Coordination Frictions and Economic Growth

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Abstract

In practice, firms face a mass of scarce innovation projects. They choose a particular research avenue towards which to direct their effort, but do not coordinate these choices. This gives rise to coordination frictions. Our paper develops an expanding-variety endogenous growth model to study the impact of these frictions on the economy. The coordination failure generates a mass of foregone innovation and reduces the economy-wide research intensity. Both of these effects decrease the growth rate. Because of this, the frictions also amplify the fraction of wasteful simultaneous innovation. A numerical exercise suggests that the impact of coordination frictions on both the growth rate and welfare is substantial. This paper also analyzes firm-level data on patents which provide an estimate of the severity of the coordination problems and further evidence in favor of the hypothesis that research avenues are scarce.

Keywords: Growth, Frictions, Coordination, Simultaneous Innovation, Search for Ideas.

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1 Introduction

Innovators have technological access to many distinct research avenues (ideas).[1] At the same time, (quasi) simultaneous innovation of the same idea by several researchers is a well-documented empirical regularity.[2] Thus, often many firms engage in an innovation race for the exact same idea, i.e. research avenues are scarce. Furthermore, coordination of research efforts by firms (firm A directs its effort towards project 1, firm B towards project 2, and so on) is very unlikely in this setting because of two main reasons. First, the size of the "market" for ideas makes coordination very hard to achieve. Second, such coordination requires each firm to know the portfolio of research projects of all of its rivals. This is particularly implausible in the current context given that firms actively employ secrecy as an intellectual property protection mechanism.[3]

Motivated by these observations, we develop an expanding-variety endogenous growth model that features scarce research avenues and lack of research effort coordination. Our paper examines the impact of these coordination frictions on firms' decision to undertake R&D activities as well as their aggregate consequences. We also study the implications of the frictions for the planner's constraint-efficient allocation. Furthermore, we gauge the importance of the coordination problems for growth and welfare in a numerical exercise. Lastly, we analyze firm-level data on patents, which provide estimates for the severity of the coordination frictions and further evidence in favor of the hypothesis that ideas are scarce.

In our model, R&D firms direct their research efforts towards a particular project out of an endogenously determined mass of ideas. If innovated, each idea is transformed into one new variety. Firms which secure a patent over a variety produce. We focus on the symmetric equilibrium where firms use identical mixed strategies when directing their R&D

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[1] For example, during 2015 the U.S. Patent and Trademark Office granted more than quarter of a million patents.

[2] Perhaps the most famous example is that of the Alexander Bell and Elisha Gray telephone controversy. On February 14, 1876 Bell filed a patent application for the telephone and only hours later Gray submitted a similar application for the exact same innovation. Lemley (2011) presents ample evidence that virtually every important innovation from history has been simultaneously innovated. Cohen and Ishii (2005) documents the same phenomenon in patent examples which are not limited to major innovations. Section six of this paper details more recent examples.

[3] For a survey of the evidence see, for example, Hall *et al.* (2014).

efforts, so as to highlight their inability to coordinate. Thus, each idea is innovated by a random number of firms with mean equal to the tightness in the market for ideas (the ratio of firms to ideas). Knowledge is cumulative — each innovated idea allows firms to "stand on the shoulders of giants" and gain technological access to a number of new research projects. This intertemporal spillover effect is the ultimate source of growth in our economy — an expanding mass of ideas permanently alleviates future congestion problems, thus, reducing the cost of discovering new varieties. Along the balanced growth path (BGP henceforth), the growth rate of the economy is determined by the growth rate of the mass of ideas, which is in turn endogenously determined by the market tightness and the coordination problems.

The frictions in our model have a direct impact on the growth rate. Firms cannot coordinate their efforts, so they unintentionally gravitate towards the same research projects. This leaves a mass of profitable ideas uninnovated each period. As a consequence the growth rate of the decentralized frictional economy (DE henceforth) is reduced, as compared to a hypothetical economy in which firms can coordinate their efforts (CE henceforth). At the same time, due to a general equilibrium effect, the frictions amplify the fraction of wasteful simultaneous innovation.[4] Due to the lower growth rate firms discount future profit streams less. This increases the value of holding a patent and, in equilibrium, induces more congestion in the market for ideas. This higher congestion, in turn translates to a higher fraction of wasteful innovation. Furthermore, for any market tightness, the coordination frictions reduce firms' probability of securing a monopoly position. Given a market tightness, the ratio of innovations to ideas is the same for both the DE and the CE. In the DE, however, there is a mass of foregone innovation. Hence, a lower fraction of these innovations are distinct and as a consequence there is a lower number of patents to be distributed among firms. This reduced probability of securing a patent induces firms to decrease their entry into the R&D sector, leaving the DE with a lower R&D intensity (market tightness). As a result, the DE growth rate is decreased even further.

The decentralized equilibrium is inefficient as compared to the second-best allocation (SB

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[4] Since only one firm can obtain a patent over a particular variety, the R&D investment by all other rivals who innovate simultaneously represents wasteful duplication of effort.

henceforth). In addition to the usual appropriability externality, there are two others in the model. First, due to the possibility of simultaneous innovation, there is a "business-stealing" effect that leads to a congestion externality. The marginal R&D entrant finds innovation profitable even if a rival has already directed its research efforts towards the corresponding idea, as long as the entrant receives a patent for the innovation. In that event the rival is denied a patent and the entrant effectively steals the monopoly rents. The planner, on the other hand, finds the marginal entry beneficial only if no other firm has directed its research effort towards the corresponding idea. That is, on the margin, she values only the sole inventor. Thus, the congestion externality induces firms to over-invest as compared to the SB level. Second, there is a learning externality — the planner values innovation in part because it leads to an increase in the mass of ideas. Firms, on the other hand, do not have a mechanism through which to appropriate these extra ideas so they do not value them. Thus, in equilibrium the learning externality pushes firms towards under-investment as compared to the SB level. The size of the congestion externality is larger than that of the learning one, so implementing the SB requires the government to impose a tax on R&D spending.

The frictions in our model impact welfare negatively through two channels: they (i) generate a mass of foregone innovation and (ii) amplify the fraction of wasteful innovation. In the benchmark calibration, eliminating the frictions in the DE leads to a 13% welfare gain (in consumption equivalent terms). The DE growth rate is only 2/3 of the CE one, so the welfare cost of foregone innovation is 10.35%. Coordination problems increase the fraction of wasteful innovation by 8pp (to 39%), which translates to a 2.65% welfare cost. Moreover, if the planner could eliminated the frictions and assign the first-best allocation (FB henceforth), she would achieve welfare 16.15% higher than that in the SB. However, only 5.66pp of the gain is due to eliminating foregone innovation. This is because of two reasons. First, the SB features a much smaller fraction of foregone innovation than the DE. Second, removing the frictions in the SB reduces the fraction of wasteful duplication of effort from 52% to 0 since the FB does not suffer from the over-investment present in CE.

Our paper also tests the hypothesis that ideas are scarce using firm-level data on patents granted between 1976 and 2006. Given the intuition from the theoretical model, if research avenues are scarce, then an increase in the market tightness should be accompanied by a decrease in each firm's probability of securing a patent. To test this prediction, we combine firm-level data on patents and firm characteristics with aggregate-level data on patents. We find that the data provides strong support in favor of this prediction. Furthermore, our analysis allows us to estimate the level of congestion in the market for ideas.

1.1 Relationship to the Literature

Our paper models firms' choice of direction for their R&D efforts and the coordination problems inherent in this decision. As such, it is related to a recent literature on economic growth which emphasizes matching, and other, frictions in the innovation process (see, for example, Perla and Tonetti (2014), Lucas and Moll (2014), Benhabib *et al.* (2014), Chiu *et al.* (2015), and Akcigit *et al.* (2016)). The work here complements that literature by examining a different source of friction. In particular, to the best of my knowledge, this is the first growth paper to emphasize search frictions in the market for ideas which take the form of a coordination failure. Previous growth models have focused instead on a search process which takes the form of arrival rate of innovations, a McCall-type search for innovations, or frictions in the market for innovations.[5]

The theoretical model in this paper differs from the existing literature on economic growth in a number of additional dimensions. First, our analysis emphasizes firms' choice of research avenues by explicitly modeling the mass of available ideas. In particular, we make a distinction between potential innovations (ideas) and actual innovations.[6] Second, our model

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[5] For papers which feature search as arrival rate of innovations see, for example, Aghion and Howitt (1992), Grossman and Helpman (1991), and Klette and Kortum (2004). For papers that feature a McCall-type search for heterogeneous technologies see, for example, Kortum (1997), Perla and Tonetti (2014), and Lucas and Moll (2014). For papers which focus on frictions in the market for innovations see, for example, Chiu *et al.* (2015) and Akcigit *et al.* (2016). It is worth noting that Chiu *et al.* (2015) and Akcigit *et al.* (2016) do not make a distinction between ideas and innovations. In particular, the market for ideas in our paper (firms searching for a potential R&D project) is different from the "market for ideas" in Chiu *et al.* (2015) and Akcigit *et al.* (2016) where firms search for opportunities to trade the property rights over an innovation.

[6] This is in contrast to the previous literature on economic growth (Jones, 1995, 2002; Jones and Kim, 2014; Chiu *et al.*, 2015; Akcigit *et al.*, 2016; Bloom *et al.*, 2016) which has used ideas and innovations

features a scarce mass of potential research projects such as, for example, Grossman and Helpman (1991) and Klette and Kortum (2004).[7] Unlike those studies, our paper explicitly models the decision of firms to direct their R&D activities and emphasizes the coordination frictions inherent in this problem.[8] Third, in contrast to the previous literature, this paper features an endogenously determined mass of ideas. Fourth, in our paper firms compete for ideas through their choice of research avenue. This competition is different than the competition firms face at the product market or the innovation race which the previous literature has examined.[9]

Within the literature on industrial organization the two closest papers to ours are Kultti *et al.* (2007) and Kultti and Takalo (2008) which also feature search frictions in the market for ideas. In these papers there is the possibility of simultaneous innovation due to a matching technology which is the same as the equilibrium one in our paper. Kultti *et al.* (2007) and Kultti and Takalo (2008) focus on intellectual property rights in a partial equilibrium framework with a fixed mass of ideas and without free entry into the innovation sector. In contrast, our model focuses on a general equilibrium framework with growth, an endogenously determined mass of ideas, and an endogenously determined market tightness through free entry in the R&D sector.

The rest of the paper is organized as follows. Section two introduces the environment and characterizes the decentralized equilibrium. Section three examines the social planner's second-best allocation. Section four highlights the impact of coordination frictions in our model. Section five presents a numerical exercise. Section six details the empirical analysis. Section seven concludes.

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interchangeably.

[7] In contrast, some previous studies (Romer, 1990; Corriveau, 1994, 1998; Kortum, 1997) have examined models which feature an abundance of research avenues, whereas some others (Aghion and Howitt, 1992; Segerstrom *et al.*, 1990) have examined models where a single avenue of research is available. For a recent review of the literature see, for example, Aghion *et al.* (2014).

[8] In contrast, these papers do not focus on this decision and assume that firms can either perfectly coordinate their efforts (Grossman and Helpman, 1991) or cannot choose the direction of their research altogether (Klette and Kortum, 2004).

[9] See, for example, Segerstrom *et al.* (1990), Aghion and Howitt (1992), Corriveau (1994), Corriveau (1998), Aghion *et al.* (2005), and Acemoglu and Akcigit (2012)

2 The Economy

The environment is an augmented, discrete time version of the textbook model in Barro and Sala-i Martin (2003) Chapter 6 (BSM henceforth). There are three types of agents — a final good producer, a unit measure of consumers, and a continuum of R&D firms. The only point of departure from BSM is in the R&D sector, so as to emphasize the novel features of the model. In particular, R&D projects are scarce and R&D entrants can direct their efforts towards a particular project, but they cannot coordinate their research activities.

2.1 Final Good Sector

The final good is produced by a single price taker, using the following technology

$$Y_t = AL^{1-\lambda} \int_0^{N_t} X_t^{\lambda}(n)dn, \quad 0 < \lambda < 1 \tag{1}$$

where Y_t is output, L is the fixed labor supply of households, N_t is the mass of intermediate varieties, and $X_t(n)$ is the amount of a particular variety n employed in production. The price of the final good is normalized to unity. The final good firm faces a competitive market for labor, which is hired at the wage w_t, and a monopolistically competitive market for varieties, where a unit of each variety n is bought at the price $P_t(n)$. As in BSM, the firm's maximizing behavior yields the wage $w_t = (1-\lambda)Y_t/L$ and the inverse demand function for varieties $P_t(n) = \lambda AL^{1-\lambda}X_t^{\lambda-1}(n)$.

2.2 R&D Sector

The novel features of our model are contained in the R&D sector of the economy. The innovation process has three stages and makes a distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter the R&D sector at a cost $\eta > 0$ units of the final good. The mass of R&D entrants is denoted by μ_t and is to be determined in equilibrium. At stage two firms direct their innovative effort towards a particular R&D project from a finite mass ν_t of ideas. The choice is private knowledge and firms cannot coordinate their efforts. To capture this coordination failure, we follow the previous literature on coordination frictions and focus on a symmetric equilibrium where firms use identical mixed strategies.[10] Ideas are identical and, if innovated, transform into exactly one new variety. Innovation takes one period — a firm which enters at time t innovates the chosen project at time $t+1$. Thus, the only source of uncertainty in our model is the random realization of firms' equilibrium mixed strategies — some ideas may be innovated by many firms simultaneously, while others may not be innovated at all. Innovators apply for a patent which grants perpetual monopoly rights over the variety. Each innovation is protected by exactly one patent — if several firms simultaneously apply for the same patent, then each has an equal chance of receiving it. Stage three is as in BSM. Patent holders supply their variety in a monopolistically competitive market. Both the average and marginal costs of production are normalized to unity so profits are given by $\pi_t(n) = (P_t(n) - 1)X_t(n)$. Furthermore, the value of holding a monopoly over a variety n at time t, V_t, is given by

$$V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi_i(n)$$

where d_{it} is the stochastic discount factor.

A necessary condition for positive long term growth in our model is that the mass of ideas, ν_t, grows at a positive rate. We follow Kortum (1997) and Romer (1990), among others, and assume that knowledge is cumulative. Patenting an idea at time t allows firms to "stand on the shoulders of giants" and gain access to $M > 1$ new research avenues at $t + 1$. Thus, unlike previous growth models, in ours the mass of ideas is endogenously determined. Once an idea is innovated, it is no longer a potential R&D project and so it is removed from the pool.[11] Thus the net increase in the pool of ideas from innovating one new variety is $M - 1$. Due to the frictions in our model, there is a chance that an idea is not innovated, i.e. no firm directs its research efforts towards the idea in question. Let us denote this probability

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[10] See, for example, Julien *et al.* (2000), Burdett *et al.* (2001), and Shimer (2005).

[11] Each innovation is protected by a patent, so no firm has an incentive to imitate at a late date. Thus, the idea no longer represents a profitable R&D project and as a consequence it is no longer in ν_{t+1}.

by ζ_t, then the law of motion for ideas is given by

$$\nu_{t+1} = \nu_t + (1-\zeta_t)(M-1)\nu_t$$

As each innovated idea is transformed into a new variety, it follows that

$$N_{t+1} = N_t + (1-\zeta_t)\nu_t$$

2.3 Households

Consumers are endowed with a discount factor β and a per-period utility function $U(C_t) = \ln C_t$. They can save by accumulating assets, which in this economy are claims on intermediate firms' profits. In particular, households have access to a mutual fund that covers all intermediate good firms. Let a_t denote the amount of shares held by the representative household at the beginning of period t. Each period all profits are redistributed as dividends, thus, the total assets of the household entering period t are $a_t \int_0^{N_t}(\pi_t(n)+V_t(n))dn$. At time t households decide on the shares they would like to hold at $t+1$, a_{t+1}. The mutual fund at that time covers all firms which exist at time $t+1$, N_{t+1}. Hence, the household's budget constraint is given by

$$a_{t+1}\int_0^{N_{t+1}} V_t(n)dn = a_t \int_0^{N_t} (\pi_t(n)+V_t(n))dn + w_t L - C_t$$

The household's first order conditions imply the Euler equation below

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}}\Big(\int_0^{N_{t+1}} (\pi_{t+1}(n)+V_{t+1}(n))dn\Big)\Big(\int_0^{N_{t+1}} V_t(n)dn\Big)^{-1}$$

The intuition is standard — consumers equate the marginal utility at time t with the discounted marginal utility at time $t+1$, times the gross rate of return on their assets.

2.4 Equilibrium

We restrict the analysis to a set of parameter values which ensures that firms have an incentive to enter the R&D sector, i.e. $\eta \leq (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)}L/[\lambda(M-\beta)]$. The usual profit maximization of intermediate good firms along with the demand function imply that $P_t(n) = 1/\lambda$ and $X := X_t(n) = (\lambda^2 A)^{1/(1-\lambda)}L$. Thus, every intermediate good firm yields the same per period profits of $\pi := \pi_t(n) = X(1-\lambda)/\lambda$. This implies that $V_t := V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi$ — every firm is equally valuable. Since each variety carries the same amount of profits, the stage two equilibrium strategy of firms is to direct their R&D effort towards each idea with equal probability.[12] This implies the following equilibrium outcome.

Proposition 1. *The number of firms which direct their R&D effort towards a particular idea follows a Poisson distribution with mean* θ_t, *where* $\theta_t \equiv \mu_t/\nu_t$.

A proof is in Appendix C. The random realization of firms' equilibrium strategies gives rise to the standard urn-ball matching technology.[13] The ratio of firms to ideas, θ_t, represents the tightness in the market for ideas and captures the level of congestion in the economy. An R&D firm becomes a monopolist with probability $\sum_{m=0}^{\infty} Pr($ exactly m rival firms direct their research effort towards the particular idea$)/(m+1) = \sum_{m=0}^{\infty} e^{-\theta_t}\theta_t^m/(m+1)! = (1-e^{-\theta_t})/\theta_t$. This probability captures the business-stealing effect in the model. An innovator faces the threat that a rival directs its research efforts towards the exact same idea. If that is the case, then the rival has a chance of securing a patent over the innovation, effectively stealing the innovator's monopoly rents. Thus, higher congestion increases the expected number of rivals, which lowers each firm's chance of securing a patent. Given free entry, it follows that

$$\eta = \frac{1-e^{-\theta_t}}{\theta_t} V_t \tag{2}$$

The level of congestion firms are willing to tolerate is governed by the net present value of

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[12] We follow the literature on coordination frictions (see, for example, Julien *et al.* (2000)) and derive the optimal behavior for firms when there are finite number of ideas. The result is then obtained by taking the limit as $\nu_t \to \infty$, keeping the ratio μ_t/ν_t constant.

[13] See, for example, Wolinsky (1988), Lu and McAfee (1996), Julien *et al.* (2000), and Burdett *et al.* (2001).

profits and the entry cost. Higher profits (or lower costs) induce firms to tolerate a lower chance of securing a monopoly position and as a consequence higher tightness. The matching technology implies that $\zeta_t = e^{-\theta_t}$. Hence,

$$\begin{aligned} \nu_{t+1} &= \nu_t + (1 - e^{-\theta_t})(M - 1)\nu_t \\ N_{t+1} &= N_t + (1 - e^{-\theta_t})\nu_t \end{aligned}$$

Furthermore, the frictions in our model induce an economy-wide varieties production function (New Varieties $= (1 - e^{-R_t/(\eta\nu_t)})\nu_t$) which is concave in the aggregate research effort, $R_t \equiv \eta\mu_t$. A higher aggregate research effort is associated with higher mass of firms which, in turn, increases the congestion in the market. Thus, the marginal entrant has a higher chance of duplicating an innovation, rather than innovating a distinct new variety. In particular, the higher level of congestion increases the fraction of wasteful duplicative innovation, $\omega \equiv 1 - (1 - e^{-\theta_t})/\theta_t$.[14]

Since all firms receive the same profits, the Euler equation simplifies to

$$V_t = \beta \frac{C_t}{C_{t+1}} \Big(\pi + V_{t+1}\Big) \tag{3}$$

Hence, the stochastic discount factor is $d_{it} = \beta^i C_t / C_{t+i}$. Given consumers' budget constraint, free entry, and the law of motion for varieties it is straightforward to derive the economy-wide resource constraint which takes the usual form — output is distributed towards consumption, production of intermediate inputs, and investment in R&D.

$$Y_t = C_t + N_t X + \mu_t \eta \tag{4}$$

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[14] Only one firm can hold a patent over a certain variety. Hence, whenever $m \geq 1$ firms innovate the same idea, $m - 1$ of them make a wasteful duplicative innovation. Each entrant makes an innovation, so the total number of innovations is μ_t. The total number of useful innovations equals the total number of new varieties, $(1 - e^{-\theta_t})\nu_t$. Thus, the fraction of innovations which represent wasteful duplication of effort is simply $1 - (1 - e^{-\theta_t})/\theta_t$.

2.5 Balanced Growth Path

Our analysis focuses on the BGP of the economy, where output, consumption, varieties, ideas, and the mass of entrants all grow at constant (but possibly different) rates. Denote the growth rate of any variable x along the BGP by g_x. It is straightforward to establish that output, varieties, consumption, entry into R&D, and the stock of ideas all grow at the same rate along the BGP. Namely, $g \equiv g_Y = g_C = g_N = g_\mu = g_\nu = (1 - e^{-\theta})(M - 1)$, where θ is the value of the market tightness along the BGP.[15] As in BSM Y_t, C_t, N_t, and μ_t all grow at the same rate. In our model, the mass of ideas, ν_t, also grows at this rate. In fact, the expansion of ν_t is the ultimate source of growth in the economy. Due to learning, innovation today increases the mass of ideas in the future. This permanently reduces the severity of the coordination problems and subsequently the cost of securing a monopoly position.[16] This lower cost in turn induces higher entry into R&D up to the point where congestion reaches its BGP level. Furthermore, the fraction of foregone innovation, $e^{-\theta}$, directly impacts the growth rate, as only innovated ideas at time t contribute to the expansion of ν_{t+1}.

It is convenient to solve the model by looking at the stable ratios θ, $\frac{\nu}{N}$ and $\frac{C}{N}$. From the law of motion of ideas and varieties, and from $g_N = g_\nu$, it follows that $\frac{\nu}{N} = M - 1$. Next, the resource constraint implies that

$$\frac{C}{N} = \frac{1 + \lambda}{\lambda}\pi - \eta\theta(M - 1) \tag{5}$$

Lastly, we can use the fact that $g_C = g_\nu$, the Euler equation, the law of motion for ν_t, and the free entry condition to find an implicit solution for the market tightness.

$$\eta = \left(\frac{1 - e^{-\theta}}{\theta}\right)\frac{\beta\pi}{1 + (1 - e^{-\theta})(M - 1) - \beta} \tag{6}$$

Even though we cannot explicitly solve for θ, it is straightforward to establish that the solution is unique. Intuitively, as θ increases the market for ideas gets more congested and

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[15] A proof is available upon request.

[16] The average cost of securing a monopoly position is $\eta/Pr(\text{monopoly}) = \eta\theta/(1-e^{-\theta})$, which is decreasing in ν_t.

each firm's chance of becoming a monopolist decreases. At the same time, higher market tightness implies a higher growth rate. This, in turn, increases the rate with which firms discount future profit streams and as a consequence decreases the value of holding a patent. Both of these effects decrease the incentives to enter the R&D sector when the market tightness is high and vice versa.

3 Second-Best Allocation

This section examines the planner's second best allocation — the planner chooses the optimal BGP allocations subject to the coordination frictions in the market for ideas. Without loss of generality, we impose symmetry in the intermediate varieties, i.e. $X_t(n) = X_t(n')$ for any varieties n and n'. Thus, the planner faces the problem of choosing production of varieties, consumption, a mass of varieties, a mass of ideas, and the market tightness in order to maximize welfare subject to the resource constraint, the laws of motion for ideas and varieties, and the coordination frictions.

$$\max_{\{C_t, X_t, \theta_t, N_t, \nu_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$AL^{1-\lambda} N_t X_t^{\lambda} = N_t X_t + C_t + \eta \theta_t \nu_t \tag{7}$$

$$N_{t+1} = N_t + (1 - e^{-\theta_t})\nu_t \tag{8}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\theta_t})(M - 1)\nu_t \tag{9}$$

Maximizing with respect to X_t yields the usual solution for varieties $X^* := X_t = (\lambda A)^{1/(1-\lambda)} L$. As in BSM the difference between the planner's solution and the decentralized outcome comes from the monopoly pricing of intermediate goods. Let $\pi^* = X^*(1-\lambda)/\lambda$ denote the implied per period monopoly profits at efficient level of intermediate varieties.

Then, the rest of the first order conditions are

$$[C_t]: \quad \beta\frac{C_t}{C_{t+1}} = \frac{\phi_{t+1}}{\phi_t} \tag{10}$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \phi_{t+1}\pi^* \tag{11}$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1}\Big(e^{-\theta_{t+1}} + (1-e^{-\theta_{t+1}})M\Big) + h_{t+1}(1-e^{-\theta_{t+1}}) - \phi_{t+1}\eta\theta_{t+1} \tag{12}$$

$$[\theta_t]: \quad \eta = e^{-\theta_t}\Big(\frac{h_t}{\phi_t} + \frac{\lambda_t}{\phi_t}(M-1)\Big) \tag{13}$$

where ϕ_t, h_t, λ_t are the multipliers associated with (7), (8), and (9), respectively. From (10) and (11), it follows that

$$\frac{h_t}{\phi_t} = \beta\frac{C_t}{C_{t+1}}\Big(\pi^* + \frac{h_{t+1}}{\phi_{t+1}}\Big) \tag{14}$$

The above equation characterizes the planner's valuation of varieties: the value of a variety equals the discounted sum of per period profits, π^*, and the continuation value h_{t+1}/ϕ_{t+1}. There are only two differences as compared to the DE — the level of profits is higher and the planner chooses a different tightness.

The value of an idea is the discounted sum of several terms.

$$\frac{\lambda_t}{\phi_t} = \beta\frac{C_t}{C_{t+1}}\Big(-\eta\theta_{t+1} + (1-e^{-\theta_{t+1}})\Big(\frac{h_{t+1}}{\phi_{t+1}} + \frac{\lambda_{t+1}}{\phi_{t+1}}(M-1)\Big) + \frac{\lambda_{t+1}}{\phi_{t+1}}\Big) \tag{15}$$

First, there is the dividend, $-\eta\theta_{t+1}$, which represents the average cost of R&D per idea. It captures the intuition that unlike other assets, which carry positive returns, an idea is only valuable if it is innovated. Hence, the planner finds it costly to keep a stock of ideas because it diverts resources away from consumption and into R&D. The second term represents the capital gain from innovation — the probability an idea is innovated, $(1-e^{-\theta_{t+1}})$, times the social benefit from innovating. This benefit is the value of the extra variety, h_{t+1}/ϕ_{t+1}, plus the value of the extra ideas that would be added to the pool because of innovation, $\lambda_{t+1}/\phi_{t+1}(M-1)$. Lastly, the idea carries its continuation value λ_{t+1}/ϕ_{t+1}.

The frictions in our model are linked to two externalities, which are illustrated in equation (13). First, the congestion externality manifests through the difference in the fraction of socially and privately beneficial innovations. The planner finds the marginal entry beneficial only if the firm is the sole inventor, i.e. with probability $e^{-\theta_t}$. Firms, on the other hand, value entry even if they duplicate an innovation, as long as they receive the patent for it. In particular, due to the business-stealing effect, the probability of a privately beneficial innovation is $(1 - e^{-\theta_t})/\theta_t > e^{-\theta_t}$. Hence, the congestion exterlaity induces firms to over-invest in R&D as compared to the SB.[17] Second, there is the learning externality — firms cannot appropriate the benefit of any ideas that come about from their innovations, so they do not value them.[18] The planner, on the other hand, does because they permanently alleviate future coordination problems. Specifically, more innovation today increases the amount of future research avenues, which allows the economy to innovate more varieties without increasing the congestion problems. Thus, the extra ideas permanently reduce the cost of discovering new varieties.[19] As a result the learning externality creates incentives for firms to under-invest as compared to the SB.

It is straightforward to establish that along the BGP the SB allocations are characterized by[20]

$$\left(\frac{\nu}{N}\right)^{SB} = M - 1 \tag{16}$$

$$\left(\frac{C}{N}\right)^{SB} = \pi^* - \eta\theta^{SB}(M-1) \tag{17}$$

$$1 + (1 - e^{-\theta^{SB}})(M-1) = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\theta^{SB}} + (1 - e^{-\theta^{SB}} - \theta^{SB}e^{-\theta^{SB}})(M-1)\Big) \tag{18}$$

The difference between the SB solution for the market tightness, (18), and the DE one, (6), comes from the aforementioned externalities. To see this clearly, let us define the implied

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[17] The business-stealing effect in the model is a consequences of firms' choice of R&D project and the coordination frictions inherent in this decision. It is thus different than the business-stealing effect examined in the previous literature (see, for example, Corriveau (1994) and Corriveau (1998)).

[18] This externality is similar in spirit to the inter-temporal spillover effects present in previous models (see, for example, Romer (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991)). In the present paper, the externality operates through the market for ideas — the planner values ideas because they alleviate the coordination problems in the economy.

[19] The average cost of discovering one new variety is $\eta/Pr(\text{sole inventor}) = \eta e^{\theta_t}$, which is decreasing in the mass of ideas.

[20] A proof is available upon request.

rate of return in the DE by

$$r := \frac{C_{t+1}}{\beta C_t} - 1 = \frac{\pi}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big) \tag{19}$$

which is nothing but the rate of return on a unit investment in R&D — π is the flow of profits and $(1-e^{-\theta})/\theta$ is the probability of securing a monopoly position. The implied rate of return in the SB represents the social rate of return on a unit of investment on R&D and is defined by

$$r^{SB} := \frac{C^{SB}_{t+1}}{\beta C^{SB}_t} - 1 = e^{-\theta^{SB}}\Big(\frac{\pi^*}{\eta} - \theta^{SB}(M-1)\Big) + (1-e^{-\theta^{SB}})(M-1) \tag{20}$$

First, the planner eliminates the monopoly distortion, so the flow of profits is π^*. Second, she values the marginal innovation only when the firm is the sole inventor, which occurs with probability $e^{-\theta^{SB}}$. In that event, the net return is given by the normalized profits, π^*/η, less the normalized "storage cost" of the new research avenues, $\theta^{SB}(M-1)$. Third, each innovation increases the mass of ideas, so the permanent decrease in future congestion yields the return of $(1-e^{-\theta^{SB}})(M-1)$.

In BSM the externalities can be eliminated by using a subsidy on the purchases of intermediate goods. In our model such a subsidy is still necessary to eliminate the dead-weight loss from monopoly and the appropriability externality, but it is not sufficient to achieve the SB. This is due to the congestion and learning externalities. To implement the SB, the planner needs to impose a tax on the entry into R&D. This is because the congestion externality is larger than the learning one, so the over-investment effect of the former dominates the under-investment effect of the latter. In particular, suppose that the government imposes a subsidy on the purchases of intermediate varieties at a rate s and a tax on R&D activities at a rate τ. Furthermore, if the government keeps a balanced budget through the means of lump-sum transfers, then the optimal policy is summarized below.

Proposition 2. *The optimal subsidy on the purchase of intermediate varieties is given by*

$s^* = 1 - \lambda$. *The optimal tax rate on R&D entry is given by*

$$\tau^* = \frac{\beta \pi^* (1 - e^{-\theta^{SB}})}{\eta \theta^{SB} (e^{-\theta^{SB}} + (1 - e^{-\theta^{SB}}) M - \beta)} - 1$$

Furthermore, $\tau^* > 0$ *because the magnitude of the congestion externality is larger than that of the learning externality.*

A proof is included in Appendix C. The optimal subsidy, s^*, is the same rate as in BSM. Unlike in BSM, however, this subsidy needs to be supplemented by a tax on R&D investment. The optimal tax rate is devised such that firms internalize the inefficiencies associated with the frictions in our model. Lastly, even though it is optimal to impose a tax on R&D spending, it may be the case that the decentralized economy suffers from under-investment, i.e. $\theta < \theta^{SB}$. This is due to the appropriability externality. Whether or not there will be under-investment in equilibrium depends on parameter values.

4 The Impact of Coordination Frictions

4.1 Decentralized Economy

A goal of the analysis is to study the impact of coordination frictions in our economy. To this end we compare the DE's BGP to the BGP of a hypothetical CE. In particular, the only difference between the latter economy and the DE one is that firms can coordinate their research efforts at stage two of the innovation process.[21] Let superscript c denote the value of any variable in the CE along the BGP. Evidently, when firms can coordinate their research efforts, all research avenues are undertaken and subsequently all ideas are innovated. At the same time, the CE may feature a positive fraction of wasteful duplication of effort due to the usual "over-grazing" problem.[22] However, this waste, ω^c, is smaller than the one in the DE. Furthermore, this is the case, even though the CE features a higher market tightness.

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[21] The proof of Proposition 3 explicitly defines the process of coordination.

[22] For a survey of the literature see, for example, Reinganum (1989).

Proposition 3. *In the coordination economy all ideas are innovated and the growth rate equals $M - 1$. Furthermore, $\omega > \omega^c$ and $\theta < \theta^c$.*

A proof is included in Appendix C. Intuitively, when firms can coordinate their R&D activities all ideas are innovated because each of them represents an opportunity to gain a profitable monopoly position. Thus, in the CE there is no foregone innovation. This, results in a higher growth rate as compared to the DE. Because of this the foregone innovation in the DE generates a general equilibrium effect which induces firms to tolerate a higher congestion than firms in the CE. In particular, the lower growth rate increases the stochastic discount factor, which in turn raises the value of holding a patent. Since, in both economies, the probability of making a wasteful innovation is simply the probability of not receiving a patent, it follows that $\omega > \omega^c$.

Moreover, $\theta < \theta^c$, even though the DE features a higher fraction of wasteful simultaneous innovation. This is the case because, for a given market tightness, the coordination frictions reduce an entrant's chance of securing a monopoly position. In particular, the probability of securing a patent in the DE for a given tightness $\tilde{\theta}$, $(1 - e^{-\tilde{\theta}})/\tilde{\theta}$, is only a fraction $1 - e^{-\tilde{\theta}}$ of the one in the CE, $1/\tilde{\theta}$. As firms cannot coordinate their efforts, in the DE only a fraction $1 - e^{-\tilde{\theta}}$ of ideas are patented. Thus, even though the number of patent applications per idea, $\tilde{\theta}$, is the same in both economies, in the DE there are relatively less patents to be distributed among innovators. This decreases each entrant's chance of securing a monopoly position and subsequently reduces the incentives to enter the R&D sector. This is true even though the DE features a higher value of holding a patent. In other words, the decrease in the probability of securing a patent dominates the increase in the net present value of profits, ultimately reducing incentives to enter the R&D sector and decreasing the market tightness. Furthermore, the effect on the market tightness provides an indirect channel through which the presence of foregone innovation reduces the growth rate in the DE. A lower tightness decreases each idea's chance of being innovated which results in a lower aggregate mass of innovation.

4.2 Planner's Allocation

To highlight the impact of coordination frictions in the planner's allocation we compare the BGP in the SB to that in the FB. In the FB the planner can directly assign firms to projects. Thus, it is straightforward to establish that $\theta^{FB} = 1$ and that the FB does not feature any foregone innovation, nor any wasteful duplication of effort.[23] Thus, it is readily observable that the frictions amplify both the fractions of foregone and wasteful innovation. Unlike in the decentralized case, however, the coordination failure does necessarily reduce the research intensity in the economy. This is so because in the SB the planner faces a trade-off when deciding on the market tightness (as depicted in equation (13)). On the one hand, a higher tightness increases congestion and subsequently the cost of wasteful innovation, $\eta \times Pr(\text{duplication of effort}) = \eta(1 - e^{-\theta_t})$. On the other hand, a higher tightness decreases the fraction of foregone innovation. The benefit from this decrease is given by the probability the marginal firm is the sole inventor, $e^{-\theta_t}$, times the the social benefit of the innovation net of the entry cost, η. Thus, the planner chooses θ^{SB} that, on the margin, strikes a balance between these two opposing effects. In the FB, however, she faces no such trade-off so the decision of setting the market tightness is independent of the parameters which govern the welfare costs of wasteful duplication of effort and foregone innovation.

5 Numerical Exercise

We gauge the importance of the frictions in our model for growth and welfare through the means of a numerical exercise. Our calibration matches key moments of the U.S. economy and is set at annual frequency. The discount factor, β, is set to 0.95, the productivity parameter, A, and labor supply, L, are both normalized to unity. We set the markup to 17.43% ($\lambda = 0.8516$) to match the average R&D share of non-farm GDP, $\eta\mu_t/Y_t = 3.1194\%$, for the period between 1966 and 2011.[24] To calibrate η and M we use two additional

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[23] Furthermore, $(\nu/N)^{FB} = M - 1$, $(C/N)^{FB} = \pi^* - \eta(M - 1)$, and $g^{FB} = M - 1$. A proof is available upon request.

[24] The data on non-farm GDP is in 2009 chained dollars and taken from NIPA table 1.3.6. The data on nominal R&D expenditures is from NIPA table 5.6.5 and includes private fixed investment in R&D (including

moments. First, we match the average growth rate of non-farm GDP for the same period of 1.7546%. Second, in our model the ratio of patent grants to patent applications is $(1-e^{-\theta})/\theta$. Matching this fraction to its empirical counterpart, 0.60957, results in a market tightness $\theta = 1.0876$.[25] Together these two moments yield $\eta = 0.1715$ and $M = 1.0265$.

The calibrated DE features a fraction of wasteful innovation $\omega = 39\%$. This is about 25% larger than that in the CE, $\omega^c = 31\%$, even though θ is about 25% smaller than $\theta^c = 1.4491$. At the same time the DE features a large fraction of research avenues which are not undertaken — 33.7%. This implies that the growth rate is about 2/3 of the CE growth rate $g^c = 2.65\%$. Eliminating the frictions generates a welfare gain of 13% in consumption equivalent terms.[26] About 10.35pp of the gain is due to the increased growth rate and the rest is due to the reduction in the fraction of wasteful innovation.

The DE exhibits too little innovation — the SB market tightness, θ^{SB}, is 1.7154. Thus, in the SB the percentage of innovations which represent a wasteful duplication of effort, ω^{SB}, is 52%. The SB features a fraction of uninnovated research avenues of 18%. While this is still quite sizable, it is about half of that in the DE. As a consequence, the SB growth rate (of 2.17%) is considerably larger than the one in the DE. Eliminating the frictions in the planner's allocation results in a 16.15% welfare gain. Of this 5.6pp is the gain due to eliminating the fraction of foregone innovation and the rest is due to eliminating the fraction of wasteful innovation.

The relative welfare costs of foregone innovation and wasteful duplication of effort are different in the decentralized equilibrium and the planner's allocation. This is the case because of two reasons. First, the planner chooses θ^{SB} which, on the margin, strikes a balance between these two welfare costs. As a result the fraction of foregone innovation in the SB is much smaller. Thus, eliminating this fraction leads to a relatively smaller welfare gain. Second, eliminating the frictions in the DE does not fully eliminate the fraction of

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software). To obtain the series on real R&D investment, we deflate the nominal series using the implicit GDP price deflator from NIPA table 1.1.9.

[25] The data is taken from the U.S. Patent and Trademark Office. The data on patent grants is by year of application.

[26] A detailed explanation of the welfare calculations is included in Appendix A.

wasteful innovation. In particular, since the CE features $w^c = 31\%$, the reduction in the waste is only 8pp. On the other hand, if the planner could achieve the first-best, then all of the waste would be eliminated, leading to a reduction of 52pp.

Insert Figure 1 here

To explore the robustness of the quantitative results, we repeat the numerical exercise for different calibrated values of the DE market tightness, θ. Two alternative calibration strategies allow us to discipline the analysis. First, we turn to estimates on the return to R&D expenditure from the existing literature.[27] The majority of these estimates are consistent with a rate of return for the U.S. economy between 20% to 40%. This yields a market tightness $\theta \in [0.6471, 1.8039]$.[28] Second, we use firm-level data on patents to estimate the elasticity of the probability of securing a monopoly position with respect to the mass of R&D firms, $-(1 - e^{-\theta} - \theta e^{-\theta})/(1 - e^{-\theta})$. This yields $\theta \geq 1$.[29] We consider the interval $\theta \in [1, 1.8]$, which is consistent with both calibration strategies.

Figure 1 illustrates the quantities of interest for the different values of the calibrated market tightness. The welfare gain is substantial for all considered values — it is at least 4.7% for the decentralized economy and 10.8% for the planner's allocation (Figure 1d). Furthermore, the gain is decreasing in the calibrated value of the tightness. Intuitively, higher θ implies a lower fraction of foregone innovation, $e^{-\theta}$, which in turn decreases the difference between the CE and DE growth rates, $g^c - g = e^{-\theta}(M - 1)$. At the same time, the reduced growth rate gap implies that the amplification in the fraction of wasteful innovation, $(\omega - \omega^c)$ is smaller. Both of these effects serve to mitigate the impact of the coordination frictions and as a consequence the welfare gain from eliminating these frictions. The intuition

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[27] For a survey see, for example, Hall *et al.* (2010).

[28] The return to R&D investment in our model is given by

$$\frac{\partial Y_{t+1}}{\partial R_t} = \Big(\frac{g}{\eta\mu_t/Y_t}\Big)\frac{\theta e^{-\theta}}{1 - e^{-\theta}}$$

As the growth rate, g, and the R&D share of GDP, $\eta\mu_t/Y_t$, are matched to the aggregate data, the above allows us to match the tightness, independent of other model parameters.

[29] Further details are in section six.

for the case of the planner's allocation is slightly different and it serves to explain why the welfare gain decreases relatively less than the gain in the decentralized case. Firstly, a higher calibrated market tightness implies a lower parameter value for the entry cost, η, and for the number of new ideas generated per new variety, M. Thus, the planner finds it optimal to set a higher SB market tightness, since innovation is cheaper and the net present value of implied profits higher. However, this increase is relatively smaller than the corresponding increase in θ. Thus, the response in $e^{-\theta^{SB}}$ and the SB growth rate is relatively smaller. At the same time a higher θ^{SB} leads to a larger fraction of wasteful duplication of effort, ω^{SB}. This effect puts an upward preassure on the welfare gain as θ^{SB} increases. Nonetheless, the welfare cost of wasteful innovation decreases because of the lower entry cost and ratio of ideas to varieties, $(\nu_t/N_t)^{SB}$. The resulting net effect on the welfare gain due to eliminating wasteful innovation is negative. Yet, this effect is smaller than the one in the DE and as a result the welfare gain in question is larger for all considered values of θ.

Lastly, we explore the robustness of the quantitative results in an extension of our baseline model. In particular, our augmented model features uncertainty in the innovation process and endogenous firm-level research intensity. Upon choosing a direction for their effort, R&D firms decide on an intensity i and incur the cost ϕi $(\phi > 0)$. The amount of effort devoted affects their probability of successfully innovating the idea according to $Pr(\text{success}) = 1-e^{-\gamma i}$ $(\gamma > 0)$. The welfare cost of frictions in this extension is virtually the same as in the baseline model, so the results are presented in Appendix B.

6 Empirical Analysis

We use firm-level panel data on patents in order to test the hypothesis that ideas are scarce and to provide an alternative estimate of the market tightness. If firms do face a common pool of scarce research avenues, then we should expect to see firms simultaneously developing the exact same innovation relatively often. If they do not, then simultaneous innovation should be a rare occurrence. Indeed, the phenomenon of simultaneous innovation is well documented in the literature. For example, Lemley (2011) details anecdotal evidence that virtually every major historical innovation (such as the cotton gin, the steam engine, the computer, and the laser) has been simultaneously innovated by several groups of researchers. Cohen and Ishii (2005) find that a positive fraction of patents for the period between 1988 and 1996 were declared in interference.[30] In a more recent example, Siemens applied for a patent for a positron emission tomography scanner on April 23, 2013 (application number 13/868,256). Most claims are rejected because Philips (application number 14/009,666 filed on March 29, 2012 and application number 14/378,203 filed on February 25, 2013) had simultaneously made similar innovations.[31] In another recent case Google Inc. filed a patent application on November 1 2012 (number 13/666,391) for methods, systems, and apparatus that provide content to multiple linked devices. All twelve claims contained in the application are rejected because of simultaneous innovations made by Yahoo! Inc. (application number 13/282,180 with filing date October 26, 2011), Microsoft Corporation (application number 13/164,681 with filing date June 20, 2011), and Comscore Inc. (application number 13/481,474 with filing date May 25, 2012).[32]

Our analysis takes a different, complementary approach from the one in the aforementioned literature. We estimate the resulting increase in the congestion due to an increase in the number of innovators. If ideas are indeed scarce, then an increase in the number of firms would suggest that, on average, more innovators apply for the same patent and, as a consequence, each of them has a lower probability of securing that patent. If ideas are not scarce, then we would expect the marginal entrants to work on distinct projects and as a result not observe any change in firms' probability of securing a patent. Furthermore, our approach allows us to recover the market tightness from the empirical estimates.

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[30] Patents are declared in interference if two innovators file for the same patent within three months of each other (six months for major innovations).

[31] The information on the patent applications is taken from the U.S. Patent and Trademark Office Patent Application Information Retrieval. Philips's applications were made public on January 23, 2014 and January 15, 2015. Siemens' patent application was rejected on September 10, 2015. The examiner rejected most claims under 35 U.S.C. 103 citing the two patent applications in the text, as well as a patent held by the National Institute of Radiological Sciences in Japan (patent application number 12/450,803).

[32] Yahoo! Inc.'s application was made public on May 2, 2013, Microsoft's application was made public on December 20, 2012, and Comscore Inc.'s application was made public on December 20, 2012. Google's patent application was rejected on November 20, 2014. The examiner rejected the application under pre-AIA 35 U.S.C. 103(a) citing the three patent applications in the text.

6.1 Empirical Methodology

We augment the empirical model of the patent production function (see, for example, Hall *et al.* (1986) and Fabrizio and Tsolmon (2014)) to account for the number of innovators as captured by the aggregate number of patent applications. If ideas are scarce, then a higher aggregate number of patent applications would reduce each firm's probability of securing a patent and, controlling for firm-level quality and quantity of patent applications, each firm's number of patents granted. In addition, we control for the number of patents in force because of two reasons. First, an increase in the aggregate number of patent applications does not necessarily imply a higher market tightness, instead it might simply be a response to a higher mass of ideas. Thus, motivated by the cumulative nature of knowledge, we use past innovation, captured by the number of patents in force, as a proxy for the current stock of research avenues. Second, patents might have a strategic aspect that allows their owner to block rivals from innovating, and subsequently patenting related innovations.[33] Since a higher number of patent applications in the aggregate translates to a higher number of patents, we include the number of patents in force to ensure that the estimated effect captures only the congestion we are interested in and not the reduction in the number of patents because of the aforementioned strategic effect. Since firm-level data on the quantity and quality of patent applications is not available, we proxy for these using a set of firm-level controls.

As the number of patents granted to each firm is a count variable, we estimate the following equation using the Poisson quasi-maximum likelihood estimator (Wooldridge, 1999):

$$E[P_{i,t}|I_t] = \exp\big[\sum_{i=0}^{L_{\text{Apps}}} \beta_i \Delta\ln(\text{Apps}_{t-i}) + \sum_{i=0}^{L_{\text{PatsInForce}}} \gamma_i \Delta\ln(\text{PatsInForce}_{t-i}) \\ + \alpha_i + t + \sum_{j=1}^{k}\sum_{i=0}^{L_j} \delta_{j,i} X_{j,t-i}\big] \tag{21}$$

where $P_{i,t}$ is the number of successful patent applications filed by firm i at time t, $\Delta\ln(\text{Apps}_t)$

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[33] See, for example, Bessen and Meurer (2006), Hall *et al.* (2014), and Choi and Gerlach (2017).

is the growth rate of aggregate patent applications at time t, $\Delta\text{ln}(\text{PatsInForce}_t)$ is the growth rate of the number of patents in force at time t, $X_{j,t}$ represents firm-level controls, and α_i represents firm-level fixed effects. L_{Apps} and $L_{\text{PatsInForce}}$ denote the maximum lags of $\Delta\text{ln}(\text{Apps}_{t-i})$ and $\Delta\text{ln}(\text{PatsInForce}_{t-i})$ included in the estimation.

We include the growth rate of Apps_t, rather than its natural log because of unit root considerations.[34] The equation includes lags of $\Delta\text{ln}(\text{Apps}_t)$ to account for the patent grant lag observed in the data. In particular, the relevant mass of rivals that work on the same ideas simultaneously is all firms that apply for a patent after the original innovator but before she has received the patent.[35] Since only 28.26% of all patents granted in the U.S. between 1976 and 2006 have a grant lag of less than two years, the appropriate measure should include Apps_t and Apps_{t-1}. In contrast, most of the successful patent applications filed at time $t-2$ or earlier were granted by time t, so we do not include further lags in the measure of the mass of innovators. Nonetheless, we do include $\Delta\text{ln}(\text{Apps}_{t-2})$ in the regression equation for robustness and to capture any possible learning not absorbed by the number of patents in force. Thus, L_{Apps} is set to two. This implies that

$$\begin{aligned}\sum_{i=0}^{L_{\text{Apps}}} \beta_i \Delta\text{ln}(\text{Apps}_{t-i}) &= \beta_0 \Delta\text{ln}(\text{Apps}_t) + \beta_1 \Delta\text{ln}(\text{Apps}_{t-1}) + \beta_2 \Delta\text{ln}(\text{Apps}_{t-2}) \\ &= \beta_0 \text{ln}(\text{Apps}_t) + (\beta_1 - \beta_0)\text{ln}(\text{Apps}_{t-1}) + (\beta_2 - \beta_1)\text{ln}(\text{Apps}_{t-2})\end{aligned}$$

The coefficient β_1 captures the relevant congestion in the market — the percentage response of $E[P_{i,t}|I_t]$ to a one percent increase in the relevant mass of innovations, $(\text{Apps}_t + \text{Apps}_{t-1})$. Thus, the null hypothesis that ideas are not scarce corresponds to $\beta_1 = 0$. Through the lens

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[34] An augmented Dickey-Fuller test on $\text{ln}(\text{Apps}_t)$ over the period $1964 - 2014$ yields a test statistic of 1.813 with a 10% critical value of -2.6, whereas the same test on $\Delta\text{ln}(\text{Apps}_t)$ over the period $1965 - 2014$ yields a test statistic of -5.991 with a 1% critical value of -3.587.

[35] If the original innovator has received a patent prior to the application filing date of a rival, then the change in the number of patents by the rival may represent learning or strategic blocking. Also, if the rival observes the patent prior to filing her application then there is the chance that she made a purposeful imitation of an existing innovation rather than an unintentional simultaneous innovation. Furthermore, prior to November 2000 the USPTO did not have a policy of making most patent applications public. Thus, it is reasonable to assume that firms which apply for a patent at time t do not observe pending patent applications filed prior to t and as a result cannot learn from nor purposefully imitate the innovations described in these applications.

of our model, we can interpret β_1 as the elasticity of the probability of securing a monopoly position, $(1 - e^{-\theta})/\theta$, with respect to the mass of R&D firms, μ_t. To see this clearly, observe that we can decompose the average number of successful patent applications as $E[P_{i,t}|I_t] = \Pr(\text{grant}) \times \text{Applications}_{i,t} = ((1 - e^{-\theta})/\theta) \times \text{Applications}_{i,t}$, where $\text{Applications}_{i,t}$ is the number of firm's innovations, i.e. patent applications which are of high enough quality to warrant a patent. As the firm-level quantity and quality of applications is independent of the aggregate number of patent applications, it follows that $\beta_1 = -(1 - e^{-\theta} - \theta e^{-\theta})/(1 - e^{-\theta})$. This relationship allows us to recover θ from the empirical estimates.

Lastly, we include lags of PatsInForce_t to better proxy for the mass of ideas. For consistency, the equation includes the growth rate of PatsInForce_t rather than its natural log.[36] The number of lags, $L_{\text{PatsInForce}}$, is set to six.[37]

6.2 Data and Variables

To construct the sample we start with the NBER Patent Data (Hall *et al.* (2001), HJT henceforth) which consists of $3,279,509$ unique patent-assignee observations and covers all utility patents granted by the USPTO between 1976 and 2006. To mitigate truncation problems, we drop all applications filed prior to 1975 and post 2002. Additionally, after dropping all observations for which information on assignees is not available, the sample size reduces to $2,550,892$ observations. We use this sample to calculate the total number of patents per year of patent application filling date per assignee. The data is then matched with Compustat using the unique company identifier, gvkey. This results in a panel of $11,957$ firms covering $333,193$ observations and $1,061,995$ patents. After dropping observations which have missing firm-level control variables there remain $49,913$ observations on $5,901$ firms and $967,820$ patents. Lastly, we drop 609 observations with only one firm-year observation

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[36] Also, there is strong evidence that $\ln(\text{PatsInForce}_t)$ contains a unit root. Its first difference appears to be $I(0)$, however. As the series appears to have a prominent break in its level, we apply a Zivot-Andres unit root test (Zivot and Andrews, 1992) to $\Delta\ln(\text{PatsInForce}_t)$ for the period of $1965 - 2014$. The minimum $t-$statistic is -5.111 while the 5% critical value is -4.80.

[37] Table 8 in Appendix D shows the results for different values of $L_{\text{PatsInForce}}$. The benchmark estimation does not include lags of $\Delta\ln(\text{PatsInForce}_t)$ higher than sixth, since they are insignificant and do not significantly affect the estimates of β_1.

and $1,335$ firms ($7,783$ observations) which have zero total patents in the sample years. This results in a data set of $41,566$ observations covering $966,688$ patents by $3,957$ firms.

Insert Table 1 here

Table 1 describes the variables we use and their sources. The dependent variable in the regressions, $\text{NumPats}_{i,t}$, is the count of patented inventions by firm and year of patent application. The firm-level controls include current and one-period-lagged natural log of (i) firm's real expenditures in R&D, $\ln(\text{R\&D})_{i,t}$; (ii) company size, measured by the total number of employees, $\ln(\text{Emp})_{i,t}$; (iii) firm's real value of property, plant, and equipment, $\ln(\text{PPE})_{i,t}$; (iv) firm's real net sales, $\ln(\text{Sales})_{i,t}$. We deflate all real variables using the implicit GDP deflator. The aggregate number of patent applications, Apps_t represents all utility patent applications submitted to the USPTO. We take the data on the numbers of patents in force, PatsInForce_t, from the USPTO Historical Patent Data Files (Marco *et al.*, 2015). Tables 6 and 7 in Appendix D provide summary statistics for the key variables, and the correlations of firm-level variables.

6.3 Empirical Results

The benchmark estimates (Table 2, column (1)) provide strong support in favor of the hypothesis that ideas are scarce. The coefficient β_1 is negative and highly significant. Moreover, the results are robust to including further lags of $\Delta\ln(\text{Apps}_t)$ (Table 2). Including the third and fourth lags does not affect the significance of β_1, although its magnitude increases slightly. The results are also robust to changes in the firm-level controls (Table 3). The significance of the coefficient of interest, β_1, does not change and its point estimate varies only slightly. This is true even in column 1 where we do not include any firm-level controls. The results are robust to reasonable changes in the sample period as well (Table 4). In all cases considered β_1 remains negative and significant and its magnitude changes by only a little. Overall, the results suggest the point estimate of the market tightness is around $\theta = 3$ and the lower bound of the 95% confidence interval is around $\theta \geq 1$.

Insert Table 2 here

Insert Table 3 here

Insert Table 4 here

As a last robustness check, we estimate the elasticity by patent category. To this end, we construct the variables NumPatsCat$j_{i,t}$, for $j = 1, \ldots, 6$, which represent the total number of successful patent applications in technological category j filed in year t by firm i. Our decomposition uses the classification in HJT, where the six technological categories are "Chemical", "Computers & Communications", "Drugs & Medical", "Electrical & Electronic", "Mechanical", and "Others". Table 5 presents the results. Column 1 is the benchmark specification and each of the other 6 columns uses NumPatsCat$j_{i,t}$ as the dependent variable. The results provide further support for the hypothesis that research avenues are scarce. The coefficient β_1 is negative and significant at the 5% level for all categories except "Chemical" and "Mechanical".

Insert Table 5 here

It should be noted that, to the best of my knowledge, there is no data on the total number of patent applications filed within a year in a given technological category. Thus, in each specification we have to use the total number of patent applications. Hence, it is plausible that we cannot reject the null hypothesis in these two categories because of data limitation. Another plausible explanation is that most firms in these industries do not rely on patents to secure a monopoly position, but instead resort to other mechanisms such as secrecy or complexity. If this is the case, then patents do not capture most of the innovations made in these categories, and as a consequence most of the congestion. Survey data from Cohen *et al.* (2000), in particular, favors this explanation.[38]

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[38] When asked for what percentage of product innovations are patents considered an effective property rights protection mechanism the average response across all manufacturing firms is 34.83%. The response of firms in the "Food" and "Textiles" industries (both in HJT subcategory 11 of the "Chemical" technological category) is 18.26% and 20%, respectively. The response of "Mineral Products", "Metal", and "Steel" (subcategories 51 and 52 of HJT "Mechanical") is 21.11%, 20%, and 22%, respectively. Thus, if these industries do not find patents as effective it stands to reason that they do not rely heavily on patents.

7 Conclusion

We develop an expanding-variety endogenous growth model in which firms direct their investment towards a specific research avenue (out of a scarce mass of potential R&D projects), but cannot coordinate their efforts. Due to the coordination frictions, the equilibrium number of firms which innovate the exact same idea is a random variable with mean given by the tightness in the market for ideas. Because of the frictions in our model, a fraction of research avenues remain uninnovated. This foregone innovation reduces the growth rate which in turn generates a general equilibrium effect that amplifies the fraction of wasteful simultaneous innovation. Furthermore, these frictions reduce the equilibrium level of research intensity. Implementing the second-best allocation in our model requires the government to impose a tax on R&D investment. This is because the two externalities affected by the frictions in our model, the congestion and learning ones, are such that the incentives to over-invest due to the former dominate the incentives to under-invest due to the later.

Our paper gauges the impact of coordination frictions on the growth rate and welfare. Eliminating the coordination failure in the decentralized economy results in a 13% welfare gain, whereas the gain in the planner's allocation is 16.15%. Furthermore, the majority of the welfare gain is due to eliminating the welfare cost of foregone innovation in the decentralized case and due to eliminating the welfare cost of wasteful simultaneous innovation in the planner's allocation.

We also analyze firm-level data on patents granted between 1976 and 2006. The data strongly favor the hypothesis that research avenues are scarce. Moreover, our empirical analysis provides an estimate of the market tightness and subsequently allows us to discipline the numerical analysis in the paper.

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8 Appendix

8.A Appendix A: Welfare Comparison

We follow Akcigit *et al.* (2016) and compare the welfare difference between any two economies A and B in consumption equivalent terms. In particular, consider the welfare in economy A, W^A, and economy B, W^B, along their BGPs. Suppose at time $t = 0$, both economies start at the same initial position with $N_0^A = N_0^B$. Now, welfare in economy i is given by

$$W^i = \sum_{t=0}^{\infty} \beta^t \ln C_t^i = \ln\left((1+g^i)^{\frac{\beta}{(1-\beta)^2}} C_0^{i\,\frac{1}{1-\beta}} \right)$$

Then, let $\alpha^{A,B}$ measure the fraction with which initial consumption in economy A, C_0^A, must be increased for consumers to have the same welfare as people in economy B. Thus, α is given by

$$\alpha^{A,B} = e^{(1-\beta)(W^B - W^A)} - 1$$

This measure of welfare is used throughout the text. In particular, the welfare gain from eliminating frictions in the DE is given by $\alpha^{DE,CE}$ and the gain from eliminating frictions in the planner's allocation is given by $\alpha^{SB,FB}$.

We decompose the welfare gain from eliminating frictions into the gain from eliminating foregone innovation and the gain from eliminating wasteful innovation. The welfare gain from eliminating foregone innovation in the DE is given by $\alpha^{DE,DEF}$, where DEF is a hypothetical decentralized economy that features no foregone innovation but the same level of wasteful innovation as the DE. In particular, $g^{DEF} = g^c$, $C_0^{DEF}/N_0 = \pi(1+\lambda)/\lambda - \eta\theta^{DEF}(M-1)$, and $\theta^{DEF} = \beta\pi/(\eta(1+g^{DE}-\beta)$. Thus, the welfare cost of wasteful innovation in the decentralized economy is given by $\alpha^{DE,CE} - \alpha^{DE,DEF}$. Similarly, the welfare cost of foregone innovation in the SB is given by $\alpha^{SB,SBF}$, where SBF is a hypothetical allocation in which the planner can assign firms to projects but has to keep the fraction of wasteful innovation as in the SB. In particular, $g^{SBF} = g^{FB}$, $C_0^{SBF}/N_0 = \pi^* - \eta\theta^{SBF}(M-1)$, and $\theta^{SBF} = \theta^{SB}/(1-e^{-\theta^{SB}})$. Thus, the welfare cost of wasteful duplication of effort in the SB is given by $\alpha^{SB,FB} - \alpha^{SB,SBF}$.

8.B Appendix B: Augmented Model

We explore the robustness of the quantitative results from section five in an extension of our baseline model. The economy in this extension features uncertainty in the innovation process and endogenous research effort intensity. In the interest of consistency, the only difference with the baseline model is in the innovation sector. At stage one firms still enter at a cost $\eta > 0$ and at stage two firms still choose a direction for their R&D effort. However, now at stage three entrants choose a research intensity i which affects their probability of successfully innovating. In particular, the cost of exerting effort i is ϕi and the probability of successfully innovating the chosen project is $1 - e^{-\gamma i}$, where $\phi, \gamma > 0$. Stage four is as in the baseline model.

8.B.1 Decentralized Economy

The final good sector and the final stage of the innovation process are as in the baseline model. Hence, $P_t(n) = 1/\lambda$ and $X_t(n) = X$. At stage three, firms choose effort i that maximizes the expected reward from the R&D stage, $R_t(i) \equiv Pr(\text{patent})V_t - \phi i$. Since $Pr(\text{patent}) = Pr(\text{success})Pr(\text{patent}|\text{success}) = (1 - e^{-\gamma i})Pr(\text{patent}|\text{success})$, it follows that the optimal research effort solves

$$Pr(\text{patent}|\text{success})V_t = \frac{\phi}{\gamma}e^{\gamma j} \tag{22}$$

where j is the level of research effort in a symmetric equilibrium. The second stage is analogous to the one in the baseline model, except now there is a chance firms are not successful in innovating. Let the effective market tightness be denoted by $\tilde{\theta}_t \equiv (1 - e^{-\gamma j})\theta_t$. Then, it is straightforward to establish that the number of firms that successfully innovate a particular idea follows a Poisson distribution with mean $\tilde{\theta}_t$.[39] Thus, the probability of receiving a patent conditional on innovating is given by $Pr(\text{patent}|\text{success}) = (1 - e^{-\tilde{\theta}_t})/\tilde{\theta}_t$. Hence,

$$\frac{1 - e^{-\tilde{\theta}_t}}{\tilde{\theta}_t}V_t = \frac{\phi}{\gamma}e^{\gamma j} \tag{23}$$

Free entry implies that $\eta = R(j)$. Thus,

$$\eta + \phi j = \frac{\phi}{\gamma}(e^{\gamma j} - 1) \tag{24}$$

which yields an implicit solution for the equilibrium research intensity j.

The laws of motion for varieties and ideas are analogous to the baseline model, with the

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[39] A proof is available upon request.

only exception that now the probability an idea is innovated is given by $1 - e^{-\tilde{\theta}_t}$. Hence, $\nu/N = M - 1$ and $g = (1 - e^{-\tilde{\theta}})(M - 1)$. Furthermore, consumers face the same problem as in the baseline model. Thus, $V_t = \beta\pi/(1 + g - \beta)$. Using the economy's resource constraint and (24) it follows that along the BGP

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \tilde{\theta}(M-1)\frac{\phi}{\gamma}e^{\gamma j} \tag{25}$$

Finally, using (23) and the expression for V_t, it follows that the effective market tightness solves

$$\frac{\beta\pi}{1 + (1 - e^{-\tilde{\theta}})(M-1) - \beta} = \frac{\phi}{\gamma}e^{\gamma j}\frac{\tilde{\theta}}{1 - e^{-\tilde{\theta}}} \tag{26}$$

8.B.2 Coordination Economy

As in the baseline version of the model, the only difference between the DE and the CE is that at stage two of the innovation process — in the CE, a Walrasian auctioneer coordinates firm's research efforts. Thus, $P_t(n) = 1/\lambda$ and $X_t(n) = X$. Next, as in the DE, the optimal research effort in equilibrium solves

$$Pr(\text{patent}|\text{success})V_t^c = \frac{\phi}{\gamma}e^{\gamma j} \tag{27}$$

Then, let us focus on stage two. Whenever there are $\mu_t < \nu_t$ firms in the R&D sector, the auctioneer assigns a unique idea to each firm and $Pr(\text{patent}|\text{success}) = 1$. When $\theta^c \geq 1$, however, the auctioneer distributes firms to ideas as equally as she can, subject to assigning integer number of firms to each research avenue. In the event that $l \geq 1$ firms successfully innovate the same idea, they each receives the patent with probability $1/l$.[40] For example, if $\theta^c = 8.2$, then a fraction 0.2 of ideas are matched with 9 firms and a fraction 0.8 of ideas are matched with 8 firms. Thus, a fraction 1.8/8.2 of firms face 8 rivals and a fraction 6.4/8.2 face 7. In general, a fraction $\lceil\theta^c\rceil(\theta^c - \lfloor\theta^c\rfloor)/\theta^c$ of firms face $\lfloor\theta^c\rfloor$ rivals and a fraction

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[40] This process of coordination is different from the one in the baseline model where innovation is certain, so the auctioneer can effectively assign patents to entrants. This is because firms do not innovate for sure in our augmented model.

$\lfloor\theta^c\rfloor(\lceil\theta^c\rceil-\theta^c)/\theta^c$ face $\lfloor\theta^c\rfloor-1$, where $\lfloor x\rfloor$ is the largest integer less than x and $\lceil x\rceil$ is the smallest integer larger than x. Hence,

$$\begin{aligned}Pr(\text{patent}|\text{success}) =& \frac{\lfloor\theta^c\rfloor(\lceil\theta^c\rceil-\theta^c)}{\theta^c}\sum_{l=0}^{\lfloor\theta^c\rfloor-1}\binom{\lfloor\theta^c\rfloor-1}{l}(1-e^{-\gamma j})^l e^{-\gamma j(\lfloor\theta^c\rfloor-1-l)}\frac{1}{l+1}\\ &+\frac{\lceil\theta^c\rceil(\theta^c-\lfloor\theta^c\rfloor)}{\theta^c}\sum_{l=0}^{\lfloor\theta^c\rfloor}\binom{\lfloor\theta^c\rfloor}{l}(1-e^{-\gamma j})^l e^{-\gamma j(\lfloor\theta^c\rfloor-l)}\frac{1}{l+1}\\ =&\frac{\lceil\theta^c\rceil-\theta^c}{(1-e^{-\gamma j})\theta^c}(1-e^{-\gamma j\lfloor\theta^c\rfloor})+\frac{\theta^c-\lfloor\theta^c\rfloor}{(1-e^{-\gamma j})\theta^c}(1-e^{-\gamma j\lceil\theta^c\rceil}) \end{aligned} \tag{28}$$

The case relevant for out numerical exercise is $\theta^c \geq 1$, so we restrict our attention to it.

Next, free entry and (27) imply that

$$\eta+\phi j^c = \frac{\phi}{\gamma}(e^{\gamma j^c}-1) \tag{29}$$

which yields the same equilibrium research effort as in the DE.

The laws of motion for varieties and ideas is the same as in the DE with the exception that now the probability an idea is innovated is given by $(\theta^c-\lfloor\theta^c\rfloor)(1-e^{-\gamma j^c\lceil\theta^c\rceil})+(\lceil\theta^c\rceil-\theta^c)(1-e^{-\gamma j^c\lfloor\theta^c\rfloor})$. The consumer's optimization problem yields $V_t^c=\beta\pi/(1-\beta+g^c)$, where $g^c=(\theta^c-\lfloor\theta^c\rfloor)(1-e^{-\gamma j^c\lceil\theta^c\rceil})+(\lceil\theta^c\rceil-\theta^c)(1-e^{-\gamma j^c\lfloor\theta^c\rfloor})(M-1)$. Lastly, the resource constraint and the expression for the value of holding a patent yield

$$\left(\frac{C}{N}\right)^c = \frac{1+\lambda}{\lambda}\pi-\theta^c(M-1)(\eta+\phi j^c) \tag{30}$$

$$\left(\frac{\lceil\theta^c\rceil-\theta^c}{(1-e^{-\gamma j})\theta^c}(1-e^{-\gamma j\lfloor\theta^c\rfloor})+\frac{\theta^c-\lfloor\theta^c\rfloor}{(1-e^{-\gamma j})\theta^c}(1-e^{-\gamma j\lceil\theta^c\rceil})\right)^{-1}\frac{\phi}{\gamma}e^{\gamma j^c} = \frac{\beta\pi}{1-\beta+g^c} \tag{31}$$

8.B.3 Second-Best Allocation

Analogously to the baseline model, the planner solves

$$\max_{\{C_t, X_t, \tilde{\theta}_t, N_t, \nu_t, j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{32}$$

$$AL^{1-\lambda} N_t X_t^{\lambda} = N_t X_t + C_t + \tilde{\theta}_t \nu_t \frac{\eta + \phi j}{1 - e^{-\gamma j}} \tag{33}$$

$$N_{t+1} = N_t + (1 - e^{-\tilde{\theta}_t}) \nu_t \tag{34}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\tilde{\theta}_t})(M - 1)\nu_t \tag{35}$$

The first order condition with respect to X_t yields $X_t = X^*$ as in the baseline model. Furthermore, the first order condition with respect to the research effort, j, yields

$$\eta + \phi j^{SB} = \frac{\phi}{\gamma}(e^{\gamma j^{SB}} - 1) \tag{36}$$

which is the same level of research effort as in the DE. Let $\tilde{\eta} \equiv \phi e^{\gamma j^{SB}} / \gamma$, hence, the rest of the first order conditions are

$$[C_t]: \quad \beta \frac{C_t}{C_{t+1}} = \frac{\tilde{\phi}_{t+1}}{\tilde{\phi}_t} \tag{37}$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \tilde{\phi}_{t+1} \pi^* \tag{38}$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1}\Big(e^{-\tilde{\theta}_{t+1}} + (1 - e^{-\tilde{\theta}_{t+1}})M\Big) + h_{t+1}(1 - e^{-\tilde{\theta}_{t+1}}) - \tilde{\phi}_{t+1} \tilde{\eta} \tilde{\theta}_{t+1} \tag{39}$$

$$[\tilde{\theta}_t]: \quad \tilde{\eta} = e^{-\tilde{\theta}_t}\Big(\frac{h_t}{\tilde{\phi}_t} + \frac{\lambda_t}{\tilde{\phi}_t}(M - 1)\Big) \tag{40}$$

where $\tilde{\phi}_t$, h_t, and λ_t and the multipliers associated with (33), (34), and (35), respectively. Thus, the planner's problem reduces to the one in the baseline model. Hence,

$$\left(\frac{\nu}{N}\right)^{SB} = M - 1 \tag{41}$$

$$\left(\frac{C}{N}\right)^{SB} = \pi^* - \tilde{\eta}\tilde{\theta}^{SB}(M-1) \tag{42}$$

$$1 + (1 - e^{-\tilde{\theta}^{SB}})(M-1) = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\tilde{\theta}^{SB}} + (1 - e^{-\tilde{\theta}^{SB}} - \tilde{\theta}^{SB}e^{-\tilde{\theta}^{SB}})(M-1)\Big) \tag{43}$$

8.B.4 First-Best Allocation

Without loss of generality we impose symmetry in the production of varieties and the research effort intensity. Observe that by symmetry the planner assigns the same number of firms per idea. Hence, the probability an idea is innovated is given by $1 - e^{-\gamma\tilde{j}_t}$, where $\tilde{j}_t = j\theta_t$ is the effective research effort per idea. Now, the planner can achieve an additional unit of effective research by either increasing θ_t by $1/j$ units or increasing j by $1/\theta_t$ units. Furthermore, the cost of the former is $\nu_t\phi + \nu_t\eta/j$ units of the final good and the cost of the latter is $\nu_t\phi$. Thus, it is always cheaper to induce higher effective research effort by increasing the research intensity, j. Hence, $\theta^{FB} = 1$. Then, the planner's problem reduces to

$$\max_{\{C_t, X_t, N_t, \nu_t, j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t \tag{44}$$

$$AL^{1-\lambda}N_tX_t^{\lambda} = N_tX_t + C_t + \nu_t\eta + \nu_t\phi j \tag{45}$$

$$N_{t+1} = N_t + (1 - e^{-\gamma j})\nu_t \tag{46}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\gamma j})(M-1)\nu_t \tag{47}$$

The first order condition for X_t implies that the level of intermediate varieties is still given by X^*. Taking the rest of the first order conditions and applying straightforward algebra yields

$$\left(\frac{C}{N}\right)^{FB} = \pi^* - (\eta + \phi j^{FB})(M-1) \tag{48}$$

$$\frac{\phi}{\gamma} e^{\gamma j^{FB}} = \frac{\beta\pi^*}{1-\beta+g^{FB}} + \frac{\beta^2\pi^*(1-e^{-\gamma j^{FB}})(M-1)}{(1-\beta+g^{FB})(1+g^{FB})(1-\beta)} - \frac{\beta(\eta+\phi j^{FB})(M-1)}{(1+g^{FB})(1-\beta)} \tag{49}$$

where $g^{FB} = (1-e^{-\gamma j^{FB}})(M-1)$.

8.B.5 Numerical Exercise

As in the baseline case, we calibrate the model at annual frequency, so the discount factor is set at $\beta = 0.95$. Furthermore, we normalize $\gamma = L = A = 1$. To calibrate η, M, and λ we use the same three moments as in the baseline case. In addition, we set the elasticity of firm-level output with respect to R&D investment at 0.05. This value is consistent with most firm-level estimates for the U.S.[41] The elasticity in our model is given by $\gamma j e^{-\gamma j}/(1-e^{-\gamma j})$, hence, the equilibrium research effort of firms is $j = 4.5139$.[42] Setting the R&D share of GDP to its empirical value yields $\lambda = 0.8516$, as in the baseline model. Next, the fraction of patents to patent applications is $(1-e^{-\tilde{\theta}})/\tilde{\theta}$. Matching this expression to its empirical counterpart yields $\tilde{\theta} = 1.0876$. Hence, $\theta = 1.0997$. Lastly, setting $g = (1-e^{-\tilde{\theta}})(M-1) = 1.7546\%$ and using (24), (26) yields $M = 1.0265$, $\eta = 0.1611$, and $\phi = 0.0019$. The resulting welfare cost of coordination frictions in the DE is 12.76% and in the SB is 15.97%.

Insert Figure 2 here

As in the baseline model, we explore the robustness of our quantitative results by varying the effective market tightness in the interval $\tilde{\theta} \in [1, 1.8]$.[43] The magnitude of the welfare costs is virtually the same as in the baseline model for all considered values of $\tilde{\theta}$ (Figure 2).

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[41] For a survey see Hall *et al.* (2010).

[42] In our model firm-level output corresponds to $O(\tilde{c}) \equiv (1-e^{-\gamma\tilde{c}/\phi})(1-e^{-\tilde{\theta}})V_t/\tilde{\theta}$, where $\tilde{c} \equiv \phi j$ is the firm's R&D investment.

[43] In our augmented model the two alternative calibration strategies set the bounds on $\tilde{\theta}$. In particular the return of R&D is now given by $\partial Y_{t+1}/\partial R_t = g\tilde{\theta}e^{-\tilde{\theta}}/((1-e^{-\tilde{\theta}})\eta\mu_t/Y_t)$ and the elasticity of the probability of securing a monopoly position with respect to the mass of R&D firms is $-(1-e^{-\tilde{\theta}}-\tilde{\theta}e^{-\tilde{\theta}})/(1-e^{-\tilde{\theta}})$.

8.C Appendix C: Proofs Omitted from the Text

Proof of Proposition 1:

Proof. We follow previous literature (see, for example, Julien *et al.* (2000)) and threat the mass of entrants, μ_t and ideas, ν_t, as finite. Then the resulting equilibrium outcome is evaluated at the limit as $\mu_t, \nu_t \to \infty$ (keeping θ_t constant), so as to characterize the behavior in a market with continuum of firms and ideas.

First, by assumption, the firm's probability of securing a monopoly position given that there are exactly n rivals, $Pr(\text{monopoly}|n) = 1/(n+1)$. In a symmetric equilibrium all firms place the same probability s_i of directing their effort towards a particular idea i. Then, the chance that a firm would face exactly n rivals is

$$Pr(n) = \binom{\mu_t - 1}{n} s_i^n (1-s_i)^{\mu_t - 1 - n}$$

Hence, the probability of securing a monopoly position is given by

$$Pr(\text{monopoly}) = \sum_{n=0}^{\mu_t - 1} Pr(\text{monopoly}|n) P(n) = \sum_{n=0}^{\mu_t - 1} \binom{\mu_t - 1}{n} s_i^n (1-s_i)^{\mu_t - 1 - n} \frac{1}{n+1} =$$

$$= \frac{1}{\mu_t} \sum_{n=0}^{\mu_t - 1} \binom{\mu_t}{n+1} s_i^n (1-s_i)^{\mu_t - 1 - n} = \frac{1}{\mu_t s_i} \left(\sum_{n=0}^{\mu_t} \binom{\mu_t}{n} s_i^n (1-s_i)^{\mu_t - n} - (1-s_i)^{\mu_t} \right) = \frac{1-(1-s_i)^{\mu_t}}{\mu_t s_i}$$

Next, we show that $s_k = s_j$ for all $k, j \in \nu_t$. Suppose not. Then, there exists some k, j such that $s_k > s_j$. But for any $i \in \nu_t$, we have that

$$\frac{\partial Pr(\text{monopoly})}{\partial s_i} = \frac{\mu_t^2 s_i (1-s_i)^{\mu_t - 1} - \mu_t [1-(1-s_i)^{\mu_t}]}{(\mu_t s_i)^2}$$

For any $s_i \in (0,1)$, it follows that $Pr(\text{monopoly})$ is decreasing in s_i if and only if $(1 - s_i)^{\mu_t - 1} < Pr(\text{monopoly})$ which clearly holds since $\mu_t \geq 2$. Now, for $s_i = 1$, we have that $\partial Pr(\text{monopoly})/\partial s_i = -1/\mu_t < 0$. Furthermore, it is easy to see that $\lim_{s_i \to 0} \partial Pr(\text{monopoly})/\partial s_i = -(\mu_t - 1)/2 < 0$. Hence, $Pr(\text{monopoly})$ is decreasing in s_i everywhere in its domain. Then, $s_k > s_j$ implies that $Pr_k(\text{monopoly}) < Pr(\text{monopoly})_j$, which then implies that

$Pr_k(\text{monopoly})V_{k,t} < Pr_j(\text{monopoly})V_{j,t}$ since all varieties are equally profitable. Thus, $s_k > s_j$ cannot be an equilibrium. Hence, we must have $s_i = s_j$ for all $i, j \in \nu_t$. Thus, $s_i = 1/\nu_t$.

Then, it follows that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) = \binom{\mu_t}{n}\Big(\frac{1}{\nu_t}\Big)^n\Big(1 - \frac{1}{\nu_t}\Big)^{\mu_t - n}$$

Taking the limit as $\mu_t, \nu_t \to \infty$ (keeping the ratio θ_t constant) we get that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) \to \frac{\theta_t^n e^{-\theta_t}}{n!}$$

■

Proof of Proposition 2:

Proof. First, let us prove the following lemma

Lemma 1. *The magnitude of the congestion externality is larger than that of the learning externality.*

Proof. First, we can decompose the difference between the planner's valuation of the benefit of entry and the firm's valuation of this benefit. At the SB this difference is given by

$$\mathcal{A} + \mathcal{L} + \mathcal{C} = \eta - \Big(\frac{1 - e^{-\theta^{SB}}}{\theta^{SB}}\Big)V^{SB} \tag{50}$$

where $\mathcal{A}$, $\mathcal{L}$, and $\mathcal{C}$ denote the appropriability, learning, and congestion externalities; $V^{SB} := \beta\pi/(e^{-\theta^{SB}} + (1 - e^{-\theta^{SB}})M - \beta)$ is the value of having a monopoly position at the second best level of the market tightness. The right hand side of (50) gives the difference between the planner's valuation of the benefit of entry, η, and the firm's, V^{SB} times the probability of securing a patent. Then, one can decompose the sum of the three externalities in the following manner

$$\mathcal{A} := \Big(\Big(\frac{h}{\phi}\Big)^{SB} - V^{SB}\Big)\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) \tag{51}$$

$$\mathcal{L} := \Big(\frac{\lambda}{\phi}\Big)^{SB}\Big(e^{-\theta^{SB}}(M-1)\Big) \tag{52}$$

$$\mathcal{C} := -\Big(\frac{h}{\phi}\Big)^{SB}\left(\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) - e^{-\theta^{SB}}\right) \tag{53}$$

Thus, $\mathcal{A}$ is the measure of how much more would the planner value entry than the firm if the appropriability externality was the only one in the model. $\mathcal{L}$ and $\mathcal{C}$ measure the same difference if the only externality in the model was learning and congestion, respectively.

From equations (52) and (53), it follows that the magnitude of the congestion externality is larger than that of the learning externality if and only if

$$\Big(\frac{h}{\phi}\Big)^{SB}\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) > e^{-\theta^{SB}}\left(\Big(\frac{h}{\phi}\Big)^{SB} + \Big(\frac{\lambda}{\phi}\Big)^{SB}(M-1)\right) \tag{54}$$

From equations (13) and (14), it then follows that (54) holds if and only if

$$\frac{(1-e^{-\theta^{SB}})\beta\pi^*}{\theta^{SB}\eta} > e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta \tag{55}$$

Next, from the planner's solution, (18), it follows that $|\mathcal{C}| > \mathcal{L}$ if and only if $\pi^* - \eta\theta^{SB}(M-1) > 0$. But this has to hold, from equation (17), as the SB must feature $C_t > 0$.

■

Now, let us turn back to the problem of implementing the SB. The government imposes a tax on R&D activities at a rate τ and subsidizes the purchase of intermediate varieties at a rate s. Furthermore, it keeps a balanced budget through the means of lump-sum transfers to households in the amount T_t. Thus, the government's budget constraint is given by

$$T_t = \int_0^{N_t} sP_t(n)X_t(n)dn - \tau\eta\mu_t$$

The final good firm chooses labor and intermediate inputs to maximize profits, now given by $Y_t - w_t L - \int_0^{N_t} (1-s) P_t(n) X_t(n) dn$. The first order conditions yield the same labor demand equation as in the DE, $w_t = (1-\lambda) Y_t / L$, and an inverse demand function for intermediaries given by $P_t(n) = \lambda A L^{1-\lambda} X_t^{\lambda-1}(n)/(1-s)$.

At stage three of the innovation process, the monopolist faces an analogous problem as in the DE. The only difference now is in the inverse demand function. Hence, in equilibrium, $P = 1/\lambda$, $X = [A\lambda^2/(1-s)]^{1/(1-\lambda)} L$, $\pi = (1-\lambda)X/\lambda$, $Y_t = [A(\lambda^2/(1-s))^\lambda]^{1/(1-\lambda)} L N_t$.

As in the economy without government intervention, all ideas are equally profitable, so the matching technology is as in the DE. The free entry condition is now given by

$$\eta(1+\tau) = \frac{1-e^{-\theta_t}}{\theta_t} V_t$$

where the value of the monopoly position, V_t, is defined as in the DE.

The laws of motion for ideas and varieties, and the Euler equation are as in the DE. Hence, the value of the monopoly position is still given by (3). Furthermore, the resource constraint is still given by (4).

Along the BGP, we still have that $\nu_t / N_t = M - 1$, as the laws of motion for ideas and varieties are as in the DE. Thus, from the resource constraint, (4) it follows that

$$\frac{C}{N} = \frac{1-s-\lambda^2}{(1-\lambda)\lambda}\pi - \eta\theta(M-1)$$

Next, (3), the law of motion for ideas, and the free entry condition imply that

$$1 + (1-e^{-\theta})(M-1) = \beta\Big(1 + \frac{\pi}{\eta(1+\tau)}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big)$$

Then, setting $s = s^{SB}$ implies that $\pi = \pi^*$ and setting $\tau = \tau^*$ implies that $\theta = \theta^{SB}$. Thus, $C/N = (C/N)^{SB}$. Furthermore, τ^* is given by

$$\tau^* = \frac{\beta\pi^*(1-e^{-\theta^{SB}})}{\eta\theta^{SB}(e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta)} - 1$$

To see that the optimal tax rate is positive because the congestion externality dominates the learning one, observe that

$$\begin{aligned}
-\mathcal{C}-\mathcal{L} = & \left(\frac{h}{\phi}\right)^{SB}\left(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\right) - e^{-\theta^{SB}}\left(\left(\frac{h}{\phi}\right)^{SB} + \left(\frac{\lambda}{\phi}\right)^{SB}(M-1)\right) \\
= & \left(\frac{h}{\phi}\right)^{SB}\left(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\right) - \eta \\
= & \eta\tau^*
\end{aligned}$$

where the first equality follows from (13) and the second equality from (14) and the fact that the SB growth rate is given by $(1-e^{-\theta^{SB}})(M-1)$. Hence, $|\mathcal{C}| > |\mathcal{L}| \Rightarrow \tau^* > 0$.

■

Proof of Proposition 3:

Proof. First, let us explicitly characterize the environment in the CE. The only difference to the DE is at the second stage in the innovation process. Coordination is achieved through the means of a centralized allocation of firms to ideas. In particular, upon entry, a Walrasian auctioneer directs firms' research efforts and assigns patents in the following way. If $\mu_t \leq v_t$, then each firm is directed towards a distinct project and each firm receives a patent. If $\mu_t > \nu_t$, the auctioneer chooses ν_t firms at random, assigns each a distinct project, and grants each a patent over the corresponding variety. The rest $\mu_t - \nu_t$ firms are randomly assigned a project, but none of them receives a patent.

The assumption we have placed on the parameter vales ensures that firms find all research avenues profitable. Hence, in equilibrium, all ideas are innovated, i.e. $\mu_t \geq \nu_t$, and each firm secures a patent with probability $Pr(\text{monopoly}) = 1/\theta_t$. Hence, the laws of motion for ideas and varieties are given by

$$\nu_{t+1} = M\nu_t \tag{56}$$

$$N_{t+1} = N_t + \nu_t \tag{57}$$

Since the final good sector and the intermediate varieties production technology are as in the DE, it follows that in equilibrium it is still the case that $P_t(n) = 1/\lambda$, $X = (\lambda^2 A)^{1/(1-\lambda)} L$, $Y_t = (\lambda^{2\lambda} A)^{1/(1-\lambda)} L N_t$, $\pi = X(1-\lambda)/\lambda$, $V_t^c = \sum_{i=t+1}^{\infty} d_{it}\pi$. As all ideas are equally productive, the free entry condition is now given by

$$\eta = \frac{1}{\theta_t} V_t^c \tag{58}$$

Moreover, consumers face the same problem as in the DE, so the Euler equation is analogous to (3):

$$V_t^c = \beta \frac{C_t}{C_{t+1}} \left(\pi + V_{t+1}^c \right) \tag{59}$$

Furthermore, the resource constraint is still given by (4).

One can establish in a manner analogous to that in the DE case that have $g_Y = g_C = g_N = g_\mu = g_\nu$. However, now from the law of motion for ideas, it follows that $g_\nu = M - 1$.

Next, using the laws of motion for ideas and varieties, it follows that along the BGP we still have, $\nu/N = M - 1$. Furthermore, from the resource constraint, it follows that

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \eta\theta^c(M-1) \tag{60}$$

Lastly, using the free entry condition and the Euler equation, it follows that the market tightness is given by

$$\theta^c = \frac{\beta\pi}{\eta(M-\beta)} \tag{61}$$

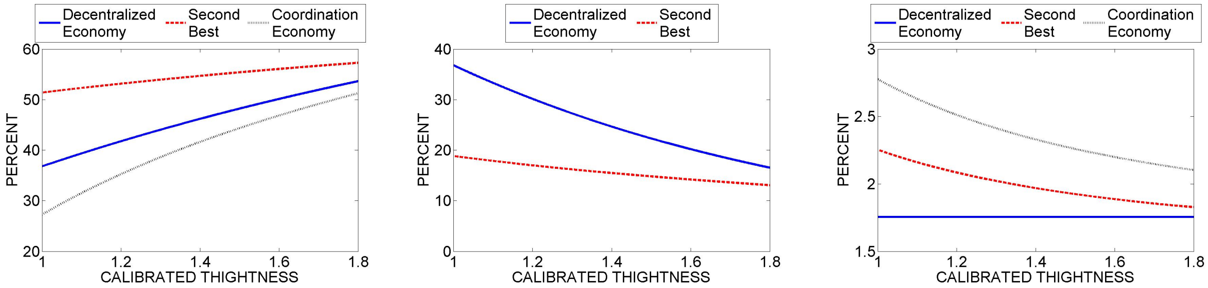
Next, we can compare the percent of wasteful innovations in the two economies. In the CE there are μ_t innovations and ν_t of those are beneficial. Hence, $\omega^c = 1 - 1/\theta^c$. Then, observe that $V_t > V_t^c$ because the DE growth rate, $(1 - e^{-\theta})(M-1)$, is always smaller than the CE growth rate, $M - 1$. Then, using the two free entry conditions, it follows that $\omega = 1 - \eta/V_t > 1 - \eta/V_t^c = \omega^c$.

Next, from (61) it follows that

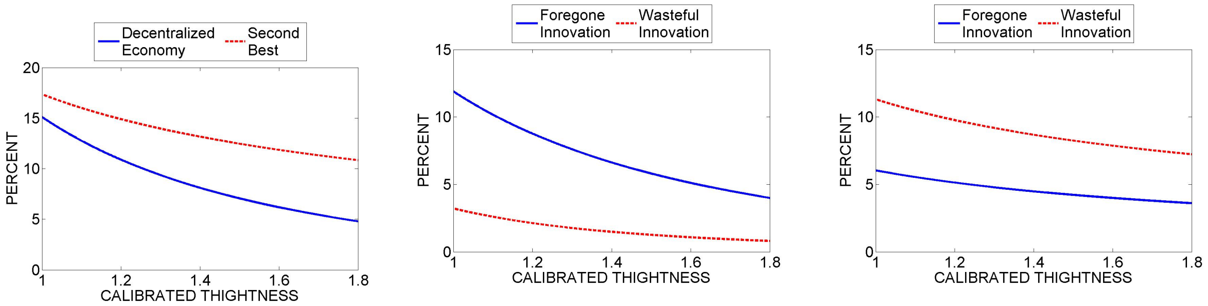
$$\frac{\theta^c}{1-e^{-\theta}} = \frac{\beta\pi}{\eta(M-\beta)(1-e^{-\theta})} > \frac{\beta\pi}{\eta(1+(1-e^{-\theta})(M-1)-\beta)} = \frac{\theta}{1-e^{-\theta}} \tag{62}$$

where the inequality follows because $\beta < 1 \Rightarrow 1+(1-e^{-\theta})(M-1)-\beta > (M-\beta)(1-e^{-\theta})$. Hence, $\theta^c > \theta$. ■

8.D Appendix D: Figures and Tables from the Text



(a) Wasteful Innovation (b) Foregone Innovation (c) Growth Rate



(d) Welfare Gain (e) Welfare Gain: Decentralized Economy (f) Welfare Gain: Planner's Allocation

Figure 1: Alternative Calibration Values

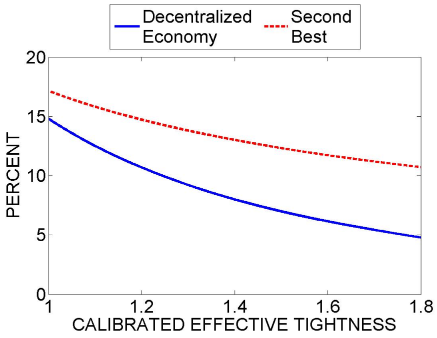


Figure 2: Welfare Gain in the Augmented Model

Table 1: Variable Description and Sources

Variable	Description	Level	Data Source
Endogenous Variables			
$\text{NumPats}_{i,t}$	Count of patented inventions by application year	Firm	NBER Patent Data
Exogenous Variables			
Deflator_t	Implicit GDP deflator	Aggregate	U.S. Bureau of Economic Analysis (NIPA Table 1.1.9)
Apps_t	Total utility patent applications	Aggregate	USPTO (U.S. Patent Statistics Chart Calendar Years $1963-2015$)
PatsInForce_t	Number of patents in force	Aggregate	USPTO (Historical Patent Data Files)
$\text{NomR\&D}_{i,t}$	Nominal private R&D expenditures	Firm	Compustat
$\text{NomSales}_{i,t}$	Nominal net sales	Firm	Compustat
$\text{Emp}_{i,t}$	Number of employees	Firm	Compustat
$\text{NomPPE}_{i,t}$	Nominal gross value of property, plant, and equipment	Firm	Compustat
$\text{R\&D}_{i,t}$	(Real) Private R&D expenditures	Firm	$100\times\text{NomR\&D}_{i,t}/\text{Deflator}_t$
$\text{Sales}_{i,t}$	(Real) Net sales	Firm	$100\times\text{NomSales}_{i,t}/\text{Deflator}_t$
$\text{PPE}_{i,t}$	(Real) Gross value of property, plant, and equipment	Firm	$100\times\text{NomPPE}_{i,t}/\text{Deflator}_t$

Table 2: $\Delta\ln(\text{Apps}_t)$ Lags table

	(1)	(2)	(3)
$\Delta\ln(\text{Apps}_{t-1})$	-0.875^{***}	-0.971^{***}	-1.001^{***}
	(0.216)	(0.271)	(0.278)
$\Delta\ln(\text{Apps}_{t-3})$	No	Yes	Yes
$\Delta\ln(\text{Apps}_{t-4})$	No	No	Yes
Fixed Effects	Yes	Yes	Yes
Trend	Yes	Yes	Yes
Applications	Yes	Yes	Yes
Controls	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes
N(Firms)	$3,957$	$3,957$	$3,957$
N	$41,566$	$41,566$	$41,566$
χ^2	704.3	732.1	772.2

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. PatsInForce indicates the inclusion of the growth rate of patents in force and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Firm-Level Controls Table

	(1)	(2)	(3)	(4)	(5)
$\Delta\ln(\text{Apps}_{t-1})$	-0.863^{***}	-0.873^{***}	-0.875^{***}	-0.752^{***}	-0.716^{***}
	(0.228)	(0.220)	(0.216)	(0.203)	(0.204)
$\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$	No	Yes	Yes	Yes	Yes
$\ln(\text{R\&D}_{i,t-1})$, $\ln(\text{Sales}_{i,t-1})$, $\ln(\text{Emp}_{i,t-1})$, $\ln(\text{PPE}_{i,t-1})$	No	No	Yes	Yes	Yes
$\ln(\text{R\&D}_{i,t-2})$, $\ln(\text{Sales}_{i,t-2})$, $\ln(\text{Emp}_{i,t-2})$, $\ln(\text{PPE}_{i,t-2})$	No	No	No	Yes	Yes
$\ln(\text{R\&D}_{i,t-3})$, $\ln(\text{Sales}_{i,t-3})$, $\ln(\text{Emp}_{i,t-3})$, $\ln(\text{PPE}_{i,t-3})$	No	No	No	No	Yes
Fixed Effects	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes	Yes	Yes
N(Firms)	$6,247$	$4,333$	$3,957$	$3,508$	$3,089$
N	$173,778$	$46,234$	$41,566$	$37,232$	$33,585$
χ^2	354.5	577.0	704.3	706.5	788.7

PatsInForce indicates the inclusion of the growth rate of patents in force and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 4: Sample Period Table

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta\ln(\text{Apps}_{t-1})$	-0.875^{***}	-0.776^{***}	-0.884^{***}	-0.839^{***}	-0.747^{***}	-0.777^{***}	-0.735^{***}
	(0.216)	(0.221)	(0.229)	(0.214)	(0.214)	(0.220)	(0.220)
Sample	$1975-2002$	$1975-2001$	$1975-2000$	$1976-2002$	$1977-2002$	$1976-2001$	$1977-2000$
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N(Firms)	$3,957$	$3,776$	$3,541$	$3,879$	$3,790$	$3,698$	$3,374$
N	$41,566$	$39,278$	$37,038$	$40,187$	$38,804$	$37,901$	$34,286$
χ^2	704.3	664.8	682.3	680.3	660.9	662.2	673.8

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. PatsInForce indicates the inclusion of the growth rate of patents in force and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 5: Patents By Technological Category

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\Delta\ln(\text{Apps}_{t-1})$	-0.875^{***}	0.0711	-1.946^{***}	-0.936^{*}	-0.578^{*}	-0.245	-0.586^{*}
	(0.216)	(0.221)	(0.353)	(0.447)	(0.287)	(0.230)	(0.278)
HJT Category	All	Chemical	Computers & Communications	Drugs & Medical	Electrical & Electronic	Mechanical	Others
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes
PatsInForce	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N(Firms)	$3,957$	$1,734$	$1,932$	$1,215$	$1,898$	$1,813$	$1,869$
N	$41,566$	$22,397$	$22,463$	$14,632$	$24,726$	$24,270$	$24,586$
χ^2	704.3	226.4	1036.9	285.5	502.7	315.0	235.9

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. HJT designates the technological category as defined in Hall *et al.* (2001). PatsInForce indicates the inclusion of the growth rate of patents in force in the corresponding technological category and its first six lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. $\Delta\ln(\text{Apps}_{t-3})$ and $\Delta\ln(\text{Apps}_{t-4})$ are also included in the estimation of column (4). Robust standard errors, clustered by firm are in parentheses.
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

8.E Appendix E: Supplementary Tables

Table 6: Descriptive Statistics

Variable	N	N(Firms)	Mean	SD	Min	Median	Max
			Aggregate Variables				
$\Delta\ln(\text{Apps}_t)$	28	N/A	0.042	0.052	-0.084	0.045	0.121
$\Delta\ln(\text{PatsInForce}_t)$	28	N/A	0.013	0.024	-0.017	0.008	0.072
			Firm-Level Variables				
$\text{NumPats}_{i,t}$	41,566	3,957	23.257	120.708	0	1	4,344
$\ln(\text{R\&D}_{i,t})$	41,566	3,957	2.112	2.224	-6.468	2.041	9.359
$\ln(\text{Sales}_{i,t})$	41,566	3,957	5.226	2.546	-6.283	5.160	12.356
$\ln(\text{Emp}_{i,t})$	41,566	3,957	0.103	2.257	-6.908	-0.023	6.809
$\ln(\text{PPE}_{i,t})$	41,566	3,957	4.394	2.595	-3.468	4.131	12.981

Sample period: 1975 − 2002

Table 7: Correlations

Variables	$\text{NumPats}_{i,t}$	$\ln(\text{R\&D}_{i,t})$	$\ln(\text{R\&D}_{i,t-1})$	$\ln(\text{Sales}_{i,t})$	$\ln(\text{Sales}_{i,t-1})$	$\ln(\text{Emp}_{i,t})$	$\ln(\text{Emp}_{i,t-1})$	$\ln(\text{PPE}_{i,t})$
$\text{NumPats}_{i,t}$	1.000							
$\ln(\text{R\&D}_{i,t})$	0.378	1.000						
$\ln(\text{R\&D}_{i,t-1})$	0.382	0.977	1.000					
$\ln(\text{Sales}_{i,t})$	0.321	0.743	0.749	1.000				
$\ln(\text{Sales}_{i,t-1})$	0.317	0.726	0.741	0.984	1.000			
$\ln(\text{Emp}_{i,t})$	0.323	0.748	0.753	0.960	0.951	1.000		
$\ln(\text{Emp}_{i,t-1})$	0.322	0.738	0.754	0.955	0.958	0.991	1.000	
$\ln(\text{PPE}_{i,t})$	0.337	0.774	0.786	0.941	0.937	0.947	0.946	1.000
$\ln(\text{PPE}_{i,t-1})$	0.334	0.757	0.778	0.935	0.940	0.940	0.949	0.992

Sample period: $1975 - 2002$. $N = 41,566$. All values are significant at the 0.1% level.

Table 8: $\Delta\ln(\text{PatsInForce}_t)$ Lags table

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
$\Delta\ln(\text{Apps}_{t-1})$	-0.436^{**}	-0.991^{***}	-1.026^{***}	-1.080^{***}	-1.081^{***}	-1.015^{***}	-1.007^{***}	-0.875^{***}	-0.894^{***}	-0.868^{***}	-0.855^{***}
	(0.168)	(0.200)	(0.213)	(0.223)	(0.228)	(0.233)	(0.232)	(0.216)	(0.230)	(0.217)	(0.221)
$\Delta\ln(\text{PatsInForce}_t)$		2.618^{**}	3.167^{***}	3.400^{***}	3.396^{***}	3.363^{***}	4.121^{***}	4.158^{***}	4.193^{***}	4.262^{***}	4.515^{***}
		(0.867)	(0.843)	(0.876)	(0.859)	(0.862)	(0.918)	(0.923)	(0.917)	(0.953)	(0.922)
$\Delta\ln(\text{PatsInForce}_{t-1})$			-0.684	2.007^{***}	2.017^{***}	1.960^{***}	0.265	0.222	0.236	0.106	-0.105
			(0.718)	(0.528)	(0.554)	(0.569)	(0.640)	(0.651)	(0.680)	(0.621)	(0.723)
$\Delta\ln(\text{PatsInForce}_{t-2})$				-3.364^{***}	-3.357^{***}	-3.989^{***}	-3.830^{***}	-4.069^{***}	-4.091^{***}	-4.045^{***}	-4.083^{***}
				(0.822)	(0.788)	(0.800)	(0.785)	(0.840)	(0.802)	(0.783)	(0.773)
$\Delta\ln(\text{PatsInForce}_{t-3})$					-0.0229	-0.182	-1.040^{**}	-1.146^{**}	-1.149^{**}	-1.154^{**}	-1.101^{**}
					(0.380)	(0.354)	(0.379)	(0.390)	(0.387)	(0.389)	(0.410)
$\Delta\ln(\text{PatsInForce}_{t-4})$						1.828^{**}	2.789^{***}	2.427^{***}	2.391^{***}	2.379^{**}	2.340^{**}
						(0.607)	(0.684)	(0.654)	(0.726)	(0.733)	(0.729)
$\Delta\ln(\text{PatsInForce}_{t-5})$							5.885^{***}	6.134^{***}	6.140^{***}	6.189^{***}	6.250^{***}
							(1.041)	(1.114)	(1.099)	(1.103)	(1.089)
$\Delta\ln(\text{PatsInForce}_{t-6})$								3.096^{***}	3.079^{***}	2.940^{***}	2.911^{**}
								(0.881)	(0.931)	(0.887)	(0.893)
$\Delta\ln(\text{PatsInForce}_{t-7})$									-0.171	-0.260	-0.338
									(0.742)	(0.730)	(0.751)
$\Delta\ln(\text{PatsInForce}_{t-8})$										0.835	1.041
										(0.863)	(0.714)
$\Delta\ln(\text{PatsInForce}_{t-9})$											-0.642
											(0.717)
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Trend	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Applications	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
N(Firms)	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957	3,957
N	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566	41,566
χ^2	495.5	532.6	567.7	603.1	607.4	655.8	682.9	704.3	707.8	857.4	860.9

Controls indicates the inclusion of $\ln(\text{R\&D}_{i,t})$, $\ln(\text{Sales}_{i,t})$, $\ln(\text{Emp}_{i,t})$, $\ln(\text{PPE}_{i,t})$, and their first lags. Applications indicates the inclusion of $\Delta\ln(\text{Apps}_t)$ and $\Delta\ln(\text{Apps}_{t-2})$. Robust standard errors, clustered by firm are in parentheses.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$