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Abstract

For any intransitive community preference, we construct a non-convex economy where all the marginal cost pricing general equilibria are Pareto inefficient (theorem 3.2). The result is valid without requiring a fixed income distribution rule (corollary 3.3). Intransitive community preferences are a frequent occurrence (theorem 3.1): necessary and sufficient conditions for transitivity of the community preference fail in a set which is open and dense in the space of individual preferences with a standard topology.

1. Introduction

Rational individuals are deemed to have transitive preferences, so that if α is preferred to β and β is preferred to χ, then α is also preferred to χ. However, examples of intransitive preferences emerge readily when instead of individual preferences one considers group preferences or aggregate preferences. Typically, the problem emerges when aggregating individual into social preferences. De Condorcet [11] provided a well-known example of three individuals with transitive preferences who give rise to a majority with intransitive preferences. Many other examples exist in social choice theory, which seeks to define the preferences of a group under different assumptions and ethical axioms about how individual preferences should be represented in a society [17]. Other examples are found in demand theory [14,9].

A less explored phenomenon is the impact of intransitivity on market behaviour. Markets aggregate preferences after a fashion, but do not follow explicit welfare or ethical considerations. The market's main virtue is, instead, efficiency. This paper shows how the intransitivity of preferences may prevent market efficiency in much the same way as it interferes with ethical axioms in the aggregation of individual into social preferences. The results are presented in the context of marginal cost price equilibrium in non-convex markets, as defined in Brown and Heal [2,4,5], and Guesnerie [13]. These are markets with economies of scale in production. The issue is that with intransitive community preferences, all of the *marginal cost pricing* (mcp) *equilibria*, a standard notion of equilibrium in such markets, may fail to be Pareto efficient. This means that transferring goods among the parties away from the equilibrium distribution improves the welfare of some without decreasing that of anyone else. When community preferences are instead transitive, at least one mcp equilibrium is always Pareto efficient (see Brown and Heal [1,2])*. The intransitivity of community preferences causes the loss of market efficiency.

Necessary and sufficient conditions on individual preferences have been found under which community preferences are always transitive (see Chichilnisky and Heal [10]), thus guaranteeing the efficiency of at least one market equilibrium. We prove here that these necessary and sufficient conditions for transitivity of community preferences are seldom satisfied: they fail in a set which is open and dense in the space of individual preference profiles. We adopt a standard definition of preferences and their topology (see Debreu [12] and Chichilnisky [6,7]).

The first result is that for an open and dense set of individual preference profiles, community preferences are intransitive (theorem 3.1). The second result is that whenever individual preferences give rise to intransitive community preferences, an economy can be constructed where all the marginal cost pricing equilibria are Pareto inefficient (theorem 3.2), and we do so. This construction is valid for any distribution of income, provided this allocates a given production vector in a pre-assigned fashion. It does not require a fixed distribution of income rule (corollary 3.3), as is frequently required in the literature on equilibria with non-convexities.

In sum: for an open and dense set of individual preferences there exist economies where transfers away from the mcp equilibria are Pareto improving.

2. Definitions

2.1. PREFERENCES

Consumption vectors belong to P, the interior of the positive cone of $\mathbb{R}^n$. Following Debreu [12] and Chichilnisky [6,7], a preference g is defined by specifying for every ξ in P a non-zero vector $g(\xi)$ in $\mathbb{R}^n$, the intended interpretation of which is that the hyperplane $H(\xi)$ through ξ orthogonal to $g(\xi)$ is tangent at ξ to the indifference hypersurface of ξ, and $g(\xi)$ indicates a direction of preference. We normalize $g(\xi)$ by requiring that $||g(\xi)|| = 1$, where the vertical bars denote the Euclidean norm.We assume that there exists a real valued positive utility function u defined on P, with hypersurfaces contained in P, and having a derivative Du which is everywhere a strictly positive multiple of g, i.e. such that everywhere in P,

$$Du = \lambda g, \tag{2.1.1}$$

where λ is a function from P to the set of strictly positive real numbers. This definition follows Debreu [12]. When the utility u is C^2 (a twice continuously differentiable

*This result is stated in Brown and Heal [2] and is proved in Brown and Heal [1]; it establishes that if an mcp equilibrium exists, and community preferences are transitive, at least one mcp equilibrium is Pareto efficient. In a similar context, Beato and Mas-Colell (J. Econ. Theory (1985)) presented an example of a non-convex economy where all marginal cost pricing equilibria are productively inefficient.

function with bounded derivatives on P), which we now assume, preferences are elements of the space $C^1(P, \mathbb{R}^n)$ of continuously differentiable bounded functions from P to $\mathbb{R}^n$ with bounded derivatives*, and are therefore topologized with the standard topology they inherit from $C^1(P, \mathbb{R}^n)$. This topological space of preferences is denoted S. A preference g in S is *monotone* when for all ξ in P, $g(\xi) \in P$, and in this case the utility u in (2.1.1) is *regular* since its derivative never vanishes. The space $\mathcal{P}$ of all monotone preferences in S is an open subset of S.

A *profile of preferences* is an ordered m-tuple of preferences $(g_1, \ldots, g_m)$ in the product space $\mathcal{P}^m$ representing the preferences of m individuals in the economy. The space $\mathcal{P}^m$ is endowed with the product topology it inherits from $\mathcal{P}$.

2.2. COMMUNITY PREFERENCES

A preference can also be identified by its graph as a subset of $P \times P$, where the graph of a preference is its graph as a relation: (α, β) is in the graph of a preference when β is preferred to α. We now define the set of choices preferred to a given ξ in P by a community preference associated to the profile $(g_1, \ldots, g_m)$:

$$G(\xi) = \bigcup_{\substack{\xi_i:\ \Sigma \xi_i = \xi \\ g_i(\xi_i) = g_j(\xi_j),\ \forall ij}} \sum_i G_i(\xi_i), \quad i, j = 1, \ldots, m, \tag{2.2.1}$$

where $g_i(\xi_i)$ is the normal to i's indifference surface at ξ_i, and $G_i(\xi_i)$ is the set of choices in P which are preferred by g_i to ξ_i. The graph of a *community preference* $\mathbf{G}$ is constructed by defining for each ξ in P the choices preferred to ξ: those in the set $\mathbf{G}(\xi)$, which is the union of the sum of certain preferred sets of individual preferences at choices ξ_i which add up to choice ξ. The union is taken not over all possible combinations of points that sum to ξ, but *only over those points which sum to ξ and at which the various individuals have the same marginal rate of substitution*. Equivalently, the sum is over those points where all individuals have identical normals to their indifference surfaces [ref. [16], p. 8; refs. [9,10,14]). This corresponds to an assumption that we aggregate the preferences which are revealed when all consumers face the same prices.

For each distribution (ξ_i^1), (ξ_i^2), etc. of endowments adding up to ξ at which the various individuals have the same tangents to their indifference hypersurfaces, there are sum sets $G^1(\xi) = \sum_i G_i(\xi_i^1)$, $G^2(\xi) = \sum_i G_i(\xi_i^2)$, etc. The sets $G^k(\xi)$ may be different, in particular each $G^k(\xi)$ could have a different tangent at ξ. Therefore, the different

*Since the cone P is unbounded, the condition that functions and their derivatives be bounded is needed to define the C^1 norm of a function $\|f\|_1 = \sup_{x \in P} \| f(x), Df(x) \|$. Such boundedness is not needed when the function f is defined over a compact space.

distributions of the endowment ξ may give rise to different sum sets $G^k(\xi)$ at ξ. Necessary and sufficient conditions were given in ref. [10] to ensure that the different distributions of ξ among the individuals do not give rise to different sum sets and therefore to contradictory statements about the community preference G. When different sets $G^1(\xi)$ and $G^2(\xi)$ arise from different distributions of a given ξ, there is a loss of transitivity of the community preferences in the following sense: There are choices β and γ near ξ such that for one distribution D_1 of ξ, γ is indifferent to ξ and both ξ and γ are strictly preferred to β according to the corresponding preferred set $G^1(\xi)$, while for another distribution D_2 at ξ, β is strictly preferred to ξ according to the corresponding preferred set $G^2(\xi)$, thus violating transitivity. Therefore, when the different distributions of an endowment ξ give rise to different sum sets $G^1(\xi)$ and $G^2(\xi)$, the community preference is called *intransitive*.

2.3. MARGINAL COST PRICING EQUILIBRIUM

We consider an economy consisting of m consumers and $k+1$ producers; consumers are indexed by i, each of them has a consumption set $X^i = P$, and a preference g_i defined over X^i. Each producer has a production set Y^j where j runs over I and 1 to k, and where I denotes the increasing returns to scale industry. The aggregate endowment $W \in \mathbb{R}^n$ is assumed to be strictly positive. As usual, individual income I_i is defined as the value of endowments and profits. For any non-empty closed subset C of $\mathbb{R}^n$, and for $x \in C$, the *tangent cone* $T_C(x)$ consists of all $y \in \mathbb{R}^n$ such that for all sequences (x_k) chosen from C which tend to x, $x_k \xrightarrow{C} x$, and any sequence (t_k) of positive numbers converging to zero, $t_k \downarrow 0$, there exists a sequence $y_k \to y$ such that for all k large enough, $x_k + t_k y_k \in C$. The *normal cone* is given by $N_C(x) = \{y \in \mathbb{R}^n: (z, y) \leq 0, \forall z \in T_C(x)\}$. The definition of the normal cone formalizes the intuitive concept of the set of vectors which are normal to those in the tangent cone. Tangent and normal cones are used, for example, by Brown et al. [3]. Their tangent cone is defined somewhat differently from, but can be shown to be equivalent to, Clarke's tangent cone (see, for example, ref. [15]).

A *marginal cost pricing equilibrium* (mcp equilibrium) is a triple $((\xi_i), (y^j), p)$ such that $p \in \Delta$, the unit simplex in $\mathbb{R}^n$, and

2.3(i)

For all i, $(\xi_i) \in A^i(p, y^j) = \{\sigma^i \in X^i \mid p\sigma^i \leq I_i\}$ and $g_i((\xi_i))$ (also denoted $g_i(\xi_i)) \succeq g_i(\sigma_i)$ for all $\sigma^i \in A^i(p, y^j)$ unless $I_i = 0$, where g_i is the preference of the ith consumer.

2.3(ii)

For all $j = 1, \ldots, k, I$, $y^j \in \mathrm{Bdry}\,(Y^j)$ and p is in the normal cone $N_{Y^j}(y^j)$.

2.3(iii)

$$\sum \xi_i \leq \sum y^i + W.$$

This definition follows Brown et al. [3]*. Condition (i) ensures that the equilibrium consumption allocation ξ^i is in the ith budget for all i, and that it maximizes i's preferences. Condition (ii) ensures that the equilibrium production vector y^j is in the boundary of the corresponding production set Y^j. Condition (iii) ensures that $\sum \xi^i$ is in the attainable set of the economy given its endowments and technology. This definition corresponds to a quasi-equilibrium, but it agrees with the usual equilibrium concept under the following assumption which we now adopt:

2.3(A)

For all $y^l \in \mathrm{Bdry}\,(Y^l)$, $p \in N_{Y^l}(y^l)$ implies that $p(\sum y^j + W) > 0$, where for $j = 1, \ldots, k$, $y^j \in \mathrm{Arg\,max}_{y \in Z^j}\, py$ and Z^j is the jth attainable production set☆.

This assumption is attributed to James Meade and ensures that the sum of the aggregate revenue from profit maximization in the convex sectors and marginal cost pricing in the non-convex sector, plus the value of the endowment, is non-negative.

3. Results: Transitivity and the loss of market efficiency

THEOREM 3.1

For an open and dense set of individual preference profiles in $\mathcal{P}^m$, community preferences are intransitive.

Proof

Consider an economy with m consumers, and let $(g_1, \ldots, g_m)$ be their preference profile in $\mathcal{P}^m$. Since the indifference hypersurfaces of the individual preferences are contained in P, a necessary and sufficient condition for the community preference corresponding to this profile to be transitive is that the preferences* g_i, $i = 1, \ldots, m$, be all *homothetic* and *identical*, i.e. that for all $i, j = 1, \ldots, m$ and all $\lambda > 0$,

$$g_i(\xi) = g_j(\xi) = g_j(\lambda \xi), \tag{3.1}$$

(see Chichilnisky and Heal [10], theorem 4, p. 47).

*Their definition in [3] requires also a fixed distribution of income, a condition which is used in their proof of existence of an mcp equilibrium. We do not prove existence, and we do not require their condition of a fixed income distribution.

☆The set of attainable states $Z = \{(x^i)(y^j) : \sum x^i + \sum y^j = W, \text{ and } x^i \in X^i, y^j \in Y^j \text{ for all } i, j\}$. Let the projection of Z on Y^j be denoted Z^j, called the jth attainable production set.

*Weaker conditions for the transitivity of the community preference are provided in Chichilnisky and Heal [10], in those cases where the indifference surfaces may not be contained in the positive cone. The resulting class of preferences yielding transitive community preferences is larger than that described in theorem 3.1; however, the space of profiles satisfying these weaker conditions is still residual in $\mathcal{P}$.

We shall now show that this necessary and sufficient condition is satisfied on the complement of an open and dense set within the space $\mathcal{P}^m$ of preference profiles endowed with the product topology it inherits from $\mathcal{P}$.

First, note that the space Q of homothetic preferences in $\mathcal{P}$ is a closed subset of $\mathcal{P}$: by definition, Q is contained in the space of homothetic C^1 vector fields $\mathcal{H}$, namely C^1 vector fields h satisfying $h(\xi) = h(\lambda\xi)$, for all ξ in P and all $\lambda > 0$*. This is clearly a closed condition, one which is satisfied on a closed subspace of this C^1 space. It follows, therefore, that Q, which is the intersection of $\mathcal{H}$ with $\mathcal{P}$, is a closed subset of $\mathcal{P}$. Similarly, the space Q^m of profiles of homothetic preferences is contained in a closed set of the space of preference profiles $\mathcal{P}^m$, namely the product of m closed sets within the space $\mathcal{P}$. Finally, the condition that all homothetic preferences be identical is a closed condition in Q^m, and therefore the set ΔQ^m consisting of profiles of identical and homothetic preferences is a closed subset of $\mathcal{P}^m$. It remains to show that the complement of ΔQ^m is dense. For this, it suffices to note that ΔQ^m is contained in the diagonal $\Delta^m = \{(g_1, \ldots, g_m)$ in $\mathcal{P}^m: \forall i, j, g_i = g_j\}$, since all preferences of a profile in Q^m are identical, and that the diagonal Δ^m is a nowhere dense set in the product space $\mathcal{P}^m$.□

THEOREM 3.2

For any profile of individual preferences giving rise to an intransitive community preference, there exist economies where all mcp equilibria are Pareto inefficient.

Proof

The proof consists of two parts. In the first part, we prove that intransitivity of the community preference implies the existence of two points ξ and β and corresponding distributions $D_1(\xi)$ and $D_4(\beta)$ such that the community preference prefers β to ξ in one of these distributions, while it prefers ξ to β in the other. The second part of the proof uses this property of the community preference to construct an economy in which the only possible production-efficient vectors are β and ξ, and then shows that neither β nor ξ are Pareto efficient, thus proving that no Pareto-efficient marginal cost equilibrium exists.

Step 1: Consider a profile $(g_1, \ldots, g_m) \in \mathcal{P}^m$ not contained in ΔQ^m, thus violating the necessary and sufficient condition (3.1). There is therefore a vector ξ in P where the corresponding community preference is intransitive, the intransitivity arising from the fact that there exist two distributions of ξ, $D_1(\xi) = (\xi_1, \ldots, \xi_m)$ and $D_2(\xi) = (\xi'_1, \ldots, \xi'_m)$, with $\sum\xi_i = \sum\xi'_i = \xi$, giving rise to two different sum sets $G^1(\xi) = \sum_i G_i(\xi_i)$ and $G^2(\xi) = \sum_i G_i(\xi'_i)$. We say that α is strictly preferred, or indifferent, to ξ according to $G^1(\xi)$ when α belongs to the interior, or the boundary, in P of the set $G^1(\xi)$, respectively. Since $G^1(\xi)$ is different from

*Note that some of the homothetic vector fields in $\mathcal{H}$ are not integrable, i.e. may not satisfy for any u condition (2.1.1) in the definition of $\mathcal{P}$.

$G^2(\xi)$, we may choose a point μ in P such that μ is indifferent to ξ according to $G^2(\xi)$, while according to $G^1(\xi)$, μ is strictly preferred to ξ. Since μ is indifferent to ξ according to $G^2(\xi)$, there exists a distribution $D_3(\mu) = (\mu_1, \ldots, \mu_m)$, $\Sigma\mu_i = \mu$, for which the corresponding sum set $G^3(\mu) = \Sigma_i G_i(\mu_i)$ equals $G^2(\xi)$*.

Since under the distribution $D_3(\mu)$, $G^3(\mu) = G^2(\xi)$, it follows that μ is also indifferent to ξ according to $G^3(\mu)$. We now consider a new point β and a distribution $D_4(\beta) = (\beta_1, \ldots, \beta_m)$, $\Sigma\beta_i = \beta$, where $\beta_i = \mu_i$ for $i = 1, \ldots, m-1$, and where β_m satisfies $g_m(\mu_m) = g_m(\beta_m)$, and μ_m is strictly preferred to β_m by the preference g_m☆. Let $G^4(\beta) = \Sigma_i G_i(\beta_i)$. By construction, μ is strictly preferred to β according to $G^4(\beta)$; ξ is also strictly preferred to β according to $G^4(\beta)$, because for all $i = 1, \ldots, m-1$, $G_i(\beta_i) = G_i(\mu_i)$, while $G_m(\beta_m)$ strictly contains $G_m(\mu_m)$ since μ_m is strictly preferred by m to β_m. By continuity of g_1, we may choose β_m sufficiently close to μ_m so that, as μ is strictly preferred to ξ according to $G^1(\xi)$, β is also strictly preferred to ξ according to $G^1(\xi)$. We have now constructed a pair of points ξ and β satisfying the following property: there exists a distribution $D_1(\xi)$ of ξ in which the corresponding sum set $G^1(\xi)$ strictly prefers β to ξ, and there is a distribution $D_4(\beta)$ of β in which the corresponding sum set $G^4(\beta)$ strictly prefers ξ to β (fig. 1).

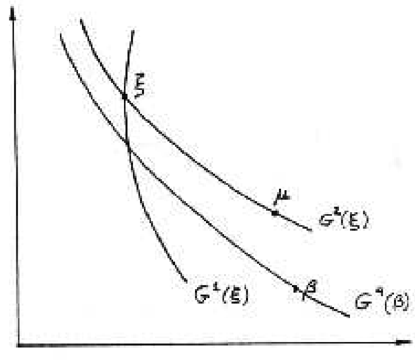

Fig. 1. Illustration of step 1.

*The proof that $G^3(\mu) = G^2(\xi)$ is as follows: Since μ is indifferent to ξ, by definition μ is in the boundary of the set $G^2(\xi)$. By definition of $G^2(\xi)$, μ is the sum of m vectors $\mu_1, \ldots, \mu_m$ such that for all i, μ_i is preferred to or indifferent to ξ_i according to individual ith preference g_i. It follows that: (1) for all i, μ_i is in the boundary of $G_i(\xi_i)$, where $G_i(\xi_i)$ is the set preferred to ξ_i according to the preference g_i, and (2) for all $i, j = 1, \ldots, m$, $g_i(\mu_i) = g_j(\mu_j)$; if either (1) or (2) were violated, then μ would be interior to $G^2(\xi)$. In other words, for all i, μ_i is indifferent to ξ_i, and the indifference surface of g_i at μ_i has the same tangent the indifference surface of g_j at μ_j, for all $i, j = 1, \ldots, m$. It follows that $\mu = \Sigma\mu_i$ such that for all i, $g(\mu_i) = g(\xi_i)$ and μ_i is indifferent to ξ_i. By definition, therefore, $G^3(\mu) = G^2(\xi)$.

☆The existence of such a vector β follows from the regularity and monotonicity of the utility u_m defining the preference g_m: regularity of u_m ensures that the set $\{v \in P : g_m(v) = g_m(\mu_m)\}$ is a manifold of dimension $n-1$, and therefore there exists a point different from μ_m belonging to this manifold in every ε neighborhood of μ_m for small ε. Monotonicity ensures that such a point can be chosen so that it is strictly less preferred to μ_m.

Step 2: The next step is to construct an economy which has ξ and β as the only possible production-efficient points. Consider the two rays from the origin ending at β and ξ, respectively, and consider the set generated by the free disposal hull of these two rays, namely the subset of P consisting of points which are inferior to points in these two rays in the vector order of $\mathbb{R}^n$ (see fig. 2). This hull is denoted $\mathcal{T}$ and it describes the sum of the set given by the production technology of our economy and the total initial endowments. In this technology, the only production-efficient vectors are ξ and β.

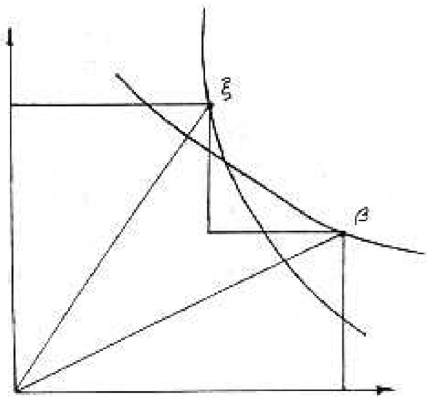

Fig. 2. Illustration of step 2.

We now define an economy $\mathcal{E}$ as follows. There are m individuals with preferences $g_1, \ldots, g_m$. There is one firm, and n commodities (inputs and outputs) with a production technology described by the set $\mathcal{T} - W$ and an initial endowment vector $W = \xi$. W is distributed among the individuals according to the distribution $D_1(\xi) = (\xi_1, \ldots, \xi_m)$, $\Sigma \xi_i = \xi$, with a pattern of ownership of production under which when $\beta - \xi$ is produced, the distribution of β (total endowment plus production) is $D_4(\beta)$. In this economy $\mathcal{E}$, the only two possible production-efficient vectors are ξ and β. Therefore, these two vectors are the only candidates for Pareto-efficient mcp equilibria, because of monotonicity of preferences. However, β is not Pareto efficient in the economy $\mathcal{E}$ because $\mathcal{E}$'s distribution of the vector β is $D_4(\beta)$, and there exists another vector ξ which is feasible in $\mathcal{E}$ and which is Pareto superior (as it is interior to the set $G^4(\xi)$), while for $\mathcal{E}$'s distribution of the initial endowment ξ, $D_1(\xi)$, there exists a feasible vector β which is Pareto superior (as it is interior to $G^1(\xi)$). This proves that neither β nor ξ can be Pareto efficient mcp equilibria. □

Note that the lack of Pareto efficient mcp equilibria is not caused by the lack of existence of mcp equilibria: examples of economies possessing several mcp equilibria of which none are Pareto efficient have been provided by Brown and Heal [2].

COROLLARY 3.3.

For any profile of individual preferences giving rise to an intransitive community preference, and for any distribution of income which allocates one endowment vector in a preassigned fashion, there exist economies where all mcp equilibria are Pareto inefficient.

Proof

It suffices to note that in the proof of theorem 3.2, the only requirement on the distribution of income is that it should allocate the endowment β according to the distribution $D_4(\beta)$. The result is therefore valid for any distribution of income in which β is allocated according to $D_4(\beta)$. This contrasts with the restriction of a fixed income distribution rule which is used frequently in the literature of equilibria with increasing returns (see, for example, Brown and Heal [2] and Guesnerie [13]). □

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