Can we Identify the Fed's Preferences?

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Abstract

Durbin (1970) pre-tests of Ramsey optimal policy versus time-consistent policy rejects time-consistent policy and optimal simple rule for the U.S. Fed during 1960 to 2006, assuming the reference new-Keynesian Phillips curve transmission mechanism with auto-correlated cost-push shock, including or not working capital. Estimates of a structural VAR shows that Ramsey optimal policy models the persistence of inflation, output gap and federal funds rate without requiring two additional parameters for inflation indexation and habit persistence. The number of reduced form parameters is larger with Ramsey optimal policy than with time-consistent policy although the number of structural parameters, including central bank preferences, is the same. The new-Keynesian Phillips curve model is under-identified with Ramsey optimal policy (one identifying equation missing) and hence under-identified for time-consistent policy (three identifying equations missing).

JEL classification numbers: C61, C62, E31, E52, E58.
Keywords: Ramsey optimal policy, Time-consistent policy, Identification, Central bank preferences, New-Keynesian Phillips curve.

1 Introduction

Can we *pre-test* if the Fed follows Ramsey optimal policy under quasi-commitment (Debortoli and Nunes (2014)) or a time-consistent policy (Cohen and Michel (1988), Oudiz and Sachs (1985), Gali (2015) chapter 5)? Are the Fed's preferences facing the same *identification* problem as Taylor rule parameters? Cochrane (2011) found that the simple Taylor rule parameters are not identified in new-Keynesian models including forward-looking inflation, but only the auto-correlation parameters of non-observable shocks.

Beginning with Simon (1956) certainty equivalence property of the linear quadratic regulator, the quadratic loss function describing policy-maker's preferences is used for modelling stabilization policy (Duarte (2009)). It took more than two decades for the estimation of Fed's preferences to begin with Salemi (1995), using inverse control (Salemi (2010)). Salemi (1995) took into account two structural breaks (1970-1, 1979-10) for three monetary policy regime during the period 1947-1992 using monthly data. Salemi (1995)

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includes a careful investigation of identification issues when the monetary policy transmission mechanism is a backward-looking vector auto-regressive (VAR) model. These identification issues are the same for Ramsey optimal policy under quasi-commitment (Debortoli and Nunes (2014)) with forward-looking variables, besides the optimal initial jump anchoring forward-looking variables (Ljungqvist and Sargent (2012)).

Following Salemi (1995), there are around thirty papers estimating Fed's preferences since the 1960's with a structural break since the 1980's (Volcker-Greenspan period) assuming inflation is a predetermined variable or a forward-looking variable, assuming the accelerationist Phillips curve or or new-Keynesian Phillips curve as a monetary policy transmission mechanism: e.g. Cechetti and Ehrmann (2002), Ozlale (2003), Favero and Rovelli (2003), Castelnuovo and Surico (2004), Castelnuovo (2006), Soderstrom et al. (2005), Juillard et al. (2006), Salemi (2006), Kara (2007), Adjemian and Devulder (2011), Adolfson et al. (2011), Ilbas (2012), Levieuge and Lucotte (2014), Paez-Farrell (2015), Debortoli and Lakdawala (2016) and many others.

Contrasting estimations of Ramsey optimal policy with Oudiz and Sachs (1985) time-consistent policy is recent (Givens (2012), Matthes (2015)). This paper highlights two general theoretical issues when contrasting Ramsey optimal policy and time-consistent policy. Distinct numbers of parameters can be identified for each policy. Distinct numbers of parameters are required by each policy to fit the auto-correlation of inflation, output gap and Federal funds rate.

Firstly, both policies has the same number of structural parameters. But there is a smaller number of reduced form parameters for time-consistent policy with respect Ramsey optimal policy. The policy instrument and policy maker's Lagrange multiplier are predetermined variables in Ramsey optimal policy under quasi-commitment (Debortoli and Nunes (2014)). Policy instruments are forward-looking variables in time-consistent policy. According to Blanchard and Kahn's (1980) determinacy condition, Ramsey optimal policy has *richer dynamics* than time-consistent policy. It includes *more predetermined variables*, *more stable eigenvalues (bifurcation)*, *more linearly independent variables* in its stable vector auto-regressive (VAR) representations and *more reduced form parameters* which are elements of its larger size VAR matrix than time-consistent policy. For example, if Ramsey optimal policy structural parameters are exactly identified, they are under-identified for time-consistent policy.

Secondly, this paper highlights that, with respect to time-consistent policy, the richer dynamics of Ramsey optimal policy implies a considerable advantage to model the persistence of macroeconomic time-series. Ramsey optimal policy does not require additional *ad hoc exogenous* auto-correlation parameters at least equal to 0.9 for inflation indexation or for consumption habit. By contrast, the auto-correlation of time-series is *endogenously* changed by the negative-feedback mechanism of Ramsey optimal policy rule. Our idea is to use *a Durbin (1970) pre-test of the auto-correlation of disturbances for time-consistent and for Ramsey optimal policy, in order to compare the misspecification of persistence in both models.*

In order to limit the opacity related to the numerical approximations of optimal policy in large scale models, we pre-test and test an explicit reference model with closed form solution which is widely taught. It includes inflation as a single policy target and the new-Keynesian Phillips curve with an auto-regressive cost-push shock (Debortoli and Nunes (2014), Gali (2015), chapter 5). We demonstrate that time-consistent policy is based on positive-feedback mechanism and corresponds to optimal simple rules in this model (proposition 1 and 2). Kollmann (2002) and (2008) is a precursor for optimal simple rules simulations in models including the new-Keynesian Phillips curve.

The closed form solution allows to check exactly the identification of parameters. As expected from the general case, three identifying equations are missing in time-consistent policy and only one for Ramsey optimal policy (proposition 3). Only the auto-correlation parameter of the non-observable cost-push shock can be identified for time-consistent policy, but not the Fed's preferences nor the slope of the Phillips curve. This is consistent with Cochrane (2011) where the Taylor rule parameters are not identified, but only the auto-correlation parameter of the forcing variable. By contrast, the Fed's preferences (the relative cost of changing the policy instrument) and the slope of the new-Keynesian Phillips curve can be identified with Ramsey optimal policy if the Fed's discount factor is exogenously given. We propose an alternative estimation method for Ramsey optimal policy, which avoids an identification problem between the auto-correlation of the dependent variable and auto-regressive disturbances (cost-push shock) (Feve, Matheron and Poilly (2007), Griliches (1967), Blinder (1986), McManus et al. (1994)).

As expected from the general case, the time-consistent policy rule is misspecified with a very large auto-correlation of residuals (a strong rejection of Durbin (1970) pre-test), whereas the auto-correlation of residuals is small in the case of Ramsey optimal policy. Unconstrained VAR parameter estimates are very close to the values of reduced form parameters computed from structural parameters estimates of Ramsey optimal policy, with very close residuals. The inflation equation of the VAR has identical estimates in both cases, including the inflation auto-correlation parameter. *An additional inflation indexation parameter in the new-Keynesian Phillips curve is useless for Ramsey optimal policy.* In the policy instrument equation of the VAR, its auto-correlation parameter shifts from an unconstrained estimate equal to 0.9 to a Ramsey optimal policy reduced form estimate close to its maximal value (a unit root). Ramsey optimal policy predicts slightly too much persistence of the policy instrument (here the output gap). *An additional consumption habit parameter in the Euler consumption equation is useless for Ramsey optimal policy.* The number of parameters of Ramsey optimal policy required to fit the auto-correlation of output gap and inflation is smaller than time-consistent policy. Time-consistent policy with positive-feedback mechanism for stabilization policy requires *epicycles on epicycles to be saved to fit the auto-correlation of observed time-series*, adding two ad hoc parameters for inflation indexation and consumption habit persistence.

In the test of Ramsey optimal policy, if the policy instrument is the output gap (targeting rule), the relative weight of the output gap with respect to the weight of inflation (4 to 1) is very large, although not unheard of in previous estimations, and the slope of the new-Keynesian Phillips curve is slightly too high. If the policy instrument is the Federal funds rate, assuming labor cost is financed by working capital (Christiano, Trabandt, Walentin (2014), Bratsiotis and Robinson (2016)), the relative weight on the interest rate variance with respect to the weight of inflation is plausible (1.2 to 1) in order to avoid the zero lower bound and for modelling interest rate smoothing. The slope of the new-Keynesian Phillips curve is also plausible.

We provide a Durbin (1970) pre-test on the reduced form of the Taylor rule for optimal policy taking into account the Euler consumption equation with an auto-regressive forcing variable in addition to the new-Keynesian Phillips curve (Giannoni and Woodford (2003), Kara (2007)). Durbin (1970) pre-test rejects time-consistent policy with respect to Ramsey optimal policy with quasi-commitment.

Section 2 presents issues on persistence and identification in the general case. Section 3 presents our estimation methods of Ramsey optimal policy and time-consistent policy.

Section 3 presents the bifurcation related to the shift from Ramsey policy to time consistent policy, the null hypothesis of related the pre-test and its implementation on US data during 1960-2006. Section 4 tests Ramsey optimal policy when output gap is the policy instrument. Section 5 presents results when the Federal funds rate is the policy instrument and inflation is the policy target. Section 6 pretests in the case of two policy targets (output gap and inflation) with one policy instrument (Federal funds rate). The last section concludes.

2 Persistence and Identification Issues in the General Case

When inventing infinite horizon time-consistent policies, Oudiz and Sachs (1985) and Backus and Driffil (1986) assumed that the number of stable eigenvalues for time-consistent policy is exactly equal to the number of predetermined variables ($\mathbf{u}_t$) of the private sector. By contrast, the number of stable eigenvalues for Ramsey optimal policy is equal to the number of predetermined variables ($\mathbf{u}_t$) and forward looking variables (π_t) of the private sector. Each policies corresponds to a stable vector auto-regressive model of order one (with matrix $\mathbf{A}$) and of policy rules equations (with matrix $\mathbf{F}$):

$$\begin{pmatrix} \mathbf{u}_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_{uu} & \mathbf{A}_{u\pi} \\ \mathbf{A}_{\pi u} & \mathbf{A}_{\pi\pi} \end{pmatrix} \begin{pmatrix} \mathbf{u}_t \\ \pi_t \end{pmatrix} : \text{Ramsey optimal policy} \quad (1)$$

$$\mathbf{i}_t = \mathbf{F}_\pi \pi_t + \mathbf{F}_u \mathbf{u}_t: \text{Ramsey optimal policy rule (Sargent and Ljungqvist (2013))} \quad (2)$$

$$\mathbf{u}_{t+1} = \mathbf{A}_{uu}\mathbf{u}_t: \text{Time consistent policy, with } \pi_t = \mathbf{N}_{\pi u}\mathbf{u}_t \quad (3)$$

$$\mathbf{i}_t = \mathbf{F}_{u,TC}\mathbf{u}_t: \text{Time-consistent policy rule} \quad (4)$$

In the general case, when the transmission mechanism includes m forward-looking variables and n backward-looking (predetermined) variables, determinacy for time-consistent policy and simple rule implies exactly n stable eigenvalues, whereas Ramsey optimal policy implies exactly $n+m$ stable eigenvalues. Shifting from time-consistent policy or simple rule (with determinacy) to Ramsey optimal policy is a *bifurcation where m eigenvalues shift from instability to stability.*

Firstly, for estimating the auto-correlation and cross-correlations of forward-looking variables, the matrix $\mathbf{A}_{\pi\pi}$ gives a *considerable advantage* to Ramsey optimal policy with respect to time-consistent policy. The policy maker's quasi-commitment anchors the expectations of forward-looking variables of the private sector over time.

Secondly, the number of reduced form parameters (elements) of matrices $\mathbf{A}_{\pi\pi}$ and $\mathbf{F}_\pi$ *adds* to the number of reduced form parameters of time-consistent policy in $\mathbf{A}_{uu}$, $\mathbf{N}_{\pi u}$ and $\mathbf{F}_u$. The matrices $\mathbf{A}_{\pi u}$ and $\mathbf{N}_{\pi u}$ have the same number of reduced form parameters (elements). If endogenous predetermined variables (the stocks of debt and capital) are assumed to be zero at all periods, $\mathbf{u}_t$ corresponds to exogenous auto-regressive forcing variables, so that $\mathbf{A}_{u\pi} = \mathbf{0}$. However, the number of structural parameters corresponding to the central bank preferences (weights in the loss function) and to structural parameters of the equations of the private sector monetary policy transmission mechanism (new-Keynesian Phillips curve, consumption Euler equation) are exactly the same in both Ramsey optimal policy and time-consistent policy. As a consequence, only one of the two model can be at least exactly identified.

For time-consistent policy, a parameter identification problem occurs as soon as a policy target or a policy instrument (the policy rule) is a linear function of more than n variables of the model (hence with more than n parameters), where n is the number of predetermined variable of the private sector. All variables are evolving in a stable subspace of dimension n: they are exact linear combinations of n linearly independent variables of the model. If one write the policy rule with more variables $\mathbf{z}_t$ (which could be lags of the policy instruments), these variables are necessarily a linear combination of n linearly independent variables of the model, such as π_t and $\mathbf{u}_t$. This is an identification restriction that should not be missed.

$$\mathbf{i}_t = \mathbf{F}_\pi \pi_t + \mathbf{F}_u \mathbf{u}_t \text{ is observationally equivalent to:} \tag{5}$$
$$\mathbf{i}_t = \mathbf{F}_{i\pi,z}\pi_t + \mathbf{F}_{iu,z}\mathbf{u}_t + \mathbf{F}_{iz}\mathbf{z}_t = \mathbf{F}_{i\pi,z}\pi_t + \mathbf{F}_{iu,z}\mathbf{u}_t + \mathbf{F}_{iz}\left(\mathbf{G}_{z\pi}\pi_t + \mathbf{G}_{zu}\mathbf{u}_t\right) \tag{6}$$
$$\mathbf{i}_t = \left(\mathbf{F}_{i\pi,z} + \mathbf{F}_{iz}\mathbf{G}_{z\pi}\right)\pi_t + \left(\mathbf{F}_{iu,z} + \mathbf{F}_{iz}\mathbf{G}_{zu}\right)\mathbf{u}_t \tag{7}$$
$$\text{Then } \mathbf{F}_{i\pi,z},\ \mathbf{F}_{iu,z} \text{ and } \mathbf{F}_{iz} \text{ are not identified.} \tag{8}$$

The failure of time-consistent policy to model persistence led to assume that lagged values of forward-looking variables as additional predetermined variables which enters into the dynamics of current forward-looking variables ($\pi_t = \gamma\pi_{t-1} + E_t(\pi_{t+1}) + \mathbf{u}_t$) with additional persistence parameters γ. These are inflation indexation and consumption habit parameters, for example. The projection of forward-looking variables on predetermined variables models the auto-correlation of forward-looking variables:

$$\pi_t = \mathbf{N}_{\pi_{t-1}}\left(\gamma, \mathbf{A}_{uu}\right)\pi_{t-1} + \mathbf{N}_{\pi u}\left(\gamma, \mathbf{A}_{uu}\right)\mathbf{u}_{t-1} \tag{9}$$

This specification faces Griliches (1969), Feve, Matheron, Poilly (2007) identification issue for modelling persistence when the parameters of the lagged dependent variable are competing with the auto-correlation of the disturbances ($\mathbf{u}_t$). This identification problem is eliminated in assuming zero auto-correlation ($\mathbf{A}_{uu} = \mathbf{0}$) for predetermined variables so that $\mathbf{u}_t$ is white-noise (Clarida, Gali and Gertler (1999), section 6). In this case, Simon (1956) certainty equivalence implies that the private sector and policy makers rule does not depend on white noise disturbances.

$$\pi_t = \mathbf{N}_{\pi u}(\gamma)\,\pi_{t-1} \text{ and } \mathbf{i}_t = \mathbf{F}_{u,TC}(\gamma)\,\pi_{t-1} \text{ with } \pi_{t-1} \text{ predetermined} \tag{10}$$

Time-consistent policy requires to add free exogenous persistence parameters γ which are not required by Ramsey optimal policy to obtain model the auto-correlation of forward-looking variables, for an observationally equivalent model:

$$\pi_t = \mathbf{A}_{\pi\pi}\pi_{t-1} \text{ and } \mathbf{i}_t = \mathbf{F}_\pi \pi_t \tag{11}$$

Between observationally equivalent model, Ockham's razor selects the model with the lowest number of parameters, that is the model where free parameters are all set to zero $\gamma = \mathbf{0}$. For example, the micro-economic evidence for habit persistence is weak: Havranek *et al.* (2017) meta-analysis of 597 estimates in 81 published studies found on average 0.1 estimates in micro-level data and on average 0.6 for macroeconomic dynamic stochastic general equilibrium model estimates. This micro/macro inconsistency is worrisome for macroeconomic models based on micro-economic theoretical foundations of consumer's behaviour. In Duhem's (1908) wording, adding free exogenous persistence parameters ($\gamma \neq \mathbf{0}$) may amount to add epicycles on epicycles to save the unlikely phenomena of *positive-feedback mechanism for stabilization policy*, which is the hallmark of infinite horizon time-consistent policy.

Finally, if $\mathbf{u}_t$ includes at least one endogenous controllable variables (such as public debt) instead of only exogenous auto-regressive forcing variables, there is identification issue related to multiple equilibria similar to the fiscal theory of the price level for time-consistent policy. Oudiz and Sachs (1985) time-consistent algorithm will seek one among several equilibria depending on its initial conditions. A model with two forward-looking variables and two predetermined and controllable variables (public debt and private debt) has six equilibria (the number of combinations for selecting two stable eigenvalues among four controllable eigenvalues). By contrast, Ramsey optimal policy has a unique equilibrium taking into account endogenous controllable variables.

3 The Optimal Monetary Policy Problem

We follow exactly Gali's (2015, chapter 5) reference model for Ramsey optimal policy considers the case of an efficient steady state. The welfare losses experienced by the representative household are, up to a second-order approximation, proportional to:

$$v(\pi_0, u_0) = \max_{\{x_t, \pi_t\}} -\frac{1}{2} E_0 \left\{ \sum_{t=0}^{+\infty} \beta^t \left(\pi_t^2 + \alpha_x x_t^2 \right) \right\} \tag{12}$$

where x_t represents the welfare-relevant output gap, i.e. the deviation between (log) output and its efficient level. π_t denotes the rate of inflation between periods $t-1$ and t. u_t denotes a cost-push shock. β denotes the discount factor. E_t denotes the expectation operator. $v(\pi_0, u_0)$ denotes the optimal value function. Coefficient $\alpha_x > 0$ represents the weight of the fluctuations of the marginal cost of the firm (measured by the output gap) relative to inflation in the loss function. Coefficient $\alpha_x > 0$ is the relative cost of the changing the policy instrument with respect to the costs of fluctuations of the policy target, which is inflation. It is given by $\alpha_x = \frac{\kappa}{\varepsilon}$ where κ is the coefficient on the marginal cost of the firm x_t in the New Keynesian Phillips curve, and ε is the representative household's elasticity of substitution between each differentiated goods. More generally, and stepping beyond the welfare-theoretic justification for (1), one can interpret α_x as the weight attached by the central bank to deviations of output from its efficient level (relative to price stability) in its own loss function, which does not necessarily have to coincide with the household's.

The reference new-Keynesian Phillips curve is the monetary policy transmission mechanism:

$$\pi_t = \beta E_t \left[\pi_{t+1} \right] + \kappa x_t + u_t \text{ where } \kappa > 0,\ 0 < \beta < 1 \tag{13}$$

The reduced-form parameter (denoted κ) of the slope of the new-Keynesian Phillips curve relates inflation to marginal cost or to the output gap. It depends on four structural parameters: the representative household discount factor β, the household's elasticity of substitution between each differentiated goods ε, the measure of decreasing returns to scale of labor in the production functions of the firms η, and the proportion of firms who do not reset their price each period θ (Gali (2015), chapter 3):

$$\kappa\left(\beta,\varepsilon,\eta,\theta\right)=\frac{\left(1-\theta\right)\left(1-\beta\theta\right)}{\theta}\frac{\left(1-\eta\right)}{\left(1-\eta+\eta\varepsilon\right)}.$$

Econometricians estimate the slope κ and may estimate the discount factor β. But three parameters $(\varepsilon,\eta,\theta)$ face an under-identification problem. An infinite number of values of $(\varepsilon,\eta,\theta)$ lead to an observationally equivalent given estimate of $\widehat{\kappa}$. Hence ε is not identified and welfare preferences $\alpha_x=\frac{\kappa}{\varepsilon}$ are not identified with respect to observationally distinct Fed's preferences $\alpha_x\neq\frac{\kappa}{\varepsilon}$. One may assume arbitrarily an identification restriction for the elasticity of substitution ε that the Fed's preferences are identical to the household's preferences $\alpha_x=\frac{\kappa}{\varepsilon}$. The welfare preferences $\alpha_x=\frac{\kappa}{\varepsilon}$ is a useful theory for normative economics. But it is a useless under-identified model for positive economics (Chatelain and Ralf (2017)).

Christiano, Trabandt and Walentin (2010) introduce a working capital hypothesis, where labor cost is finance by working capital. This is a simple way to include the cost of capital in the new-Keynesian Phillips curve, knowing that in the reference new-Keynesian model, labor (and not capital) is the only input in the production function. The marginal cost in the new-Keynesian Phillips curve is then a linear combination of labor and capital cost (Christiano, Trabandt and Walentin (2010), equation 35):

$$\gamma\left(1+\phi\right)x_t+\frac{\psi}{\left(1-\psi\right)\beta+\psi}i_t \tag{14}$$

We estimate the two polar cases. When material inputs are not used in production ($\gamma=1$) and when no labor cost is financed by working capital ($\psi=0$), the cost of labor is the only cost taken into account and the output gap is the policy instrument. When material inputs are all used in production ($\gamma=0$) and when all labor cost is financed by working capital ($\psi=1$), the cost of capital is the only cost taken into account and the Federal funds rate is the policy instrument.

The central bank minimizes (1) subject to the sequence of constraints given by (2). The cost push shock u_t includes an exogenous auto-regressive component:

$$u_t=\rho u_{t-1}+\varepsilon_{u,t}\text{ where }0<\rho<1\text{ and }\varepsilon_{u,t}\text{ i.i.d. normal }N\left(0,\sigma_u^2\right) \tag{15}$$

where ρ denotes the auto-correlation parameter and ε_t is identically and independently distributed (i.i.d.) according to a normal distribution with constant variance σ_u^2.

The assumption of a non-observable auto-regressive exogenous forcing variable is facing Romer's (2016) critique. It is not a necessary assumption for Ramsey optimal policy (Sargent and Ljunqvist (2012), chapter 15). Assuming that predetermined variables are only non-observable auto-regressive exogenous forcing variables allows to avoid multiple equilibrium for time-consistent policy, such as the fiscal theory of the price level (Leeper (1991)). In our estimations, it constrains the eigenvalues of the stable VAR to be real. Whenever the unconstrained VAR includes two complex conjugate eigenvalues, the maximum likelihood of the structural VAR of Ramsey optimal policy does not converge with the assumption of the auto-regressive parameter ρ to be a real number.

3.1 Ramsey Optimal Policy with Quasi-Commitment

3.1.1 Solution using Lagrange Multipliers

A policy maker with a mandate for a new policy regime revised on date $t = 0$ (corresponding to a structural break in econometrics) commits to Ramsey optimal policy from the current date *until* a given known date T where the optimal policy is optimized again (Debortoli and Nunes (2014), appendix 4). The duration T of commitment ranges from the duration between each Federal open market committee meetings (eight times a year) up to the four years mandate of the chairman of the Fed or up to ten to twenty years of a stable monetary policy regime.

The policy maker could also re-optimize on each future period with exogenous probability ("stochastic replanning" (Roberds, 1987), "quasi commitment" (Schaumburg and Tambalotti, 2007; Kara 2007) or "loose commitment" (Debortoli and Nunes, 2014)). This assumption is observationally equivalent to Chari and Kehoe (1990) optimal policy under sustainable plans facing a punishment threat at a given horizon in case of deviation of an optimal plan (Fujiwara, Kam, Sunakawa (2016)).

Ramsey optimal policy can be solved directly using Bellman's equation, substituting the law of motion of the economy into the policy-maker's loss function without Lagrange multipliers. With the Lagrange intermediate computations, the Lagrangian of Ramsey optimal policy includes a sequence of Lagrange multipliers γ_{t+1}.

$$\mathcal{L} = -E_0 \sum_{t=0}^{t=T} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x x_t^2 \right) + \gamma_{t+1} \left(\pi_t - \kappa x_t - \beta \pi_{t+1} \right) \right] \tag{16}$$

The law of iterated expectations has been used to eliminate the condition expectations that appeared in each constraint. Because of the certainty equivalence principle for determining optimal policy in the linear quadratic regulator including additive normal random shocks (Simon (1956)), the expectations of random variables u_t are set to zero and do not appear in the Lagrangian.

The program includes given initial u_0 and final boundary conditions for the predetermined forcing variable variable $\lim_{t \to +\infty} \beta^t u_t = 0$. It also includes optimal initial and final boundary values of the forward-looking variable inflation. These transversality conditions minimize the optimal value of the central bank's loss function at the initial and the final date:

$$\frac{\partial v(\pi_t, u_t)}{\partial \pi_t} = 0 = \beta^t \gamma_t \text{ predetermined for } t = \{0, T\} \Leftrightarrow \pi_t = \pi_t^* \text{ for } t = \{0, T\} \tag{17}$$

$$\lim_{T \to +\infty} \frac{\partial v(\pi_T, u_T)}{\partial \pi_T} = 0 = \lim_{T \to +\infty} \beta^T \gamma_T \Leftrightarrow \lim_{t \to +\infty} \pi_T = \lim_{t \to +\infty} \pi_T^* \text{ if } T \to +\infty \tag{18}$$

In this paper, the estimated regimes last between 78 to 108 quarters, which may be considered as a long horizon. We follow Gali (2015) and we consider the limit case where the revision for a new policy regime happens in the infinite horizon. Differentiating the Lagrangian with respect to the policy instrument (output gap x_t) and to the policy target (inflation π_t) yields the first order optimality conditions:

$$\frac{\partial \mathcal{L}}{\partial x_t} = 0 \Rightarrow \alpha_x x_t - \kappa \gamma_{t+1} = 0 \tag{19}$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = 0 \Rightarrow \pi_t + \gamma_{t+1} - \gamma_t = 0 \tag{20}$$

$$\gamma_0 = 0 \Rightarrow x_{-1} = -\frac{\kappa}{\alpha_x}\gamma_0 = 0 \text{ and } \pi_0 = -\gamma_1 = -\frac{\kappa}{\alpha_x}x_0 \tag{21}$$

that must hold for $t = 1, 2, ...$ The *natural boundary condition* $\gamma_0 = 0$ minimizes the loss function with respect to inflation at the initial date. It predetermines the policy instrument which allows to anchor the forward-looking policy target (inflation). The inflation Euler equation corresponding to period 0 is not an effective constraint for the central bank choosing its optimal plan in period 0. The former commitment to the value of the policy instrument of the previous period x_{-1} is not an effective constraint. The policy instrument is predetermined at the value zero $x_{-1} = 0$ at the period preceding the commitment.

Combining the two optimality conditions to eliminate the Lagrange multipliers yields the optimal initial anchor of forward inflation π_0 on the predetermined policy instrument x_0:

$$\pi_0 = -\frac{\alpha_x}{\kappa}x_0 \tag{22}$$

and the central bank's Euler equation for the periods following period 0, for $t = 1, 2, 3...$

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x}\pi_t. \tag{23}$$

The central bank's Euler equation links recursively the future or current value of central bank's policy instrument x_t to its current or past value x_{t-1}, because of the central bank's relative cost of changing her policy instrument is strictly positive $\alpha_x > 0$. This non-stationary Euler equation adds an unstable eigenvalue in the central bank's Hamiltonian system including three laws of motion of one forward variable (inflation π_t) and of two predetermined variables (u_t, x_t) or (u_t, γ_t).

Ljungqvist and Sargent (2012, chapter 19) seek the stationary equilibrium process using the augmented discounted linear quadratic regulator (ADLQR) solution of the Hamiltonian system (Anderson, Hansen, McGrattan and Sargent (1996)) as an intermediate step. Using the method of undetermined coefficients, this solution seeks optimal negative-feedback rule parameters $\mathbf{F}_R = (F_{\pi,R}, F_{u,R})$ function of structural parameters $(\alpha_x, \beta, \kappa, \rho)$ satisfying the infinite horizon transversality conditions. The policy instrument should be exactly correlated with private sectors variables:

$$x_t = F_{\pi,R}(\alpha_x, \beta, \kappa)\,\pi_t + F_{u,R}(\alpha_x, \beta, \kappa, \rho)\,u_t. \tag{24}$$

Ljungqvist and Sargent (2012, chapter 19) first step basis vectors (π_t, u_t) of the stable subspace or Ljungqvist and Sargent (2012, chapter 19) final step basis vectors (γ_t, u_t) or Gali's (2015, chapter 5) basis vectors (x_t, u_t) include the non-observable predetermined cost-push shock u_t in their VAR(1). How to derive one representation from the other is described in the appendix.

3.1.2 Circumventing Feve, Matheron, Poilly (2007) identification issue

A risky estimation strategy is to write in the likelihood the new-Keynesian Phillips curve with auto-correlated disturbances and the Gali (2015) representation of the optimal policy rule with the same auto-correlated disturbances. In this case, each of these equations face Feve, Matheron and Poilly (2007), Griliches (1967), Blinder (1986), McManus et al. (1994) identification issue, where auto-correlated disturbances are competing to model persistence with the lag of the dependent variable.

We eliminate the non-observable auto-correlated cost-push shock using the optimal policy rule. The auto-correlation coefficient then is into the VAR matrix of inflation and the policy instrument. It does no longer appear into the disturbances, which are now white noise. We using the basis vectors (π_t, x_t) of the stable subspace for the VAR(1) representation within the Hamiltonian system, using the mathematical equivalence of systems of equations for $t = 1, 2, 3...$:

$$(H)\left\{\begin{array}{c}\begin{pmatrix}\pi_{t+1}\\ u_{t+1}\end{pmatrix} = (\mathbf{A}+\mathbf{BF}_R)\begin{pmatrix}\pi_t\\ u_t\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\varepsilon_t\\ x_t = F_{\pi,R}\pi_t + F_{u,R}u_t\\ \pi_0 = -\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given}\end{array}\right.$$

$$\Leftrightarrow\left\{\begin{array}{c}\begin{pmatrix}\pi_{t+1}\\ x_{t+1}\end{pmatrix} = \mathbf{M}^{-1}(\mathbf{A}+\mathbf{BF})\,\mathbf{M}\begin{pmatrix}\pi_t\\ x_t\end{pmatrix} + \mathbf{M}^{-1}\begin{pmatrix}0\\ 1\end{pmatrix}\varepsilon_t\\ u_t = \frac{1}{F_{u,R}}x_t - \frac{F_{\pi,R}}{F_{u,R}}\pi_t\\ \pi_0 = -\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given}\end{array}\right. \quad (25)$$

with:

$$\mathbf{A}+\mathbf{BF}_R = \begin{pmatrix}\frac{1}{\beta} - \frac{\kappa}{\beta}F_{\pi,R} & -\frac{1}{\beta} - \frac{\kappa}{\beta}F_{u,R}\\ 0 & \rho\end{pmatrix}$$

$$\begin{pmatrix}\pi_t\\ x_t\end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix}\pi_t\\ u_t\end{pmatrix} \text{ with } \mathbf{M}^{-1} = \begin{pmatrix}1 & 0\\ F_{u,R} & F_{\pi,R}\end{pmatrix}$$

$F_{u,R}$ is eliminated using $-\frac{1}{\beta}-\frac{\kappa}{\beta}F_{u,R} = (1-\rho)\,\lambda_R\frac{F_{u,R}}{F_{\pi,R}}$ and $F_{\pi,R} = \frac{\lambda_R}{1-\lambda_R}\frac{\kappa}{\alpha_x}$ (see appendix 1):

$$\mathbf{M}^{-1}(\mathbf{A}+\mathbf{BF})\,\mathbf{M} = \begin{pmatrix}\rho\lambda_R & (1-\rho)\,\lambda_R\frac{1}{F_{\pi,R}}\\ \rho(\lambda_R-1)\,F_{\pi,R} & \rho+(1-\rho)\,\lambda_R\end{pmatrix} = \begin{pmatrix}\rho\lambda_R & (1-\rho)(1-\lambda_R)\frac{\alpha_x}{\kappa}\\ -\rho\lambda_R\frac{\kappa}{\alpha_x} & \rho+(1-\rho)\,\lambda_R\end{pmatrix}$$

with:

$$\lambda_R\left(\underset{-}{\beta}, \underset{+}{\alpha_x}, \underset{-}{\kappa}\right) = \frac{1-\kappa F_\pi}{\beta} = \frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}\right) - \sqrt{\frac{1}{4}\left(1+\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}\right)^2 - \frac{1}{\beta}} = \delta$$

where the two invariant stable eigenvalues of the stable subspace are λ_R denoted δ by Gali (2015) and ρ (appendix 2).

Structural parameters are estimated with feasible generalized non-linear least squares for a system of equations. Theory-based constraints on the four reduced form parameters of the matrix $\mathbf{M}^{-1}(\mathbf{A}+\mathbf{BF})\mathbf{M}$ imply that only three structural parameters can be identified: ρ,λ_R, $F_{\pi,R}$ or ρ,λ_R, $\frac{\alpha_x}{\kappa}$ or ρ, $\alpha(\beta), \kappa(\beta)$ for a given value of the discount factor β:

$$\kappa(\beta) = \frac{1-\lambda_R \beta}{F_{\pi,R}} \Rightarrow \alpha_x(\beta) = \left(\frac{\lambda_R}{1-\lambda_R}\right)\frac{1}{F_{\pi,R}}\kappa(\beta). \tag{26}$$

If initial values of inflation and of the policy instrument (in deviation from their equilibrium values) were perfectly measured at the date of commitment, the ratio $\frac{\alpha_x}{\kappa}$ would be over-identified by the optimal initial anchor of forward inflation on the predetermined policy instrument equation:

$$\frac{\alpha_x}{\kappa} = \frac{-\pi_0}{x_0}. \tag{27}$$

The semi-reduced form cost-push shock rule parameter $F_{u,R}$ requires an identification restriction, for example, setting a value for β (see appendix 2):

$$F_{u,R}(\beta) = \frac{-1}{1-\beta\rho\lambda_R}F_{\pi,R} < 0. \tag{28}$$

The standard error σ_u of cost-push shock is computed using the standard error of residuals $\sigma_{\varepsilon,x}$ of the output gap rule equation in the VAR(1). It requires an identification restriction, because it depends on $F_{u,R}$:

$$\sigma_u(\beta) = \frac{\sigma_{\varepsilon,x}}{F_{u,R}(\beta)}. \tag{29}$$

The standard error of the measurement of the inflation equation σ_π (which is theoretically predicted to be zero) and its covariance with the cost push shock $\sigma_{x\pi} = F_{u,R}\sigma_{xu}$ are also available.

One identifying equation is missing in order to identify the remaining four structural parameters $(\alpha_x, \kappa, \beta, \sigma_u)$ and the negative feedback rule parameter $F_{u,R}$. We set an identification restriction on the discount factor to a given value: $\beta = 0.99$ or $\beta = 1$ in the estimations.

3.2 Infinite Horizon Time-Consistent Policy

Cohen and Michel (1988), Oudiz and Sachs (1985) and Backus and Driffill (1986) invented an infinite horizon time-consistent policy, which holds if the policy maker optimizes at all periods. Time-consistent policy is applied by Gali (2015) using the new-Keynesian Phillips curve transmission mechanism with a very minor change with respect to Cohen and Michel (1988) and Oudiz and Sachs (1985) detailed in the appendix. The central bank minimizes its loss function subject to the new-Keynesian Phillips curve *and* subject to two additional constraints. These constraints forces the marginal value of the loss function with respect to inflation (the policy maker's Lagrange multiplier on inflation) to stick to the value zero at all periods. Hence, this rule does not change if the policy maker optimizes at the initial date or at any future date.

These constraints *assume* that both the private sector and the central bank commit for ever to *restricted* policy rules where their policy instrument reacts only to the contemporary *predetermined variable* u_t at all periods t, with a *perfect correlation*. These time-consistent rules are determined by time-invariant rule parameters N_{TC} and $F_{u,TC}$ to be optimally chosen for all periods, assuming common and complete knowledge of structural parameters including preferences of both agents:

$$\pi_t = N_{TC} u_t \text{ and } x_t = F_{u,TC} u_t \tag{30}$$

The central bank policy time-consistent rule has a representation where its policy instruments responds only to current inflation, after substitution of private sector time-consistent rule:

$$x_t = F_{u,TC} u_t = F_{\pi,TC} \pi_t \text{ with } F_{\pi,TC} = \frac{F_{u,TC}}{N_{TC}} \tag{31}$$

The Central Bank *commits for ever* to a *restricted time-consistent rule* where the policy instrument responds only to current inflation or only to the current non-observable cost-push shock *with a perfect correlation.*

Substituting the private sector's inflation rule and the policy rule in the loss function:

$$\max_{\{\pi, x_t\}} -\frac{1}{2} E_0 \sum_{t=0}^{+\infty} \beta^t \left(\pi_t^2 + \alpha_x x_t^2\right) = \max_{\{F_{u,TC}, N_{TC}\}} -\frac{1}{2}\left(N_{TC}^2 + \alpha_x F_{u,TC}^2\right) \frac{u_0^2}{1-\beta\rho^2}$$

The central bank first order condition is:

$$\begin{aligned} 0 &= N_{u,TC} \frac{\partial N_{u,TC}}{\partial F_{u,TC}} + \alpha_x F_{u,TC} \\ F_{\pi,TC} &= \frac{F_{u,TC}}{N_{TC}} = -\frac{1}{\alpha_x} \frac{\partial N_{u,TC}}{\partial F_{u,TC}} \end{aligned}$$

Substituting the private sector's inflation rule and the policy rule in the inflation law of motion leads to the following relation between N_{TC} on date t, $N_{TC,t+1}$ and $F_{u,TC}$:

$$\begin{aligned} \pi_t &= \beta E_t \left[\pi_{t+1}\right] + \kappa x_t + u_t \Rightarrow \\ N_{TC} u_t &= \beta N_{TC,t+1} \rho u_t + \kappa F_{u,TC} u_t + u_t \\ N_{TC} &= \beta \rho N_{TC,t+1} + \kappa F_{u,TC} + 1 \end{aligned}$$

In the reference Oudiz and Sachs' (1985) dynamic Nash equilibrium, the central bank foresees that $N_{TC,t+1} = N_{TC}$ in its optimization (see appendix):

$$N_{TC} = \frac{\kappa F_{u,TC} + 1}{1-\beta\rho} = \frac{\kappa F_{\pi,TC} N_{TC} + 1}{1-\beta\rho} \Rightarrow \frac{\partial N_{u,TC}}{\partial F_{u,TC}} = \frac{\kappa}{1-\beta\rho}$$

The endogenous rule parameters are increasing function of the central bank cost of changing the policy instrument α_x. They are bounded by limit values of $\alpha_x \in]0, +\infty[$:

$$0 < \frac{\pi_{t,TC}}{u_t} = N_{TC}(\underset{+}{\alpha_x}) = \frac{\alpha_x (1-\beta\rho)}{\alpha_x (1-\beta\rho)^2 + \kappa^2} < N = \frac{1}{1-\beta\rho}$$
$$-\frac{1}{\kappa} < \frac{x_{t,TC}}{u_t} = F_{u,TC}(\underset{+}{\alpha_x}) = \frac{-\kappa}{\alpha_x (1-\beta\rho)^2 + \kappa^2} < 0$$
$$-\infty < \frac{x_{t,TC}}{\pi_{t,TC}} = F_{\pi,TC}(\underset{+}{\alpha_x}) = \frac{F_{u,TC}}{N_{TC}} = \frac{-\kappa}{\alpha_x (1-\beta\rho)} < \frac{-\kappa}{\alpha_x} < 0$$

For an infinite cost of changing the policy instrument $\alpha_x \to +\infty$, we label this equilibrium as "laissez-faire" because two policy rule parameters are both equal to zero $F_{\pi,TC} = 0 = F_{u,TC}$. The policy instrument x_t is set to zero at all dates: it is eliminated in the model. It corresponds to the maximal initial response of inflation (in absolute values) to cost-push shock $Nu_t = \frac{1}{1-\beta\rho} u_t$ for time-consistent policy.

For the limit case of a zero cost of changing the policy instrument ($\alpha_x \to 0$), the policy instrument (output gap) has its largest response to cost-push shock $x_0 = -\frac{1}{\kappa} u_0$ so that the policy target (inflation) does not respond to the cost-push shock (N_{TC} is zero).

The policy instrument (the output gap) x_t is exactly *negatively* correlated ($F_{\pi,TC} < 0$) with the policy target (inflation) π_t. When increasing the central bank's preferences (α_x) for the relative cost of changing the output gap from zero to infinity, the strictly negative rule parameter $F_{\pi,TC}$ increases from minus infinity to zero. There is one stable eigenvalue and one unstable eigenvalue:

$$0 < \rho < 1 < \frac{1}{\beta} \leq \lambda_{TC} = \frac{1 - \kappa F_{\pi,TC}}{\beta} < +\infty. \tag{32}$$

The welfare loss of time-consistent policy v_{TC} as a proportion of the limit maximal value of the welfare loss with the largest volatility of inflation (laissez-faire) v_{LF} turns to be equal to the ratio of inflation under time-consistent policy to inflation under laissez-faire. It increases from zero to one when the cost of changing the policy instrument increases from zero to infinity:

$$0 < \frac{v_{TC}}{v_{LF}} = \frac{N_{TC}^2 + \alpha_x F_{u,TC}^2}{N^2} = \frac{\alpha_x (1-\beta\rho)^2}{\alpha_x (1-\beta\rho)^2 + \kappa^2} = \frac{N_{TC}}{N} = \frac{\pi_{t,TC}}{\pi_{t,LF}} < 1$$

4 Pre-test of Ramsey versus Time-Consistent Policy and Optimal Simple Rule

4.1 A Bifurcation from negative-feedback to positive feedback mechanism

This section adds two new results to Gali (2015) model.

Proposition 1: *In Gali's (2015) model, there is a saddle-node bifurcation when shifting from negative-feedback of Ramsey optimal policy under commitment or quasi-commitment to positive-feedback mechanism of infinite horizon time-consistent, with opposite sign of the policy rule parameter responding to inflation.*

Proof: The reduced form inflation rule parameters $F_{\pi,TC}$ of time-consistent policy (respectively $F_{\pi,R}$ of Ramsey optimal policy) is an *affine negative function* of the inflation eigenvalue λ_{TC} (respectively λ_R). The inflation rule parameters satisfies the following inequalities:

$$F_{\pi,TC} = -\frac{1}{1-\beta\rho}\frac{\kappa}{\alpha_x} < 0 \ < \frac{1-\beta}{\kappa} < F_{\pi,R} = \frac{1-\beta\lambda_R}{\kappa} < \frac{1}{\kappa} \tag{33}$$

The inflation eigenvalues satisfies the following inequalities:

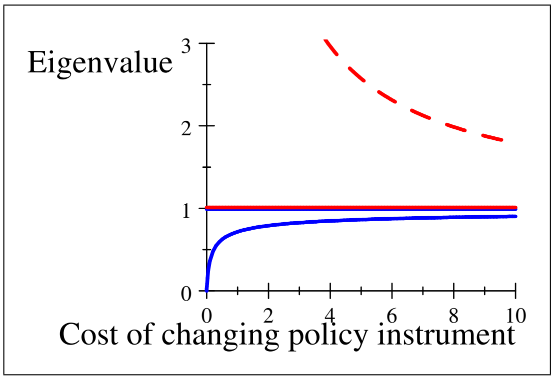
$$0 < \lambda_R = \frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}\right) - \sqrt{\frac{1}{4}\left(1+\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}\right)^2 - \frac{1}{\beta}} < 1 \tag{34}$$

$$1 \leq \frac{1}{\beta} < \lambda_{TC} = \frac{1-\kappa F_{\pi,TC}}{\beta} = \frac{1}{\beta} + \frac{1}{\beta}\frac{1}{1-\beta\rho}\frac{\kappa^2}{\alpha_x} \tag{35}$$

The shift from the stable eigenvalue of the inflation equation (λ_R) for Ramsey optimal policy to the unstable eigenvalue (λ_{TC}) larger than one for time-consistent policy is a saddle-node bifurcation, because $0 < \lambda_R < 1 < \lambda_{TC}$. The policy instrument is predetermined for Ramsey optimal policy whereas it is forward-looking with infinite horizon time-consistent policy. This implies an additional stable eigenvalue for Ramsey optimal policy with respect to time-consistent policy (Blanchard and Kahn (1980)). The stable inflation eigenvalue is related to negative-feedback policy rule parameter for Ramsey optimal policy whereas the unstable inflation eigenvalue is related to positive-feedback policy rule parameter. QED.

These results are new because Gali (2015) only computes the strictly negative rule parameter $F_{\pi,TC}$ for time-consistent policy, but not the positive rule parameter $F_{\pi,R}$ for Ramsey optimal policy. Gali (2015) only computes the inflation eigenvalue λ_R (denoted δ in his book) for Ramsey optimal policy but not λ_{TC} for time-consistent policy.

Figures 1 and 2: Inflation eigenvalues λ and inflation rule parameters F_π function of α_x for Ramsey optimal policy (solid line) and time-consistent policy (dash line)



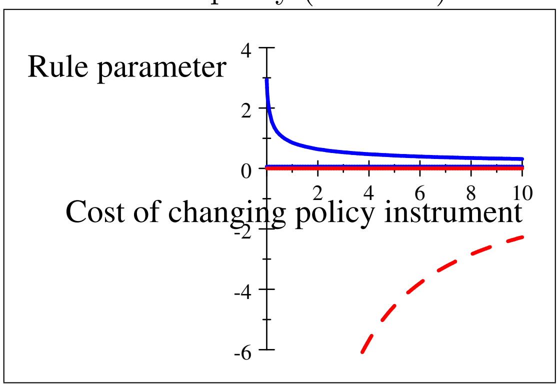


Figure 1 plots the eigenvalue λ_{TC} of time-consistent policy (and respectively the eigenvalue λ_R of Ramsey optimal policy) as non-linear decreasing (respectively increasing) function of the relative cost of changing the policy instrument α_x for the estimated parameters $\rho = 0.995$, $\kappa = 0.340$ for a given $\beta = 0.99$ of the Ramsey optimal policy model during Volcker-Greenspan's Fed starting 1979q3-2006q2 (see estimation section, with estimated $\lambda_R = 0.856$ and $\alpha_x = 4.552$).

- For a near-zero cost of changing the policy instrument (the Fed is an inflation nutter), the inflation eigenvalue λ_R tends to zero for Ramsey optimal policy and the inflation eigenvalue λ_{TC} tends to infinity for time-consistent policy.

- For an infinite cost of changing the policy instrument (the Fed has maximal inertia):

the inflation eigenvalue λ_R tends to one for Ramsey optimal policy and λ_{TC} tends to $1/\beta > 1$ for time-consistent policy.

Figure 2 plots inflation rule parameter $F_{\pi,TC}$ of time-consistent policy (and respectively $F_{\pi,R}$ of Ramsey optimal policy) as non-linear decreasing (respectively increasing) function of the relative cost of changing the policy instrument α_x for the same estimated parameters than for figure 1 (with estimated $F_{\pi,R} = 0.447$ and $\alpha_x = 4.552$).

- For a near-zero cost of changing the policy instrument, the inflation rule parameter $F_{\pi,R}$ tends to $1/\kappa$ and $F_{\pi,TC}$ tends to minus infinity for time-consistent policy.

- For an infinite cost of changing the policy instrument, the inflation rule parameter $F_{\pi,R}$ tends to $(1-\beta)/\kappa$ for Ramsey optimal policy and $F_{\pi,TC}$ tends to zero for time-consistent policy.

Proposition 2: *In Gali's (2015) model:*

(i) *An optimal simple rule minimizing the central bank loss function is a reduced form of time-consistent policy, with negative inflation rule parameter $F_{\pi,S} < 0$ corresponding to a unique Central Bank preferences $\alpha_x > 0$ for a given discount factor β and a given monetary policy transmission parameter κ.*

(ii) *Simple rules with positive inflation rule parameters $F_{\pi,S} \in \left[0, \frac{1-\beta}{\kappa}\right[\cup \left]\frac{1+\beta}{\kappa}, +\infty\right[$ cannot be the reduced form of an optimal simple rule.*

Proof:

Simple rule assumes that inflation and the policy instrument are forward-looking variables. Only the cost-push auto-regressive shock is a predetermined variable with an exogenous stable eigenvalue ρ. Blanchard and Kahn's (1980) determinacy condition implies that the controllable eigenvalue, indexed by S for "simple rule": $\lambda_S = \frac{1-\kappa F_{\pi,S}}{\beta}$ should be unstable ($|\lambda_S| > 1$). This implies that the inflation rule parameter satisfies: $F_{\pi,S} = \frac{1-\beta\lambda_s}{\kappa} < \frac{1-\beta}{\kappa}$ or $F_{\pi,S} = \frac{1-\beta\lambda_s}{\kappa} > \frac{1+\beta}{\kappa}$.

(i) For a given monetary policy transmission mechanism $(\beta, \kappa, \rho, \sigma_u)$, a simple rule with a *strictly negative* inflation parameter $F_{\pi,S}$, forcing an unstable eigenvalue $\lambda_S \in \left]\frac{1}{\beta}, +\infty\right[$ by positive feedback, is the reduced form of time-consistent policy with a unique central bank preference parameter α_x given by:

$$F_{\pi,S} = F_{\pi,TC} = -\frac{\kappa}{\alpha_x}\frac{1}{1-\beta\rho} < 0 \Longrightarrow \alpha_x = -\frac{\kappa}{F_{\pi,S}}\frac{1}{1-\beta\rho} \quad (36)$$

(ii) The remaining cases of simple rules with *positive* rule parameter $F_{\pi,S} \in \left[0, \frac{1-\beta}{\kappa}\right[\cup \left]\frac{1+\beta}{\kappa}, +\infty\right[$ forcing an unstable eigenvalue $\lambda_S \in \left[1, \frac{1}{\beta}\right[\cup]-\infty, -1[$ by positive feedback do not minimize a central bank loss function in time-consistent policy. For $F_{\pi,S} \in \left[0, \frac{1-\beta}{\kappa}\right[$, these simple rules imply a jump of inflation larger than in laissez-faire ($N_S > N$). For $F_{\pi,S} \in \left]\frac{1+\beta}{\kappa}, +\infty\right[$, these simple rules imply a jump of inflation with an opposite sign with respect to laissez-faire ($N_S < 0 < N$). Those simple rules are sub-optimal because they "overshoot" the initial anchor of inflation with "too much effort" of the central bank. Kollmann (2002 and 2008) is a precursor for computing optimal simple rules with new-Keynesian Phillips curve models. Q.E.D.

4.2 Durbin (1970) Pre-Test of Time-consistent

4.2.1 Pre-test of time-consistent policy

Time-consistent policy is described by a permanent anchor of inflation on the output gap and by two AR(1) processes of inflation and of the output gap. Variables, such as inflation ($\pi_t + \pi^*$) are not computed as deviations of equilibrium (already denoted π_t). Estimates of equilibrium values (π^*, x^*) are then sample mean values found in the estimates of intercepts. The reduced form time-consistent policy policy rule to be tested (which corresponds to a permanent anchor of inflation on the output gap) allows to estimate the reduced form time-consistent policy rule parameter $F_{\pi,TC}$:

$$x_t + x^* = F_{\pi,TC}(\pi_t + \pi^*) + (x^* - F_{\pi,TC}\pi^*) + \varepsilon_{x\pi,t} \text{ with} \\ \varepsilon_{x\pi,t} = 0 \text{ for all dates, } R^2 = 1 \text{ and } F_{\pi,TC} < 0 \tag{37}$$

The simple correlation between the output gap and inflation provides another estimate of the time-consistent policy rule parameter $F_{\pi,TC} = r_{x\pi}\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi} < 0$. It is equal to the one found using the ratio of standard errors of residuals of the AR(1) estimations for inflation and for the output gap : $F_{\pi,TC} = -\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi}$ only if the following condition is satisfied: $r_{x\pi} = -1$. A preliminary test of time-consistent policy against Ramsey optimal policy amounts to test the negative sign of correlation between the output gap and inflation, which is predicted to be perfect: $r_{x\pi} = -1$ (stochastic singularity). Because of test of a simple correlation exactly equal to -1 cannot be performed, we can perform a one-sided test of a composite null hypothesis of a simple correlation very close to minus one (subscript TC is for time-consistent policy):

$$\text{Stochastic singularity (SS): } H_{0,TC,SS} : r_{x\pi} < -0.99 \tag{38}$$

Indeed, the rejection of stochastic singularity is expected due to measurement errors. A test of statistical significance with negative sign is less demanding. A positive sign of the time-consistent rule leads to either sub-optimal simple rule or suggests Ramsey optimal policy as an alternative:

$$\text{Negative sign (NS): } H_{0,TC,NS} : r_{x\pi} < 0 \tag{39}$$

The acid test of time-consistent policy versus Ramsey optimal policy is a Durbin (1970) test of the auto-correlation of residuals of the time-consistent positive-feedback policy rule. The measurement errors of the time-consistent policy rule should not be auto-correlated according to the theory of time-consistent policy. Else, the private sector agents should take into account this additional predetermined forcing variable in their decision, the way they do already account for the non-observable auto-correlation parameter of the cost-push shock. This would add another stable eigenvalue required for determinacy. The time-consistent policy rule would be such that the policy instrument is a function of the non-controllable auto-regressive cost-push shock and of this additional non-observable auto-regressive forcing variable.

$$\text{No unnoticed forcing variable: } H_{0,TC,Durbin} : \rho_{\varepsilon,x\pi} = 0 \text{ for } \varepsilon_{x\pi,t} = \rho_{\varepsilon,x\pi}\varepsilon_{x\pi,t} + \eta_t \text{ with } \eta_t \text{ i.i.d.} \tag{40}$$

If measurement errors are correlated, this alternative hypothesis suggests that at least one lagged policy instrument are missing in the regression of the policy rule. This is exactly a reduced form of the *Ramsey optimal policy rule* which also depends on inflation. Finally, we can perform another test. In the time-consistent feedback rule, the policy instrument is a linear function of the policy target. The two AR(1) process for inflation and the output gap have common auto-correlation parameters:

$$\pi_t + \pi^* = \rho\left(\pi_{t-1} + \pi^*\right) + (1-\rho)\,\pi^* + N_{TC}\varepsilon_{u,t} \text{ with } \varepsilon_{u,t} \text{ i.i.d.} \tag{41}$$

$$x_t + x^* = \rho\left(x_{t-1} + x^*\right) + (1-\rho)\,x^* + F_{\pi,TC}N_{TC}\varepsilon_{u,t} \text{ with } \varepsilon_{u,t} \text{ i.i.d.} \tag{42}$$

We can test the hypothesis of a common auto-correlation parameter for the policy target (inflation) and for the policy instrument (output gap):

$$\text{Common auto-correlation (CA): } H_{0,TC,CA} : \rho_\pi = \rho_x \tag{43}$$

Proposition 3: *In Gali's (2015) model, for time-consistent policy, only the auto-correlation coefficient of the cost-push shock ρ can be identified. But the four other structural parameters are not identified: the Fed's preferences parameter α_x, the slope of the new-Keynesian Phillips curve κ, the discount factor β and the variance of the cost-push shock $\sigma^2_{\varepsilon,u}$. If the Fed's preferences is a welfare loss function, where Fed's preferences parameter is endogenous ($\alpha_x = \frac{\kappa}{\varepsilon}$), the representative household's elasticity of substitution between each differentiated goods ε is not identified.*

Proof. If common auto-correlation hypothesis is not rejected, the AR(1) estimates of the policy target and of the policy instrument identify the auto-correlation parameter of the non-observable cost-push shock: ρ. The ratio of the standard errors of residuals of each AR(1) estimations of inflation and output gap provides another estimate of the time-consistent reduced form rule parameter $F_{\pi,TC}$, (if $r_{x\pi} = 1$), if the hypothesis of a negative sign is not rejected:

$$F_{\pi,TC} = -\sigma_{\varepsilon,x}/\sigma_{\varepsilon,\pi} \tag{44}$$

The variance $\sigma^2_{\varepsilon,\pi}$ of perturbations of the inflation AR(1) process is:

$$\sigma^2_{\varepsilon,\pi} = N^2_{TC}\sigma^2_{\varepsilon,u} \Rightarrow N^2_{TC} = \frac{\sigma^2_{\varepsilon,\pi}}{\sigma^2_{\varepsilon,u}} \tag{45}$$

The cross equations covariance $\sigma_{\varepsilon,\pi x}$ between the residuals of both AR(1) process of inflation and of the output gap does not allow to identify either the private sector reduced form parameter N_{TC} anchoring inflation on the cost-push shock or the variance of the cost-push shock $\sigma^2_{\varepsilon,u}$. The simple correlation between the two residuals is predicted to be exactly negatively correlated ($r_{\varepsilon,\pi x} = -1$):

$$\sigma_{\varepsilon,\pi x} = -\frac{\sigma_{\varepsilon,x}}{\sigma_{\varepsilon,\pi}}\frac{\sigma^2_{\varepsilon,\pi}}{\sigma^2_{\varepsilon,u}}\sigma^2_{\varepsilon,u} = -\sigma_{\varepsilon,x}\sigma_{\varepsilon,\pi} < 0. \tag{46}$$

It is not possible to identify at least one of these four remaining structural parameters separately, because the identified parameter $F_{\pi,TC}$ does not depend only on one of these four structural parameters:

$$F_{\pi,TC} = \frac{-1}{1-\beta\rho}\frac{\kappa}{\alpha_x} < 0. \tag{47}$$

Three identifying equations are missing in the case of time-consistent policy. QED.

Finally, the tests of reduced form parameters of bivariate VAR(1) of time-consistent policy versus Ramsey optimal policy are not feasible. The exact multicollinearity (exact correlation) between regressors (current output gap and inflation) imply a bivariate VAR(1) with infinite coefficients with denominator including the term $1 - r_{x\pi}^2$ equal to zero:

$$\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} +\infty & -\infty \\ -\infty & +\infty \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} N_{TC} \\ F_{\pi,TC} N_{TC} \end{pmatrix} \varepsilon_t \quad (48)$$

The time-consistent policy equilibrium predicts that out-of-equilibrium behavior corresponds to a non-stationary bivariate VAR including one unstable eigenvalue λ_{TC} and one stable eigenvalue ρ, which cannot be estimated. By contrast, the stationary structural VAR(1) of output gap and inflation with Ramsey optimal policy allows to identify a larger number of structural parameters.

$$\begin{pmatrix} x_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} F_{u,R} \\ 0 \end{pmatrix} \varepsilon_t$$

Two additional reduced form parameters (b, c) are available, because the stable subspace of the VAR process is of dimension two with Ramsey optimal policy instead of dimension one with time-consistent policy.

5 Tests with Output Gap as Policy Instrument

5.1 Pre-test

The annualized quarter-on-quarter rate of inflation and the congressional budget office (CBO) measure of the output gap are taken from Mavroeidis' (2010) online appendix (detailed information at the end of this paper's appendix). The pre-Volcker sample covers the period 1960q1 to 1979q2 and the Volcker-Greenspan sample runs until 2006q2. The period of Paul Volcker's tenure is 1979q3 to 1987q2. The period of Alan Greenspan's tenure is 1987q3 to 2006q1.

According to Debortoli and Nunes (2015), a structural break corresponds to an new initial anchor of forward inflation on the output gap for Ramsey optimal policy with finite horizon. Clarida, Gali and Gertler (2000) and Mavroeidis (2010) consider the beginning of Paul Volcker's mandate 1979q3 as a structural break. Givens (2012) considers 1982q1 as a structural break, after 1981 fall of inflation and before the 1982 recession. Matthes (2015) estimation of the private sectors beliefs regarding central bank regimes also points to 1982q1 as a structural break. Table 2 presents summary statistics before and after the 1979q3 and 1982q1 structural breaks.

Table 2: Summary statistics of inflation and output gap

	dates	obs.	mean	min	max	after:	obs.	mean	min	max
π_t	$< 79q3$	78	4.39 (2.71)	0.59	11.79	$\geq 79q3$	108	3.18 (2.03)	0.64	10.93
x_t	$< 79q3$	78	0.47 (2.59)	−4.97	6.10	$\geq 79q3$	108	−1.11 (2.07)	−7.95	3.01
π_t	$< 82q1$	88	4.86 (2.90)	0.59	11.79	$\geq 82q1$	98	2.64 (1.08)	0.64	5.61
x_t	$< 82q1$	88	0.20 (2.59)	−4.97	6.10	$\geq 79q3$	98	−1.03 (2.12)	−7.95	3.01

Standard deviations are in parentheses.

The mean of inflation and of output gap are lower during Volcker-Greenspan than before Volcker. Excluding the period 1979q3 to 1981q4, in particular the sharp disinflation which occurred during 1981 (figure 3), the standard error of inflation decreases by half from 2.03 to 1.08 .

Table 3: Pre-tests of time-consistent policy rule

Dates	obs	$r_{x\pi}$	Low 95% r	p	$R^2_{x\pi}$	$F_{\pi,TC}$	c	$\rho_{\varepsilon,x\pi}$
$< 79q3$	78	-0.13	-0.21	< 0.001	0.02	-0.13 (0.11)	1.03 (0.56)	**0.92** (0.04)
$< 82q1$	108	-0.30	-0.42	< 0.001	0.09	-0.30 (0.09)	-0.14 (0.35)	**0.91** (0.04)
$\geq 79q3$	88	-0.24	-0.31	< 0.001	0.06	$\mathbf{-0.22}$ (0.09)	1.25 (0.53)	**0.92** (0.05)
$\geq 82q1$	98	-0.40	-0.53	< 0.001	0.16	-0.78 (0.18)	1.03 (0.52)	**0.89** (0.05)

The pre-tests of the null hypothesis of a quasi perfect negative correlation $H_0 : r_{x\pi} < -0.99$ between observed inflation and observed output gap are rejected. The test uses Fisher's Z transformation using the procedure corr with the software SAS. The threshold of the composite null hypothesis -0.99 is far away from the 95% single tail confidence interval, where the lowest 95% confidence limit reported in table 3 is at most equal to -0.53 for the period beginning from 1982q1 (figure 4). The opposite null hypothesis $H_0 : r(x_t, \pi_t) = 0$ is not rejected before 1979q3. time-consistent policy predicts a perfect correlation for the anchor of inflation expectations with the output gap. If the time-consistent policy equilibrium occurred before 1979q3, we do not reject the null hypothesis $H_0 : r(E_{t-1}(\pi_t), \pi_t) = 0$ that the rational expectations of inflation are orthogonal to observed inflation.

The pre-tests of the null hypothesis of the auto-correlation of residuals $H_0 : \rho_{\varepsilon,x\pi} = 0$ are strongly rejected, with a point estimate at least equal to 0.89 (figure 5). These tests gives a hint of model misspecification. They suggest an omitted lagged policy instrument in the policy rule. When it is included in Ramsey optimal policy rule, the R^2 increases from 16% (table 3, last line) to 93% (table 6, last line) beginning in 1982q1.

Table 4: Auto-correlation of inflation and output gap

dates	obs.	var.	r	R^2	ρ	c	σ_ε	ρ_ε	DF	PP
$< 79q3$	78	π_t	0.86	0.74	0.88 (0.06)	0.62 (0.30)	1.38	-0.22 (0.12)	0.55	0.41
$< 79q3$	78	x_t	0.93	0.86	0.93 (0.04)	0.03 (0.11)	0.99	0.26 (0.11)	0.20	0.33
$< 82q1$	88	π_t	0.88	0.78	0.88 (0.05)	0.63 (0.28)	1.35	-0.19 (0.11)	0.34	0.23
$< 82q1$	88	x_t	0.92	0.85	0.93 (0.04)	-0.03 (0.11)	1.03	0.23 (0.11)	0.27	0.37
$\geq 79q3$	108	π_t	0.89	0.79	0.85 (0.04)	0.42 (0.16)	0.93	-0.27 (0.09)	0.04	0.01
$\geq 79q3$	108	x_t	0.94	0.88	0.94 (0.03)	-0.07 (0.08)	0.70	0.34 (0.09)	0.11	0.25
$\geq 82q1$	98	π_t	**0.64**	0.41	**0.59** (0.07)	1.06 (0.21)	0.83	$\mathbf{-0.20}$ (0.10)	**0.00**	**0.00**
$\geq 82q1$	98	x_t	**0.96**	0.92	**0.95** (0.03)	-0.02 (0.07)	0.60	**0.35** (0.09)	**0.07**	**0.35**

Table 4 investigates the auto-correlation and unit roots of inflation and output gap. The output gap and inflation are highly auto-correlated (respectively 0.93 and 0.86), except when inflation excludes the 1981 disinflation for the period after 1981q4. For the period 1982q1 to 2006q2, the inflation auto-correlation coefficient falls in the 95% confidence interval 0.6 ± 0.14 and it is statistically different from the output gap auto-correlation coefficient in the 95% confidence interval 0.95 ± 0.06 (figures 4 and 5). As the time-consistent policy equilibrium predicts that the auto-correlation of the output gap and of inflation should be the same, this is an additional test against time-consistent policy, which holds for the period 1982q1 to 2006q2.

There is a negative auto-correlation of residuals ρ_ε for inflation and a (statistically significant at the 5% level) positive auto-correlation of residuals for the output gap. The column DF reports the p-value of the Dickey-Fuller test of unit root with one lag without trend. The column PP reports the p-value of the Phillips-Perron test of unit root, which takes into account auto-correlation, with one lag without trend. The null hypothesis of a unit root is rejected for inflation after 1979q2 and after 1981q4.

6 Tests of Ramsey Optimal Policy

Table 5 presents estimates of structural parameters, Table 5 report estimates of three structural VAR estimations for $(\rho, \lambda_R, F_{\pi,R})$, $\left(\rho, \lambda_R, \frac{\alpha_x}{\kappa}\right)$ and $(\rho, \kappa(\beta), \alpha_x(\beta))$ with two given values for the discount factor $\beta = 1$ or $\beta = 0.99$ for the Volcker-Greenspan period. With these three estimations, delta method is not necessary to compute the standard errors of parameters in each case. Maximum likelihood did not converge for pre-Volcker period (unconstrained VAR corresponds to complex conjugate eigenvalues, which are excluded because of exogenous real auto-correlation of cost-push shock). Post 1982q1 estimations converged to unlikely estimates.

Table 5: Ramsey optimal policy structural parameters

Dates	ρ	λ_R	$F_{\pi,R}$	$\frac{\alpha_x}{\kappa} = \frac{1}{\varepsilon}$	β	$\kappa(\beta)$	$\alpha_x(\beta)$	$F_{u,R}(\beta)$	$\sigma_u(\beta)$
$\geq 79q3$	0.995* (0.024)	0.857* (0.054)	0.447 (0.292)	13.375* (6.627)	1	0.321 (0.303)	4.296 (5.447)	−3.027	0.229
$\geq 79q3$	0.995* (0.024)	0.857* (0.054)	0.447 (0.292)	13.375* (6.627)	0.99	0.340 (0.314)	4.552 (5.703)	−2.861	0.242

The cost-push shock faces is extremely persistent, close to a unit root, with ρ estimate close to one. The ratio $\frac{\alpha_x}{\kappa}$ is statistically significant. If the Fed's preferences are identical to (welfare) household's preferences), then $\frac{\alpha_x}{\kappa} = \frac{1}{\varepsilon}$ and $\widehat{\varepsilon} = 0.07$. The new-Keynesian Phillips curve parameter κ is relatively large. The Fed's preference parameter α_x is relatively large (although not unheard of in previous estimations). for the period 1979-2006.

Table 6: Inflation and output gap structural (S) versus unconstrained (U) VAR

dates	obs.	var.	S/U	π_{t-1}	x_{t-1}	c	ρ_ε	R^2	λ
$\geq 79q3$	108	π_t	S	0.85	0.009	0.428 (0.16)	−0.25 (0.09)	0.793	0.857* (0.054)
$\geq 79q3$	108	π_t	U	0.85 (0.04)	0.009 (0.04)	0.43 (0.16)	−0.25 (0.09)	0.79	0.85
$\geq 79q3$	108	x_t	S	−0.064	**0.999**	0.198 (0.121)	0.29 (0.09)	0.888	**0.995*** (0.024)
$\geq 79q3$	108	x_t	U	−0.084 (0.03)	0.917 (0.03)	0.17 (0.11)	0.29 (0.09)	0.89	0.92

Table 6 compares the reduced form parameters of the VAR of Ramsey optimal policy with parameters of an unconstrained VAR. The inflation equation of the VAR are the same up to the third decimal of all statistics. There is no Granger causality from output gap to inflation. For the output gap equation, *Ramsey optimal policy slightly over-estimates the persistence of the output gap: its VAR auto-correlation parameter shifts from* 0.93 *to* 1. There is Granger causality from inflation to output gap. For the output gap rule, the auto-correlation of residuals fell from 0.92 with time-consistent policy to 0.29 with Ramsey optimal policy, but it remains statistically significant. As well, the inflation equation has a statistically significant auto-correlation (−0.25).

The reduced form Ramsey optimal policy rule of the structural VAR is observationally equivalent to the LQR first step representation including the non-observable cost-push shock, when taking into account the other equations of the Hamiltonian system:

$$\text{Ramsey}: x_t = 0.995x_{t-1} - 0.064\pi_{t-1} - 2.861\varepsilon_{u,t} \text{ or } x_t = 0.447\pi_t - 2.861u_t \text{ for } \beta = 0.99 \quad (49)$$

The policy instrument responds to two variables for Ramsey stable subspace of dimension two. The policy instrument responds to one variable in the time-consistent policy stable subspace of dimension one. The reduced form policy rule for time-consistent policy is:

$$\text{Time-consistent: } x_t = -0.22\pi_t \quad (50)$$

7 Tests with Federal Funds Rate

7.1 Pre-test of time-consistent policy

Assuming the polar case where all labor cost is financed by working capital, the federal funds rate is used here as a policy instrument. Figure 2 represents the time series of inflation and federal funds rate. Table 2 presents summary statistics before and after the 1979q3 and 1982q1 structural breaks.

Table 2B: Summary statistics of inflation and federal funds rate

	dates	obs.	mean	min	max	after:	obs.	mean	min	max
π_t	$< 79q3$	78	4.39 (2.71)	0.59	11.79	$\geq 79q3$	108	3.18 (2.03)	0.64	10.93
i_t	$< 79q3$	78	5.47 (2.42)	1.68	12.09	$\geq 79q3$	108	6.56 (3.76)	1.00	17.78
π_t	$< 82q1$	88	4.86 (2.90)	0.59	11.79	$\geq 82q1$	98	2.64 (1.08)	0.64	5.61
i_t	$< 82q1$	88	6.84 (3.74)	1.68	17.78	$\geq 79q3$	98	5.76 (2.84)	1.00	14.51

Standard deviations are in parentheses below the mean.

The means of inflation are lower after Volcker than before Volcker. Excluding the period 1979q3 to 1981q4, in particular the sharp disinflation which occurred during 1981 (figure 3), the standard error of inflation decreases by half from 2.03 to 1.08. The difference of means between the policy interest rate and inflation increased after Volcker.

Table 3B: Pre-test of time-consistent policy rule

dates	obs	$r_{i\pi}$	t	$r_{i\pi} = 0 : p$	$R^2_{i\pi}$	$F_{\pi,D}$	c	$\rho_{\varepsilon,i\pi}$
$< 79q3$	78	0.83	12.85	< 0.001	0.68	0.74 (2.22)	2.22 (0.30)	0.61 (0.09)
$< 82q1$	88	0.79	11.83	< 0.001	0.62	1.01 (0.08)	1.55 (0.48)	0.73 (0.08)
$\geq 79q3$	108	0.75	11.63	< 0.001	0.56	1.39 (0.12)	1.39 (0.11)	0.76 (0.06)
$\geq 82q1$	98	0.53	6.08	< 0.001	0.28	1.39 (0.22)	2.09 (0.65)	0.80 (0.06)

The tests of the null hypothesis of a quasi perfect negative correlation $H_0 : r_{i\pi} < -0.99$ between inflation and federal funds rate have been replaced by the usual tests of the null hypothesis: $H_0 : r_{i\pi} = 0$, because $r_{i\pi} > 0$. The negative sign of the time-consistent policy rule is rejected, with large t statistics. The Durbin and Breusch-Godfrey tests strongly reject the lack of serial correlation (for one or two lags) with p-value below 10^{-4}. Tests of the null hypothesis of the first order auto-correlation of residuals $H_0 : \rho_{\varepsilon,i\pi} = 0$ are rejected, with a point estimate at least equal to 0.60 and at most 0.80. The infinite horizon time-consistent positive-feedback mechanism *does not fit the data when the cost of capital is taken into account into the monetary transmission mechanism.* Finally, the Taylor principle (an inflation coefficient F_π larger than one) is not satisfied before Volcker and satisfied after Volcker.

Table 4B investigates the auto-correlation of inflation and federal funds rate. Inflation and Federal funds rate are highly auto-correlated (respectively 0.93 and 0.86), except for the period 1982q1 to 2006q2, the inflation auto-correlation coefficient falls in the 95% confidence interval 0.6 ± 0.14 and it is statistically different from the federal funds rate auto-correlation coefficient in the 95% confidence interval 0.95 ± 0.02. Infinite horizon time-consistent policy predict that the auto-correlation should be the same, which is not the case after 1982q1.

Table 4B. Auto-correlation of inflation and federal funds rate

dates	obs.	var.	r	R^2	ρ	c	σ_ε	ρ_ε	DF	PP
$< 79q3$	78	π_t	0.86	0.74	0.88 (0.06)	0.62 (0.30)	1.38	−0.22 (0.12)	0.55	0.41
$< 79q3$	78	i_t	0.93	0.87	0.95 (0.04)	0.33 (0.25)	0.89	0.43 (0.10)	0.17	0.54
$< 82q1$	88	π_t	0.88	0.78	0.88 (0.05)	0.63 (0.28)	1.35	−0.19 (0.11)	0.34	0.23
$< 82q1$	88	i_t	0.94	0.89	0.96 (0.04)	0.36 (0.27)	1.26	0.21 (0.11)	0.42	0.65
$\geq 79q3$	108	π_t	0.89	0.79	0.85 (0.04)	0.42 (0.16)	0.93	−0.27 (0.09)	0.04	0.01
$\geq 79q3$	108	i_t	0.94	0.92	0.96 (0.03)	0.23 (0.20)	1.05	0.15 (0.09)	0.38	0.44
$\geq 82q1$	98	π_t	**0.64**	0.41	0.59 (0.07)	1.06 (0.21)	0.83	−0.20 (0.10)	0.00	0.00
$\geq 82q1$	98	i_t	0.96	0.95	0.94 (0.02)	0.26 (0.14)	0.63	0.45 (0.09)	0.18	0.08

There is a negative auto-correlation of residuals ρ_ε for inflation and a positive auto-correlation of residuals for federal funds rate. The column DF reports the p-value of the Dickey-Fuller test of unit root with one lag without trend. The column PP reports the p-value of the Phillips-Perron test of unit root, which takes into account auto-correlation, with one lag without trend. The null hypothesis of a unit root is rejected at the 5% threshold for inflation after 1979q2 and after 1981q4. It is rejected for federal funds rate at the 10% level after 1981q4 only for the Phillips-Perron test.

7.2 Tests of Ramsey optimal policy

Table 5B reports estimates using four structural VAR estimations for $(\rho, \lambda_C, F_{\pi,C})$, for $\left(\rho, \lambda_C, \frac{\alpha_i}{\kappa}\right)$ and for $(\rho, \kappa(\beta), \alpha_i(\beta))$ with the identification restrictions for the discount factor $\beta = 1$ or $\beta = 0.99$. We check that point estimates satisfy the theoretical constraints of Ramsey optimal policy. When the federal funds rate is the policy instrument, the estimations only converged to plausible values for the pre-Volcker period, before 1979q3.

Table 5B: Ramsey optimal policy structural parameters

Dates	ρ	λ_R	$F_{\pi,R}$	$\frac{\alpha_i}{\kappa}$	β	$\kappa\,(\beta)$	$\alpha_i\,(\beta)$	$F_{u,C}\,(\beta)$	$\sigma_u\,(\beta)$
$< 79q3$	0.550^* (0.092)	0.947^* (0.047)	0.873^* (0.131)	20.83 (18.268)	1	0.060 (0.06)	1.24^* (0.34)	-1.822	0.483
$< 79q3$	0.550^* (0.092)	-	-	-	0.99	0.071 (0.06)	1.47^* (0.36)	-1.802	0.489

Estimates of structural parameters are plausible values. The Fed's preference parameter α_i and the reduced form rule parameter $F_{\pi,R}$ are significantly different from zero at the 5% level, as well as the reduced form rule parameter $F_{\pi,C}$. The new-Keynesian Phillips curve parameter κ is not significantly different from zero at the 5% level.

Table 6B shows that the reduced form parameters of the structural VAR(1) (rows S) are *identical* to the unconstrained VAR(1) estimates (rows U) up to the third decimal.

Table 6B: Inflation and federal funds rate structural (S) versus unconstrained (U) VAR

dates	obs.	var.	S/U	π_{t-1}	i_{t-1}	c	ρ_ε	R^2	ΔR^2	λ
$< 79q3$	77	π_t	S	0.521	0.488	-0.471 (0.340)	-0.12 (0.16)	0.804	0.06	0.550^* (0.092)
$< 79q3$	77	π_t	U	0.52 (0.09)	0.49 (0.10)	-0.471 (0.343)	-0.12 (0.16)	0.804	0.06	0.550
$< 79q3$	77	i_t	S	-0.025	0.976	0.316 (0.226)	0.55 (0.12)	0.869	0.01	0.947^* (0.047)
$< 79q3$	77	i_t	U	-0.025 (0.065)	0.976 (0.07)	0.317 (0.252)	0.55 (0.12)	0.869	0.01	0.947

(1) For the inflation equation, there is Granger causality from lagged federal funds rate to inflation. The exogenous cost-push shock auto-correlation is close to the auto-correlation of inflation in the VAR. The residuals are not auto-correlated controlling for endogenous lagged inflation and federal funds according to Durbin's test. The autocorrelation estimate is -0.12, It not statistically different from zero (Durbin's test: $t = -0.77$, $p = 0.44$).

(2) For the federal funds rate rule equation, the auto-correlation 0.976 is relatively close to a unit root. The endogenous stable eigenvalue corresponds to the auto-correlation of the federal funds rate. There is no Granger causality from lagged inflation to the federal funds rate. The residuals are auto-correlated controlling for endogenous lagged inflation and federal funds according to Durbin's test. The autocorrelation estimate is 0.55. It is statistically different from zero (Durbin's test: $t = 4.66$, $p = 0.000$). For the residuals of both equations of the VAR, the joint LM test reject the null of no auto-correlation of order one of the residuals. It does not reject the null of no auto-correlation of order two of the residuals. This suggests a lag of order two is missing for rule of the federal funds rate. We explore this issue in the next section.

8 Pre-test with Consumption Euler Equation

Following Gianonni and Woodford (2003) and Kara (2007), the monetary transmission mechanism includes the consumption Euler equation including a forcing auto-regressive variable. In the representative household's intertemporal substitution (IS) consumption Euler equation, current output gap x_t is positively correlated with *expected* output gap and negatively correlated x_t with real rate of interest, equal to the nominal rate i_t minus *expected* inflation $E_t\pi_{t+1}$. The intertemporal elasticity of substitution (IES) $\gamma = 1/\sigma$ is a measure of the responsiveness of the growth rate of consumption to the interest rate, usually considered to be smaller than one. It is the inverse or the relative degree of resistance to intertemporal substitution of consumption (RISC) of σ (the relative fluctuation aversion), which measure the strength of the preference for smoothing consumption over time, usually considered to be larger than one.

$$x_t = E_t x_{t+1} - \gamma \left(i_t - E_t \pi_{t+1}\right) + z_{x,t} \text{ where } \gamma > 0 \tag{51}$$

A non-controllable exogenous stationary and predetermined variable $z_{x,t}$ is auto-regressive of order one ($0 < |\rho_{z,x}| < 1$) where $\varepsilon_{g,t}$ are zero-mean, normally, independently and identically distributed additive disturbances. Initial values of predetermined forcing variable are given.

$$z_{x,t} = \rho_{z,x} z_{x,t-1} + \varepsilon_{x,t} \text{ where } \varepsilon_{x,t} \text{ is i.i.d. } N\left(0, s_x^2\right), \; z_{x,0} \text{ given,} \tag{52}$$

The policy maker minimizes the expectation of the expected discounted present value of a discounted quadratic loss function over a finite horizon of duration $T \geq 3$. The weight on inflation is normalized to one, the weight on output gap is $\alpha_x \geq 0$ and the weight on the policy instrument (the federal funds rate) is $\alpha_i > 0$.

$$-E_t \sum_{t=0}^{T} \beta^t \left\{ \frac{\pi_t^2}{2} + \alpha_x \frac{x_t^2}{2} + \alpha_i \frac{i_t^2}{2} \right\}, T \geq 3 \tag{53}$$

subject to the private sector's new-Keynesian Phillips curve and the consumption Euler equation, with initial conditions for predetermined state variables and natural boundary conditions for private sector's forward variables. In this model, shifting from time-consistent policy and optimal simple rule to Ramsey optimal policy is a *Hopf bifurcation* (Chatelain and Ralf (2017)).

Gianonni and Woodford (2003) and Kara (2007) found a reduced form for Ramsey optimal policy which depends on four predetermined variables, including two lags of the federal funds rate (table 7, row 1).

Time-consistent reduced form policy rule has identified representations that depends only on two variables, because the number of predetermined variables is equal to two (the two auto-regressive shocks). Four representations of the time-consistent rule depending only on two variables and on disturbances corresponding to white-noise measurement errors are estimated (table 7, rows 2 to 5). Only the rule which depends on two lags of the federal funds rate pass the Durbin test (the auto-correlation of measurement errors is not statistically different from zero). The three other representations of time-consistent policy rule are also predicted to have zero auto–correlation of disturbances, but they do not pass the Durbin test.

Table 7: Durbin test on Ramsey (R) versus time-consistent (TC) reduced form Taylor rules

dates	obs.	var.	R/TC	i_{t-1}	i_{t-2}	π_{t-1}	x_{t-1}	c	ρ_ε	R^2	$R_R^2 - R_{TC}^2$
$< 79q3$	76	i_t	R	1.17 (0.12)	−0.43 (0.11)	0.16 (0.07)	0.12 (0.04)	0.69 (0.23)	0.11 (0.11)	0.910	−
$< 79q3$	76	i_t	TC	−	−	0.74 (0.06)	0.38 (0.06)	2.11 (0.29)	0.49 (0.10)	0.732	0.178
$< 79q3$	76	i_t	TC	1.38 (0.10)	−0.47 (0.10)	−	−	0.54 (0.23)	0.12 (0.12)	0.896	0.014
$< 79q3$	76	i_t	TC	0.976 (0.08)	−	−0.025 (0.07)	−	0.32 (0.25)	0.42 (0.11)	0.869	0.041
$< 79q3$	76	i_t	TC	0.928 (0.04)	−	−	0.13 (0.04)	0.41 (0.23)	0.37 (0.11)	0.888	0.022

With habit persistence parameter and inflation indexation parameter, two additional predetermined variables (lagged output and lagged inflation) are taken into account for both policies. Time-consistent policy shifts to four predetermined variables and Ramsey optimal policy shifts to six predetermined variables. Then, the reduced form policy rule including four variables in table 7 corresponds to time-consistent policy. Even before checking exact parameter identification issues, time-consistent policy with habit persistence and inflation indexation is less parsimonious with respect to the number of parameters than Ramsey optimal policy without habit persistence and without inflation indexation: it includes two more free parameters.

9 Conclusion

Using closed form solutions of Ramsey optimal policy and time-consistent policy from Gali (2015) model, we take *exactly* into account the identification restrictions related to the dimension of the stable subspace of each policy. Ramsey optimal policy with quasi-commitment has a comparative advantage with respect to infinite horizon time-consistent policy and optimal simple rules for modelling persistence *with fewer parameters.* Durbin tests allows to test Ramsey optimal policy with respect to time consistent policy.

References

[1] Adjemian S. and Devulder A. (2011). Evaluation de la politique monétaire dans un modèle DSGE pour la zone euro. *Revue française d'économie*, 26(1), 201-245.

[2] Adolfson, M., Laséen, S., Lindé, J., & Svensson, L. E. (2011). Optimal Monetary Policy in an Operational Medium-Sized DSGE Model. *Journal of Money, Credit and Banking*, 43(7), 1287-1331.

[3] Anderson E.W., Hansen L.P., McGrattan E.R. and Sargent T.J. (1996). Mechanics of Forming and Estimating Dynamic Linear Economies. in Amman H.M., Kendrick D.A. and Rust J. (editors) *Handbook of Computational Economics*, Elsevier, Amsterdam, 171-252.

[4] Backus, D., & Driffill, J. (1986). The consistency of optimal policy in stochastic rational expectations models (No. 124). CEPR Discussion Papers.

[5] Blanchard O.J. and Kahn C. (1980). The solution of linear difference models under rational expectations. *Econometrica,* 48, pp. 1305-1311.

[6] Blinder A.S. (1986). More on the speed of adjustment in inventory models. *Journal of Money, Credit and Banking,* 18, 355–365.

[7] Bratsiotis, G. J. and Robinson, W. A. (2016). Unit Total Costs: An Alternative Marginal Cost Proxy for Inflation Dynamics. *Macroeconomic Dynamics*, 1-24.

[8] Castelnuovo E. and Surico P. (2004). Model uncertainty, optimal monetary policy and the preferences of the Fed. *Scottish Journal of Political Economy,* 51(1), pp. 105-126.

[9] Castelnuovo E. (2006). The Fed's preferences for Policy Rate Smoothing: Overestimation due to misspecification? *The B.E. Journal in Macroeconomics (Topics),* 6(2), article 5.

[10] Cecchetti S. and Ehrmann M. (2002), "Does Inflation Targeting Increase Output Volatility? An International Comparison of Policymakers Preferences and Outcomes," in Loayza, Norman, and Klaus Schmidt-Hebbel (eds.), *Monetary Policy: Rules and Transmission Mechanisms*, vol. 4 of Series on Central Banking, Analysis, and Economic Policies, Central Bank of Chile, 247—274.

[11] Chatelain, J. B., and Ralf, K. (2017a). Publish and Perish: Creative Destruction and Macroeconomic Theory. Available at SSRN: https://ssrn.com/abstract=2915860.

[12] Chatelain, J. B., and Ralf, K. (2017b). Hopf bifurcation from new-Keynesian Taylor rule to Ramsey optimal policy (2017). Available at SSRN: https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2971227.

[13] Chatelain, J. B., and Ralf, K. (2017c). A Simple Algorithm for Solving Ramsey Optimal Policy with Exogenous Forcing Variables. Econstor working paper.

[14] Chari, V. V., & Kehoe, P. J. (1990). Sustainable plans. *Journal of political economy*, 98(4), 783-802.

[15] Christiano, L. J., & Trabandt ■, Walentin. K. (2011). DSGE models for monetary policy analysis in *Handbook of monetary economics*, editors BM Friedman, M. Woodford, volume 3A.

[16] Clarida R., Gali J. and Gertler M. (1999). The Science of Monetary Policy: a new Keynesian Perspective. *Journal of Economic Literature,* 37(4), 1661-1707.

[17] Clarida R., Gali J. and Gertler M. (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. *Quarterly Journal of Economics,* 115(1), pp. 147-180.

[18] Cochrane J.H. (2011). Determinacy and Identification with Taylor Rules. *Journal of Political Economy,* 119(3), pp. 565-615.

[19] Cohen D. and Michel P. (1988). How Should Control Theory Be Used to Calculate a Time-Consistent Government Policy? *Review of Economic Studies,* 55, 119(3), 263-274.

[20] Debortoli, D., and Nunes, R. (2014). Monetary regime switches and central bank preferences. *Journal of Money, Credit and Banking*, 46(8), 1591-1626.

[21] Debortoli, D., & Lakdawala, A. (2016). How credible is the Federal Reserve? A structural estimation of policy re-optimizations. *American Economic Journal: Macroeconomics*, 8(3), 42-76.

[22] Duarte, P. G. (2005). A Feasible and Objective Concept of Optimality: the quadratic loss function and US monetary policy in the 1960s. *In* Brazilian Association of Graduate Programs in Economics in its Series Proceedings of the 33th Brazilian Economics Meeting with (No. 016).

[23] Duhem, P. (1908). *Sauver les phénomènes. Essai sur la notion de théorie physique de Platon à Galilée.* Vrin, Paris, 1ère edition. Hermann, Paris.

[24] Durbin, J. (1970). Testing for serial correlation in least-squares regressions when some of the regressors are lagged dependent variables. *Econometrica* 38: 410–421.

[25] Favero, C. A., & Rovelli, R. (2003). Macroeconomic stability and the preferences of the Fed: A formal analysis, 1961-98. *Journal of Money, Credit, and Banking*, 35(4), 545-556.

[26] Fève P., Matheron J. and Poilly C. (2007). Monetary Policy Dynamics in the Euro Area. *Economics Letters,* 96(1), 97-102.

[27] Fujiwara, I., Kam, T., & Sunakawa, T. (2016). A note on imperfect credibility. SSRN working paper.

[28] Gali J. (2015). *Monetary Policy, Inflation, and the Business Cycle,* (2nd edition) Princeton University Press.

[29] Giannoni, M., & Woodford, M. (2004). Optimal inflation-targeting rules. In *The Inflation-Targeting Debate* (pp. 93-172). NBER, University of Chicago Press.

[30] Giordani and Söderlind (2004). Solution of Macromodels with Hansen-Sargent Robust Policies: some extensions. *Journal of Economic Dynamics and Control*, 12, 2367-2397.

[31] Givens, G. E. (2012). Estimating central bank preferences under commitment and discretion. *Journal of Money, credit and Banking*, 44(6), 1033-1061.

[32] Griliches, Z. (1967). Distributed lags: a survey. *Econometrica* 35, 16–49.

[33] Hansen L.P. and Sargent T. (2008). *Robustness*, Princeton University Press, Princeton.

[34] Havranek, T., Rusnak, M., & Sokolova, A. (2017). Habit formation in consumption: A meta-analysis. *European Economic Review*, 95, 142-167.

[35] Ilbas, P. (2012). Revealing the preferences of the US Federal Reserve. *Journal of Applied Econometrics*, 27(3), 440-473.

[36] Juillard M., Karam P.D., Laxton D., Pesenti P.A. (2006). Welfare-based monetary policy rules in an estimated DSGE model of the US. ECB working paper 613.

[37] Kara, H. (2007). Monetary policy under imperfect commitment: Reconciling theory with evidence. *International Journal of Central Banking.* 3(1), 149-177.

[38] Kollmann R. (2002). Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics*, 49(5), 989-1015.

[39] Kollmann, R. (2008). Welfare-maximizing operational monetary and tax policy rules. *Macroeconomic dynamics*, 12(S1), 112-125.

[40] Kydland F. and Prescott E.C. (1980). Dynamic Optimal Taxation, Rational Expectations and Optimal Control. *Journal of Economic Dynamics and Control*, 2, pp.79-91.

[41] Leeper, E. M. (1991). Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. *Journal of monetary Economics*, 27(1), 129-147.

[42] Levieuge G., Lucotte Y. (2014). A Simple Empirical Measure of Central Banks’ Conservatism, *Southern Economic Journal*, 81(2). 409-434.

[43] Ljungqvist L. and Sargent T.J. (2012). *Recursive Macroeconomic Theory.* 3rd edition. The MIT Press. Cambridge, Massaschussets.

[44] Matthes, C. (2015). Figuring Out the Fed—Beliefs about Policymakers and Gains from Transparency. *Journal of Money, credit and Banking*, 47(1), 1-29.

[45] Mavroeidis S. (2010). Monetary Policy Rules and Macroeconomic Stability: some new Evidence. *American Economic Review.* 100(1), pp. 491-503.

[46] Mavroeidis S., Plagbord-Moller M., Stock J.M. (2014). Empirical Evidence on Inflation Expectations in the New Keynesian Phillips Curve. *Journal of Economic Literature.* 52(1), pp.124-188.

[47] McManus, D.A., Nankervis, J.C., Savin, N.E. (1994). Multiple optima and asymptotic approximations in the partial adjustment model. *Journal of Econometrics* 62, 91–128.

[48] Oudiz G. and Sachs J. (1985). International Policy Coordination in Dynamic Macroeconomic Models, in W.H. Buiter and R.C. Marston (eds), *International Economic Policy Coordination*, Cambrdige, Cambridge University Press.

[49] Ozlale, U. (2003). Price stability vs. output stability: tales of federal reserve administrations, *Journal of Economic Dynamics and Control*, 27(9), 1595-1610.

[50] Paez-Farrell J. (2015). Taylor rules, central bank preferences and inflation targeting. Working Paper, university of Sheffield.

[51] Roberds, W. (1987). Models of policy under stochastic replanning. *International Economic Review*, 731-755.

[52] Romer, P. (2016). The trouble with macroeconomics. *The American Economist*, forthcoming.

[53] Salemi, M. K. (1995). Revealed preference of the Federal Reserve: using inverse-control theory to interpret the policy equation of a vector autoregression. *Journal of Business & Economic Statistics*, 13(4), 419-433.

[54] Salemi, M. K. (2006). Econometric policy evaluation and inverse control. *Journal of Money, Credit, and Banking*, 38(7), 1737-1764.

[55] Salemi, M. K. (2010). It’s what they do, not what they say. *The New International Monetary System: Essays in Honour of Alexander Swoboda*, chapter 11, p;162.

[56] Schaumburg, E., and Tambalotti, A. (2007). An investigation of the gains from commitment in monetary policy. *Journal of Monetary Economics*, 54(2), 302-324.

[57] Sims C.A. (1980). Macroeconomics and Reality. *Econometrica*, 48, 1-48.

[58] Simon H.A. (1956). Dynamic Programming under Uncertainty with a Quadratic Criterion Function. *Econometrica*, 24(1), 74-81.

[59] Söderlind P. (1999). Solution and estimation of RE macromodels with optimal policy. *European Economic Review*, 43(4), 813-823.

[60] Söderström, U., Söderlind, P., & Vredin, A. (2005). New-Keynesian Models and Monetary Policy: A Re-examination of the Stylized Facts. *The Scandinavian journal of economics*, 107(3), 521-546.

9.1 Appendix 1: Augmented Discounted Linear Quadratic Regulator

The new-Keynesian Phillips curve can be written as a function of the Lagrange multiplier:

$$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \kappa\frac{\kappa}{\alpha_x}\gamma_{t+1} + u_{\pi,t} \text{ where } \kappa > 0,\ 0 < \beta < 1$$

It can be written:

$$E_t\left[\pi_{t+1}\right] + \frac{\kappa^2}{\beta\alpha_x}\gamma_{t+1} = \frac{1}{\beta}\pi_t - \frac{1}{\beta}u_{\pi,t} \text{ where } \kappa > 0,\ 0 < \beta < 1$$

The solution of the Hamiltonian system are based on the demonstrations of the augmented discounted linear quadratic regulator in Anderson, Hansen, McGrattan and Sargent [1996], following the steps in Chatelain and Ralf (2017c):

$$\mathbf{L}^a \begin{pmatrix} \pi_{t+1} \\ \gamma_{t+1} \\ u_{t+1} \end{pmatrix} = \mathbf{N}^a \begin{pmatrix} \pi_t \\ \gamma_t \\ u_t \end{pmatrix}$$

where

$$\mathbf{L}^a = \begin{pmatrix} 1 & \frac{\kappa^2}{\beta\alpha_x} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mathbf{N}^a = \begin{pmatrix} \frac{1}{\beta} & 0 & \frac{-1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix}$$

As $\mathbf{L}^a$ is non singular:

$$\left(\mathbf{L}^a\right)^{-1}\mathbf{N}^a = \mathbf{M}^a = \begin{pmatrix} \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix} = \begin{pmatrix} \frac{1}{\beta a} - 1 & 1 + \frac{1}{\beta} - \frac{1}{\beta a} & -\frac{1}{\beta} \\ -1 & 1 & 0 \\ 0 & 0 & \rho \end{pmatrix}$$

where Gali (2015) denotes $a = a\left(\beta, \kappa, \alpha_x\right) = \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2} = \frac{1}{1+\beta+\frac{\kappa^2}{\alpha_x}}$. The characteristic polynomial of matrix $\mathbf{M}^a$:

$$(X-\rho)\left(X^2 - \frac{1}{\beta a}X + \frac{1}{\beta}\right) = 0$$

Matrix $\mathbf{M}^a$ has two stable roots with bounded discounted quadratic loss function (below $\sqrt{\frac{1}{\beta}}$): ρ and $\lambda_R = \frac{1-\sqrt{1-4\beta a^2}}{2\beta a}$ (λ_R is denoted δ in Gali (2015)) and one unstable root $\lambda_U = \frac{1+\sqrt{1-4\beta a^2}}{2\beta a}$ because the determinant of the matrix $\mathbf{M}^a$ is $\rho\lambda_R\lambda_U = \rho\sqrt{\frac{1}{\beta}}\sqrt{\frac{1}{\beta}}$ and $\lambda_R < \sqrt{\frac{1}{\beta}}$ imply $\lambda_U = \frac{1+\sqrt{1-4\beta a^2}}{2\beta a} = \frac{1}{\beta\lambda_R} > \sqrt{\frac{1}{\beta}}$.

$$\lambda_R\left(\beta,\kappa,\alpha_x\right) = \frac{1}{2}\left(1+\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}-\sqrt{\left(1+\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}\right)^2-\frac{4}{\beta}}\right)$$

$$\frac{\partial\lambda_R}{\partial\alpha_x} > 0,\ \lim_{\alpha_x\to 0}\lambda_R = 0 \text{ and } \lim_{\alpha_x\to+\infty}\lambda_R = 1 < \frac{1}{\sqrt{\beta}}$$

Identification of $\frac{\kappa}{\alpha_x}$: The ratio $\frac{\kappa}{\alpha_x}$ is identified using the following two equalities defining the inflation rule parameter $F_{\pi,R}$, which are found for the characteristic polynomial equal to zero:

$$F_{\pi,R} = \frac{1-\beta\lambda_R}{\kappa} = \left(\frac{\lambda_R}{1-\lambda_R}\right)\frac{\kappa}{\alpha_x} \Rightarrow$$
$$0 = \beta\lambda_R - \left(1+\beta+\frac{\kappa^2}{\alpha_x}\right)\lambda_R + 1$$

Positive sign restriction of $F_{\pi,R}$: The eigenvalue λ_R is a linear decreasing function of the inflation rule parameter $F_{\pi,R}$. It varies between zero (for the relative cost of changing the interest rate tending to zero: $\alpha_x \to 0$) and the inverse β of the laissez-faire eigenvalue $\frac{1}{\beta}$ (for the relative cost of changing the interest rate tending to infinity: $\alpha_x \to +\infty$). This sets boundaries restrictions of the inflation rule parameter $F_{\pi,R}$, which is strictly positive (see appendix):

$$F_{\pi,R} = \frac{1}{\kappa} - \frac{\beta}{\kappa}\lambda_R = \left(\frac{\lambda_R}{1-\lambda_R}\right)\frac{\kappa}{\alpha_x} \in \left]\frac{1-\beta^2}{\kappa},\frac{1}{\kappa}\right[\ . \tag{54}$$

Ricatti equation solution: P_π is the slope of eigenvectors of the stable eigenvalue λ_R of the matrix $\mathbf{H}$ of the Hamiltonian system when $u_0 = 0 = u_t$

$$\begin{pmatrix} \frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ P_\pi & P_U \end{pmatrix}\begin{pmatrix} \lambda_R & 0 \\ 0 & \lambda_U \end{pmatrix}\begin{pmatrix} 1 & 1 \\ P_\pi & P_U \end{pmatrix}^{-1}$$
$$\begin{pmatrix} \frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x} & -\frac{\kappa^2}{\beta\alpha_x} \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{1-\lambda_R} & \frac{1}{1-\frac{1}{\beta\lambda_R}} \end{pmatrix}\begin{pmatrix} \lambda_R & 0 \\ 0 & \frac{1}{\beta\lambda_R} \end{pmatrix}\begin{pmatrix} 1 & 1 \\ \frac{1}{1-\lambda_R} & \frac{1}{1-\frac{1}{\beta\lambda_R}} \end{pmatrix}^{-1}$$

The stable eigenvalue λ_R is the stable solution of the characteristic polynomial of the hamiltonian matrix $\mathbf{H}$:

$$\lambda_R = \frac{1}{2}\left(\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}+1-\sqrt{\left(\frac{1}{\beta}+\frac{\kappa^2}{\beta\alpha_x}+1\right)^2-\frac{4}{\beta}}\right)$$

The slope P_π of eigenvectors of the stable eigenvalue λ_R is given by:

$$P_\pi = \frac{\lambda_R - a_{11}}{a_{12}} = \frac{a_{21}}{\lambda_R - a_{22}} = \frac{1}{1-\lambda_R} \in [1,+\infty[\text{ where } \begin{pmatrix} a_{11} = \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} & a_{12} = -\frac{\kappa^2}{\beta\alpha_x} \neq 0 \\ a_{21} = -1 & a_{22} = 1 \end{pmatrix} \text{ or}$$

$$P_\pi = \frac{1}{2}\frac{\beta\alpha_x}{\kappa^2}\left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - 1 + \sqrt{\left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} + 1\right)^2 - \frac{4}{\beta}}\right) \text{ or}$$

$$P_\pi = \frac{1}{2}\left(\frac{\alpha_x}{\kappa^2} + 1 - \frac{\beta\alpha_x}{\kappa^2} + \sqrt{\left(\frac{\alpha_x}{\kappa^2} + 1 - \frac{\beta\alpha_x}{\kappa^2}\right)^2 + 4\beta\frac{\alpha_x}{\kappa^2}}\right)$$

P_π is also the positive solution of a scalar Ricatti equation (demonstration using undetermined coefficients in proposition 1):

$$\frac{P_\pi - 1}{P_\pi} = \frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - \frac{\kappa^2}{\beta\alpha_x}P_\pi$$

$$0 = -\frac{\kappa^2}{\beta\alpha_x}P_\pi^2 + \left(\frac{1}{\beta} + \frac{\kappa^2}{\beta\alpha_x} - 1\right)P_\pi + 1$$

$$0 = P_\pi^2 - \left(\frac{\alpha_x}{\kappa^2} + 1 - \frac{\beta\alpha_x}{\kappa^2}\right)P_\pi - \frac{\beta\alpha_x}{\kappa^2}$$

Proposition 1: Rule parameters F_u and P_u of the cost-push shock u_t satisfy:

$$\frac{P_u}{P_\pi} = \frac{-\lambda_R}{1-\lambda_R\rho\beta} = \frac{-\frac{1}{\beta}}{\frac{1}{\beta\lambda_R} - \rho} \text{ and } \frac{F_u}{F_\pi} = -1 + \beta\rho\frac{P_u}{P_\pi} = \frac{-1}{1-\lambda_R\rho\beta} = \frac{1}{\lambda_R}\frac{P_u}{P_\pi} \tag{55}$$

$$P_u = \frac{-\lambda_R}{1-\lambda_R\rho\beta}\frac{1}{1-\lambda_R} \text{ and } F_u = \frac{-1}{1-\lambda_R\rho\beta}\left(\frac{1}{\kappa} - \frac{\beta}{\kappa}\lambda_R\right) = \frac{-1}{1-\lambda_R\rho\beta}\left(\frac{\lambda_R}{1-\lambda_R}\right)\frac{\kappa}{\alpha_x} = P_u\frac{\kappa}{\alpha_x} \tag{56}$$

and the optimal initial anchor of inflation on the cost-push shock is:

$$\pi_0\left(\underset{+}{\lambda_R}, \underset{-}{\rho}\right) = \frac{-P_u}{P_\pi}u_0 = \frac{\frac{1}{\beta}}{\frac{1}{\beta\lambda_R} - \rho}u_0 = \frac{-\alpha_x}{\kappa}x_0 \text{ with } \rho < 1 < \frac{1}{\beta} < \frac{1}{\beta\lambda_R} = \lambda_U \tag{57}$$

Demonstration: It uses the method of undetermined coefficients of Anderson, Hansen, McGrattan and Sargent's (1996), section 5, on Gali's (2015) Ramsey optimal policy. Using the infinite horizon transversality conditions, the solution is the one that stabilizes the state-costate vector for any initialization of inflation π_0 and of the exogenous shock u_0 in a stable subspace of dimension two within a space of dimension three (π_t, γ_t, u_t) of the Hamiltonian system. We seek a characterization of the Lagrange multiplier γ_t of the form:

$$\gamma_t = P_\pi\pi_t + P_u u_t.$$

To deduce the control law associated with matrix (P_π, P_u), we substitute it into the Hamiltonian system:

$$\mathbf{L}^a \begin{pmatrix} \pi_{t+1} \\ P_\pi \pi_{t+1} + P_u u_{t+1} \\ u_{t+1} \end{pmatrix} = \mathbf{N}^a \begin{pmatrix} \pi_t \\ P_\pi \pi_t + P_u u_t \\ u_t \end{pmatrix}$$

If we write the three equations in this system separately,

$$\begin{aligned} \left(1 + \frac{\kappa^2}{\beta\alpha_x} P_\pi\right) \pi_{t+1} + \frac{\kappa^2}{\beta\alpha_x} P_u u_{t+1} &= \frac{1}{\beta}\pi_t - \frac{1}{\beta} u_t \\ P_\pi \pi_{t+1} + P_u u_{t+1} &= (P_\pi - 1)\pi_t + P_u u_t \\ u_{t+1} &= \rho u_t \end{aligned}$$

Substitute the last equation into the first and solve for π_{t+1}:

$$\pi_{t+1} = \left(1 + \frac{\kappa^2}{\beta\alpha_x} P_\pi\right)^{-1} \left(\frac{1}{\beta}\pi_t + \left(-\frac{1}{\beta} - \frac{\kappa^2}{\beta\alpha_x} P_u \rho\right) u_t\right)$$

It is straightforward to verify that:

$$\frac{1}{1 + \frac{\kappa^2}{\beta\alpha_x} P_\pi} = 1 - \frac{\frac{\kappa^2}{\beta\alpha_x} P_\pi}{1 + \frac{\kappa^2}{\beta\alpha_x} P_\pi} = 1 - \frac{\frac{\kappa^2}{\beta} P_\pi}{\alpha_x + \frac{\kappa^2}{\beta} P_\pi}$$

The policy instrument evolves in the stable subspace of the Hamiltonian. We seek a characterization of the policy rule of the form:

$$x_t = F_\pi \pi_t + F_u u_t.$$

The evolution equation of inflation can be rewritten with a feedback rule as:

$$\pi_{t+1} = \left(\frac{1}{\beta} - \frac{\kappa}{\beta} F_\pi\right) \pi_t + \left(-\frac{1}{\beta} - \frac{\kappa}{\beta} F_u\right) u_t$$

where F_π is given by:

$$F_\pi = \frac{\frac{\kappa}{\beta} P_\pi}{\alpha_x + \frac{\kappa^2}{\beta} P_\pi} = \frac{P_\pi}{\beta + \frac{\kappa^2}{\alpha_x} P_\pi} \frac{\kappa}{\alpha_x} = \left(\frac{\lambda_R}{1 - \lambda_R}\right) \frac{\kappa}{\alpha_x} \tag{58}$$

where F_u is given by (demonstration (1) below):

$$\frac{F_u}{F_\pi} = -1 + \beta\rho \frac{P_u}{P_\pi}$$

where $\frac{P_u}{P_\pi}$ is given by (demonstration (2) below):

$$\frac{P_u}{P_\pi} = \frac{-\lambda_R}{1 - \lambda_R \rho \beta}$$

so that F_u is given by:

$$\frac{F_u}{F_\pi} = -1 - \frac{\beta\rho\lambda_R}{1 - \beta\rho\lambda_R} = \frac{-1}{1 - \lambda_R \rho \beta} = \frac{1}{\lambda_R} \frac{P_u}{P_\pi}$$

Demonstration (1) is:

$$\left(1-\frac{\frac{\kappa^2}{\beta}P_\pi}{\alpha_x+\frac{\kappa^2}{\beta}P_\pi}\right)\left(-\frac{1}{\beta}-\frac{\kappa^2}{\beta\alpha_x}P_u\rho\right)$$

$$=-\frac{1}{\beta}-\frac{\kappa^2}{\beta\alpha_x}P_u\rho-\frac{\frac{\kappa^2}{\alpha_x\beta}P_\pi}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}\left(-\frac{1}{\beta}-\frac{\kappa^2}{\beta\alpha_x}P_u\rho\right)$$

$$=-\frac{1}{\beta}+\frac{-\frac{\kappa^2}{\beta\alpha_x}P_u\rho+\frac{\kappa^2}{\alpha_x\beta}P_\pi\frac{1}{\beta}}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}=-\frac{1}{\beta}-\frac{\kappa}{\beta}\frac{\frac{\kappa}{\alpha_x}P_u\rho-\frac{\kappa}{\alpha_x\beta}P_\pi}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}\Rightarrow$$

$$F_u=\frac{-\frac{\kappa}{\alpha_x\beta}P_\pi+\frac{\kappa}{\alpha_x}P_u\rho}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}=\frac{\frac{\kappa}{\alpha_x\beta}P_\pi}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}\left(-1+\beta\rho\frac{P_u}{P_\pi}\right)$$

$$\frac{F_u}{F_\pi}=-1+\beta\rho\frac{P_u}{P_\pi}.$$

For demonstration (2), substitute the auto-regressive equation of the forcing variable u_t into the law of motion of the Lagrange multiplier remaining in stable subspace and solve for $P_\pi\pi_{t+1}$:

$$P_\pi\pi_{t+1}+P_u u_{t+1}=(P_\pi-1)\,\pi_t+P_u u_t$$

$$P_\pi\pi_{t+1}=(P_\pi-1)\,\pi_t+(P_u-\rho P_u)\,u_t$$

The coefficient on u_t is $P_u-\rho P_u$. To obtain an alternative formula for this coefficient, premultiply the evolution equation for inflation including the feedback rule by $\frac{1}{\beta}P_\pi$:

$$\frac{1}{\beta}P_\pi\pi_{t+1}=\frac{1}{\beta}P_\pi\left(\frac{1}{\beta}-\frac{\kappa}{\beta}F_\pi\right)\pi_t+\frac{1}{\beta}P_\pi\left(-\frac{1}{\beta}-\frac{\kappa}{\beta}F_u\right)u_t$$

Using both formulas of the feedback rule, we rewrite the coefficient on u_t. First:

$$\left(\frac{1}{\beta}-\frac{\kappa}{\beta}F_\pi\right)\left(P_\pi\frac{-1}{\beta}+P_u\rho\right)$$

$$=\frac{1}{\beta}P_\pi\frac{-1}{\beta}+\frac{1}{\beta}P_u\rho-\frac{\kappa}{\beta}\frac{\frac{\kappa}{\alpha_x\beta}P_\pi}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}\left(P_\pi\frac{-1}{\beta}+P_u\rho\right)$$

$$=\frac{1}{\beta}P_\pi\left(\frac{-1}{\beta}-\frac{\kappa}{\beta}\frac{\frac{\kappa}{\alpha_x\beta}\left(-P_\pi+P_u\beta\rho\right)}{1+\frac{\kappa^2}{\alpha_x\beta}P_\pi}\right)+\frac{1}{\beta}P_u\rho$$

Hence:

$$\frac{1}{\beta}P_\pi\left(-\frac{1}{\beta}-\frac{\kappa}{\beta}F_u\right)=\left(\frac{1-\kappa F_\pi}{\beta}\right)\left(P_\pi\frac{-1}{\beta}+P_u\rho\right)-\frac{1}{\beta}P_u\rho$$

That is:

$$-\frac{1}{\beta}-\frac{\kappa}{\beta}F_u=\lambda_R\frac{\beta}{P_\pi}\left(P_\pi\frac{-1}{\beta}+P_u\rho\right)-\frac{\beta}{P_\pi}\frac{P_u}{\beta}\rho$$

That is:

$$-\frac{1}{\beta}-\frac{\kappa}{\beta}F_u=\lambda_R\left(-1+\beta\rho\frac{P_u}{P_\pi}\right)-\frac{P_u}{P_\pi}\rho=\lambda_R\frac{F_{u,R}}{F_{\pi,R}}-\lambda_R\frac{F_{u,R}}{F_{\pi,R}}\rho$$
$$-\frac{1}{\beta}-\frac{\kappa}{\beta}F_u=(1-\rho)\,\lambda_R\frac{F_{u,R}}{F_{\pi,R}}$$

Equating coefficients on u_t in the two equations results in a scalar Sylvester equation:

$$P_u-P_u\rho=\left(\frac{1-\kappa F_\pi}{\beta}\right)(-P_\pi+P_u\beta\rho)-P_u\rho$$
$$P_u=\lambda_R\left(-P_\pi+P_u\beta\rho\right)$$
$$P_u=\frac{-\lambda_R P_\pi}{1-\lambda_R\rho\beta}\Longrightarrow\frac{P_u}{P_\pi}=\frac{-\lambda_R}{1-\beta\rho\lambda_R}$$

Hence:

$$\frac{F_{u,R}}{F_{\pi,R}}=-1+\beta\rho\frac{P_u}{P_\pi}=-1+\beta\rho\left(\frac{-\lambda_R}{1-\lambda_R\rho\beta}\right)=\frac{-1}{1-\beta\rho\lambda_R}$$

Q.E.D.

9.2 Appendix 2: A representation of the optimal policy rule function of the non-observable AR(1) cost-push shock.

Gali (2015) stationary equilibrium process for the output gap and the cost-push shock, using basis vectors (u_t, x_t):

$$u_t=\rho u_{t-1}+\varepsilon_{u,t} \tag{59}$$
$$x_t=\lambda_R x_{t-1}-\frac{\lambda_R}{1-\beta\rho\lambda_R}\frac{\kappa}{\alpha_x}u_t \tag{60}$$

corresponds to a change of basis vectors (u_t, x_t) of the ADLQR representation:

$$\begin{pmatrix}u_t\\x_t\end{pmatrix}=\mathbf{N}^{-1}\begin{pmatrix}u_t\\\pi_t\end{pmatrix}\text{ with }\mathbf{N}^{-1}=\begin{pmatrix}1&0\\F_{u,R}&F_{\pi,R}\end{pmatrix}$$

implying Gali (2015) observationally and mathematically equivalent third representation of the VAR(1) of Ramsey optimal policy:

$$\left\{\begin{array}{c}\left(\begin{array}{c}u_{t+1}\\ \pi_{t+1}\end{array}\right)=(\mathbf{A}+\mathbf{BF}_C)\left(\begin{array}{c}u_t\\ \pi_t\end{array}\right)+\left(\begin{array}{c}1\\ 0\end{array}\right)\varepsilon_t\\ x_t=F_{\pi,R}\pi_t+F_{u,R}u_t\\ \pi_0=-\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given}\end{array}\right.$$

$$\Leftrightarrow\left\{\begin{array}{c}\left(\begin{array}{c}u_{t+1}\\ x_{t+1}\end{array}\right)=\mathbf{N}^{-1}(\mathbf{A}+\mathbf{BF})\,\mathbf{N}\left(\begin{array}{c}u_t\\ x_t\end{array}\right)+\mathbf{N}^{-1}\left(\begin{array}{c}1\\ 0\end{array}\right)\varepsilon_t\\ \pi_t=\frac{1}{F_{\pi,R}}x_t-\frac{F_{u,R}}{F_{\pi,R}}\pi_t\\ \pi_0=-\frac{\alpha_x}{\kappa}x_0 \text{ and } u_0 \text{ given}\end{array}\right.$$

with Gali (2015) representation of the Ramsey optimal policy rule as the second line of the VAR(1). The output gap rule depends on its lagged value and on the lagged value of the cost-push shock u_t:

$$\mathbf{N}^{-1}(\mathbf{A}+\mathbf{BF})\,\mathbf{N}=\left(\begin{array}{cc}\rho & 0\\ -\frac{\lambda_R}{1-\beta\rho\lambda_R}\frac{\kappa}{\alpha_x}\rho & \lambda_R\end{array}\right)$$

for $t=1,2,3...$ where the two stable eigenvalues of the stable subspace ρ and λ_R are invariant to changes of basis vectors. This is obtained with intermediate computations:

$$\left(\begin{array}{cc}1 & 0\\ AF_{\pi,R} & F_{\pi,R}\end{array}\right)\left(\begin{array}{cc}\rho & 0\\ (1-\rho)\,A\lambda_R & \lambda_R\end{array}\right)\left(\begin{array}{cc}1 & 0\\ AF_{\pi,R} & F_{\pi,R}\end{array}\right)^{-1}=\left(\begin{array}{cc}\rho & 0\\ (1-\lambda_R)\,AF_{\pi,R}\rho & \lambda_R\end{array}\right)$$

where:

$$(1-\lambda_R)\,AF_{\pi,R}=(1-\lambda_R)\frac{-1}{1-\beta\rho\lambda_R}\left(\frac{\lambda_R}{1-\lambda_R}\right)\frac{\kappa}{\alpha_x}$$

9.3 Appendix 3: Identification issue for reduced form including a non-observable AR(1) shock.

Because the auto-correlation of the policy instrument x_t and the auto-correlation of the cost-push shock are competing to explain the persistence of the policy instrument x_t, this partial adjustment model with serially correlated shocks has a problem of identification and multiple equilibria (Griliches (1967), Blinder (1986), McManus et al. (1994), Fève, Matheron Poilly (2007)). This VAR(1) can be written as:

$$x_t=\lambda_R x_{t-1}+\eta_t \text{ and } \eta_t=\rho\eta_{t-1}+\varepsilon_{\eta,t}$$

where $\eta_t=-\frac{\kappa}{\alpha_x}\frac{\lambda_R}{(1-\lambda_R\rho\beta)}u_t$. It is an AR(2) model of the policy instrument rule:

$$\begin{array}{l}x_t=\lambda_R x_{t-1}+\rho\,(x_{t-1}-\lambda_R x_{t-2})+\varepsilon_{\eta,t}\\ x_t=b_1x_{t-1}+b_2x_{t-2}+\varepsilon_{\eta,t} \text{ with } b_1=\lambda_R+\rho \text{ and } b_2=-\lambda_R\rho.\end{array}$$

The structural parameter ρ and the semi-structural parameter λ_R are functions of reduced form parameters b_1 and b_2 solutions of:

$$X^2-b_1X-b_2=0$$

which are given by:

$$\lambda_R = \frac{b_1 \pm \sqrt{b_1^2 + 4b_2}}{2} \text{ and } \rho = \beta b - \lambda_R$$

where $\Delta = b_1^2 + 4b_2 = (\rho - \lambda_R)^2$. If $\Delta \neq 0$ and $\rho \neq \lambda_R$, two sets of values for λ_R and ρ are observationally equivalent. The first solution is such that $\lambda_R > \rho$ and the second solution is such that $\lambda_R < \rho$. The larger Δ, the larger the identification issue, because it increases the gap between a more inertial monetary policy with lower correlation of monetary policy shocks and a less inertial monetary policy, that we cannot distinguish. The ADLQR representation and Gali (2015) representation of the stationary solution of the VAR(1) of optimal policy are not useful to identify parameters, because they include the cost-push shock u_t which is not observable.

The reduced form estimated variance σ_η provides another equation with a theoretical positive sign restriction $\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1-\lambda_R \rho\beta)} > 0$ for five unknowns structural parameters $(\alpha_x, \kappa, \rho, \beta, \sigma_u)$:

$$\frac{\kappa}{\alpha_x} \frac{\lambda_R}{(1 - \lambda_R \rho\beta)} \sigma_u = \sigma_\eta$$

9.4 Appendix 4: Oudiz and Sachs (1985) vs Gali (2015) time consistent policy

Substituting the private sector's inflation rule (8) and policy rule (9) in the inflation law of motion (1) and comparing it with the forcing variable law of motion (2) leads to the following relation between N_{TC} on date t, $N_{TC,t+1}$ and $F_{u,TC}$:

$$\begin{aligned} \pi_t &= \beta E_t [\pi_{t+1}] + \kappa x_t + u_t \Rightarrow \\ N_{TC} u_t &= \beta N_{TC,t+1} \rho u_t + \kappa F_{u,TC} u_t + u_t \\ N_{TC} &= \beta\rho N_{TC,t+1} + \kappa F_{u,TC} + 1 \end{aligned}$$

A myopic central bank does not notice that $N_{TC,t+1} = N_{TC}$ (Gali (2015)) in its optimization:

$$\begin{aligned} N_{TC,Gali} &= \beta\rho N_{TC,t+1} + \kappa F_{u,TC} + 1 \; \Rightarrow \frac{\partial N_{TC,Gali}}{\partial F_{u,TC}} = \kappa \\ F_{\pi,TC} &= \frac{F_{u,TC}}{N_{TC}} = -\frac{\kappa}{\alpha_x} < 0 \end{aligned}$$

This first order condition of the central bank optimization is substituted into the new-Keynesian Phillips curve equation, where, only at this stage, players of the game discover that it is assumed $N_{TC,t+1} = N_{TC,t} = N_{TC}$. Gali's (2015) solutions are:

$$\begin{aligned} F_{u,TC,Gali} &= -\frac{\kappa}{\kappa^2 + \alpha_x (1 - \beta\rho)} = -\frac{\kappa}{\alpha_x} N_{TC} \\ N_{TC,Gali} &= \frac{\alpha_x}{\kappa^2 + \alpha_x (1 - \beta\rho)} \to \frac{1}{1 - \beta\rho} = N \text{ when } \alpha_x \to +\infty \end{aligned}$$

In time-consistent equilibrium (Oudiz and Sachs (1985)), the central bank does foresees that $N_{TC,t+1} = N_{TC}$ in its optimization, with the following solutions, that we consider for the remaining part of the paper:

$$\begin{aligned}
N_{TC} &= \frac{\kappa F_{u,TC}+1}{1-\beta\rho} = \frac{\kappa F_{\pi,TC} N_{TC}+1}{1-\beta\rho} \Rightarrow \frac{\partial N_{u,TC}}{\partial F_{u,TC}} = \frac{\kappa}{1-\beta\rho} \\
F_{\pi,TC} &= \frac{F_{u,TC}}{N_{TC}} = -\frac{\kappa}{\alpha_x}\frac{1}{1-\beta\rho} = -\frac{\kappa}{\alpha_x} < 0 \\
F_{u,TC} &= -\frac{\kappa}{\kappa^2+\alpha_x(1-\beta\rho)^2} \\
N_{TC} &= \frac{\alpha_x(1-\beta\rho)}{\kappa^2+\alpha_x(1-\beta\rho)^2} \rightarrow \frac{1}{1-\beta\rho} = N \text{ when } \alpha_x \rightarrow +\infty
\end{aligned}$$

In Oudiz and Sachs' (1985) general solution, this is the condition after substitutions of the private sector's rule (matrix N_{TC}) and the policy maker's rule (matrix $F_{u,TC}$) for *both* dates t and $t+1$ into the law of motion of the private sector dynamics:

$$\begin{aligned}
N_{TC,t} &= J - KF_{u,TC} \\
J &= (A_{22} + N_{TC,t+1}A_{12})^{-1}(N_{TC,t+1}A_{11} + A_{21}) \\
K &= (A_{22} + N_{TC,t+1}A_{12})^{-1}(N_{TC,t+1}B_1 + B_2)
\end{aligned}$$

with general notations and equalities with Gali's (2015) transmission mechanism:

$$\begin{pmatrix} u_{t+1} \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} A_{11}=\rho & A_{12}=0 \\ A_{21}=-\frac{1}{\beta} & A_{22}=\frac{1}{\beta} \end{pmatrix} \begin{pmatrix} u_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} B_1 = 0 \\ B_2 = -\frac{\kappa}{\beta} \end{pmatrix} x_t$$

In Oudiz and Sachs (1985), $N_{TC,t+1} = N_{TC,t}$ at all dates, whereas Gali (2015) assumes myopia (or $N_{TC,t+1} = 0$) for the policy maker. This assumption changes the initial jump of inflation, impulse response functions of inflation and the output gap and welfare. It does not change the identification problem of discretion raised in this paper, because the stable subspace of discretion have the same dimension (one) using the reference Oudiz and Sachs (1985) discretion equilibrium or Gali (2015) and Clarida, Gali, Gertler (1999) myopia assumption.

9.5 Appendix 5: Definition of data variables

Mavroeidis data are running from 1960-Q1 to 2006-Q2.

Inflation is annualized quarter-on-quarter rate of inflation, 400 * LN(GDPDEF/ GDPDEF(-1)) with GDPDEF: Gross Domestic Product Implicit Price Deflator, 2000=100, Seasonally Adjusted. Released in August 2006. Source: U.S. Department of Commerce, Bureau of Economic Analysis.

GAPCBO is the output gap measure: 100 * LN(GDPC1/GDPPOT) with GDPC1: Real Gross Domestic Product, Billions of Chained 2000 Dollars, Seasonally Adjusted Annual Rate, Released in August 2006. Source: U.S. Department of Commerce, Bureau of Economic Analysis and GDPPOT: Real Potential Gross Domestic Product, Billions of Chained 2000 Dollars. Source: U.S. Congress, Congressional Budget Office.

Federal Funds Rate : Averages of Daily Figures - Percent, Source: Board of Governors of the Federal Reserve System

Figure 3: Time series of inflation, output gap (gapcbo), federal funds rate (fyff), Volcker's 1979q3 and 1982q1

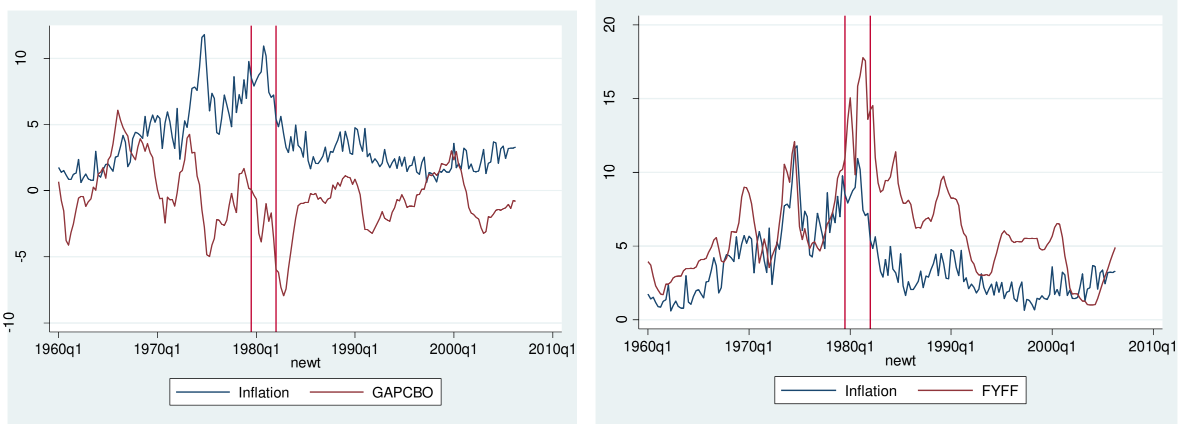


Figure 4: Time-consistent rule null hypothesis: perfect negative correlation (all dots should be on a regression line with negative slope). Output gap rule r=-0.11 (t=-1.51), Working capital Federal funds rate rule r=0.66>0 (t=11.9).

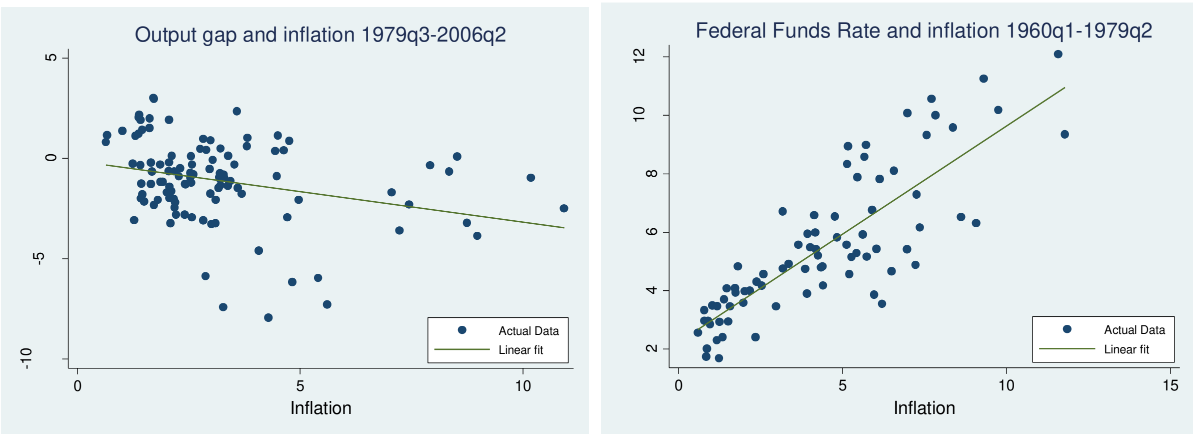
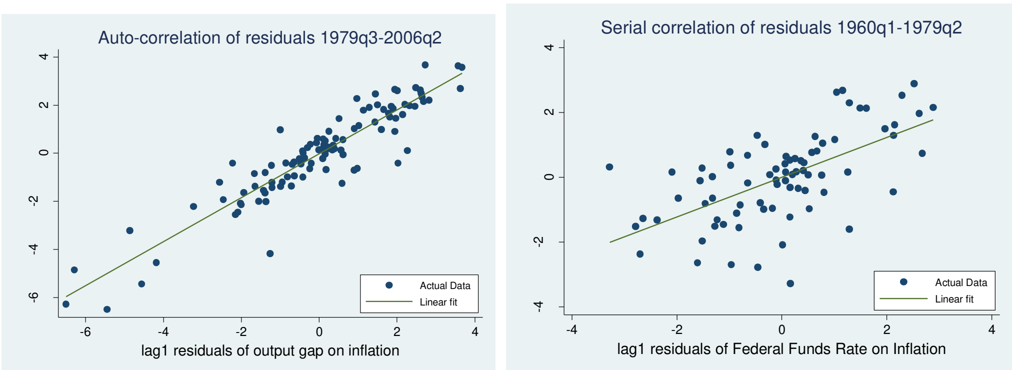


Figure 5: Time-consistent rule null hypothesis: zero serial correlation of residuals (horizontal regression line) is rejected for both rule. Output gap rule: r=0.92 (t=24.5). Federal funds rate rule r=0.61 (t=6.7).



Figures 6: Time-consistent null hypothesis: identical slopes (auto-correlation) of inflation (r=0.64) and of output gap (r=0.958) or federal funds rate (r=0.974) is rejected for 1982q1 to 2006q2.