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Abstract

We investigate the optimal privatization policy in mixed oligopolies with shadow cost of public funds (excess burden of taxation). The government is concerned with both the total social surplus and the revenue obtained by the privatization of a public firm. We find that the relationship between the shadow cost of public funds and the optimal privatization policy is non-monotone. When the cost is moderate, then higher the cost is, the lower is the optimal degree of privatization. However, this does not hold when the cost is high. A further increase of the cost might drastically increase the optimal degree of privatization.

JEL classification H42, L33

Keywords Shadow cost of public funds; free entry; state-owned public enterprises; foreign competition

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1 Introduction

For more than 50 years, we have observed a worldwide wave of the privatization of state-owned public enterprises. Nevertheless, many public and semi-public enterprises (i.e., firms owned by both public and private sectors) are still active in planned and market economies in developed, developing, and transitional countries. While some public enterprises are traditional monopolists in natural monopoly markets, a considerable number of public (including semi-public) enterprises compete with private enterprises in a wide range of industries.[1] Optimal privatization policies in such mixed oligopolies have attracted extensive attention from economics researchers in such fields as industrial organization, public economics, financial economics, international economics, development economics, and political economy.[2]

Specifically, the literature on mixed oligopolies has investigated optimal privatization policy in different situations. Matsumura (1998) showed that the optimal degree of privatization is never zero unless full nationalization yields a public monopoly. Lin and Matsumura (2012) found that the optimal degree of privatization increases with the number of private firms and decreases with the foreign ownership share in private firms. Matsumura and Okamura (2015) showed that more intensive competition might reduce the optimal degree of privatization, using the relative profit maximization approach of Matsumura *et al.* (2013). In free-entry markets, Matsumura and Kanda (2005) showed that the optimal degree of privatization is zero when private competitors are domestic, while Cato and Matsumura (2012) found that the optimal degree is strictly positive when private competitors are foreign and increases with the foreign ownership share in private firms. In addition, Chen (2017) showed that the optimal degree of privatization is positive even in free-entry markets if privatization improves production efficiency. Fujiwara (2007) showed a monotonic (non-monotonic) relationship

[1] Examples include United States Postal Service, Deutsche Post AG, Areva, Nippon Telecom and Telecommunication, Japan Tobacco, Volkswagen, Renault, Électricité de France, Japan Postal Bank, Kampo, Korea Development Bank, and Korea Investment Corporation.

[2] The idea of mixed oligopoly dates at least to Merrill and Schneider (1966). Recently, the literature on mixed oligopoly has become richer and more diverse. For examples of mixed oligopolies and recent developments in this field, see Ishibashi and Matsumura (2006), Ishida and Matsushima (2009), Colombo (2016), Chen (2017), and the papers cited therein.

between the degree of product differentiation and the optimal degree of privatization in a free-entry (non-free-entry) market. Cato and Matsumura (2015) discussed the relationship between the optimal trade and privatization policies in free-entry markets, showing that a higher tariff rate reduces the optimal degree of privatization. Wu *et al.* (2016) investigated a vertically related market in which an upstream foreign monopolist sells an essential input to public and private firms. The authors showed that full privatization can (cannot) be optimal with the domestic (foreign) ownership in private firms.

One common assumption in these studies is the equal weight of the profit of public firms and the consumer surplus in welfare. If there is excess burden of taxation (or shadow cost of public funds), the profit of the public firms might be more valuable for welfare than the consumer surplus is, because the government can use the profits of the public firms or the revenue from selling the stocks of the public firms for tax reduction in other markets and thereby reduce the dead weight loss in the markets. Compensation of government deficit is an important motive for privatization (Vickers and Yarrow, 1988), and it is argued that putting appropriate welfare weights on consumer surplus, firms' profits, and government revenues is important in cost–benefit analysis of privatization (Jones *et al.*, 1990). Thus, models that neglect the shadow cost of public funding fail to incorporate this important factor.

Recently, some studies have incorporated the shadow cost of public funds into mixed oligopolies. Capuano and De Feo (2010) showed that simultaneous-move outcome appears in the observable delay game formulated by Hamilton and Slutsky (1990) when the shadow cost is high. Matsumura and Tomaru (2013) showed that the privatization neutrality theorem shown by White (1996) does not hold unless the shadow cost is zero. Matsumura and Tomaru (2015) showed that locations of both public and private firms are distorted unless the shadow cost is zero. Xu *et al.* (2016) showed that privatization might play the role of a commitment device to agree to FTA and improve both domestic and global welfare. However, none of these studies has investigated the relationship between the optimal privatization policy (the optimal degree of privatization) and the shadow cost.

In this study, we investigate the relationship between the optimal degree of privatization and the shadow cost of public funds. We show that the relationship is non-monotone, and the results crucially depend on whether the shadow cost of public funds is larger than one.[3]

First, we investigate a free-entry market. Owing to recent deregulation and liberalization, entry restrictions in mixed oligopolies have significantly weakened (Lee *et al.* 2017). As a result, private enterprises have newly entered many mixed oligopolies, such as the banking, insurance, telecommunications, and transportation industries. The literature on mixed oligopolies has intensively investigated the optimal privatization policy in free-entry markets, as mentioned above. We find that as long as the shadow cost is lower than one, the optimal degree of privatization can be zero (i.e., full nationalization can be optimal) and it is more likely when the shadow cost is higher. When the solution is interior (partial privatization is optimal), the optimal degree of privatization decreases with the shadow cost. An increase in the shadow cost makes the public firm less aggressive, because the weight of its own profit becomes larger. This less aggressive behavior makes the private firms more aggressive through the strategic interaction between public and private firms, which results in a lower profit of the public firm. In order to keep the public firm aggressive, the government decreases the degree of privatization.

However, when the shadow cost is higher than one, the results change drastically. Full nationalization is never a unique optimal solution, and the optimal degree of privatization can be discontinuously increasing with the shadow cost. As explained above, an increase in the shadow cost makes the public firm less aggressive and the government should keep the public firm aggressive by adjusting the privatization policy. When the shadow cost is larger than one and the degree of privatization is small, an increase in the degree of privatization makes the public firm more aggressive, in contrast to the case in which the shadow cost is lower than one. Therefore, the government should increase the degree of privatization to keep the public firm aggressive. This is why the results depend on whether the shadow cost is larger than one.

Our results have important policy implications. The shadow cost of public funds is usually higher

[3] The shadow cost of public funds is quite popular in many fields of economics. Meade (1944) undertook a pioneering work, which was developed in Laffont and Tirole (1986). According to Laffont (2005), λ is estimated to be around 0.3 in developed countries and more than 1 in developing countries. In addition, recent analysis has suggested that it can be higher than one even in developed countries. Furthermore, as Kleven and Kreiner (2006) showed, the cost of public funds would be larger if endogenous labor force participation by workers were considered. This suggests that the shadow costs of public funds in both developed and developing countries could be larger than the past estimates, which ignore this effect. Thus, it is important to discuss the cases in which λ is larger and smaller than one.

when the government deficit is larger, because the deadweight loss of taxation is proportional to the square of the tax rate whereas tax revenue is less than proportional to the tax rate.[4] Therefore, our results suggest that as long as the government deficit is moderate (and thus, the shadow cost is smaller than one), the more deficit the government has, and the less the government should privatize public firms. However, if the government deficit is quite serious (and thus, the shadow cost is larger than one), this does not hold. When the shadow cost is larger than one, the government should sell substantial shares in public firms.

Next, we investigate a non-free-entry market. Although the results are less clear than those with free-entry markets, we find some important results. We find that when the shadow cost is lower than one, full nationalization can be the unique optimal privatization policy, whereas it is not when the cost is higher than one. In addition, we find that full nationalization is much less likely to be optimal when the shadow cost is larger than one.

The rest of the paper is organized as follows. In Section 2, we present a model in a free-entry market. Section 3 presents the equilibrium analysis and main results. Section 4 discusses a non-free-entry case. Section 5 concludes.

2 Model

Consider a market in which one state-owned public firm, firm 0, competes against n private firms.[5] Firms produce perfectly substitutable commodities for which the inverse demand function is denoted by $p(Q)$, where p is the price and Q is the total output. We assume that p is twice continuously differentiable and $p' < 0$ as long as $p > 0$. Firm 0's cost function is $c_0(q_0) + K_0$ where q_0 is the output of firm 0. Each private firm i $(= 1, \ldots, n)$ has an identical cost function, $c(q_i) + K$, where q_i is the

[4] See Browning (1976) and papers cited therein.

[5] Our result holds even when multiple public firms exist. For a discussion on multiple public firms, see Matsumura and Shimizu (2010), Matsumura and Okumura (2013), and Haraguchi and Matsumura (2016). However, we do not allow government nationalization of all firms (the whole industry). As pointed out by Merrill and Schneider (1966), the most efficient outcome is achieved by the nationalization of all firms in the case in which nationalization does not change firms' costs (i.e., there is no X-inefficiency in the public firm). The need for an analysis of mixed oligopoly lies in the fact that it is impossible or undesirable, for political or economic reasons, to nationalize an entire sector. For example, without competitors, public firms might lose the incentive to improve their costs, resulting in a loss of social welfare. Thus, we neglect the possibility of nationalizing all firms.

output of firm i, $c(q_i)$ is the production cost, and K is the entry cost.[6] We assume that c_0 and c are twice continuously differentiable. To ensure an interior solution, we further assume that $c_0', c' \geq 0$, $c_0'', c'' > 0$, $c_0'(0) = c'(0) = 0$, and $\lim_{q\to\infty} c_0'(q), c'(q) = \infty$.[7]

The profit of firm 0 is given by $\pi_0 = p(Q)q_0 - c_0(q_0) - K_0$, and that of firm i $(= 1, \ldots, n)$ is given by $\pi_i = p(Q)q_i - c(q_i) - K$. Domestic social welfare is defined as

$$W = \int_0^Q p(q)dq - p(Q)Q + \pi_0 + (1-\theta)\sum_{i=1}^{n} \pi_i + \lambda(D + R), \tag{1}$$

where $\lambda > 0$ is the shadow cost of public funds, D is the revenue from firm 0's dividends, R is the revenue from privatization, and θ is the foreign ownership share in private firms.[8] Private firms are foreign (domestic) when $\theta = 1$ $(\theta = 0)$. The shadow cost of public funds is the deadweight loss from collecting a unit of tax (excess burden of taxation). Thus, a unit of government revenue from firm 0 yields λ welfare gain, because it saves an excess burden of taxation in the other markets.[9]

We assume that the financial market is perfect. In other words, the government sells its share in firm 0 at the fundamental value of the firm. The fundamental value of firm 0, V, equals π_0. Therefore, at the beginning of the game, the government obtains $R = \alpha V$ if it sells α shares of firm 0. In addition,

[6]In this study, we allow a cost difference between public and private firms, although we do not allow a cost difference among private firms. While some readers might consider the public firm to be less efficient than the private firm, not all empirical studies support this view. See Megginson and Netter (2001) and Stiglitz (1988). In addition, Martin and Parker (1997) suggested that corporate performance can either increase or decrease after privatization, based on their study in the United Kingdom. See Matsumura and Matsushima (2004) for a theoretical discussion of the endogenous cost differences between public and private enterprises.

[7]In the literature on mixed oligopolies, the model with quadratic production costs is popular and satisfies these assumptions (De Fraja and Delbono, 1989; Matsumura and Shimizu, 2010). However, another popular model in the literature, the model with constant marginal costs and cost disadvantage of the public firm, does not satisfy these assumptions (Pal, 1998; Matsumura, 2003a). The model with constant marginal costs yields a problem in free-entry markets. For example, suppose that $\theta = 0$. As discussed in Matsumura and Kanda (2005), when the marginal cost of firm 0 is constant, firm 0's production level is zero if $c_0' > p(Q^*)$ and the number of entering firms is zero if $c_0' < p(Q^*)$. Therefore, mixed oligopolies do not appear (either a public monopoly or a private oligopoly appears). To avoid this technical problem, most studies on mixed oligopolies analyzing free-entry markets of homogeneous products assume increasing marginal costs. Therefore, increasing marginal costs are crucial in our analysis of a free-entry market. However, we can drop the assumption of increasing marginal costs in the analysis of a non-free-entry market discussed in Section 4.

[8]For discussions on the nationality of private enterprises in mixed oligopolies, see the literature starting with Corneo and Jeanne (1994) and Fjell and Pal (1996). See also Pal and White (1998) and Bárcena-Ruiz and Garzón (2005a,b), and Xu *et al.* (2016). We assume that the public firm, firm 0, is domestic. For the effect of foreign ownership in semi-public firm on the optimal privatization policy, see Lin and Matsumura (2012).

[9]See Matsumura and Tomaru (2013). Introducing the shadow cost of public funding is popular in many contexts. See studies mentioned in footnote 3.

at the end of the game, the government obtains $D = (1-\alpha)\pi_0$.

Let $(1-\alpha)$ denote the government's ownership share in firm 0. Following the standard formulation in the literature on mixed oligopolies, we assume that firm 0 maximizes the weighted average of social welfare and its own profit, whereas private firms maximize their own profits (Matsumura, 1998). Firm 0 maximizes $(1-\alpha)W + \alpha\pi_0$ ($\alpha \in [0,1]$). In the case of full nationalization ($\alpha = 0$), firm 0 maximizes social welfare. In the case of full privatization ($\alpha = 1$), firm 0 maximizes its own profit.

The three-stage game runs as follows. In the first stage, the government chooses the degree of privatization, α. In the second stage, each private firm chooses whether to enter the market. In the third stage, firms entering the market compete in quantities. We use the subgame perfect Nash equilibrium as the equilibrium concept. Throughout this study, we restrict our attention to the case in which K is small such that the number of entering private firms, n, is larger than one.

3 Equilibrium

We solve the game by backward induction. We discuss the competition in the third stage. Note that the government has already sold firm 0's share. Therefore, when firm 0 chooses q_0, R was given exogenously. Firm 0 maximizes $(1-\alpha)W + \alpha\pi_0$. The first-order condition of firm 0 is

$$(1+(1-\alpha)^2\lambda)p + (1-(1-\alpha)(1-\theta)+(1-\alpha)^2\lambda)p'q_0 - (1+(1-\alpha)^2\lambda)c_0'(q_0) - (1-\alpha)\theta p'Q = 0. \quad (2)$$

The first-order conditions of the private firms are

$$p + p'q_i - c'(q_i) = 0 \text{ for } i = 1, \ldots, n. \quad (3)$$

We assume that the second-order conditions,

$$(1+(1-\alpha)^2\lambda)(p''q_0 + 2p' - c_0'') + (1-\alpha)(-p' - \theta p''Q - (1-\theta)p''q_0) < 0 \quad (4)$$

and

$$2p' + p''q_i - c'' < 0, \quad (5)$$

are satisfied. A sufficient but not necessary condition is that c_0'' and c'' are sufficiently large. We also assume

$$p' + p''q_i < 0. \tag{6}$$

This implies that the strategies of private firms in the quantity competition stage are strategic substitutes.[10] A sufficient but not necessary condition is $p'' \leq 0$. These are standard assumptions in the literature.

Henceforth, we focus on the symmetric equilibrium wherein all private firms produce the same output level q (i.e., $q_i = q_j = q$ for all $i, j = 1, \ldots, n$). Solving equations (2) and (3) as well as the following equation (7) leads to the equilibrium outputs in the third stage, given α and n:

$$Q = q_0 + nq. \tag{7}$$

Let $q_0(\alpha, n, \lambda)$, $q(\alpha, n, \lambda)$, and $Q(\alpha, n, \lambda)$ be the equilibrium firm 0's output, each private firm's output, and the total output in the third stage subgame, respectively.

Lemma 1 *Suppose that $\lambda \leq 1$. (i) $q_0(\alpha, n, \lambda)$ and $Q(\alpha, n, \lambda)$ are decreasing in α, (ii) $q(\alpha, n, \lambda)$ is increasing in α.*

Proof See the Appendix.

Lemma 2 *Suppose that $\alpha \neq 1$. (i) $q_0(\alpha, n, \lambda)$ and $Q(\alpha, n, \lambda)$ are decreasing in λ, (ii) $q(\alpha, n, \lambda)$ is increasing in λ.*

Proof See the Appendix.

Lemma 3 *Suppose that $\lambda > 1$. (i) $q_0(\alpha, n, \lambda)$ and $Q(\alpha, n, \lambda)$ are increasing in α for $\alpha \in [0, 1 - 1/\sqrt{\lambda})$ and decreasing in α for $\alpha \in (1 - 1/\sqrt{\lambda}, 1]$. (ii) $q(\alpha, n, \lambda)$ is decreasing in α for $\alpha \in [0, 1 - 1/\sqrt{\lambda})$ and increasing in α for $\alpha \in (1 - 1/\sqrt{\lambda}, 1]$.*

Proof See the Appendix

Lemma 1 is known in the literature when λ is zero (Matsumura, 1998). A decrease in α increases

[10] We do not assume that the strategy of the public firm is a strategic substitute because the public firm can be a strategic complement under plausible assumptions when private firms are foreign. See Matsumura (2003b).

the weights of consumer surplus and of the gain of reducing excess burden of taxation.

When λ is small, the former (consumer surplus) effect dominates the latter (excess burden of taxation) effect, and a decrease in α makes firm 0 more aggressive. Because the strategies of the private firms are strategic substitutes, this decreases the output of each private firm through the strategic interaction. The direct effect (the effect on firm 0's output) dominates the indirect effect (the effect thorough strategic interaction), and thus, the total output is decreasing in α (Lemma 2).

We now explain the intuition behind Lemma 3. When λ is larger and α is smaller, firm 0 is more concerned with the gain of reducing excess burden of taxation. When λ is large and α is small, a decrease in α makes the public firm more concerned with its own profit to reduce the excess burden of taxation, and thus, a decrease in α makes the public firm less aggressive. Therefore, the output of firm 0 can be increasing in α, which is in sharp contrast to the result of Matsumura (1998). Because of the strategic interaction, this increases the output of each private firm. Again, the direct effect (the effect on firm 0's output) dominates the indirect effect (the effect thorough strategic interaction), and thus, the total output can be increasing in α.

However, when α is large, an increase in firm 0's profit is less likely to reduce the excess burden of taxation because R is given exogenously at this stage and the gain is limited to $\lambda(1-\alpha)\pi_0$. Thus, a marginal decrease in α makes firm 0 more aggressive, and similar results to Lemma 1 are obtained.

In the second stage, private firms enter the market as long as profit is positive, which yields the free-entry condition

$$p(Q)q - c(q) - K = 0. \tag{8}$$

Let the superscript "L" be the equilibrium outcome of the free-entry equilibrium (long-run equilibrium), $q^L(\alpha), q_0^L(\alpha), Q^L(\alpha), n^L(\alpha)$ be the output of individual private firms, the output of the public firm, the total output, and the number of private firms, which satisfy the equations (2), (3), (7), and (8), respectively.

Lemma 4 *(i) q^L and Q^L are independent of α, λ, and θ. (ii) Suppose that $\lambda \leq 1$. q_0^L is decreasing in α. (iii) Suppose that $\lambda > 1$. q_0^L is increasing in α for $\alpha \in [0, 1-1/\sqrt{\lambda})$ and decreasing in α for*

$\alpha \in (1 - 1/\sqrt{\lambda}, 1]$. *(iv)* q_0^L *is increasing in* θ *and decreasing in* λ. *(v) If* q_0^L *is increasing (decreasing) with a parameter, then* n^L *is decreasing (increasing) with this parameter.*

Proof See the Appendix.

Lemma 4(i) is a familiar result in free-entry models (Matsumura and Kanda, 2005; Cato and Matsumura, 2012) . Lemmas 4(ii)–(iv) are similar to Lemmas 1–3.

Using Lemma 4, we analyze the optimal privatization policy for the government. Consider the first stage. Let $W^L(\alpha)$ be domestic welfare. The government maximizes $W^L(\alpha)$ with respect to α. From Lemma 4(i), we find that the price (and thus, the consumer surplus) is independent of α. In a free-entry market, the private firms' profits are zero. Therefore, W^L is maximized when π_0 is maximized. Because the price is independent of α, π_0 is maximized when the price is equal to its marginal cost. This yields the following proposition.

Proposition 1 *Domestic welfare is increasing (decreasing) in* $p - c_0'$ *if* $p - c_0' < 0$ $(p - c_0' > 0)$.

The derivative of W with respect to α is

$$\begin{aligned}
\frac{dW}{d\alpha} &= \frac{dQ^L}{d\alpha}\left(-p'(Q^L)Q^L + (1+\lambda)p'(Q^L)q_0^L + (1-\theta)np'(Q^L)q\right) + \frac{dq_0^L}{d\alpha}(1+\lambda)(p(Q^L) - c_0'(q_0^L)) \\
&\quad + \frac{dq^L}{d\alpha}(1-\theta)n(p(Q^L) - c'(q^L)) + \frac{dn^L}{d\alpha}(1-\theta)\left(p(Q^L)q^L - c(q^L) - K\right) \qquad (9) \\
&= \frac{dq_0^L}{d\alpha}(1+\lambda)(p(Q^L) - c_0'(q_0^L)), \qquad (10)
\end{aligned}$$

where the first term and the third term in (9) are zero from Lemma 4(i), and the last term in (9) is zero from the free-entry condition. From this equation, we can observe Proposition 1.

We now present the results on the optimal degree of privatization. We find that the properties of optimal degree of privatization drastically change depending on whether $\lambda \leq 1$ or $\lambda > 1$. First, we discuss the case in which the shadow cost of public funds is moderate (i.e., $\lambda \leq 1$). Let α^L be the optimal degree of privatization.

Proposition 2 *Suppose that* $\lambda \leq 1$. *(i)* $\alpha^L < 1$ *for any* $\lambda \in [0, 1]$. *(ii) There exists* $\theta_a(\lambda) \in [0, 1]$ *such that* $\alpha^L = 0$ *for* $\theta \leq \theta_a(\lambda)$. *(iii)* $\theta_a(\lambda)$ *is increasing in* λ. *(iv) If* $\theta > \theta_a(\lambda)$, α^L (> 0) *is decreasing in*

λ.

Proof See the Appendix.

Proposition 2(i) states that full privatization is not optimal for any λ. Propositions 2(ii)–(iii) state that full nationalization is more likely optimal when λ is larger. Proposition 2(iv) states that when the solution is interior, the optimal degree of privatization is decreasing in λ. Overall, Propositions 2(i)–(iv) suggest that the government should choose a lesser degree of privatization when λ is larger as long as $\lambda \leq 1$.

This result might be counterintuitive. Usually, the larger the government's deficit is, the higher is the marginal cost of public funds, because the deadweight loss of taxation might be proportional to the square of tax rate and a large deficit requires a higher tax rate. Therefore, Proposition 2 suggests that the greater is the government's deficit, the more the government should hold a share in the public firm. An increase in λ makes firm 0 less aggressive, because it increases the weight of its own profit in the objective of firm 0. However, this less aggressive behavior induces the additional entry of the private firms and reduces firm 0's resulting profit. To offset this effect (to keep the public firm choosing the marginal cost pricing), the government chooses a smaller degree of privatization when λ is larger.

However, this result depends on the property that q_0 is decreasing in α. This holds when $\lambda \leq 1$ but does not always hold when $\lambda > 1$. This yields a different result. The following proposition presents a relationship between α^L and λ when $\lambda > 1$.

Proposition 3 *Suppose that $\lambda > 1$. (i) $\alpha^L < 1$ for any λ. (ii) $\alpha = 0$ and $\alpha = 1 - 1/\lambda$ yields the same outcomes, q_0, q, Q and n. (iii) If $\alpha^L = 0$ for $\lambda = \lambda'$, then $\alpha^L > 0$ for any $\lambda \neq \lambda'$. (iv) There exists $\theta_b > 0$ such that $\alpha^L = 1 - 1/\sqrt{\lambda}$ for $\theta < \theta_b$. (v) There exists $\theta_c \in (\theta_b, 1)$ such that there are two optimal degrees of privatization for $\theta \in [\theta_b, \theta_c]$. One lies on $[0, 1 - 1/\sqrt{\lambda}]$ and the other on $[1 - 1/\sqrt{\lambda}, 1 - 1/\lambda]$. The former is increasing in λ and the latter is decreasing in λ. (vi) $\alpha^L > 1 - 1/\lambda$ if $\theta > \theta_c$.*

Proof See the Appendix.

Proposition 3(iii) states that full nationalization is optimal only in the single value of λ. Moreover, Proposition 3(ii) implies that full nationalization is never the unique solution, and $\alpha = 1 - 1/\lambda$ is another solution if $\alpha^L = 0$ is a solution. These results are in sharp contrast to Propositions 2(ii)–(iii), which states that full nationalization is optimal for a wide range of parameters, and this range is wider when λ is larger, as long as $\lambda \leq 1$.

We explain the intuition. When $\lambda \leq 1$, the output of firm 0 is decreasing in α. If q_0 is too small for domestic social welfare when $\alpha = 0$, an increase in α is harmful for welfare. In such cases, $\alpha^L = 0$. However, when $\lambda > 1$, the output of firm 0 is increasing in α for $\alpha \in [0, 1 - 1/\sqrt{\lambda}]$. Thus, a marginal increase in α from zero always improves domestic welfare. If q_0 is too large for domestic welfare when $\alpha = 0$, a marginal increase in α from zero reduces domestic welfare. However, $\alpha = 0$ and $\alpha = 1 - 1/\lambda$ yields the same outcomes, and the output of firm 0 is decreasing in α for $\alpha \geq 1 - 1/\lambda$. Thus, a marginal increase in α from $\alpha = 1 - 1/\lambda$ improves welfare. Therefore, $\alpha^L \neq 0$ even if q_0 is too large for domestic welfare under full nationalization. Under these conditions, $\alpha^L = 0$ only if q_0 is neither too large nor too small for domestic welfare under full nationalization, which holds for measure zero events.

4 Non-Free-Entry Case

In this section, we investigate the model in which the number of firms is given exogenously. For simplicity, we consider the case with $n = 1$. We slightly relax the assumption of the cost functions. We assume that c_0 and c are twice continuously differentiable. To ensure an interior solution, we further assume that $c_0', c' \geq 0$, $c_0'', c'' \geq 0$, $c_0'(0) \leq P^M$, and $c'(0) \leq P_0^M$ where P^M (P_0^M) is the monopoly price of the private firm. The assumptions on the demand function are common with the free-entry case. We again assume that the solution at the quantity competition stage is interior.

The game runs as follows. In the first stage, the government chooses α. In the second stage, firms 0 and 1 face Cournot competition. The first-order conditions for firms 0 and 1 are common with those in Section 3.

Let the superscript ‘S’ be the equilibrium outcome of this game (short-run equilibrium). Let $q^S(\alpha), q_0^S(\alpha)$, and $Q^S(\alpha)$ be the output of the private firm, the output of the public firm, and the total output, respectively, given α.

Let α^S be the equilibrium α in this game. The first-order condition for the short-run optimal degree of privatization α^S is

$$\begin{aligned}\frac{dW^S}{d\alpha} &= \frac{\partial q_0}{\partial \alpha}(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)p'q_1) \\ &\quad + \frac{\partial q_1}{\partial \alpha}(-p'Q + (1+\lambda)p'q_0 + (1-\theta)(p + p'q_1 - c')) = 0 \qquad (11)\end{aligned}$$

Let $R_1(q_0)$ be the reaction function of firm 1. Define

$$\begin{aligned}W^R(q_0) &:= \int_0^{q_0+R_1(q_0)} p(Q)dQ - p(q_0 + R_1(q_0))(q_0 + R_1(q_0)) \\ &\quad +(1+\lambda)(p(q_0 + R_1(q_0))q_0 c_0(q_0)) + (1-\theta)(p(q_0 + R_1(q_0))q_1 c(q_1)).\end{aligned}$$

We do not discuss the Stackelberg model with public leadership but discuss the Cournot model. However, this exposition is useful for subsequent analysis because the government can control q_0 by choosing α and can indirectly control q_1 through strategic interaction between two firms. We assume that $W^R(q_0)$ is concave.

Proposition 4 *If $\lambda \in (0,1]$, $\alpha^S = 0$ if and only if $q_1^S(0) \leq \lambda q_0^S(0)$.*

Proof See the Appendix.

We now specify the demand and cost functions and present a result suggesting that $\alpha^S = 0$ for a wide range of parameters.[11]

Proposition 5 *Suppose that $p(Q) = a - Q$, $c_0(q_0)k_0 q_0^2/2$, and $c_1(q_1) = k_1 q_1^2/2$, where k_0 and k_1 are positive constants. (i) $\alpha^S = 0$ if and only if*

$$g(\theta, \lambda, k_0, k_1) := \lambda((1+\lambda)(1+k_1) + \theta) - \frac{(1+\lambda)(2+k_0)(1+k_1)}{2+k_1} \geq 0.$$

(ii) $g(\theta, \lambda, k_0, k_1)$ is increasing in θ and decreasing in k_0. (iii) $g(\theta, \lambda, k_0, k_1)$ is increasing in λ if and only if $k_0 \leq k_1 + (2+k_1)(2\lambda + \theta/(1+k_1))$.

Proof See the Appendix.

[11] The linear demand and quadratic cost functions used in Propositions 5 and 7 are popular in the literature on mixed oligopolies. See De Fraja and Delbono (1989) and Matsumura and Shimizu (2010). The assumption of concavity of $W^R(q_0)$ is satisfied under linear demand and quadratic cost functions.

Proposition 5(ii) states that full nationalization is more likely optimal when the foreign ownership share in private firms is larger and the public firm's cost condition is better. Proposition 5(iii) states that full nationalization is more likely to take place when λ is high unless the public firm is highly less efficient than the private firm.

Figure 1 describes the region where full nationalization ($\alpha = 0$) is optimal for $\lambda \leq 1$, with $k_0 = 2/15$ and $k_1 = 1/15$. Figure 1 indicates that full nationalization is optimal for a wide range of parameters.[12]

We now discuss the case with $\lambda > 1$.

Proposition 6 *Suppose that $\lambda > 1$. (i) $\alpha = 0$ is an optimal privatization policy, then $\alpha = 1 - 1/\lambda$ is another optimal privatization policy. (ii) $\alpha = 0$ is an optimal privatization policy if and only if $q_1^S(0) = \lambda q_0^S(0)$. (iii) If $c_0 = c$ (i.e., both firms share a common cost function), then $\alpha = 0$ is not an optimal privatization policy.*

Proof See the Appendix.

If λ exceeds one, full nationalization is never the unique optimal policy (Proposition 6(i)), and is rarely one of the optimal policies (Proposition 6(ii)). Proposition 6(ii) is in sharp contrast to Proposition 4. When λ is less than one, $\alpha^S = 0$ if and only if $q_1^S(0) \leq \lambda q_0^S(0)$, and it might hold for a wide range of parameters, as suggested by Proposition 5 and Figure 1. When λ exceeds one, $\alpha^S = 0$ if and only if $q_1^S(0) = \lambda q_0^S(0)$, which is a much stricter condition.

We explain the intuition behind these results. When λ is less than one, the output of firm 0 is maximized when $\alpha = 0$. In other words, the government can reduce but cannot increase the output of firm 0 by increasing α. Thus, the government chooses $\alpha = 0$ if $q_0^S(0)$ is optimal or is too small for welfare. When λ exceeds one, the output of firm 0 is neither maximized nor minimized when $\alpha = 0$. In other words, the government can reduce as well as increase the output of firm 0 by controlling α. Thus, the government chooses $\alpha = 0$ only if $q_0^S(0)$ happens to be optimal. Therefore, the government rarely chooses $\alpha = 0$ when λ exceeds one.

These results suggest that at least partial privatization should be undertaken if the shadow cost

[12] If we set $k_0 = k_1 = 1/15$, the range is further wider.

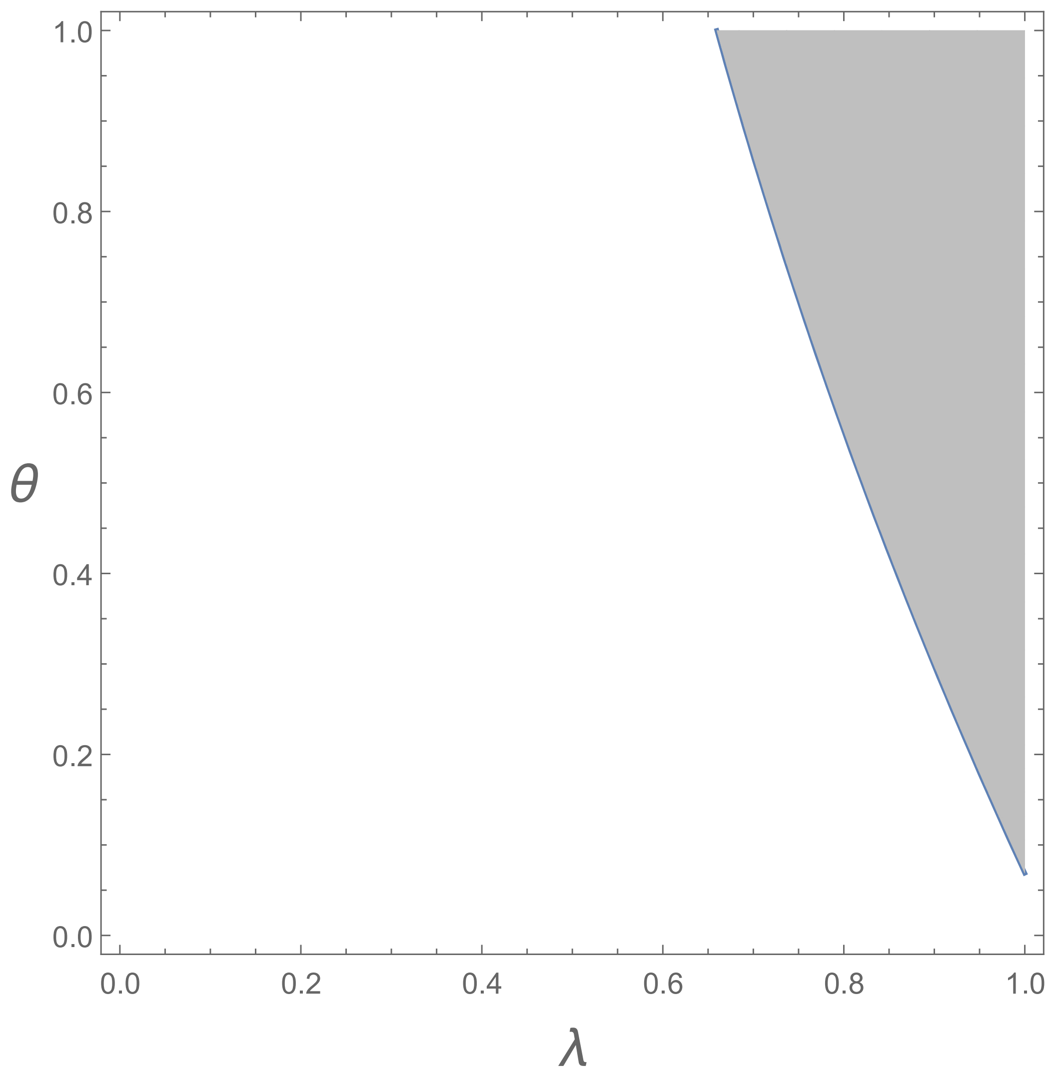

Figure 1: The region where full nationalization is optimal for $\lambda \leq 1$.

of public funding is high, while full nationalization can be a reasonable option when the shadow cost is moderate.

Finally, we again specify the demand and cost functions and present a result suggesting that full nationalization is optimal at best for measure zero events when λ exceeds one.

Proposition 7 *Suppose that* $p(Q) = a - Q$*,* $c_0(q_0) = k_0 q_0^2/2$*, and* $c_1(q_1) = k_1 q_1^2/2$*, where* k_0 *and* k_1 *are positive constants.* $\alpha^S = 0$ *if and only if* $g(\theta, \lambda, k_0, k_1) = 0$*.*

Proof See the Appendix.

Figure 2 describes the region where full nationalization ($\alpha = 0$) is optimal for $\lambda > 1$, with $k_0 = 2/15$ and $k_1 = 1/15$. Figure 2 indicates that full nationalization rarely occurs, at best for measure zero events.[13]

5 Concluding Remarks

In this study, we investigate the relationship between the optimal degree of privatization and the shadow cost of public funds. We find that when the shadow cost of public funds is moderate, full nationalization is optimal for a wide range of parameters, and the range is wider when the cost is higher. In addition, we find that when partial privatization is optimal, the optimal degree of privatization is decreasing in the shadow cost of public funds. When the cost is significant, however, these properties do not hold. Full nationalization is optimal except for measure zero events, and the optimal degree of privatization can be increasing in the shadow cost of public funds if it exceeds one. These results suggest that the society with high public fund costs should privatize firms more.

The shadow cost of public funding might affect many choices of public firms, such as R&D, product positioning, and environmental activities. Incorporating these activities into the discussion with the shadow cost of public funding and investigating the optimal privatization policy remains for future research.

[13] If we set $k_0 = k_1 = 1/15$, full nationalization is never optimal.

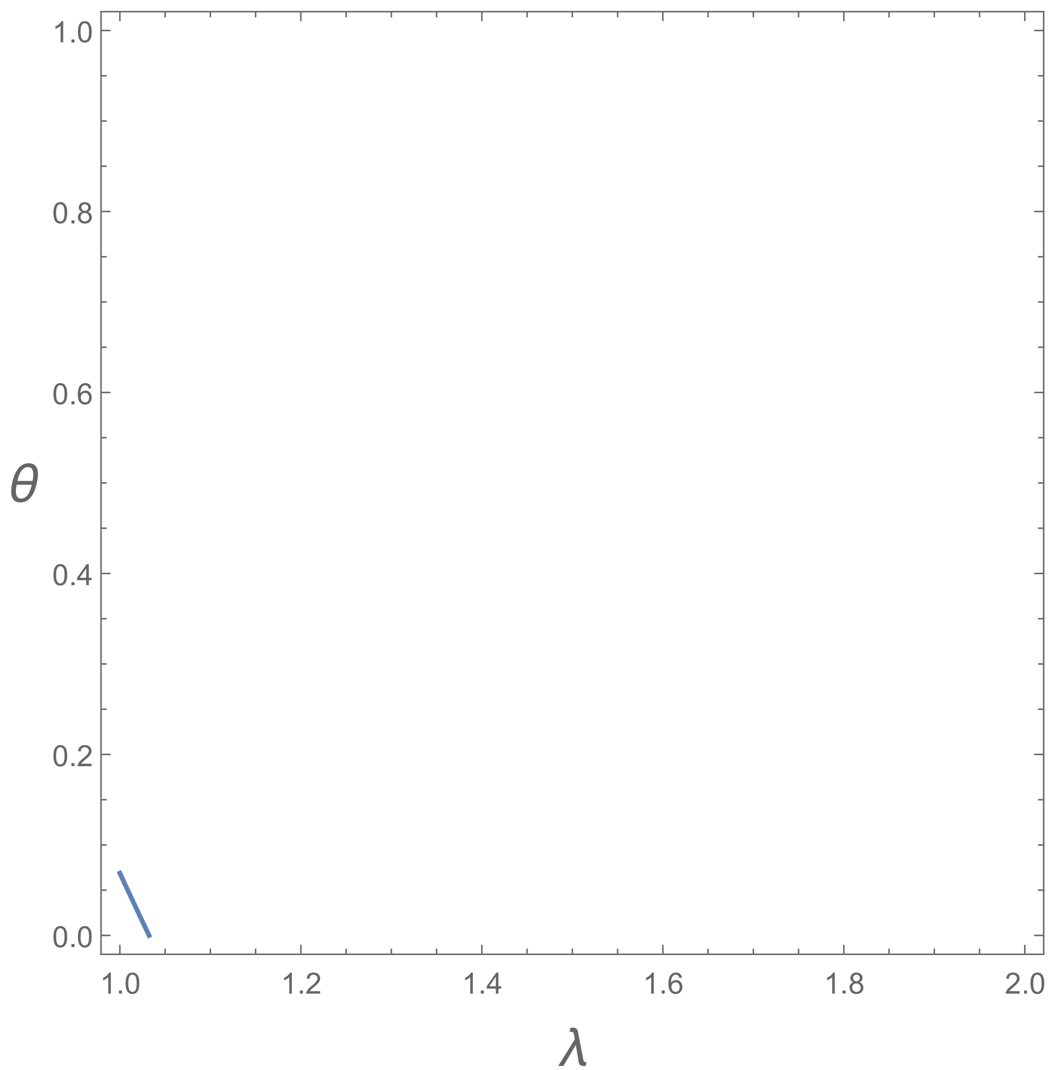

Figure 2: The region where full nationalization is optimal for $\lambda > 1$.

Appendix

In the following proofs, we suppress the arguments of the functions.

Proofs of Lemmas 1–3

Let

$$H := \begin{pmatrix} A & 0 & B \\ 0 & p' - c'' & p' + p''q \\ -1 & -n & 1 \end{pmatrix},$$

where $A := (1 - (1-\alpha)(1-\theta) + (1-\alpha)^2\lambda)p' - (1 + (1-\alpha)^2\lambda)c_0'' < 0$ and $B := (1 + (1-\alpha)^2\lambda)(p''q_0 + p') + (1-\alpha)(-p' - \theta p''Q - (1-\theta)p''q_0)$. Let

$$\begin{aligned} f_1 &:= (1 + (1-\alpha)^2\lambda)p(Q) + (1 - (1-\alpha)(1-\theta) + (1-\alpha)^2\lambda)p'(Q)q_0 \\ &\quad -(1 + (1-\alpha)^2\lambda)c_0'(q_0) - (1-\alpha)\theta p'(Q)Q \\ f_2 &:= p(Q) + p'(Q)q - c'(q) \\ f_3 &:= Q - nq - q_0. \end{aligned}$$

By differentiating (2), (3), and (7) with respect to $x \in \{n, \alpha, \lambda\}$, we obtain

$$H \begin{pmatrix} dq_0 \\ dq_1 \\ dQ \end{pmatrix} = -\begin{pmatrix} \partial f_1/\partial x \\ \partial f_2/\partial x \\ \partial f_3/\partial x \end{pmatrix} dx. \tag{12}$$

We obtain

$$\begin{pmatrix} \partial f_1/\partial \alpha \\ \partial f_2/\partial \alpha \\ \partial f_3/\partial \alpha \end{pmatrix} = \begin{pmatrix} \frac{(1-\alpha)^2\lambda - 1}{1+(1-\alpha)^2\lambda} p'(-\theta Q - (1-\theta)q_0) \\ 0 \\ 0 \end{pmatrix},$$

$$\begin{pmatrix} \partial f_1/\partial \lambda \\ \partial f_2/\partial \lambda \\ \partial f_3/\partial \lambda \end{pmatrix} = \begin{pmatrix} (1-\alpha)^2(p + p'q_0 - c_0') \\ 0 \\ 0 \end{pmatrix}.$$

By applying Cramer's rule to (12), we obtain

$$\frac{dq_0}{d\alpha} = -\frac{\frac{(1-\alpha)^2\lambda - 1}{1+(1-\alpha)^2\lambda} p'(-\theta Q - (1-\theta)q_0)(p' - c'' + n(p' + p''q))}{|H|}, \tag{13}$$

$$\frac{dq}{d\alpha} = \frac{\frac{(1-\alpha)^2\lambda - 1}{1+(1-\alpha)^2\lambda} p'(-\theta Q - (1-\theta)q_0)(p' + p''q)}{|H|}, \tag{14}$$

$$\frac{dQ}{d\alpha} = -\frac{\frac{(1-\alpha)^2\lambda - 1}{1+(1-\alpha)^2\lambda} p'(-\theta Q - (1-\theta)q_0)(p' - c'')}{|H|}, \tag{15}$$

$$\frac{dq_0}{d\lambda} = -\frac{(1-\alpha)^2(p+p'q_0-c_0')(p'-c''+n(p'+p''q))}{|H|}, \tag{16}$$

$$\frac{dq}{d\lambda} = \frac{(1-\alpha)^2(p+p'q_0-c_0')(p'+p''q)}{|H|}, \tag{17}$$

$$\frac{dQ}{d\lambda} = -\frac{(1-\alpha)^2(p+p'q_0-c_0')(p'-c'')}{|H|}, \tag{18}$$

where $|H| = (p'-c'')(A+B)+nA(p'+p''q)$.

From the second-order condition of q_0, we obtain $A+B<0$. From (6), we obtain $(p'+p''q)<0$. Thus, $|H|>0$.

Under these conditions, (13) and (15) are negative (positive), and (14) is positive (negative) if $(1-\alpha)^2\lambda-1<\ (>)\ 0$. If $\lambda<1$, $(1-\alpha)^2\lambda-1<0$. This implies Lemma 1. Suppose that $\lambda>1$. Then $(1-\alpha)^2\lambda-1<\ (>)\ 0$ if $\alpha>1-1/\sqrt{\lambda}]$ $(\alpha\in[0,1-1/\sqrt{\lambda}])$. This implies Lemma 3.

Finally, (16) and (18) are positive and (17) is negative. These imply Lemma 2 Q.E.D.

Proof of Lemma 4

The following two equations determine the values of q^L and Q^L,

$$p(Q^L)+p'(Q^L)q^L-c'(q^L)=0,$$

$$p(Q^L)q^L-c(q^L)-K=0.$$

These are independent of α, λ, and θ. This implies Lemma 4(i).

The values q_0^L and n^L are determined by the equations

$$(1+(1-\alpha)^2\lambda)p(Q^L)+(1-(1-\alpha)(1-\theta)+(1-\alpha)^2\lambda)p'(Q^L)q_0^L-(1+(1-\alpha)^2\lambda)c_0'(q_0^L)-(1-\alpha)\theta p'(Q^L)Q^L=0$$

and

$$Q^L-nq^L-q_0=0.$$

The comparative statics with respect to α is

$$\begin{pmatrix} (1-(1-\alpha)(1-\theta)+(1-\alpha)^2\lambda)p'(Q^L)-(1+(1-\alpha)^2\lambda)c_0''(q_0^L) & 0 \\ -1 & -q^L \end{pmatrix}\begin{pmatrix} \frac{dq_0^L}{d\alpha} \\ \frac{dn^L}{d\alpha} \end{pmatrix}$$
$$= -\begin{pmatrix} \frac{(1-\alpha)^2\lambda-1}{1+(1-\alpha)^2\lambda}p'(-\theta Q^L-(1-\theta)q_0^L) \\ 0 \end{pmatrix}.$$

Then, we obtain

$$\frac{dq_0^L}{d\alpha} = \frac{-\left(\frac{(1-\alpha)^2\lambda-1}{1+(1-\alpha)^2\lambda}p'(-\theta Q^L-(1-\theta)q_0^L)\right)}{(1-(1-\alpha)(1-\theta)+(1-\alpha)^2\lambda)p'(Q^L)-(1+(1-\alpha)^2\lambda)c_0''(q_0^L)},$$

which is negative (non-negative) if $(1-\alpha)^2\lambda-1<0$ ($(1-\alpha)^2\lambda-1\geq 0$). This implies Lemma 4(ii) and Lemma 4(iii).

Similarly, the comparative statics with respect to λ and θ is

$$\frac{dq_0^L}{d\lambda} = \frac{-(1-\alpha)^2(p(Q^L)-p'(Q^L)q_0^L-c_0'(q_0^L)))}{(1-(1-\alpha)(1-\theta)+(1-\alpha)^2\lambda)p'(Q^L)-(1+(1-\alpha)^2\lambda)c_0''(q_0^L)}<0,$$

$$\frac{dq_0^L}{d\theta} = \frac{-p'(Q^L)(1-\alpha)(q_0+Q))}{(1-(1-\alpha)(1-\theta)+(1-\alpha)^2\lambda)p'(Q^L)-(1+(1-\alpha)^2\lambda)c_0''(q_0^L)}>0,$$

which implies Lemma 4(iv).

Finally, differentiating $Q^L-n^Lq^L-q_0^L=0$ with respect to any parameter $x\in\{\alpha,\theta,\lambda\}$, we obtain

$$\frac{dn^L}{dx}q^L = -\frac{dq_0^L}{dx},$$

which implies Lemma 4(v). Q.E.D.

Proof of Proposition 2

By substituting $\alpha=1$ into (2), we obtain

$$p-c=-p'q_0>0.$$

In addition, $\partial q_0^L/\partial\alpha<0$ at $\alpha=1$ by Lemma 3(i). Thus, we obtain

$$\left.\frac{dW}{d\alpha}\right|_{\alpha=1}<0.$$

which implies Proposition 2(i).

From Proposition 1, we obtain that α^L is determined by

$$(\partial q^L/\partial\alpha)(p(Q^L)-c_0'(q^L))=0$$

for the interior solution. We define $\theta_a(\lambda)$ as the value of θ, which satisfies the following equation:

$$\left[p(Q^L)-c_0'\right]_{\alpha=0}=0.$$

Then, for $\theta \leq \theta_a(\lambda)$, we obtain

$$\left[p(Q^L) - c_0'\right]_{\alpha=0} \geq 0$$

since q_0^L is increasing in θ and c_0 is convex. Thus, $\alpha = 0$ is optimal for $\theta \in [0, \theta_a(\lambda)]$, which implies Proposition 2(ii).

Moreover, we obtain

$$\frac{d\theta_a}{d\lambda} = -c_0'' \frac{\partial q_0^L}{\partial \lambda} \Big/ c_0'' \frac{\partial q_0^L}{\partial \theta} = -\frac{\partial q_0^L}{\partial \lambda} \Big/ \frac{\partial q_0^L}{\partial \theta} > 0,$$

which implies Proposition 2(iii).

Next, consider the case of the interior solution, that is, $\theta > \theta_a$. In this case, the optimal degree of privatization α^L is determined by the equation

$$\left[p(Q^L) - c_0'\right]_{\alpha=\alpha^L} = 0.$$

By differentiating this equation with respect to λ, we obtain

$$\frac{d\alpha^L}{d\lambda} = -c_0'' \frac{\partial q_0^L}{\partial \lambda} \Big/ c_0'' \frac{\partial q_0^L}{\partial \alpha} = -\frac{\partial q_0^L}{\partial \lambda} \Big/ \frac{\partial q_0^L}{\partial \alpha} < 0,$$

which implies Proposition 2(iv).

Proof of Proposition 3

The proof of Proposition 2(i) can be applied to Proposition 3(i).

Next, we show that $\alpha = 0$ and $\alpha = 1 - 1/\lambda$ yields the same outcomes. At $\alpha = 0$, (2) becomes

$$(1 + \lambda)(p + p'q_0 - c_0') - p'(\theta Q + (1 - \theta)q_0).$$

At $\alpha = 1 - 1/\lambda$, (2) becomes

$$(1 + 1/\lambda)(p + p'q_0 - c_0') - \frac{1}{\lambda}p'(\theta Q + (1 - \theta)q_0) = \frac{1}{\lambda}\Big((1 + \lambda)(p + p'q_0 - c_0') - p'(\theta Q + (1 - \theta)q_0)\Big) = 0.$$

Thus, $\alpha = 0$ and $\alpha = 1 - 1/\lambda$ give the same first-order condition to firm 0. Since the other equations are independent of α, $\alpha = 0$ and $\alpha = 1 - 1/\lambda$ yield the same outcomes.

We now consider λ such that $\alpha^L = 0$. Then, we must have

$$\left.\frac{dW}{d\alpha}\right|_{\alpha=0} = \left[\frac{dq_0^L}{d\alpha}(p - c_0')\right]_{\alpha=0} = 0.$$

Since $dq_0^L/d\alpha > 0$ at $\alpha = 0$, we must have $p - c_0' = 0$ at $\alpha = 0$. From (2), this requires

$$(1+\lambda)q_0^L - (\theta Q^L + (1-\theta)q_0^L) = 0$$

and $q_0^L = c_0^{'-1}(p(Q^L))$. Putting these together must yield

$$(\lambda + \theta)c^{'-1}(p(Q^L)) - \theta Q^L = 0.$$

Since the left-hand side of the above equation is monotone in λ, there is only one λ that satisfies the equation. Under these conditions, we obtain Proposition 3(iii).

We now show Proposition 3(iv). Consider θ_b such that

$$\left[p(Q^L) - c_0'(q_0^L)\right]_{\alpha=1-1/\sqrt{\lambda}} = 0.$$

For $\theta < \theta_b$, we obtain

$$\left[p(Q^L) - c_0'(q_0^L)\right]_{\alpha=1-1/\sqrt{\lambda}} > 0.$$

because q_0^L is decreasing in θ, which increases $p - c_0'$. This implies Proposition 3(iv).

We then show Proposition 3(v). Consider θ_c such that

$$\left[p(Q^L) - c_0'(q_0^L)\right]_{\alpha=0} = 0.$$

For $\theta \in (\theta_b, \theta_c)$, we obtain

$$\left[p(Q^L) - c_0'(q_0^L)\right]_{\alpha=0} > 0 \text{ and } \left[p(Q^L) - c_0'(q_0^L)\right]_{\alpha=1-1/\sqrt{\lambda}} < 0.$$

Thus, α^L lies on $[0, 1-1/\sqrt{\lambda}]$. In conjunction with (ii), we observe that there is another $\alpha \in [1-1/\sqrt{\lambda}, 1-1/\lambda]$ that achieves the same outcome with $\alpha \in [1-1/\sqrt{\lambda}]$. Thus, this is also an optimal degree of privatization. $d\alpha^L/d\lambda$ is given by

$$\frac{d\alpha^L}{d\lambda} = -c_0''\frac{\partial q_0^L}{\partial \lambda}\Big/ c_0''\frac{\partial q_0^L}{\partial \alpha} = -\frac{\partial q_0^L}{\partial \lambda}\Big/\frac{\partial q_0^L}{\partial \alpha},$$

which is negative (positive) if $\partial q_0^L/\partial\alpha < 0$ (> 0). This implies Proposition 3(v).

Finally, we show Proposition 3(vi). If $\theta > \theta_c$, we obtain

$$\frac{dW}{d\alpha}\bigg|_{\alpha=1-1/\lambda} = \left[\frac{dq_0^L}{d\alpha}(p - c_0')\right]_{\alpha=1-1/\lambda} > 0,$$

which implies that $\alpha^L > 1 - 1/\lambda$. Q.E.D.

Proof of Proposition 4

We obtain

$$\begin{aligned}
\frac{dW^S}{d\alpha}\bigg|_{\alpha=0} &= \frac{\partial q_0}{\partial \alpha}(-p'Q + (1+\lambda)(p + p'q_0 - c_0') + (1-\theta)p'q_1) + \frac{\partial q_1}{\partial \alpha}(-p'Q + (1+\lambda)p'q_0 + p + p'q_1 - c') \\
&= \frac{\partial q_1}{\partial \alpha}(-p'Q + (1+\lambda)p'q_0 + (1-\theta)(p + p'q_1 - c')) \\
&= \frac{\partial q_1}{\partial \alpha}p'(-Q + (1+\lambda)q_0) \\
&= \frac{\partial q_1}{\partial \alpha}p'(\lambda q_0 - q_1), \qquad (19)
\end{aligned}$$

which is non-positive if $\lambda q_0(0) \geq q_1(0)$. Q.E.D.

Proof of Proposition 5

By Proposition 4, we examine the condition under which $\lambda q_0(0) \geq q_1(0)$ holds. $q_0(0)$ and $q_1(0)$ are

$$q_0(0) = \frac{(1+\lambda)(1+k_1) + \theta}{(1+\lambda)((2+k_0)(2+k_1) - 1) + \theta}a$$

$$q_1(0) = \frac{(1+\lambda)(2+k_0)(1+k_1)}{(1+\lambda)((2+k_0)(2+k_1) - 1) + \theta}\frac{a}{2+k_1}.$$

$\alpha^S = 0$ if and only if

$$g(\theta, \lambda, k_0, k_1) := \lambda((1+\lambda)(1+k_1) + \theta) - \frac{(1+\lambda)(2+k_0)(1+k_1)}{2+k_1} \geq 0.$$

This implies Proposition 5(i).

$g(\theta, \lambda, k_0, k_1)$ is obviously increasing in θ and decreasing in k_0 (Proposition 5(ii)).

We obtain

$$\frac{\partial g}{\partial \lambda} = (1+k_1)\frac{(k_1 - k_0 + 2\lambda(2+k_1))}{2+k_1} + \theta,$$

which is positive if and only if

$$k_0 \leq k_1 + (2+k_1)\left(2\lambda + \frac{\theta}{1+k_1}\right).$$

This implies Proposition 5(iii). Q.E.D.

Proof of Proposition 6

The proof of Proposition 3(ii) can be applied to the proof of Proposition 6(i).

We then show Proposition 6(ii). Suppose that $dW^R(q_0)/dq_0 > 0$ at $\alpha = 0$. Then a marginal increase in α from $\alpha = 0$ increases q_0 and improves welfare. Thus, $\alpha = 0$ is not optimal. Suppose that $dW^R(q_0)/dq_0 < 0$ at $\alpha = 0$. Proposition 6(i) states that $\alpha = 0$ and $\alpha = 1 - 1/\sqrt{\lambda}$ yield the same outcome. Then a marginal increase in α from $\alpha = 1 - 1/\sqrt{\lambda}$ decreases q_0 from $q_0^S(0)$ and improves welfare. Thus, $\alpha = 0$ is not optimal. Therefore, $\alpha = 0$ is optimal if and only if $dW^R(q_0)/dq_0 = 0$ at $\alpha = 0$, and holds if and only if (19) is zero. (19) is zero if and only if $\lambda q_0^S(0) = q_1^S(0)$. These imply Proposition 6(ii).

Finally, we show Proposition 6(iii). If $c_0 = c_1$, $q_0^S(1) = q_1^S(1)$. Because $q_0^S(\alpha)$ is decreasing in α for $\alpha \in [1 - 1/\sqrt{\lambda}, 1]$, is increasing in $\alpha \in [0, 1 - 1/\sqrt{\lambda}]$, and $q_0^S(0) = q_0^S(1 - 1/\lambda)$, we obtain $q_0^S(\alpha) > q_0^S(1)$ for all $\alpha \in [0, 1]$. Because $q_1^S(\alpha)$ is increasing in α for $\alpha \in [1 - 1/\sqrt{\lambda}, 1]$, is decreasing in $\alpha \in [0, 1 - 1/\sqrt{\lambda}]$, and $q_1^S(0) = q_1^S(1 - 1/\lambda)$, we obtain $q_1^S(\alpha) < q_0^S(1)$ for all $\alpha \in [0, 1]$. Therefore, $q_0^S(0) > q_1^S(0)$. Because $\lambda > 1$, we obtain $\lambda q_0 > q_1$. From Proposition 6(ii), we obtain Proposition 6(iii). Q.E.D.

Proof of Proposition 7

In the same way as the proof of Proposition 5, $q_0(0)$ and $q_1(0)$ are

$$q_0(0) = \frac{(1+\lambda)(1+k_1)+\theta}{(1+\lambda)((2+k_0)(2+k_1)-1)+\theta} a$$

$$q_1(0) = \frac{(1+\lambda)(2+k_0)(1+k_1)}{(1+\lambda)((2+k_0)(2+k_1)-1)+\theta} \frac{a}{2+k_1}.$$

By Proposition 6 (ii), $\alpha = 0$ is optimal if and only if $\lambda q_0(0) = q_1(0)$, and holds if and only if $g(\theta, \lambda, k_0, k_1) = 0$. Q.E.D.

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