Identifying Bubbles in Latin American Equity Markets: Phillips-Perron-based Tests and Linkages

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Abstract

The identification of periods of price exuberance in equity markets is of great interest to policy makers and financial investors. In this paper, we identify financial bubble periods within the major equity markets in Latin America. We use the recently developed recursive Augmented Dickey-Fuller methods and propose similar recursive procedures based on Phillips-Perron. We find that conditional on bubbles in the S&P 500, there are strong links between bubble episodes across equity markets in Latin America. In addition, the financial bubble periods in Latin America begin earlier and last longer than bubble periods in the United States during the 2008 financial crisis. Price bubbles were identified prior to the establishment of the Integrated Latin American Market (MILA).

Keywords: GSADF; Latin America; MILA; Price bubbles; Price exuberance

JEL classification: C22; C58; F30; G12

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1 Introduction

The identification of financial bubbles has become a critical endeavor for policy makers and financial professionals given the now common agreement that the most recent financial crisis was originated form a bubble in real estate prices. Previous episodes of bubbles, e.g. the dot-com bubble, also had its impact on economic growth, employment and the overall health of the financial system. The timely identification of financial bubbles would provide policy makers and investors with a window of opportunity to prevent losses to investments and damage to the greater economy. Moreover, bubble detection is of particular interest to developing economies where the economic structure is more fragile and contagion may be present.

In this paper, we initially use the recently developed methods by Phillips, Wu and Yu (2011, PWY henceforth) and Phillips, Shi and and Yu (2015, PSY henceforth) to identify the beginning and the end of bubble periods in six developed Latin American equity markets. Moreover, following the same structure as the recursive procedures in PWY and PSY, which are based on Augmented Dickey-Fuller (ADF) tests, we proposed similar recursive procedures based on Phillips-Perron (PP) tests. The benefit is that the PP statistics that we employ use the Newey and West (1987) heterocedasticity- and autocorrelation-consistent covariance matrix estimator. With the estimated bubble periods we set to analyze the links between bubbles across different equity markets. We do this by estimating a simple dynamic conditional correlation model based on Engle (2002). With the ADF-based and PP-based statistics we also seek to identify which Latin American equity markets exhibited price bubbles during the recent financial crisis.

Latin America has been known for a particular tendency to display erratic growth rates, combined with political transitions and poor macroeconomic performance (Bittencourt, 2012). Furthermore, Latin America has been characterized to have capital markets that commonly fall below growth expectations (De la Torre et al., 2007). Domestic stock markets in developing countries are described to have different performance results across nations. Some countries have experienced stock market growth, but in most cases, the growth was not significantly greater that in developed countries. Other countries have experienced a deterioration of their domestic capital markets. Furthermore, this difference becomes apparent when comparing the development of domestic capital markets across the region. For example, Latin America is characterized by delisting and lack of liquidity. On the other hand, for example, capital markets in East Asia have developed relatively well (Poitras, 2012).

Identification of bubble periods in Latin American equity markets is also important due the classification of these economies as emerging. According to the Organization for Economic Co-operation and Development (OECD), the global economy has shifted wealth towards emerging economies over the last decades. This has been reflected in the increasing contribution of emerging economies to the world Gross Domestic Product (GDP) growth (OECD, 2013). Developing markets have increased their share in global GPD from 40% in 2000 to 49% in 2010 and it is estimated that it will be 57% in 2030 (OECD, 2010).

More recently, De Gregorio (2013) presented evidence that Latin American countries showed an unprecedented resilience to the global financial crisis because of their macroeconomic conditions. Latin America has shown a steady growth in the sizes of its equity markets in the last two decades. Moreover, its market capitalization has increased from an average of 28% in the late 1990s to 52% during the 2006-2010 period (see OECD, 2013). According to market capitalization reports by The World Bank for 2012 and the 2014 Fact Book of the Federación Iberoamericana de Bolsas, Latin America's market capitalization represents 24.1% of the U.S. market and is equivalent to 99.8% of the United Kingdom and Germany markets combined.

The Integrated Latin American Market (MILA) was designed to capitalize on this resilience with the world's first virtual integration of multiple equity markets for Chile, Colombia and Peru (Mellado and Escobari, 2015). MILA allows traders to have direct access to the other exchanges and removes a host country intermediary from the transaction. This integrated market, established in May 2011, was structured to maintain the independence of each country's equity market, but tear down the barriers that disallow traders to easily facilitate the purchase of equities in the neighboring markets. This virtual market enabled each country to encourage trading and increase diversification without giving up its own economic autonomy. The MILA infrastructure was interesting enough to persuade Mexico to join in August 2014, making the MILA the largest integrated market in Latin America. In this paper, we seek to identify if price bubbles occurred in and around the establishment of the MILA in order to describe the pricing characteristics surrounding market integrations.

Using the ADF non-cointegration tests, Tran (2016) did not find rational bubbles in Argentina, Brazil, Chile, Colombia, Mexico, and Peru for the period of 1990-2009.[1] However using the tests of periodically collapsing bubbles he finds that these are periodically collapsing bubbles on Latin America countries over the same period. He also finds that the more stock market opens to the foreign investors, the more speculative bubbles are likely to arise. Johansen and Sornette (2001) for the period of 1990 to 1999 using a combination of parametric fits and of non-parametric log-frequency analysis they identify four bubbles for Argentina, one for Brazil, two for Chile, two for Mexico, two for Peru, and one for Venezuela, with a subsequent large crash/decrease. This is important for emerging stock markets because crises have become less severe in developed market than in emerging stock markets over time (Patel and Sarkar, 1998). Sarno and Taylor (2003) examine asset price bubbles in the stock markets of six Latin American countries. They find strong evidence for the existence of stock price bubbles in each of the markets examined.

Our results find strong evidence of bubble periods for Brazil, Chile, Colombia, Mexico, and Peru. Only for the Argentinean equity market we find no evidence of explosive behavior. The findings are fairly consistent across the ADF-based and the PP-based tests—the labeling of periods as bubble or no-bubble coincide in 92.9% of the cases. The results show a clear overlap of bubble periods across markets prior to the 2007 financial crisis. Moreover, bubbles for the Chilean, Colombian and Peruvian markets match with the months leading up to the establishment of the MILA in May 2011. Interestingly, for Mexico and Brazil, who are not part of MILA, there is no evidence of bubble episodes during the same period. Overall, periods of price exuberance in Latin American equity markets appear to begin earlier and stay in price exuberance for a longer period of time than for the S&P 500. When analyzing the correlations between bubbles across equity markets, we find that conditional

[1] Tran (2016) points out that there are several econometric techniques that could be used to detect speculative bubbles. He classifies five categories of tests: (1) tests for excess volatility (Kleidon, 1986; LeRoy and Porter, 1981; Marsh and Merton, 1986; Shiller, 1980), (2) tests for bubble premiums (Hardouvelis, 1988; Rappoport and White, 1993), (3) specification test (West, 1987), (4) tests for the cointegration of dividends and prices (Diba and Grossman, 1988), and (5) duration dependence tests (McQueen and Thorley, 1994).

on the existence of bubbles in the S&P 500, there are strong links between equity markets in Latin America. Overall, the correlations are higher when all equity markets are in a bubble period or none of them are in a bubble period. This can be interpreted as evidence that the ability to diversify across equity markets is higher during the periods when different countries experience the beginning and end of speculative bubbles. In addition, the results also suggest the existence of stronger links between larger equity markets (i.e., between Brazil and Mexico).

The rest of the paper is structured as follows. Section 2 presents the data, while section 3 describes the bubble periods identification strategy, along with the proposed PP-based tests. Section 4 presents and discusses the results, with a particular emphasis on the links between bubbles across equity markets. Finally, section 5 concludes.

2 Data

The data used in this study spans for 14 years containing monthly observations from July 2000 through June 2014. The inflation-adjusted stock indices we have are from Argentina, Brazil, Chile, Colombia, Mexico, Peru, in addition to the S&P 500 Composite Index, all obtained from Datastream International. We include data from the S&P 500 to be able to compare the bubble episodes in Latin America with the more commonly employed S&P 500.

[Table 1, about here]

Multiple major economic events that affected financial markers are present in this time period, including the real estate bubble, the financial crisis that officially lasted from December 2007 to June 2009, and the advent of the MILA. Panel A in Table 1 reports the summary statistics for each of the country indices in levels (P_t), while Panel B reports the summary statistics for the series expressed as returns (r_t), following the conventional approach of taking first differences of the natural logarithm of the levels, $r_t = \ln(P_t) - \ln(P_{t-1})$. To visualize the dynamics, Figures 1 and 2 present the time series graphs for six of the series (left-hand side axes). A common element across these indices in the pronounced drop during the 2007-09 financial crisis as well as the run-up immediately thereafter.

3 Identification Strategy

3.1 The Link between Explosive Behavior and Bubbles

To be able to test for the existence and to identify the periods of explosive pricing behavior, including obtaining the dates of the beginning and the end of explosive behavior, we employ the Supremum Augmented Dickey-Fuller ($SADF$) and General Supremum Augmented Dickey-Fuller ($GSADF$) test statistics proposed in PWY and PSY, as well as the Supremum Augmented Phillips-Perron (SPP) and General Supremum Augmented Phillips-Perron ($GSPP$) that we propose in this article. Perhaps the most important feature of using the $SADF$, $GSADF$, SPP, and $GSPP$ to test for explosive behavior is that we do not need to have information on market fundamentals. On the other hand, one limitation is that finding empirical evidence of explosive behavior is not necessarily empirical evidence of bubbles. For example, if a particular market fundamental that is positively correlated with the price of the asset is growing unexpectedly faster than previously, the $SADF$, $GSADF$, SPP and $GSPP$ methods may lead to mistakenly conclude that there is an asset bubble. Following the standard definition of bubble (see, e.g., Flood and Hodrick, 1990, Escobari et al., 2015, Escobari and Jafarinejad, 2016, and Harvey et al., 2015), let B_t denote the bubble. Then the bubble is just the difference between the after-dividend price P_t of an asset and the market fundamental P_t^f, i.e., $B_t = P_t - P_t^f$. Moreover, define r_f as the risk-free interest rate, D_t as the dividend received or payoff from the asset, and let U_t represent the unobserved market fundamentals. Then, we can then write the following asset pricing equation for the market fundamentals:

$$P_t^f = \sum_{i=0}^{\infty} \left(\frac{1}{1+r_f}\right)^i E_t(D_{t+i} + U_{t+i}) \tag{1}$$

If there are no bubbles, the degree of stationarity of P_t^f entirely determines the degree of stationarity of P_t. This means that following equation (1), it would depend on the character of the D_t series and the U_t series. For example, if D_t is $I(1)$ and the fundamentals are either $I(0)$ or $I(1)$, then the P_t is at most $I(1)$. If the bubble series satisfy the submartingale property $E_t(B_{t+1}) = (1+r_f)B_t$, in the presence of bubbles P_t will be explosive. Therefore, if D_t is $I(0)$ after differencing and U_t is at most $I(1)$, then empirical evidence of explosive behavior in the P_t series, as obtained with the $SADF$, $GSADF$, SPP, $GSPP$, may be used to conclude the existence of asset bubbles.

3.2 Identifying Explosive Behavior

The crux of the $GSADF$ methodology is its ability to identify explosive behavior in a random walk and assign dates to the beginning and the end of those periods. In addition, the $GSADF$ can identify multiple periods of explosive behavior within the historical pricing of a financial asset. Moreover, the recursive nature of the $GSADF$ allows for this identification to occur in real time as data becomes available. Obtaining the beginning and end dates of these periods is of substantial interest to policy makers and finance professionals.

Both, the $SADF$ and the $GSADF$, are based on the following Augmented Dickey-Fuller structure,

$$\Delta P_t = \alpha_{r_1,r_2} + \beta_{r_1,r_2} P_{t-1} + \sum_{i=1}^{k} y^i_{r_1,r_2} \Delta P_{t-i} + \epsilon_t, \tag{2}$$

where P_t denotes the price of the asset or a market index, $\epsilon \overset{iid}{\sim} N(0, \sigma^2_{r_1,r_2})$, and r_1 and r_2 denote fractions of the total sample size that specify the starting and ending points of each subsample period. The k lagged difference terms are included to control for autocorrelation, with k being selected by minimizing the Akaike information criterion. We are interested in the following test statistic

$$ADF^{r_2}_{r_1} = \frac{\hat{\beta}_{r_1,r_2}}{s.e.(\hat{\beta}_{r_1,r_2})}. \tag{3}$$

The challenge of creating windows where structural breaks could be measured in terms of time/date was tackled by PWY when they proposed a forward recursive ADF test to identify the origination and end of explosive time periods. This test was evaluated by Homm and Breitung (2012) who described that the PWY procedure performed appropriately in comparison with other models and was effective in identifying the windows of the NASDAQ's dot.com bubble. The PWY identification strategy is structured in a two-step process. First, identify if there is a period of price exuberance in the series by using the ADF statistic. Once we know that there is exuberance in the series we can identify the windows in which this period exists.

PWY's statistic is constructed by making the process recursive over the entire sample with a defined initial minimum window size. The $SADF$ statistic is then obtained by taking the supremum from all of the independent, and forwardly recursive, ADF statistics. That is, the $SADF$ statistic is defined as

$$SADF(r_0) = \sup_{r_2 \in [r_0, 1]} ADF_0^{r_2}. \tag{4}$$

PWY explain that there is explosive behavior in the series when the $SADF$ statistic is greater than the right tailed critical values from its limit distribution.[2]

Following the same structure as PWY, PSY introduce the $GSADF$ statistic, which is a double recursive procedure to compliment the forward recursive $SADF$ statistic. The important difference between the two methodologies is that PSY is designed to identify multiple bubbles within a series. PSY takes the *sup ADF* from each shift in end-period, as in PWY, but then constructs a series of statistics by shifting the beginning point of each period and running the first loop each time. From this series of *sup ADF* statistics, PSY takes the greatest value and assigns that as the $GSADF$ statistic. PSY suggest that we can identify explosive behavior when the $GSADF$ test statistic is greater than its right tail critical values. The $GSADF$ statistic is formally defined by

$$GSADF(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} ADF_{r_1}^{r_2}, \tag{5}$$

where r_1 and r_2 are the beginning and ending points of each sample in the recursive process.[3] The null hypothesis is that there are no explosive periods within the series such that the presence of any $SADF$ or $GSADF$ greater than its own right-tail critical values, respectively, confirms that there is at least one period where the series exhibits price exuberance.

[2]The limit distribution of the $SADF$ statistic given by

$$\sup_{r_2 \in [r_0, 1]} \frac{\int_0^1 W dW}{\int_0^1 W^2}$$

where W is a standard Wiener process.

[3]The limit distribution of the $GSADF$ statistic is

$$\sup_{r_2 \in [r_o, 1], r_1 \in [0, r_2 - r_0]} \left\{ \frac{\frac{1}{2} r_w [W(r_2)^2 - W(r_1)^2 - r_w] - \int_0^1 W(r) dr [W(r_2) - W(r_1)]}{r_w^{\frac{1}{2}} \left[r_w \int_{r_w}^{r_2} W(r)^2 dr - [\int_{r_w}^{r_2} W(r)^2 dr]^2 \right]^2} \right\}$$

where $r_w = r_2 - r_1$.

Once we identify that a series has an explosive behavior within its selected observations, we use a backward *sup ADF* ($BSADF$) series to identify the windows where this price exuberance exists. The $BSADF$ process is constructed by moving r_1 backward instead of r_2 forward and provides consistent estimates of the origination and termination points of each bubble (Phillips et al., 2015). The $BSADF$ statistic is defined as

$$BSADF_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} ADF_{r_1}^{r_2}. \tag{6}$$

The dates of the beginning and closing periods of price exuberance are identified as the first and last dates within each window where the $BSADF$ statistic is greater than the right tail critical values of its own distribution. Furthermore, note the actual limit distributions of each test, because they are not standard, must be calculated via Monte Carlo simulations (see, e.g., Pavlidis et al., 2016).

3.3 Phillips-Perron-based Tests

The estimation of equation (2) involves including k lags of the first differences of P_t to account for the potential serial correlation in ϵ_t. An alternative approach to deal with serial correlation involves setting $k = 0$ in equation (2) and then using one of the proposed test statistics in Phillips and Perron (1988).[4] The Phillips-Perron statistic that we employ can be viewed as an ADF statistic that has been made robust to serial correlation by using the Newey and West (1987) heteroscedasticity- and autocorrelation-consistent covariance matrix estimator. In particular, we use the following PP statistic

$$PP_{r_1}^{r_2} = \sqrt{\frac{\hat{\gamma}_{0,T}}{\hat{\lambda}_T^2}} \frac{\hat{\beta}_{r_1,r_2}}{s.e.(\hat{\beta}_{r_1,r_2})} - \frac{1}{2}\left(\hat{\lambda}_T^2 - \hat{\gamma}_{0,T}\right)\frac{1}{\hat{\lambda}_T}\frac{T \cdot s.e.(\hat{\beta}_{r_1,r_2})}{s_T} \tag{7}$$

where T is the (sub)sample size.[5] The critical values for the Phillips-Perron test are the same as those of the ADF test (Hamilton, 1994). Moreover, note that the $PP_{r_1}^{r_2}$ statistic in equation (7) follows the same structure as the $ADF_{r_1}^{r_2}$ statistic in equation (2). Both of

[4] We thank an anonymous referee for suggesting us this approach.

[5] $\hat{\gamma}_{0,T}$, $\hat{\lambda}_T^2$, and s_T are given by

$$\hat{\gamma}_{j,T} = \frac{1}{T}\sum_{t=j+1}^{T} \hat{u}_t\hat{u}_{t-j}; \quad \hat{\lambda}_T^2 = \hat{\gamma}_{0,T} + 2\sum_{j=1}^{q}\left(1 - \frac{j}{q+1}\right)\cdot\hat{\gamma}_{j,T}; \quad s_T = \frac{1}{T-2}\sum_{t=1}^{T}\hat{u}_t^2.$$

Moreover, u_t is the OLS residual and q is the number of Newey-West lags used when calculating $\hat{\lambda}_T^2$.

these statistics depend on the beginning and ending points of each subsample. Hence, we can analogously define our PP-based statistics, the *sup PP* (SPP),

$$SPP(r_0) = \sup_{r_2 \in [r_0, 1]} PP_0^{r_2}, \tag{8}$$

while the generalized SPP ($GSPP$) is given by

$$GSPP(r_0) = \sup_{\substack{r_2 \in [r_0, 1] \\ r_1 \in [0, r_2 - r_0]}} PP_{r_1}^{r_2}. \tag{9}$$

Lastly, the backwards *sup PP* ($BSPP$) is defined as

$$BSPP_{r_2}(r_0) = \sup_{r_1 \in [0, r_2 - r_0]} PP_{r_1}^{r_2}. \tag{10}$$

We will use these statistics as an alternative approach to identify bubbles, and compare the results with the ones obtained with the ADF-based tests.

4 Results

4.1 Date-stamping Bubble Periods

In the first phase to identify periods of price bubbles, we test for the presence of explosive behavior in each of our price series using the $SADF$, $GSADF$, SPP, and $GSPP$ statistics. Panel A in Table 2 reports these results for each of our series. The corresponding critical values, reported in Panel B, are constructed with Monte Carlo simulations using 2,000 replications and a smallest window size of 24 months. Note that the smallest window size in the recursive procedures is determined by selecting r_0. PSY explain that for a given sample size there is a trade-off when selecting r_0. A larger r_0, which corresponds to a larger 'smallest window', will make sure that there are enough observations for adequate initial estimation. On the other hand, a smaller r_0 will help to avoid missing any opportunity to detect an early explosive episode. A small r_0 may cause potential problems with structural breaks in the data generating process as well as issues related with small samples. Hence, the selection of r_0 needs to balance these potential issues. Based on simulation findings, PSY recommend a rule for choosing r_0 that follows a simple functional form, $r_0 = 0.01 + 1.8/\sqrt{T}$. Following this rule we obtain $r_0 = 0.154$, which corresponds to a smallest window of 24 months.

The results based on the supremum statistics, reported in columns 1 and 2, show that in five markets—Brazil, Chile, Colombia, Mexico and Peru—there is evidence of at least one episode of explosive behavior. These results hold for any of the reported critical levels and are consistent across the *sup* ADF and the *sup* PP. For the S&P 500 the table reports that the $SADF$ and the SPP statistics are both smaller than all reported critical values, but the $GSADF$ in column 3 is greater than its 95% critical value, and the $GSPP$ in column 4 is greater that its 99% critical value. We interpret this as evidence of multiple bubbles in the S&P 500. We observe that the Argentine market is the only one where our empirical approach does not identify the existence of any bubbles—all reported statistics lie below the 90% critical values. While they use a different sample period, our findings of explosive behavior in these equity markets are consistent with documented non-statinarity found for the same markets in Chen et al. (2002).

[Table 2, about here]

Note that from the structure of equations (4) and (5), we know that the $SADF$ is always less of equal to the $GSADF$. This is because the subsamples in the $SADF$ recursive procedure are a subset of the subsamples in the $GSADF$ double recursive procedure. The results in Table 2 show that the $SADF$ is equal to the $GSADF$ for Brazil, Chile, Colombia, and Mexico. This is the case when the largest ADF statistics in the double recursive $GSADF$ occurs when $r_1 = 0$ in equation (5). The same is true for Colombia, Mexico, and Peru when comparing the SPP with the $GSPP$.

We know that the $SADF$ is particularly effective when there is a single bubble in the series, but as PSY explain the $SADF$ may suffer from reduced power and can be inconsistent when multiple bubbles occur. For the S&P 500, the $SADF$ is failing to identify the dip in the equity market following the collapse in 2008. The benefit in using the $GSADF$ algorithm is that it works with subsamples of the data with different initializations in the recursions. The observed $SADF < GSADF$ for the S&P 500 is consistent with the $GSADF$ capturing the dip following the collapse in 2008. The $GSADF$ captured the dip during one of the recursions with r_1 being around the end of 2007. Then, the recorded supremum ADF statistic is greater than the supremum ADF statistic recorded with $r_1 = 0$. The PP-based test statistics for the S&P 500 in columns 2 and 4 are consistent with this explanation.

[Figure 1, about here]

Etienne et al. (2015) explain that one potential drawback from the estimates reported in Table 2, is that they do not appropriately show the results beyond finding evidence of explosive behavior. To further our analysis Figure 1 presents more details for Brazil and Chile, while Figure 2 for Colombia, Mexico, Peru, and the S&P 500. The left-hand side axes capture the indices in levels, while on the right-hand side axes we have the $BSADF$ and the $BSPP$ sequences as obtained using equations (6) and (10) for different values of r_2. Moreover, also measured on the right-hand side axes we present the 95% critical value sequences based on Monte Carlo simulations with 2,000 replications that take into account the sample size of 156 and the smallest window of 24 observations. The shaded areas are the bubble periods as identified by the $GSADF$—when the $BSADF$ statistics is above the corresponding 95% critical value. Figures 1 and 2 include only the markets in which the statistics in Table 2 showed evidence of explosive behavior.

To assess whether the ADF-based results are substantially different from the PP-based results, we initially obtain the pair-wise correlation coefficients between the $BSADF$ and the $BSPP$ sequences. The results, reported in Panel A of Table 3, show that the highest correlations are for Colombia (0.972) and Peru (0.957), while the lowest is for Chile (0.788). Even though these correlations can be viewed as relatively high, they might not be necessarily too informative as the key element is whether the $BSADF$ and $BSPP$ sequences lie above or below their critical values. When we compare, one by one, every month for each of the equity markets in the sample, we observe that the ADF-based and the PP-based tests coincide to label the month as a bubble or not bubble in 92.9% of the cases.

[Figure 2, about here]

Figures 1 and 2 show that the overlap for the 2007-2009 financial crisis is evident for all markets. The S&P 500 appears to break out of the 2007 bubble before Latin American countries. Moreover, the second bubble period in the United States, that appears early in 2009, can be described as a left tail bubble and visible in Figure 2 as the down dip of the equity market following the bubble collapse in 2008. It notably occurs well after Latin American bubbles have ended and should not be confused for price exuberance.

Interestingly, the bubble periods late in 2010 and at the beginning of 2011 for the Chilean, Colombian and Peruvian markets match with the months prior to the establishment of the MILA in May 2011. Chile's bubble begins in August 2010 and is prolonged through the MILA initialization until July 2011. Colombia's bubble lasts a single month, January 2011, while Peru's bubble is during October and November, 2010. These results show that for the three markets, bubbles are present during the months leading up to the establishment of this common integrated equity market. On the other hand, note that the results for Mexico and Brazil—who are not part of MILA—do not show evidence of bubble episodes during the same period. This result suggests that investors believed that prices of stocks in each index would increase with the development of the MILA. This perception may have created a period of overvaluation that began with the announcement of the future formation of the MILA and ended with its operational initiation.

The recent work of Harvey et al. (2015) analyzed the relative local asymptotic and finite sample power performance of the PWY test, and compared it to the Homm and Breitung (2012) test to detect explosive behavior. They find that the relative performance of these two tests depends on the location and timing of the bubble periods. While they do not consider the PSY test for multiple bubbles, their findings show that the PWY test is better suited to detecting explosive regimes than the Homm and Breitung (2012) test when the bubble occurs early or towards the middle of the sample. The Homm and Breitung (2012) is better when the bubble occurs towards the end.

Recent work has also progressed the statistics to detect bubble periods. Harvey et al. (2016) focus on the performance of the PWY test when volatility of the innovation process is subject to non-stationarity, for example, when structural breaks occur in the unconditional variance of the innovation process. They explain that non-stationary volatility can result in spurious rejection of the unit root null hypothesis in favor of explosive behavior. Astill et al. (2016) propose a wild bootstrap implementation of the PWY test to replicate the pattern of non-stationary volatility. Following the same line, Astill et al. (2016) consider a wild bootstrap variant of the PSY to obtain a statistic that is asymptotically robust to non-stationary volatility. Note that non-stationary volatility can affect the results during the recursive procedures as the unconditional variance of P_t can change within the same window and across windows. Our statistics based on the PP that uses the Newey and West (1987) heteroscedasticity-consistent covariance matrix estimator should help controlling for changes in conditional volatility within the same window, but not across windows.

4.2 Links between Bubbles across Equity Markets

To allow for an easier comparison between bubble periods across equity markets, Figure 3 displays the summary of the time periods when each of the indices is in a $GSADF$ defined bubble period. The lower panel shows the overlap of periods of exuberance with darker areas illustrating that bubbles existed in more markets. From this figure there appears to be a strong link between bubbles across markets. Most of the bubbles are clustered around 2005 and 2007. For example, between September 2005 and May 2006 (with the exception of February, 2005), all five equity markets were experiencing a bubble. The same is true for the period between May, 2007 and August, 2007.[6] Moreover, around these dates different equity markets experienced booms and bursts of bubbles, with nearly all of them occurring between mid 2003 and mid 2008.

[Figure 3, about here]

A simple approach to further study the link between bubble periods is to calculate the pair-wise correlation between bubbles across equity markets. Calculating the correlation between bubble periods is interesting as this gives information on the degree of synchronization between bubbles in different countries. Let the dummy variable BU_t^i be equal to one if the $BSADF$ statistic of country i at time t lies above its 95% critical value, zero otherwise. The countries we use in the sample are the same ones where the $GSADF$ found evidence of multiple bubble periods, as reported in Table 2. These are Brazil, Chile, Colombia, Mexico, and Peru; hence, we have $i = (\mathrm{Bra}, \mathrm{Chi}, \mathrm{Col}, \mathrm{Mex}, \mathrm{Per})$. Note that these dummy variables are illustrated as the shaded areas in Figures 1 and 2, as well as in Figure 3.

[6] Specifically, the month in which all five countries experienced as bubbles are: March, 2005; September, 2005 through January, 2006; March, 2006 through May, 2006; November 2006; January, 2007; February, 2007; and May, 2007 through August, 2007.

Panel B in Table 3 presents the unconditional pair-wise correlation coefficients of between bubbles in different equity markets. The strongest link is between Mexico and Brazil, both the largest economies and the largest equity markets in the region. Figures from January 2013, show that from our sample of countries, Mexico is the second largest in market capitalization ($706,098) with Brazil ($1,257,888) being the first (Mellado and Escobari, 2015). The smallest correlation coefficients are associated with Peru, which represent the smallest equity market in our sample of countries. One concern with these observed unconditional correlations in Panel B is that they might be the result of a contagion effect from bubbles occurring in a different country. For example, bubbles in the U.S. stock market. To control for this effect and to study the changes in the correlations over time, we employ the multivariate GARCH methods developed in Engle (2002) and estimate dynamic conditional correlations (DCC) between bubbles.

[Table 3, about here]

We model the vector of bubbles $BU_t = (BU_t^{\text{Bra}}, BU_t^{\text{Chi}}, BU_t^{\text{Col}}, BU_t^{\text{Per}}, BU_t^{\text{Mex}})'$ in the mean equation as a function of bubbles in the S&P 500, $BU_t^{\text{S\&P500}}$. In particular the specification is:

$$BU_t = \delta_0 + \delta_1 BU_t^{\text{S\&P500}} + \varepsilon_t, \tag{11}$$

where $\varepsilon_t = (\varepsilon_t^{\text{Bra}}, \varepsilon_t^{\text{Chi}}, \varepsilon_t^{\text{Col}}, \varepsilon_t^{\text{Per}}, \varepsilon_t^{\text{Mex}})$, and $\varepsilon_t|\Omega_{t-1} \sim N(0, H_t)$.[7] Given the discrete nature of our dependent variables, our emphasis is on having a simple parsimonious model. Hence, we only include $BU_t^{\text{S\&P500}}$ as control, which serves as a factor that can potentially affect bubbles in the other five equity markets. The main idea is to model the time-variation of the variance-covariance matrix H_t. We use the following specification for this conditional variance,

$$H_t = G_t C_t G_t, \tag{12}$$

where both G_t and C_t are time-varying. C_t is the (5×5) correlation matrix of interest, while G_t is a (5×5) diagonal matrix. In addition to Engle (2002), further details on the estimation methods can be found in Engle and Sheppard (2001) and in Chiang et al. (2007).

[Table 4, about here]

[7] With Ω_{t-1} being the information set available at the end of period $t-1$.

The estimated DCC-GARCH estimated model is presented in Table 4, while the resulting conditional correlations are reported in Panel C of Table 3. The first thing to notice from these estimates is that conditional on the observed bubbles in the S&P 500, the correlations between bubbles in equity markets in Latin America are greater than the unconditional correlations. We interpret this as even stronger evidence of interdependence between bubble periods across these equity markets. These results on linkages between bubble episodes are consistent with the findings in Pagán and Soydemir (2000) and Chen et al. (2002), who study linkages on equity markets in Latin America. Pagán and Soydemir (2000) use simple vector autoregressive models, while Chen et al. (2002) employs cointegration analysis and error-correction vector autoregressions. The strong linkages that we find can be explained by common macroeconomic shocks across these countries. This is in line with Araújo (2009), who finds that for Latin America, cross-country co-movements in stock returns can be the result of macroeconomic shocks. Clustering of equity markets can as well be a feasible explanation (see, e.g., Mendes et al., 2007).

Note that because we have five equity markets, there are ten dynamic conditional correlations that come from C_t in equation (12). Because we are not interested in any particular market-pair conditional correlations, to visualize the dynamics we take the average of the ten correlations at each time t. The resulting average conditional correlations are presented as the solid black line in Figure 3. The strong link between bubbles can be best appreciated when the correlation is at its highest. This occurs when all equity markets are either during a bubble period (darker shaded areas) or none of them is in a $GSADF$ defined bubble. The correlation drops when equity markets experience the beginning or end of bubbles.

5 Conclusion

This paper begins by using the recently developed $SADF$ of Phillips et al. (2011) and the $GSADF$ of Phillips et al. (2015) to estimate random walk breaks in major equity indices in Latin America. By applying these recursive methods, based on the Augmented Dickey-Fuller statistic, we are able to identify the beginning and end dates of periods of price exuberance. A key benefit from the double recursive methods that we employ is that they allow us to identify multiple bubble periods within each equity market. To help control for serial correlation and heteroscedasticity, we follow the structure in the $SADF$ and the $GSADF$ to propose similar recursive procedures based on Phillips-Perron. The results show that for our sample of equity markets, the ADF-based and the PP-based tests coincide 92.9% of the times when labeling specific time periods as bubble or not bubble.

Our findings, consistent across the ADF-based and the PP-based tests, show that bubbles in Latin American equity markets appear to begin earlier and stay in price exuberance for a longer period of time than the S&P 500. This characteristic may serve as an indicator for other international equity markets, particularly those highly correlated to the U.S. markets. Our results identified a period of exuberance common to equity markets in Chile, Colombia and Peru that coincides to the period prior to the establishment of the Integrated Latin American Market (MILA) between these markets. Interestingly, during the same periods we do not observe bubbles for the countries that did not participate in MILA.

To further study the links between bubble periods across equity markets, we estimated the correlations between bubbles conditional on the observed bubbles in the S&P 500. The results from the dynamic conditional correlations show strong links between bubbles across equity markets. The correlations are higher when all equity markets are either during a bubble period or none of them is in a bubble.

References

Araújo, E. (2009). Macroeconomic shocks and the co-movement of stock returns in Latin America. *Emerging Markets Review*, 10(4):331–344.

Astill, S., Harvey, D. I., Leybourne, S. J., and Taylor, A. R. (2016). Tests for an end-of-sample bubble in financial time series. *Econometric Reviews*, Forthcoming.

Bittencourt, M. (2012). Financial development and economic growth in Latin America: Is Schumpeter right? *Journal of Policy Modeling*, 34(3):341–355.

Chen, G.-m., Firth, M., and Rui, O. M. (2002). Stock market linkages: Evidence from Latin America. *Journal of Banking & Finance*, 26(6):1113–1141.

Chiang, T. C., Jeon, B. N., and Li, H. (2007). Dynamic correlation analysis of financial contagion: Evidence from Asian markets. *Journal of International Money and Finance*, 26(7):1206–1228.

De Gregorio, J. (2013). Resilience in Latin America: Lessons from macroeconomic management and financial policies. International Monetary Fund, Working Paper WP/13/259.

De la Torre, A., Gozzi, J. C., and Schmukler, S. L. (2007). Innovative experiences in access to finance: Market friendly roles for the visible hand? World Bank, Policy Research Working Paper 4326.

Diba, B. T. and Grossman, H. I. (1988). The theory of rational bubbles in stock prices. *The Economic Journal*, 98(392):746–754.

Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, 20(3):339–350.

Engle, R. F. and Sheppard, K. (2001). Theoretical and empirical properties of Dynamic Conditional Correlation Multivariate GARCH. NBER Working Paper No. 8554.

Escobari, D., Damianov, D., and Bello, A. (2015). A time series test to identify housing bubbles. *Journal of Economics and Finance*, 39(1):136–152.

Escobari, D. and Jafarinejad, M. (2016). Date stamping bubbles in Real Estate Investment Trusts. *The Quarterly Review of Economics and Finance*, 60:224–230.

Etienne, X. L., Irwin, S. H., and Garcia, P. (2015). Price explosiveness, speculation, and grain futures prices. *American Journal of Agricultural Economics*, 97(1):65–87.

Flood, R. P. and Hodrick, R. J. (1990). On testing for speculative bubbles. *Journal of Economic Perspectives*, 4(2):85–101.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton University Press.

Hardouvelis, G. A. (1988). Evidence on stock market speculative bubbles: Japan, the United States, and Great Britain. *Federal Reserve Bank of New York Quarterly Review*, 13(2):4–16.

Harvey, D. I., Leybourne, S. J., and Sollis, R. (2015). Recursive right-tailed unit root tests for an explosive asset price bubble. *Journal of Financial Econometrics*, 13(1):166–187.

Harvey, D. I., Leybourne, S. J., Sollis, R., and Taylor, A. R. (2016). Tests for explosive financial bubbles in the presence of non-stationary volatility. *Journal of Empirical Finance*, 38(B):548–576.

Homm, U. and Breitung, J. (2012). Testing for speculative bubbles in stock markets: A comparison of alternative methods. *Journal of Financial Econometrics*, 10(1):198–231.

Johansen, A. and Sornette, D. (2001). Bubbles and anti-bubbles in Latin-American, Asian and Western stock markets: An empirical study. *International Journal of Theoretical and Applied Finance*, 4(6):853–920.

Kleidon, A. W. (1986). Variance bounds tests and stock price valuation models. *Journal of Political Economy*, 94(5):953–1001.

LeRoy, S. F. and Porter, R. D. (1981). The present-value relation: Tests based on implied variance bounds. *Econometrica*, 49(3):555–574.

Marsh, T. A. and Merton, R. C. (1986). Dividend variability and variance bounds tests for the rationality of stock market prices. *American Economic Review*, 76(3):483–498.

McQueen, G. and Thorley, S. (1994). Bubbles, stock returns, and duration dependence. *Journal of Financial and Quantitative Analysis*, 29(3):379–401.

Mellado, C. and Escobari, D. (2015). Virtual integration of financial markets: A dynamic correlation analysis of the creation of the Latin American Integrated Market. *Applied Economics*, 47(19):1956–1971.

Mendes, B. V., Leal, R. P., and Carvalhal-da Silva, A. (2007). Clustering in emerging equity markets. *Emerging Markets Review*, 8(3):194–205.

Newey, W. K. and West, K. D. (1987). A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. *Econometrica*, 55(3):703–708.

OECD (2010). Perspectives on global development 2010: Shifting wealth. OECD, Paris, France.

OECD (2013). Trends and factors impacting on latin american equity market development. Latin American Corporate Governance Roundtable. OECD, Quito, Ecuador.

Pagán, J. A. and Soydemir, G. (2000). On the linkages between equity markets in Latin America. *Applied Economics Letters*, 7(3):207–210.

Patel, S. A. and Sarkar, A. (1998). Crises in developed and emerging stock markets. *Financial Analysts Journal*, 54(6):50–61.

Pavlidis, E., Yusupova, A., Paya, I., Peel, D., Martinez-Garcia, E., Mack, A., and Crossman, V. (2016). Episodes of exuberance in housing markets: In search of the smoking gun. *Journal of Real Estate Finance and Economics*, 53(4):419–449.

Phillips, P. C. and Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, 75(2):335–346.

Phillips, P. C., Shi, S., and Yu, J. (2015). Testing for multiple bubbles: Historical episodes of exuberance and collapse in the S&P 500. *International Economic Review*, 56(4):1043–1078.

Phillips, P. C., Wu, Y., and Yu, J. (2011). Explosive behavior in the 1990s NASDAQ: When did exuberance escalate asset values? *International Economic Review*, 52(1):201–226.

Poitras, G. (2012). *Handbook of Research on Stock Market Globalization*. Edward Elgar Publishing.

Rappoport, P. and White, E. N. (1993). Was there a bubble in the 1929 stock market? *Journal of Economic History*, 53(3):549–574.

Sarno, L. and Taylor, M. (2003). An empirical investigation of asset price bubbles in Latin American emerging financial markets. *Applied Financial Economics*, 13(9):635–643.

Shiller, R. (1980). Do stock prices move too much to be justified by subsequent changes in dividends? *American Economic Review*, 71(3):421–436.

Tran, T. B. N. (2016). Speculative bubbles in emerging stock markets and macroeconomic factors: A new empirical evidence for Asia and Latin America. *Research in International Business and Finance*, Forthcoming.

West, K. D. (1987). A specification test for speculative bubbles. *Quarterly Journal of Economics*, 102(3):553–580.

Table 1: Descriptive Statistics

	(1) Obs	(2) Mean	(3) Std Dev	(4) Min	(5) Max
Panel A. Levels (P_t):					
Argentina	156	139.321	55.667	22.743	254.809
Brazil	156	366.700	219.447	40.829	756.296
Chile	156	303.039	158.028	75.238	591.19
Colombia	156	1036.085	745.570	84.218	2377.36
Mexico	156	268.167	129.106	71.552	480.914
Peru	156	1014.475	636.538	81.395	2011.63
S&P 500	156	102.001	20.023	57.480	157.883
Panel B. Returns (r_t):					
Argentina	155	0.603	11.569	-49.432	30.573
Brazil	155	0.880	11.296	-39.103	25.548
Chile	155	0.913	6.515	-25.833	20.561
Colombia	155	1.746	9.312	-40.019	39.936
Mexico	155	0.943	7.715	-34.832	25.339
Peru	155	1.817	8.767	-34.071	20.815
S&P 500	155	0.295	4.924	-18.361	14.612

Notes: Panel A presents the descriptive statistics for the series in levels (P_t), while Panel B for the series expressed as returns (r_t), with r_t calculated as $r_t = \ln(P_t) - \ln(P_{t-1})$.

Table 2: ADF-based and PP-based Test Statistics

	(1)	(2)	(3)	(4)
	Supremum		Generalized Supremum	
	$SADF$	SPP	$GSADF$	$GSPP$
Panel A. Test Statistics:				
Argentina	0.198	0.762	0.905	0.963
Brazil	3.447*	3.778*	3.447*	4.273*
Chile	2.608*	3.675*	2.608*	4.436*
Colombia	11.334*	10.480*	11.334*	10.480*
Mexico	3.398*	3.530*	3.398*	3.530*
Peru	6.647*	6.243*	6.670*	6.243*
S&P 500	0.521	0.752	2.038†	4.373*
Panel B. Finite Sample Critical Values:				
90%	0.934		1.540	
95%	1.243		1.882	
99%	1.907		2.359	

Notes: The $SADF$ and $GSADF$ statistics follow PWY and PSY, while the SPP and $GSPP$ are proposed in this article. The 95% critical values based on Monte Carlo simulations with 2,000 replications (sample size 156). * significant at 1%; † significant at 5%; ‡ significant at 10%.

Table 3: Unconditional and Conditional Correlations

	(1) Brazil	(2) Chile	(3) Colombia	(4) Mexico	(5) Peru
Panel A. $BSADF$ versus $BSPP$ Correlations:					
ADF vs. PP	0.902	0.788	0.972	0.899	0.957
Panel B. Unconditional Correlations:					
Brazil	1.000				
Chile	0.499	1.000			
Colombia	0.577	0.648	1.000		
Mexico	0.811	0.583	0.708	1.000	
Peru	0.282	0.541	0.673	0.436	1.000
Panel C. Conditional Correlations:					
Brazil	1.000				
Chile	0.767	1.000			
Colombia	0.787	0.853	1.000		
Mexico	0.903	0.807	0.830	1.000	
Peru	0.607	0.659	0.813	0.646	1.000

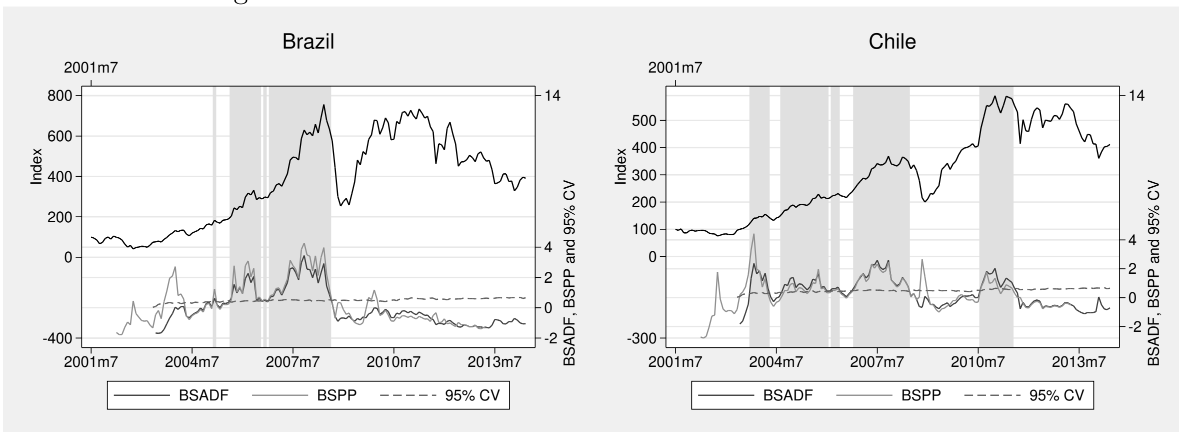
Notes: In Panel A, the correlation is between the $BSADF$ and the $BSPP$ sequences. In Panels B and C, bubble periods as defined by the $GSADF$ at the 95% critical level. In Panel C, correlations are conditional on bubbles in the S&P 500, estimated with the methods in Engle (2002).

Table 4: Estimation Results DCC-GARCH Model

	(1) Brazil	(2) Chile	(3) Colombia	(4) Peru	(5) Mexico
Panel A. Mean Equations:					
δ_0	0.0440*	0.161*	0.185*	0.0908‡	0.158*
	(0.0109)	(0.0526)	(0.0505)	(0.0379)	(0.0582)
δ_1	-0.0399	-0.135*	-0.148*	-0.0669	0.105
	(0.0510)	(0.0789)	(0.0864)	(0.0591)	(0.113)
Panel B. Variance Equations:					
c	0.00364*	0.0606*	0.141*	0.0202*	0.0288‡
	(0.000805)	(0.0192)	(0.0417)	(0.00539)	(0.0120)
a	0.500*	0.592*	0.890*	0.554*	0.256*
	(0.110)	(0.150)	(0.192)	(0.112)	(0.0739)
b	0.578*	0.337‡	-0.0540	0.543*	0.728*
	(0.0416)	(0.135)	(0.128)	(0.0532)	(0.0658)
Panel C. Multivariate DCC Equation:					
θ_1			0.494*		
			(0.0570)		
θ_2			0.141*		
			(0.0444)		
Observations			156		
χ^2			16.49		
χ^2 (p-value)			0.00559		

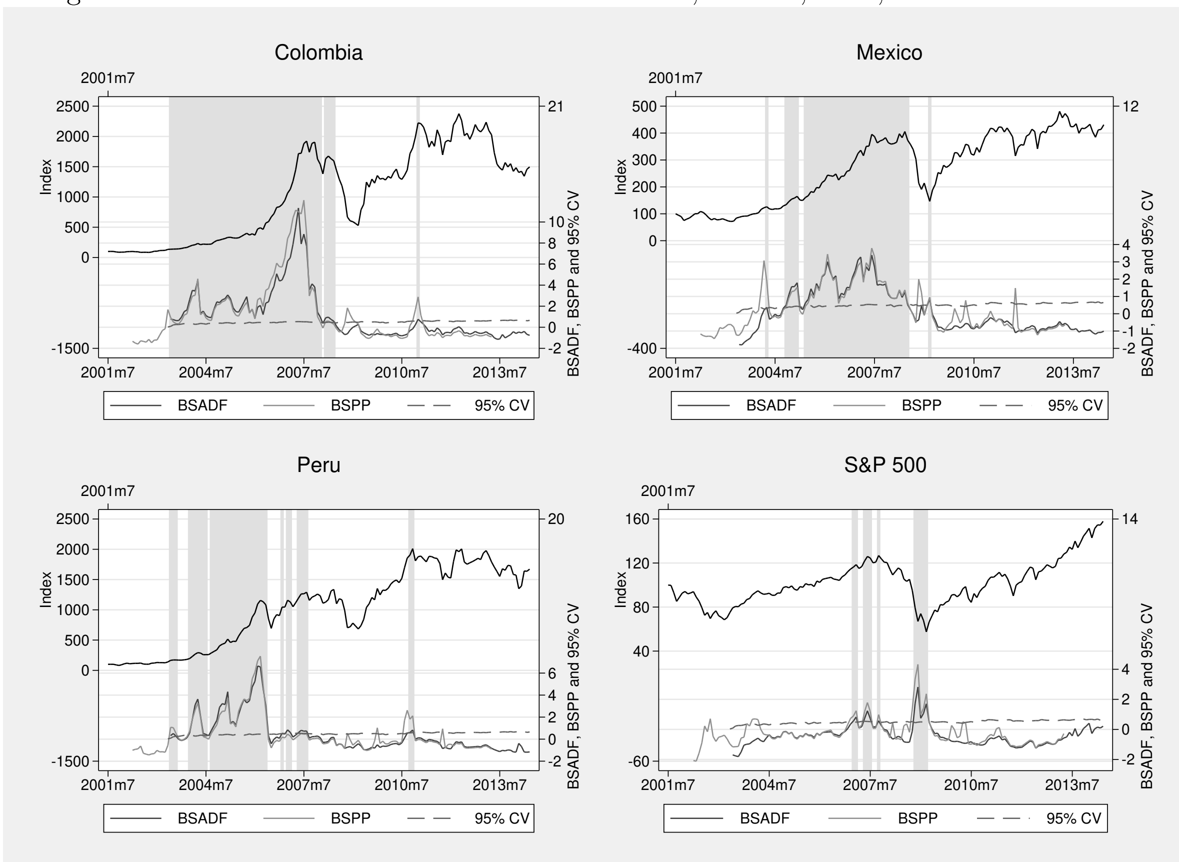
Notes: The figures in parentheses are standard errors. * significant at 1%; † significant at 5%; ‡ significant at 10%. The mean bubble equation is: $BU_t = \delta_0 + \delta_1 BU_t^{\text{S\&P500}} + \varepsilon_t$, with $BU_t = (BU_t^{\text{Bra}}, BU_t^{\text{Chi}}, BU_t^{\text{Col}}, BU_t^{\text{Per}}, BU_t^{\text{Mex}})'$, $\varepsilon_t = (\varepsilon_t^{\text{Bra}}, \varepsilon_t^{\text{Chi}}, \varepsilon_t^{\text{Col}}, \varepsilon_t^{\text{Per}}, \varepsilon_t^{\text{Mex}})$, and $\varepsilon_t|\Omega_{t-1} \sim N(0, H_t)$. The variance equations: $h_t^i = c^i + a^i h_{t-1}^i + b^i(\varepsilon_{t-1}^i)^2$ for $i = (\text{Bra}, \text{Chi}, \text{Col}, \text{Per}, \text{Mex})$.

Figure 1: GSADF and GSPP Results for Brazil and Chile



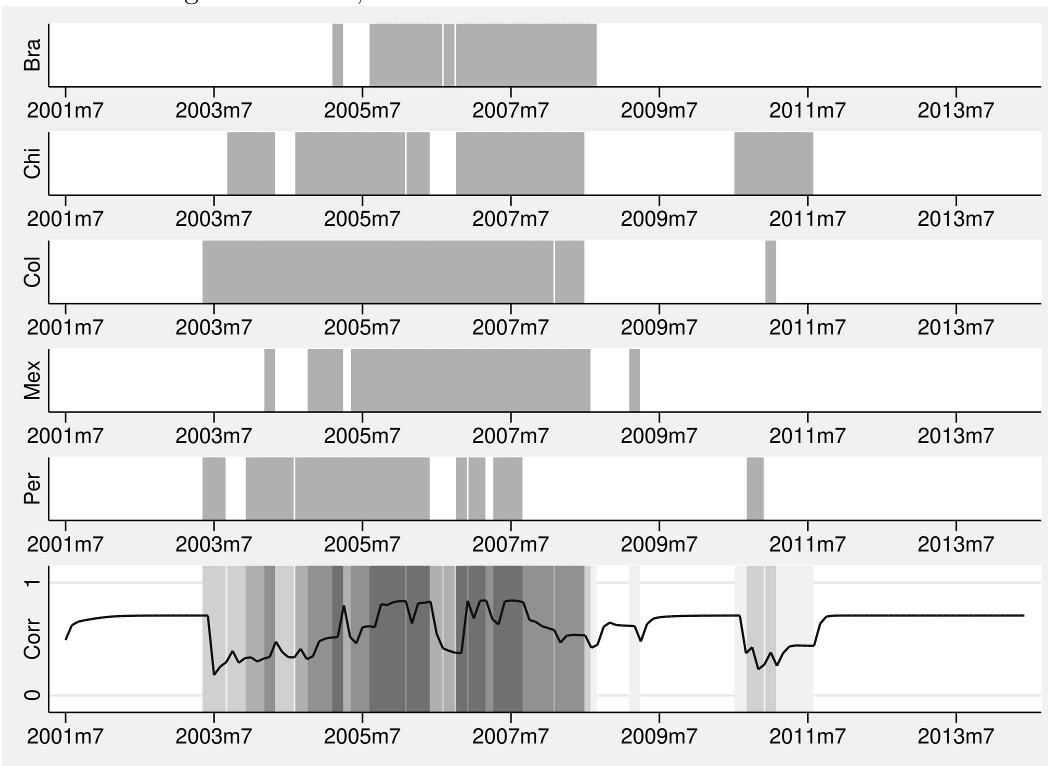
Notes: The inflation-adjusted stock indices are obtained from Datastream International (left axes). The sample spans from July 2000 to June 2014 with the total number of monthly observations being 156. The Backward Supremum Augmented Dickey-Fuller ($BSADF$, right axes) follows Phillips et al. (2015), while the Backward Supremum Phillips-Perron ($BSPP$, right axes) is proposed in this article. The shaded areas are the bubble periods identified by the $BSADF$. The 95% critical value sequence (right axes) based on Monte Carlo simulations with 2,000 replications (the sample size is 156 and the smallest window has 24 observations).

Figure 2: GSADF and GSPP Results for Colombia, Mexico, Peru, and the S&P 500



Notes: The inflation-adjusted stock indices are obtained from Datastream International (left axes). The sample spans from July 2000 to June 2014 with the total number of monthly observations being 156. The Backward Supremum Augmented Dickey-Fuller ($BSADF$, right axes) follows Phillips et al. (2015), while the Backward Supremum Phillips-Perron ($BSPP$, right axes) is proposed in this article. The shaded areas are the bubble periods identified by the $BSADF$. The 95% critical value sequence (right axes) based on Monte Carlo simulations with 2,000 replications (the sample size is 156 and the smallest window has 24 observations).

Figure 3: Index, GSADF and Critical Values for Colombia



Notes: Bubble periods as defined by the $GSADF$ at the 95% critical level for the equity markets in which the $GSADF$ identified at least one episode of exuberance. The lower panel shows the overlap of periods of exuberance with darker areas illustrating that bubbles existed in more markets. The solid black line is the average dynamic conditional correlations between bubble periods across markets, estimated using the methods in Engle (2002).