# The Patent System as a Tool for Eroding Market Power

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**Abstract**

The conventional viewpoint on the patent system is that it allocates market power in order to stimulate disclosure of information and create incentives for firms to innovate. This paper develops a dynamic equilibrium search model to show that, in sharp contrast to this traditional view, the patent system can erode, rather than allocate market power. This result can be obtained, regardless of whether or not it provides prior user rights, by incentivizing firms to patent and, at the same time, delivering a sufficiently weak patent protection. The patent system delivers incentives by punishing firms that choose not to patent (when there are no prior user rights) and by providing a strategic advantage to firms that patent (when there are prior user rights). I find that it may be optimal to set weak patent protection so as to induce only some firms to patent while others keep identical innovations secret. This result suggests that the patent system's ability to erode market power may be central to its capability to increase welfare.

*Keywords*: Patent, Secrecy, Competition, Market Power, Simultaneous Innovation, Search

*JEL Classification*: O30, O34, L13.

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# 1 Introduction

The prevailing view on the patent system is that it creates temporary monopolies in order to incentivize firms to innovate (the reward theory of patents) and disclose information (the contract theory of patents). According to the reward theory, on the one hand, patents allocate market power (appropriability) to firms, allowing them to yield profits from their innovative activity, which creates incentives to innovate. On the other hand, the generated monopoly produces a dead weight loss (Arrow (1962), Nordhaus (1969)).[1] The contract theory of patents claims that, by patenting, firms disclose useful information which allows others to "avoid duplication of research, possibly acquire useful knowledge and, when the patent expires, quickly imitate the innovation" — Hall *et al.* (2014, p. 3). The benefit to society from disclosure is then a justification for bearing the social cost of the dead weight loss due to monopoly.[2]

Both the reward and contract theory of patents have the salient feature that the patent system has to allocate market power (reduce ex-post competition).[3] The rationale behind this traditional view goes as follows. In the absence of a patent system firms can secure a certain level of appropriability through informal intellectual property protection mechanisms (IPPMs henceforth) such as secrecy and complexity. With a patent system, they have an extra mechanism at their disposal — a patent. If patenting reduces their appropriability, then firms opt for the outside option of informal IPPMs and the patent system does not affect the degree of competition. If, on the other hand, patenting increases appropriability, firms choose to patent and the patent system reduces competition. Hence, the patent system must (weakly) reduce competition. This rationale, however, ignores two important features of the patent system. First, an innovator's outside option of not patenting has different

[1] The resulting trade off between these two effects has been a major focus for a large body of literature (for surveys see Denicolo (1996) among others).

[2] See, for example, Denicolo and Franzoni (2003) and Denicolò and Franzoni (2008). For a recent paper on the disclosure aspect of patents see Hopenhayn and Squintani (2016).

[3] Throughout the paper I refer to competition as a measure of the ex-post number of innovating firms. In particular, I say that the patent system increases competition if the number of firms which can commercially exploit an innovation is larger when there is a patent system as compared to the case when a patent system is absent. Given the stochastic nature of the model, the criterion I use for the comparison of the degree of competition is first order stochastic dominance that I define formally in section four.

values with and without a patent system — with a patent system a potential duplicator can patent the innovation, whereas she cannot do so when there is no patent system. Second, patents provide a strategic advantage — a patent legally precludes rivals who have innovated simultaneously from commercially exploiting the innovation. In the US, for example, the patent holder can block all rivals that have began commercially exploiting the innovation less than one year prior to the patent filling date. Hence, it is easier to secure an initial monopoly position for innovators that patent, as compared to those who do not.

To capture the importance of these two aspects of the patent system, I develop a dynamic equilibrium model of innovation with features that are consistent with the empirical findings. First, firms try to secure a lead time advantage and can choose to protect their innovations with either a patent or secrecy.[4] Second, firms search simultaneously for ideas (potential R&D projects) as in Kultti *et al.* (2006) and Kultti *et al.* (2007).[5] Third, the dynamic nature of the model allows for duplicative innovation — there is a chance that a firm will independently innovate a previously developed innovation.[6]

I find that, in sharp contrast to the traditional view, the patent system can erode, rather than allocate market power to firms, i.e. it can increase competition, under a very general market structure. Moreover, the patent system can achieve this outcome, regardless of whether or not it provides prior user rights (PUR henceforth). To see the intuition behind this result, notice that if the patent system is to increase competition, then it has to provide weak patent protection so that it decreases firms' appropriability. At the same time, it also needs to deliver incentives for firms to patent, even though secrecy would give them

[4]For a discussion of the evidence see, for example, Cohen *et al.* (2000) and Arundel (2001), for a recent survey see Hall *et al.* (2014). For other papers that allow firms to choose between patenting and secrecy see, for example, Anton and Yao (2004), Denicolo and Franzoni (2003), Denicolo and Franzoni (2004), Denicolò and Franzoni (2008), and Kultti *et al.* (2007).

[5]For example, Granstrand (1999) and Lemley (2011) point out strong evidence that most significant innovations have been (nearly) simultaneously innovated. This assumption is in part motivated by the famous example of Alexander Bell and Elisha Gray telephone controversy — on February 14, 1876 Bell filed a patent application for the telephone and two hours later Gray filed a similar application for the same innovation.

[6]For models where firms innovate sequentially (duplicate previous innovations) see, for example Denicolo and Franzoni (2003), Denicolo and Franzoni (2004), Green and Scotchmer (1995), Zhang (2012), Reinganum (1982) and Reinganum (1983).

stronger protection. When the patent system provides no PUR, the result is driven by its ability to punish those firms which keep their innovations secret. If an innovator chooses to opt for secrecy, then a duplicator can patent the innovation at a later time and block the original innovator from receiving any revenue from the innovation. In the absence of a patent system, however, the duplicator cannot patent and subsequently block the innovator. Thus, the innovator can earn positive profits even if the innovation is duplicated. Hence, by allowing the duplicator to patent and not providing PUR to the innovator, the patent system redistributes the profits of the original innovator, effectively punishing it for not patenting the innovation. By reducing the option value of secrecy, the patent system can induce firms to use patent protection, even when it results in a lower expected duration of monopoly than what they would have had in its absence, effectively increasing competition. In the presence of PUR, the patent system can increase competition by providing a strategic advantage to firms that patent. Given the assumption of lead time advantage, firms' investment decisions are tightly linked to the strategic aspect of patents. Most importantly, the strategic aspect allows firms that patent to not only have a higher chance of securing an initial monopoly position, but also to do so with lower investment in R&D, on average, as compared to secrecy using firms. Because of this, it is possible that the benefit due to the strategic advantage outweighs the loss in appropriability due to weak patent protection. If this is the case, some firms choose to patent, which leads to lower market power on average, i.e. increased competition.

To the best of my knowledge, this is the first paper which finds that the patent system can increase competition. There are a number of previous studies which feature a patent system with the potential of increasing competition (due to duplicative innovation and no PUR), yet they have not commented on this possibility.[7] However, none of the previous papers in the literature examine a model with the property that a patent system can increase competition when it provides PUR. Most previous studies abstract from simultaneous innovation, which is a necessary condition for both the strategic aspect of patents and the result when the

[7] For example, Zhang (2012) develops a model of sequential innovation where the patent system may increase competition for some parameter values, but the author does not examine this feature of the model.

patent system provides PUR. Even previous papers that have allowed firms to innovate simultaneously, however, find that the patent system increases market power. For example, Kultti *et al.* (2007) develop an equilibrium search model of simultaneous innovation, but do not take into account that firms will "race" against each other. In their model, investment decisions can only affect the probability of successfully innovating — they do not give firms lead time advantage. As the strategic aspect of patents is not linked to investment in R&D, it cannot deliver large enough benefits to firms for them to choose patenting when it yields lower appropriability than secrecy. Hence, their model underestimates the degree of the strategic benefit.

The ability of the patent system to increase competition has important practical applications. This paper analyses the planner's problem of choosing a patent protection strength in order to maximize welfare in the case of Bertrand competition and a patent system which provides PUR. I find that the patent system is always welfare improving — it can induce all firms to patent and subsequently disclose their innovations without increasing the expected duration of monopoly.[8] Depending on the size of the strategic advantage, however, the planner may find it optimal to provide weak patent protection so as to induce only a fraction of all firms to patent. When the protection is weak, patented innovations have a lower expected duration of monopoly as compared to innovations developed under secrecy. This reduces the average expected duration of monopoly in the economy and, hence, increases competition and welfare. At the same time, however, weak patent protection implies that some firms have an incentive to not disclose their innovations and resort to secrecy, instead. In the model, disclosing an innovation allows firms to acquire useful knowledge about new potential R&D projects. Thus, lower overall disclosure implies a lower steady state mass of ideas and innovations, which decreases welfare. The planner chooses the strength of patent protection which strikes a balance between these two opposing effects. Larger strategic benefits from patents induces a higher fraction of firms to use patent protection, for a given protection strength, which leads to higher competition and disclosure overall in the economy. Thus, if

[8] This paper abstracts from firms' incentives to innovate. Incorporating firms' incentives to innovate might be an interesting avenue for future research.

the strategic benefit is small, reducing the market power of firms is too costly, so the planner sets a patent protection strength which induces all firms to patent. If, on the other hand, it is large enough the welfare gains due to higher competition outweigh the costs from reduced disclosure and the planner induces an equilibrium in which patent protection is weak and only some firms use patents. This result suggests that the patent system's ability to increase competition may be a key driver of its capability to increase welfare.

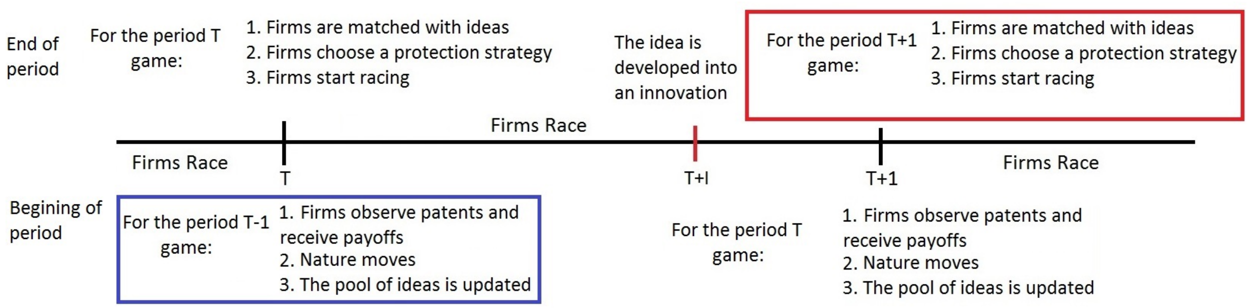
The result that the planner may find it optimal to induce only a fraction of all firms to patent is novel to this paper. Most previous studies do not feature an equilibrium where some firms patent and others protect identical innovations with a secret. Even previous papers that have this feature, however, find it optimal for the planner to induce all firms to patent (see, for example, Kultti *et al.* (2007)). The reason is that the planner has an incentive to provide such weak patent protection only if the patent system can increase competition. Since previous studies do not feature such a patent system, they find that it is not socially optimal to induce some firms to use secrecy protection.

The rest of the paper is structured as follows. Section two explains the timing and assumptions of the model. Section three studies the equilibrium behavior of firms. Section four presents the main results by comparing the degree of competition with and without a patent system. Section five studies welfare and the optimal patent protection strength. Section six concludes.

## 2 The Model

The model is a dynamic innovation game, $\mathcal{G}_P$. Time is discrete, runs from 1 to infinity, and every period has three stages. At stage one firms are matched with ideas. At stage two firms observe which ideas they are matched with and decide on a protection strategy of patenting or secrecy. At stage three, they "race" by choosing an investment strategy (or a bid). The winner of the race introduces the innovation as a new product on the consumer market and yields a certain per period profit.[9] The assumption that firms commit to a protection strategy prior to innovating is not unreasonable. In practice, firms may need to take specific steps, prior to innovating, to ensure that the protection strategy is effective. For example, patenting firms may need to hire a team of lawyers to help the firm navigate through the patent system and represent it in potential law suits. Secrecy using firms, on the other hand, may need to make the innovation as complex as possible to deter competitors from reverse engineering. They may also need to pay their workers higher wages in order to provide incentives for them to protect the secret. Nevertheless, the main results in the paper would hold even if firms are to chose a protection strategy after they innovate. In particular, in the appendix I show that theorems one and two hold even if firms choose bids at stage two and a protection strategy at stage three. Figure 1 illustrates the timing of the model.[10]

**Figure 1**




Let us consider an arbitrary period $T$. There is a fixed measure $\mu$ of ex-ante identical firms which, at stage one, are randomly matched with ideas. The measure of ideas is $\nu$ (I also refer to $\nu$ as the "pool" of ideas) and each idea is equally productive if developed into an innovation. As in Kultti *et al.* (2007), the innovation process distinguishes between ideas (potential innovations) and actual innovations (ideas which have been brought to fruition through costly investment in R&D). Each firm is paired with exactly one idea, but the number of firms matched with a given idea is a random variable that follows a Poisson distribution

[9]An alternative interpretation is that the innovation constitutes a quality improvement of an existing product or a cost reducing technology.

[10]In this paper, both secrecy and patent protection have a chance to fail at the end of each period. Nature's move is to choose which patents remain valid and which secrets do not leak.

with parameter $\theta := \mu/\nu$.[11] This matching technology would arise if, for example, firms could direct their search towards ideas. In particular, I interpret $\nu$ as the pool of all possible R&D projects that firms might choose to undertake and a match as the particular project each firm chooses to devote effort to. The ratio of firms to ideas, $\theta$, captures the congestion in the "market" for ideas: higher $\theta$ means, on average, a higher number of firms race for the same idea. Matches are private knowledge and, in particular, firms cannot observe how many rivals (if any) are matched with the same idea. They observe, however, exactly how many competitors have previously innovated the idea they are matched with. Furthermore, firms can exert R&D effort only towards developing ideas they are matched with in the current period.

At stage two firms choose between patenting or secrecy and the choice is private knowledge.[12] Both IPPMs provide imperfect protection. Each period $T + l$, $l \geq 1$, the patent fails with probability $1 - \alpha$ and the secret leaks with probability $1 - \beta$. In the event that the patent fails (secret leaks) all firms can costlessly imitate the innovation which is then produced under perfect competition.[13] Firms cannot switch protection strategies, i.e. a firm which innovated under secrecy cannot apply for a patent at a later point in time. However, this assumption is innocuous — in the appendix I prove that, in equilibrium, no firm has an incentive to switch protection strategies.

When a firm chooses to patent it goes to the patent office and discloses all of the relevant information, which then becomes public knowledge. An innovation protected by secrecy, on the other hand, becomes public knowledge only if the secret leaks. This difference in disclosure has important effects on duplicative innovation that takes place in the model, as well as on the welfare analysis in section five. I assume that whenever an innovation becomes public knowledge, the corresponding idea is removed from the pool. This assumption captures the intuition that no firm will engage in costly innovative activity if it has the know-how to

[11] For a derivation in the context of the labor market see, for example, Julien *et al.* (2000). In this model, the economic interpretation of $\theta$ is that it is a measure of the market tightness in the market for ideas.

[12] I follow the majority of the existing literature by assuming an innovation can be protected by either a patent or a secret, but not both (see, for example, Anton and Yao (2004) and Kultti *et al.* (2007)).

[13] I follow previous research (see, for example Kultti *et al.* (2007)) and assume that the leakage of the secret and a failure of the patent have the same effect.

imitate the innovation. Since ideas protected by a valid secret remain in $\nu$, there is the potential of duplicative innovation. I refer to duplicative innovation as the possibility that at time $T$ a firm is matched with and independently innovates an idea previously developed by a rival at some $T' < T$.

The potential of duplicative innovation requires us to make two further assumptions. First, I assume the patent system does not provide PUR. The important implication for this model is that a patent holder excludes from the market all previous innovators. Thus, a patented innovation is produced under either monopoly or perfect competition, whereas innovations protected with a secret may be produced by any number of firms. I refer to this assumption as A1a and relax it later on.

**Assumption A1a.** *The patent system provides no prior user rights. That is, if a firm duplicates an innovation and patents it, then all previous innovators receive zero profits from then on.*

Second, we must make an assumption on the structure of the consumer market. To keep it as general as possible, the only two restrictions are that firms' profits are not strictly increasing in the number of producing firms and that for a large enough number of firms (strictly larger than some $\bar{n}$), no firm makes positive profits. In particular, the assumption allows for an arbitrary degree of product differentiation. Formally, the assumption is given by A2 below.

**Assumption A2.** *Take a sequence $(d_n)_{n\in\mathbb{N}}$ such that $d_n \leq d_{n'}$ for all $n > n'$, $0 \leq d_n \leq 1$ for all $n$, $d_1 = 1$, and $d_n = 0$ for $n > \bar{n} \geq 1$. If exactly $n \in \mathbb{N}$ firms produce an innovation, each receives a fraction $d_n$ of the monopoly profits. Otherwise, if the protection mechanism has failed, the innovation is produced under perfect competition and all firms earn zero profits.*

I assume exactly one new idea enters in $\nu$ whenever an innovation becomes public knowledge, i.e. it is patented or its secret leaks. Since the idea corresponding to such innovation is removed from the pool, the measure $\nu$ is constant. This leads to constant $\theta$ which makes the model much simpler without sacrificing any of the insights. Alternatively, if there were convergent dynamics the model's implications would still hold at the steady state.[14]

At stage three firms compete in a winner-take-all race to develop the idea into an innovation. The race is a generalized version of the symmetric game in Kaplan *et al.* (2003). However, it is worth noting that the specific structure of the race is not important for the main results in this paper. What is key is lead time advantage and the strategic aspect of patents that I introduce in assumption A3.

Firms race by choosing an investment strategy, or a bid, that specifies at what time the firm will innovate. Investment is sunk, only the winner receives a positive reward, and the reward and cost structures, which I explicitly discuss in the next subsection, ensure that all races end by time $T + 1$. Every firm innovates for sure at the chosen time and bids are private knowledge. The winner in the race is granted the right to produce the innovation which is protected by the chosen IPPM — patenting or secrecy. To capture an important feature of the patent system (a patent holder can, in practice, legally block all rivals who have innovated nearly simultaneously from commercially exploiting the innovation), the winner is not necessarily the firm which innovates first. In particular, I make the following assumption:

**Assumption A3.** *The firm which innovates first among all firms that use patent protection wins the race. In the event that all firms use secrecy, the firm which innovates first is the winner.*

The assumption captures what I refer to as the *strategic aspect of patents*. Whenever a firm chooses patent protection, it can block all rivals that have innovated simultaneously under secrecy from producing.[15] Most importantly, patents allow firms to block rivals even if the rival has innovated slightly sooner than the patent holder. For the purposes of the model, the most important implication of this assumption is that firms which use patent protection are guaranteed to win against rivals that use secrecy, regardless of the chosen bids. The assumption also captures the importance of lead time advantage which, as a

[14]However, it is key to have a positive $\theta$ at the steady state. If $\theta \longrightarrow 0$, then there would be no simultaneous search asymptotically.

[15]Firms that participate in the same race are said to innovate simultaneously.

myriad of studies suggests, is a key IPPM for firms.[16] When a firm uses secrecy protection it "races" to be the first innovator. Innovating first allows the firm to secure a monopoly position through lead time advantage. If a firm decides to patent, then it must (in a first-to-file patent system) "race" to the patent office to ensure it is the first one to do so.

The model allows for lead time advantage to die out over time. In particular, several firms can have an innovation based on the same idea, but only if these firms innovate in different periods. The rationale behind this is that later innovators could potentially differentiate their innovation from the earlier ones. This product differentiation allows the later innovators to circumvent the lead time advantage and gain a certain share of the market. If several firms innovate at the same period, then they cannot observe each other's innovations and, hence, cannot differentiate them. However, the model allows for the lead time advantage to persist over time, as well. In particular, in the case of Bertrand competition ($\bar{n} = 1$) no firm is willing to duplicate an innovation under secrecy. Then, the lead time advantage persists until the secret leaks or a competitor duplicates the innovation using patent protection.

At the beginning of $T + 1$, when all matched ideas have been developed into innovations the following events happen. First, firms receive their profits and observe all patents filed between dates $T$ and $T+1$. Second, a fraction $1-\alpha$ of all patents fail and a fraction $1-\beta$ of all secrets leak. Firms observe which patents fail and all secrets that leak. Third, the pool of ideas is updated in the aforementioned way. After all these events have occurred, at the end of $T + 1$, the three stages repeat.

## 2.1 The Innovation Race

Consider an innovation race that starts at $T$ and suppose that the idea has already been developed by $n - 1$ firms in previous periods. Participants in the race observe how many firms have previously innovated the idea, i.e. how many firms are currently selling the corresponding product. However, prior to innovating, they do not know the details on how

[16]For a survey of the evidence see, for example, Hall *et al.* (2014). I implicitly assume that lead time advantage is so strong as to discourage firms other than the first innovator from producing (provided, of course, the other firms cannot block the first innovator by patenting).

to produce and commercialize the product in question. Each firm that participates in the race bids a time $t \in [0, 1]$ which means the firm will innovate at time $T + t$.[17] Innovating is costly. At time $T$ firms pay a cost of $c(t)$, with $c'(t) < 0$ for $t \in [0, 1)$.

The reward from the race, which also depends on $t$ (due to discounting), is comprised of all expected future profits. In particular, let $\pi(t)$ be the per period monopoly profit, where $t$ stands or the innovation time and $\pi'(t) < 0$ on $t \in [0, 1)$.[18] Then, let us consider a firm that chooses patent protection. Define $R_P(t)$ to be the reward from the race, conditional on the firm under study winning, and $V_P$ the value of holding a valid patent.[19] Both are independent of how many firms have already innovated, by assumption A1a. Then

$$R_P(t) = \pi(t) + \alpha\gamma V_P \tag{1}$$

$$V_P = \pi(0) + \alpha\gamma V_P, \tag{2}$$

where $\gamma$ is the discount factor. When a firm innovates at time $T + t$, the $T$ period profits are $\pi(t)$ because the firm could not produce between times $T$ and $T + t$. As is evident from (2), the profits for any period $T' > T$ are given by $\pi(0)$, since innovation took place at a previous period. The patent is valid next period with probability $\alpha$ and has a discounted value of $\gamma V_P$.

Analogously, let $R_S^n(t)$ be the reward from the race when the firm shares a secret with $n - 1$ others and $V_S^n$ be the value of sharing that secret. Then

$$R_S^n(t) = d_n\pi(t) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1 - e^{-\theta})\beta\gamma V_S^{n+1} \tag{3}$$

The period $T$ profits are shared with the rivals who also know the secret, so the firm receives only a fraction $d_n$ of the monopoly profits. It keeps the value $V_S^n$ next period if the secret does not leak and no one duplicates the idea at $T + 1$, i.e. with probability

[17] Firms can bid any $t \in [0, \infty)$, however, a normalizing assumption on the profit function ensures that in equilibrium no firm will bid $t > 1$.

[18] I further impose that $\pi(t)$ is strictly decreasing over $[0, 1]$.

[19] Firms receive their profits for period $T$ at the beginning of $T + 1$. As a result, $\pi(t)$, $R_P(t)$ and $V_P$ are time $T$ discounted quantities.

$\beta \times Pr(\text{the idea is matched with no firms}) = \beta e^{-\theta}$. The firm receives the value $V_S^{n+1}$ if one more rival begins commercially exploiting the innovation under secrecy protection next period.[20] This happens with probability $Pr(\text{at least one firm is matched with the idea}) \times Pr(\text{the } n+1\text{-st innovator has chosen secrecy}) = (1-e^{-\theta})\zeta_{n+1}$.[21] In the events that the secret leaks or the next innovator patents, the information about the innovation becomes public knowledge and the value of knowing the secret is 0. $V_S^n$ is defined similarly.

$$V_S^n = d_n\pi(0) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1-e^{-\theta})\beta\gamma V_S^{n+1} \tag{4}$$

Lastly, I make three technical assumptions: i) $\pi(t)-c(t)$ is increasing over $[0,1]$ to ensure the support of the equilibrium CDF is connected; ii) $\pi(t)=0$ for $t>1$ to ensure all races will end by $T+1$; iii) for all $n$ either $e^{-\theta}d_n\pi(1)-c(1)\geq 0$ or $d_n=0$, which implies that either all firms choose to innovate or none of them do. I make this assumption to abstract the analysis from firms' incentives to innovate.

# 3 Equilibrium Behavior

The equilibrium is an infinite sequence of matches, protection strategies, and bids (investment strategies). At stage one firms make no decisions and the equilibrium outcome in this stage plays no role in the analysis of the main results. Thus, for the sake of clarity in exposition, the characterization of the equilibrium stage one outcome is placed in the appendix. In general the protection and investment strategies firms decide on at time $T$ will depend on all past decisions and outcomes. However, one should observe that the payoffs from each innovation race are independent of any previous decisions or outcomes. Hence, the focus in this paper is

[20]By assumption A3 at most one new firm can begin commercially exploiting the innovation each period. Also, at $T+1$, the $n+1$-st innovator receives $d_{n+1}\pi(t)$ while the other $n$ firms each receive $d_{n+1}\pi(0)$, as they have innovated previously. Hence, $V_S^{n+1}$ is independent of $t$.

[21]The $n$-th innovator develops the innovation under secrecy if all firms at the innovation race choose secrecy protection, due to assumption A3. Let $s_n$ be the probability firms place on playing secrecy in a symmetric equilibrium, when they are the $n$-th innovator. Then, $\zeta_n := P(\text{all matched firms choose secrecy}|\text{the idea is matched with at least 1 firm}) = \sum_{i=1}^{\infty}\frac{1}{1-e^{-\theta}}\frac{e^{-\theta}\theta^i}{i!}s_n^i = \frac{e^{-\theta}}{1-e^{-\theta}}\left(e^{\theta s_n}-1\right)$.

on stationary equilibria where strategies are independent of the history. Moreover, firms are ex-ante identical and, as is customary in the search and matching literature, I will focus on symmetric equilibria in which firms do not collude. The equilibrium concept I use is subgame perfect Nash equilibrium, and I solve for the equilibrium using backward induction.

### 3.1 Equilibrium Behavior with a Patent System

Let us first characterize the equilibrium behavior at stage three. Take an innovation race which starts at time $T$ and is associated with an idea that has been developed by $n-1$ firms in previous periods. Define the equilibrium CDFs to be $F_j^n(t)$ on $[\underline{S}_j^n, \bar{S}_j^n]$, where $j = P, S$ stands for the protection strategy of patenting or secrecy, respectively.[22] In anticipation of the stage two results, let $s_n$ denote the probability firms place on using secrecy protection. I follow Kultti *et al.* (2007) and, also, refer to $s_n$ as the fraction of firms which choose secrecy protection. Then, the equilibrium behavior in stage three is given by the following lemma.

**Lemma 1.** *At stage 3 firms randomize their innovation time using the CDFs*

$$F_P^n(t) = \frac{1}{\theta(1-s_n)} ln\Big(\frac{R_P(t)}{c(t) + e^{-\theta(1-s_n)} R_P(1) - c(1)}\Big)$$
$$F_S^n(t) = \frac{1}{\theta s_n} ln\Big(\frac{R_S^n(t)}{c(t) + e^{-\theta} R_S^n(1) - c(1)}\Big) - \frac{1-s_n}{s_n}$$

The proof is fairly standard and, therefore, in the appendix.

Firms use a different investment strategy, depending on the IPPM, because there is a substitution effect between investment in R&D and patents' strategic advantage. To see the intuition behind this result, first observe that as a straightforward result of the assumptions, it follows that $\bar{S}_j^n = 1$. Second, firms choose to innovate sooner ($t < 1$) only for the sake of winning the race, as $\pi(t) - c(t)$ is increasing in $t$. Third, the strategic advantage of patents depends on how many firms use patent protection. The more firms use patents, the lower the fraction of rivals that a patenting firm can block, hence, the lower the strategic advantage.

[22] In equilibrium, firms use a mixed strategy when they choose an innovation time. This can be seen by applying a standard argument.

The result in Lemma 1 implies that when patenting provides a higher strategic advantage ($s_n$ increases) a firm that patents decreases its investment (chooses a higher $t$) because it faces less competition from rivals. In the extreme case, as $s_n \longrightarrow 1$ the firm can block all rivals, so there is no incentive to bid any $t < 1$, which leads to $\underline{S}_P^n \longrightarrow 1$. Firms that use secrecy condition their investment strategies on the level of the strategic advantage as well. A firm that uses secrecy protection cannot win against a rival that patents, so lower $s_n$ implies that secrecy using firms get "discouraged" and bid less aggressively. In particular, as $s_n \longrightarrow 0$ innovating sooner is not beneficial because all rivals can block the firm, hence, $\underline{S}_S^n \longrightarrow 1$.

This result differs from what previous research has found, because I explicitly account for the innovation race, as well as simultaneous innovation. For example, in Kultti *et al.* (2007) firms cannot exclude rivals from commercially exploiting the innovation by innovating sooner — they invest solely to increase their chance of successfully developing an idea into an innovation. Because of this the level of investment, in their model, does not react to changes in the level of strategic advantage of patents. More importantly, the strategic benefit is smaller than that in my model. To see this clearly, let $s_n \to 1$. Then a secrecy-using firm expects to win the race with probability $(1 - e^{-\theta})/\theta$ and makes, on average, an investment of $c(t^*) > c(1)$.[23] A firm that patents, on the other hand, wins the race for sure and makes the minimum investment of $c(1)$. Hence, the benefit from the strategic advantage consists of both a higher chance to win the race and a lower investment in R&D. In Kultti *et al.* (2007), however, investment decisions are independent of the chosen IPPM and the strategic advantage affects only the probability of securing a monopoly position. Thus, Kultti *et al.* (2007) underestimate the importance of patents to firms because, in their model, it does not include the benefit of lower average investment.

At stage two, firms choose between patenting and secrecy in the following way.

[23] Here $t^*$ is such that $c(t^*)$ equals the expected investment of secrecy using firms in equilibrium.

**Lemma 2.** *At stage 2 firms choose secrecy with probability $s_n$, where*

$$s_n = \begin{cases} 0 & \text{if } \alpha \geq \bar{\alpha}_n, \\ \frac{1}{\theta} ln\left(\frac{R_S^n(1)}{R_P(1)}\right) & \text{if } \alpha \in (\max\{0, \underline{\alpha}_n\}, \bar{\alpha}_n), \\ 1 & \text{if } \alpha \leq \underline{\alpha}_n. \end{cases}$$

*where $\bar{\alpha}_n$ is such that $V_P - V_S^n = \pi(0)(1-\gamma)(1-d_n)$ and $\underline{\alpha}_n$ is such that $V_P = e^{-\theta} V_S^n + (1-\gamma)\pi(0)(1-e^{-\theta} d_n)$.*

Proof is included in the appendix. The intuition behind the result is the same as in Kultti *et al.* (2007). When $\alpha \geq \bar{\alpha}_n$ patenting provides higher appropriability ($R_P(t) \geq R_S^n(t)$), so all firms choose patent protection in equilibrium. For intermediate patent strengths there is a randomizing behavior because of patents' strategic advantage. In particular, the expected payoff for a firm when it uses patenting is $e^{-\theta(1-s_n)} R_P(1) - c(1)$, which is simply the chance that no other patenting firm participates in the innovation race ($e^{-\theta(1-s_n)}$) times the expected profits ($R_P(1)$) minus the cost of innovating ($c(1)$) when the firm bids $t = 1$. Analogously, when the firm uses secrecy protection, its expected payoff is $e^{-\theta} R_S^n(1) - c(1)$. When $\alpha \in (\max\{0, \underline{\alpha}_n\}, \bar{\alpha}_n)$ there is a trade-off between the reward and strategic aspect of patents. As patent protection decreases, the reward $R_P(t)$ decreases as well and more firms opt for secrecy. However, as $s_n$ increases, firms that still use patent protection have a higher chance of winning the race ($e^{-\theta(1-s_n)}$) because they can block a higher fraction of their rivals. Thus, as $\alpha$ decreases patents provide lower appropriability, but higher benefit from the strategic advantage. This randomizing behavior persists until $\alpha$ is so low that the strategic gain from patenting cannot compensate for the loss in appropriability, i.e. $\alpha < \underline{\alpha}_n$. In particular, $\underline{\alpha}_n$ is the protection strength at which the net gain from the strategic advantage of blocking all rivals, $R_S^n(1) - e^{-\theta} R_S^n(1) = (1-e^{-\theta})(V_S^n - d_n(1-\gamma)\pi(0))$, equals the loss in appropriability from patenting, $R_S^n(1) - R_P(1) = V_S^n - V_P + (1-\gamma)(1-d_n)\pi(0)$.[24] Unlike in

[24] From equations (1), (2), (3), (4) we have that $R_P(t) = V_P - (\pi(0) - \pi(t))$ and $R_S^n(t) = V_S^n - d_n(\pi(0) - \pi(t))$. When the firm can block all rivals, it wins for sure and its expected payoff from the race is $R_S^n(1) - c(1)$. If the firm does not have the strategic advantage of blocking rivals, then it receives positive payoffs only when it faces no rivals (i.e. with probability $e^{-\theta}$) and the expected payoff is $e^{-\theta} R_S^n(1) - c(1)$. Thus, the net gain

Kultti *et al.* (2007), however, in the present paper an equilibrium where $s_n = 0$ may not be achievable, regardless of how low is $\alpha$. This is due to the assumption of lead time advantage — first, the strategic benefit of patents is higher in the present paper than it is in Kultti *et al.* (2007) and, second, firms that patent can receive some profits even if $\alpha = 0$. If it is the case that $\underline{\alpha}_n < 0$, then the strategic aspect of patents is strong enough to always induce at least some firms to patent, regardless of the strength of patent protection.

It is important to note that the cutoff values $\underline{\alpha}_n$ and $\bar{\alpha}_n$ depend on how many times the innovation has been duplicated. In particular, firms require less of an incentive to patent the larger the number of previous innovators. Corollary 1 establishes the result.

**Corollary 1.** $e^{-\theta}\beta = \bar{\alpha}_1 \geq \bar{\alpha}_2 \geq \cdots \geq \bar{\alpha}_{\bar{n}} \geq \bar{\alpha}_{\bar{n}+1} = 0$

The proof is in the appendix. As in Kultti *et al.* (2007), firms patent even if patent protection is lower than secrecy protection. In particular, all firms patent as long as the patent strength, $\alpha$, is weakly higher than the effective protection under secrecy, $e^{-\theta}\beta$. This is key to establishing the main results in this paper and is due to the patent system's ability to punish innovators that choose secrecy. Intuitively, at $\alpha = e^{-\theta}\beta$ the expected duration of monopoly under patenting and secrecy is the same, but a firm that uses secrecy protection can potentially receive positive profits even if it looses its monopoly position (whenever a firm duplicates under secrecy). However, this potential is never realized in equilibrium, because the patent system incentivizes the duplicator to always patent by redistributing the original innovator's profits.[25] Since the innovator always receives those potential profits in its absence, the patent system effectively punishes the firm.

due to the strategic advantage is simply the difference given in the text. The loss in appropriability is simply the difference of the values of sharing the secret with $n-1$ other firms and the value of holding a valid patent, adjusted for the difference in per period profits under secrecy, $d_n\pi(0)$, and under patent protection, $\pi(0)$.

[25] I say the patent system redistributes the profits, because if the duplicator uses secrecy, then both it and the original innovator receive $d_2 \leq 1$ of the monopoly profits. If the duplicator patents, however it receives the monopoly profits and the original innovator receives 0. Moreover, the patent system provides an additional incentive for the $n$-th innovator to patent — the threat that the $n+1$-st innovator will patent.

## 3.2 Equilibrium Behavior without a Patent System

Without a patent system firms play the dynamic game $\mathcal{G}_N$, which is the same as $\mathcal{G}_P$ with the only exception that now firms cannot use patent protection. I again present the equilibrium outcome of stage one in the appendix as it plays no role in the analysis of the main results. The behavior of firms at stages two and three is summarized in the proposition below.

**Proposition 1.** *$\mathcal{G}_N$ has a unique stationary symmetric equilibrium, where*

- *At stage 2: Firms trivially choose secrecy protection*
- *At stage 3: Firms randomize their innovation time using the CDF*

$$F_s^n(t) = \frac{1}{\theta} ln\Big(\frac{R_S^n(t)}{c(t) + e^{-\theta} R_S^n(1) - c(1)}\Big)$$

*for $n \leq \bar{n}$, and they stay out of the race, if $n \geq \bar{n} + 1$.*

The proof is analogous to that of Lemma 1 when $s_n = 1$, for all $n$.

# 4 Competition

This section presents the main results in the paper: the patent system can increase competition, regardless of whether or not it provides PUR. It is natural to think of competition in terms of how many firms are producing the innovation.[26] Given the stochastic nature of the model, an unambiguous criterion for comparing the degree of competition between different equilibria is the notion of first order stochastic dominance. This paper abstracts from uncertainty in the innovation process and incentives to innovate. Because of this, I compare the distribution of firms conditional on the innovation having been innovated. By symmetry, these distributions will be the same for all innovations regardless of which period

[26] For example, under Cournot competition with no fixed costs the market output and price converge to their perfect competition levels as the number of firms increases. In this sense, a higher degree of competition translates to a market outcome which is "closer" to perfect competition.

the idea was originally innovated. Thus, without loss of generality I will only focus on ideas innovated at period one.

Let $P_i(n)$ be the probability an innovation is produced by exactly $n$ firms in equilibrium $i$, given that the corresponding idea was originally developed in period 1, where $i = P$ stands for the equilibrium with a patent system and $i = N$ for the equilibrium without one. Notice that $P_i(n)$ will, in general, depend on $T$, but I suppress it from the notation to simplify the exposition. Also, I slightly abuse notation by letting $n = \infty$ denote the case of perfect competition.

Next, let us define the distribution of firms producing the innovation by $G_i^T(k) := \sum_{n=1}^{k} P_i(n)$. Then, the following definition formally introduces the criterion for comparing the degree in competition across equilibria.

**Definition 1.** *We say that equilibrium $i$ provides higher competition than equilibrium $i'$ if $G_i^T$ first order stochastically dominates $G_{i'}^T$. Formally, $G_i^T(k) \leq G_{i'}^T(k)$ for all $k \geq 1$, $T \geq 1$ and the inequality is strict for at least some $k$ for all $T \geq \max\{2, k\}$.*

## 4.1 No Prior User Rights

Intuitively, the patent system would increase competition if patents provide weaker protection than secrecy. Weaker protection increases competition and reduces firms' appropriability — the more firms produce, the higher the degree of competition and the lower the per firm profits. Yet, this raises a question: Why would firms use patent protection if this reduces appropriability? The answer is that when the patent system does not provide PUR, it can punish firms that opt for secrecy by incentivizing duplicators to patent, effectively reducing the option value of secrecy. In particular, for high enough patent strengths ($\alpha \geq \bar{\alpha}_1$) a firm that chooses secrecy protection yields positive profits next period with probability $e^{-\theta}\beta$. Since in the absence of a patent system a secrecy using firm receives positive profits next period with probability $\beta$, it follows that the presence of a patent system reduces the attractiveness of secrecy. Thus, in order for firms to patent, patents must provide stronger protection than secrecy in the presence of a patent system, which might still be weaker than the protection secrecy provides in the absence of a patent system. The theorem below formalizes this intuition.

**Theorem 1.** *For all $\gamma, \beta \in (0,1)$ and all $\theta > 0$, there exists a patent strength $\alpha$ such that the patent system increases competition.*

The proof is straightforward and carries much of the intuition behind the result, so I provide it in the text below.

*Proof.* Let us fix $\alpha = \bar{\alpha}_1 = e^{-\theta}\beta$, so that the equilibrium with a patent system is such that all firms patent for sure. If $T = 1$ the innovation is produced under monopoly with certainty regardless of which equilibrium we are in, so $P_P(n) = P_N(n)$ for all $n$. Then, fix $T \geq 2$ for the remainder of the proof.

Let us first consider the case when there is a patent system. With a patent, an innovator protects its monopoly position by blocking all potential duplicators from securing any market power. Hence, the innovation is produced under monopoly or perfect competition, i.e. $P_P(n) = 0$ for $n \neq 1, \infty$. Moreover, the innovation is produced under monopoly at time $T$, if the patent has held for all previous periods, i.e. $P_P(1) = (e^{-\theta}\beta)^{T-1}$ and $P_P(\infty) = 1 - (e^{-\theta}\beta)^{T-1}$.

Next, consider the case without a patent system. It will be useful to distinguish between two possibilities: i) $\bar{n} \geq 2$ and ii) $\bar{n} = 1$. From corollary 1, the patent system punishes secrecy-using firms by redistributing their profits when the innovation is duplicated. The difference between the two possibilities is that under $\bar{n} \geq 2$ the patent system exacerbates this threat, while under $\bar{n} = 1$ — creates it.

If $\bar{n} = 1$ no firm is willing to become a duopolist, so the innovator does not face the threat of duplication when there is no patent system. In particular, any firm matched with a previously developed idea chooses to opt out of the race, so $P_N(n) = 0$ for all $n \geq 2$. Then it follows that, a monopolist loses its market power only when the secret leaks, i.e. $P_N(1) = \beta^{T-1}$. This implies that there is a strictly higher expected duration of monopoly when there is no patent system, $1/(1-\beta)$, as compared to the case with a patent system, $1/(1-e^{-\theta}\beta)$. The patent system can achieve this and at the same time induce all firms to patent because the punishment for firms that opt for secrecy is much stronger when $\bar{n} = 1$. Thus, we have that $G_N^T(k) = P_N(1) = \beta^{T-1} > (e^{-\theta}\beta)^{T-1} = P_P(1) = G_P^T(k)$ for all $1 \leq k < \infty$. Moreover, under both scenarios (with and without a patent system) an innovation can only be produced under perfect competition or monopoly, hence, $P_N(\infty) = 1-\beta^{T-1} < 1-(e^{-\theta}\beta)^{T-1} = P_P(\infty)$.

If $\bar{n} \geq 2$ firms find it profitable to share a secret with at least one rival, so they will duplicate whenever given the chance. Thus, $P_N(n) > 0$ for at least one $n \geq 2$. Moreover, an innovator would lose its monopoly position if the innovation is duplicated, hence $P_N(1) = Pr(\text{the secret has not leaked}) \times Pr(\text{no rival has duplicated}) = (e^{-\theta}\beta)^{T-1}$. Then, it follows that $G_P^T(1) = G_N^T(1)$ and $G_P^T(k) < G_N^T(k)$ for any $2 \leq k < \infty$. This concludes the proof.

□

To the best of my knowledge, this is the first paper which finds that the patent system can increase competition, even though some previous studies may have that feature under no PUR. For example, Zhang (2012) looks at the optimal patent protection strength in a model of sequential innovation that allows for duplicative innovation and no PUR. Yet, he does not compare the degree of competition between the presence and absence of a patent system. One can show that, unlike this paper, in the framework of Zhang (2012) the patent system can increase competition only for a certain range of parameter values. Furthermore, in Zhang (2012) the patent system cannot increase competition under PUR since the paper abstracts away patents' strategic aspect.

It is worth noting that the conclusion in Theorem 1 is not unique to the particular matching technology and innovation race. It is straightforward to see that the only requirement we have placed on the matching function is that $e^{-\theta} < 1$, i.e. there is a possibility of duplication. Moreover, the assumption on the existence of a pool of ideas, $\nu$, can be easily relaxed. In particular, a model where there is exactly one idea that can be innovated and firms enter the innovation race each period stochasticly according to $Pr(\text{exactly } n \text{ firms enter}) = \frac{e^{-\theta}\theta^n}{n!}$ leads to the exact same results. Furthermore, the assumptions behind the innovation race are not critical either. The key aspect is that $\bar{\alpha}_1 = e^{-\theta}\beta$, which is mainly a feature of the assumption on lead time advantage.

## 4.2 Prior User Rights

The result in Theorem 1 (and its intuition) relies on assumption of no PUR. In practice however, the patent laws of most countries provide some PUR.[27] Then, it is relevant to ask if the patent system can increase competition, given that it provides PUR. In what follows I relax the assumption of no PUR and show that the patent system can, indeed, increase competition.

Assume that if a later innovator patents, then all firms that benefit from the prior user defense make the same profits as the patent holder.[28] Formally,

**Assumption A1b.** *The patent system provides prior user rights. That is, suppose that $n-1$ firms have independently developed an innovation and kept it secret. If the $n$-th innovator develops the innovation and patents it, then all $n$ firms receive the fraction $d_n$ of monopoly profits.*

From now on, I replace assumption A1a with assumption A1b. Since the patent system cannot exacerbate the threat of sequential innovation, it can only increase competition by providing a strategic advantage to firms which patent. Theorem 2 gives the result.

**Theorem 2.** *If*

$$\frac{1}{\gamma} > \Big(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\Big)\Big(\frac{\beta}{1-\beta\gamma}\Big) \tag{5}$$

*then the patent system can increase competition.*

The proof is included in the appendix. The intuition behind the result goes as follows. Take an innovation developed by $n-1$ innovators, who all chose secrecy protection. With a

[27]The US patent system did not provide PUR for most patents until 2011. The America Invents Act, however, increased the scope of "prior user rights" defense to infringement.

[28]Even with PUR, there are some restrictions on what can the first innovator do. For example, in the US the original innovator cannot license, assign, or transfer the prior user defense. Moreover, the defense is geographically limited to sights where the innovation has been commercially exploited for at least one year prior to the patent filling date. Hence, in reality, the second innovator could "partially" exclude (or block from expanding) the original one, even if the patent system provides PUR. From the anteceding analysis it will become clear that the results hold for any arbitrary degree of partial exclusion. Intuitively, the threat of duplication depends on the degree of exclusion –the higher the degree of exclusion, the more the patent system exacerbates the threat. By the logic of Theorem 1, it will be "easier" for the patent system to increase competition under partial exclusion than under full exclusion.

patent system the $n$-th innovator can choose to patent. In this case, the innovation will be produced by $n$ firms next period with probability $\alpha$. With the complimentary probability, $1 - \alpha$, it will be produced under perfect competition. When there is no patent system, the $n$-th innovator has no choice but to keep the innovation secret. Then, with probability $e^{-\theta}\beta$ the innovation will be produced by $n$ firms next period, with probability $(1 - e^{-\theta})\beta$ by $n+1$ firms, and with probability $1 - \beta$ under perfect competition. Hence, a necessary condition for the patent system to increase competition is $\alpha \leq e^{-\theta}\beta$, otherwise it will increase the chance an innovation is produced by $n$ firms next period. With PUR, however, the patent holder receives only a fraction $d_n \leq 1$ of monopoly profits, as opposed to $d_1 = 1$ without PUR. Thus, the option of patenting with PUR is not as attractive as the one without PUR. Hence, with PUR, it is no longer true that $\bar{\alpha}_1 = e^{-\theta}\beta$. When $\alpha \leq e^{-\theta}\beta$ patenting provides strictly lower expected reward than secrecy does, and the only way to incentivize firms to patent is by providing them with strategic benefits.

Then, we need to see under what conditions this patent strength ($\alpha = e^{-\theta}\beta$) is consistent with $s_n \in (0, 1)$, that is, the gain in the expected payoff from the race due to the strategic aspect of patents can exceed the loss in appropriability. The strategic advantage is largest when all other firms use secrecy protection, i.e. $s_n = 1$, hence, a sufficient condition for $s_n \in (0, 1)$ is $R_P^n(1) > e^{-\theta}R_S^n(1)$.[29] The inequality can be rewritten to separate the strategic and reward aspects of patents.

$$V_S^n - V_P^n < (1 - e^{-\theta})(V_S^n - d_n\pi(0)(1 - \gamma)) \tag{6}$$

On the left hand side we have the net loss in appropriability due to patenting — the difference between the value of sharing the secret with $n - 1$ other firms and the value of patenting when the firm is the $n$-th innovator. The right hand side captures the strategic benefit of patenting. If the firm could block rivals (when it uses secrecy) in the innovation race, then its reward would be $R_S^n(1)$, which is nothing but $V_S^n - d_n\pi(0)(1-\gamma)$. Because secrecy protection

[29]Since the patent system provides PUR, the expected profits when patenting depend on the number of innovators. Also, $R_P^n(1) \leq e^{-\theta}R_S^n(1)$ whenever $s_n = 1$.

does not aid the firm in blocking its rivals, its expected reward is $e^{-\theta}R_S^n(1)$. The difference between the two is the net gain in expected profits from the ability to block rivals.

Theorem 2 provides a sufficient condition for equation (6) to hold for at least one $n$ for any market structure (captured by $(d_n)_{n\in\mathbb{N}}$).[30] To see the intuition behind (5) we can interpret $1-\gamma$ as the rate with which innovations become obsolete. Then, the right hand side of (5) is the product of $Pr$(Keeping a monopoly position next period, given that the innovation does not become obsolete) $\times$ (Expected duration of monopoly) when firms are willing to become a duopolist, $e^{-\theta}\beta/(1-e^{-\theta}\beta\gamma)$, and when firms are not willing to become a duopolist, $\beta/(1-\beta\gamma)$. Then, the theorem says that if the inverse of the probability the innovation does not become obsolete next period, $1/\gamma$, is larger than the right hand side, the benefit from the strategic aspect of patents (when no other firm patents) would dominate the loss in appropriability for $\alpha = e^{-\theta}\beta$.

The condition is more likely to hold when $\theta$ is higher. Larger congestion implies that a patenting firm can block, on average, a higher number of rivals, which directly translates into a higher strategic benefit from patenting. Similarly, the patent system is more likely to have the ability to increase competition when $\beta$ is lower. The reason is that low secrecy protection strength implies a lower loss in appropriability due to patenting.

This intuition helps explain why, to the best of my knowledge, no previous studies were able to find that with PUR the patent system can increase competition: it can only do is if patenting provides enough strategic benefits. Denicolo and Franzoni (2003), for example, develop a two-firm game without simultaneous innovation. Because of this, patenting does not grant a strategic advantage and cannot increase competition.

Even in previous models, with simultaneous innovation, the patent system cannot increase competition, as they do not explicitly consider the firms' incentive to secure lead time advantage. For example, in Kultti *et al.* (2007) patents can only affect the probability of becoming a monopolist. In their model, the patent system cannot incentivize firms to patent when this provides lower appropriability, because the strategic aspect of patents and investment in R&D are not linked, i.e. in the absence of lead time advantage, the strategic

[30] In particular, $\bar{n}$ could be any positive integer, so the theorem provides a sufficient condition for $s_1 \in (0,1)$.

benefit is too low.[31]

The patent system can increase competition even if (5) does not hold — the condition is only sufficient. For example, if $\bar{n} = 1$ (Bertrand competition), no firm is willing to duplicate the innovation. Then, without a patent system, a developed innovation will be produced under monopoly next period with probability $\beta$ and under perfect competition with probability $1 - \beta$. In an equilibrium where all firms patent the corresponding probabilities are $\alpha$ and $1-\alpha$. Thus, the patent system increases competition whenever $s_1 < 1$ and $\alpha < \beta$. This is the case for all patent strengths consistent with $s_1 \in (0, 1)$. Hence, the patent system can always increase competition when $\bar{n} = 1$. The reason is that secrecy cannot allocate market power to more than one firm under Bertrand competition. Thus, the patent system will increase competition whenever it decreases the chance that an innovation is produced under monopoly.

The analysis so far has only looked at a single industry. In practice, however, there are many different industries, say $j \in [1, 2, \ldots, J]$, which have different strengths of secrecy protection, $\beta(j)$, market tightness, $\theta(j)$, and discount factors $\gamma(j)$. It is, however, easy to see that the results can be generalized, so that the patent system may increase the degree of competition in some industries without adversely affecting other industries. If one follows the logic behind Theorem 2, this will be true if (5) holds for at least one industry $j$, such that $j \in \operatorname{argmin}\{e^{-\theta(j')}\beta(j') \mid j' \in [1, 2, \ldots, J]\}$. The intuition behind the condition is analogous to the single industry case, with the only exception that now the necessary condition for the patent system to increase competition is not $\alpha \leq e^{-\theta}\beta$, but rather $\alpha \leq e^{-\theta(j')}\beta(j')$ for all $j' \in [1, 2, \ldots, J]$.

[31] It should be noted that the assumption of lead time advantage affects the market structure as well. For example, in this model if two secrecy using firms innovate simultaneously, only the one which innovated first would begin commercially exploiting the innovation. On the other hand, in Kultti *et al.* (2007) both of these firms would exploit the innovation.

# 5 Welfare and the Optimal Patent Strength

The main results of the paper are not only of theoretical interest, but of practical as well. This section applies the insights of the preceding analysis in the context of the planner's problem to choose a patent strength which maximizes welfare. I find that the patent system is always welfare improving — it can induce all firms to disclose their innovations without increasing their market power. Depending on parameter values, however, it may be optimal for the planner to induce only some firms to patent. She can increase competition and, hence, welfare by setting a low patent protection strength. At the same time, however, this may decrease welfare because some firms might have incentives to not disclose their innovations and opt for secrecy, instead. Ideas which correspond to innovations protected by a secret remain in the pool, thus, lower disclosure implies a lower mass of new products is introduced to the consumer market each period, which leads to a decrease in welfare. Because of this the planner sets an optimal patent strength to strike a balance between increasing competition and inducing disclosure. If congestion is low, then the welfare loss due to reduced disclosure outweighs the gain due to higher competition and the planner chooses a patent strength that induces all firms to patent. If it is high, on the other hand, she finds it optimal to induce some firms to use secrecy protection.

To highlight the features of this trade off, I abstract the analysis from the impact of firms' equilibrium investment choices on welfare. Formally, $\pi(t) = \pi$ for $t \in [0, 1]$. Furthermore, for the purposes of this section, I follow Kultti *et al.* (2007), among others, and assume Bertrand competition in the consumer market. Formally, I set $d_n = 0$ for all $n \geq 2$, which implies that no firm is willing to share a secret with someone else. Hence, in equilibrium, all innovations are produced under monopoly or perfect competition. I denote the corresponding per period consumer surplus by $S_M$ and $S_C$, where $S_C > S_M + \pi$.[32] Also, the analysis focuses on a patent system which provides prior user rights, as this is the empirically more relevant case. Since profits are time independent and no firm has an incentive to duplicate an innovation,

[32]For consistency I assume that the per period consumer surplus is received at the same time firms receive their profits — after all races in a given period have ended. Hence, the surplus for period $T$ is received at $T + 1$ and, similarly to $\pi$, $S_M$ and $S_C$ are period $T$ discounted quantities.

I write $R_S$, $R_P$, $s$, and $\zeta$ instead of $R_S^n(t)$, $R_P^n(t)$, $s_n$, and $\zeta_n$.

## 5.1 Welfare

When there is a patent system, depending on the chosen patent strength, the planner can induce three equilibria — patenting equilibrium ($s = 0$), secrecy equilibrium ($s = 1$), and mixed equilibrium ($s \in (0, 1)$). It will be useful to consider each of these equilibria separately when we look at the planner's problem, so I denote $i = P, S, M, N$ for $s = 0$, $s = 1$, $s \in (0, 1)$, and the equilibrium without a patent system, receptively.

The planner's problem is the maximize the steady state value of welfare, which is equivalent to maximizing the expected net present value (ENPV henceforth) of welfare generated by all innovations made in a given period. I denote this quantity by $\bar{W}^i$, where $i$ stands for the equilibrium under study. By symmetry, $\bar{W}^i$ is the ENPV of welfare generated by a single innovation, denoted by $W^i$, times the mass of innovations made in a given period. As firms do not find it profitable to duplicate innovations, they innovate only if matched with a new (previously not innovated) idea. I also refer to such ideas as profitable and denote their steady state fraction in the pool by $\bar{N}^i$. Given the matching technology, it follows that each period the mass of innovations is $\nu(1 - e^{-\theta})\bar{N}^i$ (the mass of new ideas matched with at least one firm). If we denote the ENPV of per innovation profits and consumer surplus by $\Pi^i$ and $\mathbb{S}^i$, it follows that

$$\bar{W}^i = \nu(1 - e^{-\theta})\bar{N}^i W^i = \nu(1 - e^{-\theta})\bar{N}^i \Big(\Pi^i + \mathbb{S}^i\Big)$$

The above representation of welfare helps illustrate the intuition behind the analysis. $W^i$ captures the patent system's effect on welfare due to competition. Lower(higher) competition implies longer(shorter) expected duration of monopoly, which leads to lower(higher) ENPV of per innovation welfare, and ultimately to lower(higher) $\bar{W}^i$. At the same time, $\bar{N}^i$ captures the patent system's effect on welfare due to disclosure. Lower(higher) disclosure implies lower(higher) steady state fraction of new ideas, which leads to a lower(higher) mass of innovations made each period, and ultimately to lower(higher) $\bar{W}^i$.

The structure of welfare under $i = N, S, P$ is very similar, so it is useful to characterize it separately from the welfare under the mixed equilibrium. First, it is easy to see that the reward firms receive conditional on winning the race is $R_S = \pi/(1-\beta\gamma)$ for $i = N, S$, whereas it is $R_P = \pi/(1-\alpha\gamma)$ in the patenting equilibrium. Then, the ENPV of profits from participating in a race is given by $e^{-\theta}\pi/(1-p_i\gamma)-c(1)$, where $p_i$ stands for the strength of the chosen protection strategy, i.e. $p_P = \alpha$ and $p_S = p_N = \beta$. Given the matching technology, only a fraction $\bar{N}^i$ of firms choose to participate in a race, so the ENPV of profits from all innovations made in a given period is $\mu\bar{N}^i(e^{-\theta}\pi/(1-p_i\gamma) - c(1))$. Hence, the ENPV of profits per innovation is given by

$$\Pi^i = \frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-p_i\gamma} - c(1)\Big)$$

An innovation is produced under monopoly for as long as the protection strategy holds and under perfect competition every period after that. So, if an innovation is developed in period $T$ and its protection strategy fails in period $T+j$ the NPV of consumer surplus generated by that innovation is $((1-\gamma^j)S_M + \gamma^j S_C)/(1-\gamma)$. The probability that the protection strategy fails in period $T+j$ is simply $Pr$(the protection strategy has held for all previous periods) $\times$ $Pr$(the protection strategy fails in $T+j$) $= p_i^{j-1}(1-p_i)$. Then, we have that

$$\mathbb{S}^i = \sum_{j=1}^{\infty} p_i^{j-1}(1-p_i)\Big(\frac{S_M + \gamma^j(S_C - S_M)}{1-\gamma}\Big) = \frac{S_M}{1-p_i\gamma} + \frac{\gamma(1-p_i)}{(1-\gamma)(1-p_i\gamma)}S_C$$

Whenever a firm patents an innovation its specifications are disclosed and it becomes public knowledge. In that case, the corresponding idea is replaced with a new one in the pool. Thus, in the patenting equilibrium all ideas are profitable, i.e. $\bar{N}^P = 1$. In the secrecy and no patent system equilibria, however, firms do not disclose their innovations, so they become public knowledge only when the secret leaks. Thus, for $i = S, N$ we have the following law of motion.

$$N^i_T = e^{-\theta} N^i_{T-1} + (1-\beta)\Big((1-e^{-\theta})N^i_{T-1} + 1 - N^i_{T-1}\Big)$$

An idea is new in some period $T$ in one of two cases. First, if it was new at $T-1$ and no firm was matched with it, as captured by the term $e^{-\theta}N^i_{T-1}$. Second, if it was produced under monopoly during $T-1$ and secrecy protection failed, as captured by the second term. Thus, $\bar{N}^S = \bar{N}^N = (1-\beta)/(1-\beta+(1-e^{-\theta})\beta)$.

Then, $\bar{W}^i$ under the three equilibria is given by

$$\begin{aligned}\bar{W}^N = \bar{W}^S =& \nu(1-e^{-\theta})\frac{1-\beta}{1-\beta+(1-e^{-\theta})\beta}\left(\frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\beta\gamma} - c(1)\Big) + \right.\\ &\left. + \frac{S_M}{1-\beta\gamma} + \frac{\gamma(1-\beta)S_C}{(1-\gamma)(1-\beta\gamma)}\right)\end{aligned}$$

$$\bar{W}^P = \nu(1-e^{-\theta})\left(\frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\alpha\gamma} - c(1)\Big) + \frac{S_M}{1-\alpha\gamma} + \frac{\gamma(1-\alpha)S_C}{(1-\gamma)(1-\alpha\gamma)}\right)$$

Next, let us characterize the structure of welfare for $i = M$. In a mixed equilibrium firms make the same expected profits from participating in a race, regardless of which protection strategy they use. Hence, similarly to the other equilibria

$$\Pi^M = \frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\beta\gamma} - c(1)\Big)$$

The NPV of the consumer surplus for an innovation which was made in period $T$ and whose protection strategy fails at $T+j$ is still given by $((1-\gamma^j)S_M + \gamma^j S_C)/(1-\gamma)$. In the mixed equilibrium, however, some innovations are protected by a secret and some by a patent. Hence, the ex-ante probability that an innovation's protection fails in period $T+j$ is given by $Pr$(the innovation is protected by a secret) $\times$ $Pr$(secrecy protection fails in $T+j$) $+Pr$(the innovation is protected by a patent) $\times$ $Pr$(patent protection fails in $T+j$) $= \zeta\beta^{j-1}(1-\beta) + (1-\zeta)\alpha^{j-1}(1-\alpha)$. Thus, $\mathbb{S}^M = \zeta\mathbb{S}^S + (1-\zeta)\mathbb{S}^P$. This implies that the ENPV of welfare generated by a single innovation in the mixed equilibrium is given by

$$\begin{aligned}W^M =&\frac{\theta}{1-e^{-\theta}}\Big(\frac{e^{-\theta}\pi}{1-\beta\gamma}-c(1)\Big)+(1-\zeta)\frac{S_M}{1-\alpha\gamma}+\zeta\frac{S_M}{1-\beta\gamma}+\\&+(1-\zeta)\frac{\gamma(1-\alpha)S_C}{(1-\gamma)(1-\alpha\gamma)}+\zeta\frac{\gamma(1-\beta)S_C}{(1-\gamma)(1-\beta\gamma)}\end{aligned}$$

The law of motion for $\bar{N}^M$ is similar to the one in the secrecy and no patent system equilibria. The only difference is that in the mixed equilibrium a fraction $1-\zeta$ of all innovations are patented and become public knowledge right away. Hence,

$$N_T^M = e^{-\theta}N_{T-1}^M + (1-\zeta)(1-e^{-\theta})N_{T-1}^M + (1-\beta)\Big(\zeta(1-e^{-\theta})N_{T-1}^M + 1 - N_{T-1}^M\Big)$$

Then, the steady state fraction is given by $\bar{N}^M = (1-\beta)/(1-\beta+(1-e^{-\theta})\beta\zeta)$.

## 5.2 The Optimal Patent Strength

It is instructive to approach the planner's problem in the following manner. First, I find the optimal patent strength consistent with each equilibrium. After that, I compare the resulting optimized welfare. To this end, one first need to find the range of patent strengths consistent with each equilibrium. The next lemma gives the result.

**Lemma 3.** *At stage 2 firms choose secrecy with probability $s$, where*

$$s = \begin{cases} 0 & \text{if } \alpha > \beta,\\ \frac{1}{\theta}ln\Big(\frac{1-\alpha\gamma}{1-\beta\gamma}\Big) & \text{if } \alpha \in [\max\{0, \frac{\beta\gamma+e^{-\theta}-1}{e^{-\theta}\gamma}\}, \beta],\\ 1 & \text{if } \alpha < \frac{\beta\gamma+e^{-\theta}-1}{e^{-\theta}\gamma}.\end{cases}$$

The proof is immediate from the proof of lemma 4 in the appendix. As in the general model from the previous sections, all firms patent if patent protection is at least as strong as the effective protection under secrecy. Since no firm is willing to duplicate an innovation, this effective protection is simply $\beta$. Furthermore, if the strategic advantage of patenting is large enough, a secrecy equilibrium may not be achievable: when $e^{-\theta} < 1-\beta\gamma$ congestion is very high, so the strategic aspect of patenting is always large enough to induce at least some firms to use patent protection.

Next, we turn to the planner's problem. Proposition 2 establishes that it is never optimal to have a patent system which does not provide incentives for any firm to patent.

**Proposition 2.** *The optimal patent strength consistent with a patenting equilibrium is* $\alpha_P = \beta$*. Moreover, a secrecy equilibrium is never optimal.*

The proof is in the appendix. The intuition behind the result is straightforward. In the patenting equilibrium all firms disclose their innovations, so the planner can increase competition at no cost. Thus, she sets $\alpha = \beta$ — the patent strength which maximizes competition subject to the equilibrium being patenting. At $\alpha_P$ both the patenting and secrecy equilbria provide the same expected duration of monopoly, namely $1/(1-\beta)$. Hence, the ENPV of per innovation welfare is the same in both equilibria, $W^P(\alpha_P) = W^S$.[33] Welfare in the patenting equilibrium, however, can be strictly larger than that in the secrecy equilibrium because of disclosure. In the secrecy equilibrium not all innovations are public knowledge, because of this only a fraction $\bar{N}^S < 1$ of ideas are new. This lower mass of profitable R&D projects effectively reduces the total mass of innovations made each period, as compared to the patenting equilibrium.

**Corollary 2.** *The maximum welfare when there is a patent system is always strictly larger than the welfare when there is no patent system.*

*Proof.* This is immediate from the fact that the maximum welfare with a patent system is at least as large as $\bar{W}^P(\alpha_P)$, proposition 2, and $\bar{W}^N = \bar{W}^S$.

□

As in most previous works (see for example, Denicolo and Franzoni (2003) and Kultti *et al.* (2007)) the patent system is welfare improving. The intuition behind the result is the same as in the previous paragraph. At $\alpha = \beta$ the expected duration of monopoly is the

[33] $\bar{W}^N$ and $\bar{W}^S$ are independent of the patent strength. However, to be explicit, I use $\bar{W}^M(\alpha)$ and $\bar{W}^P(\alpha)$ when the ENPV of welfare is evaluated at a particular patent strength in the mixed and patenting equilibria.

same under the patenting equilibrium and the equilibrium without a patent system, hence, $W^P(\alpha_P) = W^N$. However, disclosure and, hence, the mass of innovations made each period is lower in the equilibrium without a patent system.

The intuition behind the result differs somewhat from previous papers, however. For example, Denicolo and Franzoni (2003) find that the patent system is welfare improving because of two reasons. First, in their model the dead weight loss under patenting may be smaller than the dead weight loss under secrecy (even though the expected duration of monopoly is longer under patenting than under secrecy). Second, because of disclosure, patents allow society to avoid wasteful duplication of R&D effort. In contrast, in this paper the result is driven by the diffusion aspect of disclosure: whenever an innovation is patented, it is disclosed and a new idea, which can potentially be innovated and generate welfare, enters the pool. When an idea is developed in secrecy, however, the innovation is not disclosed and society bares the cost of foregone welfare.

The present paper features the novel result that the optimal patent strength may lie in the region consistent with a mixed equilibrium. In that region the planner faces a trade off between increasing competition and providing incentives for firms to disclose their innovations. As the patent strength decreases there is the potential of reducing the expected duration of monopoly and, hence, increasing the ENPV of per innovation welfare. At the same time, however, more firms use secrecy protection, which reduces disclosure and, hence, the mass of profitable innovations. Thus, the planner aims to strike a balance between these two opposing effects when she decides on the optimal patent strength.

The welfare loss from reduced disclosure depends on the speed with which firms switch from patenting to secrecy as the patent strength decreases ($\partial s/\partial\alpha$). If the firms' marginal response is large, so is the impact of $\alpha$ on $\bar{N}^M$. The gain from competition, on the other hand, depends on both this speed and the dead weight loss from monopoly. The welfare gain from reduced patent strength is higher for larger dead weight loss, but more importantly, it is smaller for large $\partial s/\partial\alpha$. To see the intuition clearly, observe that in the mixed equilibrium the ex-ante expected duration of monopoly is $\zeta/(1-\beta)+(1-\zeta)/(1-\alpha)$. As $\alpha$ decreases, so does the expected duration of monopoly of patented innovations. At the same time, however, more firms switch to secrecy protection which decreases the ex-ante expected duration of monopoly (because $\alpha < \beta$ for $s \in (0, 1)$). The larger $\partial s/\partial\alpha$ is, the larger $\partial\zeta/\partial\alpha$ is, and the larger is the marginal decrease in ex-ante expected duration of monopoly.

Thus, if firms switch too fast, the planner may find it optimal to induce a patenting equilibrium and not sacrifice disclosure. If they switch slow enough, however, she may find it optimal to provide weak patent protection and induce the mixed equilibrium, as the welfare gains from higher competition are larger than the costs of reduced disclosure. The next proposition gives a sufficient condition for the gains from competition to outweigh the loss from reduced disclosure for at least some patent strengths in the mixed equilibrium as compared to the patenting equilibrium.

**Proposition 3.** *If*

$$\frac{S_C - S_M}{S_C} \geq \frac{e^{-\theta}\beta}{1-\beta}\left(1 + \frac{\gamma(1-\beta)}{1-\gamma}\right) \tag{7}$$

*then the optimal patent strength is such that the equilibrium is mixed.*

The proof is in the appendix. The left hand side of (7) captures the monopoly dead weight loss in per period consumer surplus. The right hand side represents the relative cost of of destroying that monopoly. The term in the brackets represents an upper bound on the welfare loss from decreasing disclosure: lower disclosure implies a lower mass of profitable ideas and, hence, a lower mass of innovations made each period. The term $e^{-\theta}\beta/(1-\beta)$ captures the speed with which firms switch protection strategies. When secrecy protection is low, firms are slow to switch because secrecy allows for only minor increases in appropriability. Similarly, firms are slow to switch to secrecy when there is higher congestion. This is due to the strategic aspect of patents. When $\theta$ is higher, so is the expected number of competitors any given firm would face at the race. This implies a higher strategic benefit because an innovator that patents can block a higher number of rivals, on average. Thus, the strategic aspect of patents is important not only for the patent system's ability to increase competition, but also for its ability to allow the planner to turn higher competition into welfare gains. More precisely, in the current setting the patent system provides a strictly higher degree of competition (as compared to $i = N$) when $s \in (0, 1)$. Thus, the strategic advantage is always large enough to allow the patent system to erode market power. If (7) does not hold, however, the strategic benefit from patenting is relatively low and firms switch protection strategies relatively fast. So, the planner finds it too costly to increase competition. Only when congestion is high enough, the strategic advantage of patents is sufficient to allow the planner to exploit the patent system's ability to increase competition for welfare gains.

The result is interesting because of a couple of reasons. First, it suggests that the analysis in the preceding sections may have important practical applications for policy makers. Second, the result suggests that it may be optimal to provide weak incentives to firms, so that only a fraction of them use patent protection. To the best of my knowledge, this is the first paper which finds a mixed equilibrium might be optimal. The majority of previous studies do not feature an equilibrium where some firms would patent and some would keep identical innovations secret. Even studies which feature such an equilibrium, for example Kultti *et al.* (2007), have found that it is socially optimal to incentivize all firms to use patent protection.[34] This is the case because, to the best of my knowledge, previous studies do not feature a patent system that provides prior user rights and can increase competition and, as the preceding analysis suggests, the patent system's ability to increase competition is the key reason why the planner might want to induce a mixed equilibrium.

# 6 Conclusion

The traditional view of the patent system is that it creates temporary monopolies in order to stimulate disclosure of information and/or create incentives for firms to innovate. This paper develops a dynamic equilibrium search model of innovation which aims to show that

[34] In their model the dead weight loss due to monopolies is the same in the equilibrium without a patent system, the secrecy equilibrium, and the mixed equilibrium. Moreover, in the patenting equilibrium it is at least as large as in the other equilibria. Hence, the planner has no welfare gains from eliciting the mixed equilibrium. Kultti *et al.* (2007) also consider firms' incentives to innovate. In their model, however, firms make the same equilibrium investment in the equilibrium without a patent system, the secrecy equilibrium, and the mixed equilibrium. Moreover, this level of investment is achievable in the patenting equilibrium as well. Thus, the planner cannot induce any gains in welfare due to changing incentives to innovate from eliciting a mixed equilibrium.

this traditional view does not hold when one takes into account the possibility of duplication, simultaneous innovation, and the importance of lead time advantage. In fact, the patent system can reduce the market power of innovators, i.e. increase competition, while providing incentives for at least some firms to patent.

The patent system can increase competition, regardless of whether or not it provides PUR. This is achieved by setting weak enough patent protection and providing incentives for firms to patent given the reduced appropriability. Without PUR, the patent system can incentivize firms by reducing the option value of secrecy through the threat of duplicative innovation. With a patent system, a firm can independently duplicate and patent the innovation, thus, excluding the original innovator from any profits. In contrast, without a patent system, the duplicator uses secrecy and both the original innovator and the duplicator yield duopoly profits. With PUR, the patent system can provide incentives for firms to patent, because patents have a strategic aspect (a firm that patents can block all rivals who innovate simultaneously and opt for secrecy from commercially exploiting the innovation). When firms want to secure a lead time advantage, the benefit due to the strategic advantage of patents could outweigh the loss in appropriability. Then, at least some firms choose patent protection even if this provides them with lower market power as compared to secrecy.

The results of this paper are not only of theoretical interest, but of practical as well. In a series of works Boldrin and Levine make a case against the patent system by pointing out the "huge" social costs due to temporary monopolies induced by patents.[35] In light of the results of this paper, however, this argument can be viewed in favor of the patent system, as it can erode temporary monopolies and, in fact, reduce the aforementioned social costs.

Furthermore, this paper analyses the welfare implications of a patent system which can increase competition. I find that the patent system is always welfare improving and that it can induce all firms to patent (and disclose their innovations) without imposing any addi-

[35]See, for example, Boldrin and Levine (2005), Boldrin and Levine (2008), Boldrin *et al.* (2008), and Boldrin and Levine (2013). More precisely, they argue that the rationale behind the traditional view of patents implies that the presence of a patent system decreases welfare in practice. This is the case since the social loss due to monopolies is too large and cannot be compensated for by the diffusion of information and increased incentives to innovate (which the authors argue yield very low social benefit).

tional social costs due to temporary monopolies. I also find that, depending on parameter values, the patent system can, in fact, improve welfare by directly reducing these costs. If the strategic advantage of patenting is large enough, then the planner finds it optimal to induce an equilibrium in which only a fraction of all firms use patent protection. If this is the case, then the patent system induces disclosure of at least some innovations and at the same time increases competition. This result suggests that the patent system's ability to erode market power may be a key factor when it comes to its capacity to improve welfare.

# 7 Appendix

## 7.1 Proofs Omitted from the Text

**Proof of Lemma 1:**

*Proof.* Consider firms matched with an idea that has been previously developed by $n-1$ innovators and look at stage three. First, notice that there is no pure equilibrium where firms bid some $t \in [0,1]$ with certainty.[36] Observe that $F_j^n(t)$ has no atoms and the support is a connected interval.[37] Let $k_n(p)$ be the firm's expected payoff from the race if it has chosen to patent and $k_n(s)$ if it has chosen secrecy. Also, let $N$ represent the number of competitors a firm faces in the race. Now, consider a firm that chooses to patent. In equilibrium, a firm bidding any $t$ in the support of $F_P^n(t)$ receives an expected payoff of

$$P(\text{win}|t)R_P(t) - c(t) = k_n(p)$$

The probability a firm will win the race when there are exactly $m$ competitors is the chance that exactly $i$ competitors choose patent protection and the firm bid a lower innovation time

[36] If this were the case, then a firm can do strictly better by bidding $t'$ just before $t$, as this means it will win the race for sure.

[37] This can be seen easily by applying standard arguments.

than all of them, summed across $0 \leq i \leq m$. Then one can solve for $F_P^n(t)$:

$$\sum_{m=0}^{\infty} P(N=m) \sum_{l=0}^{m} \binom{m}{l} (1-s_n)^l (1-F_P^n(t))^l s_n^{m-l} R_P(t) - c(t) = k_n(p)$$

$$F_P^n(t) = \frac{1}{\theta(1-s_n)} \ln\Big(\frac{R_P(t)}{c(t)+k_n(p)}\Big) \tag{8}$$

where the second equation follows by applying the binomial theorem, substituting for the probabilities using the matching function, and using the fact that $e^{\theta} = \sum_{m=0}^{\infty} \theta^m/m!$. Now, suppose $\bar{S}_P^n$ is the upper bound of the support of $F_P^n(t)$. At $t = \bar{S}_P^n$ the firm will win the race only when it does not face a competitor that patents (which happens with probability $e^{-\theta(1-s_n)}$). Then, it follows that, from (8)

$$e^{-\theta(1-s_n)} R_P(\bar{S}_P^n) - c(\bar{S}_P^n) = k_n(p) \tag{9}$$

Also, let the lower bound of the support be $\underline{S}_P^n$, then at $t = \underline{S}_P^n$ the firm will always win the race and, hence,

$$k_n(p) = R_P(\underline{S}_P^n) - c(\underline{S}_P^n) \tag{10}$$

(9) and (10) uniquely determine $k_n(p)$ and $\underline{S}_P^n$ as a function of $\bar{S}_P^n$. One can determine $\bar{S}_P^n$ from firms' maximization behavior:

**Claim 1.** *The upper bound of the support of $F_P^n(t)$ is given by*

$$\bar{S}_P^n = 1 = \operatorname*{argmax}_t e^{-\theta(1-s_n)} R(\bar{S}_P^n) - c(\bar{S}_P^n)$$

*Proof.* Let us prove the claim by contradiction. Suppose $\bar{S}_P^n < t_m$ where $t_m$ is the unique solution to the maximization problem. Then, a firm can deviate by choosing $t_m$ as the upper bound of the support instead and yield strictly higher expected profits since the chance of winning will be unaffected (there are no atoms), but the net reward will be higher. On the other hand if $\bar{S}_P^n > t_m$, a deviant can choose to place the upper bound of the support at $t_m$

and then she will earn strictly higher expected profits as

$$
\begin{aligned}
P(\text{win}|t_m)R_P(t_m) - c(t_m) \geq & \sum_{m=0}^{\infty} P(N=m)(s_n)^m R_P(t_m) - c(t_m) \\
> & \sum_{m=0}^{\infty} P(N=m)(s_n)^m R_P(\bar{S}_P^n) - c(\bar{S}_P^n)
\end{aligned}
$$

The first inequality follows by the fact that the chance of winning cannot be less when the deviant bids a lower $t$ and the second inequality by the fact that $t_m$ is the unique maximizer. Thus, the upper bound must solve the maximization problem. Moreover, by assumption the unique maximizer is 1 since

$$
e^{-\theta(1-s_n)} R_P'(\bar{S}_P^n) - c'(\bar{S}_P^n) = e^{-\theta(1-s_n)} \pi'(\bar{S}_P^n) - c'(\bar{S}_P^n) \geq \pi'(\bar{S}_P^n) - c'(\bar{S}_P^n) > 0
$$

for all $\bar{S}_P^n < 1$ and the payoff is negative for all $\bar{S}_P^n > 1$.

□

Then, equations (8), (9), (10), and Claim 1 uniquely characterize the expected payoff in equilibrium, the equilibrium CDF, and its bounds, when the firm patents.

Analogously, when the firm chooses secrecy to protect its innovation the equilibrium CDF could be solved for by observing that

$$
\begin{aligned}
\sum_{m=0}^{\infty} P(N=m)(1-F_S^n(t))^m s_n^m R_S^n(t) - c(t) &= k_n(s) \\
F_S^n(t) &= \frac{1}{\theta s_n} \ln\Big(\frac{R_S^n(t)}{c(t) + e^{-\theta} R_S^n(\bar{S}_S^n) - c(\bar{S}_S^n)}\Big) - \frac{1-s_n}{s_n}
\end{aligned} \tag{11}
$$

Since the firm will win the race only if all competitors in the race choose secrecy as well and it bid a lower time than all of them. At $t = \bar{S}_S^n$ the firm will win only if it faces no competitors (which happens with probability $e^{-\theta}$), hence,

$$
e^{-\theta} R_S^n(\bar{S}_S^n) - c(\bar{S}_S^n) = k_n(s) \tag{12}
$$

Similarly, if $\underline{S}_S^n$ is the lower bound of the support, at $t = \underline{S}_S^n$ the firm will win the race as long as no competitor chooses patent protection. Then,

$$e^{-\theta(1-s_n)} R_S^n(\underline{S}_S^n) - c(\underline{S}_S^n) = k_n(s) \tag{13}$$

Thus, analogously to the case when the firm patents, the following claim holds.

**Claim 2.** *The upper bound of the support of $F_p(t)$ is given by*

$$\bar{S}_s^n = 1 = \underset{t}{\operatorname{argmax}}\, e^{-\theta} R_S^n(\bar{S}_s^n) - c(\bar{S}_s^n)$$

I omit the proof as it is analogous to that of Claim 1. Then, equations (11), (12), (13), and Claim 2 uniquely characterize the expected payoff in equilibrium, the equilibrium CDF, and its bounds, when the firm chose secrecy. At this point one needs to show the proposed strategies follow valid CDFs and are indeed an equilibrium. However, I omit this as it is straightforward.

□

**Proof of Lemma 2:**

*Proof.* Let us now derive $s_n$, $\bar{\alpha}_n$, and $\underline{\alpha}_n$. From Claim 1 and Claim 2, the upper bounds of the support for $F_S^n(t)$ and $F_P^n(t)$ are the same. Then, using (9) and (12), the probability firms will place on using secrecy protection, $s_n$, in the mixed equilibrium is given by

$$s_n = \frac{1}{\theta} \ln\left(\frac{R_S^n(1)}{R_P(1)}\right) \tag{14}$$

Next, I will show that for high enough patent strength, $\alpha \geq \bar{\alpha}_n$, the equilibrium is such that $s_n = 0$ and for low enough patent strength, $\alpha \leq \underline{\alpha}_n$, it is such that $s_n = 1$. The argument follows by induction. First, take $n = \bar{n}$ and observe that by equations (1) and (2), it follows that $R_P(1)$ is strictly increasing in $\alpha$. Second, by assumption $d_{\bar{n}+1} = 0$, so the $\bar{n} + 1$-st innovator chooses patenting regardless of the patent strength. Hence, $s_{\bar{n}+1} = 0$ and $\zeta_{\bar{n}+1} = 0$. Thus, from equations (3) and (4), it follows that $R_S^{\bar{n}}(1)$ is independent of $\alpha$.

Hence, $R_S^{\bar{n}}(1)/R_P(1)$ is strictly decreasing in $\alpha$. Thus, for $\alpha \geq \bar{\alpha}_{\bar{n}}$ we have that $s_{\bar{n}} = 0$ and for $\alpha \leq \underline{\alpha}_{\bar{n}}$ we have that $s_{\bar{n}} = 1$. Moreover, $s_{\bar{n}}$ is weakly decreasing in $\alpha$ and so is $\zeta_{\bar{n}}$.

Next, take any $n$ such that $1 \leq n < \bar{n}$ and suppose that $\zeta_{n+i}$ is weakly decreasing in $\alpha$ for $i > 0$. Then, by equations (3) and (4), it follows that $R_S^n(1)$ is weakly decreasing in $\alpha$ as well. Hence, $R_S^n(1)/R_P(1)$ is strictly decreasing in $\alpha$. Thus, for $\alpha \geq \bar{\alpha}_n$ we have that $s_n = 0$ and for $\alpha \leq \underline{\alpha}_n$ we have that $s_n = 1$. Moreover, $s_n$ is weakly decreasing in $\alpha$ and so is $\zeta_n$.

From (14) we can implicitly solve for the upper and lower bounds $\bar{\alpha}_n$ and $\underline{\alpha}_n$. In particular, $\bar{\alpha}_n$ is such that[38]

$$V_P - V_S^n = \pi(0)(1-\gamma)(1-d_n) \tag{15}$$

And $\underline{\alpha}_n$ is such that

$$V_P = e^{-\theta}V_S^n + \pi(0)(1-e^{-\theta}d_n)(1-\gamma) \tag{16}$$

The three equations — (14),(15), and (16) characterize the equilibrium protection strategies and the bounds of the patent strength consistent with each equilibrium.

□

**Equilibrium stage one outcome:**

Here I characterize the equilibrium outcome in stage one. First, suppose that there is a patent system. Let $N_T^n$ be the fraction of ideas innovated by $n$ firms at time $T$, just before matches are made. Then, recall that nature matches firms with ideas at random. Thus, it follows that the probability of matching a firm with an idea previously developed by $n$ innovators is just the fraction of these ideas in the pool of readily available ideas. Then, for all $T$ in the patenting equilibrium $N_T^0 = 1$ and $N_T^n = 0$ for all $n \neq 0$, because all ideas in the pool are new ideas.

Next, let us look at the case where some firms might use secrecy protection, i.e. $s_n \in (0, 1]$ for at least one $n$. Take any $T$ and $n \neq 0$ and observe that ideas are developed by $n$ firms at

[38]Since $\pi(1) = \gamma\pi(0)$.

period $T$ in one of two cases. First, if at the beginning of period $T-1$ they were developed by $n$ firms, at $T-1$ no firm was matched with them, and secrecy held at the beginning of $T$. Second, if at the beginning $T-1$ they had been developed by $n-1$ firms, at $T-1$ they were matched with at least one firm which innovated under secrecy, and at the beginning of $T$ secrecy protection held.

Then, for any $n \neq 0$ we have that

$$N_T^n = e^{-\theta}\beta N_{T-1}^n + (1-e^{-\theta})\beta\zeta_n N_{T-1}^{n-1} \tag{17}$$

Similarly, new ideas ($n=0$) at time $T$ come from two sources — ideas which are new and stay new, and ideas which replace previously developed ones in the pool. If an idea has not been previously developed and no firm was matched with it at time $T-1$, it is still new at $T$. On the other hand, new ideas replace old ones whenever the secret has leaked or the idea is patented. It is worth nothing that if an idea was new in the beginning of $T-1$ and it was matched with at least one firm that innovated under patent protection at $T-1$, then it is replaced by a new idea at $T$. Similarly, if an idea was new in the beginning of $T-1$, it was matched with at least one firm that innovated under secrecy at $T-1$, and secrecy protection failed at $T$, then the idea is replaced by a new one at $T$. Hence,

$$\begin{aligned} N_T^0 =& e^{-\theta}N_{T-1}^0 + (1-e^{-\theta})\sum_{i=0}^{\bar{n}}(1-\zeta_{i+1})N_{T-1}^i + \\ &+ (1-e^{-\theta})\sum_{i=0}^{\bar{n}}(1-\beta)\zeta_{i+1}N_{T-1}^i + (1-\beta)\sum_{i=1}^{\bar{n}}N_{T-1}^i \end{aligned} \tag{18}$$

Equations (17) and (18) characterize nature's moves for all $T$.

When there is no patent system, the laws of motion are given by

$$N_T^n = e^{-\theta}\beta N_{T-1}^n + (1-e^{-\theta})\beta N_{T-1}^{n-1} \quad \text{for } n \neq 0, \bar{n} \tag{19}$$

$$N_T^{\bar{n}} = \beta N_{T-1}^{\bar{n}} + (1-e^{-\theta})\beta N_{T-1}^{\bar{n}-1} \tag{20}$$

$$N_T^0 = e^{-\theta} N_{T-1}^0 + (1-e^{-\theta}) \sum_{i=0}^{\bar{n}-1} (1-\beta) N_{T-1}^i + (1-\beta) \sum_{i=1}^{\bar{n}} N_{T-1}^i \tag{21}$$

The intuition and derivation is similar to the case when there is a patent system, so I omit them.

**Proof of Corollary 1:**

*Proof.* First, observe that $\bar{\alpha}_{\bar{n}+1} = s_{\bar{n}+1} = \zeta_{\bar{n}+1} = 0$ as an immediate consequence of assumption A2. Also, it must be the case that $\bar{\alpha}_n \geq 0$ for all $n \leq \bar{n}$. Hence, $\bar{\alpha}_n \geq \bar{\alpha}_{\bar{n}+1}, \forall n \leq \bar{n}$. Thus, what is left to show is that $e^{-\theta}\beta = \bar{\alpha}_1 \geq \bar{\alpha}_2 \geq \bar{\alpha}_3 \geq \cdots \geq \bar{\alpha}_{\bar{n}}$. I will first show the inequalities hold and then prove that $e^{-\theta}\beta = \bar{\alpha}_1$.

First, observe that by equation (4), it follows that

$$\begin{aligned}
V_S^n = & \frac{d_n \pi(0)}{1-e^{-\theta}\beta\gamma} + \sum_{i=1}^{\infty} \frac{d_{n+i}\pi(0)(1-e^{-\theta})^i \beta^i \gamma^i}{(1-e^{-\theta}\beta\gamma)^{i+1}} \prod_{k=1}^{i} \zeta_{n+k} \\
\leq & \frac{d_n \pi(0)}{1-e^{-\theta}\beta\gamma} + \sum_{i=1}^{\infty} \frac{d_n \pi(0)(1-e^{-\theta})^i \beta^i \gamma^i}{(1-e^{-\theta}\beta\gamma)^{i+1}} \\
\leq & \frac{d_n \pi(0)}{1-e^{-\theta}\beta\gamma} \Big[1 + \sum_{i=1}^{\infty} \Big(\frac{(1-e^{-\theta})\beta\gamma}{1-e^{-\theta}\beta\gamma}\Big)^i\Big] \\
\leq & \frac{d_n \pi(0)}{1-\beta\gamma}
\end{aligned}$$

Hence, by equation (3), it follows that

$$R_S^n(1) \leq d_n \Big(\frac{\pi(0)}{1-\beta\gamma} - \pi(0)(1-\gamma)\Big)$$

Thus, by equations (1) and (2), it follows that at $\alpha = 1$, $R_P(1) > R_S^n(1)$, for all $n$. Hence, by Lemma 2, it follows that $1 > \max_n\{\bar{\alpha}_n | n \leq \bar{n}\}$. Now fix any $m$ such that $1 \leq m \leq \bar{n}$, I will show that $\bar{\alpha}_m = \max_n\{\bar{\alpha}_n | m \leq n \leq \bar{n}\}$. For $m = \bar{n}$, the claim holds trivially. So, take $m < \bar{n}$ and proceed the proof by contradiction. Suppose, to the contrary, that there exists some $k$, such that $m < k \leq \bar{n}$ with $\bar{\alpha}_k = \max_n\{\bar{\alpha}_n | m \leq n \leq \bar{n}\}$ and $\bar{\alpha}_k > \bar{\alpha}_m$. Then, equations (1) and (2) together with Lemma 2 imply that $R_S^k(1; \bar{\alpha}_k) = R_P(1; \bar{\alpha}_k) > R_P(1; \bar{\alpha}_m) = R_S^m(1; \bar{\alpha}_m)$, where I have explicitly denoted the values of $\alpha$ at which the reward functions are evaluated. But, from Lemma 2 it follows that at $\alpha = \bar{\alpha}_k$, it is the case that $s_n = 0$ for all $n$ such that $m \leq n \leq \bar{n}$. Then, from equations (3) and (4) it follows that $R_S^n(1; \bar{\alpha}_k) = d_n\gamma\pi(0) + d_n e^{-\theta}\beta\gamma\pi(0)/(1 - e^{-\theta}\beta\gamma)$, for $n$ such that $m \leq n \leq \bar{n}$. Thus, $R_S^m(1; \bar{\alpha}_k) \geq R_S^k(1; \bar{\alpha}_k)$, since $d_m = \max_n\{d_n | m \leq n \leq \bar{n}\}$ by assumption A2. Hence, $R_S^m(1; \bar{\alpha}_k) > R_S^m(1; \bar{\alpha}_m)$. But this is a contradiction since $R_S^m(1)$ is weakly decreasing in $\alpha$ by the arguments in the proof of Lemma 2. Thus, it must be the case that $\bar{\alpha}_m = \max_n\{\bar{\alpha}_n | m \leq n \leq \bar{n}\}$. Hence, $\bar{\alpha}_1 \geq \bar{\alpha}_2 \geq \bar{\alpha}_3 \geq \cdots \geq \bar{\alpha}_{\bar{n}}$.

Lastly, we need to show that $e^{-\theta}\beta = \bar{\alpha}_1$. By Lemma 2, it follows that, $R_P(1; \bar{\alpha}_1) = R_S^1(1; \bar{\alpha}_1)$. Since, at $\alpha = \bar{\alpha}_1 = \max_n\{\bar{\alpha}_n | n \leq \bar{n}\}$, we have that $s_n = \zeta_n = 0, \forall n \leq \bar{n}$, it follows that $R_S^1(1; \bar{\alpha}_1) = d_1\gamma\pi(0) + d_1 e^{-\theta}\beta\gamma\pi(0)/(1 - e^{-\theta}\beta\gamma)$. As $d_1 = 1$ by assumption A2, straightforward algebra implies that $e^{-\theta}\beta = \bar{\alpha}_1$.

□

**Proof of Theorem 2:**

*Proof.* First, we need the following lemma.

**Lemma 4.** *If* $\frac{1}{\gamma} > \left(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\right)\left(\frac{\beta}{1-\beta\gamma}\right)$, *then* $\alpha = e^{-\theta}\beta$ *implies that* $s_1 \in (0, 1)$.

*Proof.* With PUR the repeated game firms place is exactly the same with the only difference that now $R_P^n(t)$ depends on $n$. Thus, it is straightforward to establish that the investment decisions at stage three will follow the CDFs

$$F_p^n(t) = \frac{1}{\theta(1 - s_n)}\ln\Big(\frac{R_P^n(t)}{c(t) + k^n(p)}\Big)$$
$$F_s^n(t) = \frac{1}{\theta s_n}\ln\Big(\frac{R_S^n(t)}{c(t) + k^n(s)}\Big) - \frac{1 - s_n}{s_n}$$

Where $k^n(p) = e^{-\theta(1-s)} R_P^n(\bar{S}) - c(\bar{S})$, $k^n(s) = e^{-\theta} R_S^n(\bar{S}) - c(\bar{S})$ and $\bar{S} = 1$.

Similarly, at stage two if firms place a probability of $s_n \in (0,1)$ on playing secrecy, then

$$s_n = \frac{1}{\theta} \ln\Big(\frac{R_S^n(1)}{R_P^n(1)}\Big)$$

The difference with PUR comes from the reward structure. In particular, $R_P^n(t)$ has the following form

$$R_P^n(t) = d_n \pi(t) + \alpha\gamma V_P^n \tag{22}$$

where $V_P^n = d_n \pi(0) + \alpha\gamma V_P^n$. If a firm chooses secrecy, for $n < \bar{n}$, the reward will be

$$\begin{aligned} R_S^n(t) =& d_n \pi(t) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1 - e^{-\theta})\beta\gamma V_S^{n+1} + \\ &+ (1 - \zeta_{n+1})(1 - e^{-\theta})\beta\gamma V_P^{n+1} \end{aligned} \tag{23}$$

where $V_S^n = d_n \pi(0) + e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1 - e^{-\theta})\beta\gamma V_S^{n+1} + (1 - \zeta_{n+1})(1 - e^{-\theta})\beta\gamma V_P^{n+1}$. For $n = \bar{n}$, the reward is given by

$$R_S^{\bar{n}}(t) = d_{\bar{n}} \pi(t) + \beta\gamma V_S^{\bar{n}} \tag{24}$$

where $V_S^{\bar{n}} = d_{\bar{n}} \pi(0) + \beta\gamma V_S^{\bar{n}}$.

Next, observe that if $\alpha = e^{-\theta}\beta$, then $R_P^1(1) < R_S^1(1)$. Hence, $s_1 > 0$. Also, $s_1 < 1$ if and only if

$$V_S^1 - V_P^1 < (1 - e^{-\theta})(V_S^1 - \pi(0)(1 - \gamma)) \tag{25}$$

From the reward structure, it is fairly straightforward to establish that $V_S^1 \leq \frac{\pi(0)}{1-\beta\gamma}$. Hence, a sufficient condition for $s_1 < 1$ is

$$\frac{\pi(0)}{1 - \beta\gamma} - V_P^1 < (1 - e^{-\theta})\Big(\frac{\pi(0)}{1 - \beta\gamma} - \pi(0)(1 - \gamma)\Big)$$

which simplifies to the condition in the lemma.

□

Let us now prove Theorem 2. Again, if $T = 1$ the innovation is produced under monopoly for sure and $P_P(n) = P_N(n)$ for all $n$. So, fix $T \geq 2$.

When $\bar{n} = 1$, it follows that $P_N(1) = \beta^{T-1}$ and $P_N(n) = 0$ for $n \neq 1, \infty$. Since no firm is willing to duplicate the innovation (even if it has the option to patent), it follows that $\bar{\alpha}_1 = \beta$ and $P_P(n) = P_N(n)$ for $n \neq 1, \infty$. Then, for any $\underline{\alpha}_1 < \alpha < \beta$, we have that $P_P(1) = \zeta_1 \beta^{T-1} + (1 - \zeta_1)\alpha^{T-1} < P_N(1)$. Thus, the patent system increases the degree of competition when $\bar{n} = 1$.

Now, suppose that $\bar{n} \geq 2$ and fix $\alpha = e^{-\theta}\beta$. When there is no patent system we have that $P_N(1) = e^{-\theta}\beta$ and also $P_N(n) > 0$ if and only if $n \leq \min\{T, \bar{n}\}$ or $n = \infty$. Next, consider the equilibrium with a patent system. By Lemma 4, $s_1 < 1$, hence, $\zeta_1 < 1$. With $\alpha = e^{-\theta}\beta$ a firm has the same chance of keeping its monopoly position next period, regardless of whether it has chosen secrecy or patent protection, i.e. $P_P(1) = (e^{-\theta}\beta)^{T-1}$. Moreover, $P_P(n) = 0$ for $n > \min\{T, \bar{n}\}$ and $n \neq \infty$.

Then, take any $n$ such that $1 < n \leq \min\{T, \bar{n}\}$, $n \neq \bar{n}$ and look at $P_N(n)$. An innovation originally developed in $T = 1$ in an equilibrium without a patent system is produced by exactly $n$ firms in period $T$ if and only if it has been matched with at least one firm $n$ times and secrecy has held for all $T$ periods. Hence, $P_N(n) = \binom{T-1}{n-1}(1 - e^{-\theta})^{n-1}e^{-\theta(T-n)}\beta^{T-1}$. For $n = \bar{n} \leq T$, the innovation is produced by exactly $\bar{n}$ firms if the idea has been matched at least $\bar{n}$ times and secrecy has held for all $T$ periods. Thus, $P_N(n) = P_N(\bar{n}) = \sum_{i=0}^{T-\bar{n}} \binom{T-1}{i+\bar{n}-1}(1 - e^{-\theta})^{i+\bar{n}-1}e^{-\theta(T-\bar{n})}\beta^{T-1}$.

In an equilibrium with a patent system, an innovation can be produced under $n$ firms only if the first $n-1$ innovators have chosen secrecy. Hence, for $n$ such that $1 < n \leq \min\{T, \bar{n}\}$, $n \neq \bar{n}$ we have that $P_P(n) \leq \binom{T-1}{n-1}(1 - e^{-\theta})^{n-1}e^{-\theta(T-n)}\beta^{T-1}\prod_{i=1}^{n-1}\zeta_i = P_N(n)\prod_{i=1}^{n-1}\zeta_i$, where the inequality holds because a secret has a lower chance to leak than a patent has to fail, i.e. $\beta > \alpha = e^{-\theta}\beta$. Since $\zeta_1 < 1$, it follows that $P_P(n) < P_N(n)$ for all such $n$. Similarly, for $n = \bar{n} \leq T$ we have that $P_P(n) = P_P(\bar{n}) \leq \sum_{i=0}^{T-\bar{n}} \binom{T-1}{i+\bar{n}-1}(1 - e^{-\theta})^{i+\bar{n}-1}e^{-\theta(T-\bar{n})}\beta^{T-1}\prod_{i=1}^{\bar{n}-1}\zeta_i =$

$P_N(\bar{n}) \prod_{i=1}^{\bar{n}-1} \zeta_i$. Hence, $P_P(\bar{n}) < P_N(\bar{n})$.

Thus, for all finite $n \geq 2$, it follows that $G_P^T(n) < G_N^T(n)$. Hence, the equilibrium with a patent system provides higher competition than the equilibrium without, when $\bar{n} \geq 2$. This concludes the proof.

$\square$

**Claim 3.** *Suppose that in the beginning of ever period, just before firms observe which patents fail and which secrets leak, innovators are given the choice whether or not to switch their protection strategies. Then, in equilibrium, no firm has an incentive to switch protection strategies.*

*Proof.* Whenever an innovation is patented all the relevant information becomes public knowledge. Hence, no firm has an incentive to give up the patent and switch to secrecy. Thus, we only need to show that no firm has an incentive to switch from secrecy to patenting.

First, suppose that the patent system provides no PUR. Take an idea developed by $n$ firms at the beginning of period $T$, just before firms observe which patents fail and which secrets leak. Each of the $n$ firms faces the following decision. It can either keep using secrecy protection, which yields an expected payoff of $e^{-\theta}\beta\gamma V_S^n + \zeta_{n+1}(1-e^{-\theta})\beta\gamma V_S^{n+1} = R_S^n(1) - d_n\pi(1)$, or it can switch to patenting which yields an expected payoff of $\alpha\gamma V_P = R_P(1) - \pi(1)$. An innovator would have an incentive to switch protection strategies if and only if $R_P(1) - \pi(1) > R_S^n(1) - d_n\pi(1) \Leftrightarrow R_P(1) - R_S^n(1) > (1-d_n)\pi(1)$. This inequality, however, can never hold. To see this, observe that if there are $n$ innovators that produce the innovation under secrecy, it must be the case that $s_n > 0$. Hence, by lemma 2, it follows that $R_S^n(1) > R_P(1)$. Thus, $R_P(1) - R_S^n(1) < 0 \leq (1-d_n)\pi(1)$. Then, no firm has an incentive to switch from secrecy to patent protection.

Next, suppose that the patent system provides PUR. Again, take an idea developed under secrecy by $n$ firms at the beginning of period $T$, just before firms observe which patents fail and which secrets leak. Observe that we do not need to consider an idea where the $n$-th innovator has patented, because no two firms can hold a patent over the same innovation.

Hence, the $n-1$ firms that have innovated under secrecy do not have the opportunity to switch to patent protection. So, consider an idea developed by $n$ firms all of which use secrecy protection. Analogously to the case with no PUR, if an innovator keeps using secrecy they payoff is given by $R_S^n(1) - d_n\pi(1)$, and if she chooses to switch to patent protection it is $R_P^n(1) - d_n\pi(1)$. Thus, a firm would have an incentive to switch protection strategies if and only if $R_P^n(1) > R_S^n(1)$. As all firms have developed the idea under secrecy, it must be the case that $s_n > 0$. Hence, $R_P^n(1) < R_S^n(1)$ and no firm has an incentive to switch protection strategies.

□

**Proof of Proposition 2:**

*Proof.* First, observe that at $\alpha = \beta$ we have that $W^P(\beta) = W^S$. Next, look at the optimal patent strength consistent with a patenting equilibrium.

$$\begin{aligned}\frac{\partial W^P(\alpha)}{\partial \alpha} &= \frac{\theta e^{-\theta}\pi\gamma}{(1-e^{\theta})(1-\alpha\gamma)^2} + \frac{\gamma S_M}{(1-\alpha\gamma)^2} - \frac{\gamma S_C}{(1-\alpha\gamma)^2} \\ &= -\frac{\gamma}{(1-\alpha\gamma)^2}\Big(S_C - S_M - \frac{\theta e^{-\theta}}{1-e^{-\theta}}\pi\Big) \\ &< 0\end{aligned}$$

where the inequality holds because $S_C > S_M + \pi$ and $\theta e^{-\theta}/(1-e^{-\theta}) < 1$. Since $\bar{W}^P(\alpha) = \nu(1-e^{-\theta})W^P(\alpha)$, it follows that the optimal patent strength consistent with a patenting equilibrium is $\alpha_P := \beta$. As $W^P(\beta) = W^S$, it follows that $\bar{W}^P(\alpha_P) > \bar{W}^S$, since $\bar{N}^S < 1$. Thus, a secrecy equilibrium is never optimal.

□

**Proof of Proposition 3:**

*Proof.* First, straightforward algebra implies that

$$\frac{\partial W^M(\alpha)}{\partial \alpha} = (S_C - S_M)\frac{\gamma}{1-\alpha\gamma}\Big(\frac{\zeta}{1-\beta\gamma} - \frac{1-\zeta}{1-\alpha\gamma}\Big)$$

Then, suppose that the condition in the statement of proposition 3 holds. Hence,

$$
\begin{aligned}
\frac{\partial \bar{W}^M(\alpha)}{\partial \alpha}|_{\alpha=\beta} = & \quad \nu(1-e^{-\theta})\bar{N}^M \frac{\partial W^M(\alpha)}{\partial \alpha}|_{\alpha=\beta} + \\
& +\nu(1-e^{-\theta})W^M(\alpha)\frac{\partial \bar{N}^M}{\partial \alpha}|_{\alpha=\beta} \\
= & \quad \frac{\nu(1-e^{-\theta})\gamma}{1-\beta\gamma}\Bigg(-\frac{S_C-S_M}{1-\beta\gamma}+\frac{e^{-\theta}\beta}{1-\beta}W^P(\beta)\Bigg) \\
< & \quad \frac{\nu(1-e^{-\theta})\gamma}{(1-\beta\gamma)^2}\Bigg(-(S_C-S_M)+ \\
& +\frac{e^{-\theta}\beta}{1-\beta}\Big(\frac{\theta e^{-\theta}\pi}{1-e^{-\theta}}+S_M+\frac{\gamma(1-\beta)S_C}{1-\gamma}\Big)\Bigg) \\
< & \quad \frac{\nu(1-e^{-\theta})\gamma}{(1-\beta\gamma)^2}\Bigg(-(S_C-S_M)+\frac{e^{-\theta}\beta}{1-\beta}\Big(S_C+\frac{\gamma(1-\beta)S_C}{1-\gamma}\Big)\Bigg) \\
< & \quad 0
\end{aligned}
$$

Hence, by continuity, $\bar{W}^M(\alpha)$ is decreasing in $\alpha$ in the neighborhood of $\beta$. Thus, $\max_\alpha\{\bar{W}^M(\alpha)\} > \bar{W}^M(\beta) = \bar{W}^P(\alpha_P)$. Thus, by propoition 2, it follows that the optimal patent strength lies in the region consistent with a mixed equilibrium.

□

## 7.2 Alternative Timing

In this section I prove that the main results in the paper hold under an alternative timing of stages two and three. In particular, I prove that the statements in theorems one and two hold even if firms choose investment strategies and stage two and protection strategies at stage three.

**Assumption A4.** *Assume that the order of stages two and three is reversed. That is, at stage one firms are matched with ideas, at stage two firms choose an investment strategy, at stage three firms choose a protection strategy.*

### 7.2.1 Equilibrium Behavior

First, observe that when there is no patent system the game effectively consists of only two stages: first firms are matched with ideas, second — firms choose an investment strategy. Hence, the equilibrium behavior without a patent system is the same, regardless of whether or not we use the alternative timing. In what follows I focus on the equilibrium with a patent system. Since the goal is to show that the main results still hold under the alternative timing, I do not fully characterize the equilibrium. Instead, I characterize only what is needed to prove the results.

**Stage 3:** At stage three firms choose the protection strategy which gives the higher expected payoff. Let $\text{Payoff}^n(i|t) = P_i^n(\text{win}|t)R_i^n(t)$ be the expected payoff from choosing a protection strategy $i = S, P$ given that the firm has innovated at time $t$, where $P_i^n(\text{win}|t)$ is the probability of winning the race, $R_i^n(t)$ is the reward conditional on winning (as defined in the text), and $n \geq 1$ means that $n-1$ firms have already innovated the idea at previous periods.

Given assumption A3, a firm that chooses secrecy wins the race only if no other firm innovates before the firm under study and no firm innovates at a later time under patent protection. Hence, $P_S^n(\text{win}|t) = Pr(\text{no firm has innovated at } t' < t \cap \text{ no firm will innovate and choose to patent at } t' > t|n)$. Similarly, a firm that chooses patent protection wins whenever no firm patents at an earlier time. Thus, $P_P^n(\text{win}|t) = Pr(\text{no firm has innovated at } t' < t \text{ under patent protection } |n)$. Notice that I have left out the possibility that more than one firm chooses the same $t$, which is the case in equilibrium. However, if more than one firm innovate at the same time and choose the same protection strategy, the usual tie braking rule is applied: whenever $m > 1$ firms innovate at the same time under the same protection strategy, each firm has a $1/m$ chance of winning the race.

**Stage 2:** At stage two firms choose $t$ to maximize $\Pi^n(t) - c(t)$, where $\Pi^n(t) = \max\{\text{Payoff}^n(S|t), \text{Payoff}^n(P|t)\}$. The following four lemmas establish some useful properties of the equilibrium behavior. These are rather standard, nonetheless I provide the proofs for completeness.

**Lemma 5.** *For any $1 \leq n \leq \bar{n}+1$, there is no equilibrium where firms choose some $\bar{t}_n$ for* *sure at stage two.*

*Proof.* First, suppose that $n = \bar{n} + 1$. If the patent system provides PUR, then all firms choose to stay out of the race. If, on the other hand, the patent system provides no PUR, then firms choose patenting for sure at stage three. Since, under no PUR the reward under patenting is independent of how many firms have previously innovated, it follows that the behavior of firms when $n = \bar{n} + 1$ is the same as the behavior when $1 \leq n \leq \bar{n}$ and firms patent for sure at stage three. Hence, it is enough to consider $1 \leq n \leq \bar{n}$.

Next, Fix $1 \leq n \leq \bar{n}$ and suppose, to the contrary, that there exists some $\bar{t}_n$ that all firms choose when matched with an idea previously developed by $n - 1$ firms. Look at a deviant who considers bidding $t' \in (\bar{t}_n - \epsilon, \bar{t}_n)$ for some small $\epsilon > 0$. We will distinguish between two cases.

Case 1: Suppose that it is optimal to choose patent protection with positive probability at stage three, i.e. $\Pi^n(\bar{t}_n) = \text{Payoff}^n(P|\bar{t}_n) \geq \text{Payoff}^n(S|\bar{t}_n)$. Let $1 - s_n$ be the probability a firm places on patenting at stage three in a symmetric equilibrium. Then, the probability that there are exactly $m$ firms that innovate at time $\bar{t}_n$ and choose to patent is given by

$$P(N = m) = \sum_{i=m}^{\infty} \frac{e^{-\theta}\theta^i}{i!} \binom{i}{m} (1 - s_n)^m s_n^{i-m} = \frac{e^{-(1-s_n)\theta}[(1 - s_n)\theta]^m}{m!}$$

Hence, $P_P^n(\text{win}|t') = 1 > P_P^n(\text{win}|\bar{t}_n) = \sum_{i=0}^{\infty} e^{-(1-s_n)\theta}[(1-s_n)\theta]^i/(i+1)! = (1-e^{-(1-s_n)\theta})/[(1-s_n)\theta]$. Then, since $R_P^n(t)$ and $c(t)$ are continuous in $t$, it follows that for small enough $\epsilon$, $\Pi^n(t') - c(t') > \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation.

Case 2: Suppose that firms find it optimal to choose secrecy protection, i.e. $\Pi^n(\bar{t}_n) = \text{Payoff}^n(S|\bar{t}_n) > \text{Payoff}^n(P|\bar{t}_n)$ and $s_n = 1$. Analogously, if a firm chooses $t' \in (\bar{t}_n - \epsilon, \bar{t}_n)$, then $P_S^n(\text{win}|t') = 1 > P_S^n(\text{win}|\bar{t}_n) = \sum_{i=0}^{\infty} e^{-\theta}\theta^i/(i+1)! = (1-e^{-\theta})/\theta$. Hence, by continuity, for small enough $\epsilon$, $\Pi^n(t') - c(t') > \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation.

$\square$

**Lemma 6.** *With no PUR, for any $1 \leq n \leq \bar{n} + 1$, the upper bound of the support for the equilibrium CDF is $\bar{S}_n = 1$. With PUR, for any $1 \leq n \leq \bar{n}$, the upper bound of the support* *for the equilibrium CDF is* $\bar{S}_n = 1$.

*Proof.* First, from the argument in lemma 5, to show that the statement holds when the patent system provides no PUR and $n = \bar{n} + 1$, it is enough to show that it holds under no PUR when $1 \leq n \leq \bar{n}$.

Then, fix any $1 \leq n \leq \bar{n}$ and suppose to the contrary that $\bar{S}_n < 1$. Hence, $P_i^n(\text{win}|\bar{S}_n) = P_i^n(\text{win}|1)$, for $i = S, P$. Since $\pi(t) - c(t)$ is increasing over $[0, 1]$, it follows that $\pi(1) - c(1) > \pi(\bar{S}_n) - c(\bar{S}_n)$. Hence, $c(\bar{S}_n) - c(1) > \pi(\bar{S}_n) - \pi(1) > 0$, which implies that $c(\bar{S}_n) - c(1) > A\pi(\bar{S}_n) - A\pi(1) > 0$, for any $A \in [0, 1]$. Hence, $\text{Payoff}^n(i|1) - c(1) > \text{Payoff}^n(i|\bar{S}_n) - c(\bar{S})$. Thus, there exists a profitable deviation.

If, on the other hand, $\bar{S}_n > 1$, then $P_i^n(\text{win}|\bar{S}_n) \leq P_i^n(\text{win}|1)$. As $\pi(t) = 0$ for $t > 1$, it follows that $\pi(1) - c(1) > \pi(\bar{S}_n) - c(\bar{S}_n)$. Hence, by the arguments in the preceding paragraph, it follows that there exists a profitable deviation.

□

**Lemma 7.** *With no PUR, for any* $1 \leq n \leq \bar{n} + 1$*, the support of the equilibrium CDF is a connected interval. With PUR, for any* $1 \leq n \leq \bar{n}$*, the support of the equilibrium CDF is a connected interval.*

*Proof.* First, from the argument in lemma 5, to show that the statement holds when the patent system provides no PUR and $n = \bar{n} + 1$, it is enough to show that it holds under no PUR when $1 \leq n \leq \bar{n}$.

Let the equilibrium CDF be $F^n(t)$. Fix $1 \leq n \leq \bar{n}$ and suppose, to the contrary that there is a gap between some $t_1$ and $t_2$. Hence, for any $t' \in (t_1, t_2)$, it is the case that $P_i^n(\text{win}|t') = P_i^n(\text{win}|t_1)$, for $i = S, P$. By assumption, $\pi(t) - c(t)$ is increasing in over $[0, 1]$. Hence, $c(t_1) - c(t') > \pi(t_1) - \pi(t') > 0$. Then, $c(t_1) - c(t') > A\pi(t_1) - A\pi(t')$, for any $A \in [0, 1]$. Thus, $\text{Payoff}^n(i|t') - c(t') > \text{Payoff}^n(i|t_1) - c(t_1)$, for $i = S, P$. Hence, $\Pi^n(t') - c(t') > \Pi(t_1) - c(t_1) = \Pi(t) - c(t)$ for all $t$ in the support. Thus, there exists a profitable deviation.

□

**Lemma 8.** *With no PUR, for any $1 \leq n \leq \bar{n}+1$, the support of the equilibrium CDF has no atoms. With PUR, for any $1 \leq n \leq \bar{n}$, the support of the equilibrium CDF has no atoms.*

*Proof.* First, from the argument in lemma 5, to show that the statement holds when the patent system provides no PUR and $n = \bar{n}+1$, it is enough to show that it holds under no PUR when $1 \leq n \leq \bar{n}$.

Then, fix any $1 \leq n \leq \bar{n}$ and suppose to the contrary that there is an atom at some $\bar{t}_n$. Again, we would need to distinguish between two cases.

Case 1: Suppose that firms place some positive probability $1 - s_n$ of playing patenting, in a symmetric equilibrium, given that they have innovated at $\bar{t}_n$. Hence, $\text{Payoff}^n(P|\bar{t}_n) \geq \text{Payoff}^n(S|\bar{t}_n)$. Then, look at a firm that considers deviating to $t' \in (\bar{t}_n - \epsilon, \bar{t}_n)$. Because there is a strictly positive probability a firm would innovate at $\bar{t}_n$ under patenting, it follows that $P^n_P(\text{win}|t') > P^n_P(\text{win}|\bar{t}_n)$, $\forall \epsilon > 0$. Thus, by continuity, it follows that for small enough $\epsilon$, $\text{Payoff}^n(P|t') - c(t') > \text{Payoff}^n(P|\bar{t}_n) - c(\bar{t}_n) \geq \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation.

Case 2: Suppose that firms that innovate at $\bar{t}_n$ use secrecy for sure. Then $\text{Payoff}^n(S|\bar{t}_n) > \text{Payoff}^n(P|\bar{t}_n)$. Moreover, since there is a positive probability a firm would innovate at $\bar{t}_n$ under secrecy, it follows that $P^n_S(\text{win}|t') > P^n_S(\text{win}|\bar{t}_n)$, $\forall \epsilon > 0$. Hence, by continuity, for small enough $\epsilon$, it follows that $\text{Payoff}^n(S|t') - c(t') > \text{Payoff}^n(S|\bar{t}_n) - c(\bar{t}_n) \geq \Pi^n(\bar{t}_n) - c(\bar{t}_n)$. Thus, there exists a profitable deviation.

□

### 7.2.2 Competition

Given the results in the previous four lemmas, we can now state and prove the results under the alternative timing.

**Theorem 3.** *Suppose that the patent system provides no prior user rights. For all $\gamma, \beta \in (0,1)$ and all $\theta > 0$, there exists a patent strength $\alpha$ such that the patent system increases competition.*

*Proof.* First, observe that the key feature of the equilibrium with a patent system that we have used in the proof of theorem 1 is that when $\alpha = e^{-\theta}\beta$, the first innovator patents for sure. I proceed by establishing the analogous result under the alternative timing. More specifically, the following two lemmas prove that if $\alpha = e^{-\theta}\beta$, then it is an equilibrium strategy for firms to patent following any innovation time $t \in [0, 1]$.

**Lemma 9.** *If $\alpha = \tilde{\alpha}_n$, then it is a symmetric equilibrium strategy for firms to use patent protection following any innovation time $t \in [0, 1]$, where $\tilde{\alpha}_n$ is such that $R_S^n(1) = R_P^n(1)$.*

*Proof.* Suppose that firms choose patent protection following any innovation time $t \in [0, 1]$. Look at a deviant that considers choosing secrecy, instead. She has an incentive to deviate if and only if $\text{Payoff}^n(S|t) > \text{Payoff}^n(P|t)$. When all firms patent, it follows that $P_S^n(\text{win}|t) = Pr^n(\text{no other firm is matched with the idea } |n) = e^{-\theta}$ and $P_P^n(\text{win}|t) = Pr^n(\text{all other firms matched with the same innovation choose } t' > t|n) = \sum_{k=0}^{\infty} \theta^k e^{-\theta}(1-F^n(t))^k/(k!) = e^{-\theta F^n(t)}$, where $F^n(t)$ is the equilibrium CDF. Hence, the deviant would have no incentive to choose secrecy if and only if

$$e^{-\theta} R_S^n(t) \le e^{-\theta F^n(t)} R_P(t) \tag{26}$$

From the definition of $R_S^n(t)$, it follows that (26) is equivalent to

$$\begin{aligned} e^{-\theta}(d_n\pi(t) + V_S^n - d_n\pi(0)) \le e^{-\theta F^n(t)} R_P(t) \quad &\text{iff} \\ e^{-\theta F^n(t)} R_P(t) + e^{-\theta} d_n(\pi(0) - \pi(t)) \ge e^{-\theta} V_S^n& \end{aligned} \tag{27}$$

Now, look at the left hand side of (27). Let $t_1 < t_2$, then

$$\begin{aligned} e^{-\theta F^n(t_2)} R_P(t_2) + e^{-\theta} d_n(\pi(0) - \pi(t_2)) - \Big(e^{-\theta F^n(t_1)} R_P(t_1) + \\ + e^{-\theta} d_n(\pi(0) - \pi(t_1))\Big) = e^{-\theta F^n(t_2)}(R_P(t_2) - R_P(t_1)) + \\ + R_P(t_1)(e^{-\theta F^n(t_2)} - e^{-\theta F^n(t_1)}) - e^{-\theta} d_n(\pi(t_2) - \pi(t_1)) = \\ = R_P(t_1)(e^{-\theta F^n(t_2)} - e^{-\theta F^n(t_1)}) + e^{-\theta}(\pi(t_2) - \pi(t_1))(e^{\theta(1-F^n(t_2))} - d_n) < 0 \end{aligned}$$

The inequality follows because, by lemmas 7 and 8, $F^n(t)$ is strictly increasing in $t$. Also, by assumption $\pi(t)$ is strictly decreasing over $[0,1]$. Moreover, $d_n \leq 1$ and $F^n(t) \leq 1$, hence, the left hand side of (27) is strictly decreasing in $t$. Thus, if (27) holds for $t = 1$, it holds for all $t \in [0,1]$. Thus, if $R_S^n(1) \leq R_P(1)$, then $e^{-\theta} R_S^n(t) \leq e^{-\theta F^n(t)} R_P(t)$ for all $t \in [0,1]$. Hence, if $R_S^n(1) = R_P(1)$, then following any innovation time $t$, firms have no incentive to deviate and choose secrecy protection.

□

**Lemma 10.** $e^{-\theta}\beta = \tilde{\alpha}_1 \geq \tilde{\alpha}_2 \geq \cdots \geq \tilde{\alpha}_{\bar{n}} \geq \tilde{\alpha}_{\bar{n}+1} = 0$

*Proof.* The proof is completely analogous to the proof of corollary 1.

□

By lemmas 9 and 10, it follows that at $\alpha = e^{-\theta}\beta$, $P_P(1) = (e^{-\theta}\beta)^{T-1}$, $P_P(\infty) = 1 - (e^{-\theta}\beta)^{T-1}$, and $P_P(n) = 0$, for $n \neq 1, \infty$, $\forall T \geq 2$.Thus, the rest of the proof is completely analogous to the proof of theorem 1.

□

**Theorem 4.** *Suppose the patent system provides prior user rights. If*

$$\frac{1}{\gamma} > \Big(\frac{e^{-\theta}\beta}{1 - e^{-\theta}\beta\gamma}\Big)\Big(\frac{\beta}{1 - \beta\gamma}\Big)$$

*then the patent system can increase competition.*

*Proof.* Recall that the key feature in the proof of theorem 2 was that when $\alpha = e^{-\theta}\beta$, there is a strictly positive chance that the first innovator would use patent protection. Thus, if we can prove that the same is true under the alternative timing the proof of the theorem would follow directly from the proof of theorem 2. The next lemma establishes just that.

**Lemma 11.** *If* $\frac{1}{\gamma} > \left(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\right)\left(\frac{\beta}{1-\beta\gamma}\right)$, *then, in a symmetric equilibrium, a positive fraction of firms that innovate an idea which has not been previously innovated do so under patent protection, i.e.* $\zeta_1 < 1$.

*Proof.* By lemma 4,

$$\frac{1}{\gamma} > \Big(\frac{e^{-\theta}\beta}{1-e^{-\theta}\beta\gamma}\Big)\Big(\frac{\beta}{1-\beta\gamma}\Big)$$

holds if and only if

$$e^{-\theta}R^1_S(1) < R^1_P(1) \tag{28}$$

Hence, we need to prove that if equation (28) holds, then in equilibrium a positive fraction of firms that innovate a previously undeveloped idea would do so under patent protection.

Suppose that firms use secrecy protection following any $t \in supp(F)$ (except possibly over a set with measure 0). I will now show that this can never be an equilibrium because firms have a profitable deviation.

Since all firms use secrecy, it follows that if a deviant is to patent her payoff is $R^1_P(t)$, as she wins the race for sure. On the other hand, if she is to use secrecy protection, then $P^n_S(\text{win}|t) = \sum_{k=0}^{\infty}\theta^k e^{-\theta}(1-F^n(t))^k/(k!) = e^{-\theta F^n(t)}$ and her payoff is $e^{-\theta F^n(t)}R^1_S(t)$. By lemmas 7 and 8 and by continuity of $R^n_P(t)$ and $R^n_S(t)$, since (28) holds, it follows that there exists some $\delta > 0$ such that for all $t \in (1-\delta, 1)$, $R^1_P(t) > e^{-\theta F^n(t)}R^1_S(t)$. Hence, firms that innovate at such $t$ would have an incentive to deviate to patenting. Since there is a strictly positive fraction of such firms, we have a contradiction.

□

Hence, if $\alpha = e^{-\theta}\beta$, it follows that $\zeta_1 < 1$.Thus, the rest of the proof is analogous to the proof of theorem 2.

□

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