Nonneutrality of Money in Dispersion: Hume Revisited

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Abstract

For a class of standard and widely-used preferences, a one-shot money injection in a standard matching model can induce a significant and persistent output response by dispersing the distribution of wealth. Decentralized trade matters for both persistence and significance. In the presence of government bonds the injection has a liquidity effect and the inflation rate right following the injection may be below the steady-state rate level.

JEL Classification Number: E31, E40, E50

Key Words: Nonneutrality, Money Injection, Phillips Curve, Nominal Rigidity

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1 Introduction

First published in 1752, *Of Money* articulates Hume's view of nonneutrality of money that has been influential for centuries:

> At first, no alteration is perceived; by degrees the price rises, first of one commodity, then of another, till the whole at last reaches a just proportion with the new quantity of specie which is in the kingdom....When any quantity of money is imported into a nation, it is not at first dispersed into many hands; but is confined to the coffers of a few persons, who immediately seek to employ it to advantage....It is easy to trace the money in its progress through the whole commonwealth; where we shall find, that it must first quicken the diligence of every individual, before it encrease the price of labour. [Hume [20, p 172]]

In this heavily-cited passage,[1] Hume seemed to relate a stimulating effect of a money injection to (a) a limited participation in the market from which money is injected and (b) a dispersion (i.e., diffusion) process in the market from which injected money gradually reaches all people in the economy. But is there any mechanism by which a limited participation and a dispersion process can make an injection stimulating? Here we explore such a mechanism against the familiar matching model of Trejos and Wright [30] and Shi [27] with general individual money holdings, a model that accommodates a limited participation by nondegenerate wealth distributions and a gradual dispersion process by decentralized trade.[2] We use a class of standard and widely-used preferences for quantitative exercises and concentrate on one-shot money injections.

Our parameterized model has two salient features. First, aggregate output would increase in a steady state if people's incentives to trade were not changed but the distribution of wealth were more dispersed. The main force behind is simple and intuitive: a reduction in a poor seller's wealth results in a much larger increment in production than the reduction in a rich seller's. Secondly, people's incentives along a transitional path to the steady state are very close to their incentives in the steady state. These two features imply that if a redistributional shock disperses (stretches) the steady-state distribution of money but maintains the quantity of money, there is an immediate significant output response. An obvious role of decentralize trade is to add persistence by slowing down the dispersion (diffusion) of redistributed money.

[1] For its influence on contemporary monetary economics, see, e.g., Friedman [13], Lucas [24], and Wallace [32].

[2] The canonical form of the model, one with divisible money and with no upper bound on the individual holdings, has a central role in the New Monetarism literature (see Williamson and Wright [36]) in that much of the literature is built on its tractable versions, e.g., Trejos and Wright [30] and Shi [27] with indivisible money and a unit upper bound on the individual money holdings, and Lagos and Wright [21] and Shi [28] with different new ingredients.

There is a more critical role of decentralized trade. Suppose that a competitive market substitutes for decentralized trade as in a Bewley model. If there is no change in the price level, then the wealth redistribution may still have the same output effect as above; but for the market to clear, nominal prices must fall below the steady-state level in the transitional path, which, in turn, dilutes sellers' incentives to produce and dampens the output response. In other words, a suitable wealth redistribution is able to exploit the steady-state incentives to trade—in particular, poorer sellers' much stronger incentives to produce—because these incentives are preserved by decentralized trade in the transitional path. Apparently, such a redistribution can be done by a regressive money injection.

It is certainly not new that a money injection is nonneutral when it redistributes wealth (see Friedman [12]) and redistribution effects have drawn a fair amount of attention.[3] Our contribution is to reveal a mechanism by which some wealth redistribution may induce significant and persistent output responses. Such output responses, as is well known, are hard to come by in a large class of models absent of imposed nominal rigidity (see, e.g., Chari, Kehoe, and McGrattan [8]); while one may therefore appeal to nominal rigidity, there is always criticism of the assumption that people cannot change prices when they want.[4] In this context, relevance of our contribution is to show that price flexibility can be consistent with the output-response pattern in concern and, as noted below, with observed nominal rigidity.

To discipline our exercises, we endogenize regressiveness of each money injection as we endogenize a limited participation to the injection. With a unitary CRRA coefficient and a unitary Frisch elasticity of labor supply, a 1% accumulative increase in the money stock can induce a more than 3% accumulative increase in output over 20 periods, a period interpreted as a quarter. There is a Phillips curve in that the output and price responses are proportional to the increase in the money stock.

When the model includes nominal government bonds (issued before pairwise meetings), inflation in a steady state is driven by interest payments to these bonds. In the steady state people carry only a small portion of nominal wealth in money into pairwise meetings; as implied by the second salient feature of the model, most of injected money must go to the bond market so there is a liquidity effect.[5] The output response remains significant and persistent. The inflation rate may first drop below the steady-state level, a phenomena analogous to what is referred to as the price puz-

[3]Some familiar studies come from the limited-participation literature. Redistribution effects are explored by the early and some late contributions in this literature; see, e.g., Grossman and Weiss [16], Rotemberg [26], Alvarez, Atkeson, and Kehoe [2] and Williamson [34, 35].

[4]A defense to nominal rigidity is that there may be some costs to changing prices, e.g., menu costs (Mankiw [19]) or nominal contracts. It is a nontrivial task to deal with such a defense; one may refer to critics of Chari, Kehoe, and McGrattan [8] and Golosov and Lucas [15] to sticky price models built on the Calvo [7] pricing in the New Keynesian literature.

[5]For comparison, a liquidity effect arises in models of limited participation (see, e.g., Grossman and Weiss [16], Rotemberg [26], and Lucas [23]) because some money in circulation cannot reach the bond market when money is injected.

zle in some VAR studies (see, e.g., Christiano, Eichenbaum, and Evans [9]). The key is that the injection increases the value of money at the present period by reducing interest payments and, hence, the money growth in the next period. In addition to this initial fall in inflation, one may observe nominal rigidity in the usual sense.

We spell out the basic model, parameterization, and the procedure for quantitative exercises in section 2. Section 3 demonstrates the two salient features of the model and the critical role of decentralized trade. The one-shot regressive injection is introduced in section 4. The model with bonds is presented in section 5. In section 6, we offer some discussion of our model, findings, and future works.

2 The basic model

The model is the one formulated by Trejos and Wright [30] and Shi [27] with general individual money holdings. Time is discrete, dated as $t \geq 0$. There is a unit mass of infinitely lived agents. At each period, each agent has the equal chance to be a buyer or a seller. Each buyer is randomly matched with a seller. In each pairwise meeting, the seller can produce a consumption good only consumed by the buyer. The good is divisible and perishes at the end of the period. By exerting l units of the labor input, each seller can produce

$$y = A\left(\frac{1}{L}\right)^{1-\alpha} l \tag{1}$$

units of goods, where $A > 0$ and $\alpha \in (0, 1]$, and L is the aggregate labor input in the economy.[6] If an agent consumes y units of goods and exerts l units of labor in a period, his period utility is

$$U(y,l) = u(y) - c(l) = \frac{(y+\epsilon)^{1-\sigma} - \epsilon^{1-\sigma}}{1-\sigma} - \frac{l^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}, \tag{2}$$

where $\sigma > 0$, $\eta > 0$, and ϵ is small but positive to make each agent's reservation value well defined in case $\sigma \geq 1$. Each agent maximizes his expected utility with a discount factor $\beta \in (0, 1)$. There exists a durable and intrinsically useless object, called money. Money is indivisible and its smallest unit is Δ; the initial average holdings of money is M; and there is a finite but arbitrarily large upper bound B on the individual money holdings.[7] The initial distribution of money, denoted π_0, is public information.

[6] So $Y = AL^{\alpha}$ is the aggregate production function and on the aggregate level the marginal product of labor is decreasing if $\alpha < 1$. At each pairwise meeting the marginal product of labor is constant for a given L. As a rationale, one may think of a seller as a taxi driver; the driving distance he serves for the buyer is proportional to his working time while the proportional coefficient is affected by the number of occupied taxis on the road. This setting separates the influence of the decreasing marginal product of labor (captured by α) from the influence of a seller's preference (captured by η in (2)) on his labor supply; see footnote 14 for a related discussion.

[7] If $(\Delta, B) = (0, \infty)$, that is, money is divisible and there is no upper bound on money holdings, then the choice of M only affects prices. But numerical analysis still needs some $B' < \infty$ to

In each pairwise meeting, each agent can observe his meeting partner's money holdings, but not past trading histories, which rules out credits between the two agents. Following a convention introduced by Berentsen, Molico and Wright [4] into matching models with indivisible money, we allow stochastic trade so that a meeting outcome is a lottery on the feasible transfers of goods and money. Because $u(.)$ is strictly concave and $c(.)$ is strictly convex, it is not optimal for agents to choose a lottery which randomizes on the transfer of goods. So it is without loss of generality to represent a generic meeting outcome by a pair (y, μ), where $y \geq 0$ is the transfer of goods and μ is a probability measure on $\left\{0, \ldots, \min\left(m^b, B - m^s\right)\right\}$, meaning that the probability for the buyer to transfer d units of money to the seller is $\mu(d)$.

To define equilibrium, let v_t be the value function and π_t be the distribution of money at the start of period t; that is, for $m \in B_\Delta \equiv \{0, \Delta, \ldots, B\}$, $v_t(m)$ is the expected discounted utility for an agent holding m units of money and $\pi_t(m)$ is the proportion of agents holding m at the start of period t. Consider a seller with $m^s \in B_\Delta$ meets a buyer with $m^b \in B_\Delta$ at period t. To preserve concavity of value functions, we follow some recent treatment in matching models of money to let the outcome in the meeting be determined by the weighted egalitarian solution of Kalai [17];[8] that is, the equilibrium meeting outcome is

$$\left(y\left(m^b, m^s, v_{t+1}, L_t\right), \mu\left(m^b, m^s, v_{t+1}, L_t\right)\right) = \arg\max_{y,\mu} S^b\left(y, \mu, m^b, m^s, v_{t+1}, L_t\right) \quad (3)$$

subject to

$$\theta S^s\left(y, \mu, m^b, m^s, v_{t+1}, L_t\right) = (1-\theta) S^b\left(y, \mu, m^b, m^s, v_{t+1}, L_t\right), \quad (4)$$

where

$$S^b\left(y, \mu, m^b, m^s, v_{t+1}, L_t\right) = u(y) + \beta \sum_d \mu(d)\left[v_{t+1}\left(m^b - d\right) - v_{t+1}\left(m^b\right)\right] \quad (5)$$

is the buyer's surplus from trading (y, μ),

$$S^s\left(y, \mu, m^b, m^s, v_{t+1}, L_t\right) = -c\left(yL_t^{1-\alpha}/A\right) + \beta \sum_d \mu(d)\left[v_{t+1}(m^s + d) - v_{t+1}(m^s)\right] \quad (6)$$

is the seller's surplus, and θ is the buyer's share of surplus. Let $f^b\left(m^b, m^s, v_{t+1}, L_t\right)$ and $f^s\left(m^b, m^s, v_{t+1}, L_t\right)$, respectively, denote the buyer's surplus and the seller's surplus implied by the equilibrium meeting outcome. Then given (v_{t+1}, π_t), v_t satisfies

$$v_t(m) = \beta v_{t+1}(m) + 0.5 \sum_{m'} \pi_t(m')\left[f^b(m, m', v_{t+1}, L_t) + f^s(m', m, v_{t+1}, L_t)\right]; \quad (7)$$

approximate $B = \infty$; and the grid method needs some positive Δ' to approximate $\Delta = 0$. In this regard, the present model saves us from a few layers of approximation. Analytically, Zhu [39] gives a sense that $\Delta' > 0$ approximates $\Delta = 0$ when $B < \infty$ and M are fixed. In our numerical analysis below, computed equilibria seem to converge as B increases when Δ and M are fixed.

[8] See, e.g., Aruoba, Rocheteau, and Waller [3], Lester, Postlewaite and Wright [22] and Venkateswaran and Wright [31]. In Trejos and Wright [30] and Shi [27], the trade in the meeting is determined by the generalized Nash bargaining solution, treated as the limit of equilibria from an alternating-offer game in the meeting. This Nash solution does not guarantee concavity of value functions in equilibrium; in numerical analysis it certainly does not preserve concavity in iterations.

π_{t+1} satisfies

$$\pi_{t+1}(m) = \sum_{m'} \lambda(m', m, v_{t+1}, L_t)\, \pi_t(m'), \tag{8}$$

where $\lambda(m', m, v_{t+1}, L_t)$ is the proportion of agents with m' units of money leaving with m after pairwise meetings in period t implied by money transfer lotteries $\left\{\mu\left(\cdot; m^b, m^s, v_{t+1}, L_t\right) : \left(m^b, m^s\right) \in B_\Delta \times B_\Delta\right\}$ and is explicitly described in the appendix; and L_t satisfies

$$AL_t^\alpha = 0.5 \sum_{(m,m')} y(m, m', v_{t+1}, L_t)\, \pi_t(m)\, \pi_t(m'). \tag{9}$$

Definition 1 *Given π_0, a sequence $\{v_t, \pi_{t+1}\}_{t=0}^\infty$ is an equilibrium in the economy if it satisfies* (3)-(9). *An equilibrium is a monetary equilibrium if $v_t(B) > 0$ for some t. A pair (v, π) is a steady state if $\{v_t, \pi_{t+1}\}_{t=0}^\infty$ with $v_t = v$ and $\pi_t = \pi$ for all t is an equilibrium.*

Given parameter values, numerical analysis in the next two sections pertain to the output and price responses to an unanticipated shock. The analysis involves two steps.

Step 1. We first compute a steady state (v, π) such that v is strictly increasing and strictly concave; a function v on B_Δ is concave if $2v(m) \geq v(m+\Delta) + v(m-\Delta)$ for $B - \Delta \geq m \geq \Delta$. When θ is sufficiently close to one, we can adapt the proof in Zhu [38] to show existence of such a steady state if ϵ is sufficiently small; we cannot extend that proof for a general θ. Nonetheless, existence holds if we perturb the model so that money yields arbitrarily small direct utility. When perturbation goes to zero, a limit of steady states in perturbed models is a steady state in the (original) model and it is a desired steady state if the limit value function is not a zero function. We test whether a computed steady state (v, π) is an object that truly exists by testing whether it is approximated by steady states in perturbed models.

After we obtain numerical values of (v, π) we also check its local stability as follows. First, we obtain a dynamic system $(v_{t+1}, \pi_{t+1}) = \Phi(v_t, \pi_t)$ in a neighborhood of (v, π) from the definition of equilibrium. For this system, we need that v_{t+1} is solvable from the equilibrium condition (7), treating (v_t, π_t) as parameters; this is up to checking whether the relevant Jacobian is of full rank. Next we compute eigenvalues of the Jacobian of $\Phi(\cdot, \cdot)$ evaluated at (v, π). Based on the number of eigenvalues inside the unit circle, we are able to determine whether the steady state is locally stable. We leave details of the procedure into the online appendix.[9]

Step 2. Here we let the economy reach (v, π) at period 0 and let it be hit by an unanticipated shock so that before period-1 pairwise meetings it has a distribution of money different than π. Next we compute a *transitional equilibrium* $\{v_t, \pi_{t+1}\}_{t=1}^\infty$ starting from that period-1 distribution and approaching a post-shock steady state (v', π') (i.e., $(v_t, \pi_t) \to (v', \pi')$ as $t \to \infty$). If the post-shock average money holdings

[9] The website is taozhu.people.ust.hk/nonneutrality.htm.

M' are equal to M then $(v', \pi') = (v, \pi)$; otherwise we seek (v', π') so that neutrality applies to (v', π') and (v, π). Because of indivisibility of money, *neutrality* means that $\Pi'(m) \equiv \sum_{x \leq m} \pi'(x) \approx \bar{\Pi}(mM/M')$ and $v'(m) \approx \bar{v}(mM/M')$, where $\bar{\Pi}(.)$ is the linear interpolation of the mapping $m \mapsto \Pi(m) \equiv \sum_{x \leq m} \pi(x)$ and $\bar{v}(.)$ is the linear interpolation of the mapping $m \mapsto v(m)$; we choose sufficiently large M/Δ and B/Δ (as detailed below) so that neutrality applies well, i.e., these approximations are sufficiently accurate.

Between a buyer with m^b and a seller with m^s, let $\left(y_t\left(m^b, m^s\right), \mu_t\left(.; m^b, m^s\right)\right)$ be the meeting outcome at period t in the transitional equilibrium, $d_t\left(m^b, m^s\right) = \sum_d \mu_t\left(d; m^b, m^s\right) d$, and $p_t\left(m^b, m^s\right) = d_t\left(m^b, m^s\right) / y_t\left(m^b, m^s\right)$; then aggregate output at t is

$$Y_t = 0.5 \sum_{m^b, m^s} \pi_t\left(m^b\right) \pi_t\left(m^s\right) y\left(m^b, m^s\right) \tag{10}$$

and the average price at t is

$$P_t = \sum_{m^b, m^s} \pi_t\left(m^b\right) \pi_t\left(m^s\right) p\left(m^b, m^s\right). \tag{11}$$

Let $(y(.), \mu(.;.))$, $d(.)$, $p(.)$, Y, and P be counterparts at the steady state (v, π). For the output and price responses, we compare Y_t and P_t along the transitional equilibrium with Y and P, respectively; specifically, *the output response* at period t is measured by *the output increase* $Y_t/Y - 1$ and *the price response* at period t is measured by *the price increase* $P_t/P - 1$.

In the two-step analysis, our algorithm to find a steady state is essentially an iteration on the mappings implied by (3)-(8). The algorithm is standard and, as all other algorithms, details and related codes are given in the online appendix.[10] Our algorithm to find a transitional equilibrium uses an approximation treatment; that is, (v', π') is reached after T periods for a sufficiently large T. This algorithm makes sense only if (v', π') is locally stable so it is applied only after we confirm local stability of (v', π').

Regarding parameter values, we set $\beta = 1/(1 + 0.01)$, implying an annual discount rate of 4% when a period is treated as a quarter. The term A in (1) is a free parameter and we simply set $A = 1$. Because the term ϵ in (2) is to make each agent's reservation value well defined when $\sigma \geq 1$, we set $\epsilon = 10^{-4}$ (by various experiments, making it smaller has negligible influence on computation results). In nominal objects (Δ, M, B), M/Δ and B/Δ should be sufficiently large so that when M is changed to some nearby M', neutrality applies well to pre-change and post-change steady states; by various experiments, we find that $M/\Delta = 30$ and $B/\Delta = 90$ serve the purpose well and we maintain $(\Delta, M, B) = (1, 30, 90)$ throughout.

In our benchmark, we set $\alpha = 2/3$, implying a curvature of the aggregate production function commonly used in the business cycle literature; $\sigma = 1$ ($u(y) = \ln(y + \epsilon) - \ln \epsilon$); $\eta = 1$ (the unitary Frisch elasticity of labor supply); and $\theta = 1$.

[10]For iterations in all algorithms, we stop when the two-round difference is less than 10^{-8}.

Without indicated otherwise, these are benchmark values for $(\sigma, \eta, \alpha, \theta)$ used in computation.

3 Critical roles of a steady state property and decentralized trade

In this section we illustrate critical roles of a steady-state property and decentralized trade in determining the output response after a shock hits the economy. Along this line, we illustrate how the output response may be affected by certain parameters. For our purpose, let (v, π) be the steady state reached by the economy at period 0 and suppose that some imaginary shock does not change the stock of money but hits the economy to one of two distributions before period-1 pairwise meetings, denoted $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$.

The following construction of $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$ is borrowed from Wallace [33]. Provided that the initial distribution of money is π, first assign each agent some additional money. An agent with m units of money is assigned $\min\{\lceil a(m)\rceil - 1, B - m\}$ with probability $p(m) = \lceil a(m)\rceil - a(m)$ and $\min\{\lceil a(m)\rceil, B - m\}$ with probability $1 - p(m)$, where $C \geq 0$ and C_0 are fixed numbers, $a(m) = \max\{0, C_0 + C \cdot m\}$, and $\lceil a(m)\rceil$ is the smallest integer not less than $a(m)$. Next, given that $M' - M$ is the total amount of assigned money, remove each unit of money independently from the economy with probability $1 - M/M'$. We associate $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$ with some $C_0 < 0$ and $C_0 > 0$, respectively; that is, the imaginary shock hits the economy to $\pi_1^1(\pi)$ by dispersing (stretching) the distribution π and to $\pi_1^2(\pi)$ by squeezing π. For exercises below, we set $(C_0, C) = (-2, 0.1)$ for $\pi_1^1(\pi)$ and $(C_0, C) = (2, 0)$ for $\pi_1^2(\pi)$.

We first present computed results for $(\sigma, \eta, \alpha, \theta) = (1, 1, 1, 1)$ ($\alpha = 1$ is not our benchmark value). In Figure 1, the first row displays the steady-state value function v and the steady-state distribution π.[11] The second row displays the output responses along the transitional equilibria starting from $\pi_1^1(\pi)$ and from $\pi_1^2(\pi)$, respectively. Starting from $\pi_1^1(\pi)$, the output response is positive, significant, and persistent. Starting from $\pi_1^2(\pi)$, the output response is negative and much less significant.

Now we note an important observation: substituting v for v_{t+1} one gets a good approximation to $\mathbf{y}_t = \{y_t(m^b, m^s) : 0 \leq m^b, m^s \leq B\}$, $\mathbf{d}_t = \{d_t(m^b, m^s) : 0 \leq m^b, m^s \leq B\}$, and $\mathbf{p}_t = \{p_t(m^b, m^s) : 0 \leq m^b, m^s \leq B\}$ because v_{t+1} is very close to v. In other words, people's incentives to trade in the transitional equilibrium are very close to their incentives to trade in the steady state. In fact, when $\alpha = 1$, $\mathbf{y}_t$, $\mathbf{d}_t$, and $\mathbf{p}_t$ (calculated when the future value of money is given by v_{t+1}) are all well approximated by their steady-state counterparts, denoted $\mathbf{y}$, $\mathbf{d}$, and $\mathbf{p}$ (calculated when the future value of money is given by v). In the present exercise, for example,

[11] There is no result for uniqueness of a steady state. However, even though we choose many different initial values, our algorithm always converges to the same steady state.

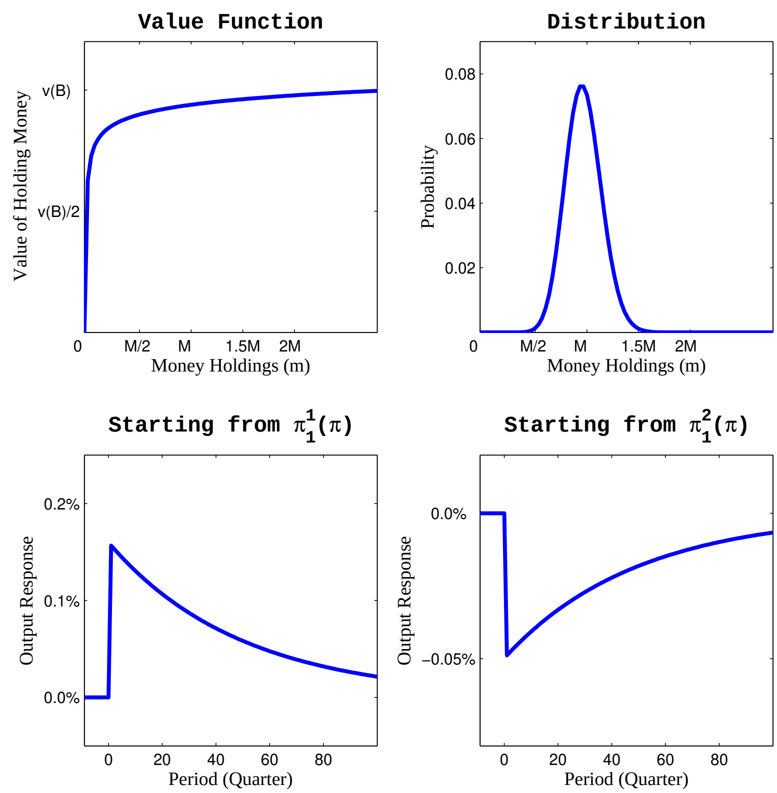

Figure 1: First row: steady-state value function and distributions; second row: output paths starting from $\pi_1^1(\pi)$ and $\pi_1^2(\pi)$.

the largest deviation of pairwise output $y_1\left(m^b, m^s\right)$ from $y\left(m^b, m^s\right)$ is around 0.004%; a deviation of x_1 from x is defined as $|x_1/x-1|$.[12] Hence, the output responses in Figure 1 are essentially driven by the differences between post-shock distributions and π.

To see why a distribution different from π may drive a significant output response, we display the set $\mathbf{y}$ in Figure 2. The top graph shows $\mathbf{y}$ in the three-dimension space. The two graphs at the bottom show two sorts of output curves that help to better visualize $\mathbf{y}$: the left graph consists of curves of the first sort each of which tells how $y\left(m^b, m^s\right)$ varies with m^s for a fixed m^b; the right graph consists of curves of the second sort each of which tells how $y\left(m^b, m^s\right)$ varies with m^b for a fixed m^s. Each curve of the first sort exhibits strong convexity, saying that when there is a marginal reduction in the seller's wealth, the increment in consumption received by a buyer is much larger if the seller is poorer. Because $\pi_1^1(\pi)$ is obtained from dispersing π, it follows that $Y\left(m^b, \pi_1^1(\pi)\right) > Y\left(m^b, \pi\right)$ all m^b, where $Y\left(m^b, h\right) = \sum h(m^s)\, y\left(m^b, m^s\right)$ is the average consumption for a buyer holding m^b under the distribution h provided that pairwise output is determined by $\mathbf{y}$. This need not imply $Y\left(\pi_1^1(\pi)\right) > Y(\pi)$, where $Y(h) = 0.5 \sum h\left(m^b\right) Y\left(m^b, h\right)$ is the aggregate consumption or output under the distribution h. Indeed, the buyer's average consumption is largely concave in his money holdings for a given h, i.e., $Y\left(m^b-1, h\right) + Y\left(m^b+1, h\right) < 2Y\left(m^b, h\right)$, as suggested by curves of the second sort.[13] But strong convexity is the dominant factor because of asymmetry in curvatures of two sorts of curves. That is, aggregate output would increase in the steady state if people's incentives to trade were not changed but π were dispersed to some h. This is the critical steady-state property that dictates the output-response patterns in display.

Next we conduct two exercises: (a) vary η in $\{1, 2, 4\}$ but maintain $(\sigma, \alpha, \theta) = (1, 1, 1)$; and (b) vary α in $\{1, 5/6, 2/3\}$ but maintain $\{\sigma, \eta, \theta\} = (1, 1, 1)$. Figure 3 displays output responses starting from $\pi_1^1(\pi)$ for these two exercises; responses from $\pi_1^2(\pi)$ remain negative and are skipped.

In exercise (a), a smaller η gives rise to a weaker output response. This finding may be explained by the observation that a smaller η reduces asymmetry between curvatures of two sorts of output curves, as shown in Figure 4 (but one should keep in mind that the output curves might not capture all the structure of the set $\mathbf{y}$). In terms of the global structure, a smaller η would imply that there is a smaller degree of variation in $\mathbf{y}$ because it is more costly for a seller to increase his labor supply in a meeting.

In exercise (b), a smaller α gives rise to a weaker output response. As noted above, substituting v for v_{t+1} gives a good approximation to $\mathbf{y}_t$, $\mathbf{d}_t$, and $\mathbf{p}_t$. When $\alpha < 1$, $\mathbf{y}_t$ and $\mathbf{y}$, viewed as surfaces in the three-dimension space, share similar shapes but

[12]The largest deviations of the pairwise payment $d_1\left(m^b, m^s\right)$ from $d\left(m^b, m^s\right)$ and of the pairwise price $p_1\left(m^b, m^s\right)$ from $p\left(m^b, m^s\right)$ are 0.008% and 0.004%, respectively.

[13]Although shapes of two sorts of curves are quite intuitive, they are formed by many general-equilibrium forces. We cannot prove why one curve is convex and another is concave.

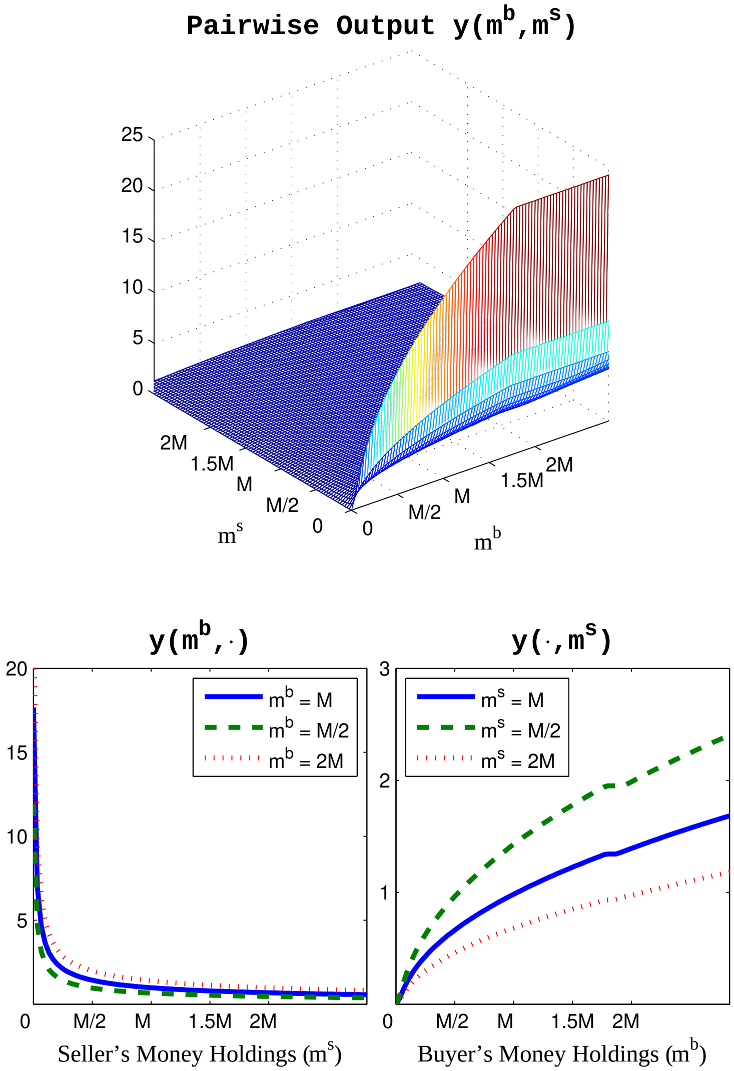

Figure 2: Steady-state pairwise output $\mathbf{y}_t = \left\{ y_t \left(m^b, m^s \right) : 0 \leq m^b, m^s \leq B \right\}$.

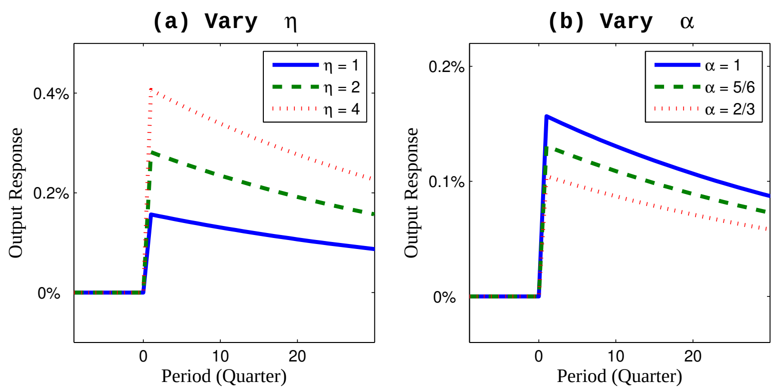

Figure 3: Transitional paths starting from $\pi_1^1(\pi)$ under (a) different η and (b) different α.

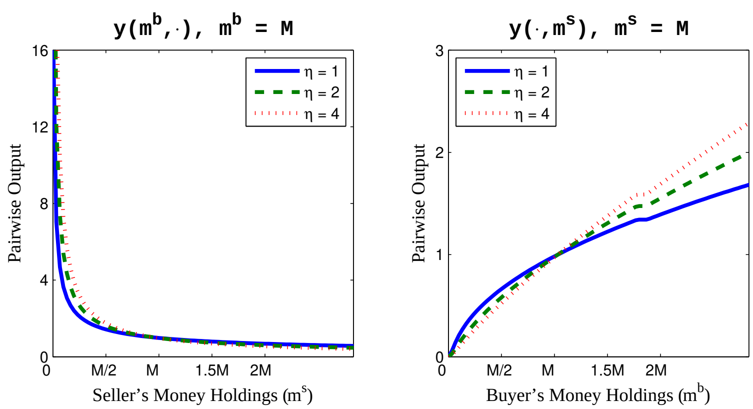

Figure 4: Output curves under different η, holding $(\sigma, \alpha, \theta) = (1, 1, 1)$.

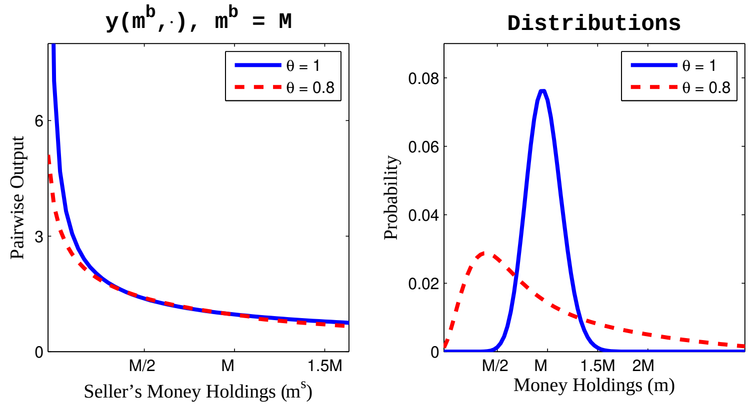

Figure 5: Steady-state output curves and distributions under different θ, holding $(\sigma, \eta, \alpha) = (1, 1, 1)$.

because the aggregate labor input L_t is greater than the steady-state aggregate labor input, the individual-level productivity $AL_t^{\alpha-1}$ is below the steady-state productivity, shifting $\mathbf{y}_t$ down from $\mathbf{y}$. Conceivably, a smaller α is accompanied with a larger systematic shifting-down effect. At $\alpha = 2/3$, $\mathbf{y}_1$ deviates from $\mathbf{y}$ by 0.05% universally on all positive points.[14]

Findings in exercises (a) and (b) seem robust. Indeed, we observe the same monotonic patterns when we use other values of (σ, θ) and vary the way to construct $\pi_1^1(\pi)$. Most importantly, we can obtain a stronger output response by a money injection in the next section if we choose a larger η or α. The same does not apply to parameters θ and σ.

Regarding θ, we find two forces in play when changing its value. A smaller θ leads to a less curvature in an output curve of the first sort displayed in the left graph of Figure 5 (this tends to dampen the output response), and a more dispersed steady-state distribution as displayed in the right graph of Figure 5 (this tends to amplify the output response). The interaction of these two forces makes the output response sensitive to how the underlying shock disperses the distribution of money. Varying θ from 1 to 0.8 and holding (σ, η, α) at $(1, 1, 1)$, we find that a smaller θ leads to a stronger output response; at $\theta = 0.8$, the peak output increase reaches 1.3% (it is 0.16% at $\theta = 1$). But with a $\pi_1^1(\pi)$ constructed differently,[15] a smaller θ leads to a

[14]The largest deviation of $d_1(m^b, m^s)$ from $d(m^b, m^s)$ is no greater than 0.005% at $\alpha = 2/3$, indicating that $\mathbf{p}_t$ rises above $\mathbf{p}$ by 0.05% universally on all positive points.

If we replace the seller's production in (1) with $y = Al^{\alpha}$, then all externality caused by the aggregate labor input disappears. Also in (6) the argument inside $c(.)$ becomes $y^{1/\alpha}$; we may transform this argument into y (so, in particular, it is linear in y as in the original setting) and accordingly transform the coefficient η for $c(.)$ into $\frac{\eta - \eta\alpha + 1}{\eta\alpha}$. So if $\eta = 1$ and $\alpha = 2/3$, setting $y = Al^{\alpha}$ certainly amplifies the output response.

[15]For this $\pi_1^1(\pi)$, let each agent with m units of money be hit by a shock: with probability $1 - \vartheta$, the agent keeps m; with probability ϑ, he draws $\varsigma(m)$ from a discrete uniform distribution

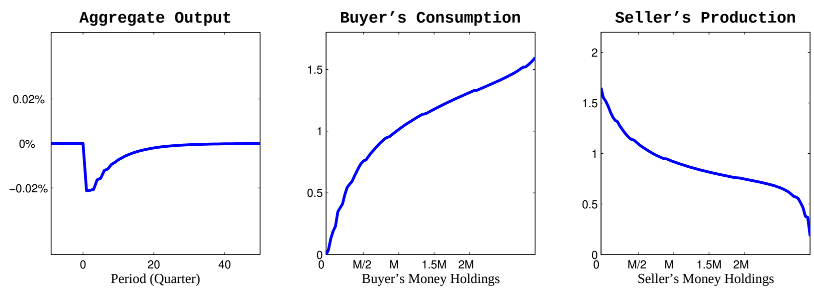

Figure 6: Left: centralized market's output response from $\pi_1^1(\pi)$; middle and right: buyer's consumption curve and seller's production curve.

weaker response; the peak output increase is 1.1% at $\theta = 1$ and 0.4% at $\theta = 0.8$. For the former $\pi_1^1(\pi)$, $var(\pi_1^1(\pi)) - var(\pi)$ increases from 4.6 to 39.1 when θ falls from 1 to 0.8; for the latter $\pi_1^1(\pi)$, $var(\pi_1^1(\pi)) - var(\pi)$ ranges from 6 to 7 under various θ. That is, when π itself is more dispersed, the imaginary shock underlying the former $\pi_1^1(\pi)$ induces a much stronger dispersing effect than the shock underlying the latter $\pi_1^1(\pi)$.

A change in σ may also involve two offsetting forces.[16] Given η in the range from 1 to 2, the overall effect of increasing σ around unity is to amplify the output response. But this effect is almost ignorable under the money injection in the next section.

Now we turn to the role of decentralized trade. Decentralized trade certainly matters for persistence of the output response because it slows down the dispersion (diffusion) of money redistributed by the shock. For its contribution to significance of the output response, we modify the basic model by replacing pairwise meetings with a centralized meeting. In this modified version, each agent has the equal chance to be a buyer or a seller in a competitive market. Each agent takes the price of money ϕ_t as given. He trades with the market a lottery μ for monetary payments and a quantity y for goods such that the expected monetary payment implied by the lottery μ is y/ϕ_t. Let v_t and π_t be the same as in the basic model. Given π_0, an equilibrium is a sequence $\{v_t, \phi_t, \pi_{t+1}\}_{t=0}^{\infty}$ satisfying standard conditions on the law of motion, the recursive relation between value functions, and the market clearing; details of these conditions are given in the appendix. A triple (v, ϕ, π) is a steady state if $\{v_t, \phi_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $(v_t, \phi_t, \pi_{t+1}) = (v, \phi, \pi)$ all t is an equilibrium.

As in the basic model, we compute a steady state (v, ϕ, π) and a transitional equilibrium from $\pi_1^1(\pi)$. From the left graph in Figure 6, we see a transient, negative and insignificant output response. To understand this pattern, we refer to the middle

on $\{-q, \ldots, 0, \ldots q\}$, where $q = \min(m, B - m)$, and his money holdings become $m + \varsigma(m)$. We use $\vartheta = 0.02$ in construction.

[16]Increasing σ toward unity is equivalent to two combined effects: decreasing η (which tends to decrease output) and a convex transformation of y in $u(.)$ (which tends to increase output). Similarly, increasing σ away from unity is equivalent to two combined effects: increasing η (which tends to increase output) and a concave transformation of y in $u(.)$ (which tends to decrease output).

and right graphs in the figure obtained from (v, ϕ, π): the middle graph has a *buyer's consumption curve* that tells how the buyer's consumption varies with his money holdings and the right graph has a *seller's production curve* that tells how the seller's production varies with his money holdings. According to curvatures of these two curves, if ϕ_1 is the same as ϕ, then dispersing π to $\pi_1^1(\pi)$ tends to raise the period-1 aggregate production above the steady-state level and reduce the period-1 aggregate consumption below the steady-state level; the influence from agents with holdings greater than $2M$ may be ignored because the proportion of these agents is very small. To clear the market, ϕ_1 must rise above ϕ (the goods price must fall) so that the period-1 buyer's consumption curve is shifted up and the period-1 seller's production curve is shifted down. Examining either curve, the dispersing effect and the shifting effect offset each other, leading to an insignificant output effect. Starting from $\pi_1^2(\pi)$, the output response is insignificant by the same reason (while it turns to be positive).

4 One-shot regressive money injection

As in the last section, the economy reaches a monetary steady state (v, π) at period 0; but here we replace the imaginary shock with a monetary shock. Specifically, the government injects money from period 1 to N, raising the stock of money from M to M'. As noted above, we look for the post-injection steady state (v', π') such that neutrality applies to (v, π) and (v', π'). Given this neutrality, aggregate output Y' and the average price P' at the steady state (v', π') are very close to Y and PM'/M, respectively.[17] As in the last section, one gets a good approximation to $\mathbf{y}_t$, $\mathbf{d}_t$, and $\mathbf{p}_t$ by substituting the post-shock steady state value function, namely v', for v_{t+1}. So if the injection induces a distribution before period-1 pairwise meetings more dispersed than π', then Y_1 increases relative to Y' and, hence, relative to Y. Recall that $\Pi'(m) \approx \bar{\Pi}(mM/M')$, implying that we should consider a regressive money injection.

We endogenize regressiveness of the injection by way of an endogenized limited participation as follows. At period $t \in \{1, \ldots, N\}$, agents are entitled to buy lotteries with money before pairwise meetings: if an agent pays x units of money, then he receives $2x$ units of money with probability $\chi_t \in (0.5, 1]$ that is set by the government; otherwise he receives no money. Given χ_1,

$$(\chi_{t+1} - 0.5) / (\chi_t - 0.5) \equiv \rho_t \in (0, 1)$$

for $t \in \{1, \ldots, N-1\}$ if $N > 1$. Strictly concave value functions and nondegenerate distributions imply a limited participation in that richer agents tend to spend more on lotteries and, hence, receive a larger portion of injected money. Recall that there is an upper bound B on the individual holdings. This bound affects the lottery-purchasing decision for agents whose holdings are close to the bound (an agent with holdings

[17]For exercises in this section, $M' = 1.01M$, the deviation of Y' from Y is no greater 0.007%, and the deviation of P' from PM'/M is no greater than 0.09%.

B does not buy any lottery). But these effects have little influence on the output response because the measure of agents with such holdings are very small.[18]

Because the injection takes only N periods, the equilibrium conditions at period $t \geq N+1$ are the same as those in section 2. At period $t \in \{1, \ldots, N\}$, the equilibrium conditions involve (v_t, π_t) as usual and the distribution $\breve{\pi}_t$ of money after the injection but before pairwise meetings; details of these conditions are given in the appendix.

We consider $N \in \{1, 5\}$ in computing the transitional equilibrium from $\breve{\pi}_1$ to the steady state (v', π'): $N = 1$ is the benchmark while $N = 5$ may mimic the familiar AR(1) process of monetary shocks. When $N = 1$ we seek a suitable lottery-winning probability χ_1 to meet $M' = 1.01M$; when $N = 5$, we set $\rho_t = \rho = 0.65$ and again we seek a χ_1 to meet $M' = 1.01M$ ($\rho = 0.65$ turns out to imply that the amount of money injected at $t+1 < N$ is around half of the amount of money injected at $t \geq 1$).

Figure 7 displays the output and price responses along the transitional equilibrium for benchmark parameter values $(\sigma, \eta, \alpha, \theta) = (1, 1, 2/3, 1)$. The output response for $N = 1$ conforms to the pattern of the output response along the transitional equilibrium starting from $\pi_1^1(\pi)$ in the last section. For $N = 5$, output expands with a declining rate as the total money stock increases; the reason is simple—the injection at $t+1$ reinforces the dispersion on the distribution of money made by the injection at t but the reinforcing effect declines as the injection rate declines.

In Figure 7, the price responses are rapid and slightly less than proportional to the change in the money supply. But the price response may be more sluggish for other parameters. Figure 8 displays the output and price responses when $(\sigma, \eta, \alpha) = (0.5, 4, 1)$ (holding $\theta = 1$ and $N = 1$). We observe a much stronger output response, which may be attributed to $\alpha = 1$ and $\eta = 4$ (as explained in the last section). We also observe a more sluggish price response. Recalling that one gets a good approximation to $\mathbf{p}_t$ by substituting v' for v_{t+1}, we display in Figure 9 two sorts of price curves obtained from the post-injection steady state when $(\sigma, \eta, \alpha) = (0.5, 4, 1)$ and from the post-injection steady state when $(\sigma, \eta, \alpha) = (1, 1, 2/3)$.

In the left graph, a price curve from a steady state tells how the pairwise price $p(m^b, m^s)$ varies with m^s for the fixed $m^b = M$; in the right graph, a price curve from a steady state tells how $p(m^b, m^s)$ varies with m^b for the fixed $m^s = M$. For comparison, price curves in each graph are rescaled so that the highest prices appear to be the same. Price curves from the former steady state are more concave,[19] contributing

[18]In the basic model with benchmark parameter values, for example, $\pi(B) = 1.63 \times 10^{-17}$ and $\pi(B-1) = 3.05 \times 10^{-17}$ in the steady state. In general, we need a finite B for numerical exercises. In this model, if B has any influence on our result, the influence is to dampen the output response; for, the upper bound being relaxed, the money injection would only disperse the distribution of money further.

[19]At the left figure, a more concave price curve implies a lower ratio of the increment in the price paid by a buyer caused by a rich seller's marginal increase in wealth to the increment caused by a poor seller's. This seems plausible for a larger η: a smaller price increment may give the rich seller sufficient incentive. At the right figure, a more concave price curve implies a higher ratio of the increment in the price received by a seller caused by a poor buyer's marginal increase in wealth to

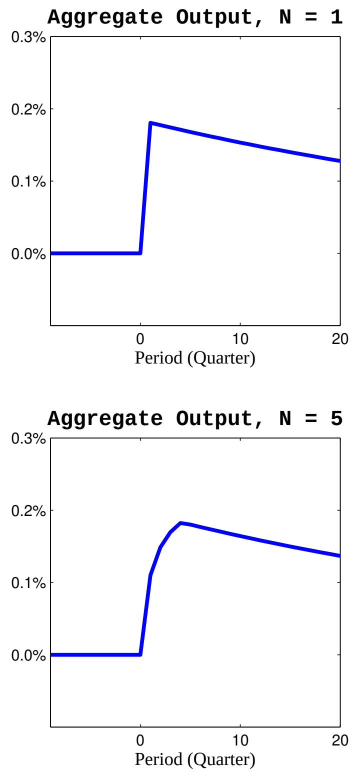

Figure 7: Output and price responses after a 1% money injection. Top row: $N = 1$; bottom row: $N = 5$.

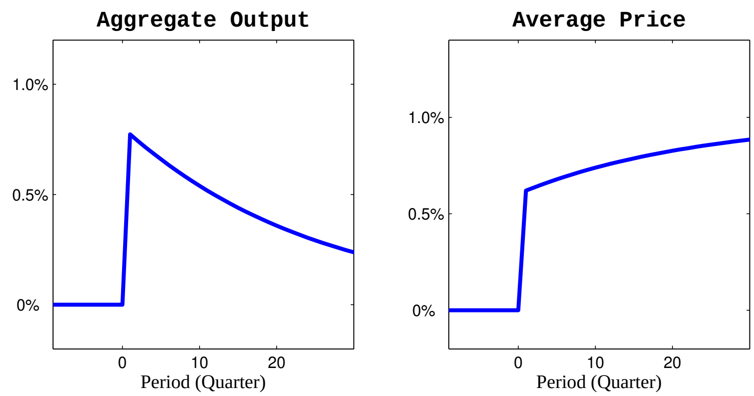

Figure 8: Output and price responses under $(\sigma, \eta, \alpha, \theta) = (0.5, 4, 1, 1)$.

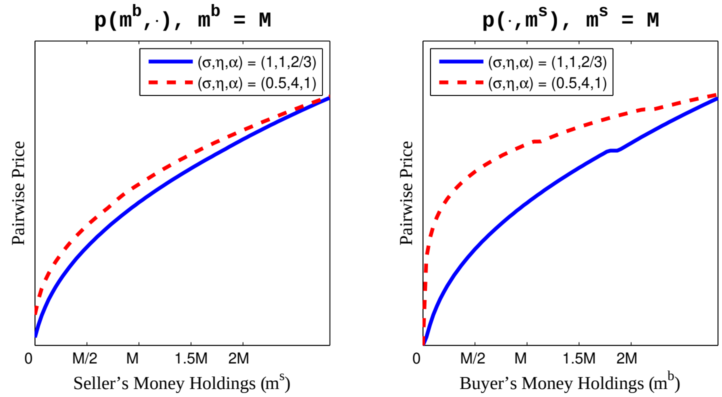

Figure 9: Steady-state price curves $p\left(m^b, m^s\right)$.

to a larger fall in P_t from P' and, hence, a less responsive price pattern following a regressive money injection; in addition, as noted in the last section, $\alpha = 1$ contributes to a stronger output response (by eliminating externality from the aggregate labor input), further dampening the price response.

Next we turn to other surplus shares of buyers. When we vary θ from 1 to 0.8 (holding σ, η, α and N at benchmark values), the degree of output response first increases as θ decreases from unity and reaches the maximum at $\theta = 0.95$; then it falls as θ falls. When θ slightly departs from 1, there is a rapid and more-than-proportional price response, i.e., price overshooting; the degree of price overshooting increases as θ decreases. The upper row in Figure 10 displays the output and price responses for $\theta \in \{1, 0.95, 0.8\}$; it is worth noting that there is not any monotonic relationship between the degree of output response and the degree of price response when θ is varied.

Regarding the output-response patterns for $\theta < 1$, we refer to the two accompanying forces when θ decreases indicated in the last section. Here an informative statistic is the lottery winning probability χ: $\chi = 0.5520$, 0.5608 and 0.5585 at $\theta = 1$, 0.95, and 0.8, respectively; a smaller χ implies a larger proportion of agents who buy lotteries, making the injection less regressive.

Regarding price overshooting, we draw price curves from post-injection steady states for $\theta \in \{1, 0.95\}$ in the bottom row of Figure 10 and, again, rescale curves in each graph so that the highest prices appear to be the same. At each graph, the curve for $\theta = 0.95$ is much different from the curve for $\theta = 1$. When $\theta = 0.95$, the curve is convex at the left graph; at the right graph, it is convex over a wide range and buyers with small holdings pay much higher prices.[20] These differences conform

the increment caused by a richer buyer's. This seems plausible for a smaller σ: a poor buyer is more aggressive in spending money.

[20]Convexity at the left figure says that when there is a marginal increase in the seller's wealth, the increment in the price paid by a buyer is much higher if the seller is richer. This seems sensible because given the decreasing marginal value of money, a higher price increment is necessary to

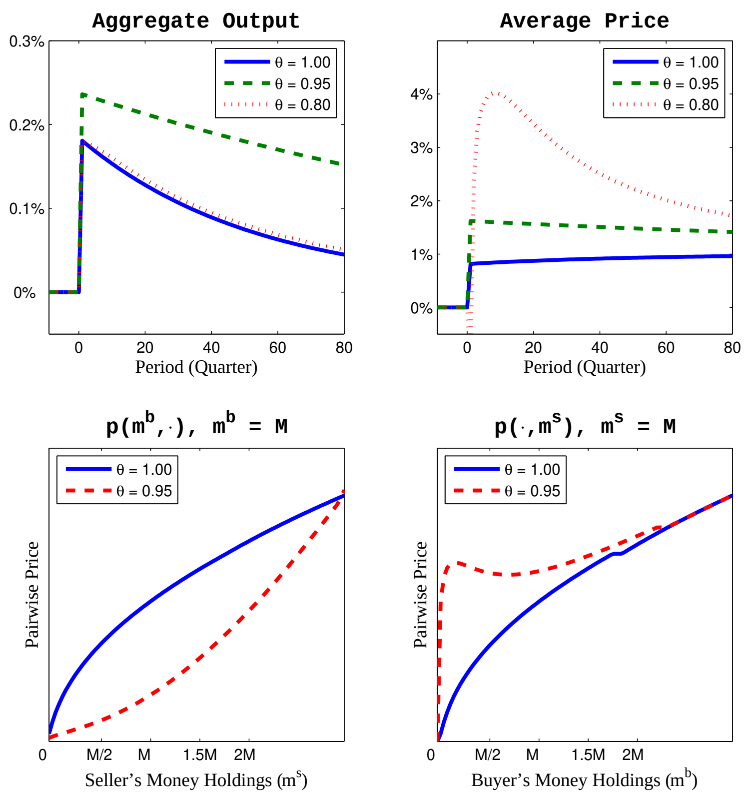

Figure 10: Transition paths and steady-state price curves, under different θ.

to the different price-responding patterns for the two values of θ.

Next we briefly comment on changing parameter values of η and σ. If we raise η to 2 (holding other parameters constant), the peak output increase is 1.7 to 1.8 times the value for $\eta = 1$; the output and price response patterns remain the same. Varying σ from 0.75 to 1.25 (holding other parameters constant), we observe little changes in degrees and patterns of the output and price responses except that there is more persistence as σ increases.

Finally, we note a short-run Phillips curve in our model. That is, if the injection increases the money stock by $a\%$ and a is not very large (say, $a \leq 5$), then the output and price responses are about a times values for the corresponding 1%-increase case.

5 Bonds and liquidity effect

In this section we modify the basic model by adding one-period government bonds as follows. Each period t consists of two stages, 1 and 2. At stage 1, the government issues D_t amount of bonds on a competitive market. An agent entering the bond market with m units of money can choose a probability measure $\hat{\mu}$ (a lottery) defined on the set $\Xi = \{\zeta = (m, w) \in B_\Delta \times B_\Delta : w \geq m\}$ that satisfies

$$\sum_{\zeta'=(m',w')} \hat{\mu}(\zeta') \cdot \left(m' + p_t^B (w' - m')\right) \leq m, \tag{12}$$

where $\hat{\mu}(\zeta')$ is the probability for the agent to leave the bond market with m' units of money and $w' - m'$ units of bonds and p_t^B, interpreted as the price of bonds, is taken as given by the agent. In equilibrium, p_t^B clears the bond market. Each unit of bonds automatically turns into one unit of money at the start of period $t + 1$. At stage 2, agents are matched in pairs as in the basic model. In pairwise meetings, each agent can observe his meeting partner's portfolio, but bonds are illiquid and money is the unique payment method.

In this modified model, the total money stock would increase over time because of interest payments. If money were divisible, then we would simply normalize the state of an agent at period t right before issuance of new bonds as mM_t/M_t^+, where m is the agent's money holdings at that time, M_t^+ is the average money holdings at that time, and M_t is the difference between M_t^+ and period-t interest payments $D_{t-1}\left(1 - p_{t-1}^B\right)$. Because mM_t/M_t^+ need not be an integer for indivisible assets, we follow Deviatov and Wallace [11] by assuming that right before new bonds are issued, each unit of money automatically disintegrates with the probability $\delta_t = 1 - M_t/M_t^+$

maintain a positive share of surplus to the seller. For the the right graph, we note that a buyer with small holdings spend very little when $\theta = 1$, largely contributed by that the marginal value of money at zero is very high. A smaller θ lowers the marginal value of money at zero, so the a higher price (with reference to $\theta = 1$) may be a better means for the buyer with small holdings to give a positive share of surplus to the seller; but the advantage of a higher price may decline rapidly when the buyer's money holdings increase because the buyer's marginal value of money gets closer to his marginal value of money under $\theta = 1$.

so that the total money stock returns to M_t.[21] Setting $M_0 = M$, we have $M_t = M$, all t.

Here we let v_t and π_t be the value function and the distribution of money after disintegration but before bonds issuance at period t (so $\sum_m \pi_t(m)\, m = M$) and let $\hat{\pi}_t$ be the distribution of portfolios right before pairwise meetings. Given π_0 and $\{D_t\}_{t=0}^{\infty}$, an equilibrium is a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}, p_t^B\}_{t=0}^{\infty}$ satisfying standard conditions on the laws of motion, the recursive relations between value functions, and the bond market clearing; details of these conditions are given in the appendix. A tuple $(v, \hat{\pi}, \pi, p^B)$ is a steady state if $\{v_t, \hat{\pi}_t, \pi_{t+1}, p_t^B\}_{t=0}^{\infty}$ with $(v_t, \hat{\pi}_t, \pi_t, p_t^B) = (v, \hat{\pi}, \pi, p^B)$ all t is an equilibrium. In a steady state, it is necessary to have $D_t = D$ for some D.

We interpret the product of the period-t gross growth rate in the price level and M_t^+/M (the period-t gross growth rate in the money stock if there is no disintegration) as *the gross inflation rate* at $t \geq 1$, denoted $1 + \varphi_t$, i.e.,

$$1 + \varphi_t = \frac{1}{1 - \delta_t} \frac{P_t}{P_{t-1}}. \tag{13}$$

We interpret $i_t \equiv 1/p_t^B - 1$ as the nominal interest rate and, hence, $i_t - \varphi_t$ as the real interest rate at t. The real interest rate at steady state is nearly zero because interests are financed by inflation.

We apply to this modified model the same one-shot money injection as in the last section. Again the economy reaches a steady state at period 0. At period $t \in \{1, ...N\}$, money is injected before issuance of new bonds. Now M_t^+ is the period-t average money holdings after injection but before issuance of new bonds and, as above, $M_t = M_t^+ - D_{t-1}\left(1 - p_{t-1}^B\right)$ and $\delta_t = 1 - M_t/M_t^+$. Starting from period 1 the supply of bonds follows the process

$$D_t = D' - \psi_t \cdot (D' - D_{t-1}), \tag{14}$$

where $D_1 = D$, D' is the supply of bonds at the post-injection steady state, and the sequence $\{\psi_t\}$ governs how quickly the bonds-money ratio returns to the steady-state level.

In exercises below, we follow the two-step procedure described in section 2. Here we choose D close to M so that the nominal interest rate at the pre-injection steady state is around 1.5%. We choose $D' \approx DM'/M$ so that neutrality applies well to pre-injection and post-injection steady states, where $M' - M$ is the total amount of money injected over the N injection periods. Other parameter values, including benchmark values for $(\sigma, \eta, \alpha, \theta)$, are the same as in the basic model.

Figure 11 presents transition paths when $D = 29.45$, $N = 1$, $M' = 1.01M$ (as in the basic model, we seek a suitable χ_1 to meet this target), $D' = 29.76$, and $\psi_t = \psi \in \{0.0, 0.5\}$ all t.[22] To facilitate the discussion, let Z_{t+1} be the average nominal wealth at the start of period $t+1$ in the transitional equilibrium; let Z be

[21] If we apply this disintegration to divisible money, an agent with m units of money before disintegration holds mM_t/M_t^+ after disintegration, exactly equivalent to the normalization without disintegration.

[22] Because of indivisibility of nominal assets, some $D' \neq DM'/M$ may bring output and the

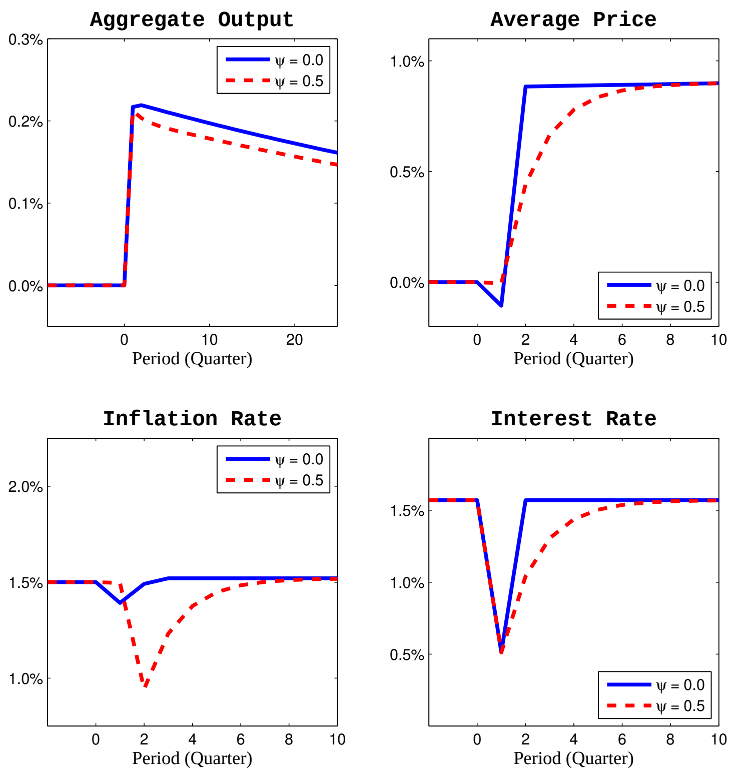

Figure 11: Transitional paths following a 1% money injection, under different ψ.

the average nominal wealth at the start of a period after the post-injection steady state is reached. As in the basic model, one gets a good approximation to period-t pairwise meeting outcomes in the transitional equilibrium by substituting the value of holding zZ/Z_{t+1} at the start of a period at the post-injection steady state for the value of holding z at the start of period $t+1$.[23]

From Figure 11, we observe a liquidity effect for each ψ: i_1 is around 0.5% while the nominal interest rate i in the pre-injection steady state is around 1.5%. This may be explained as follows. If an agent carries j units of money into pairwise meetings in the post-injection steady state, then he receives a close service from carrying $j_t = jZ_{t+1}/Z$ units of money into period-t pairwise meetings in the transitional equilibrium. So $J_t = \kappa J Z_{t+1}/Z$ for some κ not far from one, where J_t and J are the average money holdings carried into period-t and steady-state pairwise meetings, respectively. But J is slightly greater than 1 and because Z_{t+1}/Z is around 1, $J_t - J = (\kappa Z_{t+1}/Z - 1)J$ is too small to absorb most of injected money. As implied by this explanation and also observed from Figure 11, the speed of the nominal interest rate returning to the steady-state level depends on the speed of the bonds-money ratio returning to the steady-state level.

The big surprise in Figure 11 is that the injection drives down inflation: the inflation rate is down to 1.4% when $\psi = 0$ and to 0.9% when $\psi = 0.5$, while the inflation rate φ in the pre-injection steady state is around 1.5%. To understand this, it suffices to consider $\psi = 0$. One unit of money in pairwise meetings at period 1 is worth of, in the approximation sense, Z/Z_2 units in pairwise meetings at the post-injection steady state. So $P_1 = \kappa_1 P' Z_2/Z = \kappa_1 P(M'/M)(Z_2/Z)$ for some κ_1. The term κ_1 reflects the effect on P_1 caused by the difference between π' and the distribution of nominal wealth following the injection; in the corresponding situation in the basic model, it is less than but close to 1. By definition $Z_2 = M' + Di_1$ and $Z = M' + D'i'$, where $i' \approx i$ is the nominal interest rate in the post-injection steady state. The difference in interest payments $D'i' - Di_1$ is around 0.3, implying $Z/Z_2 \approx 1.013$; that is, the effect on P_1 from the increase in the nominal stock is dominated by the effect on P_1 from the increase in the real value of money. Because δ_1 is equal to the disintegration rate δ in the pre-injection steady state, it follows that φ_1 is below φ. Similarly, $P_2 = \kappa_2 P(M'/M)(Z_3/Z)$ for some κ_2 (playing the same role as κ_1). Because the difference in interest payments $D'i' - Di_2$ is nearly zero, P_2 is close to $\kappa_2 P(M'/M)$ and, hence, P_2/P_1 is close to 1.01. But φ_2 is still below

nominal interest rate in (v', π') closer to those in (v, π) than $D' = DM'/M$. In the present exercise, deviations of Y' from Y, P'/M' from P/M, and $p^{B\prime}$ from p^B are 0.001%, 0.004%, and 0.02%, respectively. If we instead choose $D' = DM'/M$, the corresponding numbers are 0.1%, 1.3%, and 0.03%.

[23] In the basic model, if the stock of money M_{t+1} at period $t+1$ in the transitional equilibrium is not equal to the stock of money M' in the post-injection steady state (e.g., $t = 2$ when $N = 5$), the value of holding m at $t+1$ is approximated by the value of holding mM'/M_{t+1} at the post-injection steady state. If mM'/M_{t+1} or zZ/Z_{t+1} is not an integer, the steady state value is taken from the linear interpolation of the relevant steady-state value function.

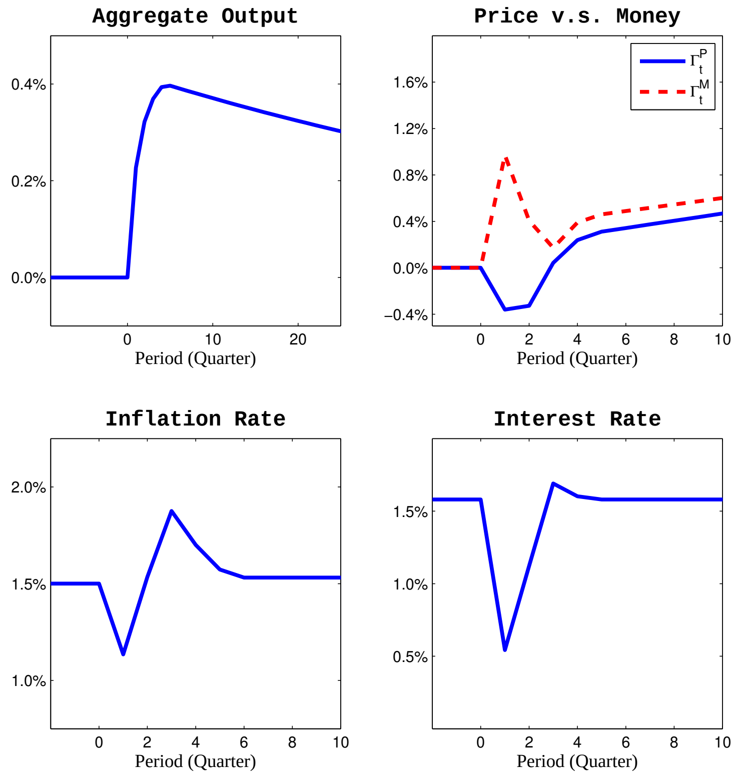

Figure 12: Transitional paths following a 1.667% money injection, $N = 5$.

φ because δ_2 is much smaller than δ, a consequence of the substantial difference in interest payments $D'i' - Di_1$.

To emphasize, the reduction in interest payments $D'i' - Di_1$ at period 2 has a direct effect on δ_2 and an indirect effect on P_1 through δ_2 so that it affects inflation rates at periods 1 and 2. The general point is that if the steady-state inflation is driven by interest payments to government bonds, then the liquidity effect of an injection may easily drive down the inflation.

There are richer and more interesting dynamics in real and nominal variables with a large N. To stay close to the basic model, we let $N = 5$ and $\rho_t = \rho = 0.65$. We seek a suitable χ_1 so that $M'/M - 1 = 1.667\%$ and money injected at period 1 is around $0.01M$ (the nominal interest at period 1 then is down by the same magnitude as above). We set $D' = 29.966$ and choose $\{\psi_t\}_{t=1}^{T}$ so that $D_t/M_{t-1} = D_{t-1}/M_{t-2}$ for $t < 3$ and $D_t = D'$ for $t \geq 3$; in this process, D_t/M_t exceeds the steady-state level at $t = 3$. Transitional paths are in Figure 12. The output response peaks at the last period of the injection (which is the same as in the basic model).

The nominal interest rate falls by 1% initially and next rises up; it exceeds the steady-state level at $t = 3$ and then slowly falls back, which makes sense because

D_t/M_t exceeds the steady-state level at $t=3$ and then falls back to the steady-state level. The inflation rate first falls below the steady state level, as in the exercise with $N=1$. Then it exceeds the steady state level at $t=3$ and then falls back. Notice that $D_t/M_t > D/M$ for $t \geq 3$ leads to $\delta_{t+1} > \delta$ (by affecting interest payments at period $t+1$), which, in turn, leads to $\varphi_{t+1} > \varphi$; $D_3/M_3 > D/M$ also leads to $\varphi_3 > \varphi$ (by affecting P_3 through $\delta_4 > \delta$).

Because the individual state is normalized, the price response does not directly reveal the money-price relationship in the usual sense. This motivates us to introduce two variables, $\Gamma_t^P \equiv (1+\tilde{\varphi}_t)/(1+\tilde{\varphi}(t)) - 1$ and $\Gamma_t^M \equiv \tilde{M}_t/\tilde{M}(t) - 1$. Here, $1 + \tilde{\varphi}_t \equiv (1+\varphi_1) \times ... \times (1+\varphi_t)$ is the gross inflation rate from period 0 to t in the transitional equilibrium and $1+\tilde{\varphi}(t) \equiv (1+\varphi)^t$ is the t-period gross inflation rate in the pre-injection steady state. Also, $\tilde{M}(t) \equiv M/(1-\delta)^t$, $\tilde{M}_t \equiv \tilde{M}_{t-1} + [\gamma_t^M + D_{t-1}\left(1-p_{t-1}^B\right)]/(1-\tilde{\delta}_t)$, γ_t^M is the amount of money injected at period t, $1-\tilde{\delta}_t \equiv (1-\delta_1) \times ... \times (1-\delta_t)$, and $\tilde{M}_0 \equiv M$. We interpret $1+\tilde{\varphi}(t)$ and $\tilde{M}(t)$, respectively, as the price level and the stock of money at period t if there were neither injection nor disintegration from period 1 to t in the pre-injection steady state. We interpret $1+\tilde{\varphi}_t$ and $\tilde{M}_t$, respectively, as the price level and the stock of money at period t if there were no disintegration from period 1 to t in the transitional equilibrium.

Hence Γ_t^P and Γ_t^M, respectively, represent departures of the non-normalized price and of the non-normalized stock of money at period t from steady-state trends. As shown in the upper-right corner of Figure 12, the non-normalized price first falls below the trend and next rises above it and gradually reaches a constant position relative to the trend. The non-normalized stock of money stays above the steady state trend and gradually reaches a constant position relative to the trend. After the non-normalized price rises above the steady state trend, its departure from the trend is still less than the departure of the non-normalized stock of money from the trend. An outside observer may view this as an evidence of nominal rigidity.

The response patterns in Figure 12 are overall consistent with responses to monetary policy shocks found by VAR studies (e.g., Christiano, Eichenbaum, and Evans [9]). Note in particular that some VAR studies find a fall in the inflation rate right after an expansionary policy shock, referred to as the price puzzle; such a fall in inflation is an equilibrium outcome in our model. Moreover, it seems possible for our model to mimic more closely some responses in VAR studies; for example, with a larger N and suitable χ_1 and ρ_t, the output response may move up more smoothly and peak before the inflation response peaks.

6 Discussion

Here we first discuss some detailed settings in our model. The first setting pertains to the lottery-purchasing scheme that endogenizes a limited participation in the money injection. Admittedly this scheme, as many modelling devices in economics, may not be directly observed in reality. But if we start from a realistic point, that is, the distribution of wealth is not degenerate, then in a realistic sense an injection is likely either to disperse or to squeeze the distribution of wealth. The lottery-purchasing scheme disciplines us in choosing the dispersion degree and it may be regarded as a parsimonious way to capture the dispersion of wealth induced by a money injection through channels not existing in the present model. In a more sophisticated model with such a channel but without that scheme, our finding would suggest that the injection may lead to a strong output response.

The second setting pertains to our parameterization of preferences. We concentrate on a class of preferences that are widely used in recent monetary economics (see, e.g., Gali [14], Woodford [37], and Christiano, Eichenbaum, and Evans [10]). With such a preference, the set of steady-state pairwise output $\mathbf{y}$ takes the shape as in Figure 2, which is critical to our findings. While this shape is quite robust when we vary underlying parameters (σ and η), it need not be so under another class of preferences. For example, the set $\mathbf{y}$ is almost concave if one adopts the disutility function of labor supply in Molico [25]. Our stand is that if the present class describe people's short-run behaviors well, then there is a whole new source for nonneutrality of money in the short run.

The third setting pertains to the buyer's surplus share. We have little knowledge about what may be a better value for this parameter. We vary it from 1 to 0.8 (moving to a lower value would imply an incredible level of price overshooting). In this range, the output response for $\theta < 1$ is at least as strong as the output response for $\theta = 1$ but there is nominal rigidity only for θ close to 1. For readers who believe that a lower buyer's share may be more realistic but also anticipate nominal rigidity, there is a simple reconciliation; that is, we may let the buyer and seller in a meeting be chosen by nature with probabilities ι and $1 - \iota$, respectively, to make a take-it-or-leave-it offer. Then the output and price responses are insensitive to the choice of ι.

Next we note that Hume seemed to believe in the long-run stimulative effect of inflation:

> ...[I]t is of no manner of consequence, with regard to the domestic happiness of a state, whether money be in a greater or less quantity. The good policy of the magistrate consists only in keeping it, if possible, still increasing; because, by that means he keeps alive a spirit of industry in the nation. [Hume [20, p 173]]

It is straightforward to modify the basic model by making permanent the regressive money injection scheme in section 4 and study the steady-state output-inflation relationship. Now a higher inflation tends to reduce the value of money and hence output between every buyer-seller pair. But the regressive injection keeps the dispersing force on the distribution of money forever so that this effect may be very powerful—one may think of a one-shot injection when N is large and the injected amount does not decline in these N periods. Offsetting this strong dispersing force by mixing the regressive injection with a repeated lump-sum money transfer, we find a positive output-inflation correlation when inflation is low and a negative correlation when inflation is high,[24] a pattern consistent with empirical findings for some economies including the U.S. (see, Ahmed and Rogers [1], Bullard [5], and Bullard and Keating [6]). Nonetheless, there is a constraint for a central bank to exploit this long-run relationship. A regressive injection always reduces average welfare in that $\sum v(m)\pi(m) > \sum v'(m)\pi'(m)$, where (v,π) is the zero-inflation steady state and (v',π') is the steady state with the stimulative injection. That is, less equality is a cost for higher output.

Finally we turn to two extension that may be taken in the future. In our model, the monetary shock is one shot. While we do not see an obvious reason that our results would be overturned if monetary shocks are recurrent, it is certainly worth of studying the extension with recurrent shocks. Apparently numerical analysis for this extension is much more challenging and may rely on some version of the method of Krusell and Smith [18]. For the second extension, we note that the output response in our model is essentially driven by people who are made poorer by an injection. There is no substantial role for people who are made richer by the injection. The latter group of people may have a much larger role if job creation is costly. The second extension therefore is to make the labor market distinguishable from the goods market and make it costly to create job in the labor market. Perceivably this is a demanding work, too.

[24] We report details in the online appendix.

Appendix A: Complete description of equilibria

A.1 The basic model

In section 2, $\lambda(m, m', v_{t+1}, L_t)$ is defined as follows. For $d^+ \in \{1, ..., B-m\}$ and $d^- \in \{1, ...m\}$,

$$\lambda\left(m, m+d^+, v_{t+1}, L_t\right) = 0.5 \sum_{m^b=1}^{B} \pi_t\left(m^b\right) \mu\left(d^+; m^b, m, v_{t+1}, L_t\right), \tag{15}$$

$$\lambda\left(m, m-d^-, v_{t+1}, L_t\right) = 0.5 \sum_{m^s=0}^{B-1} \pi_t\left(m^s\right) \mu\left(d^-; m, m^s, v_{t+1}, L_t\right),$$

$$\lambda\left(m, m, v_{t+1}, L_t\right) = 0.5 \sum_{m^b=1}^{B} \pi_t\left(m^b\right) \mu\left(0; m^b, m, v_{t+1}, L_t\right) + 0.5 \sum_{m^s=0}^{B-1} \pi_t\left(m^s\right) \mu\left(0; m, m^s, v_{t+1}, L_t\right).$$

A.2 The model with centralized market

Consider the version of the model with a centralized market in the last part of section 3. The problem for a buyer with money holdings m is

$$\max_{y,\mu} \tilde{S}^b\left(y, \mu, m, \phi_t, v_{t+1}, L_t\right) \text{ s.t. } y = \phi_t \sum_d \mu(d)\, d, \tag{16}$$

where $\tilde{S}^b\left(y, \mu, m, \phi_t, v_{t+1}, L_t\right) = u(y) + \beta \sum \mu(d)\left[v_{t+1}(m-d) - v_{t+1}(m)\right]$; and the problem for a seller with m is

$$\max_{y,\mu} \tilde{S}^s\left(y, \mu, m, \phi_t, v_{t+1}, L_t\right) \text{ s.t. } y = \phi_t \sum_d \mu(d)\, d, \tag{17}$$

where $\tilde{S}^s\left(y, \mu, m, \phi_t, v_{t+1}, L_t\right) = -c\left(yL_t^{1-\alpha}/A\right) + \beta \sum \mu(d)\left[v_{t+1}(m+d) - v_{t+1}(m)\right]$. Let $\left(\tilde{y}^a\left(m, \phi_t, v_{t+1}, L_t\right), \tilde{\mu}^a\left(m, \phi_t, v_{t+1}, L_t\right)\right)$ be the solution to the problem in (16) if $a = b$ and to the problem in (17) if $a = s$; then

$$v_t(m) = \beta v_{t+1}(m) + 0.5\left[\tilde{f}^b\left(m, \phi_t, v_{t+1}, L_t\right) + \tilde{f}^s\left(m, \phi_t, v_{t+1}, L_t\right)\right], \tag{18}$$

where $\tilde{f}^b$ and $\tilde{f}^s$, respectively, are the buyer's surplus and the seller's surplus implied by $\left(\tilde{y}^a, \tilde{\mu}^a\right)$. Also,

$$\pi_{t+1}(m) = \sum_{m'} \tilde{\lambda}\left(m', m, \phi_t, v_{t+1}, L_t\right) \pi_t\left(m'\right), \tag{19}$$

where $\tilde{\lambda}\left(m', m, \phi_t, v_{t+1}, L_t\right)$ is the proportion of agents with m' units of money leaving with m after trading that is implied by $\left(\tilde{\mu}^b, \tilde{\mu}^s\right)$; that is, for $d^+ \in \{1, ..., B-m\}$ and $d^- \in \{1, ...m\}$,

$$\tilde{\lambda}\left(m, m+d^+, \phi_t, v_{t+1}, L_t\right) = 0.5\tilde{\mu}^b\left(d^+; m, \phi_t, v_{t+1}, L_t\right),$$

$$\tilde{\lambda}\left(m, m-d^-, \phi_t, v_{t+1}, L_t\right) = 0.5\tilde{\mu}^s\left(d^-, m, \phi_t, v_{t+1}, L_t\right),$$

$$2\tilde{\lambda}\left(m, m, \phi_t, v_{t+1}, L_t\right) = \tilde{\mu}^b\left(0; m, \phi_t, v_{t+1}, L_t\right) + \tilde{\mu}^s\left(0; m, \phi_t, v_{t+1}, L_t\right).$$

Market clearing requires

$$\sum_{m} \pi_t(m)\, \tilde{y}^s(m, \phi_t, v_{t+1}, L_t) = \sum_{m} \pi_t(m)\, \tilde{y}^b(m, \phi_t, v_{t+1}, L_t). \tag{20}$$

Given π_0, a sequence $\{v_t, \pi_{t+1}, \phi_t\}_{t=0}^{\infty}$ is an equilibrium if it satisfies (18)-(20) and

$$AL_t^{\alpha} = 0.5 \sum_{m^s} \pi_t(m^s)\, \tilde{y}^s(m^s, \phi_t, v_{t+1}, L_t).$$

A.3 The basic model with one-shot injection

Turning to section 4, consider $t \in \{1, ..., N\}$. Let $\check{\pi}_t$ be as given in the main text and let $f^b(.)$, $f^s(.)$, and $\lambda(.)$ be the same as in section 2 and A.1. Then we have

$$\pi_{t+1}(m) = \sum \lambda(m', m, v_{t+1}, L_t)\, \check{\pi}_t(m'), \tag{21}$$

and the value of holding m units of money right before pairwise meetings is

$$\check{v}_t(m) = \beta v_{t+1}(m) + 0.5 \sum_{m'} \check{\pi}_t(m') \left[f^b(m, m', v_{t+1}, L_t) + f^s(m', m, v_{t+1}, L_t) \right]. \tag{22}$$

At the money injection stage, the problem for an agent with m units of money is

$$v_t(m) = \max_{x \le \min(m, B-m)} \chi_t \check{v}_t(m+x) + (1-\chi_t)\, \check{v}_t(m-x); \tag{23}$$

let $x(m, \chi_t, \check{v}_t)$ be the solution to the problem in (23). Then

$$\check{\pi}_t(m) = \sum_{m'} \check{\lambda}(m', m, \chi_t, \check{v}_t)\, \pi_t(m'), \tag{24}$$

where $\check{\lambda}(m', m, \chi_t, \check{v}_t)$ is the proportion of agents with m' units of money leaving money injection stage with m; that is, $\check{\lambda}(m, m+x, \chi_t, \check{v}_t) = \chi_t$ and $\check{\lambda}(m, m-x, \chi_t, \check{v}_t) = 1 - \chi_t$ for all m with $x = x(m, \chi_t, \check{v}_t) > 0$, and $\check{\lambda}(m, m, \chi_t, \check{v}_t) = 1$ for all m with $x = x(m, \chi_t, \check{v}_t) = 0$. Given $\{\chi_t\}_{t=1}^{N}$ and $\pi_1 = \pi$, a sequence $\{v_t, \pi_{t+1}\}_{t=1}^{\infty}$ together with $\{\check{\pi}_t\}_{t=1}^{N}$ is an equilibrium if (21)-(24) and

$$AL_t^{\alpha} = 0.5 \sum y(m, m', v_{t+1}, L_t) \check{\pi}_t(m) \check{\pi}_t(m')$$

hold for all $t \in \{1, ..., N\}$ and (3)-(9) hold for all $t > N$.

A.4 The model with bonds

For the model with bonds in section 5, let v_t, π_t, and $\hat{\pi}_t$ be as given in the main text. At period $t+1$, each unit of money disintegrates with probability $\delta(\hat{\pi}_t) = 1 - M / \left(\sum_{\zeta} \hat{\pi}_t(\zeta)\, w \right)$. Therefore the value of holding m units of nominal wealth at the start of $t+1$ is

$$\mathring{v}_{t+1}(m) = \sum_{m' \le m} \binom{m}{m'} (1 - \delta(\hat{\pi}_t))^{m'} \delta(\hat{\pi}_t)^{m-m'} v_{t+1}(m'). \tag{25}$$

At stage 2 of t, between a buyer with portfolio $\zeta^b = (m^b, w^b)$ and a seller with $\zeta^s = (m^s, w^s)$, the equilibrium meeting outcome $(y(\zeta^b, \zeta^s, \mathring{v}_{t+1}, L_t), \mu(\zeta^b, \zeta^s, \mathring{v}_{t+1}, L_t))$

is determined by (3), where we replace $\left(m^b, m^s, v_{t+1}\right)$ with $\left(\zeta^b, \zeta^s, \mathring{v}_{t+1}\right)$ and treat μ as a probability measure on $\left\{0, \ldots, \min\left(m^b, B - w^s\right)\right\}$. It follows that the value of holding portfolio $\xi \in \Xi$ before pairwise meetings is

$$\hat{v}_t(\zeta) = \beta \mathring{v}_{t+1}(w) + 0.5 \sum_{\zeta'} \hat{\pi}_t(\zeta') \left[f^b(\zeta, \zeta', \mathring{v}_{t+1}, L_t) + f^s(\zeta', \zeta, \mathring{v}_{t+1}, L_t) \right], \tag{26}$$

where f^b and f^s are the buyer's surplus and the seller's surplus from the equilibrium meeting outcome, respectively, and $\lambda(\zeta', m, \mathring{v}_{t+1}, L_t)$ is the proportion of agents carrying portfolio ζ' into pairwise meetings and leaving with m units of nominal wealth (its description is similar to the one in (15) and skipped here). Also, we have

$$\pi_{t+1}(m) = \sum_{m' \geq m} \binom{m'}{m} (1 - \delta(\hat{\pi}_t))^m \, \delta(\hat{\pi}_t)^{m'-m} \, \mathring{\pi}_{t+1}(m'), \tag{27}$$

where $\mathring{\pi}_{t+1}(m) = \sum_{\zeta'} \lambda(\zeta', m, \mathring{v}_{t+1}, L_t) \, \hat{\pi}_t(\zeta')$ is the proportion of agents who hold m units of nominal wealth at the start of $t+1$. At the bond market of t, the problem for an agent with m units of money is

$$v_t(m) = \max_{\hat{\mu}} \sum_{\zeta'} \hat{\mu}(\zeta') \cdot \hat{v}_t(\zeta') \tag{28}$$

subject to (12); let $\hat{\mu}\left(.; m, p_t^B, \hat{v}_t\right)$ be the solution to the problem in (28). Then the distribution $\hat{\pi}_t$ must satisfy

$$\hat{\pi}_t(\zeta) = \sum_{m'} \hat{\mu}\left(\zeta; m', p_t^B, \hat{v}_t\right) \pi_t(m'), \tag{29}$$

and clearing the bond market requires

$$\sum_m \left\{ \pi_t(m) \left[\sum_{\zeta' = (m', w')} (w' - m') \, \hat{\mu}\left(\zeta'; m, p_t^B, \hat{v}_t\right) \right] \right\} = D_t. \tag{30}$$

Given π_0 and $\{D_t\}_{t \geq 0}$, a sequence $\{v_t, \hat{\pi}_t, \pi_{t+1}, p_t^B\}_{t \geq 0}$ is an equilibrium if it satisfies (25)–(30) and

$$AL_t^{\alpha} = 0.5 \sum_{\zeta^b, \zeta^s} y\left(\zeta^b, \zeta^s, \mathring{v}_{t+1}, L_t\right) \hat{\pi}_t\left(\zeta^b\right) \hat{\pi}_t\left(\zeta^s\right).$$

If there is a money injection, then we can introduce $\check{\pi}_t$ as in A.3 for $t \in \{1, \ldots, N\}$.

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