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Fujisaki, Seiya

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Seiya Fujisaki*

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Abstract

We analyse the relationship between two kinds of tax and equilibrium determinacy in an economy with government expenditure used for utility. We assume that the income tax rate depends on the level of income itself and that the tax rate of consumption is constant. This describes a realistic tax system which resonates in many countries. Our model complements similar extant research, but we extend the literature by theoretically showing that the expansion of policy types can decrease the risk of instability when one condition slightly changes.

Keywords: equilibrium determinacy, progressive income tax rate, consumption tax, government expenditure on utility.

JEL Classification Numbers: E62.

*Institute of Education, Academic Assembly, Shinshu University, 6-Ro, Nishinagano, Nagano, Nagano, 380-8544, JAPAN (email: fujisaki@shinshu-u.ac.jp)

1 Introduction

We analyse the effect of two kinds of tax, income and consumption, on the stability (equilibrium determinacy) of an economy.

We consider a model in which households benefit from government expenditure. As Kamiguchi and Tamai (2011) show, an important factor of determinacy is a revenue source for providing public services rather than the presence of productive government spending, [1] and this setting has already been researched in many studies. Guo and Harrison (2008) and Hori and Maebayashi (2013) claim that externality of government expenditure on utility is significant determinants macroeconomic stability. However, they postulate constant tax rates. Chen and Guo (2016, 2017) assume endogenous growth, the government expenditure on the utility and the income tax rate can depend on the level of income itself as in Guo and Lansing (1998). They demonstrate that government expenditure on utility does not affect equilibrium determinacy. In addition, they show that regressive tax is preferable for ensuring a stable economy, which contrasts with Guo and Lansing's (1998) real business cycle model analysis with increasing returns in production.

We assume the income tax rate is a function of income as in Guo and Lansing (1998) with a constant tax rate of consumption. This is empirically credible based on the tax system as in Japan and other countries. As McKnight (2016) notes, many countries shift from direct to indirect taxation, and he shows that consumption taxes are more desirable than income taxes in view of equilibrium determinacy with interest-rate control type monetary policy. We extend the study of Chen and Guo (2014) by appending the consumption tax rate while maintaining their approach to income tax in this regard to Guo and Lansing (1998), public spending on utility and physical capital and labour in the production function. Our model can also interpret the case where the variable income tax rate is added to Hori and Maebayashi (2013). Our results complement these foregoing studies in that their results are reasonably robust even if another kind of tax is levied. For example, equilibrium is necessarily determinate if government expenditure and private consumption are substitutes. However, the expansion of policy mechanisms can decrease the risk of instability under the specific policy setting when one condition slightly changes. Under some situations, consumption taxes do not serve to satisfy the equilibrium condition and thus the self-fulfilling expectation does not hold; this results in economic stability in that non-fundamental

[1] In Bambi and Venditti (2016), consumption tax rate are time-varying and government expenditure is used for production. They show that there exists a unique balanced growth path but that sunspot equilibria based on self-fulfilling expectations emerge under countercyclical consumption taxes.

factors cannot affect economic fluctuations.

2 Economy

Representative households solve the dynamic optimisation problem, [2]

$$\max \quad \int_0^\infty \frac{[c^{\theta_c} g^{1-\theta_c}]^{1-\sigma}}{1-\sigma} - \frac{l^{1+\chi}}{1+\chi} e^{-\rho t} dt, \quad \rho > 0, \quad \sigma > 0, \quad \chi \geq 0, \quad 0 < \theta_c \leq 1 \tag{1}$$

subject to

$$\dot{k} = (1 - \tau_y(y))y - (1 + \bar{\tau}_c)c, \tag{2}$$

and the no-Ponzi condition, where ρ is the time discount rate, c private consumption, k capital, l labour, and $\tau_y(y)$ the tax rate on income which depends on income level y as below. The consumption tax rate $\bar{\tau}_c (\geq 0)$ is constant. The output technology using capital and labour is in the form of a Cobb-Douglas function,

$$y = k^\alpha l^{1-\alpha}, \quad 0 < \alpha < 1. \tag{3}$$

The amount of government expenditure g included in households' utility depends on the income and consumption taxes:

$$\tau_y(y) \cdot y + \bar{\tau}_c \cdot c = g. \tag{4}$$

The income tax rate is formulated as per Guo and Lansing (1998):

$$\tau_y(y) = 1 - \eta \left(\frac{\bar{y}}{y} \right)^\phi, \quad \eta \in (0, 1], \quad \phi < 1. \tag{5}$$

where η is the proportion of disposal income around the steady-state level of income $\bar{y}$. [3] The marginal rate of tax is

$$\tau_y(y) + \tau_y'(y)y = 1 - \eta(1-\phi) \left(\frac{\bar{y}}{y} \right)^\phi,$$

[2]We omit the case where the preference externality is enough high investigated as in Hori and Maebayashi (2013), in order to focus on the effect of fiscal policy.

[3]As in Chen and Guo (2016 and 2017), we restrict the lowest ϕ is $\max\left[-\frac{1-\alpha}{\alpha}, -\frac{1-\eta}{\eta} \right]$. In addition, we assume $\chi + \alpha + (1-\alpha)\phi > 0$. We allow regressive tax, $\phi < 0$, under these conditions. This is different from Chen and Guo (2014) who postulate $0 < \phi < 1$, which means that they do not consider regressive income tax.

and thus

$$1-\tau_y(y)-\tau_y'(y)y=\eta(1-\phi)\left(\frac{\bar{y}}{y}\right)^{\phi}, \tag{6}$$

The conditions for this optimization are as follows:

$$\theta_c c^{-[1-\theta_c(1-\sigma)]}g^{(1-\theta_c)(1-\sigma)}=\lambda[1+\bar{\tau}_c], \tag{7}$$

$$\lambda[1-\tau_y(y)-\tau_y'(y)y]\frac{(1-\alpha)y}{l}=l^{\chi}, \tag{8}$$

$$\dot{\lambda}=\left(\rho-[1-\tau_y(y)-\tau_y'(y)y]\frac{\alpha y}{k}\right)\lambda, \tag{9}$$

with the transversality condition, where λ is a shadow value of capital.

3 Reduced Dynamic System

From Eqs. (3) and (8),

$$l=[\eta(1-\phi)\bar{y}(1-\alpha)k^{\alpha(1-\phi)}\lambda]^{\frac{1}{\chi+\alpha+(1-\alpha)\phi}},$$

and thus output is

$$y=y(k,\lambda)=[\{\eta(1-\phi)\bar{y}(1-\alpha)\}^{1-\alpha}k^{\alpha(1+\chi)}\lambda^{1-\alpha}]^{\frac{1}{\chi+\alpha+(1-\alpha)\phi}}. \tag{10}$$

Combining Eqs. (4), (5), and (7), we obtain

$$\theta_c c^{-[1-\theta_c(1-\sigma)]}[y(k,\lambda)-\eta\bar{y}^{\phi}y(k,\lambda)^{1-\phi}+\bar{\tau}_c\cdot c]^{(1-\theta_c)(1-\sigma)}=\lambda[1+\bar{\tau}_c], \tag{11}$$

and thus $c=c(k,\lambda)$ which satisfies around the steady state, [4]

$$c_k=\left.\frac{\partial c}{\partial k}\right|_{ss}=(1-\theta_c)(1-\sigma)[1-\eta(1-\phi)]\frac{\alpha(1+\chi)}{\chi+\alpha+(1-\alpha)\phi}\frac{\bar{y}\bar{c}}{\bar{k}\bar{g}}\left[\sigma+(1-\theta_c)(1-\sigma)\frac{(1-\eta)\bar{y}}{\bar{g}}\right]^{-1}, \tag{12}$$

[4] From Eq. (11), the following are satisfied:

$$dc_{ss}=-\left[\sigma+(1-\theta_c)(1-\sigma)\frac{(1-\eta)\bar{y}}{\bar{g}}\right]\frac{\bar{\lambda}(1+\bar{\tau}_c)}{\bar{c}}<0,$$

$$dk_{ss}=(1-\theta_c)(1-\sigma)\frac{\bar{\lambda}(1+\bar{\tau}_c)}{\bar{g}}[1-\eta(1-\phi)]\frac{\alpha(1+\chi)}{\chi+\alpha+(1-\alpha)\phi}\frac{\bar{y}}{\bar{k}},$$

$$d\lambda_{ss}=(1-\theta_c)(1-\sigma)\frac{\bar{\lambda}(1+\bar{\tau}_c)}{\bar{g}}\frac{[1-\eta(1-\phi)](1-\alpha)}{\chi+\alpha+(1-\alpha)\phi}\frac{\bar{y}}{\bar{\lambda}}-(1+\bar{\tau}_c).$$

$$c_\lambda = \frac{\partial c}{\partial \lambda}\bigg|_{ss} = \left[(1-\theta_c)(1-\sigma)\frac{[1-\eta(1-\phi)](1-\alpha)}{\chi+\alpha+(1-\alpha)\phi}\frac{\bar{y}}{\bar{g}}-1\right]\frac{\bar{c}}{\bar{\lambda}}\left[\sigma+(1-\theta_c)(1-\sigma)\frac{(1-\eta)\bar{y}}{\bar{g}}\right]^{-1}. \tag{13}$$

The following equations constitute the dynamic system, which implies the goods-market equilibrium condition and the Euler equation:

$$\dot{k} = \eta\bar{y}^{\phi}y(k,\lambda)^{1-\phi} - (1+\bar{\tau}_c)c(k,\lambda), \tag{14}$$

$$\dot{\lambda} = \left[\rho - \frac{\alpha\eta(1-\phi)\bar{y}^{\phi}y(k,\lambda)^{1-\phi}}{k}\right]\lambda. \tag{15}$$

The coefficient matrix of the linearised system of the original (14)$-$(15) around the steady state is

$$J = \begin{bmatrix} \dot{k}_k & \dot{k}_\lambda \\ \dot{\lambda}_k & \dot{\lambda}_\lambda \end{bmatrix},$$

where

$$\dot{k}_k = \frac{\partial \dot{k}}{\partial k}\bigg|_{ss} = \eta(1-\phi)y_k - (1+\bar{\tau}_c)c_k, \quad \dot{k}_\lambda = \frac{\partial \dot{k}}{\partial \lambda}\bigg|_{ss} = \eta(1-\phi)y_\lambda - (1+\bar{\tau}_c)c_\lambda,$$

$$\dot{\lambda}_k = \frac{\partial \dot{\lambda}}{\partial k}\bigg|_{ss} = -\alpha\eta(1-\phi)\bar{\lambda}\frac{(1-\phi)\bar{k}y_k - \bar{y}}{\bar{k}^2}, \quad \dot{\lambda}_\lambda = \frac{\partial \dot{\lambda}}{\partial \lambda}\bigg|_{ss} = -\alpha\eta(1-\phi)^2\bar{\lambda}\frac{y_\lambda}{\bar{k}},$$

and thus

$$\begin{aligned} DetJ = \mu_1\mu_2 = \dot{k}_k\cdot\dot{\lambda}_\lambda - \dot{k}_\lambda\cdot\dot{\lambda}_k = \\ \frac{\alpha\eta(1-\phi)(1+\bar{\tau}_c)\bar{y}\bar{c}}{\bar{k}^2[\chi+\alpha+(1-\alpha)\phi]}\left[\sigma+(1-\theta_c)(1-\sigma)\frac{(1-\eta)\bar{y}}{\bar{g}}\right]^{-1} \\ \left[-(\chi+\phi)+\alpha\chi(1-\phi)-\sigma(1-\phi)(1-\alpha)+(1-\theta_c)(1-\sigma)(1-\alpha)\phi\frac{\bar{y}}{\bar{g}}\right], \end{aligned} \tag{16}$$

$$\begin{aligned} TraceJ &= \mu_1+\mu_2 = \dot{k}_k + \dot{\lambda}_\lambda \\ &= \rho\left\{1-\frac{(1-\theta_c)(1-\sigma)[1-\eta(1-\phi)](1+\chi)}{(1-\phi)[\chi+\alpha+(1-\alpha)\phi]}\frac{1+\bar{\tau}_c}{1-\eta+\bar{\tau}_c}\left[\sigma+(1-\theta_c)(1-\sigma)\frac{(1-\eta)\bar{y}}{\bar{g}}\right]^{-1}\right\}, \end{aligned} \tag{17}$$

because

$$\frac{\eta\bar{y}}{\bar{k}} = \frac{(1+\bar{\tau}_c)\bar{c}}{\bar{k}} = \frac{\rho}{\alpha(1-\phi)}, \quad \frac{\bar{g}}{\bar{k}} = \frac{\rho}{\alpha(1-\phi)}\frac{1-\eta+\bar{\tau}_c}{\eta(1+\bar{\tau}_c)}, \quad \frac{\bar{y}}{\bar{g}} = \frac{1+\bar{\tau}_c}{1-\eta+\bar{\tau}_c}. \tag{18}$$

Table 1: Determinacy of Equilibrium

ϕ	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$
$1-\sigma$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
$\bar{\tau}_y-(1-\alpha)(1-\theta_c)$ $+\bar{\tau}_c\{\alpha+\theta_c(1-\alpha)\}$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$
determinacy of equilibrium	D	D, NS, I	D	D	D, NS, I	D	D	D

D = Determinate, NS = Non-Stationarity, I = Indeterminate.

4 Equilibrium Determinacy and Implications

There is one jump variable, λ, and one predetermined variable, k, in the dynamic system so that the steady state satisfies local determinacy, if one eigenvalue is positive, that is, $DetJ = \mu_1\mu_2 < 0$. When all eigenvalues μ_1 and μ_2 are positive, non-stationarity holds ($TraceJ = \mu_1 + \mu_2 > 0$). Otherwise, the equilibrium is indeterminate ($TraceJ < 0$). From Eqs. (16) and (18),

$$\begin{aligned}&\text{sign}[DetJ] = \\ &\text{sign}[-(\bar{\tau}_y+\bar{\tau}_c)(1-\alpha+\alpha\phi)(\chi+\sigma)-\phi(1-\sigma)[\bar{\tau}_y-(1-\alpha)(1-\theta_c)+\bar{\tau}_c\{\alpha+\theta_c(1-\alpha)\}]],\end{aligned} \tag{19}$$

where $\bar{\tau}_y \equiv 1-\eta$, which implies the standard rate of income tax. Therefore, results are summarised as the following proposition and Table 1:

Proposition 1 *If the sign of $\phi(1-\sigma)[\bar{\tau}_y-(1-\alpha)(1-\theta_c)+\bar{\tau}_c\{\alpha+\theta_c(1-\alpha)\}]$ is non-negative and $\sigma > 1$, equilibrium is necessarily determinate. Otherwise, indeterminacy or non-stationarity may emerge.*

We intuitively interpret the results in a manner similar to Hori and Maebayashi (2013). Suppose that agents expect higher government spending. If $\sigma < 1$, that is, government expenditure and private consumption are complements, the marginal utility of private consumption increases. This accelerates capital accumulation for future consumption, and thus output and wages rise while the shadow value of capital becomes lower. This induces higher government spending. When the income tax rate is progressive to income, the tax rate increases; however, the equilibrium condition may not be violated if the base tax rates are not sufficiently high to satisfy the preferences to government spending. Therefore, equilibrium may not be determinate in that the self-fulfilling expectation as above can be realised. Consumption tax can increase the possibility of determinacy since it facilitates increased tax revenues as the source of government expenditure and then the equilibrium conditions tend to be violated. Concretely, if

$$\bar{\tau}_y - (1-\alpha)(1-\theta_c) + \bar{\tau}_c\{\alpha + \theta_c(1-\alpha)\} = (1+\bar{\tau}_c)\left[\frac{\bar{g}}{\bar{y}} - (1-\alpha)(1-\theta_c)\right],$$

in Eq. (19) is positive (resp. negative), the standard tax rates are sufficiently high (resp. low) to satisfy the preferences to government expenditure. Conversely, under the regressive income tax rate, government expenditure can be larger but the equilibrium condition may hold if the preference weight on public spending is low relative to the base tax rates, and thus equilibrium cannot be determinate.

Our results complement Hori and Maebayashi (2013) and Chen and Guo (2014), in that their results are reasonably robust even if another kind of tax is levied, but the expansion of policy mechanisms can decrease the risk of instability under the specific policy setting when one condition slightly changes. Indeed, this may be the reason why many kinds of tax exist.

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