Wendner and Goulder, 2008, Wendner, 2014). But in all those applications the strength of positional preferences is exogenous and remains constant over time, rendering these models incapable of deriving the non-monotonic evolution of savings and the distribution of wealth we observe in the historical and contemporary empirical data.

Our fundamental hypothesis is based on two elements regarding the formation of households’ preferences. First, the evolution of preferences for status is negatively related to the level of average wealth. This element has important consequences for the dynamics of savings, as discussed in this section and analyzed in subsequent sections below. Second, the dependence of status preferences on average wealth varies across countries due to different cultural and institutional characteristics. This element helps explain how cultural differences in the evolution of the concern for status can account for the divergence in wealth and
income inequality across otherwise similar economies.

Empirical studies provide support for both elements of the determinants of status preferences. In particular, Bloch et al. (2004), Chung and Fischer (2001), Banerjee and Duflo (2007), and Heffetz (2011) empirically support the ideas that people rely on relative consumption to raise their perceived status and that average income or wealth plays an important role in shaping the strength of status preferences. Charles et al. (2009), argue that since the marginal return to signaling through conspicuous consumption is decreasing in the average income of a person’s reference group, less conspicuous consumption should be observed among individuals who have richer reference groups. This observation has also been empirically formalized by Heffetz and Frank (2011) as stated in the Introduction. Boppart (2014), looking at time series data for the US and other advanced countries, states as an empirical regularity the fact that poor households spend a larger fraction of their budget on goods (as opposed to services, which are considered less positional) than do rich households (although the relative price of goods falls over time).

Across countries, Banerjee and Duflo (2007) and Clark and Senik (2010), show that in poor countries, people care more about status than in advanced countries. Moav and Neeman (2012) argue that more developed countries possess, on average, more human capital than do less developed economies. If human capital is visible (e.g. an academic title), then in more developed countries, the signaling of status (unobservable income) is pursued more with human capital than with consumption. This is not possible in less developed countries, where status signaling is done primarily via conspicuous consumption.

2.2 Dynamics of the saving rate and the real return to capital

The first implication of our hypothesis relates to the determination of the saving rate. According to the standard neoclassical growth model, for reasonable parameter values, more capital (wealth) leads to a lower rate of return on capital and, in turn, to a lower saving rate (see Barro and Sala-i-Martin 2004, pp.109ff, pp.135ff). However, empirical evidence indicates that saving rates are higher for richer countries (Loayza, Schmidt-Hebbel, and Serven, 2000). Also, examining historical data for the US, Saez and Zucman (2016) find that the saving rates tend to rise with wealth. Furthermore, Weil (2005) documents that the saving rate amounts to about 5 percent on average for countries in the lowest income decile.
(i.e. those closest to subsistence and most similar to England and other European countries in the Middle Ages). The saving rate then gradually increases with income. It amounts to about 10 percent in countries in the second decile, about 20 percent for the seventh decile and somewhat above 30 percent for the tenth decile.

The saving rate increases with wealth over time. To address this, the literature mainly considers technological factors that increase the return to capital over time and, in turn, the saving rate. However, by associating the increased saving rate with increasing returns to capital this explanation contradicts recent evidence provided by Boppart (2014) and Ledesma and Moro (2016), suggesting that the return to capital is decreasing over time.\(^6\)

On the preference side, Strulik (2012) shows that as wealth increases, the pure rate of time preference decreases. Therefore, in any given country, as capital accumulates over time, individuals become more patient, which tends to raise the saving rate. Likewise, at any given point in time, countries with patient individuals tend to experience higher saving rates. In this respect, we provide an alternative new mechanism, one also based on preferences. Following our main hypothesis, we formally argue that as a country develops people are less concerned with relative consumption. Consequently, individuals reduce their consumption growth rate over time, that is, they increase their saving rate. This mechanism is based on preferences; thus, it creates the possibility that, over time, the saving rate rises while the return on capital simultaneously declines.

### 2.3 Savings and inequality

There is an emerging literature that attributes the contemporary increase in income- and wealth inequality to differences in the saving rates across individuals. De Nardi and Fella (2017) provides an extensive review of the literature where differentials in the saving rates and income levels between wealthy and less wealthy individuals are generated by various sources. These sources include: the transmission of bequests and human capital (De Nardi, 2004) preference heterogeneity (Krusell and Smith, 1998), rates of returns heterogeneity

\(^6\)According to the Bureau of Economic Analysis (BEA), a ‘good’ is defined as “a tangible commodity that can be stored or inventoried,” which, in turn, can be a status good (as opposed to services that are positional to a lesser degree). The main (sub)categories the BEA classifies as ‘goods’ are: “motor vehicles and parts,” “furnishings and durable household equipment,” “recreational goods and vehicles,” “food and beverages purchased for off-premises consumption,” “clothing and footwear.” The goods belonging to that category typically can be positional goods because they are observable and their value depends relatively strongly on how they compare with goods owned by others.
(Benhabib, Bisin and Luo, 2015), entrepreneurship (Cagetti and De Nardi, 2006), richer earnings processes (Castaneda, Diaz-Gimenez, and Rios-Rull, 2003, and De Nardi, Fella, and Paz-Pardo, 2016), and medical expenses (De Nardi, French, and Jones, 2010). The main assumption of this literature is ex-post heterogeneity, and its theoretical underpinning is the Bewley (1977) model, which features an incomplete market environment, in which people save to self-insure against idiosyncratic earnings shocks.

These models are compelling and useful for capturing quantitatively the increase in wealth inequality in the US after 1970s. However, they do not explain: a) why those factors (the richer model structure) were less crucial during the decline in wealth and income inequality that we observe from the 1900s (Saez and Zucman, 2016); b) why the saving rate of rich individuals is higher but still declining in wealth (as capital accumulates) over time – something that we do not observe before the 1970s and in many countries even not after the 1970s; c) why wealth inequality develops differently in the US and similar developed countries (among others Sweden and Japan); d) the transitional dynamics of the wealth- or income distribution (but rather focus on contemporary data).

Our theory is consistent with the results of the previously discussed literature on savings and inequality. But in addition, it also provides explanations of the aforementioned points a) to d). To accomplish this, our model follows a different methodological approach. First, we depart from the incomplete markets assumption and from stochastic environments by assuming ex-ante rather than ex-post heterogeneity in individual wealth endowments or individual abilities. This enables us to rely on a deterministic mechanism that can explain the early decline in inequality. Second, we emphasize a behavioral mechanism according to which the saving rate is not only determined by the rate of return to capital, but also by a change in status preferences over time. This preference-based mechanism enables us to explain the contemporary differentials in wealth- and income inequality across developed countries when they are hit by the identical aggregate shock.

Our methodology follows, among others, Caselli and Ventura (2000) and García-Peñalosa and Turnovsky (2015) who assume ex-ante heterogeneity in wealth and/or abilities. In particular, Caselli and Ventura (2000) show that a technology bias (differences in the elasticity of substitution of factors of production) is able to capture the contemporary increase in inequality under a positive productivity shock. In particular, a positive productivity shock
benefits the holders of capital, if capital is or becomes more important in the production function. This mechanism is also in line with Piketty’s (2014) empirical observation of an increasing capital share in production as economies develop. However, these frameworks are less helpful in explaining the differentials in savings behavior of rich relative to poor countries, following recent evidence (Dynan, Skinner and Zeldes 2004, and De Nardi, French and Jones, 2010). Moreover, as technologies in developed countries seem to converge (e.g. according to Caselli and Feyrer, 2007 the marginal product of capital is very similar across countries), technology-based mechanisms seem less capable of explaining why inequality evolves differently in countries with the same factor shares in production. To this end, our framework complements this literature by providing a preference-based mechanism that operates through the strength in status preferences (implying differential behavior of savings) whose development is captured by cultural characteristics (Acemoglu and Robinson, 2015).

Finally, while there is agreement that the wealth-share held by the richest few is high, the extent to which this share has changed over time (and why) is still subject to debate (Piketty 2014, Saez and Zucman 2016, Bricker et al. 2016, and Kopczuk 2014). To this end, we study the transitional dynamics of the wealth distribution.

3 The model

We modify the standard neoclassical growth model with heterogeneous agents to allow for interdependence in consumption and endogenous dynamic status preferences, the strength of which declines as the country develops.

3.1 Households

The economy is populated by a continuum of individuals (households) of mass one, each of whom is endowed with one unit of labor that it supplies inelastically. They are identical in all respects except for their initial endowment of capital (wealth), $K_{i0}$.\footnote{Restricting labor supply to be inelastic has the advantage of sharpening the discussion (and intuition) of the impact of endogenous dynamic status preferences. A natural extension would allow labor to be endogenously supplied.} At each instant, $k_{i}(t) \equiv \frac{K_{i}(t)}{K(t)}$ is household $i$’s share of total wealth.\footnote{We consider a closed economy in which capital is the only asset. That is, total wealth in the economy corresponds to the aggregate capital stock $K(t)$.} Heterogeneity in wealth shares is
summarized by the cumulative distribution function, $H_t(k_i(t))$ with the standard deviation (coefficient of variation of $K_i(t)$) denoted by $\sigma_t$. The initial distribution $H_0(k_{i0})$ is exogenous, with standard deviation $\sigma_0$.

### 3.1.1 Endogenous status preferences

An individual’s utility depends both on its own consumption level, $C_i(t)$, as well as its consumption relative to some comparison group, $S_i(C_i(t), \bar{c}(t))$, where $\bar{c}(t)$ represents a consumption reference level. The status function, $S_i(t)$ is increasing in $C_i(t)$ and decreasing in the consumption reference level $\bar{c}(t)$. We represent the consumption reference level by average consumption, i.e. $\bar{c}(t) = \int_0^1 C_i(t) \, di$, where the bar indicates that individual households view the consumption reference level as exogenously given.\(^9\) A preference for relative consumption is frequently termed “position or status preference”. Our theory of endogenous dynamic status preferences focuses primarily on how intensely $S_i(t)$ is valued in a given country over (long periods of) time. We hypothesize that the valuation of $S_i(t)$ relative to own individual consumption evolves over time, as a country develops, as measured by the average capital stock $k(t) \equiv K(t).\(^{10}\)

We have already cited empirical data to suggest that the valuation of $S_i(t)$ relative to $C_i(t)$ declines over time, as a country develops. Before considering this claim more analytically, we provide a graphical intuition, as suggested by Clark et al. (2008). They argue that over time, as a country becomes wealthier, a higher-than-average income (wealth) contributes less and less to happiness.

[Figure 2 about here]

Consider the dotted lines in Fig. 2. These specify the relationship between individual incomes and happiness. The ellipse corresponding to date $t_0$ shows that a given increase in own income relative to the average raises happiness as depicted by the dotted line. Considering the other ellipses corresponding to later dates in time when the country is more developed, the same given increase in own income relative to the average raises happiness, but by less (i.e. the dotted curves become the flatter over time/with increasing wealth).

\(^9\)Clearly, the consumption reference level might differ from $\bar{c}(t)$. In this paper, however, we focus on the endogeneity of status preferences and would otherwise like to keep the setup as simple as possible.

\(^{10}\)By normalizing the population to one, averages and aggregates coincide.
The key components of our theory of endogenous dynamic status preferences comprise individual consumption, \( C_i(t) \), relative consumption, \( S_i(C_i(t), \bar{c}(t)) \), and a development-dependent \((k(t))-dependent) variable, \( \varepsilon(k(t)) \), which measures the relative strength of status preferences, the properties of which are discussed below. We let \( S_i(t) \equiv C_i(t)/\bar{c} \). Thus, instantaneous individual utility is given by

\[
U(C_i(t), S_i(t), \varepsilon(k(t))) = U(C_i(t), \left( \frac{C_i(t)}{\bar{c}(t)} \right), \varepsilon(k(t))).
\] (1)

Instantaneous utility increases in both individual and relative consumption \((U_{C_i(t)} > 0, U_{S_i(t)} > 0)\) and follows the usual concavity conditions in \( C_i(t) \) and \( S_i(t) \).

To capture the weight that is being applied to the absolute and relative consumption levels, we introduce the notion of the degree of positionality (DOP). The DOP, as defined by Johansson-Stenman et al. (2002), reflects the proportion of the total marginal utility of individual consumption that can be attributed to its impact on the increase in relative consumption. Formally, we specify this by

\[
DOP_i(t) = \frac{(\partial U/\partial S_i(t))(\partial S_i(t)/\partial C_i(t))}{(\partial U/\partial S_i(t))(\partial S_i(t)/\partial C_i(t)) + \partial U/\partial C_i(t)}.
\] (2)

Thus, if \( DOP_i(t) = 0.4 \), then 40% of marginal utility of consumption arise from an increase in relative consumption, and 60% of marginal utility of consumption arise from an increase in own absolute consumption (holding fixed \( S_i \)).

To render our analysis tractable, we introduce

**Assumption 1** The instantaneous utility function \( U(C_i(t), S_i(t), \varepsilon(k(t))) \) is homogeneous of degree \( R \) in \( C_i \). Specifically, \( U(C_i(t), S_i(t), \varepsilon(k(t))) = C_i(t)^R V(\bar{c}(t), \varepsilon(k(t))) \), where \( V_{\bar{c}(t)} < 0 \), and the elasticity \( V_{\varepsilon(k(t))} \varepsilon(k(t))/V > 0 \).

Subscripts to function \( V \) denote partial derivatives. Adopting this assumption, the utility from status, subutility \( V(\bar{c}(t), \varepsilon(k(t))) \), is decreasing in the consumption reference level and

\footnote{This specification of status preferences in relative terms – by \( \frac{C_i(t)}{\varepsilon(k(t))} \) – is prevalent throughout the literature; see, e.g., Gali (1994). A subtractive formulation, \( S_i(t) = C_i(t) - \bar{c}(t) \) is also possible and yields results equivalent to those presented in this paper.}

\footnote{As a canonical example consider the utility function \( U(C_i(t), S_i(t), \varepsilon(k(t))) = \gamma^{-1}(C_i(t)^{1-\varepsilon(k(t))} S_i(t)^{\varepsilon(k(t))}) \). Applying (2), one can immediately establish that \( DOP_i = DOP = \varepsilon(k) \).}
increasing in the strength of status concerns. Also, the degree of positionality, as shown in the Appendix, becomes

\[
DOP(\bar{c}(t), k(t)) = -\frac{V_{\epsilon(t)}(\bar{c}(t), \varepsilon(k(t)))\bar{c}(t)}{R_{\epsilon(t), \varepsilon(k(t))}}. \tag{3}
\]

The homogeneity imposed in Assumption 1 implies that the DOP is identical for all individuals. We capture the fact that the DOP declines with average wealth by endogenizing \(\varepsilon(k(t))\). As seen in (3), the degree of positionality is a function of both consumption and the stock of capital.

**Assumption 2** The properties of \(\varepsilon(t) \equiv \varepsilon(k(t))\) are:

1. \(\varepsilon(t) > 0\) is strictly positive and continuous;
2. \(\varepsilon'(t) \equiv \frac{\partial \varepsilon(t)}{\partial k(t)} < 0\);
3. \(\lim_{k(t) \to 0} \varepsilon(t) = \varepsilon_0 > 0\) and \(0 < \lim_{k(t) \to \infty} \varepsilon(t) = \varepsilon_\infty < 1\), with \(\varepsilon_0 > \varepsilon_\infty\).

Assumptions (2.i) and (2.iii) characterize the concern for status (positional preferences).\(^{13}\)

Households do not choose their individual DOP to display status. Rather, the strength of the status preference is socially determined by the society’s wealth (proxied by average wealth), which individual households take as given, and therefore treat as given, as well. Assumption (2.ii) asserts that the strength of status concerns declines with wealth (income), as suggested by Fig. 2, and the empirical evidence summarized in Section 2. That is, agents are more concerned with status in a low-wealth society than in a high-wealth society.

### 3.1.2 Household optimization

The individual household’s optimization problem is to choose a consumption stream, \(C_i(t)\), and to accumulate capital, \(K_i(t)\), so as to maximize intertemporal utility

\[
\int_0^\infty U(C_i(t), S_i(t), \varepsilon(k(t)))e^{-\beta t}, \beta > 0, \tag{4}
\]

subject to the flow budget constraint:

\[
\dot{K}_i(t) = r(t)K_i(t) + w(t) - C_i(t), \tag{5}
\]

\(^{13}\)In the canonical example in footnote 12, if \(\gamma \varepsilon < 0\), our specification implies that households keep up with the Joneses (cf., e.g., García-Peñalosa and Turnovsky, 2008).
the initial asset endowment, \( K_i(0) \), the transversality condition, and taking \( \bar{c}(t) \) and \( k(t) \) as given. In (4) and (5), \( \beta \) is the constant pure rate of time preference, \( r(t) \) is the real return on asset (capital) and \( w(t) \) is the wage rate.

Solving the intertemporal maximization problem, the individual’s equilibrium consumption growth rate is given by (see Appendix A.2):

\[
\frac{\dot{C}_i(t)}{C_i(t)} = \frac{1}{1 - R(1 - DOP(t))} \left[ r(t) - \beta + \left( \frac{V_t(\varepsilon'(k(t)))}{V(t)} \right) \dot{k}(t) \right].
\] (6)

Equation (6) represents the usual Euler equation, modified by the dynamic status effect. Consumption growth depends positively on the difference between the return on assets and the pure rate of time preference (return-to-capital effect). In the absence of positional preferences \( (DOP = 0 = \varepsilon'(k)) \), the optimal consumption growth rate (6) reduces to that of the standard neoclassical growth model.

Positional preferences modify the optimal consumption growth rate in two ways. First, they impact the intertemporal elasticity of substitution (IES), which is now given by\(^{14}\)

\[
IES(c(t), k(t)) = \frac{1}{1 - R(1 - DOP(c(t), k(t)))} > 0.
\] (7)

If \( R < 0 \), as empirical evidence overwhelmingly suggests, positionality raises the IES, relative to that of the standard neoclassical growth model, \( \frac{1}{1-R} \).\(^{15}\) For a given interest rate, individuals raise the optimal consumption growth rate, as documented by, among others, Liu and Turnovsky (2005).

Second, positional preferences introduce a dynamic status effect. If \( \dot{k} > 0 \), under Assumption 2(ii), the status effect causes the optimal consumption growth rate to decline as a country develops. The intertemporal consumption decision is affected by the degree to which people evaluate their social status over time. The more agents evaluate their relative position, the more they consume in order to raise their respective relative position. However, as the economy accumulates capital, the degree of positionality declines. That is, the marginal utility from consumption – via the rise in individual relative consumption – declines over time. As a consequence, consumption is shifted from the future to the present,

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\(^{14}\)By taking into account the impact of the consumption externality on the agent’s intertemporal substitution, (7) can be interpreted as measuring the “social intertemporal elasticity of substitution”.

\(^{15}\)See e.g. Guvenen (2006) for extensive empirical evidence on \( R \).
and the optimal consumption growth rate declines. The latter has an impact on both the saving rate level and the subsequent evolution of the saving rate. As discussed below, the level of the saving rate is lowered, and its rate of change becomes positive along transitional paths. It is this effect of positional preferences that we emphasize and focus on in this paper.

3.1.3 Production

There is a single representative firm, which produces aggregate output, \( Y(t) \), in accordance with the Cobb-Douglas production function

\[
Y(t) = F(K(t), L(t), A) = AK(t)^\alpha L(t)^{1-\alpha}, \quad 0 < \alpha < 1, \tag{8}
\]

where \( K(t), L(t) \) denote capital and labor inputs, and \( A \) represents total factor productivity (TFP). Due to the homogeneity of degree 1 of the Cobb-Douglas production function we define output per capita, \( \frac{Y(t)}{L(t)} \), capital per capita, \( \frac{K(t)}{L(t)} \), and the per capita production function \( f(k(t)) \equiv Ak(t)^\alpha \).

Labor endowment is normalized to unity, and we assume no population growth. The representative firm maximizes profit, \( \pi(t) = Y(t) - w(t)L(t) - (r(t) + \delta)K(t) \) where \( \delta \geq 0 \) is the depreciation rate of physical capital, yielding the standard first-order optimality conditions:

\[
r(t) = \alpha AL(t)^{1-\alpha}K(t)^{\alpha-1} - \delta \tag{9}
\]

\[
w(t) = (1 - \alpha)AL(t)^{-\alpha}K(t)^{\alpha}.
\]

4 Equilibrium and the dynamics of savings

4.1 Equilibrium dynamics of savings and long-run equilibrium

In this section we solve for a competitive equilibrium and analyze its properties, where we let per capita consumption \( c(t) \equiv \frac{C(t)}{L(t)} \).

**Definition 1** A competitive equilibrium is a price vector \((r(t), w(t))\) and an attainable allocation for \( t \geq 0 \) such that:
i) Individuals solve their intertemporal utility maximization problem by choosing $C_i(t)$ and $K_i(t)$, given factor prices, initial wealth endowments, aggregate capital, and the consumption reference level.

ii) Firms choose $K(t)$ and $L(t)$ in order to maximize profits, given the factor prices.

iii) All markets clear. Capital market clearing implies $k(t) = K(t)$ (total assets held by agents equal the firms capital stock). Labor market clearing implies $L(t) = 1$.

(iv) Aggregation: $K(t) = \int_0^1 K_i(t) d_i = k(t)$; $\int_0^1 k_i(t) d_i = 1$ and $C(t) = \int_0^1 C_i(t) d_i = c(t)$.

(v) Consumption reference level: $\bar{c}(t) = c(t)$.

Considering (iii), observe that the mean individual to total wealth ratio equals unity: $\int_0^1 k_i(t) d_i = \int_0^1 K_i(t) K_i(t) d_i = 1$. While individual households take the consumption reference level, $\bar{c}(t)$, as given, in equilibrium we assume that the consumption reference level is given by the economy-wide average consumption level, according to (v).

Combining equations (5)-(9), and using market clearing (and aggregation) conditions, we obtain the equilibrium dynamics of the aggregate (average) economy-wide variables:

$$\dot{k}(t) = f(k(t)) - c(t) - \delta k(t)$$

$$\dot{c}(t) = \frac{c(t)}{1 - R(1 - DOP(t))} \left[ f'(k(t)) - (\delta + \beta) + \left( \frac{V_\varepsilon(k(t))}{V(t)} \right) \dot{k}(t) \right],$$

where $DOP(t)$ is defined by (3) and $f'(k(t))$ is the first order derivative of $f(k(t))$ with respect to $k(t)$. Defining the elasticity of status-utility with respect to $k$ by $E(c, k) \equiv V_k c/V = V_\varepsilon(k) k/V \leq 0$, we can conveniently rewrite the dynamic system as

$$\dot{k}(t) = f(k(t)) - c(t) - \delta k(t)$$

$$\dot{c}(t) = c(t) IES(c(t), k(t)) \left[ f'(k(t)) - (\delta + \beta) + \left( \frac{E(c(t), k(t))}{k(t)} \right) \dot{k}(t) \right]$$

Clearly, in the absence of the endogenous dynamic status effect $\varepsilon'(k) = 0 = E(c, k)$, while in its presence $\varepsilon'(k) < 0 \Rightarrow E(c, k) < 0$. Finally, we define the saving rate by

$$s(t) = 1 - \frac{c(t)}{f(k(t))}$$
Setting $\dot{c} = \dot{k} = 0$, the steady-state per capita capital and consumption, $(k^*, c^*)$, are given by

$$k^* = \left( \frac{A\alpha}{\beta + \delta} \right)^{\frac{1}{1-\alpha}} > 0, \quad c^* = \beta \left( \frac{A\alpha}{\beta + \delta} \right)^{\frac{1}{1-\alpha}} + A(1 - \alpha) \left( \frac{A\alpha}{\beta + \delta} \right)^{\frac{\alpha}{1-\alpha}} > 0$$

which further yield the long-run capital-output ratio and saving rate $\frac{k^*}{y^*} = \frac{\alpha}{\beta + \delta}$, $s^* = \frac{\alpha \delta}{\beta + \delta}$.

The steady-state quantities are unique and positive, with the saving rate lying in the range $0 < s^* < \alpha$. They are also independent of the (dynamic) status preferences and therefore identical to those of the standard neoclassical growth model. This characteristic reflects the fact that the strength of status preferences does not affect the steady-state production process, which is the driving force behind the long-run equilibrium.

Furthermore, linearizing the dynamic system (10') around the steady state, one can easily show that the determinant of the Jacobian matrix of coefficients of the linearized system is negative implying that the unique steady state is a saddle point and is saddle-point stable. However, the dynamic status effect does affect the transitional dynamics and the distribution of income and wealth, as the change in the intensity of status matters for agents’ intertemporal decisions.\textsuperscript{16} The impact of the changing status on savings behavior is summarized by the following proposition:

**Proposition 1** Consider Assumption 2(ii), the endogenous dynamic status effect, $\varepsilon'(t) < 0$.

*During the transition associated with an increasing capital stock:*

(i) The dynamics of the saving rate are characterized by

$$\dot{s}(t) \geq 0 \text{ if and only if } s^* \geq \frac{IES(c(t), k(t))}{\xi(c(t), k(t))},$$

where $\xi(c(t), k(t)) \equiv 1 - \frac{IES(c(t), k(t)) E(c(t), k(t))}{\alpha} \geq 1$. \hspace{1cm} (12)

**Proof.** See Appendix A.3. \hspace{1cm} $\blacksquare$

\textsuperscript{16}This characteristic is identical to the conventional model where status preferences are exogenously fixed; see Liu and Turnovsky (2005). As in that model, status preferences have only long-run effects if labor supply is elastic. As we show below, isolating any long-run productive effects of status is quite helpful in facilitating comparisons between economies with similar income per capita while having different levels of income inequality (e.g. US vs. Europe).
Corollary 1  As long as the endogenous dynamic status effect is sufficiently strong, there exist plausible parameter values such that at low levels of \(k\), the saving rate increases, as capital increases. Once \(k\) reaches a threshold level, the saving rate declines, and levels out to its steady-state value, as capital increases further.

Proposition 1 shows that the transitional dynamics of the saving rate need not follow the declining pattern implied by the standard neoclassical growth model (for reasonable calibrations). The following intuition for this result applies. Consider first the (standard) neoclassical growth model. An increase in the capital stock increases the supply of capital and reduces its return. This decline in the rate of return imposes both a substitution effect and an income effect. According to the former, the price of future consumption rises relative to that of present consumption. Consequently, current consumption increases, thereby reducing the saving rate. In the case of the latter, the lower return to capital reduces income for both present and future consumption. Accordingly, individuals tend to reduce current consumption, thereby raising the saving rate. For plausible parameterization, the substitution effect dominates the income effect and thus, the neoclassical growth model predicts a declining saving rate as capital increases; see e.g. Barro and Sala-i-Martin 2004, p.136.

Consider now our augmented neoclassical growth model. The dynamic endogenous status effect introduces a third channel, whereby an increase in the capital stock impinges on the intertemporal consumption-savings decision. This effect tends to increase the saving rate over time (as capital is accumulated). When the capital stock increases, agents choose a lower rate of consumption growth, together with an initially higher level of consumption, in comparison to the standard neoclassical growth model. This is evident from (10') due to the fact that \(E(c, k) < 0\). The higher initial consumption level necessitates a lower initial saving rate (compared to the standard neoclassical growth model). (Recall that the steady-state saving rate is unaffected by status preferences.) Consequently, the presence of dynamic endogenous status preferences implies either a lower rate of decline of the saving rate or an increasing saving rate along the transitional path toward its steady state. In particular, if the dynamic status effect is sufficiently strong – that is, the absolute value \(E(c, k)\) is sufficiently large – then the consumption growth rate is lower than the output growth rate,
and the saving rate increases along its transitional path. In other words, even when the substitution effect exceeds the income effect, our extended model can produce an increasing saving rate under plausible calibrations.

More formally, in the absence of dynamic status preferences (i.e. \( E(c, k) = 0 \Leftrightarrow \xi(c, k) = 1 \)), condition (i) of Proposition 1 reduces to Barro and Sala-i-Martin’s (2004) familiar condition, \( \dot{s} \leq 0 \Leftrightarrow s^* \leq IES(c, k) \). However, in the presence of the endogenous dynamic status effect, \( \xi(c, k) > 1 \), and is unconstrained by any upper bound. For this reason, as long as \( \xi(c, k) \) is large enough, \( \dot{s} > 0 \) during transition. This holds true even when \( s^* < IES(c, k) \) – i.e. the substitution effect exceeds the income effect, following empirical evidence (among many others, see Barro and Sala-i-Martin, 2004). In this latter case, though, for large \( k \), \( E(c, k) \) becomes close to zero, thus, \( \xi \approx 1 \), and the saving rate eventually declines.

To summarize: On the one hand, the increase in capital reduces the return to capital. This lowers the rate of interest and tends to lower the saving rate, which, empirically, dominates the income effect. On the other hand, the increase in capital reduces the consumption growth rate via the endogenous dynamic status effect \( (\varepsilon'(k) < 0) \). The lowering of the consumption growth rate tends to raise the saving rate. As long as the dynamic status effect dominates the return-to-capital effect, the saving rate increases during transition.

### 4.2 An example of endogenous dynamic status preferences

In this subsection, we employ numerical simulations to provide an example of our analytical results and to illustrate the performance of our model with respect to historical data. Preferences are specified by the CES utility function which satisfies our assumptions:

\[
U = \frac{1}{\gamma} \left( [1-\varepsilon(k)]C_i^\rho + \varepsilon(k) \left( \frac{C_i}{\bar{c}} \right)^\rho \right)^{\frac{\gamma}{\rho}} = \frac{C_i^\gamma}{\gamma} \left( [1-\varepsilon(k)] + \varepsilon(k)\bar{c}^{-\rho} \right)^{\frac{\gamma}{\rho}} \tag{14}
\]

The degree of homogeneity of \( U \) is \( R = \gamma \) and the corresponding degree of positionality is

\[
DOP = \frac{\varepsilon(k)\bar{c}^{-\rho}}{1-\varepsilon(k) + \varepsilon(k)\bar{c}^{-\rho}}.
\]

17 Notice that \( s = 1 - c/f(k) \). Clearly, the proposition allows for a third pattern according to which the saving rate first increases, overshoots its steady state level, and eventually declines towards its steady state level.

18 Unless needed for clarity, we omit time indexes in what follows.
Letting $\rho \to 0$ yields the Cobb-Douglas case, and $DOP = \varepsilon(k)$. Technology is specified by the Cobb-Douglas function (8). For the evolution of the dynamic status preferences, we use an explicit function that satisfies Assumption 2:

$$\varepsilon(k(t)) = \varepsilon_\infty + (\varepsilon_0 - \varepsilon_\infty) \exp(-\kappa k(t)), \quad \kappa \geq 0, \quad \varepsilon_0 \geq \varepsilon_\infty \geq 0.$$  (15)

Parameter $\kappa$ captures the sensitivity of $\varepsilon(t)$ with respect to a change in $k$. If $\kappa = 0$, $\varepsilon(t)$ is constant over time, and the dynamic (endogenous) status mechanism is absent.\(^{19}\)

The parameterization follows standard growth literature and is largely uncontroversial. The technology parameters are assigned the following values: $\alpha = 0.4$, $A = 2$ and $\delta = 0.08$. The preference parameters assume the following values: $\beta = 0.04$, $\gamma = -3$, implying an elasticity of intertemporal substitution equal to 0.25, and $\rho = 0$ (unless otherwise stated). Finally, the status parameters are $\kappa = 0.1$, $\varepsilon_0 = 2$, $\varepsilon_\infty = 0.2$.\(^{20}\) We consider the transitional dynamics of both the saving rate and the rate of interest when the economy starts with a capital level, $k_0$, that is (far) below its steady-state level.

[Figure 3 about here]

The solid lines in Fig. 3 display the transitional dynamics of the saving rate and the return to capital in the presence of the endogenous dynamic status effect (when $\varepsilon' < 0$). As analyzed above (as well as in the Appendix), in early stages of development savings increase, and after a threshold level of the capital stock is reached, savings decline slightly, before leveling out. Thus, our model augmented to include dynamic status is able to capture both the joint historical dynamics of the savings and real interest rates (when they diverged).\(^{21}\)

The dashed lines in Fig. 3 display the transitional dynamics of the saving rate in the absence of the dynamic status effect (when $\varepsilon(t) = \varepsilon = 0$ is constant). Without this effect, the saving rate always decreases (due to the return-to-capital effect).

Three remarks merit comment. First, in contrast to the prediction of the standard neoclassical growth model, the positive correlation between the saving rate and the level of

\(^{19}\)Equivalently, if $\varepsilon_0 = \varepsilon_\infty$, $\varepsilon(t)$ is constant, and there is no dynamic status effect.

\(^{20}\)Notice that in our simulations the initial capital stock equal $k(t_0) > 0$, implying an associated $\varepsilon(t_0) < 1$ as consistent with our restrictions.

\(^{21}\)Note that Corollary 1 allows for an increasing saving rate for a low level of capital stock while a decreasing at later stages of development. Although for many countries the saving rate increases historically and contemporarily, Corollary 1 captures the inverse U-Shaped dynamic behavior of the saving rate in US as noticed, among others, by Antràs (2001).
development helps us explain the cross-country evidence, where rich countries save more than poor ones (see, among others, Dynan et al. 2004, Weil 2005). While the rate of return to capital historically falls, poor countries never seem to catch up. In this discussion, our behavioral mechanism provides an additional explanation.

Second, a non-monotonic saving rate across time plays a crucial role with respect to the speed of convergence to the long-run equilibrium. This becomes even more important in a heterogeneous agent world in which people differ in their initial wealth endowments. Below, we show how the interplay of the endogenous dynamic status- and return-to-capital effects, by affecting the speed of convergence, helps to explain the behavior of income inequality qualitatively.

Third, our model provides a preference-driven mechanism to explain the non-monotonic behavior (especially a rise followed by a decline) of the saving rate over time. Caselli and Ventura (2000) provide a technology-driven mechanism in order to explain non-monotonic behavior of the saving rate over time. They show that the elasticity of substitution between capital and labor is a key ingredient for explaining non-monotonic behavior of the saving rate (Caselli and Ventura 2000, p. 920). In contrast, we exclude a technology-driven explanation by setting the elasticity of substitution between capital and labor equal to one, according to (8). In this respect, our model provides a genuinely new foundation for the historically observed non-monotonic development of the saving rate.

5 Wealth (and income) inequality

We first characterize analytically the main mechanism underlying the evolution of income inequality. We then examine the comparative income inequality dynamics across countries that experience the identical productivity shock, but differ in the intensities of their respective status preferences responses to the productivity shock-induced change in \( k \). Our analytical results are illustrated with numerical examples, where we compare the income inequality dynamics for two cases: (i) presence of the endogenous dynamic status effect (\( \varepsilon'(t) < 0 \)), and (ii) absence of the endogenous dynamic status effect (\( \varepsilon'(t) = 0 \)). While in the former case income inequality is increased, in the latter case, the shock reduces income

\[ 22 \text{As we assume an exogenous labor supply, the evolution of wealth inequality, as measured by the coefficient of variation of the wealth distribution, is proportional to the evolution of income inequality.} \]
inequality. The bottom line is that in countries having strong dynamic status preferences, a positive productivity shock raises inequality, while in other economies with weak dynamic status preferences, a positive productivity shock does not affect- or reduces inequality. Both scenarios are consistent with the empirical evidence as illustrated in Figs. A-1 and A-2.

5.1 The dynamics of inequality

We first determine the equilibrium dynamics of individual \(i\)'s share of total capital, \(k_i(t)\). To do so, we consider the individual wealth accumulation equation (5) together with the corresponding aggregate accumulation relationship \(\dot{K}(t) = r(t)K(t) - w(t) - C(t)\), to yield:

\[
\dot{k}_i(t) = \frac{w(t)}{k(t)}(1 - k_i(t)) + \frac{c(t)}{k(t)}(-\theta_i(t) + k_i(t))
\]

where \(\theta_i(t) \equiv \frac{c_i(t)}{c(t)}\). Following the procedure described by García-Peñalosa and Turnovsky (2008, p. 463ff) the bounded solution for \(k_i(t)\) is

\[
\dot{k}_i(t) = k_i^* + h(k^*)(1 - k_i^*) \frac{k(t) - k^*}{k^*} \frac{1}{\mu^* - \beta}.
\]

where variables with an asterisk are final steady state values, \(h(k^*) = -f''(k^*) - \frac{v_1^* w^*}{c^*}, \mu^*\) is the negative eigenvalue associated with the dynamic system (16) evaluated at the final steady state, \(v_1^* = \beta - \mu^* > 0\) is the normalized part of the eigenvector associated with \(\mu^*\) and where \(f''(k(t))\) is the second order derivative of \(f(k(t))\) with respect to \(k(t)\). As the sign of \(h(k^*)\) plays a key role for the shock-induced development of income inequality, we need to investigate this term further.

First, \(h(k)\) depends only on average characteristics. Second, under Assumption 2, \(sgn h(k)\) is ambiguous. If status preferences are exogenous (\(\varepsilon'(t) = 0\)) and the technology is Cobb-Douglas, then \(h(k^*) < 0\). However, in the present case, \(sgn h(k^*)\) also depends on the change of the intensity of status concerns, \(\varepsilon'(t)\), via its impact on the negative eigenvalue. If \(\varepsilon'(t) < 0\) and large enough (in absolute terms), then \(h(k^*)\) becomes positive, governing the transitional dynamics of inequality induced by shocks.

Integrating (16) across all agents, García-Peñalosa and Turnovsky (2008) show that the

\footnote{See García-Peñalosa and Turnovsky (2008, p.455)}
dynamics of the coefficient of variation of wealth (treated as a measure of inequality) is given by
\[ \sigma_k(t) = \frac{\zeta(t)}{\zeta(0)} \sigma_k(0) \] (18)
where \( \zeta(t) \equiv 1 + \frac{h(k^*) k(t) - k^*}{\beta - \mu^*} \) and \( \zeta_0 \equiv 1 + \frac{h(k^*) k(0) - k^*}{\beta - \mu^*} \). We employ equation (17) to measure the transitional development of inequality \textit{close to a steady state}. Being a function of \( h(k) \) it is a function of average wealth only.

In the previous section, we have argued that the endogenous dynamic status effect influences the transitional dynamics of the interest rate (see Fig. 3). Due to the dynamic status effect, the rate of interest declines at a slower pace. This, in turn, impinges on the development of inequality – both during transition and in steady state – and leads us to

\textbf{Proposition 2} For any initial distribution and standard deviation of wealth, in the neighborhood of the steady state where \( k(0) < k^* \), inequality rises (falls), if \( h(k^*) > 0 \) (if \( h(k^*) < 0 \)):
\[ h(k^*) \gtrless 0 \Leftrightarrow -\frac{E(c^*, k^*)}{\alpha} \gtrless \left[ \frac{1}{s^*} - \frac{1}{IES(c^*, k^*)} \right] \] (19)

\textbf{Proof.} See Appendix A.4. \[ \square \]

Proposition 2 provides the implicit parametric condition for the evolution of inequality (increasing or decreasing) when the economy starts with an initial capital stock below (and in the neighborhood of) the steady-state equilibrium, i.e., for increasing \( k \). Consider first the case without a dynamic status effect, \( E(c, k) = 0 \). If the substitution effect is sufficiently strong, then \( s^* < IES(c^*, k^*) \), as empirical evidence suggests. Condition (18) then implies \( h(k^*) < 0 \), so that inequality declines. Intuitively, the saving rate is high and declining toward its steady-state value, in accordance with Proposition 1. As a result, the rate of capital accumulation (and speed of convergence) is high as well. In turn, the return to capital declines rapidly, which disadvantages the wealthy households more than the poor ones. As a consequence, inequality declines.

Now, consider the impact of a dynamic status effect, \( E(c, k) < 0 \). Once this effect becomes sufficiently strong, \( E(c, k) < 0 \) and \( h(k) > 0 \) in (18). In this case, the dynamic status effect induces households to reduce their consumption growth rate, \textit{ceteris paribus}. 

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In conjunction with the lower consumption growth rate, households initially raise their consumption level and reduce their saving rate. As capital increases, the saving rate rises toward its new steady-state level. Since during transition the level of the saving rate is lower compared to when \( E(c, k) = 0 \), capital is being accumulated at a lower rate (and the speed of convergence is lower). Therefore, the rate of interest declines at a slower pace. This benefits the wealthy households, whose share of income from capital is large, more than it does the poor. As a result, wealth inequality increases along the transition. Moreover, despite the fact that dynamic status preferences do not impact the steady-state levels of consumption and capital, they do impact steady-state inequality. Specifically, in the presence of dynamic status preferences, when \( E(c, k) < 0 \), inequality is higher than in their absence, when \( E(c, k) = 0 \). Consequently, differences in dynamic status preferences may cause countries that are similar with respect to aggregate measures such as per capita income, consumption, and capital, to have different degrees of wealth inequality.

To shed additional light on Proposition 2, we consider several corollaries as well as numerical simulations.

**Corollary 2** A sufficient condition for an increase in inequality is given by:

\[ s^* > IES(c, k) \Rightarrow h(k) > 0. \]  

**Proof.** Consider the right hand side of the equivalence in (18). The term in square brackets is negative, as \( s^* > IES(c, k) \). The left hand side of the inequality is greater than or equal to zero, as \( E(c, k) \leq 0 \). Consequently, (18) implies that \( h(k) > 0 \). \( \blacksquare \)

Corollary 2 builds on the result of the previous section, where under a strong income effect \( (s^* > IES(c, k)) \) the saving rate rises along transition towards its steady state level. Under the sufficient condition given in Corollary 2, the level of the saving rate is initially low and then rises. Thereby, the speed of convergence is slow as well and the rate of interest declines gradually, benefitting the wealthy households relative to the poor. Consequently, inequality rises during transition. As mentioned above, the condition \( s^* > IES(c, k) \) is not likely to be satisfied empirically. However, recall that this is a sufficient, not a necessary, condition.
Corollary 3 In a neighborhood of the steady state, conditions (18) in Proposition 2 and (12) in Proposition 1 are equivalent.

Proof. Considering $\xi(c^*,k^*) \equiv 1 - IES(c^*,k^*)E(c^*,k^*)/\alpha$ in (12) and rearranging terms immediately yields the right hand side of the equivalence in (18). As the right-hand sides of the equivalences in (12) and (18) are identical, the left hand sides are identical as well. Thus, $h(k^*) \geq 0 \Leftrightarrow \dot{s} \geq 0$. ■

Corollary 3 states that the saving rate behavior and the development of inequality are closely linked. In particular, the presence of a strong dynamic status effect can explain the joint occurrence of increasing savings together with income inequality, even when the substitution effect is high, that is when $s^* < IES(c^*,k^*)$. In the presence of a sufficiently responsive dynamic status effect (high $\xi$), for any initial capital stock the saving rate is initially lower than in its absence, and, in turn, increases toward the steady-state. The interest rate declines and generates a substitution effect that tends to reduce savings, ceteris paribus. At the same time, though, the dynamic status effect induces a behavioral change against conspicuous consumption, inducing an increase in the saving rate. The lower level of the saving rate prolongs the transition to the steady-state. As a consequence, agents that hold proportionally more capital benefit from the longer period of high interest rates.\(^2\)

We further illustrate Proposition 2 by numerical simulations. Our parameterization is identical to that of the previous section. Fig. 4 displays the transitional dynamics of wealth inequality for both cases: presence- and absence of the endogenous dynamic status effect. Due to the exogeneity of labor supply, the developments of wealth- and income inequality coincide. The vertical axis of the figure shows the growth factor of the standard deviation of wealth inequality, as given by (17), with $\sigma_k(0) \equiv 1$ normalized to unity.

[Figure 4 about here]

The solid line in Fig.4 displays the evolution of wealth- or income inequality in the presence of the endogenous dynamic status effect ($\varepsilon' < 0$) when $\xi$ is large enough so that $h(k^* > 0$, according to Corollary 3. In that case, following our analytical results, inequality

\(^2\)In other words, consider the area under the interest rate curves in Figure 3. The area is larger for the case $\varepsilon' < 0$ than for the case $\varepsilon' = 0$. The larger the area the more beneficial it is for wealthy households relative to poor ones.
increases as the dynamic status effect dominates the substitution effect. The increase in inequality is consistent with (the rising part of) the U-shaped dynamics of inequality, as displayed in Fig. A-2. The dashed line displays the case of exogenous dynamic status preferences ($\varepsilon' = 0$), where $h(k^*) < 0$ and inequality slightly falls. In this case, the substitution effect roughly balances (slightly exceeds) the dynamic status effect. This case is in line with the roughly constant part of the L-shaped dynamics of inequality, as displayed in Fig. A-1.

Three remarks are in order. First, the dynamic status effect on the aggregate economy is only transitory, in that it does not impact the aggregate steady-state level of wealth. In contrast, the dynamic status effect impacts the wealth distribution both during the transition and in the steady state. In fact, steady-state inequality is higher in the presence of the dynamic status effect than in its absence (see Fig. 4). The higher inequality during the transition carries over to the new steady state, making it path dependent.\footnote{The issue of the path dependence of long-run wealth and income inequality in response to structural changes is a general phenomenon and is discussed in detail by Atolia et al. (2012).} This enables us to capture the empirical evidence according to which countries at approximately the same level of economic development (steady-state) may nevertheless have noticeable differences in their respective wealth distributions. These may reflect cultural differences with respect to the responsiveness of status preferences to the accumulation of wealth as they have developed.

Second, this result is consistent with García-Peñalosa and Turnovsky (2008), who show that the presence of exogenous status preferences ($\varepsilon > 0$, $\varepsilon' = 0$) contribute to a lower steady-state wealth inequality. While we compare an economy with endogenous dynamic status ($\varepsilon > 0$, $\varepsilon' < 0$) to one with exogenous status ($\varepsilon > 0$, $\varepsilon' = 0$), García-Peñalosa and Turnovsky’s (2008) comparison is between exogenous status and no status ($\varepsilon = 0$). They show that the presence of status raises the intertemporal elasticity of substitution (in eq. (10) DOP becomes positive, and for $R < 0$, the IES increases). As a consequence, households desire, for any given $k$, a higher consumption growth rate, which is compatible only with an initially lower consumption level, or, equivalently, an initially higher saving rate. The higher saving rate raises the speed of convergence, thus, it reduces wealth inequality relative to a model without status. In contrast, with endogenous dynamic status, as the saving rate initially increases, $s$ is initially reduced compared to a model without status. Thus, the convergence is slower, which benefits the rich more than the poor. Consequently, the wealth
inequality increases in the presence of the dynamic status effect more than in its absence.

Third, for typical empirical parameter values \( s^* < IES(c^*, k^*) \), the neoclassical growth model predicts a decline in income inequality as the economy develops (as capital increases). Thus, it fails to explain the increase in contemporary income inequality. In contrast, in our model – in spite of a strong substitution effect – income inequality can increase or decrease, depending on the strength of the dynamic status effect. Despite the fact that the decline in the return to capital during the development process (due to diminishing returns) tends to reduce savings, the behavioral changes mitigating the consumption race for status tend to increase savings. That is, our behavioral mechanism is rich enough to account for both the historical decrease in income inequality and the contemporary increase in income inequality, as a reflection of the strength of the dynamic status effect in the process of economic development.

5.2 Evolution of inequality under universal productivity shocks

To illustrate how differences in status preferences between countries can account for the differential dynamics of income inequality as observed in contemporary data, we consider a universal productivity shock. In doing so, differences in status preferences are reflected in different values of \( E(c, k) \leq 0 \), viewed as proxying cultural differences between countries. In particular, the smaller (the more negative) \( E(c, k) \) the more responsive are a country’s status concerns with respect to an increase in its aggregate capital, \( k \). From Proposition 2, we know that the impact of the shock on wealth inequality depends critically on the strength of the dynamic status effect. Indeed, we find that the necessary and sufficient conditions for the shock to generate rising or declining wealth inequality are closely related to condition (18) in Proposition 2. The way the dynamic status effect influences the saving rate is crucial to the differential impact of a positive technology shock on inequality.

We shall demonstrate that the key mechanism explaining the impact of a positive technology shock on the development of inequality relies on the initial response of the saving rate (a jump variable) to the shock. This response, in turn, depends on whether the propensity to consume out of wealth, \( c_k \), is greater than or less than the slope of the saddle path in phase space \((k, c)\). This, in turn, is closely related to \( \text{sgn} \ h(k) \), by which the development of inequality is explained. From the solution reported in Section 4.1, it is clear that a technol-
ogy shock affects neither the steady-state saving rate nor the steady state $\frac{c}{k}$-ratio. Moreover, the slope of the saddle path is $(\beta - \mu)$, as shown in the Appendix, where we establish the following proposition.

**Proposition 3** A (positive) productivity shock, $\Delta A > 0$, impacts on both the transitional dynamics and the steady state of income and wealth inequality. The strength of the dynamic status effect is key in determining whether inequality rises or falls following a productivity shock. In countries with a strong (small) dynamic status effect, inequality rises (falls). In particular, we have:

$$(\beta - \frac{\mu^*}{s^*}) \geq \frac{c^*}{k^*} \iff h(k^*) \geq 0.$$ 

If $h(k^*) < 0$, inequality declines in response to a positive technology shock. This case applies to a situation without (or with a weak) dynamic status effect. If $h(k^*) > 0$, inequality rises in response to a positive technology shock. This case applies to a situation with a strong (enough) dynamic status effect.

**Proof.** See Appendix A.5. ■

Proposition 3 shows that a positive productivity shock increases inequality in countries having a “strong” dynamic status effect, and it reduces inequality in countries with a “weak” dynamic status effect. The main mechanism is via the initial response of the saving rate following the technology shock, and we identify two alternative scenarios.

The first is one in which the saving rate jumps up initially, and monotonically declines thereafter to its steady-state value (that is unaffected by the technology shock). In this case $h(k) < 0$, and inequality declines during transition, and is lower in the post-shock steady state than in the initial equilibrium. The high(er) saving rate implies a high rate of capital accumulation and a fast decline in the rate of interest. This fast decline disadvantages wealthy households, who derive a large share of income from capital, more so than do poor households. Consequently, inequality declines. As argued above, this situation, $h(k) < 0$, occurs without (and with weak) dynamic status preferences.

The second situation is one in which the saving rate jumps down initially, and during the subsequent transition increases toward its steady-state value. As the saving rate is low here, the rate of capital accumulation is low as well, and so is the pace at which the rate of interest declines. This benefits the wealthy households more than the poor ones, thus,
inequality rises during transition and is higher in the post-shock steady state than in the initial steady state. This situation occurs when \( h(k) > 0 \), that is, under strong dynamic status preferences. A numerical simulation illustrates the transitional dynamics of the saving rate; see Fig. A-3 in the Appendix.

Proposition 3 delivers a second result. It presents the precise conditions for which \( h(k^*) < 0 \) (or \( h(k^*) > 0 \)). It asserts that the initial response of the saving rate \( s(0) \) (positive or negative jump) to the technology shock depends on the initial response of consumption \( c(0) \). As the capital stock \( k(0) \) is fixed instantaneously, the enhanced level of technology allows for more output for given \( k(0) \). Thus, if \( c(0) \) jumps down initially, then \( s(0) \) must jump up. In the other case when \( c(0) \) jumps up initially, whether \( s(0) \) declines or increases initially depends on the magnitude of the jump in \( c(0) \). For a “small” (“large”) upward jump of \( c(0) \) the technology effect dominates (is dominated by) the consumption change, and the saving rate \( s(0) \) jumps up (down) initially. As long as \( h(k^*) < 0 \), \( s(0) \) jumps up; when \( h(k^*) > 0 \), \( s(0) \) jumps down.

Under what conditions does \( c(0) \) jump up (down) initially? Intuition is gained by considering the phase diagram representing the dynamic system \((10')\); see Fig. 5 below. Three observations are pertinent. First, a technology shock, while raising both \( c \) and \( k \), does not affect the steady-state \( \frac{c}{k} \)-ratio. That is, both the pre-shock (SS0) and the post-shock (SS1) steady states are located on a ray through the origin, with slope \( \frac{c}{k} \). The post-shock steady state, though, is located to the north-east of the pre-shock steady state. Second, the response of initial \( c(0) \) to the technology shock (given the initial stock \( k(0) \)) depends on whether or not the saddle path is steeper or flatter than the \( \frac{c}{k} \)-ray. In the first (second) case, the saddle path shifts downwards (upwards), implying a downward (an upward) jump of \( c(0) \). Third, the flatter the saddle path, the stronger the upward jump of \( c(0) \). We illustrate the argument in Fig. 5 where, following the positive technology shock, the saddle path shifts from the dotted to the solid line.

As is easily seen, whenever \( c(0) \) jumps down, as in the left pane of Fig. 5, then \( s(0) \) jumps up, implying a decline in income inequality (due to a high rate of capital accumulation and a rapid decline in the interest rate). In contrast, if \( c(0) \) jumps up, as in the right pane of Figure 5, whether the saving rate initially jumps up or down is ambiguous. Initially, \( s(0) \)
Figure 5: The impact of a positive technology shock, $\Delta A > 0$, on the initial response of consumption, $c(0)$.

jumps up (down) when $c(0)$ jumps up by little (jumps up substantially — i.e., when the saddle path is flat enough).

As long as $s(0)$ jumps up, $h(k^*) < 0$, and inequality decreases following the productivity shock. Similarly, when $s(0)$ jumps down, $h(k^*) > 0$, and inequality increases due to the productivity shock. Formally, Proposition 3 provides the necessary and sufficient condition for $c(0)$ to jump up sufficiently, so that $s(0)$ jumps down initially (cf. the proof in the Appendix).

To add intuition, consider the Euler equation in (10'). The dynamic status effect reduces the optimal consumption growth rate (as $E(c, k) < 0$). Compared to a model without dynamic status preferences, households choose a lower rate of consumption growth, together with an initially higher level of consumption. The higher initial consumption level necessitates a lower initial saving rate. If the dynamic status effect is strong enough, initial consumption jumps up so much that the initial saving rate jumps down. Proposition 3 gives rise to an immediate corollary.

**Corollary 4** Suppose $(\beta - \mu^*) > \frac{\delta}{k^*}$. Then $h(k^*) < 0$, and a positive technology shock lowers income inequality.

**Proof.** Taking into account that $s^* = \frac{\alpha \delta}{\beta + \delta} < 1$, the corollary follows directly from Proposition 3. ■

The intuition of Corollary 4 is straightforward. In $(k, c)$ phase plane, both the pre-shock and the post-shock steady states are located along a ray from the origin with slope $\frac{c}{k}$, with the
post shock steady state lying to the north-east of the initial steady state. Under the condition of the corollary, the slope of the saddle path exceeds the slope of this ray \((\frac{c}{k})\). Thus, a positive technology shock shifts the saddle path down and to the right. Consequently, consumption \(c(0)\) jumps down initially (when \(k(0)\) is fixed). Therefore the saving rate jumps up initially. In this case, capital is accumulated at a fast pace and the rate of interest declines rapidly, which disadvantages the rich more than the poor. As a consequence, inequality declines.

To illustrate further Proposition 3, we provide an example by employing the same functional forms as for the previous simulations. However, here differences in status preferences are captured by a single parameter, \(\varepsilon_\infty\), which defines the range of values the status function, \(\varepsilon(t)\), can assume. Different values of \(\varepsilon_\infty\) proxy cultural differences among countries. In particular, the lower the value of \(\varepsilon_\infty\) the more intensely a country responds to changes in aggregate wealth.\(^{26}\)

**Corollary 5** Consider our functionally specified economy (8), (13), and (14), with \(\rho = 0\) in utility function (13). Assume that in the steady state \(\varepsilon'(k) \approx 0\). Then, following a positive productivity shock, inequality evolves according to:

\[
\dot{\sigma}_k \geq 0 \iff \begin{cases} 
\varepsilon_\infty \leq \frac{\beta + \delta (1 - \alpha (1 - \gamma))}{\alpha \delta \gamma} & \text{if } \gamma < 0 \\
\varepsilon_\infty \geq \frac{\beta + \delta (1 - \alpha (1 - \gamma))}{\alpha \delta \gamma} & \text{if } \gamma > 0 
\end{cases}
\]

\(^{(21)}\)

**Proof.** If \(\rho = 0\), then \(DOP(c, k) = \varepsilon(k)\). Applying the definitions for \(E(c, k)\) and \(IES(c, k)\) to (18) and setting \(\varepsilon'(k)\) equal to zero yields (21). \(\blacksquare\)

Based on Corollary 5, we construct a simple numerical example. Consider two countries, A and B having the identical technology and initial income distribution, but with different cultural parameters in status preferences. Country A has a relatively stronger response in status concerns to the development of wealth (\(\varepsilon_\infty = 0.02\)) than has country B (\(\varepsilon_\infty = 0.25\)). For both countries, we consider \(\varepsilon_0 = 0.3\). All other parameter values are identical to those employed in the previous section. Fig. 6 illustrates the dynamics of income inequality following a positive productivity shock, where the productivity parameter \(A\) is increased

\(^{26}\)Notice that the decline in \(\varepsilon(t)\) is governed by the term \(\kappa(\varepsilon_0 - \varepsilon_\infty)\). That is, instead of specifying parameter \(\varepsilon_\infty\) as country-specific, we could have specified parameter \(\kappa\) as country-specific. The two specifications are equivalent, though, and yield the same results. Absence of the dynamic status effect, in our approach, is captured by \(\varepsilon_\infty = \varepsilon_0\).
from 2 to 3. The figure shows that in the economy where status preferences are more responsive to changes in wealth (Country A, dashed line in Fig. 6), inequality increases, while for the economy where status is less responsive to a rise in wealth (Country B, solid line in Figure 6), inequality declines in response to the same positive technology shock.

[Figure 6 about here]

The intuition follows closely the mechanism involving the convergence speed described above. In the economy with a weak dynamic status effect the return-to-capital effect dominates. Interest and the saving rate initially increase due to higher productivity (see Fig. A-3 in the Appendix). Along the transition to the steady state the saving rate falls, as the return-to-capital-effect dominates the dynamic status effect (people do not adjust their behavior towards lower status evaluation). Consequently, the convergence speed is high (due to the higher initial saving rate), and inequality declines because of the rapid decline in the interest rate. In contrast, in the economy having a strong dynamic status effect, initially, the saving rate declines. This is because, due to higher income from the productivity increase, agents consume more to display their status (see Fig. A-3 in the Appendix). However, in this case the preference for status changes rapidly, while the saving rate increases and approaches its steady-state value from below. As the level of the saving rate is low, capital is accumulated slowly, and the rate of interest remains high for a long time. Thus, inequality increases.

For the simulation displayed in Fig. 6, parameters were chosen to produce opposite effects regarding the impact of the productivity shock on the transitional dynamics of inequality. More generally, whether inequality rises or falls following a positive productivity shock depends on the respective strengths of the return-on-capital- and dynamic status effects as implied by Proposition 3.

To summarize: strong and responsive cultures for status a) encourage the society to direct productivity increases to consumption, initially lowering savings and the decline in the return on capital (capital owners that hold more capital see their returns reduced slowly), and b) increases the saving rate during the transition (capital owners accumulate relatively more capital). Both channels contribute to higher income inequality.

Two remarks merit comment. First, and more important, our mechanism whereby productivity shocks generate inequality contrasts sharply with that proposed by Caselli and
Ventura (2000). In their framework the productivity shock has a positive effect on income inequality when there is a positive technological bias towards capital returns relative to labor wages. In our approach, by assuming a Cobb-Douglas production function we isolate such a technology bias. Instead, the differential dynamics of income inequality in response to a productivity shock operate through the evolution of agents’ behavior, and specifically the sensitivity of status concerns with respect to wealth. Interestingly, this result complements the literature by providing an explanation for why countries that share the same production technology (no technology bias in the factors of production) and have the same income in the long-run (the case of many advanced countries) can nevertheless end up with a very different distribution of income after a technology- or policy shock.

Second, following Proposition 1, cultural differences in status concerns (as proxied by differences in in our numerical example) do not affect production and, in turn, do not have any long-run impact on income. This is important because the differentials in income inequality come through the dynamics of the economy rather than the level of economic development. This way we provide a framework to analyze the behavior of income distribution under a productivity shock in countries at the same stage of economic development (see for example the case of advanced countries in Figs. A1 and A2).

These two observations suggest an important policy implication. Policies targeted to increase productivity raise income but do not necessarily decrease income inequality. In fact, as argued above, such policies may raise income inequality. This may occur because in the presence of dynamic status preferences, people raise consumption in response to the positive technology (policy) shock, rather than increasing savings that generate future income and economic convergence. That is, instead of creating wealth, such a policy shock contributes to increasing both consumption and inequality. Thus, investment in institutions that induce behavioral changes away from status concerns, rather than policies of enchanting productivity increases, might turn out to be effective policy measures curtailing the inequality epidemic.
6 Conclusion and open research directions

This paper advances the hypothesis that the intensity of status preferences negatively depends on the average wealth of society. Within an otherwise standard neoclassical growth model, we provide a new mechanism to explain the saving rate dynamics (a rising- or inversely U-shaped transitional path) and the comparative development of income inequality across countries. We advance our knowledge about the development of income and wealth inequality by complementing the work of Caselli and Ventura (2000) and Piketty (2014) by introducing a dynamic behavioral factor, as opposed to their technological vehicle. The advantage of our behavioral mechanism is that it can explain the development of income inequality even after economies have technologically been converged (e.g. US vs Europe, Fig. 1 vs Fig. 2). In particular, we showed that differentials in the strength of the dynamic status effect can propagate variation of income inequality that is attributed to the differential response of agents to productivity shocks rather than a technological bias on the factor of production as in Piketty (2014). As a policy implication, our theory suggests that policies that target productivity advancement towards increasing the income of the poor countries may not be sufficient to reduce income inequality when poor countries direct their income to “unproductive” uses such as status goods consumption. Instead, institutions that support ways behavioral changes (like educational institutions) turn out to be more effective.

We believe that our theoretical framework can provide the basis for addressing a range of research questions on the inequality epidemic. Among others, first, distributive policies financed through the taxation of luxury/status goods may increase rather than decrease inequality, as per our framework, poor individuals care more about status and their consumption will be inelastic to taxes on status goods. Second, the productive effect of taxation may not produce a Laffer-curve when individual concerns for status are strong. Third, our framework extended with endogenous leisure choice can be a vehicle to explain the strong income effect of wages on labor supply. This can happen as leisure time can be used from individuals to indicate status (in turn, it can help New Keynesian models match new empirical evidence on the negative labor supply elasticity). Last, recent evidence shows that status anxiety increases in inequality. This opens another channel for (further) endogenizing the degree of positionality.
7 Appendix

A.1 Degree of positionality

Under Assumption 1,

\[ U(C_t, S_t, \varepsilon(k)) = C_i^R V(\bar{c}, \varepsilon(k)) \]

so that

\[ \frac{\partial[C_i^R V(\bar{c}, \varepsilon(k))]}{\partial C_i} = R C_i^{R-1} V(\bar{c}, \varepsilon(k)) = (\partial U/\partial S_t)(\partial S_t/\partial C_i) + (\partial U/\partial C_i), \]

which corresponds to the denominator of the definition of the DOP in (2). Next, in order to determine the numerator \((\partial U_i/\partial S_t)(\partial S_t/\partial C_i)\), we consider:

\[ \frac{\partial U_i}{\partial C_i} = \frac{\partial U_i}{\partial S_t} \frac{\partial S_t}{\partial C_i} = \left( \frac{\partial U_i}{\partial S_t} \right) \left( \frac{\partial S_t}{\partial C_i} \right) \frac{\bar{c}_t}{\bar{c}} = -C_i^{R-1} \bar{c} \]

which represents the numerator of (2). Combining the numerator and the denominator yields (3).

A.2 Derivation of (6)

Optimizing (4) subject to (5) with respect to \(C_i\) yields the first order optimality condition

\[ R C_i^{R-1} V(\bar{c}, \varepsilon(k)) = \mu_i, \]

where \(\mu_i\) is the individual’s shadow value of wealth. Taking the time derivative of this condition yields

\[ (R - 1) \frac{\dot{C}_i}{C_i} + \left( \frac{V_\dot{c}_t}{V} \right) \frac{\dot{c}}{\bar{c}} + \left( \frac{V_\dot{\varepsilon}(k)}{V} \right) \dot{k} = \frac{\dot{\mu}_i}{\mu_i}. \]

As all agents face the same rate of return, \(\dot{\mu}_i/\mu_i = -(r - \beta)\), individual consumption growth rates are independent of household characteristics, i.e., they are identical across households. Consequently, individual and average consumption growth rates coincide: \(\dot{C}_i/C_i = \dot{c}/c\). Considering \(\bar{c} = c\) in equilibrium, yields

\[ - (1 - R) \frac{\dot{c}}{c} - R DOP \frac{\dot{c}}{c} + \left( \frac{V_\dot{\varepsilon}(k)}{V} \right) \dot{k} = -(r - \beta). \]
Rearranging terms yields (6).

A.3 Proof of Proposition 1

As shown in Section 4.1, the steady-state saving rate is given by \( s^* = \alpha \delta / (\beta + \delta) \). To analyze the evolution of the saving rate \( s = 1 - c A^{-1} k^{-\alpha} \), we analyze the behavior of \( z \equiv c A^{-1} k^{-\alpha} \). Specifically, \( g_z \equiv \dot{z} = \dot{c} - \alpha \frac{k}{\dot{k}} \), and we note that the saving rate increases (decreases) when \( g_z < 0 \) (when \( g_z > 0 \)). Substituting the growth rates of \( c \) and \( k \), and employing the elasticities \( IES(c, k) = \frac{1}{1-R(1-DOP(c, k)))} \) as well as \( E(c, k) \equiv V_k k / V = V \varepsilon ' k / V \leq 0 \), we obtain

\[
g_z = IES(c, k)(r - \beta) + IES(c, k) E(c, k) \left( \frac{\dot{k}}{k} \right) - \alpha \left( \frac{\dot{k}}{k} \right) . \tag{A.1}
\]

Let \( \xi(c, k) \equiv \left[ \alpha - IES(c, k) E(c, k) \right] / \alpha \geq 1 \). Without the dynamic status effect, \( \varepsilon' = 0 = E(c, k) \), and \( \xi(c, k) = 1 \). In the presence of the dynamic status effect, \( \varepsilon' < 0 \), that is \( E(c, k) < 0 \), and \( \xi(c, k) > 1 \). Theoretically, there is no limit for the value of \( \varepsilon' \), so that \( \xi(c, k) \) can become arbitrarily large.

Considering \( \xi(c, k) \), we can re-write (A.1) as

\[
g_z = IES(c, k)(f'(k)-\delta-\beta) - \xi(c, k) \alpha \left( \frac{\dot{k}}{k} \right) .
\]

For the Cobb Douglas production technology, \( f(k)/k = (1/\alpha)f'(k) \), and \( c/k = c/f(k) f(k)/k = z f(k)/k = z(1/\alpha)f'(k) \). Therefore, \( \alpha \left( \frac{\dot{k}}{k} \right) = f'(k)(1-z) - \alpha \delta = f'(k)(1-z) - s^*(\beta + \delta) \).

Re-arranging terms yields

\[
g_z = f'(k) \left[ IES(c, k) - \xi(c, k)(1-z) \right] + (\beta + \delta) \left[ \xi(c, k) s^* - IES(c, k) \right] . \tag{A.2}
\]

Note that without the dynamic status effect – \( \varepsilon' = 0 \), \( \xi(c, k) = 1 \) – growth rate (A.2) corresponds to the standard case, as given by Barro and Sala-i-Martin (p.136, 2004).

As demonstrated by Barro and Sala-i-Martin (p.109 as well as p.135ff, 2004), for the case without the dynamic status effect, \( g_z > 0 \), implying \( \dot{s} < 0 \) for reasonable calibrations. The presence of the dynamic status effect changes this conclusion. Following the arguments by Barro and Sala-i-Martin (p.135ff, 2004), during transition the saving rate increases as long as

\[
s^* > \frac{IES(c, k)}{\xi(c, k)} . \tag{A.3}
\]

The dynamic status effect implies \( \xi(c, k) > 1 \), with no upper limit to \( \xi(c, k) \) on grounds of
theory. For this reason, (A.3) can be satisfied for reasonable calibrations. In particular, for every value of the elasticity of intertemporal substitution there exists a $\xi(c, k) > 1$ for which (A.3) is satisfied, and consequently $\dot{s} > 0$. ■

A.4 Proof of Proposition 2

Following García-Peñalosa and Turnovsky (2008), equation (17) shows that inequality rises over time ($\sigma_k(t) > \sigma_k(0)$) if $h(k) > 0$ and falls over time if $h(k) < 0$. From our Cobb-Douglas technology, it follows that $f''(k)k = -(1-\alpha)f'(k)$. Moreover, in steady state, $c = f(k) - \delta k$. Thus, $h(k) = -f''(k)k - v_1 w/c = (1-\alpha)f'(k) - (\beta - \mu)[f(k) - kf'(k)]/[f(k) - \delta k] = (1 - \alpha)f'(k)[1 - (\beta - \mu)/(f'(k) - \alpha\delta)]$. Using the steady state condition $f'(k) = \beta + \delta$, we have

$$h(k) = \frac{(1-\alpha)(\beta + \delta)}{\beta + \delta(1-\alpha)}[(1-\alpha)\delta + \mu].$$

Therefore, $\text{sgn} h(k) = \text{sgn}((1-\alpha)\delta + \mu)$.

Next, we consider the Jacobian to the dynamic system (10'), evaluated at steady state, which we can write as

$$J = \begin{bmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial c} \\ \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial c} \end{bmatrix} \begin{bmatrix} \beta & -1 \\ j_{21} & j_{22} \end{bmatrix},$$

where $j_{21} \equiv IES(c, k) c f''(k) - \beta j_{22} < 0$, $j_{22} = -IE\sigma(c, k) c E(c, k)/k \geq 0$, and $j_{22} > 0$ if and only if $\varepsilon'(k) < 0$. As the smaller eigenvalue equals

$$\mu = 2^{-1} \left\{ \beta + j_{22} - \sqrt{(\beta + j_{22})^2 - 4(\beta j_{22} + j_{21})} \right\},$$

we know that $h(k) > 0$ is equivalent to

$$2\delta(1-\alpha) + (\beta + j_{22}) > \sqrt{(\beta + j_{22})^2 - 4(\beta j_{22} + j_{21})}.$$ 

Squaring both sides of the inequality and rearranging terms yields:

$$(1-\alpha)\delta [(1-\alpha)\delta + \beta + j_{22}] > -(\beta j_{22} + j_{21}). \quad (A.4)$$

From the definitions of $j_{21}, j_{22}$ we find $\beta j_{22} + j_{21} = IES(c, k) c f''(k)$. Next, we take into account that $c f''(k) = (c/k)k f''(k) = -(c/k)(1-\alpha)f'(k) = -[(\beta + \delta)/\alpha - \delta](1-\alpha)(\beta + \delta).$
Considering this result in inequality (24) and solving for $j_{22}$ implies:

$$j_{22} > [IES(c, k)(\beta + \delta)/(\alpha \delta) - 1][\beta + \delta(1 - \alpha)] .$$  

(A.5)

Finally, we note that $j_{22} = -(c/k)IES(c, k)E(c, k)$. Taking into account that, in a steady state, $(c/k) = [(\beta + \delta)/\alpha - \delta]$ and $s^* = \alpha \delta/(\beta + \delta)$, (25) can be (after simplifying) written as

$$h(k) > 0 \Rightarrow -\frac{E(c, k)}{\alpha} > \left[\frac{1}{s^*} - \frac{1}{IES(c, k)}\right].$$  

(A.6)

All above steps can likewise be done for the reversed inequality

$$h(k) < 0 \Rightarrow -\frac{E(c, k)}{\alpha} < \left[\frac{1}{s^*} - \frac{1}{IES(c, k)}\right].$$  

(A.7)

(A.6) and (A.7) imply (18).

A.5 Proof of Proposition 3

From Propositions 1 and 2, it follows that $h(k) > 0$ ($h(k) < 0$) implies $\dot{s} > 0$ ($\dot{s} < 0$) approaching its steady state level from below (above). We need to show that

$$(\beta - \frac{\mu}{s^*}) \leq \frac{c}{k} \iff h(k) \geq 0 .$$

Consider the right hand side of the equivalence first. As argued above, $\text{sgn} h(k) = \text{sgn}((1 - \alpha)\delta + \mu)$.

$$h(k) \geq (1 - \alpha)\delta + \mu \geq 0 \iff -\mu \leq (1 - \alpha)\delta = (1 - \alpha)\frac{\beta + \delta}{\alpha} s^* ,$$

as $s^* = \alpha \delta/(\beta + \delta)$. Next, consider the left hand side of the equivalence.

$$\left(\beta - \frac{\mu}{s^*}\right) \leq \frac{c}{k} \iff -\mu \leq \left(\frac{c}{k} - \beta\right) s^* = \left(\frac{\beta + \delta}{\alpha} - (\beta + \delta)\right) s^* = (1 - \alpha)\frac{\beta + \delta}{\alpha} s^* ,$$

where the first equality follows from the fact that $c/k = (\beta + \delta)/\alpha - \delta$ in a steady state, as shown in the proof of Proposition 2.

■
A.6 Figures

[Figure A-1 about here]

[Figure A-2 about here]

[Figure A-3 about here]

References


Figures

![Graph 1](image1)

**Figure 1:** How important is it to you to compare your income with other people’s incomes? *Source: Clark and Senik (2010, p.580).*

![Graph 2](image2)

**Figure 2:** Income (wealth) and happiness as a country develops over time. *Source: Clark, Frijters and Shields (2008, p.101).*
Figure 3: The Dynamics of Savings and Return on Capital.

Figure 4: The Dynamics of Income Inequality
Figure 5: The impact of a positive technology shock, $\Delta A > 0$, on the initial response of consumption, $c(0)$.

Figure 6: Dynamics of Income Inequality to Positive Productivity Shocks

Inequality ($\sigma_k$) as of a positive productivity shock on $A$

Strong dynamic status effect:
$\epsilon_0=0.3, \epsilon_w=0.02$

Weak dynamic status effect:
$\epsilon_0=0.3, \epsilon_w=0.25$
Figure A-1: Top 1% share of total income - Europe and Japan (L-shaped) 1900 – 2011. 

Figure A-2: Top 1% share of total income ? English speaking countries (U-shaped) 1900 – 2011. Source: Roser and Ortiz-Ospina (2017).
Figure A-3: Response of the saving rate to a positive productivity shock.

The diagram shows the savings rate as a function of time, $t$, with two scenarios indicated:

- $\epsilon_0 = 0.3$, $\epsilon_\infty = 0.25$
- $\epsilon_0 = 0.3$, $\epsilon_\infty = 0.02$

The solid line illustrates the scenario with $\epsilon_\infty = 0.25$, while the dashed line represents the case with $\epsilon_\infty = 0.02$. The point marked 's (pre shock)' indicates the initial savings rate before the shock is applied.