Entry-License Tax: Stackelberg versus Cournot

Cato, Susumu and Matsumura, Toshihiro

Institute of Social Science, The University of Tokyo, Institute of Social Science, The University of Tokyo

1 May 2013

Online at https://mpra.ub.uni-muenchen.de/81297/
MPRA Paper No. 81297, posted 13 Sep 2017 08:22 UTC
Entry-License Tax: Stackelberg versus Cournot

Susumu Cato∗
Institute of Social Science, The University of Tokyo
and
Toshihiro Matsumura†
Institute of Social Science, The University of Tokyo

September 12, 2017

Abstract
This study investigates how leadership affects public policies in markets where the number of firms is endogenously determined. We focus on the relationship between the relative efficiency of an incumbent firm and the optimal entry tax (entry barrier). We find that this relationship depends on whether the incumbent can commit to the output before the entries of new firms. The optimal entry tax is decreasing (res. increasing) in the productivity of the incumbent when it takes (res. does not take) leadership. We also find that the optimal entry barrier occurring when the incumbent takes leadership is lower than that when it does not.

JEL classification numbers: L41, L51, L13

Keywords: Stackelberg, Cournot, free entry, entry tax, competition policy, beneficial concentration

∗ Corresponding author: Susumu Cato, Institute of Social Science, The University of Tokyo, 7-3-1, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. Tel: +81-3-5841-4904, Fax:+81-3-5841-4905 E-mail: susumu.cato@gmail.com
† Institute of Social Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. E-mail: matsumur@iss.u-tokyo.ac.jp
# 1 Introduction

Regulation policy is a core of public policies, and entry regulation is observed globally. Entry costs imposed by governments are significantly different among industries and countries (Djankov, La Porta, Lopez-de-Silanes, and Shleifer, 2002; Djankov, 2009). Here, one possibility is political failure. For example, the degree of corruption that affects regulation policies differs among countries (e.g., in the two papers presented). However, a similar difference can be found among industries in the same country, and we aim to explain such a difference. Our view is as follows: The optimal degree of regulation is dependent on the market structure, and thus, the degree of entry barrier differs among industries even when the government is clean and efficient.

Although various factors such as demand trends, firm technologies, and the types of firms (i.e., the market structure) in these industries affect the optimal entry fee and regulation, we mainly focus on how efficiency improvement of incumbent firms affects entry regulation. Our main question is whether the government should implement a higher or lower entry tax (implying heavier regulation of entry) when an incumbent firm successfully innovates. In other words, we examine the relationship between the relative efficiency of incumbent firms to potential new entrants and the optimal entry tax. Because the incumbent firms often make large-scale R&D investments, such a case is highly likely; thus, the question is relevant.

We find that the answer depends on the market structure and whether the incumbent firms can commit to their output level before the entries of new firms (Stackelberg) or not (Cournot).\(^1\) We have the following contrasting results. When the incumbent can (res. cannot) take leadership, the incumbent’s productivity is negatively (res. positively) correlated with the optimal entry tax.\(^2\)

---


\(^2\)For the importance of the cost difference among firms in a free-entry market, see Wang and Chen (2010) and Chen (2017) on mixed markets and Cato and Matsumura (2013) on mergers and the optimal entry tax.
the entry license tax, higher incumbent productivity reduces the number of entering firms and yields a more concentrated market. Thus, it may be natural to conclude that the entry tax should be reduced to stimulate new entries. Our results show that this is true when the incumbent firms can take leadership (Stackelberg) but not when they cannot. According to our results, the market structure matters in entry regulation policies.

We explain the intuition behind our results using the basic principle shown by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). As they show, the entry of firms is excessive in the sense that the equilibrium number of entrants is larger than the optimal one. In the Cournot case, improvement of relative efficiency of the incumbent firms intensifies this problem. A reduction of the number of entrants stimulates the production of the incumbent, improving welfare more significantly when the relative efficiency of the incumbent is higher. Thus, the government imposes a higher entry barrier when the efficiency of the incumbent is higher. In contrast, in the Stackelberg case, the incumbents sufficiently produce to reduces the number of entrants, and thus, the welfare-improving effect of stimulating the incumbent firms’ production using a higher entry barrier is weak. Thus, in order to improve consumer surplus, the government lowers the entry barrier.

Here, we mention an example of the mobile telecommunication industry in Japan. There are three first-mover leading incumbents in the industry, the three major mobile network operators (MNOs): NTT DoCoMo, KDDI, and Softbank, who have their own network infrastructure. Entry regulation of the industry was substantially relaxed from 2002 to 2007. As a result, mobile virtual network operators (MVNOs), who have no network infrastructure and use that of MNOs, entered the industry. Mobile communication technology, which is associated with leaders’ cost structure, has been drastically improved since the late 1990s. Thus, leaders’ technology has become more efficient. Therefore, deregulation in this industry is consistent with the optimal policy in our Stackelberg case.

The policy implications of our results are as follows. The aggressiveness of leaders changes the market concentration implications and drastically affects the welfare-improving effect of the entry-
license tax. Our results suggest that public policies on entry regulation are crucially dependent on the behavior of incumbent firms and that the government should pay attention to whether the incumbent takes leadership when it chooses public policies. Our result has another implication for R&D by the incumbent firms. If the government can flexibly change the entry barriers, it may stimulate the incentive for cost reduction in the Cournot case depending on the cost of the incumbent firms. This is because cost reduction results in a higher entry barrier and increases their profits. However, in the Stackelberg case, this reduces the incentive for innovation by the incumbent firms. Therefore, the government should not flexibly adjust its regulation policy in order to stimulate innovation when the incumbent firms have strong commitment power.

In this study, we consider an entry-license tax rather than direct regulation of restricting the number of entrants. We believe that direct entry regulation is not always available; the entry-license tax is an alternative policy instrument for regulation in such a case. Moreover, the optimal tax rate indicates how significant the problem of excessive entry is. A positive optimal rate indicates that the excess entry theorem by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) holds true (in our model, it is positive in both the Stackelberg and Cournot cases). A lower optimal rate indicates a lesser importance of welfare loss of excess entry. Thus, we believe that investigating the optimal tax rate property is important.

The rest of this paper is organized as follows. Section 2 presents our model. Section 3 presents our main results, where we examine the relationship between the optimal entry tax and the efficiency of the incumbent firm. Section 4 investigates the relationship between the optimal entry tax and the efficiency of new entrants, and Section 5 discusses how the demand trend affects the optimal tax. Section 6 concludes.
2 Model

We consider quantity competition models with endogenous entry. There exists an incumbent firm (firm 0), and other firms are potential new entrants. Each potential entrant (firm $i = 1, 2, \ldots$) decides whether to enter the market. Let $n$ be the number of entrants. The firms produce homogeneous products and compete in quantity. Let $q_i$ be the output of firm $i$. The inverse demand function is given by $P(Q) = \alpha - Q$, where $Q$ is the total output. We assume that $\alpha$ is sufficiently large to ensure the interior solution.

We assume that firms’ cost functions are strictly convex.\(^3\) The incumbent firm’s cost function is as follows:

$$C_0(q_0) = \frac{\hat{\beta}}{2} q_0^2 + K,$$

where $\hat{\beta} > 0$ and $K > 0$. The incumbent’s fixed cost $K$ is sunk. Each new entrant $i (= 1, 2, \ldots)$ has the following identical cost function:

$$C_i(q_i) = \frac{\beta}{2} q_i^2 + K,$$

where $\beta > 0$. The entrant’s fixed cost $K$ is not sunk before entry. Thus, the entrant earns zero profit if it does not enter the market. In this setting, $\hat{\beta}$ and $\beta$ represent the technology levels of the incumbent and of each entrant, respectively. We assume that $1 + 2\beta > \hat{\beta}$ (i.e., the incumbent is not substantially less efficient than the new entrant).

The government levies entry tax $T$ on new entrants. Each firm $i$’s profit is defined as follows:

$$\Pi_i = P(Q)q_i - C_i(q_i) - \delta_i T,$$

where $\delta_0 = 0$ and $\delta_i = 1$ for $i = 1, \ldots, n$.

\(^3\)When the marginal cost is constant, the Stackelberg leader commits to production such that all other firms are excluded from the market (see, for example, Etro (2006)). We assume strict convexity in order to make the equilibrium number of entrants positive.
Economic welfare $W$ is the sum of the consumers’ surplus, firms’ profits, and tax revenue as follows:

$$W = \int_0^Q P(z)dz - P(Q)Q + \sum_{i=1}^{m+n} \Pi_i + nT, \quad (1)$$

$$= \int_0^Q (\alpha - z)dz - \frac{\beta}{2} q_0^2 - \sum_{i=1}^n \frac{\beta}{2} q_i^2 - (n + 1)K. \quad (2)$$

The tax revenue disappears in (2) because it is a transfer from new entrants to the government.

We consider the Stackelberg and Cournot cases. In the first case, the incumbent commits to its output before the entrants’ decisions. In the second case, all firms choose their outputs simultaneously.

### 3 Main Results

#### 3.1 Stackelberg case

First, we discuss the case with incumbent leadership. The timing of the game is as follows.\(^4\) The government sets $T$ in the first stage, and the incumbent chooses $q_0$ in the second stage. In the third stage, entry occurs. In the fourth stage, each new entrant $i$ chooses $q_i$. We focus on the subgame-perfect equilibrium, where all new entrants produce the same output ($q_1 = q_i$ for all $i \in \{1, \ldots, n\}$).\(^5\)

We can solve this game using backward induction. In the fourth stage, each new entrant $i$ chooses $q_i$ simultaneously. The first-order conditions of new entrants are as follows:

$$P(Q) - P'(Q)q_i - C_i'(q_i) = 0 \quad (i = 1, \ldots, n).$$

The second-order conditions are satisfied. In the third stage, each firm enters the market as long as it earns a positive profit. The zero profit condition is as follows:

$$P(Q)q_i - C_i(q_i) - T = 0.$$  

\(^4\)We assume that entry occurs after the leader’s output is determined. This assumption is standard in the literature and is employed by a series of works by Etro (2004, 2006, 2007, 2008). See also Ino and Matsumura (2012) and Matsumura and Yamagishi (2017b).

\(^5\)We can show that no other equilibrium exists.
Because \( q_1 = q_i \) for all \( i \in \{1, \ldots, n\} \), \( q_1 \) and \( Q \) are determined using the following simultaneous equations:

\[
\begin{align*}
\alpha - Q - q_1 - \beta q_1 &= 0, \\
(\alpha - Q)q_1 - \frac{\beta}{2}q_1^2 - K - T &= 0.
\end{align*}
\]

The total output is determined regardless of the leader’s output. Then, the equilibrium price is independent of the leader’s behavior, which is a notable feature of the free-entry equilibrium (See Etro, 2006, 2007).

In the second stage, the incumbent chooses its output. Because its action does not affect the price, it becomes a price taker. Thus, the leader’s output satisfies the following equation:

\[ P(Q) - C'_0(q_0) = 0 \iff \alpha - Q - \hat{\beta}q_0 = 0. \]

That is, the firm employs the so-called marginal-cost pricing rule. From the zero profit condition and the marginal-cost pricing rule, we have the following equation:

\[ P(Q) = C_i(q_i) + T = C'_0(q_0). \] (3)

Under a free-entry equilibrium with an incumbent leadership, the leader’s marginal cost is equal to the average cost of followers. This does not rely on the assumptions of linear demand and the quadratic cost functions, which is a fundamental observation in the Stackelberg case.

Let the superscript ‘s’ denote the equilibrium outcome in the Stackelberg case given \( T \). By solving
these equations, the equilibrium outcome is characterized as follows:\(^6\)

\[
q_0^* = \frac{1 + \beta}{\hat{\beta}} \sqrt{\frac{2(K + T)}{2 + \beta}},
\]

\[
q_1^* = \sqrt{\frac{2(K + T)}{2 + \beta}},
\]

\[
n^* = \alpha \sqrt{\frac{2 + \beta}{2(K + T)}} - \frac{1}{\hat{\beta}} - \frac{\beta(1 + \hat{\beta})}{\hat{\beta}},
\]

\[
Q^* = \alpha - (1 + \beta) \sqrt{\frac{2(K + T)}{2 + \beta}}.
\]

We assume that \(n^*\) is positive.\(^7\) Both \(q_0^*\) and \(q_1^*\) are increasing in \(T\), whereas the number of entrants \(n^*\) and the total output \(Q^*\) are decreasing in \(T\). Thus, an increase in \(T\) causes a production substitution from marginal entrants to the incumbent and the other remaining entrants. Differentiating \(q_0^*\) with respect to \(T\), we obtain

\[
\frac{dq_0^*}{dT} = \frac{1 + \beta}{\hat{\beta}[2(2 + \beta)(K + T)]^{1/2}} > 0. \tag{4}
\]

This represents a production substitution from marginal new entrants to the incumbent firm. Given the entry tax \(T\), \(dq_0^*/dT\) is decreasing in \(\hat{\beta}\). Thus, the lower \(\hat{\beta}\) is, the larger the production substitution is.

We now discuss the relationship between the Herfindahl–Hirschman Index (HHI) and \(T\). The HHI is derived as follows:

\[
HHI^s = \left( \frac{q_0^*}{Q^*} \right)^2 + n^* \left( \frac{q_1^*}{Q^*} \right)^2.
\]

By differentiating this with respect to \(\hat{\beta}\), we have

\[
\frac{dHHI^s}{d\beta} = -\frac{2(1 + \beta)(2 + 2\beta - \hat{\beta})(K + T)}{\hat{\beta}^3[\alpha \sqrt{2 + \beta} - (1 + \beta) \sqrt{2(K + T)}]^2}. \tag{5}
\]

\(^6\)Throughout this paper, we assume that the number of firms is a continuous variable and the integer problem is ignored. For the analysis of an integer problem in a free-entry model, see Matsumura (2000).

\(^7\)This assumption is satisfied when \(\alpha\) is sufficiently large.
Because we assume that $1 + 2\beta > \hat{\beta}$, we have $2 + 2\beta - \hat{\beta} > 0$. This implies that $d\text{HHI}^s/d\hat{\beta} < 0$; that is, the productivity improvement of the incumbent raises the HHI in the market. Moreover, relying on the assumption that $\alpha$ is sufficiently large, we obtain $d\text{HHI}^*/dT > 0$. Thus, the entry tax enhances the market concentration.

We then derive the optimal entry tax rate $T^{**}$. Substituting $q^*_0, q^*_1, Q^*$, and $n^*$ into (2), we obtain the equilibrium economic welfare:

$$W^* = \int_0^{Q^*} (\alpha - z)dz - \frac{\hat{\beta}}{2}(q^*_0)^2 - n^*\frac{\hat{\beta}}{2}(q^*_1)^2 - (1 + n^*)K.\]$$

In the first stage, the government sets $T$ to maximize $W^*$. Differentiating $W^*$ with respect to $T$,

$$\frac{dW^*}{dT} = \frac{\alpha \hat{\beta}(2K - \beta T)\sqrt{2(2 + \beta) - 4(1 + \beta)(1 + \hat{\beta})(K + T)^\frac{3}{2}}}{4\hat{\beta}(2 + \beta)(K + T)^\frac{3}{2}}. \quad (6)$$

d$^2W^*/dT^2 < 0$ at the optimum rate, and thus, the second-order condition is satisfied. Therefore, the following first-order condition is sufficient to characterize the optimal entry tax $T^{**}$:

$$\frac{\alpha \hat{\beta}(2K - \beta T^{**})\sqrt{2(2 + \beta) - 4(1 + \beta)(1 + \hat{\beta})(K + T^{**})^\frac{3}{2}}}{4\hat{\beta}(2 + \beta)(K + T^{**})^\frac{3}{2}} = 0.$$

Moreover, it is easy to show that whenever $\alpha$ is sufficiently large, $dW^*/dT > 0$ if $T = 0$. Therefore, the optimal entry tax $T^{**}$ has a positive value: The government should exclude some entrants by imposing a tax. This result is related to the excess entry theorem presented by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). An increase of the number of entrants reduces the profits of the other firms (i.e., a business-stealing effect), which yields excessive entries without an entry tax.

By the implicit function theorem, we obtain the following equation:

$$\frac{dT^{**}}{d\hat{\beta}} = \frac{4\sqrt{2}(1 + \beta)(K + T^{**})^\frac{5}{2}}{\alpha \hat{\beta}^2 \sqrt{2 + \beta} [2(3 + \beta)K - \beta T^{**}]}.$$

Because $T^{**}$ is positive, this equation implies that

$$\frac{dT^{**}}{d\beta} > 0 \text{ if and only if } T^{**} < \frac{2(3 + \beta)K}{\beta}.$$
We can show that \( dW/dT < 0 \) if \( T \geq 2(3 + \beta)K/\beta \). Hence, \( T^{**} < 2(3 + \beta)K/\beta \) must hold. Therefore, we have the following proposition.

**Proposition 1.** Under Stackelberg competition, the optimal entry tax is increasing in \( \beta \).

According to Proposition 1, the improvement of the incumbent’s productivity decreases the optimal entry tax. From (5), we have shown that given the tax rate, the technological improvement of the incumbent firm enhances the market concentration (the HHI is increasing in \( \beta \)). Thus, a natural response of the government is to adjust the market concentration by stimulating new entries. Because the HHI is increasing in the entry tax rate, the government may reduce the entry tax to stimulate entries and to reduce the HHI. Proposition 1 implies that this is true under incumbent leadership.

However, as we show in the next subsection, this is not true in the Cournot case (Proposition 2). We explain the intuition behind these two propositions in the next subsection.

### 3.2 Cournot case

Next, we discuss the case without incumbent leadership. The timing of the game is as follows. The government sets \( T \) in the first stage, and entry occurs in the second stage. In the third stage, all firms, including the incumbent firm, simultaneously choose \( q_i \). We focus on the symmetric subgame-perfect equilibrium, where all new entrants produce the same output (\( q_1 = q_i \) for all \( i \in \{1, \ldots, n\} \)).

In the third stage, all firms simultaneously choose \( q_i \). The first-order conditions are as follows:

\[
P(Q) - P'(Q)q_i - C_i'(q_i) = 0 \quad (i = 0, 1, \ldots, n).
\]

The second-stage decision of the entrants yields the zero-profit condition:

\[
P(Q)q_i - C_i(q_i) - T = 0.
\]

---

8 We can show that the unique equilibrium is symmetric.
Because $q_1 = q_i$ for all $i \in \{1, \ldots, n\}$, $q_0$, $q_1$, and $Q$ are determined using the following equations:

\[
\alpha - Q - q_0 - \hat{\beta} q_0 = 0,
\]
\[
\alpha - Q - q_1 - \beta q_1 = 0,
\]
\[
(\alpha - Q)q_1 - \frac{\beta}{2} q_1^2 - K - T = 0.
\]

Let the superscript ‘c’ denote the equilibrium outcome in the Cournot case given $T$. By solving these equations, we have the equilibrium outcome:

\[
q_0^c = \frac{1 + \beta}{1 + \beta} \sqrt{\frac{2(K + T)}{2 + \beta}},
\]
\[
q_1^c = \sqrt{\frac{2(K + T)}{2 + \beta}},
\]
\[
n^c = \alpha \sqrt{\frac{2 + \beta}{2(K + T)}} - \frac{(1 + \beta)(2 + \beta)}{1 + \beta},
\]
\[
Q^c = \alpha - (1 + \beta) \sqrt{\frac{2(K + T)}{2 + \beta}}.
\]

We assume that $n^c$ is positive.\(^9\) As in the Stackelberg case, both the total output $Q^c$ and the entrant’s output $q^c$ are independent of the leader’s technology level $\hat{\beta}$. Furthermore, both $q_0^s$ and $q_1^s$ are increasing in $T$, and the number of entrants $n^c$ and the total output $Q^c$ is decreasing in $T$. In particular, a simple calculation yields

\[
\frac{dq_0^c}{dT} = \frac{1 + \beta}{(1 + \beta)[2 + \beta](K + T)]^{\frac{1}{2}} > 0.
\]

This represents a production substitution from marginal entrants to the incumbent firm. Here, $dq_0^c/dT$ is decreasing in $\hat{\beta}$. Moreover, from (4) and (7), we have $dq_0^c/dT > dq_0^s/dT$; that is, the production-substitution effect in the Cournot case is weaker than that in the Stackelberg case.

We again discuss the relationship between the HHI and $T$. The HHI is derived as follows:

\[
\text{HHI}^c = \left(\frac{q_0^c}{Q^c}\right)^2 + n^c \left(\frac{q_1^c}{Q^c}\right)^2.
\]

\(^9\)This assumption is satisfied when $\alpha$ is sufficiently large.
By differentiating this with respect to $\hat{\beta}$, we have

$$\frac{dHHI^C}{d\hat{\beta}} = -\frac{2(1 + \beta)(1 + 2\beta - \hat{\beta})(K + T)}{(1 + \hat{\beta})^3[\alpha\sqrt{2 + \beta} - (1 + \beta)\sqrt{2(K + T)}]^2}. \quad (8)$$

From (5) and (8),

$$0 > \frac{dHHI^C}{d\beta} > \frac{dHHI^*}{d\beta}.$$  

Thus, the HHI is less sensitive to the incumbent’s technology in the Cournot case. In addition, we can show that $dHHI^C/dT > 0$, that is, the entry tax raises the market concentration. It is easy to show the following inequalities:

$$0 < \frac{dHHI^C}{dT} < \frac{dHHI^*}{dT}. \quad (9)$$

Thus, the HHI is less sensitive to the entry tax in the Cournot case.

The equilibrium economic welfare is defined as follows:

$$W^c = \int_0^{Q^c} (\alpha - z)dz - \hat{\beta}\frac{1}{2}(q_0^c)^2 - n\beta\frac{1}{2}(q_1^c)^2 - (1 + n)K.$$  

We then derive the optimal entry tax $T^c$. The government sets $T$ so as to maximize $W^c$. Differentiating $W^c$ with respect to $T$,

$$\frac{dW^c}{dT} = \frac{\alpha(1 + \hat{\beta})^2(2K - \beta T)\sqrt{2(2 + \beta)} - 4[1 - \beta^2 + 3\hat{\beta} + \hat{\beta}^2 + \beta\hat{\beta}(3 + \hat{\beta})](K + T)^{\frac{3}{2}}}{4(1 + \hat{\beta})^2(2 + \beta)(K + T)^{\frac{3}{2}}} \quad (10)$$

d$^2W^c/dT^2 < 0$ at the optimum rate, and thus, the second-order condition is satisfied. Moreover, it is easy to show that whenever $\alpha$ is sufficiently large, $dW^c/dT > 0$ for $T = 0$. Thus, the optimal entry tax $T^{c*}$ is positive. It is determined using the following first-order condition:

$$\frac{\alpha(1 + \hat{\beta})^2(2K - \beta T^{c*})\sqrt{2(2 + \beta)} - 4[1 - \beta^2 + 3\hat{\beta} + \hat{\beta}^2 + \beta\hat{\beta}(3 + \hat{\beta})](K + T^{c*})^{\frac{3}{2}}}{4(1 + \hat{\beta})^2(2 + \beta)(K + T^{c*})^{\frac{3}{2}}} = 0.$$  

Using the implicit function theorem, we obtain

$$\frac{dT^{c*}}{d\hat{\beta}} = -\frac{4\sqrt{2}(1 + \beta)(1 + 2\beta - \hat{\beta})(K + T^{c*})^{\frac{5}{2}}}{\alpha(1 + \beta)^3\sqrt{2 + \beta}[2(3 + \beta)K - \beta T^{c*}]}.$$  

12
Because we assume that $1 + 2\beta > \hat{\beta}$, this is negative if and only if $T^* < 2(3 + \beta)K/\beta$.

Because $\alpha$ is sufficiently large, we have $dW_c/dT < 0$ if $T \geq 2(3 + \beta)K/\beta$. Hence, $T^* < 2(3 + \beta)K/\beta$ must hold. Therefore, we have the following proposition.

**Proposition 2.** Under Cournot competition, the optimal entry tax is decreasing in $\hat{\beta}$.

According to Proposition 2, productivity improvement of the incumbent increases the optimal entry tax. This result is in sharp contrast with the one with an incumbent leadership in which a technological improvement of the incumbent decreases the optimal entry tax. This contrast is counterintuitive, because the HHI is positively associated with the incumbent’s technology in both cases.

The intuition behind Proposition 2 is as follows. As shown previously, an increase in the entry tax reduces the number of entering firms, yielding production substitution from a marginal new entrant to the incumbent and other remaining entrants. Because the marginal costs of the incumbent and other entrants are smaller than the average cost of a marginal new entrant, this production substitution improves welfare.\(^{10}\) This implies that the market concentration is beneficial in the Cournot case: an increase in the HHI is welfare-improving. Because an increase in the productivity of the incumbent increases the difference between the marginal cost of the incumbent and the average cost of the marginal new entrant, it strengthens the effect of the beneficial concentration.\(^{11}\) An increase in the tax rate $T$ increases the HHI, and thus, the optimal entry tax increases when the incumbent is more efficient.

Subsequently, we explain why the effect of beneficial concentration is not amplified by the technological improvement of the incumbent in the Stackelberg case. Under incumbent leadership, the incumbent produces aggressively, and its marginal cost is equal to the average cost of each new en-

---

\(^{10}\)For a pioneering work on this welfare-enhancing production substitution, see Lahiri and Ono (1988). This effect works in many other contexts, such as international trade policies. See Lahiri and Ono (1997, 1998). For the explanation of this welfare-improving production substitution in free-entry markets, see Matsumura and Kanda (2005).

\(^{11}\)For examples of beneficial concentration, see Lahiri and Ono (1988), Daughety (1990), and Ino and Matsumura (2012).
Thus, further production substitution from a marginal new entrant to the incumbent caused by a higher entry tax does not improve welfare, whereas production substitution from a marginal new entrant to the other entrants does so. Higher incumbent productivity reduces the number of entering firms, reducing the latter production substitution effect. That is, the welfare-improving effect of concentration is reduced by the technological improvement of the incumbent. This is because the optimal entry tax is lower when the incumbent is more efficient in the Stackelberg case.

### 3.3 Comparison

Finally, we compare the optimal tax rates in the two cases. If $\hat{\beta} = 2\beta + 1$, (6) and (10) become

$$
\begin{align*}
\frac{dW^s}{dT} &= \frac{\alpha(2K - \beta T)(1 + 2\beta)\sqrt{2(2 + \beta)} - 8(1 + \beta)^2(K + T)^{\frac{3}{2}}}{4\beta(1 + 2\beta)(2 + \beta)(K + T)^{\frac{3}{2}}}, \quad (11) \\
\frac{dW^c}{dT} &= \frac{\alpha(2K - \beta T)\sqrt{2(2 + \beta)} - (5 + 4\beta)(K + T)^{\frac{3}{2}}}{4(2 + \beta)(K + T)^{\frac{3}{2}}}. \quad (12)
\end{align*}
$$

From (12), $T^{\text{cs}}$ is determined to satisfy the following equation:

$$
\alpha(2K - \beta T^{\text{cs}})\sqrt{2(2 + \beta)} - (5 + 4\beta)(K + T^{\text{cs}})^{\frac{3}{2}} = 0. \quad (13)
$$

From (11) and (13), $dW^s/dT < 0$ if $T \geq T^{\text{cs}}$. Therefore, $T^{\text{ss}} < T^{\text{cs}}$ if $\hat{\beta} = 2\beta + 1$. Propositions 1 and 2 imply that $T^{\text{cs}} - T^{\text{ss}}$ decreases in $\hat{\beta}$ given $\beta$, and thus, we obtain the following proposition.

**Proposition 3.** $T^{\text{ss}} < T^{\text{cs}}$.

According to Proposition 3, the incumbent’s leadership reduces the optimal degree of the entry barrier. The optimal entry tax is positively associated with the welfare-improving effect of production substitution. As explained in the previous subsection, the effect in the Stackelberg case is weaker than that in the Cournot case. Therefore, the optimal entry tax in the Stackelberg case is lower than that in the Cournot case.

### 4 The Efficiency of New Entrants and the Optimal Entry Tax

In this section, we briefly discuss how the incumbent’s technology affects the optimal entry tax.
We first consider the Stackelberg case. By employing the implicit function theorem to the optimality condition yielded by (6), we have the following:

$$\frac{dT^{**}}{d\beta} = \frac{(K + T^{**})[2(3 + \beta)K + (4 + 3\beta + \beta^2)T^{**}]}{(1 + \beta)(2 + \beta)[2(3 + \beta)K - \beta T^{**}]}.$$

As shown previously, $0 < T^{**} < 2(3 + \beta)K/\beta$. Thus, $dT^{**}/d\beta < 0$. That is, the technological improvement of entrants increases the optimal entry tax.

We next consider the Cournot case. A similar calculation yields the following:

$$\frac{dT^{c*}}{d\beta} = \frac{(K + T^{c*})[\alpha \sqrt{2}(1 + 4\beta + \beta^2 - 3\hat{\beta} - \hat{\beta}^2)(K + T^{c*})^{\frac{3}{2}} - \alpha \sqrt{2 + \beta}(1 + \hat{\beta})^2(2K + (4 + \beta)T^{c*})]}{\alpha(2 + \beta)^{\frac{3}{2}}(1 + \hat{\beta})^2[2(3 + \beta)K - \beta T^{c*}]}.$$

As shown before, $0 < T^{c*} < 2(3 + \beta)K/\beta$. Then, there exists an upper bound of the optimal tax, and thus, for sufficiently large $\alpha$,

$$4\sqrt{2}(1 + 4\beta + \beta^2 - 3\hat{\beta} - \hat{\beta}^2)(K + T^{c*})^{\frac{3}{2}} < \alpha \sqrt{2 + \beta}.$$

Thus, $dT^{c*}/d\beta < 0$ for sufficiently large $\alpha$. As in the Stackelberg case, the technological improvement of entrants increases the optimal entry tax.

In short, a reduction in $\beta$ increases the optimal entry tax in both the Stackelberg and Cournot cases. When $\beta$ is lower, each entrant’s output level that minimizes the entrant’s average cost is larger, and thus, the second-best number of entrants is smaller whether or not the incumbent is the leader. In other words, an increase in the entry tax induces a production substitution from a marginal new entrant to other entrants, which improves welfare more significantly when $\beta$ is smaller. Thus, a decrease in $\beta$ increases the optimal entry tax.

### 5 Demand Trends and Demand-Enhancing Innovation

We briefly discuss the relationship between the demand parameter $\alpha$ and the optimal entry tax. The large value of $\alpha$ means that there is large demand in the industry. Some industries have an upward demand trend (in the long run), whereas others have a downward trend. In this section,
we investigate how such trends affect the optimal policies. Alternatively, \( \alpha \) can change because of demand-enhancing innovation or technology shocks. We again obtain different results between the Stackelberg and Cournot cases.

In the Stackelberg case, an increase in demand increases the optimal entry tax. Using the implicit function theorem, we obtain

\[
\frac{dT^{**}}{d\alpha} = \frac{2(K + T^{**})(2K - \beta T^{**})}{\alpha [2(3 + \beta)K - \beta T^{**}]}.
\]

Because \( dW^s/dT < 0 \) for \( T = 2K/\beta \), we have \( T^{**} < 2K/\beta \). Thus, \( dT^{**}/d\alpha \) is positive. The optimal entry tax is then positively correlated with \( \alpha \) in the Stackelberg case.

In the Cournot case, the optimal entry tax can be either increasing or decreasing in \( \alpha \). By the implicit function theorem, we obtain

\[
\frac{dT^{c*}}{d\alpha} = \frac{2(K + T^{c*})(2K - \beta T^{c*})}{\alpha [2(3 + \beta)K - \beta T^{c*}]}.
\]

Here, \( \frac{2K}{\beta} < \frac{2(3+\beta)K}{\beta} \). By evaluating \( dW^c/dT \) at \( T = 2K/\beta \), we have

\[
\left. \frac{dW^c}{dT} \right|_{T=\frac{2K}{\beta}} = -\frac{1 - \beta^2 + \hat{\beta}(3 + \hat{\beta})(1 + \beta)}{(2 + \beta)(1 + \hat{\beta})^2}.
\]

When \( \beta \) is large, the value can be positive. Indeed, it is positive under the assumption that \( \hat{\beta} = 4 \) and \( \beta = 5 \). As previously discussed,

\[
\left. \frac{dW^c}{dT} \right|_{T=\frac{2(3+\beta)K}{\beta}} < 0
\]

for a sufficiently large \( \alpha \). There are two cases to be considered: (i) \( T^{c*} < 2K/\beta < 2(3 + \beta)K/\beta \) and (ii) \( 2K/\beta < T^{c*} < 2(3 + \beta)K/\beta \). In case (i), \( T^{c*} \) is increasing in \( \alpha \). In case (ii), \( T^{c*} \) is decreasing in \( \alpha \). Roughly speaking, the optimal entry tax is positively (res. negatively) correlated with \( \alpha \) when the entrants have high (low) technologies.

As explained in Section 3, in the Stackelberg case, an increase in the entry tax yields a welfare-improving production substitution from the marginal entrant to the other entrants but not to the incumbent. An increase in \( \alpha \) increases the number of entering firms and thus strengthens the former

16
effect. Therefore, the optimal entry tax is increasing in $\alpha$. In the Cournot case, the latter production-substitution effect from the marginal new entrant to the incumbent is also welfare-improving. An increase in $\alpha$ strengthens the former effect and weakens the latter one. Thus, when the latter effect dominates the former one, the optimal entry tax is decreasing in $\alpha$.

6 Concluding Remarks

The paper discussed the relationship between public policies and market structure. In particular, we examined how leadership affects the optimal entry tax in a market with free entry. When an incumbent cannot commit to its output before new entries, the welfare-improving effect of the entry tax is strong, and the technological improvement of the incumbent yields an increase in the optimal tax rate. This result changes in the presence of leadership. When the incumbent takes leadership, the welfare-improving effect of the entry tax is weak, and the technological improvement of the incumbent thus yields a reduction of the tax rate. These results can be extended to the multiple incumbent situation.

Our results are derived from two properties. One is the aggressive behavior of the incumbent with leadership, which appears in significantly more situations than those discussed in this paper and is shown to be quite robust by Etro (2004, 2006, 2007, 2008). The other property is excessive entries. A positive entry tax mitigates this problem and can improve welfare, and this property holds under a broader class of models with quantity competition as long as the strategies are strategic substitutes. Thus, we believe that our results do not depend on the specifications of our analysis, such as linear demand. However, the entries can be either excessive or insufficient if firms face Bertrand competition, and thus, the optimal entry tax can be either positive or negative.\footnote{See, for example, Matsumura and Okamura (2006a, 2006b) and Gu and Wenzel (2009).} Our results might then depend on the assumption of quantity competition, and this robustness check remains for future research.

In this paper, we focused on an entry tax policy. As shown by Etro (2004, 2006, 2007, 2008),
aggressive behaviors of the incumbent with leadership in free-entry markets prevail in many contexts, such as R&D and international trade. Thus, many other optimal policies in areas such as patents, privatization, and trade may also depend on whether the incumbent takes leadership. This also remains for future research.
Acknowledgements

This work was supported by JSPS KAKENHI (15K03347), and the Zengin Foundation for Studies on Economics and Finance.
References


