On the Degree of Scale Economies when Firms Make Technology Choice

Shintaku, Koji

Graduate School of Economics, Kyoto University

6 September 2017

On the Degree of Scale Economies when Firms Make Technology Choice

Koji Shintaku*
Graduate School of Economics, Kyoto University

Abstract

We construct a simple model to demonstrate how the firm-level degree of scale economies (D-SE) is determined when firms make technology choice. In particular, we illustrate the importance of external factors that affect the efficiency of firms' technology choice, such as public knowledge stock, when determining D-SE. A change in public knowledge stock affects D-SE both directly and indirectly through a change in the firm's output. When output is endogenized in a monopolistic competition model with a variable mark-up rate, an increase in public knowledge stock raises D-SE through technology choice if the mark-up rate is increasing in output.

Keywords: Degree of scale economies; Technology choice; Public knowledge stock; Variable mark-up rate.

JEL classification numbers : D21, D24, F10, F12, L11, L16.

1 Introduction

Economists have well recognized that firm-level scale economies are crucial in shaping various economic phenomena, such as intra-industry trade (Krugman, 1979, 1980) and firms' spatial agglomeration (Krugman, 1991). In empirical studies, the firm-level degree of scale economies (D-SE), defined as output's elasticity with respect to the total input at the firm level, is important for accurately estimating total factor productivity (TFP).[1)]

*Corresponding author. Graduate School of Economics, Kyoto University, Yoshida Honmachi, Sakyo-ku, Kyoto 606-8501, Japan. E-mail address: shintaku.shitanku@gmail.com.

1) Many empirical studies assume that the D-SE for the Cobb-Douglas production function equals one, but if it is over (under) one, TFP is over (under) estimated.

The literature, however, neglects the role of firms' technology choice in determining D-SE. In reality, firms can choose their technology level by controlling the quality and type of patents, machinery, labor, and management systems. Recent studies (Yeaple, 2005; Bustos, 2011) have shown that changes in a firm's competitive environment induces technology choice, defined as selecting both of marginal and technology adoption costs. Therefore, we need to consider the effect of technology choice on D-SE determination.

This paper examines how D-SE is determined when firms make technology choice. The essential feature of our analysis is as follows. We endogenize technology choice. As driving forces of choosing technology that exists but is new for the firm, we consider both firm size and some factors external to individual firms. The latter include public knowledge stock, firms' agglomeration, infrastructure, etc. We interpret these factors as public knowledge stock available to firms.

The main results are as follows. First, when a firm's output is given, D-SE directly depends on output, fixed costs, and public knowledge stock; i.e., an increase in output reduces D-SE, whereas an increase in public knowledge stock or fixed costs raises it, *ceteris paribus*. Second, when a firm's output is endogenized in a monopolistic competition model with a variable mark-up rate, whether an increase in public knowledge stock or fixed costs raises D-SE through technology choice depends on whether the mark-up rate is increasing in output or constant.

This paper contributes two novel findings to the literature. First, we show that external factors such as public knowledge stock affect DSE through their effects on technology choice. Second, we demonstrate that D-SE changes under variable mark-ups.

2 D-SE under exogenous firm's output

In the following model, a firm's technology choice and D-SE depend on its size in terms of output. In this section, we focus on a firm's technology choice and analyze its D-SE by assuming that its output is given. In the next section, we endogenize the firm's output choice.

2.1 Firm's technology

A firm produces outputs by inputting production factors. For analytical simplicity, we assume only one input—labor—as a numeraire. The firm has the following technology. Its production function is $y = al_P$, where y is output level, l_p is an input level of production labor, and a is the marginal product. a can be interpreted as TFP. The marginal product (a) depends on the levels of spending on technology (l_T). The relationship can be characterized by "technology choice function," F, as $a = F(l_T, \phi)$, where ϕ is a parameter external to individual firms. This parameter represents factors that affect the efficiency of firms' technology choice. Those factors may include public knowledge stock, firms' agglomeration, infrastructure, etc. We interpret this as public knowledge stock available to firms. We assume $F(0, \phi) > 0$, $F_{l_T} > 0$, and $F_\phi > 0$. $F_{l_T} > 0$ means that firms can reduce the production cost by paying a higher technology choice cost. $F_\phi > 0$ represents the whole economy's technological spillover.

2.2 Optimal technology choice

The firm faces the following cost-minimization problem. We define variable input (variable cost), l_V, as $l_V \stackrel{\text{def}}{=} l_T + l_P$. The firm minimizes l_V by selecting a pair of (l_T, l_P), given y. This problem characterizes the optimal technology choice, (l_T, l_P), conditional on y. The firm faces this problem after paying fixed cost (l_F) for entry. l_F mainly represents the costs of obtaining physical assets and constructing a distribution network.

To characterize the solution clearly, we introduce a "technology upgrading rate" (ET). ET is defined as the elasticity of marginal product (a) with respect to spending on technology (l_T), i.e., $ET \stackrel{\text{def}}{=} (\partial F / \partial l_T)(l_T / F)$.

We assume that $F_{l_T}(0, \phi) y / [F(0, \phi)]^2 \geq 1$ for arbitrary (y, ϕ) and $F_{l_T l_T} < 2(F_{l_T})^2 / F$ for arbitrary (y, ϕ, l_T). The former certifies the optimal $l_T > 0$; the latter certifies the second-order condition of the optimization. These assumptions characterize the optimal pair of (l_T, l_P), conditional on y, as follows.

Lemma 1. [2] *For arbitrary $y > 0$ and $\phi > 0$, the optimal levels of l_T and l_P are positive; these are completely characterized by $y = F(l_T, \phi) l_P$ and the following relationship:*

$$ET = \frac{F(l_T, \phi) l_T}{y}. \tag{1}$$

This lemma shows the characterization of the optimal technological choice and additionally implies that an increase in ET raises l_T, given y.

We impose two important assumptions for ET to clarify the following analysis. First, for analytical simplicity, we assume that ET depends only on ϕ: $\partial ET(l_T, \phi) / \partial l_T = 0$.[3] Second, we assume that ET is increasing in ϕ: $dET(l_T, \phi)/d\phi = \partial ET(l_T, \phi)/\partial \phi > 0$.[4] This assumption seems to be natural because of public knowledge stock's property. Now, we

2) Proofs of lemmas and propositions are provided in the appendix.

3) This condition is equivalent to $F_{l_T l_T} l_T / F_{l_T} + 1 = F_{l_T} l_T / F$, implying that $l_T = F_{l_T} F / [(F_{l_T})^2 - F F_{l_T l_T}]$. We assume $(F_{l_T})^2 - F F_{l_T l_T} > 0$ to certify $l_T > 0$.

4) This condition is equivalent to $F F_{l_T \phi} > F_{l_T} F_\phi$.

should distinguish between technology choice and technology creation. Existing knowledge reinforces technology choice while restricting technology creation.

Lemma 1 derives the following proposition.

Proposition 1. *For arbitrary $y > 0$ and $\phi > 0$, the following properties hold. (a) An increase in ϕ reduces l_P and l_V while ambiguously impacting l_T. (b) An increase in y raises l_T, l_P, and l_V.*

In (a), the impact on l_T is ambiguous because an increase in public knowledge stock has two opposite effects: raising the return on technology investment relatively to production labor (positive effect) and saving technology investment through a free ride on the existing public knowledge (negative effect).

In (b), the impact on l_T is positive, corroborating previous studies' findings.

The assumptions for ET, $\partial ET(l_T, \phi)/\partial l_T = 0$, and $\partial ET(l_T, \phi)/\partial \phi > 0$ certify the result of (a), although it does not affect the result of (b). In particular, the assumption of $\partial ET(l_T, \phi)/\partial l_T = 0$ derives $\partial l_V/\partial \phi < 0$. Both these assumptions derive $\partial l_P/\partial \phi < 0$.

We should note that the impacts on l_V are different in (a) and (b). This leads to different impacts on D-SE.

2.3 Two types of degree of scale economies

For later analysis, we define two types of degree of scale economies. One is D-SE (with fixed costs), which we denote as SE_D, defined as $SE_D \stackrel{\text{def}}{=} \partial \log y/\partial \log l$, where l represents total labor input, i.e., $l \stackrel{\text{def}}{=} l_F + l_V$. The other is the degree of scale economies without fixed costs (D-SEV). We denote D-SEV as SEV_D, defined as $SEV_D \stackrel{\text{def}}{=} \partial \log y/\partial \log l_V$. Equation (1) derives the following relationship:

$$SEV_D = 1 + ET. \tag{2}$$

(2) implies that D-SEV has a one-to-one correspondence to the technology upgrading rate for arbitrary output level.

2.4 Three channels affecting D-SE

The relationship of (2) reveals channels affecting D-SE as follows.

Lemma 2. *For arbitrary $y > 0$ and $\phi > 0$, $SE_D = SEV_D(1 + l_F/l_V)$ holds.*

This lemma means that D-SE depends on D-SEV and fixed and variable costs. Furthermore, D-SEV and variable costs depend on public knowledge stock and output. Hence, D-SE essentially depends on output, fixed costs, and public knowledge stock.

In this section, y, l_F, and ϕ are exogenous. We investigate how changes in these exogenous variables affect D-SE using Proposition 1 and Lemmas 1 and 2.

Proposition 2. *(a) An increase in y reduces SE_D. (b) An increase in l_F raises SE_D. (c) An increase in ϕ raises SE_D.*

These results can be explained as follows: (a) An increase in output raises variable costs but does not affect D-SEV, thereby reducing D-SE. (b) An increase in fixed costs directly raises SE_D but does not affect D-SEV and variable costs, thereby raising D-SE. (c) An increase in public knowledge stock raises D-SEV and reduces variable costs, thereby raising D-SE. All these results depend on the assumption of $\partial ET(l_T, \phi)/\partial l_T = 0$. The result (c) also depends on $dET(\phi)/d\phi > 0$.

As is well known, fixed costs create firm-level scale economies. Proposition 2 implies that this is true even when firms make technology choice. Furthermore, we show that some external factors, such as public knowledge stock, can affect D-SE.

3 D-SE under an endogenous firm's output

In this section, we construct a market equilibrium and endogenize y. Then, changes in l_F or ϕ affect y. We analyze how D-SE (SE_D) depends on fixed cost (l_F) and public knowledge stock (ϕ) through this effect.

3.1 Specification of technology choice function

For analytical simplicity, we specify the technology choice function $F(l_T, \phi)$. From the cost minimization problem, the following technology is chosen.

Lemma 3. *We specify $F(l_T, \phi)$ as $F(l_T, \phi) = \phi l_T^{\phi}$. Then, when the firm makes technology choice optimally, $ET = \phi$ holds, and the optimal level of l_T and l_P conditional on y is given by $l_T = y^{1/(\phi+1)}$ and $l_P = y^{1/(\phi+1)}/\phi$. The variable cost function can be uniquely specified as $l_V = [(\phi+1)/\phi]y^{1/(\phi+1)}$. Then, $SEV_D = \phi + 1$, $\partial MC/\partial y < 0$, and $\partial MC/\partial \phi < 0$ hold, where MC denotes the marginal cost in the optimal technology choice.*

In the above specification, all assumptions for technology choice function and the following new properties hold. l_T is decreasing in ϕ and MC is decreasing in y and ϕ.

3.2 Market structure

To endogenize y, we have to specify the market structure.

Consider an economy wherein a monopolistically competitive industry. Households supply labor inelastically and wage is exogenous. All these economic agents are symmetric.

The preference of the representative household is represented by $U = \int_0^n v(x_i)di$, where $v' > 0$, $v'' < 0$, and $x_i(> 0)$ is consumption of variety i (Krugman, 1979). We define $\theta(x_i)$ as $\theta(x_i) \overset{\text{def}}{=} -v'(x_i)/[v''(x_i)x_i]$. θ coincides with the demand elasticity for each variety with respect to price.

3.3 Market equilibrium

The firm's decisions are as follows. The firm decides whether to pay l_F to enter the market first, selects a pair of (l_P, l_T) second, and selects a pair of (y, p) last.

After entering the market, the firm minimizes variable cost by selecting (l_P, l_T) and then obtains a variable cost function as $l = [(\phi + 1)/\phi]y^{1/(\phi+1)} + l_F$. Next, the firm maximizes the profit, π, by selecting (y, p). Note that π is given by $\pi = py - l$. Since firms have the market power, the profit-maximization (PM) condition is given by $\mathbf{PM}: p = \mu(y)MC(y)$, where $\mu(y)$ is the mark-up and is defined as $\mu(y) \overset{\text{def}}{=} 1 + 1/[\theta(y) - 1]$. We assume $\mu(y) > 1$ and $d\mu/dy \geq 0$.

The firm can enter the market freely till its profit is zero. The free-entry (FE) condition is given by $\mathbf{FE}: p = l/y$.

The PM and FE conditions and $MC = l_V/(ySEV_D)$ give the following PM-FE condition:

$$\textbf{PM-FE}: \quad \mu(y) = SEV_D(\phi)\left(1 + \frac{l_F}{l_V(y, \phi)}\right). \tag{3}$$

(3) gives a unique inner equilibrium, y .[5)]

3.4 An increase in public knowledge stock and fixed costs

(3) derives the impacts of an increase in ϕ and l_F on (y, l_T, SE_D) as follows.

Proposition 3. *In a unique inner equilibrium, an increase in ϕ or l_F raises y and has ambiguous impacts on SE_D. It raises (or does not change) SE_D if $\mu(y)$ is increasing in y (constant).*

This proposition shows that an increase in fixed costs and public knowledge stock affects D-SE in the same way. Both raise (do not raise) D-SE through technology choice, depending on whether the mark-up rate is increasing in output (constant). Thus, it is critical whether the mark-up rate is constant or variable. The role of the mark-up rate is explained as follows. In (3), when the mark-up rate is increasing in output, changes in

5) Necessary and sufficient conditions are given in the Appendix.

public knowledge stock or fixed costs adjust not only l_V but also the mark-up rate. This then raises y moderately and weakens the negative impact of an increase in y on D-SE. Hence, the increasing mark-up rate derives a larger D-SE. The IT revolution and an increase in foreign direct investment may be interpreted as an increase in ϕ. Thus, Proposition 3 implies that these factors can increase D-SE if the mark-up rate is increasing in output.

Acknowledgments

I am grateful to Naoto Jinji for his helpful comments.

References

Bustos, P. (2011). "Trade Liberalization, Exports, and Technology Upgrading: Evidence on the Impact of MERCOSUR on Argentinian Firms." *American Economic Review*, 101(1): 304-340.

Krugman, P. R. (1979)."Increasing Returns, Monopolistic Competition, and International Trade." *Journal of International Economics*, 9(4): 469-479.

Krugman, P. R. (1980)."Scale Economies, Product Differentiation, and the Pattern of Trade." *American Economic Review*, 70(5): 950-959.

Krugman, P. R. (1991)."Increasing Returns and Economic Geography." *The Journal of Political Economy*, 99(3): 483-499.

Yeaple, S. R. (2005).“A Simple Model of Firm Heterogeneity, International Trade, and Wages.” *Journal of International Economics*, 65(1): 1-20.

Appendix

In this appendix, a hat indicates the rate of change for any variable, e.g., $\hat{x} \overset{\text{def}}{=} dx/x$. We introduce S_{l_F}, S_{l_V}, S_{l_P} and S_{l_T}, which are defined as $S_{l_F} \overset{\text{def}}{=} l_F/l$, $S_{l_V} \overset{\text{def}}{=} l_V/l$, $S_{l_P} \overset{\text{def}}{=} l_P/l_V$ and $S_{l_T} \overset{\text{def}}{=} l_T/l_V$, respectively. In subsections F and G, we introduce α and γ, which are defined as $\alpha \overset{\text{def}}{=} 1/(\phi+1)$ and $\gamma \overset{\text{def}}{=} (\phi+1)/\phi$. Hence, we can rewrite $l_V = [(\phi+1)/\phi]y^{1/(\phi+1)}$ in Lemma 3 as $l_V = \gamma y^{\alpha}$. We should note that $\alpha < 1$ holds from the definition of α and $\phi > 0$.

A. Proof of Lemma 1

First–order condition

The variable cost (l_V) minimization problem can be rewritten as maximization of $-l_V$. We construct Lagrangian, $\mathcal{L}$ as follows:

$$\mathcal{L} = -(l_P + l_T) + \lambda_y[y - F(l_T, \phi)l_P] + \lambda_P l_P + \lambda_T l_T$$

The first order Kuhn-Tucker conditions are given by

$$\frac{\partial \mathcal{L}}{\partial l_P} = -1 - \lambda_y F(l_T, \phi) + \lambda_P = 0, \tag{A.1}$$

$$\frac{\partial \mathcal{L}}{\partial l_T} = -1 - \lambda_y F_{l_T}(l_T, \phi) l_P + \lambda_T = 0, \tag{A.2}$$

$\partial \mathcal{L}/\partial \lambda_P \geq 0$, $\lambda_P \geq 0$, $(\partial \mathcal{L}/\partial \lambda_P)\lambda_P = 0$, $\partial \mathcal{L}/\partial \lambda_T \geq 0$, $\lambda_T \geq 0$, $(\partial \mathcal{L}/\partial \lambda_T)\lambda_T = 0$, and $\partial \mathcal{L}/\partial \lambda_y = 0$.

These conditions characterize (l_P, l_T, l_V) as follows. If $l_P = 0$ holds, $y = F(l_T, \phi)l_P$ does not hold for $y > 0$. Then, we obtain $l_P > 0$. $l_P > 0$ and $(\partial \mathcal{L}/\partial \lambda_P)\lambda_P = 0$ derive $\lambda_P = 0$. (A.1), (A.2) and $\lambda_P = 0$ derive

$$1 - \lambda_T = \frac{F_{l_T}(l_T, \phi) l_P}{F(l_T, \phi)}. \tag{A.3}$$

If $l_T = 0$ holds, $\lambda_T > 0$ and $l_P = y/F(0, \phi)$ hold. These properties and (A.3) derive

$$\frac{F_{l_T}(0, \phi) y}{[F(0, \phi)]^2} < 1. \tag{A.4}$$

(A.4) contradicts the assumption of $F_{l_T}(0, \phi)y/[F(0, \phi)]^2 \geq 1$. Hence, we obtain $l_T > 0$. $l_T > 0$ and $(\partial \mathcal{L}/\partial \lambda_T)\lambda_T = 0$ derive $\lambda_T = 0$. Then, $\lambda_T = 0$, (A.3) and $y = F(l_T, \phi)l_P$ derive (1). Q.E.D.

Second–order condition

We define the bordered hessian, $\tilde{H}$, as follows:

$$\tilde{H} = \begin{vmatrix} 0 & -\frac{\partial(Fl_P)}{\partial l_P} & -\frac{\partial(Fl_P)}{\partial l_T} \\ -\frac{\partial(Fl_P)}{\partial l_P} & \frac{\partial^2 \mathcal{L}}{\partial {l_P}^2} & \frac{\partial \mathcal{L}}{\partial l_P}\frac{\partial \mathcal{L}}{\partial l_T} \\ -\frac{\partial(Fl_P)}{\partial l_T} & \frac{\partial \mathcal{L}}{\partial l_T}\frac{\partial \mathcal{L}}{\partial l_P} & \frac{\partial^2 \mathcal{L}}{\partial {l_T}^2} \end{vmatrix}. \tag{A.5}$$

The second–order condition for the maximization is $\tilde{H} > 0$. From (A.5), $\tilde{H} > 0$ is equivalent to the assumption of $F_{l_T l_T} < 2(F_{l_T})^2/F$ from

$$\tilde{H} = \begin{vmatrix} 0 & -F & -F_{l_T} l_P \\ -F & 0 & -\lambda_y F_{l_T} \\ -F_{l_T} l_P & -\lambda_y F_{l_T} & -\lambda_y F_{l_T l_T} l_P \end{vmatrix} = \lambda_y l_P F[2(F_{l_T})^2 - F F_{l_T l_T}] > 0 \leftrightarrow F_{l_T l_T} < 2(F_{l_T})^2/F.$$

Q.E.D.

B. Proof of Proposition 1

Proof of property (a)

We totally differentiate $y = al_P$ by keeping y fixed and obtain

$$\hat{a} + \widehat{l_P} = 0. \tag{B.1}$$

We totally differentiate $a = F(l_T, \phi)$ and obtain

$$\hat{a} = ET\widehat{l_T} + \eta_\phi \hat{\phi}, \tag{B.2}$$

where η_ϕ is defined as $\eta_\phi \stackrel{\text{def}}{=} F_\phi \phi/F$. From $F_\phi > 0$, $\eta_\phi > 0$ holds. (B.1) and (B.2) derive

$$\widehat{l_P} + ET\widehat{l_T} + \eta_\phi \hat{\phi} = 0. \tag{B.3}$$

(1) and assumption of $\partial ET/\partial l_T = 0$ derive $ET(\phi) = l_T/l_P$. We totally differentiate this equation and obtain

$$\tau_\phi \widehat{\phi} = \widehat{l_T} - \widehat{l_P}, \tag{B.4}$$

where τ_ϕ is defined as $\tau_\phi \stackrel{\text{def}}{=} ET_\phi \phi/ET$. From assumption of $dET/d\phi > 0$, $\tau_\phi > 0$ holds.

(B.3) and (B.4) derive $(\widehat{l_P}, \widehat{l_T})$ as follows,

$$\widehat{l_P} = -\frac{\eta_\phi + ET\tau_\phi}{1+ET}\hat{\phi}, \tag{B.5}$$

$$\widehat{l_T} = \frac{\tau_\phi - \eta_\phi}{1+ET}\hat{\phi}. \tag{B.6}$$

(B.5) implies $\partial l_P/\partial\phi < 0$. (B.6) implies that the sign of $\partial l_T/\partial\phi$ is ambiguous because sign of $\tau_\phi - \eta_\phi$ is ambiguous.

We totally differentiate $l_V = l_T + l_P$ and obtain

$$\widehat{l_V} = S_{l_P}\widehat{l_P} + S_{l_T}\widehat{l_T}. \tag{B.7}$$

Equations (B.5)-(B.7) yield

$$\widehat{l_V} = -\frac{\eta_\phi}{1+\eta_{l_T}}\hat{\phi}. \tag{B.8}$$

(B.8) implies $\partial l_V/\partial\phi < 0$. Q.E.D.

Proof of property (b)

We can rewrite (1) as

$$y = \frac{F^2}{F_{l_T}}. \tag{B.9}$$

We totally differentiate (B.9) and when $d\phi = 0$ holds, we obtain

$$dl_T = \frac{(F_{l_T})^2}{F[2(F_{l_T})^2 - FF_{l_T l_T}]}dy. \tag{B.10}$$

(B.10) and the assumption of $F_{l_T l_T} < 2(F_{l_T})^2/F$ yield $\partial l_T/\partial y > 0$.

We totally differentiate $y = F(l_T, \phi)l_P$ and when $d\phi = 0$ holds, we obtain $dy = F_{l_T} l_P dl_T + F dl_P$. This equation and (1) yeild

$$dy = F(dl_T + dl_P). \tag{B.11}$$

(B,10) and (B.11) derive

$$dl_P = \frac{(F_{l_T})^2 - FF_{l_T l_T}}{F[2(F_{l_T})^2 - FF_{l_T l_T}]}dy. \tag{B.12}$$

(B.12) shows $\partial l_P/\partial y > 0$ from the assumption of $(F_{l_T})^2 - FF_{l_T l_T} > 0$ in footnote 3.

(B.10), (B.12) and $dl_V = dl_T + dl_P$ yield

$$dl_V = \frac{1}{F}dy. \tag{B.13}$$

(B.13) implies $\partial l_V / \partial y > 0$. Q.E.D.

C. Derivation of Equation (2)

We take the log of both sides of $y = al_P$ and totally differentiate it to obtain

$$\hat{y} = \hat{a} + \widehat{l_P}. \tag{C.1}$$

(B.7), (C.1), and the definition of SEV_D derive

$$SEV_D = \frac{\hat{a} + \widehat{l_P}}{S_{l_T}\widehat{l_T} + S_{l_P}\widehat{l_P}}. \tag{C.2}$$

(1) and $y = F(l_T, \phi)l_P$ derive $S_{l_T}/S_{l_P} = ET$. From $S_{l_T}/S_{l_P} = ET$ and $\hat{a}/\widehat{l_T} = ET$, (C.2) can be rewritten as

$$\begin{aligned} SEV_D =& \frac{ET\widehat{l_T} + \widehat{l_P}}{S_{l_P}(ET\widehat{l_T} + \widehat{l_P})}, \\ =& \frac{1}{S_{l_P}}, \\ =& 1 + ET. \qquad \text{by } S_{l_T}/S_{l_P} = ET \text{ and } S_{l_T} = 1 - S_{l_P} \end{aligned} \tag{C.3}$$

Hence, $SEV_D = 1 + ET$ follows. Q.E.D.

D. Proof of Lemma 2

By definition, SE_D can be rewritten as

$$\begin{aligned} SE_D &= \frac{d\log y}{d\log l}. \\ &= \left(\frac{dy}{dl}\right)\left(\frac{l}{y}\right) \\ &= \left(\frac{dy}{dl_V}\frac{dl_V}{dl}\right)\left(\frac{l}{l_V}\frac{l_V}{y}\right) \\ &= \left(\frac{d\log y}{d\log l_V}\right)\left(\frac{l}{l_V}\right) \\ &= SEV_D\left(1 + \frac{l_F}{l_V}\right). \end{aligned} \tag{D.1}$$

Hence, $SE_D = SEV_D(1 + l_F/l_V)$ directly follows. Q.E.D.

E. Proof of Lemma 3

We take the log of both sides of $F(l_T, \phi) = \phi l_T^{\phi}$ to obtain $\log F = \log \phi + \phi \log l_T$. This equation derives $\partial \log F / \partial \log l_T = \phi$. Hence, $ET = \phi$ holds. This equation and (2) derive $SEV_D = 1 + \phi$.

$F(l_T, \phi) = \phi l_T^{\phi}$ and (1) yield

$$l_T = y^{1/(\phi+1)}. \tag{E.1}$$

$y = al_P$, $a = \phi l_T^{\phi}$ and (1) yield

$$l_P = y^{1/(\phi+1)}/\phi. \tag{E.2}$$

(E.1), (E.2) and $l_V = l_P + l_T$ yield

$$l_V = \frac{\phi+1}{\phi} y^{1/(\phi+1)}. \tag{E.3}$$

We differentiate (E.3) with respect to y to obtain

$$MC = \frac{\partial l_V}{\partial y} = \frac{y^{-\phi/(\phi+1)}}{\phi}. \tag{E.4}$$

We differentiate (E.4) with respect to y and ϕ to obtain

$$\frac{\partial MC}{\partial y} = -\frac{y^{-(2\phi+1)/(\phi+1)}}{\phi+1} < 0. \tag{E.5}$$

$$\frac{\partial MC}{\partial \phi} =< 0, \tag{E.6}$$

respectively. Q.E.D.

F. Necessary and sufficient condition for a unique inner equilibrium

For the following analysis, we define AC and AVC as $AC \overset{\text{def}}{=} l/y$ and $AVC \overset{\text{def}}{=} l_V/y$, respectively.

F.1. Additional proposition

Proposition 4. *l_V is specified as $l_V = \gamma y^{\alpha}$, where γ and α are positive. Then, the following properties hold.*

(a) If an inner equilibrium, $y > 0$, exists, $\alpha < 1$ or $l_F > 0$ holds. If the inner equilibrium is unique, $\max\{l_F, d\mu/dy\} > 0$ *holds.*

(b) A unique inner equilibrium, $y > 0$, holds when $l_F > 0$ and one of the following two cases hold. Case.1: $d\mu/dy > 0$. Case.2: $d\mu/dy = 0$ and $\mu > 1/\alpha$. [6)]

F.2. (y, p) Plane

The equilibrium conditions, $PM : p = \mu MC$ and $FE : p = AC$, depict a curve in (y, p) respectively. The existence of the intersection certifies the existence of the equilibrium. We represent the right-hand sides of $PM : p = \mu MC$ and $FE : p = l/y$ as $PM(y)$ and $FE(y)$, respectively.

F.2. Proof of Property (a)

Existence of the inner equilibrium

We prove the existence of the inner equilibrium by contractive induction. We assume that under $\alpha \geq 1$ and $l_F = 0$, there is y such that y satisfies $FE(y) = PM(y)$.

From $\mu > 1$, $PM(y) > MC(y)$ holds.

On the other hand, $FE(y) \leq MC$ can shown in the following way. From $l_F = 0$, $AC = AVC$ holds. This equation and $MC = \alpha AVC$ derive $AC = (1/\alpha)MC$. Hence, $FE(y) = (1/\alpha)MC$ holds. From $\alpha \geq 1$, $FE(y) \leq MC$ holds. Hence $FE(y) \leq MC$ holds.

These properties imply $PM(y) > FE(y)$. This contradicts that there is y such that satisfies $FE(y) = PM(y)$. Hence, if the inner equilibrium exists, $\alpha < 1$ or $l_F > 0$ holds. Q.E.D.

Uniqueness of the inner equilibrium

We prove the uniqueness of the inner equilibrium by contractive induction. We assume that under $l_F = d\mu/dy = 0$, the inner equilibrium is determined uniquely.

From $l_F = 0$, $AC = AVC$ holds. From $d\mu/dy = 0$, $\mu(y) = \bar{\mu}$ holds for arbitrary y where $\bar{\mu}$ is constant. Hence, PM-FE condition is rewritten as

$$\mu = \frac{1}{\alpha}. \tag{F.1}$$

Since $l_F = 0$ is assumed, $\alpha < 1$ must be hold if the inner equilibrium exists. From $\mu > 1$, (F.1) holds under certain pairs of (μ, α).

Since both sides of (F.1) do not depend on y, a number of inner equilibrium can exist. This contradicts that under $l_F = d\mu/dy = 0$, the inner equilibrium is determined uniquely. Hence, if the inner equilibrium is determined uniquely, $\max\{l_F, d\mu/dy\} > 0$ holds. Q.E.D.

---

6) All of these cases require that PM curve intersects the FE curve only once from below in (y, p) plane, where PM and FE curves are characterized by PM and FE conditions respectively. This implies that this equilibrium is stable for an adjustment of the number of firms since $\partial\pi/\partial n < 0$ holds in the equilibrium.

F.3. Proof of Property (b)

From $FE(y) - PM(y) = AC(y) - \mu(y)MC(y)$ and $l_V = \gamma y^\alpha$, we obtain

$$FE(y) - PM(y) = \frac{l_F - (\alpha\mu - 1)\gamma y^\alpha}{y}. \tag{F.2}$$

We consider a case of $d\mu/dy = 0$. $FE(y) - PM(y) = 0$ and (F.2) derives

$$y = \left[\frac{l_F}{\gamma(\alpha\mu - 1)}\right]^{1/\alpha}. \tag{F.3}$$

Hence, under $\mu > 1/\alpha$, $y > 0$ uniquely exists.

We next consider a case of $d\mu/dy > 0$. Since $\mu(0)$ is finite, (F.2) derives $\lim_{y\to 0}[FE(y) - PM(y)] = \infty > 0$. For (F.2), from l'Hospital's rule, we obtain

$$\lim_{y\to\infty}[FE(y) - PM(y)] = -\alpha(d\mu/dy)\gamma y^\alpha - (\alpha\mu - 1)\gamma\alpha y^{\alpha-1} = -\infty.$$

Hence, $y > 0$ exists from the intermediate value theorem. The numerator on the right-hand side of (F.2) is decreasing in y. Then, we obtain $d[FE(y) - PM(y)]/dy < 0$. Hence, $y > 0$ exits uniquely. Q.E.D.

G. Proof of Proposition 3

For the following analysis, we define the elasticity of μ with respect to y, $\epsilon(y)$, as $\epsilon(y) \stackrel{\text{def}}{=} \partial \log \mu / \partial \log y$, where $\epsilon(y) \geq 0$ holds from $d\mu/d\theta < 0$ and $d\theta/dy \leq 0$.

G.1. Derivation of the rate of change of variables

From the definition of $\alpha \stackrel{\text{def}}{=} 1/(\phi + 1)$ and $\gamma \stackrel{\text{def}}{=} (\phi + 1)/\phi$, we obtain

$$\hat{\alpha} = -\frac{\phi}{\phi + 1}\hat{\phi}, \tag{G.1}$$

$$\hat{\gamma} = -\frac{1}{\phi + 1}\hat{\phi}. \tag{G.2}$$

We take the log of both sides in (3) and the total differentials are given by

$$\hat{y} = \frac{1}{\alpha S_{l_F} + \epsilon(y)}\left[S_{l_F}\widehat{l_F} + -S_{l_F}\hat{\gamma} - (\alpha S_{l_F}\log y + 1)\hat{\alpha}\right]. \tag{G.3}$$

(G.1), (G.2) and (G.3) yield $\partial y/\partial l_F > 0$ and $\partial y/\partial \phi > 0$.

From Lemma 2, $SEV_D = 1+\phi$ of Lemma 3 and the definition of α, $SE_D = 1/(\alpha S_{l_V})$ holds. Hence, we obtain $SE_D = -(\hat{\alpha} + \widehat{S_{l_V}})$. This can be rewritten as

$$\widehat{SE_D} = \frac{1}{\alpha S_{l_F} + \epsilon}\left[(\epsilon S_{l_F})\widehat{l_F} - (\epsilon S_{l_F})\hat{\gamma} - \epsilon(1 + S_{l_F}\log l_V)\hat{\alpha}\right]. \tag{G.4}$$

(G.1), (G.2) and (G.4) derive $\partial SE_D/\partial l_F > 0$ and $\partial SE_D/\partial \phi > 0$ if $\epsilon > 0$. If $\epsilon = 0$, $\partial SE_D/\partial l_F = \partial SE_D/\partial \phi = 0$ hold. Q.E.D.

G.2. Derivation of Equation (G.3)

(3) can be rewritten as

$$\mu = \frac{l}{l_V \alpha}. \tag{G.5}$$

We take the log of both sides of (G.5) and totally differentiate it to obtain

$$\hat{\mu} = \widehat{l} - \widehat{l_V} - \hat{\alpha}. \tag{G.6}$$

For the right-hand side of (G.6), the following lemma holds.

Lemma 4. $\hat{l} - \widehat{l_V} = S_{l_F}[\widehat{l_F} - (\alpha\hat{y} + \hat{\gamma}) - (\alpha \log y)\hat{\alpha}]$

On the other hand, for the left-hand side of (G.6), $\hat{\mu} = \epsilon(y)\hat{y}$ holds. Hence, we can obtain (G.3). Q.E.D.

Proof of Lemma 4

We take the log of both sides of $l = l_V + l_F$ and $l_V = \gamma y^{\alpha}$. Totally differentiate them to yield

$$\hat{l} = S_{l_V}\widehat{l_V} + S_{l_F}\widehat{l_F}, \tag{G.7}$$

$$\widehat{l_V} = \alpha\hat{y} + \hat{\gamma} + (\alpha \log y)\hat{\alpha}. \tag{G.8}$$

(G.7) and (G.8) derive the equation of Lemma 4. Q.E.D.

G.3. Derivation of Equation (G.4)

We take the log of both sides of $S_{l_V} = l_V/l$ and totally differentiate it to obtain

$$\begin{aligned}\widehat{S_{l_V}} =& \widehat{l_V} - \hat{l},\\ =& S_{l_F}[-\widehat{l_F} + \alpha\hat{y} + \hat{\gamma} + (\alpha \log y)\hat{\alpha}] \qquad \text{by Lemma 4}\end{aligned} \tag{G.9}$$

$SE_D = -(\hat{\alpha} + \widehat{S_{l_V}})$ and (G.9) derive (G.4) as follows:

$$\begin{aligned} \widehat{SE_D} =& - (\hat{\alpha} + \widehat{S_{l_V}}), \\ =& - \hat{\alpha} + S_{l_F} \left[\widehat{l_F} - \alpha \hat{y} - \hat{\gamma} - (\alpha \log y) \hat{\alpha} \right], & \text{by (G.9)} \\ =& \frac{1}{\alpha S_{l_F} + \epsilon} \left[(\epsilon S_{l_F}) \widehat{l_F} - (\epsilon S_{l_F}) \hat{\gamma} - \epsilon (1 + S_{l_F} \log l_V) \hat{\alpha} \right]. & \text{by (G.3)} \end{aligned}$$

Q.E.D.