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## **Prominence, Complexity, and Pricing**

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# Prominence, Complexity, and Pricing\*

Ioana Chioveanu<sup>†</sup>

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## Abstract

This paper analyzes prominence in a homogeneous product market where two firms simultaneously choose both prices and price complexity levels. Complexity limits competing offers' comparability and results in consumer confusion. Confused consumers are more likely to buy from the prominent firm. In equilibrium there is dispersion in both prices and price complexity. The nature of equilibrium depends on prominence. Compared to its rival, the prominent firm makes higher profit, associates a smaller price range with lowest complexity, puts lower probability on lowest complexity, and sets a higher average price. However, higher prominence may benefit consumers and, conditional on choosing lowest complexity, the prominent firm's average price is lower, which is consistent with confused consumers' bias.

**Keywords:** oligopoly markets, consumer confusion, prominence, price complexity, price dispersion

**JEL classification:** D03, D43, L13

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# 1 Introduction

Price complexity is a common feature of many markets, including those for retail financial and banking products, and retail supply of gas and electricity. It stems from the use of multi-part tariffs or partitioned prices, involved or technical language, or different price formats or information disclosure methods. A main concern is that complex pricing stifles competition by making it harder for consumers to understand firms' offers and by limiting product comparability.

The 2015 UK Competition and Market Authority investigation of the retail banking market found that “[t]here are barriers to accessing and assessing information on Personal Current Account charges” and “overdraft charges are particularly difficult to compare across banks, due to both the complexity and diversity of the banks’ charging structures”.<sup>1</sup> The 2011 report by the UK Independent Commission on Banking mentions “evidence that complexity in pricing structures makes it difficult for consumers to receive good value”. The 2007 EC study of EU mortgage credit markets and Woodward and Hall’s 2012 study of US mortgage markets echo these concerns.<sup>2</sup>

Price complexity increases the time (or effort) consumers need to make a choice and the level of cognitive abilities and sophistication required to identify the best deal. So, it may lead to consumer confusion and allow homogeneous product sellers to soften price competition and increase their profits.<sup>3</sup> Experimental research indicates that more fragmented multi-part tariffs can create confusion and lead to suboptimal consumer choices (see, for instance, Kalayci and Potters, 2011, and Kalayci, 2015). These findings are consistent with evidence from the marketing literature that partitioned (or involved) pricing makes it difficult for consumers to compare competing offers (Greenleaf et al., 2013, reviews related work).<sup>4</sup> Evidence of behavioral biases has also been found in US retail finance products (mortgage brokerage, loans, and credit card services) by Woodward and Hall (2012) and Stango and Zinman (2009a, 2009b).

In some markets where price complexity limits the comparability of competing offers, the choices of confused consumers are affected by firm prominence, which may be due to higher

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<sup>1</sup>Similarly, in the market for business current accounts “while price information is available, it is difficult for SMEs to compare fees across banks” and this is due to “complex tariff structures”, amongst other factors. See the 2015 Summary of Provisional Findings Report of the Retail Banking Market Investigation.

<sup>2</sup>Carlin (2009) discusses empirical evidence of price complexity in financial markets and concludes that “many of the households who purchase retail financial products do not understand what they are buying and how much they are paying for these goods”.

<sup>3</sup>When facing complex tariffs/markets, some consumers may rationally opt out of information processing due to its high cost. Or, they may be unable to deal with the complexity because they have poor numeracy skills and/or misjudge the information.

<sup>4</sup>See also Estelami (1997), Morwitz, Greenleaf, and Johnson (1998), and Thomas and Morwitz (2009).

brand recognition (e.g., for a pioneer or incumbent product or an intensely advertised one), to product recommendations made by an expert, agent, or other consumers, to a more salient location (at eye-level, in a display, or at the top of an online search-outcome list), or to consumers' loyalty to an already familiar brand.<sup>5</sup> For instance, consumers who shop for a mortgage or for insurance may be biased towards considering their current-account bank. In retail energy markets that were previously monopolized, consumers may favor the 'familiar' regional incumbent over new entrants (see Giulietti, Waterson, and Wildenbeest, 2014, for evidence from British electricity markets).<sup>6</sup>

This paper explores the relationship between price complexity as an obfuscation device and firm prominence and its implications in otherwise homogeneous product markets. We analyze the impact of prominence on firms' pricing and complexity choices and on market outcomes, and build on the interplay between complexity and prominence to propose a conceptual micro-foundation for consumer confusion. In our model, a prominent seller and its rival compete for a unit mass of identical consumers with unit demands. Firms simultaneously and independently choose both their prices and price-complexity levels. The timing reflects the fact that in many environments, including banking and financial markets, firms can change relatively easily the price formats or the technical language employed in their price disclosures.

We formalize price complexity by allowing each firm to select a level from a closed interval. A firm's choice of complexity affects consumers' ability to understand its price offer and, although it does not affect the complexity of the rival's price, it may limit the comparability of competing offers. More precisely, a marginal increase in a firm's complexity level increases the share of confused consumers in the market. So, complexity affects market composition: some consumers are experts, while others are confused. Confused consumers are unable to compare the firms' prices and make random choices, but are relatively more likely to select the prominent product as it enjoys higher recognition.<sup>7</sup> In Carlin (2009), confused consumers make random choices, so each firm is equally likely to be selected. In this respect, our model is an asymmetric version of his and an extension where we explore alternative confusion technologies generalizes his findings. In our benchmark model, the experts purchase the lowest-price product, but we also

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<sup>5</sup>Armstrong, Vickers, and Zhou (2009) review empirical evidence on prominence.

<sup>6</sup>In an analysis of Mexico's private social security market, Hastings, Hortaçsu, and Syverson (2017) show that firms' advertising and sales spending (which can be related to prominence) affects the choices of low-income or price-inelastic consumers. Using household-level data from the Texas residential electricity market, Hortaçsu, Madanizadeh, and Puller (2017) show that inattention and incumbent brands' advantages are sources of consumer inertia.

<sup>7</sup>Due to confusion, the confused may use intermediaries who steer them towards the prominent product, may rely on persuasive advertisements, or may have stronger default biases.

discuss a variant where they are biased towards the prominent product.

In this setting, firms have to balance conflicting incentives when setting their prices: to compete aggressively for the experts and to exploit the confused. In equilibrium, this friction rules out pure strategy pricing, so both firms randomize on prices. The prominent firm also randomizes between the lowest and the highest price complexity levels and, for moderate levels of prominence, so does the less prominent seller. However, if the prominence level is high enough, the less prominent seller chooses the lowest complexity for sure as it benefits more from market transparency. In equilibrium, whenever a firm randomizes on complexity, there is a positive relationship between prices and complexity levels.<sup>8</sup> When setting a low price, a firm benefits from a lower complexity level as this is associated with a higher fraction of experts. In contrast, when a firm sets a high price, it may benefit from choosing a high complexity level, provided that it serves a large enough fraction of confused consumers.

The firms' equilibrium mixed price and complexity strategies reflect the differences in product salience. The prominent seller makes higher profits, chooses the highest price-complexity level with higher probability than the rival, sets a lower cut-off price below which prices are associated with the lowest complexity, and chooses the monopoly price with positive probability. As it sells to a larger share of confused consumers, the salient firm is more likely to choose high complexity and also, for a given complexity level, its incentive to set a high price is stronger. The less prominent seller's price is always below the monopoly level and its average price is lower than that of the rival.

In our model, an increase in prominence may lead to lower industry profits and so consumers could be better off in a market where one firm is salient enough. Intuitively, for high enough prominence, the less salient firm chooses the lowest complexity for sure and competes more aggressively in prices. This suggests that in markets where less prominent firms (e.g. new entrants) can increase the relative prominence of their products (for instance, through advertising investments or sales efforts), this could be detrimental to consumers. Giulietti, Waterson, and Wildenbeest (2014) show that, between 2002 and 2005, in the British electricity market, the lower the share of households buying electricity through the incumbents (which enjoy higher prominence at regional level), the less competitively the market entrants behave.

Furthermore, we show that, conditional on choosing lowest complexity, the prominent firm's average price is lower. Therefore, when consumers are most able to understand the firms' prices

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<sup>8</sup>Armstrong and Chen (2009) and Chioveanu (2012) identify positive relationships between prices and product qualities in models where firms randomize on both dimensions.

(when complexity is lowest), the prominent firm appears to be offering a better deal. In this sense, confused consumers' bias for the prominent seller is consistent with the ranking of the average prices conditional on low complexity.

We show that our qualitative results are robust in a modified model where expert consumers are biased towards the prominent firm's product (i.e. willing to pay a premium for it so long as the price is below their valuation).<sup>9</sup> Using an example, we illustrate the existence of an equilibrium where firms randomize on both prices and price complexity levels, there is a positive relationship between prices, and - conditional on choosing the lowest complexity level - the prominent firm's average price is lower. We also show that such a mixed strategy equilibrium exists for more general confusion technologies whenever the marginal effect of a firm's price complexity increases in the rival's complexity choice.

In spite of their prevalence, price complexity and prominence have only recently received attention in the economics literature. Carlin (2009) analyzes a homogeneous product market where identical firms compete in both prices and price complexity levels, and where confused consumers make random choices, so each firm is equally likely to be selected. His findings are consistent with observed patterns in retail financial markets, such as price dispersion, positive mark-ups, and higher prices in more fragmented environments. Our analysis incorporates prominence into his framework and focuses on its interaction with complexity. Gu and Wenzel (2014) analyze consumer protection policy in a model where two firms compete in prices after committing to an obfuscation level. In their model, unlike ours, obfuscation is a long-run decision so it could be related, for instance, to product design rather than price format which may be changed relatively easily.<sup>10</sup> Allowing for prominence, they show that in equilibrium the salient firm chooses the highest obfuscation level for sure, while the rival's (deterministic) choice depends on the market conditions.

In a duopoly models where firms compete in prices and price-frames, Piccione and Spiegler (2012) study the impact of frame-structure on market outcomes. Chioveanu and Zhou (2013) explore in a unified framework the effects of both price complexity and price presentation format differentiation as sources of consumer confusion. We show that the nature of the equilibrium depends on the source of confusion and that in oligopoly markets a standard competition policy

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<sup>9</sup>An alternative interpretation of this extension is that consumers exhibit a default bias and, although the experts can correctly compare prices, they face a switching cost.

<sup>10</sup>See also Ellison and Wolitzky (2012), Wilson (2010), and Taylor (2017) for search-cost models of obfuscation. The latter explores an alternative rationale for obfuscation: raising 'browsing' costs allows firms to target better merchandising efforts by excluding from the market 'window-shoppers' who are unlikely to purchase.

approach may have undesired effects.

In a sequential search model where all consumers sample first one salient firm, Armstrong, Vickers, and Zhou (2009) demonstrate that, with homogeneous products, the prominent firm sets a lower price than its rivals, industry profits are higher, and consumer surplus and welfare lower than in a market where firms are equally prominent. They also show that prominence benefits both sellers and consumers when products are vertically differentiated (as the highest-quality producer has the strongest incentive to become salient) and discuss the empirical relevance of prominence. Armstrong and Zhou (2011) explore ways in which a firm can become prominent. More specifically, intermediaries may steer consumers to one firm for a fee, price advertisements may affect the order in which firms' offers are sampled, or consumers' default biases may be a source of prominence.<sup>11</sup>

In our clearinghouse setting, the order of search is irrelevant but prominence affects the behavior of consumers who are confused by price complexity. With both complexity and prominence, consumers' perceptions of prices may be biased as they may ignore the involved prices they cannot understand and take into account only those prices that are presented in less complex formats. In our model, a bias in favor of the prominent seller is consistent with such a 'myopic' assessment. We focus on environments where firms commonly employ complex prices, for example, consumer banking and energy retail markets. Prominence might be driven by default biases favouring the product under consideration or related ones or it may be due to persuasive advertising or marketing ploys which could make a firm's product salient in a consumer's mind and so more likely to be considered.

By considering the interplay between complexity and prominence in a model with consumer confusion, this study contributes to an emerging literature that explores the interaction between boundedly rational consumers and strategic firms. See Ellison (2006), Spiegler (2011), Huck and Zhou (2011), Grubb (2015), and Spiegler (2016) for related discussions and surveys of recent work. Our model is also related to the literature on price dispersion (see Baye, Morgan, and Scholten, 2006, for a review) and explores a market where firms simultaneously choose prices and complexity, and randomize in both dimensions.

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<sup>11</sup>In a model with product differentiation, Rhodes (2011) shows that a prominent firm chooses a lower price and makes higher profits, even when search is almost costless. See also Armstrong (2017) for a recent review of the ordered search literature.

## 2 Model

Consider a market for a homogeneous product with two sellers, firms 1 and 2. The firms face zero marginal costs of production. There is a unit mass of consumers, each demanding at most one unit of the product and willing to pay up to  $v = 1$ . The firms compete by simultaneously and independently choosing prices ( $p_1$  and  $p_2$ ) and price complexity levels ( $k_1$  and  $k_2$ ). The timing reflects the fact that in many cases both complexity and prices can be changed relatively easily. The level of complexity  $k_i$  captures how difficult it is for consumers to assess the price of firm  $i$  and affects the comparability of competing offers. The firms set prices  $p_i \in [0, 1]$  and can choose any complexity level  $k_i \in [\underline{k}, \bar{k}] \subset R_+$  free of cost.

Depending on firms' complexity choices, some consumers may find it difficult to correctly compare the competing price offers. More precisely, for given  $k_1$  and  $k_2$ , a fraction  $\mu(k_1, k_2) \leq 1$  of the consumers are able to accurately compare the price offers and select the best deal (we refer to these as the 'experts' or 'informed'), but the remaining  $1 - \mu(k_1, k_2)$  consumers are confused and make random choices, which may be biased due to firm prominence. Let  $\mu(k_1, k_2) \in \mathcal{C}^2$ . If one firm unilaterally increases the complexity of its price, this lowers the fraction of expert consumers in the market ( $\partial\mu/\partial k_i < 0$ , for  $i = 1, 2$ ), but does not affect the marginal impact of the rival's price complexity on consumers ( $\partial^2\mu/\partial k_1\partial k_2 = 0$ ). For simplicity, we assume that  $\mu(k_1, k_2) = 1$  iff  $k_1 = k_2 = \underline{k}$ . That is, nobody gets confused if both firms choose the lowest complexity level  $\underline{k}$ , in which case all consumers buy the cheaper product.<sup>12</sup> In section 5 we explore the robustness of our results for alternative confusion technologies with  $\partial^2\mu/\partial k_1\partial k_2 > 0$ .

We focus on the interaction between price complexity and firm prominence. In our model, prominence is exogenous (it may be due, for instance, to higher firm recognition or perceived trustworthiness) and has an impact on product choice when consumers are confused by price complexity. It also affects the choice of informed consumers if the two firms offer the same price.<sup>13</sup> More specifically, without loss of generality, firm 1 is a 'prominent' seller and the consumers who are unable to compare the prices due to complexity are more likely to purchase its product. That is, a fraction  $\sigma \in (1/2, 1)$  of the confused consumers buy from firm 1 and the remaining  $1 - \sigma$  buy from firm 2. Similarly, if both firms offer the same price, a fraction

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<sup>12</sup>This is without loss of generality so long as the monotonicity assumptions in the text are satisfied.

<sup>13</sup>Firm prominence can be itself a source of confusion. For instance, this may be the case in pharmaceutical markets where some consumers prefer branded products to generic drugs with identical composition. However, here we explore confusion due to price complexity.



$\sigma \in (1/2, 1)$  of the experts buy from firm 1 and the remaining  $1 - \sigma$  buy from firm 2. As a result, firms profits are

$$\pi_i(p_i, p_j, k_i, k_j) = p_i \cdot [q_i(p_i, p_j)\mu(k_i, k_j) + s_i(1 - \mu(k_i, k_j))] ,$$

where  $q_i(p_i, p_j)$  is given by

$$q_i(p_i, p_j) = \begin{cases} 1, & \text{if } p_i < p_j \text{ and } p_i \leq 1 \\ s_i, & \text{if } p_i = p_j \leq 1 \\ 0, & \text{if } p_i > p_j \text{ or } p_i > 1 \end{cases} \quad \text{for } i, j \in \{1, 2\} \text{ and } i \neq j ,$$

with  $s_1 = \sigma > 1/2$  and  $s_2 = 1 - \sigma$ .

We assume that the confused are unable to compare the firms' offers, however they do not pay more than their reservation price ( $v = 1$ ).<sup>14</sup> One interpretation is that consumers have a budget constraint and realize at checkout (or after purchase) if a product's price exceeds their valuation and can decline to buy or return the product. Knowing this, firms do not have incentives to set prices above consumers' valuation.<sup>15</sup> In our model, for simplicity, confused consumers' choices are affected by complexity and prominence, but are independent of how the two firms' prices rank overall. This captures the idea that confusion in price comparisons reduces consumers' price sensitivity and weakens price competition. Also, consumers do not have an opportunity to learn and infer prices from a firm's complexity choice. This is more relevant in mortgage or financial services markets, for example, where the consumers participate infrequently. Moreover, in our setting, confused consumers' bias in favour of the prominent firm is consistent with the ranking of the average prices associated with the lowest complexity.

### 3 Preliminary Analysis

We start by analyzing firms' price and complexity choices when the price format limits the comparability of competing offers and one firm is prominent. All proofs missing from the text are relegated to the appendix, unless specified otherwise. The following two results rule out the existence of pure strategy equilibria.

**Lemma 1** *There is no equilibrium where both firms choose pure price-complexity strategies.*

<sup>14</sup>Carlin (2009), Piccione and Spiegler (2012), and Chioveanu and Zhou (2013) also make this assumption.

<sup>15</sup>Nevertheless, it can be shown that our results are qualitatively robust when confused consumers may end up paying more than  $v = 1$  but less than  $1 + \varepsilon$  for  $\varepsilon < \mu(\underline{k}, \bar{k})$ .

**Proof.** Suppose firm  $i$  ( $j \neq i$ ) chooses a deterministic complexity level  $k_i$  ( $k_j$ ).

(i) If  $k_i = k_j = \underline{k}$ , all consumers are experts ( $\mu(\underline{k}, \underline{k}) = 1$ ), and firms compete à la Bertrand and make zero profits. But then firm  $i$  could profitably deviate to  $k_i^d = k' > \underline{k}$  and a price  $p_i = 1$  which would result in positive profits as there would be a non-trivial mass of confused consumers (i.e.,  $1 - \mu(k', \underline{k}) > 0$ ). Hence, it must be that in any candidate equilibrium at least one firm (w.l.o.g. let it be  $i$ ) chooses  $k_i > \underline{k}$ .

(ii) By (i) for any candidate equilibrium profile of price complexities  $(k_i, k_j)$ , some consumers are confused (i.e.,  $1 - \mu(k_i, k_j) > 0$ ). But then for any such profile  $(k_i, k_j)$ , there is a unique pricing equilibrium where firms randomize according to a c.d.f. on  $[p_0, 1]$ , with  $p_0 = \sigma(1 - \mu(k_i, k_j)) / [1 - (1 - \sigma)(1 - \mu(k_i, k_j))] > 0$  (see, for instance, Baye et al., 1992), and firm  $i$  makes profit  $\pi_i = p_0[1 - s_j(1 - \mu(k_i, k_j))]$ . But, as it must be that  $k_i > \underline{k}$ , firm  $i$  could profitably deviate to  $p_i^d = p_0$  and  $k_i^d = \underline{k}$  which would result in profit  $\pi_i^d = p_0[1 - s_j(1 - \mu(\underline{k}, k_j))] > p_0[1 - s_j(1 - \mu(k_i, k_j))]$  as  $\mu(\underline{k}, k_j) > \mu(k_i, k_j)$ . So, there can be no equilibrium where both firms choose pure price complexity strategies. ■

Lemma 1 implies that in any candidate equilibrium at least one firm randomizes on complexity levels. As a result, the firms face two different types of consumers, confused and experts.<sup>16</sup> There is a conflict between the incentive to extract all surplus from confused consumers, and the incentive to reduce price and compete for informed consumers. This intuition underlies the following result, whose proof is standard and therefore omitted (see Varian, 1980, and Rosenthal, 1980).

**Lemma 2** *There is no equilibrium where both firms use pure pricing strategies.*

Lemmas 1 and 2 show that in any duopoly equilibrium there must be dispersion in both prices and complexity levels. Firm  $i$ 's strategy space is  $[0, 1] \times [\underline{k}, \bar{k}]$ . Denote by  $\xi_i \equiv \xi_i(p_i, k_i)$  firm  $i$ 's mixed strategy for  $i = 1, 2$ .  $\xi_i$  is a bivariate c.d.f. and can be written as  $\xi_i = F_i(p_i)H_i(k_i | p_i)$ , where  $F_i(p_i)$  is the marginal c.d.f. of firm  $i$ 's random price and  $H_i(k_i | p_i)$  is the conditional c.d.f. of firm  $i$ 's complexity level.<sup>17</sup> For  $F_i(p)$  and  $H_i(k_i | p_i)$  to be well-defined c.d.f.s they should be increasing on their supports.

<sup>16</sup>We focus on a case where  $\mu(\underline{k}, \underline{k}) = 1$ . However, Lemma 1 is robust for  $\mu(\underline{k}, \underline{k}) < 1$  so long as  $\partial\mu/\partial k_i < 0$ , for  $i = 1, 2$ . In that case, even for  $k_i = k_j = \underline{k}$ , firms face both experts and confused and so in the candidate price equilibrium,  $\pi_1 = p_0[1 - (1 - \sigma)(1 - \mu(\underline{k}, \underline{k}))] = \sigma(1 - \mu(\underline{k}, \underline{k}))$ . But, firm 1 can profitably deviate to  $p_1^d = 1$  and  $k_1^d = \bar{k}$  as  $\pi_1^d = \sigma(1 - \mu(\bar{k}, \underline{k})) > \sigma(1 - \mu(\underline{k}, \underline{k}))$ . As at least one of the firms chooses  $k_i > \underline{k}$ , part (ii) in the proof of Lemma 1 applies.

<sup>17</sup>If the two random variables,  $p_i$  and  $k_i$  are independent,  $H_i(k_i | p_i) = H_i(k_i)$ .

Suppose firm  $i \neq j$  chooses a price  $p_i$  and complexity level  $k_i$ . Firm  $i$ 's expected profit, which depends on firm  $i$ 's choices and on the rival's mixed strategy  $\xi_j$ , can be written as

$$\begin{aligned} \pi_i(p_i, k_i, \xi_j) = & p_i \left[ \int_{p_i}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j | p_j > p_i) \right) dF_j(p_j) \right] + \\ & p_i s_i \left[ 1 - \int_0^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j | p_j) \right) dF_j(p_j) \right]. \end{aligned}$$

The expected base of confused consumers is presented in the second square brackets in  $\pi_i(p_i, k_i, \xi_j)$ .

The remaining consumers form the expected base of experts. But, expert consumers purchase from firm  $i$  only when it offers a lower price than its rival. The expected number of informed consumers, conditional on firm  $i$  being the low price seller, is presented in the first square brackets. Firm  $i$  serves a share  $s_i$  of the expected base of confused. The first derivative of  $\pi_i(p_i, k_i, \xi_j)$  w.r.t.  $k_i$  is presented below using Leibniz's Rule. The equality follows from the fact that  $\partial\mu(k_i, k_j)/\partial k_i$  is independent of  $k_j$ , as  $\partial^2\mu/\partial k_i \partial k_j = 0$ .

$$\begin{aligned} p_i \int_{p_i}^1 \left( \int_{\underline{k}}^{\bar{k}} \frac{\partial\mu}{\partial k_i} dH_j(k_j | p_j > p_i) \right) dF_j(p_j) - p_i s_i \int_0^1 \left( \int_{\underline{k}}^{\bar{k}} \frac{\partial\mu}{\partial k_i} dH_j(k_j | p_j) \right) dF_j(p_j) = \\ p_i \frac{\partial\mu}{\partial k_i} [(1 - F_j(p_i)) - s_i]. \end{aligned}$$

Then, as  $\partial\mu(k_i, k_j)/\partial k_i < 0$ , to maximize its expected-profit firm  $i$  chooses

$$k_i(p_i) = \begin{cases} \underline{k} & \text{if } 1 - F_j(p_i) > s_i \Leftrightarrow p_i < \hat{p}_i \\ \bar{k} & \text{if } 1 - F_j(p_i) < s_i \Leftrightarrow p_i > \hat{p}_i \\ k, \forall k \in [\underline{k}, \bar{k}] & \text{if } p_i = \hat{p}_i \end{cases},$$

where the threshold price  $\hat{p}_i$  is implicitly defined by  $F_j(\hat{p}_i) = 1 - s_i$ , whenever  $\hat{p}_i$  belongs to the support of  $F_j$ . Lemma 1 implies that at least one of the cut-off prices  $\hat{p}_i$  belongs to the support of the rival's price distribution function, as at least one firm mixes on complexity levels. The next result summarizes these findings.

**Proposition 1** *In equilibrium, a firm's complexity choice depends only on its price. Firm  $i$  chooses its price according to a c.d.f.  $F_i(p_i)$  with support  $T_i \subseteq [0, 1]$ . If  $p_i < \hat{p}_i$  ( $p_i > \hat{p}_i$ ) firm  $i$  chooses the lowest complexity  $\underline{k}$  (highest complexity  $\bar{k}$ ). If  $p_i = \hat{p}_i$ , firm  $i$  is indifferent between any complexity level  $k \in [\underline{k}, \bar{k}]$ . If the cut-off price  $\hat{p}_i \in T_j$  (for  $i \neq j$ ), then it is implicitly defined by  $F_j(\hat{p}_i) = s_j$ . If  $\hat{p}_i \notin T_j$ , firm  $i$  chooses a deterministic complexity level, but then it must be that the firm  $j$  randomizes on prices (i.e.  $\hat{p}_j \in T_i$ ).*

When a firm mixes on complexity levels in equilibrium, there is a positive relationship between prices and complexity. More specifically, if  $\hat{p}_i \in T_j$ , at all prices below the cut-off level  $\hat{p}_i$ , firm  $i$  chooses the lowest complexity and at all prices above  $\hat{p}_i$ , it chooses the highest

complexity level. Intuitively, when a firm chooses a relatively high price, its incentive to choose high complexity is stronger as it relies more on selling to confused consumers. In contrast, when setting a relatively low price, a firm has a stronger incentive to choose low complexity as this results in a larger base of experts.

Lemmas 3 - 6 in appendix A.1 explore the properties of the pricing c.d.f.s, and show that both firms choose prices according to c.d.f.s which are defined on a common interval  $T = [p_0, 1]$  and are continuous everywhere except possibly at the upper bound  $p = 1$ .<sup>18</sup> Using these properties, we first analyze a situation where both firms randomize on complexity levels, and so the cut-off prices defined in Proposition 1 must satisfy  $\hat{p}_i \in T = [p_0, 1]$  (for  $i = 1, 2$ ). This implies that firm  $i = 1, 2$  chooses complexity level  $\underline{k}$  with probability  $F_i(\hat{p}_i)$  and complexity level  $\bar{k}$  with probability  $1 - F_i(\hat{p}_i)$ . The threshold prices  $\hat{p}_i \in T$  are implicitly defined by  $s_j = F_j(\hat{p}_i)$  where  $j = 1, 2, i \neq j$ , and  $s_j$  is firm  $j$ 's share of consumers confused by complexity. Recall that  $s_1 = \sigma > 1/2$  and  $s_2 = 1 - \sigma$ . For expositional simplicity, denote:

$$\lambda_1 \equiv F_1(\hat{p}_1) \text{ and } \lambda_2 \equiv F_2(\hat{p}_2) .$$

Consistency requires that  $F_i(\hat{p}_i) \in (0, 1)$  and  $F_i(\hat{p}_j) = s_i$ . The following condition holds when both firms mix on both prices and complexity levels in equilibrium. If instead  $\hat{p}_2 < \hat{p}_1$ , then the resulting values of  $\lambda_1$  and  $\lambda_2$  are inconsistent.<sup>19</sup>

**Condition 1**

$$0 < p_0 < \hat{p}_1 < \hat{p}_2 < 1 .$$

Below we illustrate the derivation of firm 1's expected profit for  $p \in [p_0, \hat{p}_1)$ . By Proposition 1, firm 1 associates prices in this range with complexity level  $\underline{k}$ . Then, its expected profit is

$$\begin{aligned} \pi_1(p, \underline{k}) = & p\{(F_2(\hat{p}_2) - F_2(p))\mu(\underline{k}, \underline{k}) + (1 - F_2(\hat{p}_2))\mu(\underline{k}, \bar{k}) + \\ & \sigma[F_2(\hat{p}_2)(1 - \mu(\underline{k}, \underline{k})) + (1 - F_2(\hat{p}_2))(1 - \mu(\underline{k}, \bar{k}))]\} . \end{aligned} \tag{1}$$

With probability  $F_2(\hat{p}_2)$ , firm 2 chooses  $\underline{k}$ , so that there are  $\mu(\underline{k}, \underline{k})$  experts and  $1 - \mu(\underline{k}, \bar{k})$  confused consumers. A share  $\sigma$  of the confused purchases from firm 1, the prominent seller. Informed consumers purchase from firm 1 if firm 2's price is higher, which happens with probability  $F_2(\hat{p}_2) - F_2(p)$ . With probability  $1 - F_2(\hat{p}_2)$ , firm 2 chooses  $\bar{k}$  and there are  $\mu(\underline{k}, \bar{k})$  informed and  $1 - \mu(\underline{k}, \bar{k})$  confused consumers. All the informed purchase from firm 1 as it offers

<sup>18</sup>This approach is related to Narasimhan (1988) and Baye, Kovenock, and de Vries (1992), for instance.

<sup>19</sup>More specifically,  $\lambda_2 = F_2(\hat{p}_2) > 1 - \sigma = F_2(\hat{p}_1)$ .

a lower price (firm 2 associates  $\bar{k}$  with prices higher than  $\hat{p}_2$ ) and so does a share  $\sigma$  of the confused. The first two terms in curly brackets capture the expected number of experts, while the term in square brackets gives the expected number of confused consumers.

In appendix A.2, we present firm 1's expected profits at  $p_0$  and when  $p \rightarrow \hat{p}_1$ . In the same appendix, we derive firm 1's expected profit for  $p \in [\hat{p}_1, \hat{p}_2]$  and  $p \in (\hat{p}_2, 1]$ , and firm 2's expected profit over the three price ranges. Next section builds on these derivations to characterize the mixed strategy equilibrium and to identify a condition on the parameter values under which both firms randomize on both prices and complexity levels in equilibrium. When this condition does not hold - which happens when firm 1's level of prominence is relatively high - both firms mix on prices, but only the prominent firm randomizes on complexity levels.

## 4 Duopoly Equilibrium

In equilibrium, firm  $i$ 's expected profit for any price-complexity combination  $(p, k_i)$ , which is assigned positive density in equilibrium, must be constant. Then, using expressions (A1) - (A4), (A7) and (A8) in appendix A.2, we can write price ratios  $p_0/\hat{p}_1$  and  $p_0/\hat{p}_2$  as functions of  $\lambda_2 = F_2(\hat{p}_2)$ , firm 2's probability of choosing  $\underline{k}$  in equilibrium, and  $\lambda_1 = F_1(\hat{p}_1)$ , firm 1's probability of choosing  $\underline{k}$  in equilibrium. These ratios are presented in appendix A.3. We then obtain the equilibrium values of  $\lambda_1$  and  $\lambda_2$ ,

$$\lambda_1 = \frac{(1 - \sigma)[1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))]}{1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))} \quad \text{and} \quad \lambda_2 = \frac{\sigma[1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))}. \quad (2)$$

It can be checked that  $\lambda_1 \in (0, 1)$  and  $\lambda_2 > 0$ . Furthermore,  $\lambda_2 < 1$  holds iff the following condition is satisfied.

### Condition 2

$$(1 - \sigma) / [\sigma(1 - \sigma + \sigma^2)] > 1 - \mu(\underline{k}, \bar{k}).$$

Recall that  $\mu(\bar{k}, \bar{k}) = 2\mu(\underline{k}, \bar{k}) - 1$  and  $\mu(\bar{k}, \bar{k}) < \mu(\underline{k}, \bar{k})$ . As  $\mu(\bar{k}, \bar{k}) \geq 0$ , it follows that  $1 - \mu(\underline{k}, \bar{k}) \leq 1/2$ . For relatively low levels of prominence (that is, for  $\sigma \in (0.5, 0.71)$ ), this condition always holds and so firm 2 mixes between the highest and the lowest price complexity levels. More generally, for a given  $\mu(\underline{k}, \bar{k})$ , the condition is satisfied when firm 1's level of prominence is not too high. However, Condition 2 gets more stringent as firm 1's prominence increases (the LHS of the inequality in the condition is decreasing in  $\sigma$ ). When firm 1 is prominent enough, firm 2 benefits more from price transparency, as its share of confused consumers is relatively small.

In appendix A.3, we show that when  $\lambda_i \in (0, 1)$ , the consistency requirements also hold (that is,  $F_i(\hat{p}_1) < F_i(\hat{p}_2)$  for  $i = 1, 2$ , where  $F_i(\hat{p}_i) = \lambda_i$  and  $F_i(\hat{p}_j) = s_i$ ). Also there, we explore the firms' price c.d.f.s at the upper bound of the support. Using Lemma 4, we show that firm 2's price c.d.f. is continuous everywhere, while firm 1 has a mass point at the upper bound of the price c.d.f.'s support,  $p = 1$ . Then, we verify that  $p_0$ ,  $\hat{p}_1$ , and  $\hat{p}_2$  are well defined under Condition 2. Finally, we present the equilibrium cut-off prices in expressions (A10) and (A11) and the pricing c.d.f.s of the two firms. Using (A1), (A5), and (2), we obtain the equilibrium profit of firm 1,  $\pi_1^*$  and the lower bound of the price support,  $p_0$ .

$$\pi_1^* = \sigma(1 - \mu(\underline{k}, \bar{k})) \frac{2 - \sigma - \sigma(\sigma^2 - 2\sigma + 3)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} \text{ and} \quad (3)$$

$$p_0 = \sigma(1 - \mu(\underline{k}, \bar{k})) \frac{2 - \sigma - \sigma(\sigma^2 - 2\sigma + 3)(1 - \mu(\underline{k}, \bar{k}))}{\mu(\underline{k}, \bar{k}) + \sigma(1 - \sigma)(\sigma^2 - \sigma + 1)(1 - \mu(\underline{k}, \bar{k}))^2}. \quad (4)$$

Then, using  $p_0$  and (A7), we calculate firm 2's equilibrium profit,

$$\pi_2^* = \sigma(1 - \mu(\underline{k}, \bar{k})) \frac{2 - \sigma - \sigma(\sigma^2 - 2\sigma + 3)(1 - \mu(\underline{k}, \bar{k}))}{1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))}. \quad (5)$$

Note that  $\pi_1^*/\pi_2^* = \lambda_2/\sigma = (1 - \sigma)/\lambda_1$ .

Below we summarize our findings.

**Proposition 2** *Under Condition 2, in the unique mixed strategy equilibrium firm  $i$  chooses the lowest complexity  $\underline{k}$  with probability  $\lambda_i = F_i(\hat{p}_i) \in (0, 1)$ , defined in (2) and highest complexity  $\bar{k}$  with probability  $1 - \lambda_i$ . Both firms randomize on prices in  $[p_0, 1]$ , with  $p_0$  given in (4). Firm 2's price c.d.f. ( $F_2$ ) is continuous on  $[p_0, 1]$ , while firm 1's price c.d.f. ( $F_1$ ) is continuous on  $[p_0, 1)$  and has an atom at  $p = 1$ . Firm  $i$  uses  $\underline{k}$  ( $\bar{k}$ ) at prices below (above)  $\hat{p}_i \in (p_0, 1)$ . The equilibrium profits  $\pi_1^*$  and  $\pi_2^*$  are given in (3) and (5).*

When firm 1's prominence is not too high in the sense that  $\sigma > 1/2$ , but Condition 2 is satisfied, both firms randomize on complexity levels and prices in equilibrium. In this case, the difference in the firms' shares of confused consumers is not too large. In the limit, when  $\sigma \rightarrow 1/2$ ,  $\lambda_1 = \lambda_2 = 1/2$ ,  $\hat{p}_1 = \hat{p}_2$ , and both firms' pricing c.d.f.s are continuous everywhere on their common support. This is consistent with the results in Carlin (2009). The following numerical example and Figure 1 illustrate the results in Proposition 2.

**Example 1** *When  $\sigma = .6$  and  $\mu(\underline{k}, \bar{k}) = .6$ , in equilibrium firm 1 and 2 choose  $\underline{k}$  with probability  $\lambda_1 = .357$  and  $\lambda_2 = .672$ , respectively. The two firms randomize on prices according to the following c.d.f.s, which are illustrated in Figure 1,*

$$F_1(p) = \begin{cases} .846 - .284/p & \text{for } p \in [p_0, \hat{p}_2) \\ 1.171 - .474/p & \text{for } p \in [\hat{p}_2, \hat{p}_1] \\ 2.131 - 1.422/p & \text{for } p \in (\hat{p}_1, 1] \end{cases} \quad \text{and} \quad F_2(p) = \begin{cases} .948 - .319/p & \text{for } p \in [p_0, \hat{p}_2) \\ 1.313 - .531/p & \text{for } p \in [\hat{p}_2, \hat{p}_1] \\ 2.593 - 1.593/p & \text{for } p \in (\hat{p}_1, 1] \end{cases},$$

where  $p_0 = .336$ ,  $\hat{p}_1 = .582$ , and  $\hat{p}_2 = .829$ . Firm 1 makes profit  $\pi_1^* = .319$  and firm 2 makes profit  $\pi_2^* = .284$ . Firm 1's atom at  $p = 1$  is  $\phi = .108$ .

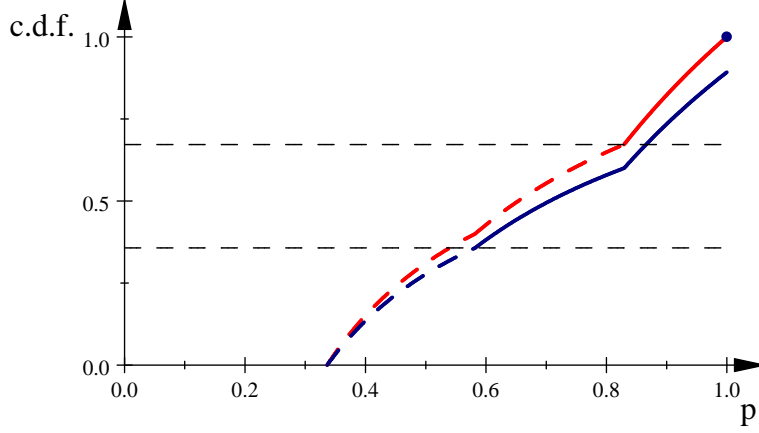


Figure 1: Firms' price c.d.f.s for  $\sigma = .6$  and  $\mu(\underline{k}, \bar{k}) = .6$ .  $F_1(p)$  is the blue line and  $F_2(p)$  is the red line. The dashed lines correspond to prices associated with  $\underline{k}$ .

When condition 2 does not hold, the results in Proposition 2 no longer apply as  $\lambda_2 \geq 1$ . In this case, because firm 1's prominence advantage is large enough, firm 2 serves a relatively small share of confused consumers. Then firm 2 relies more on expert consumers and so benefits more from market transparency than from confusion. We prove the following result in appendix A.4.

**Proposition 3** *When Condition 2 does not hold, in the unique mixed strategy equilibrium firm 2 chooses  $\underline{k}$  for sure and firm 1 chooses the lowest complexity  $\underline{k}$  with probability  $\lambda_1^h = F_1^h(\hat{p}_1^h)$  and the highest complexity  $\bar{k}$  with probability  $1 - \lambda_1^h$ , where*

$$\lambda_1^h = \frac{(1 - \sigma) [1 - \sigma(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))}.$$

*Both firms randomize on prices in  $[p_0^h, 1]$ , with  $p_0^h = \sigma(1 - \mu(\underline{k}, \bar{k}))$ . Firm 2's price c.d.f.  $F_2^h$  is continuous on  $[p_0^h, 1]$ , while firm 1's price c.d.f.  $F_1^h$  is continuous on  $[p_0^h, 1)$  and has an atom at  $p = 1$ . Firm 1 uses  $\underline{k}$  ( $\bar{k}$ ) at prices below (above)  $\hat{p}_1^h = (1 - \mu(\underline{k}, \bar{k})) \in (p_0^h, 1)$ . The equilibrium profits are given by*

$$\pi_{h1}^* = \sigma(1 - \mu(\underline{k}, \bar{k})) \quad \text{and} \quad \pi_{h2}^* = \sigma(1 - \mu(\underline{k}, \bar{k})) \frac{1 - \sigma(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))}. \quad (6)$$

Thus, when prominence is large enough, firm 2 chooses the lowest complexity for sure to minimize the number of confused buyers and reduce its disadvantage. The prominent firm, as before, associates lower prices with the lowest complexity (at those prices it benefits from more transparency) and higher prices with highest complexity (at those prices it relies more on confused consumers). More specifically, firm 1 chooses complexity  $\underline{k}$  for all prices  $p < \hat{p}_1^h \in (p_0^h, 1)$ , and  $\bar{k}$  for all prices  $p \geq \hat{p}_1^h$ . Proposition 1 then requires that firms' pricing c.d.f.s satisfy

$F_2^h(\hat{p}_1^h) = 1 - \sigma$  and  $F_1^h(1) \leq \sigma$  (that is,  $\hat{p}_2^h \geq 1$ ).<sup>20</sup> The following example and Figure 2 illustrate the results for relatively high prominence.

**Example 2** When  $\sigma = .8$  and  $\mu(\underline{k}, \bar{k}) = .6$ , in equilibrium firm 1 chooses  $\underline{k}$  with probability  $\lambda_1^h = .145$ , while firm 2 chooses  $\underline{k}$  for sure. The two firms randomize on prices according to the following c.d.f.s, which are illustrated in Figure 2,

$$F_1^h(p) = \begin{cases} .726 - .232/p & \text{for } p \in [p_0^h, \hat{p}_1^h) \\ 1.113 - .387/p & \text{for } p \in [\hat{p}_1^h, 1] \end{cases} \quad \text{and} \quad F_2^h(p) = \begin{cases} 1 - .32/p & \text{for } p \in [p_0^h, \hat{p}_1^h) \\ 1.533 - .533/p & \text{for } p \in [\hat{p}_1^h, 1] \end{cases} ,$$

where  $p_0^h = .32$  and  $\hat{p}_1^h = .4$ . Firm 1 makes profit  $\pi_{h1}^* = .32$  and firm 2 makes profit  $\pi_{h2}^* = .232$ . Firm 1's atom at  $p = 1$  is  $\phi^h = .274$ .

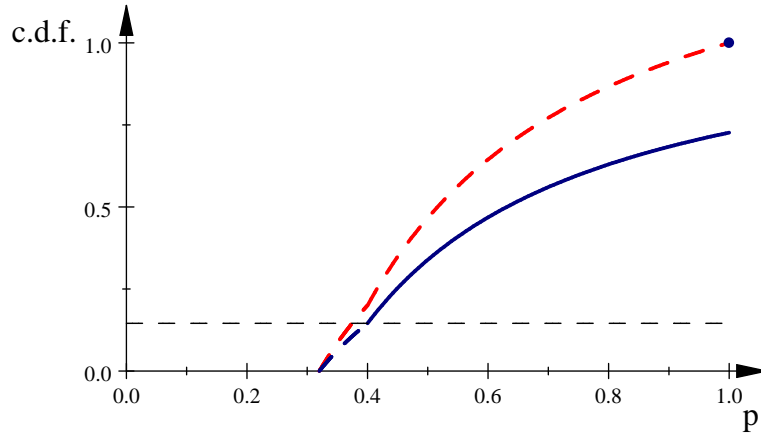


Figure 2: Firms' price c.d.f.s for  $\sigma = .8$  and  $\mu(\underline{k}, \bar{k}) = .6$ .  $F_1^h(p)$  is the blue line and  $F_2^h(p)$  is the red line. The dashed lines correspond to prices associated with  $\underline{k}$ .

Propositions 2 and 3 indicate that neither individual profits nor industry profit are globally monotonic in the level of prominence. Examples 1 and 2 show that an increase in prominence (from  $\sigma = .6$  to  $\sigma = .8$ ) might be beneficial to the consumers as industry profits decrease (from .603 to .552). When firm 1 is relatively more salient, the less prominent firm competes more fiercely, by choosing lower prices (in the first order stochastic dominance sense) and by increasing market transparency. The lower the complexity of the market, the larger the pool of potential buyers for the less prominent firm. The examples suggest that markets where new entrants compete with an incumbent firm which is prominent enough may be more competitive than markets where the differences in prominence between suppliers are relatively smaller. This is consistent with the empirical findings in Giulietti, Waterson, and Wildenbeest (2014). They show that in the British electricity markets between 2002 and 2005 new entrants have lower incentives to price aggressively as they become more prominent.

<sup>20</sup>If  $F_1^h(1) > \sigma$  then, as by Lemma 1  $F_1^h(\hat{p}_2^h) = \sigma$ ,  $\hat{p}_2^h < 1$  and the candidate  $\lambda_2^h = F_2^h(\hat{p}_2^h) < 1$ . But this is inconsistent with an equilibrium where firm 2 chooses  $\underline{k}$  for sure.



However, when Condition 2 does not hold and so firm 1 is prominent enough, both firm 1's profit ( $\pi_{h1}^*$ ) and total industry profit ( $\pi_{h1}^* + \pi_{h2}^*$ ) are strictly increasing in  $\sigma$ .<sup>21</sup> As total surplus is constant, this implies that consumer surplus decreases in  $\sigma$  in this case. When Condition 2 holds and firm 1's level of prominence is relatively low, firm 2's profit  $\pi_2^*$  is strictly decreasing in  $\sigma$  and consumer surplus is not monotonic in  $\sigma$ .<sup>22</sup> Figure 3 illustrates individual and aggregate profits as functions of the level of prominence in a numerical example where  $\mu(\underline{k}, \bar{k}) = 0.6$ ; in this case, total industry profit is lowest and consumers surplus highest at  $\sigma = 0.754$  which is the cut-off prominence value for the two types of equilibria presented in Propositions 2 and 3.

**Example 3** Suppose  $\mu(\underline{k}, \bar{k}) = 0.6$ . Then, Condition 2 holds iff  $\sigma \in (0.5, 0.754)$ .

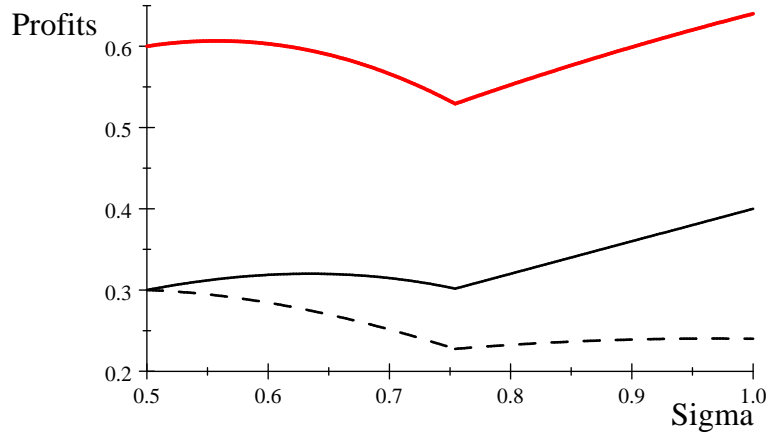


Figure 3: Profit of firm 1 (black solid), firm 2 (dashed) and total profit (red) for  $\mu(\underline{k}, \bar{k}) = .6$

Firm 1's probability of choosing the lowest complexity ( $\lambda_1$ ) decreases in  $\sigma$ . Firm 2's probability of choosing the lowest complexity ( $\lambda_2$ ) weakly increases in  $\sigma$ :  $\lambda_2$  strictly increases in  $\sigma$  when Condition 2 holds and it is constant otherwise. It can also be shown that the lower bound of the firms' price support is not monotonic in  $\sigma$ , while the cut-off prices of firm 1 and 2, respectively, are weakly decreasing and increasing in  $\sigma$ .<sup>23</sup> The likelihood that the prominent firm chooses the monopoly price strictly increases in  $\sigma$ .

Combining the results in Propositions 2 and 3, we analyze next the role of prominence.

**Corollary 1** In the mixed strategy equilibrium, (i) the more prominent firm makes higher profits than the rival; (ii) the price distribution of the prominent firm first order stochastically dominates the one of the less prominent firm; (iii) the more prominent firm's average price is

<sup>21</sup>Denote by  $\pi_h^*$  ( $= \pi_{h1}^* + \pi_{h2}^*$ ) total industry profits when Condition 2 does not hold. Then  $\partial \pi_h^* / \partial \sigma = \{2 - \sigma(1 - \mu)[4 - \sigma - [2 - \sigma(2 - \sigma)(1 - \mu)]]\} / [1 - \sigma(1 - \sigma)(1 - \mu)]^2 > 0$ .

<sup>22</sup>Numerical simulations suggest that consumer surplus is U-shaped over the range of  $\sigma$ 's where Condition 2 holds.

<sup>23</sup> $\hat{p}_1$  is strictly decreasing in  $\sigma$ , while  $\hat{p}_1^h = 1 - \mu(\underline{k}, \bar{k})$  and so independent of  $\sigma$ .  $\hat{p}_2$  is strictly increasing in  $\sigma$ , while  $\hat{p}_2^h = 1$  and so constant.

higher than that of the less prominent firm, and (iv) the less prominent firm chooses the lowest complexity ( $\underline{k}$ ) with higher probability than the rival.

The prominent firm attracts a larger share of confused consumers, and so it benefits more from market-wide confusion. For this reason, it chooses the highest level of complexity with higher probability than its rival, has lower incentives to compete for the expert consumers, and therefore it chooses a higher average price. The combined effect of charging higher prices (in the first order stochastic dominance sense) and attracting a higher share of the confused consumers allows the prominent firm to make higher profits in equilibrium. Confused consumers' bias in favor of the prominent firm appears to be inconsistent with the ranking of the average prices. However, our next result shows that their behavior is consistent with the ranking of the average prices, conditional on these being associated with the lowest complexity ( $\underline{k}$ ).

**Corollary 2 Consumer Confusion.** *In the mixed strategy equilibrium, the more prominent firm chooses a lower cut-off price - below which it uses the lowest level of price complexity  $\underline{k}$  - than its rival ( $\hat{p}_1 < \hat{p}_2$  when Condition 2 holds and  $\hat{p}_1^h < \hat{p}_2^h$  when it does not). Furthermore, conditional on choosing the lowest complexity, the more prominent firm offers a lower average price than its rival ( $E(p_1 | p_1 < \hat{p}_1) < E(p_2 | p_2 < \hat{p}_2)$  when Condition 2 holds, and  $E(p_1 | p_1 < \hat{p}_1^h) < E(p_2 | p_2 < \hat{p}_2^h)$  when it does not).*

We prove this corollary in appendix A.5 and sketch here the intuition. Conditional on pricing strictly below the monopoly level ( $p = 1$ ), the price c.d.f.s of the two firms are identical. This can be seen in Examples 1 and 2. Combined with the fact that, in equilibrium, the cut-off price below which firm 1 chooses  $\underline{k}$  is lower than the corresponding cut-off of firm 2 (that is,  $\hat{p}_1 < \hat{p}_2$ , if Condition 2 holds, and  $\hat{p}_1^h < \hat{p}_2^h$ , if it does not), this proves the corollary.

One interpretation of our model is that understanding a price associated with the high complexity level  $\bar{k}$  is costly for the consumers (e.g., requires time or effort). Consumers may opt out of this costly evaluation process, in which case they end up confused and randomize their choice. In contrast, understanding a price associated with the low complexity level  $\underline{k}$  is costless. As the cost of evaluating two prices associated with  $\bar{k}$  is higher than that of evaluating one, more consumers are confused when both firms use  $\bar{k}$  than when only one does (which is consistent with  $1 - \mu(\bar{k}, \bar{k}) > 1 - \mu(\underline{k}, \bar{k})$ ).<sup>24</sup> Consider a consumer who looks for the lowest expected price available in the market and can assess prices associated with  $\underline{k}$ , but not those associated with  $\bar{k}$ . Then, conditional on  $\underline{k}$ , the prominent firm's expected price is lower than that of the rival and so the consumer is more likely to choose its product.<sup>25</sup> If the confused

<sup>24</sup>See also Chioveanu and Zhou (2013) for a related discussion.

<sup>25</sup>Consumers may gather information on prices through recommendations on online forums or social networks. Transparent-price offers may be more likely to be recommended as more people understand them and they are

consumers are ‘myopic’ in this sense and only take into account the average price conditional on it being associated with the lowest price complexity level, the prominent firm enjoys a larger share of confused consumers. Such behavior could be further related to a stochastic utility model in which confused consumers ‘approximate’ the surplus from firm  $i$ ’s product to  $v - E(p_i | p_i < \hat{p}_i) + \varepsilon_i$ , where  $\varepsilon_i$  is a random variable that captures confusion due to complexity. For a thorough discussion of this class of models, see Anderson, De Palma, and Thisse (1992).

## 5 Extensions

### 5.1 Biased Experts

This part explores the robustness of our results in a modified model where experts’ choices are also affected by firm prominence. More specifically, although expert consumers are able to assess prices correctly, they are biased in favor of the prominent firm and purchase its product so long as  $p_1 < p_2 + d$ , where  $p_i$  is firm  $i$ ’s price, for  $i = 1, 2$ ,  $d \in (0, 1)$  is a ‘prominence premium’, and  $p_1 < 1$ . Like before, consumers’ valuation for the product is not affected by prominence (i.e., they face a budget constraint). But, so long as the price does not exceed their valuation ( $v = 1$ ), the experts are ready to pay a premium for the prominent brand.<sup>26</sup> This set-up could also be interpreted as one where consumers have a default-bias and, although they can correctly compare prices, the experts have switching cost  $d$ .

We show in appendix A.6 that if firm  $j$  employs a mixed strategy  $\xi_j^b = F_j^b(p_j)H_j^b(k_j | p_j)$ , where  $F_j^b(p_j)$  is the marginal c.d.f. of firm  $j$ ’s random price defined on support  $T_j^b$  and  $H_j^b(k_j | p_j)$  is the conditional c.d.f. of firm  $j$ ’s complexity level, then it is a best response for firm  $i$  to randomize on price complexity levels. We provide there further discussion using a numerical example which illustrates that, for some values of  $d$ , there is an equilibrium where (i) firms randomize on both prices and complexity levels, (ii) prices below (above) a cut-off level are associated with the lowest (highest) complexity, and (iii) the average price of the prominent firm conditional on using the lowest complexity is lower than that of the rival. So, in line with our main analysis, there is a positive relationship between prices and price complexity levels and consumers’ bias in favour of the prominent firm is consistent with the ranking of average prices that firms offer with the lowest complexity.

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associated with relatively low prices. Based on this, the offers of the prominent firm may appear to be better.

<sup>26</sup>However, like in our benchmark analysis, empirical evidence suggests that prominence is more likely to affect confused consumers rather than the experts. In a study of physically homogeneous products (including health products and retail food and drinks), Bronnenberg, Dubé, Gentzkow, and Shapiro (2015) find that expert consumers are considerably less likely than average consumers to pay a premium for branded products.

## 5.2 Alternative Confusion Technologies

The main analysis assumes that a marginal increase in firm  $i$ 's complexity reduces the fraction of experts in the market but does not alter the effectiveness of the rival's marginal increase in price complexity on consumers, that is,  $\partial^2\mu/\partial k_1\partial k_2 = 0$ . Below we prove that there exists an equilibrium which is qualitatively consistent with the one in the main analysis whenever  $\partial^2\mu/\partial k_1\partial k_2 > 0$ . As  $\partial\mu/\partial k_i = \mu_i < 0$ , this condition requires that the magnitude of the marginal impact of firm  $i$ 's complexity be decreasing in firm  $j$ 's complexity ( $\partial|\mu_i|/\partial k_j < 0$ ).<sup>27</sup> More specifically, we show that if the rival uses a mixed strategy with a positive relationship between price and price complexity, it is a best response for a firm to associate prices below a threshold with the lowest complexity and prices above it with the highest complexity.

Suppose firm  $j$  uses a mixed strategy  $\xi_j$  so that  $dk_j(p_j)/dp_j \geq 0$ . Consider the expected profits of firm  $i$  presented in section 3:

$$\begin{aligned} \pi_i(p_i, k_i, \xi_j) = & p_i \int_{p_i}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j | p_j > p_i) \right) dF_j(p_j) + \\ & p_i s_i \left[ 1 - \int_{p_0}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_i, k_j(p_j)) dH_j(k_j | p_j) \right) dF_j(p_j) \right]. \end{aligned}$$

The f.o.c. of firm  $i$ 's expected profit maximization w.r.t.  $k_i$  requires that

$$p_i \left( \int_{p_i}^1 \mathbb{E}(\mu_i(p_j) | p_j > p_i) dF_j(p_j) - s_i \int_{p_0}^1 \mathbb{E}(\mu_i(p_j)) dF_j(p_j) \right) = 0, \quad (7)$$

where  $\partial\mu(k_i, k_j(p_j))/\partial k_i \equiv \mu_i(k_i, k_j(p_j))$  gives the marginal impact of  $k_i$  on  $\mu$  and  $\mathbb{E}(\mu_i(p_j) | p_j > p_i) = \int_{\underline{k}}^{\bar{k}} \mu_i(k_i, k_j(p_j)) dH_j(k_j | p_j > p_i)$  is the expected marginal impact of an increase in  $k_i$  on the fraction of experts conditional on firm  $j$ 's price being higher than  $p_i$ . For given  $\xi_j$ ,  $\int_{p_0}^1 \mathbb{E}(\mu_i(p_j)) dF_j(p_j)$  - the overall expected marginal impact of an increase in  $k_i$  on the fraction of experts - is a constant. At  $p_i = p_0$ , the term in brackets becomes  $(1 - s_i) \int_{p_0}^1 \mathbb{E}(\mu_i(p_j)) dF_j(p_j) < 0$  and when  $p_i \rightarrow 1$ , it converges to  $-s_i \int_{p_0}^1 \mathbb{E}(\mu_i(p_j)) dF_j(p_j) > 0$ . So, there is at least one price  $\hat{p}_i \in (p_0, 1)$  which satisfies (7). Moreover,  $\hat{p}_i$  is unique if

$$\begin{aligned} d \left( \int_{p_i}^1 \mathbb{E}(\mu_i(p_j) | p_j > p_i) dF_j(p_j) \right) / dp_i = \\ \int_{p_i}^1 \left[ \frac{d(\mathbb{E}(\mu_i(p_j) | p_j > p_i))}{dp_i} \right] dF_j(p_j) - \mu_i^e(p_i) F_j'(p_i) > 0, \end{aligned}$$

where the equality follows from Leibniz's Rule. As  $-\mu_i^e(p_i) > 0$  and  $F_j'(p_i) > 0$ , this condition

<sup>27</sup> An example of confusion technology which satisfies this assumption is  $\mu(k_1, k_2) = (\underline{k})^2 / (k_1 k_2)$ .

holds if  $d\mathbb{E}(\mu_i(p_j) \mid p_j > p_i)/dp_i > 0$ . But, as  $dk_j(p_j)/dp_j > 0$ , a sufficient condition is then  $\partial\mu_i(k_i, k_j)/\partial k_j = \partial^2\mu(k_i, k_j)/\partial k_i\partial k_j > 0$ . Hence, whenever  $\partial^2\mu/\partial k_i\partial k_j > 0$  there exists a unique  $\hat{p}_i \in (p_0, 1)$  which satisfies (7) and it follows that firm  $i$ 's complexity level choice is

$$k_i(p) = \begin{cases} \underline{k} & \text{if } p < \hat{p}_i \\ \bar{k} & \text{if } p > \hat{p}_i \\ k, \forall k \in [\underline{k}, \bar{k}] & \text{if } p = \hat{p}_i, \end{cases}$$

whenever  $\hat{p}_i$  belongs to  $T_j$  the support of  $F_j$ . Lemma 1 implies that at least one of the cut-off prices  $\hat{p}_i$  belongs to  $T_j$ . This shows that a mixed strategy equilibrium like the one analyzed in our benchmark model exists for a more general confusion technology.

## 6 Conclusions

We analyze the interplay between consumer confusion due to price complexity and firm prominence in a model where two firms compete by simultaneously choosing prices and the complexity of their price offers. One of the firms enjoys a higher level of prominence, which may be due to higher brand recognition, industry dynamics, or advertising effort/spending. Price complexity limits the comparability of firms' price offers and so, in its presence, some consumers are informed of all prices and able to identify the best offer, while the others may get confused and are unable to assess firms' offers. Firms' price complexity choices determine the share of confused consumers. These consumers shop at random and favour the more prominent firm, in the sense that they are more likely to buy from it.

In equilibrium there is dispersion in both prices and complexity levels. The nature of the equilibrium depends on the level of prominence. For moderate levels of prominence, both firms mix on price complexity levels, while for high levels of prominence, the less prominent firm chooses the lowest price complexity. The prominent firm makes higher profits, chooses higher prices on average and the lowest complexity level with lower probability, and sets the monopoly price with positive probability. However, a decrease in prominence may increase industry profits and harm consumers. In addition, conditional on choosing the lowest complexity, the prominent firm sets a lower price, on average, which is consistent with confused consumers' behavior. The perceptions of these consumers may be biased because they ignore complex prices and consider only the most transparent ones.

We show that our results are robust in a setting where the expert consumers are also biased

towards the prominent firm and so willing to pay a premium. We also find that a qualitatively similar equilibrium exists with alternative confusion technologies if the marginal impact of an increase in one firm's price complexity increases in the rival's complexity level. Our framework may be used to endogeneize firm prominence, analyze oligopoly market outcomes, or explore the role of complexity in markets where price format differentiation, rather than complexity, is a source of confusion.

## A Appendix

### A.1 Properties of the Pricing Distribution Functions

**Lemma 3** *The supports of the pricing c.d.f.s,  $T_1$  and  $T_2$  are both connected intervals (i.e., there are no gaps in either of them).*

**Proof.** Let  $\tilde{T}_1$  and  $\tilde{T}_2$  be the convex hulls of  $T_1$  and  $T_2$ , respectively. Let  $\tilde{T} = \tilde{T}_1 \cap \tilde{T}_2$ . (i) Suppose there is a gap  $G \subset \tilde{T}$ . (a) Suppose  $G \subset T_i$  for  $i = 1, 2$ . Let  $A = \{p \in T_1 \cup T_2 \mid p \leq \inf G\}$  and  $p_a = \max A$  and  $p' = \sup G$ . Clearly,  $p_a \in T_i$  for at least one  $i$  and  $F_j(p_a) = F_j(p')$ . But then  $\pi_i(p', k, \xi_j) > \pi_i(p_a, k, \xi_j)$ . A contradiction. (b) Suppose  $G \subset T_i$  but  $G \cap T_j = \emptyset$ . Let  $p'' = \inf G$ . Clearly,  $\pi_j(p', k, \xi_i) > \pi_j(p'', k, \xi_i)$ . A contradiction. (ii) Suppose there is a gap  $Z \subset \tilde{T}_i \setminus \tilde{T}$ . Let  $B = \{p \in T_i \mid p \leq \inf Z\}$  and  $p_b = \max B$ . Let  $C = \{p \in T_i \mid p \geq \sup Z\}$  and  $p_c = \min C$ . Clearly,  $\pi_i(p_c, k, \xi_j) > \pi_i(p_b, k, \xi_j)$ . A contradiction. ■

**Lemma 4** *Neither firm can have a mass point in the interior or at the lower bound of the other one's price c.d.f. support. Moreover, firm  $i$  cannot have a mass point at the upper bound of  $T_j$  if firm  $j$  has a mass point there.*

**Proof.** Suppose firm  $j$  has a mass point at some  $p' \in T_j$  with  $p' < \max T_j$ . It must be that  $p' \in T_i$ , otherwise firm  $j$  would have incentives to move the mass point to a higher price. Then, firm  $i$  is better off deviating to  $p' - \epsilon$  as there is a discrete increase in market share and only a marginal decrease in price. Recall that  $\mu(k_1, k_2) > 0$ , that is, there are always some price aware consumers. The above argument applies also at  $p' = \max T_j$ , so that both firms cannot have a mass point at the upper bound of  $j$ 's support. ■

**Lemma 5** *In equilibrium, it must hold that  $T_1 = T_2 = [p_0, p^h]$  for  $p_0 < p^h \leq 1$ .*

**Proof.** Suppose  $\exists p' \in T_i$  such that  $p' \notin T_j$ . Let  $A = \{p \in T_j \mid p > p'\}$ . Suppose  $A \neq \emptyset$  and let  $p'' = \min A$ . Then,  $\pi_i(p'', k_i, \xi_j) > \pi_i(p', k_i, \xi_j)$  as firm  $i$  does not lose any market share when deviating from  $p'$  to  $p''$ . If  $A = \emptyset$ , then it must be that  $p' > \max T_j$ . If  $p' < 1$ , then a similar argument to the one above applies and  $\pi_i(1, k_i, \xi_j) > \pi_i(p', k_i, \xi_j)$ . If  $p' = 1$ , then

Lemma 4 implies that at least one of the firms does not have a mass point at  $\max T_j$ . Then, that firm can profitably deviate to  $p' = 1$  from  $p = \max T_j$ . ■

**Lemma 6** *In equilibrium,  $\sup T_1 = \sup T_2 = 1$ .*

**Proof.** Suppose  $\sup T_i < 1$ . By Lemma 5,  $\sup T_j = \sup T_i = p_h$ . By Lemma 4, both firms cannot have mass points at  $p_h$ . But, then, at least one firm sells only to its share of confused consumers at  $p_h$  and it is clearly better off charging a higher price  $p = 1$ . A contradiction. ■

## A.2 Expected Profits

### Derivation of Firm 1's Expected Profit

- Suppose firm 1 chooses a price  $p \in [p_0, \hat{p}_1]$ .

Using expression (1), together with  $\mu(\underline{k}, \underline{k}) = 1$  and  $F_2(\hat{p}_2) = \lambda_2$ , we obtain firm 1's expected profit at  $p = p_0$  and when  $p \rightarrow \hat{p}_1$ ,

$$\pi_1(p_0, \underline{k}) = p_0 [1 - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))] \text{ and} \quad (\text{A1})$$

$$\lim_{p \nearrow \hat{p}_1} \pi_1(p, \underline{k}) = \hat{p}_1 [\sigma - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))] . \quad (\text{A2})$$

- Suppose firm 1 chooses a price  $p \in [\hat{p}_1, \hat{p}_2]$ .

By Proposition 1, it associates prices in this range with complexity level  $\bar{k}$ . Then, its expected profit is

$$\begin{aligned} \pi_1(p, \bar{k}) = & p \{ (F_2(\hat{p}_2) - F_2(p)) \mu(\underline{k}, \bar{k}) + (1 - F_2(\hat{p}_2)) \mu(\bar{k}, \bar{k}) + \\ & \sigma [F_2(\hat{p}_2)(1 - \mu(\underline{k}, \bar{k})) + (1 - F_2(\hat{p}_2))(1 - \mu(\bar{k}, \bar{k}))] \} . \end{aligned} \quad (\text{A3})$$

The expected number of confused consumers is the term in square brackets. Firm 1 serves a fraction  $\sigma$  of this group. Firm 1 also serves the expert consumers if firm 2 chooses a higher price. With probability  $F_2(\hat{p}_2) - F_2(p)$ , there are  $\mu(\underline{k}, \bar{k})$  experts while, with probability  $1 - F_2(\hat{p}_2)$ , there are  $\mu(\bar{k}, \bar{k})$ ; this is reflected by the first two terms in curly brackets. Recall that  $\partial^2 \mu / \partial k_1 \partial k_2 = 0$ , so  $1 - \mu(\underline{k}, \bar{k}) = \mu(\underline{k}, \bar{k}) - \mu(\bar{k}, \bar{k})$  and, using  $F_2(\hat{p}_2) = \lambda_2$ , it follows that  $\pi_1(p, \bar{k}) = \lim_{p \nearrow \hat{p}_1} \pi_1(p, \underline{k})$ , as given in (A2). Also, as by Proposition 1,  $F_2(\hat{p}_1) = 1 - \sigma$ , the expected profit at  $p = \hat{p}_2$  becomes

$$\pi_1(\hat{p}_2, \bar{k}) = \hat{p}_2 \{ (1 - \lambda_2) - (1 - \mu(\underline{k}, \bar{k})) [2(1 - \sigma - \lambda_2) + \sigma \lambda_2] \} . \quad (\text{A4})$$

- Suppose firm 1 chooses a price  $p \in (\widehat{p}_2, 1]$ .

By Proposition 1, it associates prices in this range with complexity level  $\bar{k}$ . Then, its expected profit is

$$\pi_1(p, \bar{k}) = p\{(1 - F_2(p))\mu(\bar{k}, \bar{k}) + \sigma[F_2(\widehat{p}_2)(1 - \mu(\underline{k}, \bar{k})) + (1 - F_2(\widehat{p}_2))(1 - \mu(\bar{k}, \bar{k}))]\} .$$

Echoing previous reasoning, with probability  $F_2(\widehat{p}_2)$ , firm 2 chooses  $\underline{k}$ , in which case there are  $\mu(\underline{k}, \bar{k})$  informed and  $1 - \mu(\underline{k}, \bar{k})$  confused consumers. A share  $\sigma$  of the confused purchases from firm 1, the prominent seller. The experts do not purchase from firm 1 as firm 2's price is lower. With probability  $1 - F_2(\widehat{p}_2)$ , firm 2 chooses  $\bar{k}$ , so there are  $\mu(\bar{k}, \bar{k})$  informed and  $1 - \mu(\bar{k}, \bar{k})$  confused consumers. A share  $\sigma$  of confused consumers buy from firm 1. The experts purchase from firm 1 if it offers a lower price, which happens with probability  $1 - F_2(p)$ . The first term in curly brackets captures the expected number of experts, while the term in square brackets gives the expected number of confused consumers. As  $1 - \mu(\underline{k}, \bar{k}) = \mu(\underline{k}, \bar{k}) - \mu(\bar{k}, \bar{k})$  and  $F_2(\widehat{p}_2) = \lambda_2$ , firm 1's expected profit becomes

$$\pi_1(p, \bar{k}) = p\{1 - \lambda_2 - (1 - \mu(\underline{k}, \bar{k})) [2(1 - \sigma - \lambda_2) + \sigma \lambda_2]\} . \quad (\text{A5})$$

It can be checked that  $\lim_{p \searrow \widehat{p}_2} \pi_1(p, \bar{k}) = \pi_1(\widehat{p}_2, \bar{k})$  as presented in (A4).

### Derivation of Firm 2's Expected Profit

- Suppose firm 2 chooses a price  $p \in [p_0, \widehat{p}_1)$ .

By Proposition 1, it associates prices in this range with complexity level  $\underline{k}$ . Then, its expected profit is

$$\begin{aligned} \pi_2(p, \underline{k}) &= p\{(F_1(\widehat{p}_1) - F_1(p))\mu(\underline{k}, \underline{k}) + (1 - F_1(\widehat{p}_1))\mu(\underline{k}, \bar{k}) + \\ &\quad (1 - \sigma)[F_1(\widehat{p}_1)(1 - \mu(\underline{k}, \underline{k})) + (1 - F_1(\widehat{p}_1))(1 - \mu(\underline{k}, \bar{k}))]\} . \end{aligned} \quad (\text{A6})$$

With probability  $F_1(\widehat{p}_1)$ , firm 1 chooses  $\underline{k}$ , so that there are  $\mu(\underline{k}, \underline{k})$  informed and  $1 - \mu(\underline{k}, \underline{k})$  confused consumers. A share  $1 - \sigma$  ( $< \sigma$ ) of the confused purchases from firm 2, the less prominent seller. The experts purchase from firm 2 if firm 1's price is higher, which happens with probability  $F_1(\widehat{p}_1) - F_1(p)$ . With probability  $1 - F_1(\widehat{p}_1)$ , firm 1 chooses  $\bar{k}$ , so there are  $\mu(\underline{k}, \bar{k})$  informed and  $1 - \mu(\underline{k}, \bar{k})$  confused consumers. All experts purchase from firm 2 as it offers a lower price (firm 1 associates  $\bar{k}$  with prices higher than  $\widehat{p}_1$ ) and so does a share  $1 - \sigma$



of the confused consumers. The first two terms in the curly brackets capture the expected number of experts, whereas the term in square brackets gives the expected number of confused consumers. Using  $\mu(\underline{k}, \underline{k}) = 1$  and  $F_1(\widehat{p}_1) = \lambda_1$ , it follows that,

$$\pi_2(p_0, \underline{k}) = p_0[1 - \sigma(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))] \quad \text{and} \quad \lim_{p \nearrow \widehat{p}_1} \pi_2(p, \underline{k}) = \widehat{p}_1(1 - \lambda_1)[1 - \sigma(1 - \mu(\underline{k}, \bar{k}))] . \quad (\text{A7})$$

- Suppose firm 2 chooses a price  $p \in [\widehat{p}_1, \widehat{p}_2]$ .

By Proposition 1, it associates prices in this range with complexity level  $\underline{k}$ . Then, firm 2's expected profit becomes

$$\begin{aligned} \pi_2(p, \underline{k}) &= p\{(1 - F_1(p))\mu(\underline{k}, \bar{k}) + (1 - \sigma)[F_1(\widehat{p}_1)(1 - \mu(\underline{k}, \underline{k})) + (1 - F_1(\widehat{p}_1))(1 - \mu(\underline{k}, \bar{k}))]\} \\ &= p[(1 - F_1(p))\mu(\underline{k}, \bar{k}) + (1 - \sigma)(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))] . \end{aligned}$$

The logic behind the expression above is similar to the one for (A6), with the difference that when firm 1 uses  $\underline{k}$  there are  $\mu(\underline{k}, \underline{k}) = 1$  informed consumers and when it uses  $\bar{k}$  there are  $\mu(\underline{k}, \bar{k})$ . Clearly, when firm 1 uses  $\underline{k}$ , it attracts all the experts, as it offers a lower price. It is easy to check that  $\pi_2(\widehat{p}_1, \bar{k}) = \lim_{p \nearrow \widehat{p}_1} \pi_2(p, \underline{k})$  as given by (A7), and that the expected profit at  $\widehat{p}_2$  is

$$\pi_2(\widehat{p}_2, \underline{k}) = \widehat{p}_2(1 - \sigma)[1 - \lambda_1(1 - \mu(\underline{k}, \bar{k}))] . \quad (\text{A8})$$

- Suppose firm 2 chooses a price  $p \in (\widehat{p}_2, 1]$ .

By Proposition 1, it associates prices in this range with complexity level  $\bar{k}$ . Then, its expected profit becomes

$$\begin{aligned} \pi_2(p, \bar{k}) &= p\{(1 - F_1(p))\mu(\bar{k}, \bar{k}) + (1 - \sigma)[F_1(\widehat{p}_1)(1 - \mu(\underline{k}, \bar{k})) + (1 - F_1(\widehat{p}_1))(1 - \mu(\bar{k}, \bar{k}))]\} \\ &= p\{(1 - F_1(p))(2\mu(\underline{k}, \bar{k}) - 1) + (1 - \sigma)(2 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))\} . \end{aligned} \quad (\text{A9})$$

### A.3 Equilibrium Analysis

#### Price Ratios Using the Firms' Constant Profit Conditions

In equilibrium, each firm  $i$ 's expected profit for any price-complexity combination  $(p, k_i)$ , which is assigned positive density in equilibrium, must be constant.

Using (A1) - (A4), the constant profit conditions for firm 1 lead to the following price ratios expressed as functions of  $\lambda_2$ :

$$\frac{p_0}{\widehat{p}_1} = \frac{\sigma - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}{1 - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))} \text{ and } \frac{p_0}{\widehat{p}_2} = \frac{1 - \lambda_2 - [2(1 - \sigma)(1 - \lambda_2) - \sigma\lambda_2](1 - \mu(\underline{k}, \bar{k}))}{1 - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}.$$

Using (A7) and (A8), the constant profit conditions of firm 2 lead to the following price ratios expressed as functions of  $\lambda_1$

$$\frac{p_0}{\widehat{p}_1} = \frac{(1 - \lambda_1)[1 - \sigma(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))} \text{ and } \frac{p_0}{\widehat{p}_2} = \frac{(1 - \sigma)[1 - \lambda_1(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))}.$$

### Equilibrium $\lambda$ Values

In this part, we show that equilibrium  $\lambda_1$  is always well defined and that  $\lambda_2$  is well defined when Condition 2 holds. The expression for the  $\lambda$ 's is given in (2).

- (i) First, it is easy to see that  $\lambda_1 < \sigma$  and  $\lambda_2 > 1 - \sigma$  as  $1 > \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))$ .
- (ii) We now check that  $\lambda_i \in (0, 1)$ .

- As  $\mu(\underline{k}, \bar{k}) + \sigma^2(1 - \mu(\underline{k}, \bar{k})) > 0$ ,  $\lambda_1 > 0 \Leftrightarrow 1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k})) > 0 \Leftrightarrow 1/(1 - \mu(\underline{k}, \bar{k})) > \sigma(2 - \sigma)$ . This always holds as the RHS is lower than 1 and the LHS larger than 1.
- $\lambda_1 < 1 \Leftrightarrow (1 - \sigma)[1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))] < \mu(\underline{k}, \bar{k}) + \sigma^2(1 - \mu(\underline{k}, \bar{k})) \Leftrightarrow \sigma/(1 - \sigma)(1 - \sigma + \sigma^2) > (1 - \mu(\underline{k}, \bar{k}))$ , which always holds as the LHS is always larger than 1.
- $\lambda_2 > 0$ , by the same argument used to show that  $\lambda_1 > 0$ .
- $\lambda_2 < 1 \Leftrightarrow (1 - \sigma)/[\sigma(1 - \sigma + \sigma^2)] > (1 - \mu(\underline{k}, \bar{k}))$  which gives Condition 2.

### Mass Point at Upper Bound

If both firms' price c.d.f.s were continuous everywhere (that is, if  $F_1(1) = F_2(1) = 1$ ), then using (A5) and (A9), it would follow that  $\pi_1^* = \sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))$  and  $\pi_2^* = (1 - \sigma)(2 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))$ . Then, the lower bounds of the supports would be

$$p_0^1 = \frac{\sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}{1 - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))} > p_0^2 = \frac{(1 - \sigma)(2 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))}.$$

The inequality uses the fact that  $\lambda_1/(1 - \sigma) = \lambda_2/\sigma$ .<sup>28</sup> But, this contradicts Lemma 5. Suppose now that firm 2 had a mass point, so that  $F_2(1) < 1$ . By Lemma 4, it must be that  $F_1(1) = 1$

<sup>28</sup>It can then be reduced to  $1 - \mu(\underline{k}, \bar{k}) < (2\sigma - \lambda_2)/[2\sigma - \lambda_2 + \sigma(1 - \sigma)(\sigma - \lambda_2)]$ . But as  $\lambda_2 > \sigma$  for  $\sigma \geq 1/2$  the RHS is larger than 1, while the LHS is smaller than 1.

and firm 2's profit is  $\pi_2^* = (1 - \sigma)(2 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))$ . But then if firm 2 deviates to  $p_0^1$ , it makes profits  $[1 - \sigma(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))]p_0^1 > (1 - \sigma)(2 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))$ . A contradiction.

So, it must be that firm 2's price c.d.f. is continuous everywhere, while firm 1 has a mass point at  $p = 1$ . Then, at  $p = 1$ , firm 1's expected profit is

$$\pi_1(1, \bar{k}) = \sigma[1 - \lambda_2\mu(\underline{k}, \bar{k})] - (1 - \lambda_2)\mu(\bar{k}, \bar{k}) = \sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k})) .$$

## Equilibrium Profits and Boundary Prices

First we present the boundary price  $p_0$  and the cut-off prices  $\hat{p}_1$  and  $\hat{p}_2$  as functions of  $\lambda_2$  and check that they are consistent with Condition 1.

$$p_0 = \frac{\sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}{1 - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))} \quad \text{and} \quad \hat{p}_1 = \frac{\sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}{\sigma - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))} ,$$

$$\hat{p}_2 = \frac{\sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}{1 - \lambda_2 - [2(1 - \sigma - \lambda_2) + \sigma\lambda_2](1 - \mu(\underline{k}, \bar{k}))}$$

$$= \frac{\sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))}{(1 - \lambda_2)[1 - (2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))] + \sigma(1 - \mu(\underline{k}, \bar{k}))} .$$

We focus on a situation where both firms randomize on prices and complexity, so  $\lambda_2 \in (0, 1)$ . Also, by Proposition 1,  $F_2(\hat{p}_1) = 1 - \sigma$ . As  $\hat{p}_1 < \hat{p}_2$ , it must be that that  $F_2(\hat{p}_1) = 1 - \sigma < \lambda_2 = F_2(\hat{p}_2)$  (see Lemmas 3 and 4).

- $\hat{p}_1 > p_0 \Leftrightarrow 1 - \sigma > 0$ , so it clearly holds.
- $\hat{p}_2 > p_0 \Leftrightarrow -\lambda_2 - (1 - \lambda_2)(1 - \sigma)(1 - \mu(\underline{k}, \bar{k})) < 0$  which holds for  $\lambda_2 \in (0, 1)$ .
- $\hat{p}_1 < 1 \Leftrightarrow -\sigma\lambda_2\mu(\underline{k}, \bar{k}) < (2\sigma - 1)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))$  which holds for  $\lambda_2 \in (0, 1)$ .
- $\hat{p}_2 < 1 \Leftrightarrow (2\mu(\underline{k}, \bar{k}) - 1)(\lambda_2 - 1) < 0$ .
- $\hat{p}_2 > \hat{p}_1 \Leftrightarrow \sigma - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k})) > (1 - \lambda_2) - (1 - \mu(\underline{k}, \bar{k}))[2(1 - \sigma - \lambda_2) + \sigma\lambda_2] \Leftrightarrow 1 - \sigma - \lambda_2 < 0$ .

Below we check that the equilibrium profits are well defined and present the equilibrium values of the cut-off prices.

►  $\pi_1^*$  given in (3) is well defined. It is easy to see that  $1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k})) > 0$ . Also, under Condition 2,  $[2 - \sigma - \sigma(3 - 2\sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))] > 0$ . It follows that  $\pi_1^* > 0$ . Furthermore, as

$$\frac{2 - \sigma - \sigma(3 - 2\sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} < \frac{1}{\sigma(1 - \mu(\underline{k}, \bar{k}))} \Leftrightarrow$$

$$[1 - (2 - \sigma)\sigma(1 - \mu(\underline{k}, \bar{k}))]^2 + (2\sigma - 1)\sigma^2(1 - \mu(\underline{k}, \bar{k}))^2 > 0 ,$$

it follows that  $\pi_1^* < 1$ .

►  $\pi_2^*$  given in (5) is well defined. Under Condition 2, as  $\sigma > 1/2$ , it follows that  $2 - \sigma - \sigma(\sigma^2 - 2\sigma + 3)(1 - \mu(\underline{k}, \bar{k})) > 0$ . It is then straightforward that  $\pi_2^* > 0$ . Noting that  $\pi_2^* < \pi_1^*$  as  $1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k})) > 1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k})) \Leftrightarrow \sigma > 1/2$ , it follows that  $\pi_2^* < 1$ .

► The expressions for  $\hat{p}_1$  and  $\hat{p}_2$  are presented below.

$$\hat{p}_1 = \frac{\sigma(1 - \mu(\underline{k}, \bar{k}))[2 - \sigma - \sigma(3 - 2\sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))]}{\sigma + (\sigma^3 - 3\sigma^2 + 2\sigma - 1)(1 - \mu(\underline{k}, \bar{k})) + \sigma(1 - \sigma)(1 - \sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))^2} \quad (\text{A10})$$

$$\hat{p}_2 = \frac{\sigma(1 - \mu(\underline{k}, \bar{k}))[2 - \sigma - \sigma(\sigma^2 - 2\sigma + 3)(1 - \mu(\underline{k}, \bar{k}))]}{(1 - \sigma)[1 - (1 - \sigma)(2 + \sigma)(1 - \mu(\underline{k}, \bar{k})) + \sigma(1 - \sigma)(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))^2]} \quad (\text{A11})$$

### Equilibrium Pricing

Firm 2's c.d.f. is implicitly defined by the constant profit conditions of firm 1. These conditions can be written using the expected profits, which are presented in appendix A.2, and the equilibrium profit  $\pi_1^*$  defined in (3). There are three different price ranges to be considered, so that

$$F_2(p) = \begin{cases} F_2^L(p) & \text{for } p \in [p_0, \hat{p}_2) \\ F_2^M(p) & \text{for } p \in [\hat{p}_2, \hat{p}_1] \\ F_2^H(p) & \text{for } p \in (\hat{p}_1, 1] \end{cases} \quad .$$

Below we identify piece-wise the c.d.f., using the equilibrium  $\lambda_2$ , as presented in (2).

For prices in  $[p_0, \hat{p}_1)$ , the constant profit condition of firm 1 requires

$$p[1 - F_2(p) - (1 - \sigma)(1 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))] = \pi_1^* \quad .$$

After re-arranging the terms, we obtain

$$1 - F_2^L(p) = (1 - \mu(\underline{k}, \bar{k})) \frac{(1 - \sigma)[1 - \sigma - \sigma(1 - \sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} + \frac{\pi_1^*}{p} \quad .$$

For prices in the middle range  $[\hat{p}_1, \hat{p}_2]$ , the constant profit condition is

$$p\{\mu(\underline{k}, \bar{k})(\lambda_2 - F_2(p)) + \mu(\bar{k}, \bar{k})(1 - \lambda_2) + \sigma[\lambda_2(1 - \mu(\underline{k}, \bar{k})) + (1 - \lambda_2)(1 - \mu(\bar{k}, \bar{k}))]\} = \pi_1^* \quad .$$

After re-arranging the terms, we obtain

$$\begin{aligned} 1 - F_2^M(p) &= -\frac{(1 - \mu(\underline{k}, \bar{k}))}{\mu(\underline{k}, \bar{k})} [1 - (1 - \sigma)(2 - \lambda_2)] + \frac{\pi_1^*}{p\mu(\underline{k}, \bar{k})} \\ &= -\frac{(1 - \mu(\underline{k}, \bar{k}))}{\mu(\underline{k}, \bar{k})} \left[ 1 - (1 - \sigma) \frac{2 - \sigma - \sigma(3 - 2\sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} \right] + \frac{\pi_1^*}{p\mu(\underline{k}, \bar{k})} \quad . \end{aligned}$$

For prices in the high range  $(\widehat{p}_2, 1]$ , the constant profit condition is,

$$p [(2\mu(\underline{k}, \bar{k}) - 1)(1 - F_2(p)) + \sigma(2 - \lambda_2)(1 - \mu(\underline{k}, \bar{k}))] = \pi_1^* .$$

It follows that

$$\begin{aligned} 1 - F_2^H(p) &= -\frac{(1 - \mu(\underline{k}, \bar{k}))}{(2\mu(\underline{k}, \bar{k}) - 1)}\sigma(2 - \lambda_2) + \frac{\pi_1^*}{p(2\mu(\underline{k}, \bar{k}) - 1)} \\ &= -\frac{(1 - \mu(\underline{k}, \bar{k}))}{(2\mu(\underline{k}, \bar{k}) - 1)} \frac{\sigma[2 - \sigma - \sigma(3 - 2\sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))]}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} + \frac{\pi_1^*}{p(2\mu(\underline{k}, \bar{k}) - 1)} . \end{aligned}$$

It is straightforward to check that  $F_2(p)$  is continuous on  $[p_0, 1]$  and strictly increasing.

To pin down firm 1's c.d.f. we use the constant profit conditions for firm 2, the expected profits presented earlier in this appendix, and the equilibrium profit  $\pi_2^*$  defined in (5). As before, there are three different price ranges to be considered, so that

$$F_1(p) = \begin{cases} F_1^L(p) & \text{for } p \in [p_0, \widehat{p}_2) \\ F_1^M(p) & \text{for } p \in [\widehat{p}_2, \widehat{p}_1] \\ F_1^H(p) & \text{for } p \in (\widehat{p}_1, 1] \end{cases} .$$

We proceed to identify piece-wise the c.d.f., substituting the equilibrium  $\lambda_1$ , as presented in (2).

For prices in  $[p_0, \widehat{p}_1)$ , the constant profit condition of firm 2 requires

$$p[1 - F_1(p) - \sigma(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))] = \pi_2^* .$$

By re-arranging the terms, we get

$$1 - F_1^L(p) = (1 - \mu(\underline{k}, \bar{k})) \frac{\sigma[\sigma - (1 - \sigma)(\sigma^2 - \sigma + 1)(1 - \mu(\underline{k}, \bar{k}))]}{1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))} + \frac{\pi_2^*}{p} .$$

For prices in the middle range  $[\widehat{p}_1, \widehat{p}_2]$ , the constant profit condition is

$$p [\mu(\underline{k}, \bar{k})(1 - F_1(p)) + (1 - \sigma)(1 - \lambda_1)(1 - \mu(\underline{k}, \bar{k}))] = \pi_2^* .$$

It follows that

$$1 - F_1^M(p) = -\frac{(1 - \mu(\underline{k}, \bar{k}))}{\mu(\underline{k}, \bar{k})} \frac{(1 - \sigma)[\sigma - (1 - \sigma)(\sigma^2 - \sigma + 1)(1 - \mu(\underline{k}, \bar{k}))]}{1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))} + \frac{\pi_2^*}{p\mu(\underline{k}, \bar{k})} .$$

For prices in the high range  $(\widehat{p}_2, 1]$ , the constant profit condition is,

$$p\{2\mu(\underline{k}, \bar{k}) - 1)(1 - F_1(p)) + (1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))(2 - \lambda_1)\} = \pi_2^* .$$

After re-arranging the terms, we obtain

$$1 - F_1^H(p) = -\frac{(1 - \mu(\underline{k}, \bar{k})) (1 - \sigma)[1 + \sigma - (1 - \sigma)(2 + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))]}{2\mu(\underline{k}, \bar{k}) - 1} + \frac{\pi_2^*}{p(2\mu(\underline{k}, \bar{k}) - 1)} .$$

It is straightforward to check that  $F_1(p)$  is continuous on  $[p_0, 1)$  and strictly increasing. Furthermore, firm 1 has a mass point at  $p = 1$ ,

$$\phi \equiv 1 - F_1^H(1) = \frac{(2\sigma - 1)(1 - \mu(\underline{k}, \bar{k}))}{1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))} \in (0, 1) \text{ for } \sigma > 1/2 .$$

#### A.4 Equilibrium Analysis for High Prominence

In this subsection we focus on a situation where Condition 2 does not hold.

**Proof of Proposition 3.** When firm 1 chooses a price  $p \in [p_0^h, \widehat{p}_1^h)$ , it uses complexity level  $\underline{k}$ . Then, firm 1's expected profit in this range is  $\pi_1^h(p, \underline{k}) = p(1 - F_2^h(p))$ . Hence,  $\pi_1^h(p_0^h, \underline{k}) = p_0^h$  and  $\lim_{p \nearrow \widehat{p}_1^h} \pi_1^h(p, \underline{k}) = \sigma \widehat{p}_1^h$ . When firm 1 chooses a price  $p \in [\widehat{p}_1^h, 1)$ , it uses complexity level  $\bar{k}$ . Then, its expected profit is

$$\pi_1^h(p, \bar{k}) = p \left[ (1 - F_2^h(p))\mu(\underline{k}, \bar{k}) + \sigma(1 - \mu(\underline{k}, \bar{k})) \right] ,$$

and it follows that  $\pi_1(\widehat{p}_1^h, \bar{k}) = \sigma \widehat{p}_1^h$ . Note that the constant profit condition of firm 1 implies that  $p_0^h = \sigma \widehat{p}_1^h$ .

When firm 2 chooses a price  $p \in [p_0^h, \widehat{p}_1^h)$ , it uses complexity level  $\underline{k}$ . Then, as  $\mu(\underline{k}, \underline{k}) = 1$  and  $F_1^h(\widehat{p}_1^h) = \lambda_1^h$ , firm 2's expected profit becomes

$$\pi_2^h(p, \underline{k}) = p \left[ \lambda_1^h - F_1^h(p) + (1 - \lambda_1^h)\mu(\underline{k}, \bar{k}) + (1 - \sigma)(1 - \lambda_1^h)(1 - \mu(\underline{k}, \bar{k})) \right] .$$

It then follows that,

$$\pi_2^h(p_0^h, \underline{k}) = p_0^h \left[ 1 - \sigma(1 - \lambda_1^h)(1 - \mu(\underline{k}, \bar{k})) \right] \text{ and } \lim_{p \nearrow \widehat{p}_1^h} \pi_2^h(p, \underline{k}) = \widehat{p}_1^h(1 - \lambda_1^h) \left[ 1 - \sigma(1 - \mu(\underline{k}, \bar{k})) \right] .$$

Combining  $p_0^h = \sigma \widehat{p}_1^h$  with the constant profit condition of firm 2, we obtain the value for  $\lambda_1^h$  in the proposition.

When firm 2 chooses a price  $p \in [\widehat{p}_1^h, 1)$ , it still uses complexity level  $\underline{k}$ . Then, firm 2's expected

profit becomes

$$\pi_2^h(p, \underline{k}) = p \left[ (1 - F_1^h(p))\mu(\underline{k}, \bar{k}) + (1 - \sigma)(1 - \lambda_1^h)(1 - \mu(\underline{k}, \bar{k})) \right] .$$

By Lemma 4, both firms cannot have a mass point at  $p = 1$ . It can be checked that  $F_1^h(1) = 1$  leads to a contradiction. More precisely, if  $F_1^h(1) = 1$ , then  $p_0^h = \pi_{h1}^* = \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k})) / [1 - \sigma(1 - \mu(\underline{k}, \bar{k}))]$  and  $F_2^h(1) < 0$ , which is not possible. Hence, it must be that firm 1 has an atom at  $p = 1$  and firm 2's c.d.f. is continuous on  $[p_0^h, 1]$ . Then,  $F_2^h(1) = 1$  implies that, in equilibrium,  $\hat{p}_1^h = (1 - \mu(\underline{k}, \bar{k}))$  and firms' profits and  $p_0^h$  follow.

The mass point in firm 1's price c.d.f. is

$$\phi^h \equiv 1 - F_1^h(1) = \frac{\sigma^2(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} < 1 ,$$

and consistency requires that  $F_1^h(1) \leq \sigma$ , which is the case whenever

$$(1 - \sigma)/\sigma(1 - \sigma + \sigma^2) \leq (1 - \mu(\underline{k}, \bar{k})) .$$

But this is exactly the reverse of Condition 2. ■

### Equilibrium Pricing

To identify firm 2's c.d.f. we use the constant profit conditions for firm 1. More specifically, we use the expected profits presented in section 4, and the equilibrium profit  $\pi_{h1}^*$  defined in (6). There are two price ranges to be considered, so that

$$F_2^h(p) = \begin{cases} F_2^{hL}(p) & \text{for } p \in [p_0, \hat{p}_1^h) \\ F_2^{hH}(p) & \text{for } p \in [\hat{p}_1^h, 1] \end{cases} .$$

Suppose firm 1 chooses a price  $p \in [p_0, \hat{p}_1^h)$ , then

$$1 - F_2^{hL}(p) = \frac{\sigma(1 - \mu(\underline{k}, \bar{k}))}{p} .$$

Suppose firm 1 chooses a price  $p \in [\hat{p}_1^h, 1]$ , then

$$1 - F_2^{hH}(p) = \frac{\sigma(1 - \mu(\underline{k}, \bar{k}))}{\mu(\underline{k}, \bar{k})} \left( \frac{1}{p} - 1 \right) .$$

It is straightforward to check that  $F_2^{hL}(\hat{p}_1^h) = F_2^{hH}(\hat{p}_1^h) = 1 - \sigma$  as  $\hat{p}_1^h = 1 - \mu(\underline{k}, \bar{k})$ .

To pin down firm 1's c.d.f. we use the constant profit conditions for firm 2. There are two

price ranges to be considered, so that

$$F_1^h(p) = \begin{cases} F_1^{hL}(p) & \text{for } p \in [p_0, \widehat{p}_1^h) \\ F_1^{hH}(p) & \text{for } p \in [\widehat{p}_1^h, 1] \end{cases}.$$

Suppose firm 2 chooses a price  $p \in [p_0^h, \widehat{p}_1^h)$ , then

$$1 - F_1^{hL}(p) = \frac{\sigma(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} \left[ \frac{1 - \sigma(1 - \mu(\underline{k}, \bar{k}))}{p} + \sigma \right].$$

Suppose now that firm 2 chooses a price  $p \in [\widehat{p}_1^h, 1]$ , then

$$1 - F_1^{hH}(p) = \frac{\sigma(1 - \mu(\underline{k}, \bar{k}))}{\mu(\underline{k}, \bar{k})[1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))]} \left[ \frac{1 - \sigma(1 - \mu(\underline{k}, \bar{k}))}{p} - (1 - \sigma) \right].$$

It is straightforward to check that  $F_1^{hL}(\widehat{p}_1^h) = F_1^{hH}(\widehat{p}_1^h) = \lambda_1^h$ .

Firm 1's atom at  $p = 1$  is given by

$$\phi^h \equiv 1 - F_1^{hH}(1) = \frac{\sigma^2(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))}.$$

## A.5 The Role of Prominence

**Proof of Corollary 1.** (i) Suppose that Condition 2 holds and consider the equilibrium in Proposition 2. From (3) and (5),  $\pi_1^* > \pi_2^* \Leftrightarrow (2\sigma - 1)(1 - \mu(\underline{k}, \bar{k})) > 0$  which holds for  $\sigma > 1/2$ . Suppose now that Condition 2 does not hold and consider the equilibrium in Proposition 3. Using (6), it is easy to see that  $\pi_{h1}^* > \pi_{h2}^*$ .

(ii) Suppose that Condition 2 holds. Let us inspect the equilibrium price c.d.f.s in appendix A.2. (a) Consider first prices  $p \in [p_0, \widehat{p}_1)$ ,  $dF_1^L(p)/dp = \pi_2^*/p^2 < dF_2^L(p)/dp = \pi_1^*/p^2$  from point (i) above. As  $F_1^L(p_0) = F_2^L(p_0) = 0$ , it follows that  $F_1^L(p) < F_2^L(p)$  in this range. Note also that  $\lim_{p \nearrow \widehat{p}_1} F_1^L(p) < \lim_{p \nearrow \widehat{p}_1} F_2^L(p)$ ; (b) Consider now  $[\widehat{p}_1, \widehat{p}_2]$ . First note that  $dF_1^M(p)/dp = \pi_2^*/\mu(\underline{k}, \bar{k})p^2 < dF_2^M(p)/dp = \pi_1^*/\mu(\underline{k}, \bar{k})p^2$ . Moreover, point (a) and continuity of  $F_i$  on  $[p_0, 1)$  imply that  $F_1^M(\widehat{p}_1) < F_2^M(\widehat{p}_1)$ . So,  $F_1^M(p) < F_2^M(p)$  in this range. (c) Consider  $[\widehat{p}_2, 1]$ . From part (b) it follows that  $F_1^M(\widehat{p}_2) < F_2^M(\widehat{p}_2)$ . By continuity,  $\lim_{p \searrow \widehat{p}_2} F_1^H(p) < \lim_{p \searrow \widehat{p}_2} F_2^H(p)$ . Noting that  $dF_1^H(p)/dp = \pi_2^*/(2\mu(\underline{k}, \bar{k}) - 1)p^2 < dF_2^H(p)/dp = \pi_1^*/(2\mu(\underline{k}, \bar{k}) - 1)p^2$ , it follows that  $F_1^H(p) < F_2^H(p)$ . Combining (a)-(c),  $F_1(p) < F_2(p)$  on  $[p_0, 1]$ , and so the price of the prominent firm first order stochastically dominates that of the less prominent firm. Suppose that Condition 2 does not hold. Let us inspect the equilibrium price c.d.f.s in appendix A.4.



Consider first prices  $p \in [p_0, \hat{p}_1^h)$ ,  $(dF_1^{hL}(p)/dp) / (dF_2^{hL}(p)/dp) = [1 - \sigma(1 - \mu(\underline{k}, \bar{k}))] / [1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))] < 1$ . As  $F_1^{hL}(p_0^h) = F_2^{hL}(p_0^h) = 0$ , it follows that  $F_1^{hL}(p) < F_2^{hL}(p)$  in this range. Consider now prices in  $[\hat{p}_1^h, 1]$ ,  $(dF_1^{hH}(p)/dp) / (dF_2^{hH}(p)/dp) = [1 - \sigma(1 - \mu(\underline{k}, \bar{k}))] / [1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))] < 1$ . As  $F_1^h$  and  $F_2^h$  are continuous at  $\hat{p}_1^h$ , it follows that  $F_1^{hH}(p) < F_2^{hH}(p)$  in this range, too. So,  $F_1^h(p) < F_2^h(p)$  on  $[p_0^h, 1]$ , and so the price of the prominent firm first order stochastically dominates that of the less prominent firm.

(iii) The ranking of the average prices follows from (ii) as  $E(p_i) = \int_0^\infty (1 - F_i(p_i)) dp_i = \int_{p_0}^1 (1 - F_i(p_i)) dp_i + p_0$  when Condition 2 holds and  $E(p_i) = \int_{p_0^h}^1 (1 - F_i^h(p_i)) dp_i + p_0^h$  when Condition 2 does not hold.

(iv) Recall that when Condition 2 holds  $\lambda_1 = \sigma(1 - \sigma)/\lambda_2$ , so  $\lambda_1 < \lambda_2 \Leftrightarrow \sigma > (1 - \sigma)(1 - \sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))$  which holds for  $\sigma > 1/2$ . When Condition 2 does not hold, it is easy to see from Proposition (3) that  $\lambda_1^h < 1$ . ■

**Proof of Corollary 2.** First let us compare the cut-off prices. Suppose Condition 2 holds. Using (A10) and (A11) in appendix A.2, we can check that  $\hat{p}_1 < \hat{p}_2$  as

$$\frac{\hat{p}_1}{\hat{p}_2} = \frac{(1 - \sigma) - (1 - \sigma)^2(2 + \sigma)(1 - \mu(\underline{k}, \bar{k})) + \sigma(1 - \sigma)^2(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))^2}{\sigma + (\sigma^3 - 3\sigma^2 + 2\sigma - 1)(1 - \mu(\underline{k}, \bar{k})) + \sigma(1 - \sigma)(1 - \sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))^2}.$$

Then,  $\hat{p}_1 < \hat{p}_2 \Leftrightarrow$

$$-\frac{\mu(\underline{k}, \bar{k})(2\sigma - 1)[1 - (1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))]}{\sigma + (\sigma^3 - 3\sigma^2 + 2\sigma - 1)(1 - \mu(\underline{k}, \bar{k})) + \sigma(1 - \sigma)(1 - \sigma + \sigma^2)(1 - \mu(\underline{k}, \bar{k}))^2} < 0.$$

The last inequality follows from the fact that, for  $\sigma \in (.5, 1)$ , all the terms in the numerator are positive, while the denominator is positive (the last term is clearly positive, while the sum of the first two is also positive as  $(1 - \mu(\underline{k}, \bar{k})) \leq 1/2$ ).

If Condition 2 does not hold, firm 2 uses  $\underline{k}$  for all prices on  $[p_0^h, 1]$  and  $\hat{p}_1^h = 1 - \mu(\underline{k}, \bar{k}) < 1$ .

Next we compare the firms' average prices conditional on using the lowest complexity level. Suppose Condition 2 holds.  $F_2$  is continuous on  $[p_0, 1]$  so that  $F_2(p) = F_2(p | p < 1)$ , whereas  $F_1$  is continuous on  $[p_0, 1)$ , but has an atom at  $p = 1$ ,  $\phi = (2\sigma - 1)(1 - \mu(\underline{k}, \bar{k})) / [1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))]$ . Using the price c.d.f.s in appendix A.2, we can show that

$$F_1(p | p < 1) = \frac{F_1(p)}{F_1(1)} = F_1(p) \frac{1 - (1 - \sigma^2)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(2 - \sigma)(1 - \mu(\underline{k}, \bar{k}))} = F_2(p).$$

Let  $G(p) = F_1(p | p < 1)$ . Note that  $F_1(p | p < \hat{p}_1) = G(p | p < \hat{p}_1)$ . This is because

$F_1(p | p < \hat{p}_1) = F_1(p) / F_1(\hat{p}_1)$  and  $G(p | p < \hat{p}_1) = F_1(p) / F_1(1)G(\hat{p}_1)$ . But then,

$$G(p | p < \hat{p}_1) = F_2(p | p < \hat{p}_1) > F_2(p | p < \hat{p}_2) ,$$

where the inequality follows from the fact that  $\hat{p}_1 < \hat{p}_2$  and  $F_2$  is a well-defined c.d.f. Putting together the expressions above, it follows that

$$F_1(p | p < \hat{p}_1) > F_2(p | p < \hat{p}_2) .$$

Finally, note that

$$\begin{aligned} E(p_1 | p_1 < \hat{p}_1) &= \int_{p_0}^{\hat{p}_1} (1 - F_1(p | p < \hat{p}_1)) dp - p_0 \quad \text{and} \\ E(p_2 | p_2 < \hat{p}_2) &= \int_{p_0}^{\hat{p}_1} (1 - F_2(p | p < \hat{p}_2)) dp + \int_{\hat{p}_1}^{\hat{p}_2} (1 - F_2(p | p < \hat{p}_2)) dp - p_0 . \end{aligned}$$

It is then easy to see that  $E(p_1 | p_1 < \hat{p}_1) < E(p_2 | p_2 < \hat{p}_2)$ .

Suppose now that Condition 2 does not hold.  $F_2^h$  is continuous on  $[p_0^h, 1]$  so that  $F_2^h(p) = F_2^h(p | p < 1)$ , whereas  $F_1^h$  is continuous on  $[p_0^h, 1)$ , but has an atom at  $p = 1$ ,  $\phi^h = \sigma^2(1 - \mu(\underline{k}, \bar{k})) / [1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))]$ . Using the price c.d.f.s in appendix A.4, we can show that

$$F_1^h(p | p < 1) = \frac{F_1^h(p)}{1 - \phi^h} = F_1^h(p) \frac{1 - \sigma(1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))}{1 - \sigma(1 - \mu(\underline{k}, \bar{k}))} = F_2^h(p) ,$$

and an argument similar to the one above applies as  $\hat{p}_1^h < 1$  and  $\phi^h > 0$ . ■

## A.6 Expected Profits with Biased Experts

### Existence of a Mixed Strategy Equilibrium

Suppose firm 2 uses a mixed strategy  $\xi_2^b$ . Then, the expected profit of firm 1 when choosing a price  $p_1$  and price complexity  $k_1$  against firm 2's mixed strategy is

$$\begin{aligned} \pi_1^b(p_1, k_1, \xi_2^b) &= p_1 \int_{p_1-d}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_1, k_2(p_2)) dH_2^b(k_2 | p_2 > p_1) \right) dF_2^b(p_2) + \\ &\quad p_1 \sigma \left[ 1 - \int_{p_0}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_1, k_2(p_2)) dH_2^b(k_2 | p_2) \right) dF_2^b(p_2) \right] . \end{aligned}$$

The f.o.c. of the expected profit maximization problem w.r.t.  $k_1$  is

$$\begin{aligned} p_1 \left( \int_{p_1-d}^1 \int_{\underline{k}}^{\bar{k}} \frac{\partial \mu}{\partial k_1} dH_2^b(k_2 | p_2 > p_1) dF_2^b(p_2) - \sigma \int_{p_0}^1 \int_{\underline{k}}^{\bar{k}} \frac{\partial \mu}{\partial k_1} dH_2^b(k_2 | p_2) dF_2^b(p_2) \right) &= 0 \Leftrightarrow \\ p_1 \frac{\partial \mu}{\partial k_1} \left[ (1 - F_2^b(p_1 - d)) - \sigma \right] &= 0 , \end{aligned}$$

where the equivalence follows from  $\partial^2 \mu(k_1, k_2) / \partial k_1 \partial k_2 = 0$ . As  $\partial \mu / \partial k_1 < 0$ , we obtain

$$k_1(p_1) = \begin{cases} \underline{k} & \text{if } 1 - F_2^b(p_1 - d) > \sigma \Leftrightarrow p_1 < \hat{p}_1^b \\ \bar{k} & \text{if } 1 - F_2^b(p_1^b - d) < \sigma \Leftrightarrow p_1 > \hat{p}_1^b \\ k, \forall k \in [\underline{k}, \bar{k}] & \text{if } p_1 = \hat{p}_1^b \end{cases} ,$$

where the threshold price  $\hat{p}_1^b$  is implicitly defined by  $F_2^b(\hat{p}_1^b - d) = 1 - \sigma$ , whenever  $\hat{p}_1^b - d$  belongs to  $T_2^b$ , the support of  $F_2^b$ .

Suppose firm 1 uses a mixed strategy  $\xi_1^b$ . Then, the expected profit of firm 2 when choosing a price  $p_2$  and price complexity  $k_2$  against firm 1's mixed strategy is

$$\begin{aligned} \pi_2^b(p_2, k_2, \xi_1^b) &= p_2 \int_{p_2+d}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_1(p_1), k_2) dH_1^b(k_1 | p_1 > p_2) \right) dF_1^b(p_1) + \\ & p_2(1 - \sigma) \left[ 1 - \int_{p_0}^1 \left( \int_{\underline{k}}^{\bar{k}} \mu(k_1(p_1), k_2) dH_1^b(k_1 | p_1) \right) dF_1^b(p_1) \right] . \end{aligned}$$

A similar argument leads to the following result

$$k_2(p_2) = \begin{cases} \underline{k} & \text{if } 1 - F_1^b(p_2 + d) > 1 - \sigma \Leftrightarrow p_2 < \hat{p}_2^b \\ \bar{k} & \text{if } 1 - F_1^b(p_2 + d) < 1 - \sigma \Leftrightarrow p_2 > \hat{p}_2^b \\ k, \forall k \in [\underline{k}, \bar{k}] & \text{if } p_2 = \hat{p}_2^b \end{cases} ,$$

where the threshold price  $\hat{p}_2^b$  is implicitly defined by  $F_1^b(\hat{p}_2^b + d) = \sigma$ , whenever  $\hat{p}_2^b + d$  belongs to  $T_1^b$ , the support of  $F_1^b$ .

## Expected Profits and Constant Profit Conditions

In this part, we present the expected profit expressions which underlie example 4 in section 5. To do so, we adapt the main analysis to capture expert consumers' bias towards the prominent firm's product.

If firm 1 chooses  $p \in [p_0^2 + d, \hat{p}_1^b)$ , it associates this price with complexity level  $\underline{k}$ . Then, its expected profit is

$$\begin{aligned} \pi_1^b(p, \underline{k}) &= p \{ (\lambda_2^b - F_2^b(p - d)) \mu(\underline{k}, \underline{k}) + (1 - \lambda_2^b) \mu(\underline{k}, \bar{k}) + \sigma [\lambda_2^b (1 - \mu(\underline{k}, \underline{k})) + (1 - \lambda_2^b) (1 - \mu(\underline{k}, \bar{k}))] \} \\ &= p \left[ 1 - F_2^b(p - d) - (1 - \sigma)(1 - \lambda_2^b)/2 \right] . \end{aligned}$$

This expression is similar to (1), but biased expert consumers purchase from firm 1 if firm 2's price is higher than  $p - d$ . Evaluating it at  $p_0^2 + d$  and when  $p \rightarrow \hat{p}_1^b$ , we obtain

$$\begin{aligned}\pi_1^b(p_0^2 + d, \underline{k}) &= (p_0^2 + d) \left[ 1 - (1 - \sigma)(1 - \lambda_2^b)/2 \right] \text{ and} \\ \lim_{p \nearrow \hat{p}_1^b} \pi_1^b(p, \underline{k}) &= \hat{p}_1^b \left[ \sigma - (1 - \sigma)(1 - \lambda_2^b)/2 \right],\end{aligned}$$

where we use  $1 - F_2^b(\hat{p}_1^b - d) = \sigma$ .

If firm 1 chooses  $p \in [\hat{p}_1^b, 1)$ , it associates this price with complexity level  $\bar{k}$ . Then, its expected profit is

$$\begin{aligned}\pi_1^b(p, \bar{k}) &= p \{ (\lambda_2^b - F_2^b(p - d)) \mu(\underline{k}, \bar{k}) + (1 - \lambda_2^b) \mu(\bar{k}, \bar{k}) + \\ &\quad \sigma [\lambda_2^b (1 - \mu(\underline{k}, \bar{k})) + (1 - \lambda_2^b) (1 - \mu(\bar{k}, \bar{k}))] \} \\ &= p \{ (\lambda_2^b - F_2^b(p - d))/2 + \sigma [\lambda_2^b/2 + (1 - \lambda_2^b)] \}.\end{aligned}$$

As before this expression follows from adapting (A3) to reflect the fact that firm 1 serves  $\mu(\underline{k}, \bar{k})$  experts whenever the rival chooses the lowest complexity and a price higher than  $p - d$ . Evaluating this expression at  $\hat{p}_1^b$  and using  $1 - F_2^b(\hat{p}_1^b - d) = \sigma$ , it can be checked that the expected profit function is continuous at  $\hat{p}_1^b$ , i.e.  $\pi_1^b(\hat{p}_1^b, \bar{k}) = \lim_{p \nearrow \hat{p}_1^b} \pi_1^b(p, \underline{k})$ . Moreover, when  $p \rightarrow 1$ ,

$$\lim_{p \nearrow 1} \pi_1^b(p, \bar{k}) = \sigma [\lambda_2^b/2 + (1 - \lambda_2^b)].$$

The constant profit conditions of firm 1 lead to the following equations

$$\begin{aligned}(p_0^2 + d) \left[ 1 - (1 - \sigma)(1 - \lambda_2^b)/2 \right] &= \sigma [\lambda_2^b/2 + (1 - \lambda_2^b)] \text{ and} \\ \hat{p}_1^b \left[ \sigma - (1 - \sigma)(1 - \lambda_2^b)/2 \right] &= \sigma [\lambda_2^b/2 + (1 - \lambda_2^b)].\end{aligned}$$

If firm 2 chooses  $p \in [p_0^2, \hat{p}_1^b - d]$ , it associates the price with  $\underline{k}$ . In this range  $p + d < \hat{p}_1^b$ , and firm 2's expected profit is

$$\begin{aligned}\pi_2^b(p, \underline{k}) &= p \{ \mu(\underline{k}, \underline{k}) (\lambda_1^b - F_1^b(p + d)) + (1 - \lambda_1^b) \mu(\underline{k}, \bar{k}) + \\ &\quad (1 - \sigma) [\lambda_1^b (1 - \mu(\underline{k}, \underline{k})) + (1 - \lambda_1^b) (1 - \mu(\underline{k}, \bar{k}))] \} \\ &= p \left[ \lambda_1^b - F_1^b(p + d) + (1 - \lambda_1^b)/2 + (1 - \sigma) (1 - \lambda_1^b)/2 \right].\end{aligned}$$

This expression adapts (A6) with the difference that biased experts purchase from firm 2 if the

rival chooses  $\underline{k}$  and a price above  $p + d$ . Evaluating this expression at  $p_0^2$  and  $\hat{p}_1^b - d$ , we obtain

$$\begin{aligned}\pi_2^b(p_0^2, \underline{k}) &= p_0^2 \left[ \lambda_1^b + (1 - \lambda_1^b)/2 + (1 - \sigma)(1 - \lambda_1^b)/2 \right] \text{ and} \\ \pi_2^b(\hat{p}_1^b - d, \underline{k}) &= (\hat{p}_1^b - d) \left[ (1 - \lambda_1^b)/2 + (1 - \sigma)(1 - \lambda_1^b)/2 \right].\end{aligned}$$

If firm 2 chooses  $p \in [\hat{p}_1^b - d, 1 - d]$ , it associates the price with  $\underline{k}$ . In this range  $p + d > \hat{p}_1^b$ , and firm 2's expected profit is

$$\begin{aligned}\pi_2^b(p, \underline{k}) &= p \{ \mu(\underline{k}, \bar{k})(1 - F_1^b(p + d)) + (1 - \sigma) [\lambda_1^b(1 - \mu(\underline{k}, \underline{k})) + (1 - \lambda_1^b)(1 - \mu(\underline{k}, \bar{k}))] \} \\ &= p \left[ (1 - F_1^b(p + d)) + (1 - \sigma)(1 - \lambda_1^b) \right] / 2.\end{aligned}$$

Firm 2 can only serve the expert consumers when firm 1 prices above  $p + d$ , in which case there are  $\mu(\underline{k}, \bar{k})$  informed consumers. It can be checked that firm's 2 expected profit function is continuous at  $\hat{p}_1^b - d$  and, evaluating at  $1 - d$ , we obtain

$$\pi_2^b(1 - d, \underline{k}) = (1 - d) [(1 - F_1^b(1)) + (1 - \sigma)(1 - \lambda_1^b)] / 2,$$

where  $(1 - F_1^b(1)) \geq 0$  (if the inequality is strict, then firm 1 has a mass point at  $p = 1$ ).

Finally, if firm 2 chooses  $p = 1$ , it associates this price with  $\bar{k}$  and its expected profit is

$$\pi_2^b(1, \bar{k}) = (1 - \sigma) [\lambda_1^b / 2 + 1 - \lambda_1^b].$$

The constant profit conditions of firm 1 lead to the following equations

$$\begin{aligned}p_0^2 \left[ \lambda_1^b + (1 - \lambda_1^b)/2 + (1 - \sigma)(1 - \lambda_1^b)/2 \right] &= (1 - \sigma) [\lambda_1^b / 2 + 1 - \lambda_1^b] \text{ and} \\ (\hat{p}_1^b - d) \left[ (1 - \lambda_1^b) + (1 - \sigma)(1 - \lambda_1^b) \right] / 2 &= (1 - \sigma) [\lambda_1^b / 2 + 1 - \lambda_1^b].\end{aligned}$$

The equations above determine  $p_0^2$ ,  $\hat{p}_1^b$ ,  $\lambda_1^b$ , and  $\lambda_2^b$ . Although the closed-form solutions for arbitrary  $d$  and  $\sigma$  are cumbersome, it is straightforward to solve the system of equations for given values of  $d$  and  $\sigma$ , as shown in example 4.

### Numerical Example and Further Discussion

In example 4, we focus on an equilibrium where  $T_1^b = [p_0^2 + d, 1]$  and firm 1 associates  $\underline{k}$  with prices on  $[p_0^2 + d, \hat{p}_1^b)$  and  $\bar{k}$  with prices on  $[\hat{p}_1^b, 1]$ , and  $T_2^b = [p_0^2, 1 - d] \cup \{1\}$  and firm 2 associates  $\underline{k}$  with prices on  $[p_0^2, 1 - d]$  and  $\bar{k}$  with  $p = 1$ . For these mixed strategies to be part of an equilibrium, consistency requires  $\hat{p}_2^b = 1 - d$ ,  $p_0^2 + d < \hat{p}_1^b < 1 - d$ ,  $p_0^2 > 0$ ,  $F_2^b(\hat{p}_1^b - d) = (1 - \sigma)$ ,  $F_1^b(\hat{p}_2^b + d) = \sigma$ ,  $\lambda_1^b = F_1^b(\hat{p}_1^b) \in (0, 1)$  and  $\lambda_2^b = F_2^b(1 - d) \in (0, 1)$ . Using firms' expected profits

presented above and the equilibrium constant profit conditions, we obtain  $p_0^2$ ,  $\hat{p}_1^b$ ,  $\lambda_1^b$ , and  $\lambda_2^b$ . These can be used to identify firms' profits, and price c.d.f.s. For expositional simplicity, we focus on a case where  $\mu(\bar{k}, \bar{k}) = 0$ . As  $\mu(\underline{k}, \underline{k}) = 1$ , under the assumption that  $\partial^2 \mu / \partial k_1 \partial k_2 = 0$ , this implies that  $\mu(\underline{k}, \bar{k}) = 1/2$ .

**Example 4** Let  $\mu(\underline{k}, \underline{k}) = 1$ ,  $\mu(\underline{k}, \bar{k}) = .5$ , and  $\mu(\bar{k}, \bar{k}) = 0$ . Suppose  $\sigma = .7$  and  $d = .1$ . There exists an equilibrium where firms 1 and 2 choose  $\underline{k}$  with probability  $\lambda_1^b = .3$  and  $\lambda_2^b = .7$ , respectively. The two firms randomize on prices according to the following c.d.f.s, which are illustrated in Figure 4.

$$F_1^b(p) = \begin{cases} F_{1L}^b(p) = .75 - .255/(p - d) \text{ for } p \in [p_0^2 + d, \hat{p}_1^b) \\ F_{1H}^b(p) = 1.2 - .5/(p - d) \text{ for } p \in [\hat{p}_1^b, 1] \end{cases} \quad \text{and}$$

$$F_2^b(p) = \begin{cases} F_{2L}^b(p) = 1 - .45/(p + d) \text{ for } p \in [p_0^2, \hat{p}_1^b) \\ F_{2H}^b(p) = 1.6 - .9/(p + d) \text{ for } p \in [\hat{p}_1^b, 1 - d] \\ 1 \text{ for } p = 1 \end{cases} ,$$

where  $p_0^2 = .35$ ,  $p_0^2 + d = .45$ ,  $\hat{p}_1^b = .65$ , and  $1 - d = .9$ . Firm 1 makes profit  $\pi_{b1}^* = .45$  and firm 2 makes profit  $\pi_{b2}^* = .255$ .

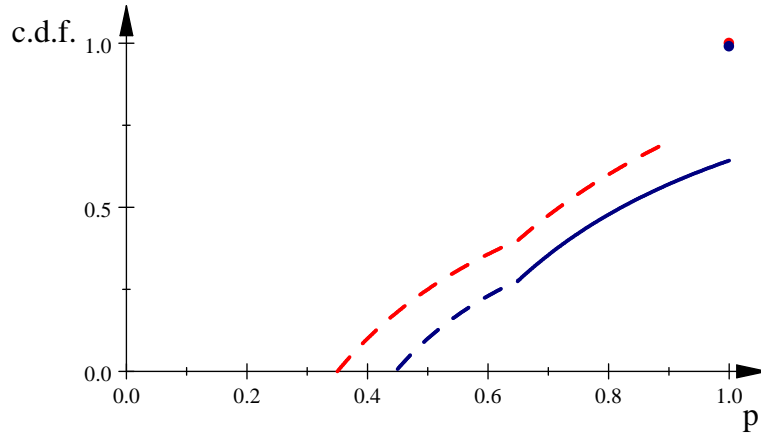


Figure 4: Firms' price c.d.f.s for  $\sigma = .7$  and  $d = .1$  with biased experts.  $F_1^b(p)$  is the blue line and  $F_2^b(p)$  is the red line. The dashed lines correspond to prices associated with  $\underline{k}$ .

It is easy to see that in the example the price c.d.f.s are well defined. The consistency requirements are satisfied,  $F_2^b(\hat{p}_1^b - d) = 1 - \sigma = .3$ ,  $F_2^b(1 - d) = \lambda_2^b = .7$ ,  $F_1^b(1) = \sigma = .7$ , and  $F_1^b(\hat{p}_1^b) = \lambda_1^b = .3$ . Furthermore, conditional on choosing the lowest complexity, the expected price of firm 1 is

$$E(p_1 \mid p_1 < \hat{p}_1^b) = \int_0^\infty (1 - F_1^b(p \mid p < \hat{p}_1^b)) dp = p_0^2 + d + \int_{p_0^2 + d}^{\hat{p}_1^b} (1 - F_{1L}^b(p) / \lambda_1^b) dp = .54 ,$$

while that of firm 2 is

$$\begin{aligned} E(p_2 \mid p_2 < 1 - d) &= \int_0^\infty (1 - F_2^b(p \mid p < 1 - d)) dp \\ &= p_0^2 + \int_{p_0^2}^{\hat{p}_1^b} (1 - F_{2L}^b(p)/\lambda_2^b) dp + \int_{\hat{p}_1^b}^{1-d} (1 - F_{2H}^b(p)/\lambda_2^b) dp = .6 . \end{aligned}$$

Like in the benchmark model, consumers' bias in favor of the prominent firm is consistent with the average prices, conditional on these being associated with the lowest complexity level,  $\underline{k}$ . This is the case although, when the experts are willing to pay a prominence premium, the prominent firm's lowest possible price is strictly larger than that of its rival. In example 4, the experts never pay more than  $p_2 = 1 - d$  for the less prominent product. The equilibrium there is consistent with an environment where prominence-biased experts are willing to pay up to  $v^1 = 1$  for the prominent product but no more than  $v^2 = 1 - d$  for the less prominent one.

Example 4 also highlights some differences from the benchmark model with unbiased experts. In particular, with biased experts, in the mixed strategy equilibrium both firms may have a mass point at the monopoly price and the less prominent firm sets this price with higher probability than the rival (i.e., in the example  $1 - F_1^b(1) = .355$  and  $1 - F_2^b(1) = .3$ ). Although there is a positive probability of a tie at price  $p = 1$ , the less prominent firm cannot improve its market share by slightly undercutting and neither can the rival. So, the reasoning in Lemma 4 does not apply when  $d > 0$ . Moreover, the supports of the price c.d.f.s are not identical and the c.d.f. of the less prominent firm has a gap.

A full characterization of the equilibria in the model with biased experts is beyond the scope of this section. However, there are also other equilibria. For instance, if  $d > \hat{d} = [1 - (1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))]$ , there exists a pure strategy equilibrium where  $k_1 = \underline{k}$ ,  $k_2 = \bar{k}$ , and  $p_1 = p_2 = 1$ . Firms' profits are then  $\pi_1(\underline{k}, \bar{k}) = 1 - (1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))$  and  $\pi_2(\underline{k}, \bar{k}) = (1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))$ . Given these equilibrium prices, firm 1 (firm 2) cannot increase its market share by increasing  $k_1$  (decreasing  $k_2$ ). If firm 2 deviates to  $p_2^d = (1 - \varepsilon) \geq (1 - d)$  and  $k_2 \in [\underline{k}, \bar{k})$ , its deviation profit is  $\pi_2^d = (1 - \varepsilon)[1 - \sigma(1 - \mu(\underline{k}, k_2))]$  and  $\pi_2^d > \pi_2(\underline{k}, \bar{k})$  iff  $\varepsilon < 1 - (1 - \sigma)(1 - \mu(\underline{k}, \bar{k}))/[1 - \sigma(1 - \mu(\underline{k}, k_2))] \leq \hat{d}$ . Consistency requires  $d < \varepsilon$ . So if  $d > \hat{d}$ ,  $\nexists \varepsilon$  s.t.  $\pi_2^d > \pi_2(\underline{k}, k_2)$  for  $k_2 \in [\underline{k}, \bar{k})$ .

## References

- ANDERSON, S. P., A. DE PALMA, AND J. F. THISSE (1992): *Discrete Choice Theory of Product Differentiation*. MIT press.
- ARMSTRONG, M. (2017): “Ordered consumer search,” *Journal of the European Economic Association*, forthcoming.
- ARMSTRONG, M., AND Y. CHEN (2009): “Inattentive Consumers and Product Quality,” *Journal of the European Economic Association*, 7(2-3), 411–422.
- ARMSTRONG, M., J. VICKERS, AND J. ZHOU (2009): “Prominence and Consumer Search,” *The RAND Journal of Economics*, 40(2), 209–233.
- ARMSTRONG, M., AND J. ZHOU (2011): “Paying for Prominence,” *The Economic Journal*, 121(556), 368–395.
- BAYE, M., D. KOVENOCK, AND C. DE VRIES (1992): “It Takes Two to Tango: Equilibria in a Model of Sales,” *Games and Economic Behavior*, 4(4), 493–510.
- BAYE, M. R., J. MORGAN, AND P. SCHOLTEN (2006): “Information, Search, and Price Dispersion,” in *Handbook of Economics and Information Systems*, ed. by T. Hendershott. Elsevier Press, Amsterdam.
- BRONNENBERG, B. J., J.-P. DUBÉ, M. GENTZKOW, AND J. M. SHAPIRO (2015): “Do pharmacists buy Bayer? Informed shoppers and the brand premium,” *The Quarterly Journal of Economics*, 130(4), 1669–1726.
- CARLIN, B. (2009): “Strategic Price Complexity in Retail Financial Markets,” *Journal of Financial Economics*, 91(3), 278–287.
- CHIOVEANU, I. (2012): “Price and Quality Competition,” *Journal of Economics*, 107(1), 23–44.
- CHIOVEANU, I., AND J. ZHOU (2013): “Price Competition with Consumer Confusion,” *Management Science*, 59(11), 2450–2469.
- COMPETITION AND MARKETS AUTHORITY (2015): “Retail Banking Market Investigation: Summary of Provisional Findings Report,” available at <https://www.gov.uk/cma-cases/review-of-banking-for-small-and-medium-sized-businesses-smes-in-the-uk>.



- ELLISON, G. (2006): “Bounded Rationality in Industrial Organization,” in *Advances in Economics and Econometrics: Theory and Applications: Ninth World Congress of the Econometric Society*, ed. by R. Blundell, W. Newey, and T. Persson. Cambridge University Press, Cambridge, UK.
- ELLISON, G., AND A. WOLITZKY (2012): “A Search Cost Model of Obfuscation,” *RAND Journal of Economics*, 43(3), 417–441.
- ESTELAMI, H. (1997): “Consumer Perceptions of Multi-Dimensional Prices,” in *Advances in Consumer Research*, ed. by M. Brucks, and D. MacInnis, pp. 392–399.
- EUROPEAN COMMISSION (2007): “Staff Working Document Accompanying the White Paper on the Integration of EU Mortgage Credit Markets,” available at <http://eur-lex.europa.eu>.
- GIULIETTI, M., M. WATERSON, AND M. WILDENBEEST (2014): “Estimation of Search Frictions in the British Electricity Market,” *The Journal of Industrial Economics*, 62(4), 555–590.
- GREENLEAF, E., E. J. JOHNSON, V. MORWITZ, AND E. SHALEV (2015): “The Price Does Not Include Additional Taxes, Fees, and Surcharges: A Review of Research on Partitioned Pricing,” *Fees, and Surcharges: A Review of Research on Partitioned Pricing (July 21, 2015)*.
- GRUBB, M. D. (2015): “Failing to Choose the Best Price: Theory, Evidence, and Policy,” *Review of Industrial Organization*, 47(3), 303–340.
- GU, Y., AND T. WENZEL (2014): “Strategic Obfuscation and Consumer Protection Policy,” *The Journal of Industrial Economics*, 62(4), 632–660.
- HASTINGS, J. S., A. HORTAÇSU, AND C. SYVERSON (2017): “Advertising and Competition in Privatized Social Security: The Case of Mexico,” *Econometrica*, forthcoming.
- HORTAÇSU, A., S. A. MADANIZADEH, AND S. L. PULLER (2017): “Power to choose? An analysis of consumer inertia in the residential electricity market,” *American Economic Journal: Economic Policy*, forthcoming.
- HUCK, S., AND J. ZHOU (2011): “Consumer Behavioural Biases in Competition: A survey,” *NYU Working Paper*.
- INDEPENDENT BANKING COMMISSION (2011): “Final Report Recommendations,” available at <http://bankingcommission.independent.gov.uk/>.
- KALAYCI, K. (2015): “Price Complexity and Buyer Confusion in Markets,” *Journal of Economic Behavior & Organization*, 111, 154–168.

- KALAYCI, K., AND J. POTTERS (2011): “Buyer Confusion and Market Prices,” *International Journal of Industrial Organization*, 29(1), 14–22.
- MORWITZ, V., E. GREENLEAF, AND E. JOHNSON (1998): “Divide and Prosper: Consumers’ Reaction to Partitioned Prices,” *Journal of Marketing Research*, 35(4), 453–463.
- NARASIMHAN, C. (1988): “Competitive Promotional Strategies,” *Journal of Business*, 61(4), 427–449.
- PICCIONE, M., AND R. SPIEGLER (2012): “Price Competition under Limited Comparability,” *The Quarterly Journal of Economics*, 127(1), 97–135.
- RHODES, A. (2011): “Can Prominence Matter even in an Almost Frictionless Market?,” *The Economic Journal*, 121(556), 297–308.
- ROSENTHAL, R. (1980): “A Model in which an Increase in the Number of Sellers Leads to a Higher Price,” *Econometrica*, 48(6), 1575–1579.
- SPIEGLER, R. (2011): *Bounded Rationality and Industrial Organization*. Oxford University Press.
- (2016): “Choice Complexity and Market Competition,” *Annual Review of Economics*, 8, 1–25.
- STANGO, V., AND J. ZINMAN (2009a): “What Do Consumers Really Pay on Their Checking and Credit Card Accounts? Explicit, Implicit, and Avoidable Costs,” *American Economic Review*, 99(2), 06–9.
- (2009b): “Exponential Growth Bias and Household Finance,” *The Journal of Finance*, 64(6), 2807–2849.
- TAYLOR, G. (2017): “Raising Search Costs To Deter Window Shopping Can Increase Profits and Welfare,” *The Rand Journal of Economics*, forthcoming.
- THOMAS, M., AND V. MORWITZ (2009): “The Ease of Computation Effect: The Interplay of Metacognitive Experiences and Naive Theories in Judgments of Price Differences,” *Journal of Marketing Research*, 46(1), 81–91.
- VARIAN, H. (1980): “A Model of Sales,” *American Economic Review*, 70(4), 651–659.
- WILSON, C. M. (2010): “Ordered Search and Equilibrium Obfuscation,” *International Journal of Industrial Organization*, 28(5), 496–506.

WOODWARD, S., AND R. HALL (2012): “Diagnosing Consumer Confusion and Sub-optimal Shopping Effort: Theory and Mortgage-Market Evidence,” *American Economic Review*, 102(7), 3249–3276.