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The Mechanism behind Product Differentiation:
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Abstract

The strategy of product differentiation has been viewed as very important in the field of business administration, but it has not necessarily been viewed as an important source of large differences in firms’ profits in the field of economics. In this paper, this apparent contradiction is examined based on the concepts of “ranking preference and value.” In the proposed model, if a product’s implicit rank is higher, households will purchase the product far more often than competing products even if their qualities are almost identical and the tastes of households are uniformly distributed. Even a slight difference in quality can result in a clear difference in implicit ranks and consequently large differences in firms’ profits. Therefore, the effects of differentiation are amplified by ranking preference, and product differentiation efforts are truly very important for firms.

JEL Classification: D42, D43, M10, M31, M37
Keywords: Product differentiation; Ranking preference; Ranking value; Monopoly power; Monopoly profits; Marketing strategy

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1 INTRODUCTION

The importance of product differentiation has been emphasized in the field of business administration, particularly by Porter (1980, 1985), who included differentiation as one of the three fundamental generic strategies of cost leadership, differentiation, and focus. Many researchers have also since studied differentiation from this perspective (e.g., Miller, 1988; De Meyer et al., 1989; Kotha and Orne, 1989; Kotha and Vadlamani, 1995; Lillis, 2002; Baines and Langfield-Smith, 2003). However, most of the arguments on differentiation in business administration are descriptive, and the mechanism behind differentiation does not seem to be sufficiently clear.

Product differentiation has also been studied in economics. The most important early example is the work of Hotelling (1929), which was extended by, among others, Salop (1979), Gabszewicz and Thisse (1979, 1980), and Shaked and Sutton (1982, 1983). Hotelling explained product differentiation based on a location model in which a linear city of length 1 is assumed and a duopoly game is played by two shops located along the linear city. Consumers are assumed to be distributed uniformly along the linear city and prices are constant and equal between the two shops. The basic result is that the two shops’ profits are identical at a Nash equilibrium because the best strategy is for the shops to be located next to each other. This is known as Hotelling’s law or the principle of minimum differentiation, and it means that it is rational for firms to make their products as similar to a rival firm’s product as possible. Because competing firms use similar strategies and positions, their profits are similar even if households’ tastes are different.

The arguments of Hotelling (1929) and those of Porter (1980, 1985) seem to contradict each other. In markets, rival firms are fiercely competitive, and therefore their products usually seem very similar, as Hotelling (1929) indicated they should be. On the other hand, Porter (1980, 1985) argued that the differentiation strategy is so important that firms should pursue it as much as possible. The model in Hotelling (1929) implies that a slight amount of product differentiation will slightly alter a firm’s profits, but such a small difference will only have small effects on a firm’s prosperity. The question arises then: If product differentiation results in only slight differences in profits, why is a differentiation strategy so important for firms?

In this paper, I examine this contradiction and show that both arguments are correct in their essences, but both equally lack an important element. The missing element is what Harashima (2016) called “ranking preference and value.” People feel, obtain, or consume “value” when using, enjoying, or consuming goods and services. Values derived from practical use have normally been considered in economics, but people will also consume value derived from ranking. For example, if a curio is evaluated to be the best among a set of similar items, its price will increase until it is much higher than that of other similar items even if it is not practically useful. That is, people obtain utility not only from practical use but also from a sense of ranking. This means that people have a “ranking preference” and consume “ranking values” (see the Appendix for a detailed discussion).

If we consider people’s ranking preferences, we can uncover the reason why product differentiation is so important. Even if the qualities of competing products are almost identical and tastes are distributed uniformly across households, the products’ ranks will clearly differentiate them and result in large differences in firms’ profits. Therefore, firms’ product differentiation efforts are truly very important.

2 RANKING PREFERENCE AND VALUE

In this section, the concepts of ranking preference and value that were first presented by Harashima (2016) are briefly summarized. These concepts are explained in detail in the Appendix.
2.1 Ranking value
Value is regarded as reflecting something useful. There are two kinds of value: practical value and ranking value. Practical value is the value that people perceive when consuming a good or service for practical purposes. Ranking value is the value that people feel from the rank of a good or service in a set of similar types of goods or services that people use, possess, or observe. Ranking value, therefore, is the value people place on goods or services on the basis of their relative place within a set of similar goods or services (e.g., the ranking of a book in a best-seller list or that of a professional sport team in a league).

2.2 Ranking preference
Goods and services have the following properties: quantity, quality, and rank. Quality is related to practical value, rank is related to ranking value, and quantity is related to both values. Suppose that the quality and rank of each good or service are given exogenously and fixed. Here, for simplicity, I assume that there is only one type of good or service in the economy, and that all goods or services belong to this type (these goods or services are hereafter called “goods”) and are substitutable for each other for households’ practical uses. Although the goods are substitutable from the point of view of practical use, they are differentiated from the point of view of rank.

Let \( R (= 1, 2, 3, \ldots) \) be the rank of the goods. Goods with rank \( R = 1 \) are those most preferred by households, \( R = 2 \) indicates the next most preferred, and so on. For simplicity, it is assumed that no goods have the same rank. A household’s utility derived from consuming the goods with rank \( R \) is

\[
u(q_{n,R} \cdot q_{l,R}, R)\]

where \( q_{n,R} \) and \( q_{l,R} \) are the quantity and quality of the goods with rank \( R \), respectively. For simplicity, the utility of the household is modified to

\[
u(\tilde{q}_R, R)\]

where \( \tilde{q}_R \) is the “quality-adjusted quantity” of the goods with rank \( R \), and \( \tilde{q}_R = q_{n,R} \cdot q_{l,R} \).

The utility function has the following conventional characteristics:

\[
\frac{\partial u(\tilde{q}_R, R)}{\partial \tilde{q}_R} > 0
\]

and

\[
\frac{\partial^2 u(\tilde{q}_R, R)}{\partial \tilde{q}_R^2} < 0 .
\]

In addition, for any \( r \in R \),

\[
u(\tilde{q}_r, r + 1) < u(\tilde{q}_r, r)
\]

and

\[
u(\tilde{q}_r, r + 2) - u(\tilde{q}_r, r + 1) > u(\tilde{q}_r, r + 1) - u(\tilde{q}_r, r).
\]
2.3 Monopoly power
The presence of ranking preference and value gives monopoly powers to the producers of high
rank products because selling ranking values to consumers requires no additional cost; that is, the
marginal cost of producing a ranking value is zero. Therefore, these producers can set the prices
of their products above their marginal costs (see the Appendix).

2.4 Implicit ranking
Although some goods and services have explicit rankings (e.g., a book on a best-seller list), most
goods and services do not because there is no open or formal competition among them. However,
it is highly likely that people still feel a sense of ranking, possibly unconsciously, from many
goods and services because they usually want to know which products most people are paying
attention to, and to buy the products that are the most popular and well known. Fame is valuable
because it provides information about “implicit rankings” and generates a sense of ranking. If
there are no explicit product rankings, households will want to discover their implicit ranking by
any means and will be alert to every chance of obtaining that information, for example, through
word-of-mouth communication, social networking sites and other websites, blogs, or TV
programs. Because implicit rankings are formed essentially on the basis of information about
which product is more preferred and sold, they do not represent an individual household’s unique
and personal rankings; rather, they are socially and widely recognized rankings. That is, implicit
rankings basically represent households’ common knowledge.

2.5 Positional goods
The concept of ranking preference and value may be seen as the same as that of positional goods
(Frank, 1985, 1991; Solnick and Hemenway, 1998). The essential nature of positional goods is
that people demand positional goods for the purpose of seeking status—that is, because they can
feel, and be seen as, special—in particular, as superior to other people. Hence, positional goods
are mostly luxury goods and services. On the other hand, non-luxury goods and services as well
as luxury goods and services can have ranking values. In essence, the theory of positional goods
reflects the rankings of consumers, whereas that of ranking preference and value reflects the
rankings of goods and services. The emotions or desires of people in these contexts are therefore
completely different.

Although the concept of positional goods is useful when studying luxury goods and
services, this paper focuses on non-luxury goods and services. The markets for luxury goods and
services (i.e., positional goods) are beyond the scope of this paper; therefore, those goods are
ignored here.

3 THE MECHANISM BEHIND PRODUCT
DIFFERENTIATION

3.1 The model
3.1.1 A household’s choice
Suppose that there are two products (Product 1 and Product 2) that are purchased for an identical
practical use, and each of them is produced by one of two competing firms (Firm 1 and Firm 2
produce Product 1 and Product 2, respectively). A household purchases one of the two products.
Before purchasing it, a household compares them and judges which product is better, or the
“winner.” Even if the two products’ prices are equal and their qualities are almost identical, a
household must judge one of the two products as the winner because it purchases only one of
them.
There are many aspects that distinguish the two products, including quality, packaging, design, label, trademark, and price. To assess the products comprehensively, a household awards weighted “points” to a product for each of those various aspects, sums the points, compares the total weighted sums for Products 1 and 2, judges the product with the greater number of points as the winner, and purchases the winner.

Households also have a ranking preference with regard to the two products; therefore, in addition to points awarded for the previously mentioned aspects, points are also given on the basis of the product’s implicit ranking. For simplicity, it is assumed that no goods have the same rank, except for a zero rank (or as yet unranked) as discussed below.

### 3.1.2 The total sum of points

Suppose that a household assigns points to products, particularly with regard to quality, taste (i.e., a preference for color, design, or any of a variety of a product’s aspects), and rank. Points for quality and taste are given by real numbers, but those for rank are given by natural numbers because, unlike quality and taste, ranks are intrinsically discrete. “Quality” indicates the degree of usefulness for a practical purpose(s). If the quality of a product is thought to be higher, more points are assigned to the product. In this context, usefulness includes many aspects, such as ease of use, durability, safety, reliability, and functionality. Therefore, quality points are awarded to a household to a product with regard to aspects of quality, where there are a total of $Q$ aspects. “Taste” indicates the degree to which a household likes or dislikes a product for a given set of quality, rank, and price. If a product is more fitted to a household’s taste, more points are awarded. Taste also consists of various aspects (e.g., color, design, smell, palate, tactile properties, etc.). Let $q_i$ ($i = 1, 2, \ldots, Q$) be points given by a household to a product with regard to $i$ aspects of quality, where there are a total of $Q$ aspects. Suppose for simplicity that points given by households to any aspect of taste are uniformly distributed in a finite interval, following the model of Hotelling (1929); that is, there is no particular aspect of taste that is unevenly preferred by households. “Rank” indicates the implicit rank of a product. Let $R_{P}$ be points given by a household to a product with regard to rank, where $R_{P} = 1$ indicates the highest rank and $R_{P} = 2$ is the second. Unlike $q_i$ and $t_j$, $R_{P}$ for each product is common across households because $R_{P}$ is a socially recognized rank. If an implicit ranking has not yet been formed socially, then $R_{P} = 0$ for any product and household. Finally, let $\Pi$ be the price of a unit of a product and be commonly known to all households.

The total number of points given by a household to a product is

$$\bar{T} = \frac{w_q \sum_{i=1}^{Q} q_i + w_t \sum_{j=1}^{T} t_j + w_{R} R_{P}}{\Pi},$$

where $w_q$, $w_t$, and $w_{R}$ are constant weights. Because higher ranks have smaller point values (i.e., $R_{P} = 1$ is the highest or best rank), $w_{R} < 0$. A household purchases the product that has the larger $\bar{T}$. The weights $w_q$, $w_t$, and $w_{R}$ will differ across households but, for simplicity, they are assumed to be identical for all households. Suppose also for simplicity that $\Pi$ is identical for the two products and equals 1.

Note that $w_{R}$ depends on the strength of households’ ranking preference. For given values of $w_q$ and $w_t$, a larger absolute value of $w_{R}$ indicates a stronger ranking preference. Note also that, in actuality, $w_{R}$ is not independent of $R_{P}$ but may be, for example, an exponentially decreasing function of $R_{P}$ with some lower limit. However, because there are only two competing products in the model, $w_{R}$ is assumed to be constant and independent of $R_{P}$.

### 3.1.3 The probability of purchase

For simplicity, the total points given by households to Product $v$ ($v = 1, 2$) are assumed to be
uniformly distributed between the interval from $T_v - 1$ to $T_v + 1$ for any $v$ where $T_v$ is the mean of the total points given by households to Product $v$ (see Figure 1). That is, the probability density function of total points for Product $v$ is

$$f(x) = \frac{1}{2}$$

for $T_v - 1 \leq x \leq T_v + 1$, and

$$\int_{T_v - 1}^{T_v + 1} f(x) dx = 1.$$ 

In addition, $T_1 > T_2$, and $T_1$ and $T_2$ are not correlated with each other.

**Figure 1: Distributions of $\tilde{T}_1$ and $\tilde{T}_2$ across households**

Let $\tilde{T}_v$ be the total points given by a household to Product $v$. Given the distribution of total points given by households (Figure 1), the probability that a household awards more total points to Product 1 ($\tilde{T}_1$) than to Product 2 ($\tilde{T}_2$) is

$$p(\tilde{T}_1 > \tilde{T}_2) = \int_{T_2 - 1}^{T_2 + 1} \frac{1}{2} dx + \int_{T_1 - 1}^{T_1 + 1} \frac{1}{2} (T_2 + 1 - x) dx$$

$$= \frac{1}{8} \left[ (T_1 - T_2)^2 + 4(T_1 - T_2) + 4 \right],$$  

(2)

where $p(\tilde{T}_1 > \tilde{T}_2) = 1$ if $T_1 - T_2 \geq 2$ (i.e., all households purchase Product 1). Equation (2) indicates
that, if $T_1 = T_2$, then $p(\bar{T}_1 > \bar{T}_2) = \frac{1}{2}$ (i.e., both products are evenly purchased by households), and if $T_1 > T_2$, then $p(\bar{T}_1 > \bar{T}_2) > \frac{1}{2}$ (i.e., Product 1 is purchased more than Product 2). As the difference between $T_1$ and $T_2$ increases, $p(\bar{T}_1 > \bar{T}_2)$ increases.

### 3.1.4 Sequence of events

Assume the two products are released in the market at the same time. At the time of the release, households do not know any information about the implicit ranks of the two products. Therefore, a household initially calculates $\bar{T}_1$ and $\bar{T}_2$ with $R_P = 0$; that is, it calculates $\bar{T}_v = \frac{w_q \sum_{i=1}^O q_i + w_i \sum_{j=1}^T t_j}{O}$. Because $O = 1$ is assumed for both products, a household initially calculates $\bar{T}_v = w_q \sum_{i=1}^O q_i + w_i \sum_{j=1}^T t_j$ and purchases the product with the higher $\bar{T}_v$.

After many households purchase the products, an implicit ranking of the products becomes socially established on the basis of information about which product is more preferred and purchased. That is, an implicit ranking is basically formed on the basis of the difference of $\bar{T}_v = \sum_{i=1}^O q_i + \sum_{j=1}^T t_j$ between the two products. After the implicit ranking becomes established socially, the term related to the rank (i.e., $R_P = 1$ or 2) is added in each household’s calculation of $\bar{T}_v$. Hence, a household calculates $\bar{T}_v = w_q \sum_{i=1}^O q_i + w_i \sum_{j=1}^T t_j + w_P R_P$ and purchases the product with the higher $\bar{T}_v$.

### 3.1.5 Ranking preference and ranking bias

The phenomenon in which a higher-ranked product is favored may indicate the existence of “ranking bias” rather than ranking preference. Ranking bias means that a household’s evaluation of the quality of product is biased because of its implicit ranking, such that the quality of the product with the higher rank is perceived to be much higher than it actually is. That is, a household’s judgement about quality is confused and biased by its implicit ranking.

It may be difficult to distinguish between ranking preference and ranking bias. Nevertheless, rational households are not likely to be repeatedly and continuously confused and biased largely by information about implicit rankings. Hence, for products that are repeatedly purchased by a household, the effect of ranking bias will be very small and possibly even negligible.

### 3.2 The decisive role of rank in households’ choices

#### 3.2.1 The difference between rank and the other elements

Because ranks are intrinsically discrete, they have a very different nature from quality and taste. There is a lower limit of the difference between ranks (i.e., 1), and thereby there is also a lower limit of the difference between $w_q R_P$ for adjacent ranks (i.e., $w_q$). On the other hand, there is no lower limit of difference between adjacent points for quality or taste because they are represented by real numbers, and there is no lower limit of difference between adjacent values of $w_q \sum_{i=1}^O q_i$ or $w_i \sum_{j=1}^T t_j$. Therefore, the means of all households’ $w_q \sum_{i=1}^O q_i + w_i \sum_{j=1}^T t_j$ in $\bar{T}_v$ can be almost
identical for the two products, but the means of all households’ \( w_i \sum_{j=1}^{T} t_j \) in \( \tilde{T}_v \) are always clearly different unless \( R_P = 0 \). Note that \( w_i \sum_{j=1}^{T} t_j \) is common for all households because the implicit ranking is common to them and no tied ranks are assumed.

Because points assigned on the basis of any aspect of taste are assumed to be uniformly distributed across a finite interval (Hotelling, 1929), the means of all households’ \( w_i \sum_{j=1}^{T} t_j \) in \( \tilde{T}_v \) are identical for the two products. In addition, because the firms are competing, the levels of technology will not differ markedly, and many of the necessary patents can be purchased. As a result, the qualities of competing firms’ products will become almost identical, and the means of all households’ \( w_i \sum_{j=1}^{T} q_i \) in \( \tilde{T}_v \) will also be almost identical in many cases. Of course, they will never be completely identical, but the difference in quality between competing products will be usually very small. Therefore, the means of all households’ \( \sum_{i=1}^{Q} \sum_{j=1}^{T} (q_i + w_i t_j) \) in \( \tilde{T}_v \) will usually be almost identical for two products, but the means of all households’ \( w_i R_P \) in \( \tilde{T}_v \) will clearly be different.

### 3.2.2 Rank as the deciding factor

If \( T_1 \) and \( T_2 \) are identical, the shares of sales of each product will be 50% by equation (2). Because the means of all households’ \( \sum_{i=1}^{Q} \sum_{j=1}^{T} q_i + w_i \sum_{j=1}^{T} t_j \) in \( \tilde{T}_v \) are usually almost identical, as shown in Section 3.2.1, if \( R_P = 0 \), the share of sales will also usually be almost 50%. In actuality, the shares of sales of many competing products are clearly different. This means that \( R_P \) is not zero for many products; that is, an implicit ranking exists for many products, and competing products can be always clearly differentiated by households even though the means of all households’ \( w_i \sum_{j=1}^{T} q_i + w_i \sum_{j=1}^{T} t_j \) in \( \tilde{T}_v \) are usually almost identical.

Let \( \overline{R}_p = R_{P,2} - R_{P,1} \), where \( R_{P,1} \) and \( R_{P,2} \) are the \( R_P \) of Products 1 and 2, respectively, and \( R_{P,1} \leq R_{P,2} \) (\( R_{P,1} = R_{P,2} \) only if \( R_{P,1} = R_{P,2} = 0 \)) and thereby \( \overline{R}_p \geq 0 \). (i.e., the rank of Product 1 is higher than that of Product 2 unless \( R_{P,1} = R_{P,2} = 0 \)). In this model, \( R_{P,1} = 1 \), \( R_{P,2} = 2 \), and \( \overline{R}_p = 1 \) once an implicit ranking has been formed. Let also \( \hat{T}_v = T_v - w_i \sum_{j=1}^{T} t_j \) for \( v = 1, 2 \) where \( \hat{T}_v \) is the total points given by an individual household without the points awarded for taste. Therefore, \( T_1 - T_2 = \hat{T}_1 - \hat{T}_2 - w_k R_P = \hat{T}_1 - \hat{T}_2 - w_k \). By equation (2),

\[
p(\hat{T}_1 > \hat{T}_2 | \overline{R}_p = 1) = \frac{1}{8} \left[ (\hat{T}_1 - \hat{T}_2 - w_k)^2 + 4(\hat{T}_1 - \hat{T}_2 - w_k) + 4 \right]. \tag{3}
\]

Because \( \hat{T}_1 - \hat{T}_2 \geq 0 \) will hold usually, as argued in Section 3.2.1, then by equation (3), usually

\[
p(\hat{T}_1 > \hat{T}_2 | \overline{R}_p = 1) \approx \frac{w_k^2 - 4w_k + 4}{8}. \tag{4}
\]

Because \( w_k < 0 \) as shown in Section 3.1.2, usually
\[ p\left( \tilde{T}_1 > \tilde{T}_2 \middle| \tilde{R}_p = 1 \right) \geq \frac{w_R^2 - 4w_R + 4}{8} > \frac{1}{2}. \]

Equation (4) indicates that the magnitude of product differentiation is usually determined by the value of \( w_R \); that is, product differentiation is usually governed by the strength of households’ ranking preference. Equation (4) also indicates that the probability that Product 1 is purchased varies largely depending on the value of \( w_R \). As the absolute value of \( w_R \) increases (i.e., as the strength of households’ ranking preference becomes stronger), the effect of \( w_R \) on product differentiation increases. If the strength of the ranking preference is sufficiently strong (i.e., the absolute value of \( w_R \) is sufficiently large), all households purchase Product 1 even if \( \tilde{T}_1 - \tilde{T}_2 \neq 0 \).

As argued above, usually \( \tilde{T}_1 - \tilde{T}_2 \approx 0 \), but \( T_1 \) and \( T_2 \) are sufficiently different because of ranks. This result indicates that a very small difference between \( \tilde{T}_1 \) and \( \tilde{T}_2 \) is significantly amplified by households’ ranking preference to a large difference between \( \tilde{T}_1 \) and \( \tilde{T}_2 \).

### 3.3 Monopoly profits

As shown in Section 2.3., ranking preference and value bring monopoly power to producers of highly ranked products. In this example, Firm 1 therefore has monopoly power. In the previous sections, it is assumed that \( \Pi = 1 \) for both products. However, Firm 1 can set the price of Product 1 higher than that of Product 2 because of its monopoly power. As discussed in Section 3.2.1, usually \( \tilde{T}_1 - \tilde{T}_2 \approx 0 \), and the marginal costs of producing Products 1 and 2 will therefore be almost identical. Hence, Firm 1 can set the price of Product 1 higher than both its marginal cost and the price of Product 2; that is, Firm 1 can exploit monopoly profits. The stronger the ranking preference of households (i.e., the larger \( w_R \)), the higher the price that Firm 1 can set for Product 1 and the larger its monopoly profits.

The amount by which Firm 1 will raise the price of Product 1 will be determined strategically, as standard duopoly models show. As the price of Product 1 increases, its sales will decrease because \( \tilde{T}_1 \) decreases by equation (1). \( p(\tilde{T}_1 > \tilde{T}_2) \) also decreases, but Firm 1’s profits increase unless it raises the price by too much. Firm 1 will consider the response of Firm 2 to its price strategy and set the price of Product 1 to maximize its expected profits.

An important reason why product differentiation is so important for firms is that even if the difference between \( \tilde{T}_1 \) and \( \tilde{T}_2 \) is very small, Firm 1 can still obtain a large amount of monopoly profits.

### 3.4 Persistence of monopoly power

Once the implicit ranking is formed, the monopoly power of Firm 1 will persist because it is not easy for Firm 2 to reverse the position \( T_1 > T_2 \) even though it may be relatively easy for Firm 2 to reverse the position \( \tilde{T}_1 > \tilde{T}_2 \). The established implicit ranking is difficult to change because the implicit ranking is greatly influenced by information about which product is the better seller. Because more of Product 1 is currently purchased than Product 2, the additional efforts of Firm 2 must be sufficient to override the effect of rank (i.e., \( w_R \) in \( T_v \)). As a result, implicit rankings are persistent and so is the monopoly power of Firm 1; this is another factor supporting the importance of product differentiation. This persistent nature may be a source of the value of a “brand” in business.

It is not necessarily completely impossible to reverse the situation however. As will be shown in Section 4, using a “category differentiation strategy” and advertising can be effective in some cases. Furthermore, Firm 1 may relax its guard or respond slowly to Firm 2’s strategies, or Firm 2 may create a breakthrough innovation.
4 DIFFERENTIATING A PRODUCT

4.1 Win by at least a narrow margin
As argued in Section 3, an implicit ranking is initially formed essentially on the basis of a comparison between $\hat{T}_1$ and $\hat{T}_2$. Therefore, it is important for a firm to first differentiate its product to achieve the position $\hat{T}_1 > \hat{T}_2$, even though $\hat{T}_1 - \hat{T}_2 \equiv 0$. The initial relative advantage is extremely important, so it is not necessary for a firm to make $\hat{T}_1$ far larger than $\hat{T}_2$; it has only to make sure that $\hat{T}_1$ is at least slightly larger than $\hat{T}_2$.

Note, however, that $\hat{T}_1$ is composed of many aspects and weights, as shown by the term $w_i \sum_{i=1}^{o} q_i + w_i \sum_{j=1}^{T} t_j$, so the allocation of the efforts is also important.

4.2 Differentiation of category
In the previous sections, it is assumed that competing products are purchased only for an identical practical use. Many products, however, can also be used for some secondary practical purposes. Even if the main practical uses of competing products are identical, their secondary practical uses may differ somewhat. Firms can use these possible differences in secondary practical uses to differentiate the product categories.

People feel ranking values only when the competing products belong to the same category. If the products are perceived to belong to different categories, people lose interest in the competition and feel little ranking value from these particular products. Firm 2 can utilize this nature of ranking preference to escape from its disadvantageous situation. If Firm 2 can add some secondary practical uses to Product 2 that are different from the secondary practical uses of Product 1 by using a “category differentiation strategy,” households may begin to think that Product 2 belongs to a different category from Product 1. As a result, households may not assign perceived implicit ranks to Products 1 and 2. Because $R_{P,1} = R_{P,2} = 0$ and $\hat{T}_1 - \hat{T}_2 \equiv 0$, Firm 2 may be able to escape from a high $p(\hat{T}_1 > \hat{T}_2)$ and the monopoly power of Firm 1. Firm 2’s category differentiation strategy may, of course, not always succeed, for a variety of reasons. For example, the demand for the newly perceived category to which Product 2 is thought to belong may be far smaller than that for the category Product 1 is thought to belong to, or adding different secondary practical uses to Product 2 may make $\hat{T}_2$ smaller than it was originally and $p(\hat{T}_1 > \hat{T}_2)$ may increase.

In addition to successfully implementing a category differentiation strategy, Firm 2 can simultaneously try to improve the quality of its product for the main practical use. By doing so, Firm 2 may be able to change the position from $\hat{T}_1 > \hat{T}_2$ to $\hat{T}_1 < \hat{T}_2$ in the original category. If the implicit ranking in the original category is reset to the initial state (i.e., $R_{P,1} = R_{P,2} = 0$) by Firm 2’s category differentiation strategy, a change in position from $\hat{T}_1 > \hat{T}_2$ to $\hat{T}_1 < \hat{T}_2$ will make households reverse the previous implicit ranking to $R_{P,1} = 2$ and $R_{P,2} = 1$ and Product 2 can take a larger share of sales than Product 1. However, Firm 1 will not sit idly by as Firm 2 pursues this strategy; it will soon counterattack Firm 2 by using the same strategy. If Firm 1 succeeds, Firm 2 will counterattack Firm 1, and these reciprocal attacks and counterattacks may be repeated for a long period. These “battles” between firms may be described as wave-like dynamic processes.

The effectiveness of a category differentiation strategy will depend on how households judge the categories of the two products. In general, classification has a hierarchical structure. Higher layers in the hierarchy include broader and more general subjects and lower layers include narrower and more specific subjects. In the higher layers, the two products may belong to the same category, but not on the lower layers. Hence, household recognition of product category
similarities varies depending on the layer on which the household judges. Conversely, a key issue for the success of a category differentiation strategy is to correctly anticipate the layer on which the households judge the product category.

4.3 Firm rank

In the previous sections, firm rank was ignored, but households give implicit ranks not only to products but also to firms, and the firm rank also influences $\hat{T}$. The implicit rank of a firm will be formed on the basis of its reputation, fame, size, history, international activities, social activities, environmental consciousness, and many other factors. Similar to product categories, if firms belong to different industries, no implicit rank is assigned. It is likely that, as the rank of a firm increases, the $\hat{T}$ of a product that the firm produces also increases. Therefore, higher ranking firms have advantages over lower ranking firms when they introduce new products into markets. It is likely that the implicit ranks of larger and more famous firms will be relatively high, and conversely, smaller and less well-known firms may generally have disadvantages from the beginning.

Note that ranking bias may also exist for firms (see Section 3.1.5). Households may perceive that the quality of products produced by a relatively large and more famous firm is basically better than that produced by a relatively small and less famous firm. In other words, households may judge that the risks associated with products produced by a relatively large and more famous firm are lower. The risks associated with products produced by unknown firms may be truly higher, but the true difference may be far smaller than generally thought. If the difference is exaggerated, this is an example of biased perception. Nevertheless, as was the case with the implicit ranking of products, it seems unlikely that rational households will persistently incorrectly evaluate these risks. In particular, for products that are repeatedly purchased by a household, the effect of firm ranking bias is most likely quite small, or even negligible if it truly exists.

4.4 Advertising

An important function of advertising is to disseminate information about the products a firm wants to sell. The model of product differentiation presented in this paper indicates that advertising has another important function—to induce households to perceive that a product’s implicit rank is high. Because implicit rankings are formed through various sources of information, there is room for a firm to manipulate the implicit rank of its product. An important element that influences implicit rankings is fame, and fame can be manipulated by advertising to some extent. If households are exposed to a large number of television commercials about a specific product, they may begin to perceive that the product’s implicit rank is high, regardless of the product’s actual rank. Once the product is successfully perceived as the higher ranked product, its $\hat{T}$ increases even if its $\hat{T}$ is lower than its rival’s.

4.5 Creating a breakthrough innovation

As shown in Section 3.4, the monopoly power of Firm 1 is persistent and not easy to change. One possible way for Firm 2 to reverse the relationship is to create a breakthrough innovation. If the effect of a breakthrough innovation can overwhelm the effect of the current implicit ranking, then Firm 2 can change the position not only from $T_1 > \hat{T}_2$ to $\hat{T}_1 < \hat{T}_2$ but also from $T_1 > \hat{T}_2$ to $T_1 < T_2$. Of course, creating a breakthrough innovation is not usually an easy task.

5 CONCLUDING REMARKS

Product differentiation has been studied in both economics and business administration, but the
arguments in the two fields seem to be contradictory. In economics, the strategy of product differentiation does not necessarily lead to large differences in firms’ profits (e.g., Hotelling, 1929), whereas in business administration, it is one of Porter’s (1980, 1985) three core strategies.

In this paper, product differentiation is examined by considering ranking preference and value—an element that is not considered by Hotelling or Porter. Households choose and purchase a product from among rival products because its implicit rank in society is higher, even if the other factors like quality and taste are almost the same. A slight difference in quality results in clear differences in implicit ranks of products and therefore in large differences in firms’ profits. The effect of differentiation is amplified by households’ ranking preferences, so firms’ efforts for product differentiation are very important.
APPENDIX

A1  Ranking value and preference

A1.1  Ranking

Ranking indicates an ordered list of relative standing. Indeed, the concept of relativity is essential in ranking. Even if the absolute abilities and performances of competitors are almost identical, only one person or group can be the best or the champion. Even if there is little difference in an absolute sense, there are significant differences in a relative sense because people will derive utility through various types of rankings in different aspects of life. On some occasions, people practically neglect the absolute performances and are interested only in the rankings. Therefore, both absolute and relative differences surely have an important influence on people’s thoughts and activities.

In general, absolute terms are used in economic studies. Even though ordinal utility is conceptually important, cardinal utility is assumed in most studies. Therefore, the concept of ranking or relativity has not typically been seen as an important element in economics. However, because people are interested in rankings and relative differences, these factors will affect many aspects of economic activity.

A1.2  Ranking value and preference

A1.2.1  Practical value and ranking value

Value is regarded as reflecting something useful. In this paper, it is assumed that there are two kinds of value: practical value and ranking value. Practical value is the value that people feel when consuming a good or service for practical purposes. Ranking value is the value that people feel from the rank of a good or service in a set of similar types of goods or services that people use, possess, or observe. Ranking value, therefore, is the value people place on goods or services on the basis of their rank (e.g., the ranking of a book in a best-seller list or that of a professional baseball team in a league). For example, people will buy a book not only because of its practical usefulness but also because of its popularity. That is, a book can have value not only on the basis of its practical usefulness (its practical value) but also on the basis of its popularity (its ranking value).

In many cases, practical value may be almost identical to the usual sense of value. If a good or service is more practically useful than another good or service, it has a higher practical value. People obtain utility from practical value through the consumption of goods and services. On the other hand, ranking value does not require practical usefulness. Even if a good or service is not practically useful, it can still have ranking value if it possesses a ranking: an example is the price of a curio that is not practically useful but is evaluated to be the best among a set of similar types of curios. If the rank of a good or service is higher than those of others in the set, its ranking value is higher. People obtain utility from ranking value through the consumption (use, possession, or observation) of goods and services.

For example, many people like to watch professional sports, even though watching them may be of little practical value. A sense of enthusiasm and fun is generated when viewing professional sports, but such emotions may not reflect any practical usefulness. Although the emotions generated may not be practically useful, the desire to watch, witness, and immediately know the winner (i.e., the best or the champion) provides ranking value. Although some people may want to see a game because they enjoy watching the performance (similar to watching a circus act), most people watch professional sports to watch, witness, and immediately know who wins: that is, they want to feel the sense of ranking and consume ranking value. People thereby obtain utility from the sense of ranking. I call people’s preference for ranking value a “ranking preference.” A mathematical expression of ranking preference is presented in Section A2.

A1.2.2  The origin of ranking value and preference
If ranking preference is a deep parameter of human behavior, it will be deeply rooted in the process of evolution of human beings; that is, it will be closely related to survival in the long history of humans. I propose the following two fundamental desires as the roots of ranking preference.

(1) Desire to win a struggle, conflict, or war
Struggles and conflicts are fundamental elements in life. In social species, they occur not only between individuals but also between rival groups. Struggles and conflicts generate intense emotions, including a strong desire to win. In an evolutionary sense, winning or losing a struggle or conflict was often a matter of life and death. Hence, people are very excited by struggles and conflicts. Interestingly, people may be excited by the occurrence of struggle or conflict itself, regardless of the eventual outcome. People therefore may “demand” the excitement of struggle, conflict, and even war (James, 1910; Cannon, 1915). If a particular group wins, the people in that group will be happy and comfortable (i.e., obtain utility). It is likely that humans have evolved to be excited by the occurrence of struggle or conflict because this response is important for their survival. Therefore, people have evolved to obtain utility from the occurrence of struggle or conflict regardless of whether they are actually involved in the conflict itself.

An important nature of struggle and conflict is that, regardless of the quality of performances in the struggle or conflict, a win is a win. Absolute performances in struggles or conflicts are basically meaningless, whereas relative performances are vitally important. For example, in a horse race, the winning time is basically meaningless, but the order of finish is valuable. If they are relatively superior, people can win even if the difference in the performances is very slight. That is, people are happy not only because they are strong but also because they are stronger, and they are happy when they are the strongest. As a result, rankings generate strong emotions in people’s minds. These emotions are among the origins of ranking value and preference.

(2) Desire to behave in accordance with dominance hierarchy
Many species—particularly social species—have dominance hierarchies (see, e.g., Landau, 1951; Bayly et al., 2006). Most primates, including humans, have hierarchical societies. Dominance hierarchy has evolved to be deeply integrated into primate societal behaviors by necessity. Under dominance hierarchy, an individual’s rank in its group is crucial. Knowing one’s own rank and the rank of others is a significantly important part of living in a societal group.

Because life strategies in a societal group vary depending on an individual’s rank, individuals must constantly reconfirm their rank. If individuals are unable to confirm their ranks, they may face adverse outcomes or even death. This confirmatory need may be accompanied by subordinate behaviors such as admiring, supporting, and following the leader and punishing members who neglect the ranking.

Therefore, it is likely that humans have evolved to possess the emotion or urge to regularly reconfirm rankings. In other words, people have evolved to obtain utility from regularly reconfirming ranking orders within groups. Conversely, people will be very uneasy and uncomfortable if they are unable to correctly assess the latest rankings. Only after they reconfirm rankings will they feel at ease and comfortable and be satisfied. It is likely that this intrinsic emotion is another origin of ranking value and preference.

A1.2.3 Importance of ranking
The emotions that underlie ranking value and preference will clearly surface on various occasions. Sports have been often seen as a substitute for war (Santayana, 1972; Fischer, 2002). Watching professional sports satisfies people’s desires and makes them feel comfortable because the games substitute for struggle, conflict, and war. Fans of a specific team may view the team as a substitute for the mother country or tribe in a war. People often form attachments to a specific sports team and maintain allegiance to it as if it gives them a sense of tribal unity. Another example of this
emotion is in people’s responses to titles in the business world. A title indicates the rank of a person in a company, organization, or group. Whatever the true quality of performance of any given person, people evaluate and judge that person on the basis of the title to some extent. The quality of performance is of course important, but the title (rank) is also important.

Ranking is therefore an important element in people's lives and economic activities. For some goods or service, people may even place higher values on rank than on practical use. It is highly likely that humans are intrinsically equipped with emotions that respond to various types of ranking. Therefore, if people do not sufficiently consider rankings in their daily activities, they may not be successful in managing their lives.

A2  A model of superstardom
A2.1  Background
As noted in the Introduction, Rosen (1981) performed early and important work on the model of superstardom. He attributed the extremely high incomes of superstars to a special market structure (i.e., “non-rivalry”). However, Adler (2006) criticized Rosen (1981) by arguing that non-rivalry results in very low prices, thus suggesting that superstars would actually be poor. Adler (1985) presented a different model and argued that the extremely high incomes of superstars can be attributed to what he called consumption capital. However, the assumption of increasing marginal utility in his model is not easily acceptable. Borghans and Groot (1998) presented a model based on the argument of Frank and Cook (1995). They argued that the extremely high incomes of superstars are attributable to “endogenous property rights” and the monopoly power that these rights generate, because people have a strong tendency to want to watch the performance of someone known to be the best. Frank and Cook (1995) argued that, in modern economies, the winner takes all of the money in many industries. However, the argument of Frank and Cook (1995) is strictly narrative and lacks a theoretical model that clearly explains why the winner takes all. The model of Borghans and Groot (1998) suffers from the same drawback, because the mechanism used to explain people’s strong tendency to want to watch the winner, the best, or the champion is not sufficiently developed and is at best only suggested.

The concept of ranking value and preference discussed in Section A1 is, however, closely related to the arguments of Frank and Cook (1995) and Borghans and Groot (1998), because they commonly emphasize that household consumption is influenced by both absolute and relative performances or qualities.

A2.2  The model of superstardom
A2.2.1  The model
Goods and services have three properties: quantity, quality, and ranking. Quality is related to practical value, and ranking is related to ranking value. Quantity is related to both values. Suppose that the quality and ranking of each good or service are given exogenously and fixed. Here, for simplicity, I assume that there is only one type of good or service in the economy, and that all goods or services belong to this type (these goods or services are hereafter called “goods”) and are substitutable for each other for households’ practical uses. Although the goods are substitutable from the point of view of practical uses, they are differentiated from the point of view of ranking.

Let $R (= 1, 2, 3, \ldots)$ be the rank of the goods. Goods with rank $R = 1$ are those most preferred by households. $R = 2$ indicates the next most preferred, and so on. It is assumed for simplicity that there is no tied rank. A household’s utility derived from consuming the goods with rank $R$ is

$$u(q, R)$$

where $q$ and $R$ are the quantity and quality of the goods with rank $R$, respectively. For
simplicity, the utility of the household is modified to

$$u(\tilde{q}_R, R)$$

where $\tilde{q}_R$ is the “quality-adjusted quantity” of the goods with rank $R$, and $\tilde{q}_R = q_{n,R} q_{l,R}$. The use of quality-adjusted quantity is based on the assumption that, given a standard (reference) quality of the goods, consuming $\alpha\%$ worse/better quality goods than the standard quality goods for practical use is equivalent to consuming $\alpha\%$ more/less of these goods than the standard quality goods for practical use. The quality-adjusted quantity $\tilde{q}_R$ therefore indicates the “real” quantity of the goods standardized by a reference quality.

The utility function has the following conventional characteristics:

$$\frac{\partial u(\tilde{q}_R, R)}{\partial \tilde{q}_R} > 0$$

and

$$\frac{\partial^2 u(\tilde{q}_R, R)}{\partial \tilde{q}_R^2} < 0 .$$

In addition, for ranking preference, the following characteristics are assumed. For any $r \in R$ ,

$$u(\tilde{q}_i, r+1)<u(\tilde{q}_i, r) \quad (A1)$$

and

$$u(\tilde{q}_i, r+2)−u(\tilde{q}_i, r+1)>u(\tilde{q}_i, r+1)−u(\tilde{q}_i, r) \quad (A2)$$

Inequality (A1) indicates that, as rank becomes lower ($R$ increases), utility decreases, and inequality (A2) indicates that, as rank becomes lower ($R$ increases), the magnitude of decrease in utility with a lowering in rank decreases.

It is assumed, furthermore, for simplicity that utilities are separable. Therefore, a household’s total utility derived from its consumption of goods of various ranks is described as

$$U = u_{\text{Quant}}(\tilde{q}) + \sum_{R=1}^{\infty} u_{\text{Rank}}(\tilde{q}_R, R) \quad (A3)$$

where $u_{\text{Quant}}(\cdot)$ and $u_{\text{Rank}}(\cdot)$ are the utility function for consumption of practical value and that of ranking value, respectively, and

$$\tilde{q} = \sum_{R=1}^{\infty} \tilde{q}_R \quad .$$

Similarly, the following conventional characteristics of the utility function are assumed:

$$\frac{\partial u_{\text{Quant}}(\tilde{q})}{\partial \tilde{q}} > 0$$
\[
\frac{\partial^2 u_{\text{Quant}}(\bar{q})}{\partial \bar{q}^2} < 0
\]

\[
\frac{\partial u_{\text{Rank}}(\bar{q}_R;R)}{\partial \bar{q}_R} > 0
\]

and

\[
\frac{\partial^2 u_{\text{Rank}}(\bar{q}_R;R)}{\partial \bar{q}_R^2} < 0
\]

In addition, with respect to ranking preference, the following characteristics are assumed. For any \(r \in R\),

\[
u_{\text{Rank}}(\bar{q}_r, r+1) < u_{\text{Rank}}(\bar{q}_r, r) \quad (A4)
\]

and

\[
u_{\text{Rank}}(\bar{q}_r, r+2) - u_{\text{Rank}}(\bar{q}_r, r+1) > u_{\text{Rank}}(\bar{q}_r, r+1) - u_{\text{Rank}}(\bar{q}_r, r) \quad . \quad (A5)
\]

The budget constraint of households is

\[
I = p_{\text{Quant}} \sum_{R=1}^{\infty} \bar{q}_R + \sum_{R=1}^{\infty} p_{\text{Rank}, R} \bar{q}_R = p_{\text{Quant}} \bar{q} + \sum_{R=1}^{\infty} p_{\text{Rank}, R} \bar{q}_R \quad (A6)
\]

where \(I\) is the budget (income) of the household and is exogenously given and constant, \(p_{\text{Quant}}\) is the price of a unit of \(\bar{q}_R\) consumed for practical value, and \(p_{\text{Rank}, R}\) is the price of a unit of \(\bar{q}_R\) consumed for ranking value. The price for practical value is identical for any \(R, q_n, R, and q_l, R\). Equation (A6) indicates that there is not only a price for ranking value but also a price for practical value, and households pay for both practical values and ranking values when they buy the goods. A household maximizes its utility (equation [A3]) subject to the budget constraint (equation [A6]).

On the other hand, the producer of the goods with rank \(R\) behaves to maximize its profits. For simplicity, costs to produce the goods are assumed to be directly proportional to \(\bar{q}_R\) and identical for any \(R, q_n, R, and q_l, R\). Let \(c\) be the cost per one unit of \(\bar{q}_R\). Therefore, the profit of the producer of the goods with rank \(R\) \((\Pi_R)\) is

\[
\Pi_R = p_{\text{Quant}} \bar{q}_R + p_{\text{Rank}, R} \bar{q}_R - c \bar{q}_R - \overline{c}_R \quad (A7)
\]

where \(\overline{c}_R\) is the fixed cost of the producer of goods with rank \(R\).

A2.2.2 The model with continuous ranking

Ranks are discrete by nature. However, for simplicity, it is assumed that rank is continuous. Let \(R \in [0, 1]\). The utility of a household is therefore changed to

\[
U = u_{\text{Quant}}(\bar{q}) + \int_0^1 u_{\text{Rank}}(\bar{q}_R;R) dR \quad (A8)
\]
where \( \bar{q} = \int_0^t \bar{q}_R dR \). The budget constraint of a household is changed to

\[
I = p_{\text{Rank}} \bar{q} + \int_0^t p_{\text{Rank}, R} \bar{q}_R dR
\]

Inequalities (A1), (A2), (A4), and (A5) are changed respectively to

\[
\frac{\partial u(\bar{q}_R, R)}{\partial R} < 0 , \quad (A9)
\]

\[
\frac{\partial^2 u(\bar{q}_R, R)}{\partial R^2} > 0 ,
\]

\[
\frac{\partial u_{\text{Rank}}(\bar{q}_R, R)}{\partial R} < 0 , \quad (A10)
\]

and

\[
\frac{\partial^2 u_{\text{Rank}}(\bar{q}_R, R)}{\partial R^2} > 0 .
\]

### A2.3 The mechanism of superstardom

#### A2.3.1 Extremely high incomes of superstars

##### A2.3.1.1 Monopoly power

Ranking value and preference provide monopoly powers to the producers of the goods because selling ranking value to consumers requires no additional cost, i.e., the marginal cost of producing a ranking value is zero, and thereby producers can set \( P_{\text{Rank}, R} \) above the marginal cost. Thanks to their monopoly powers, producers are not price-takers. Rather, they can strategically set their prices for rank \( p_{\text{Rank}, R} \) in equation (A7) so as to maximize their profits \( \Pi_r \). \( \bar{q}_R \) and \( p_{\text{Rank}, R} \) are therefore determined by producers’ strategic behaviors.

##### A2.3.1.2 The shape of the utility function

The shape of the utility function with regard to ranking (\( u_{\text{Rank}} \)) is important in determining the magnitude of the producers’ monopoly power. Depending on the values of \( \frac{\partial u_{\text{Rank}}(\bar{q}_R, R)}{\partial R} \) in inequality (A10), utility functions can take various shapes, and the strength of monopoly power depends on inequality (A10) (or inequality [A4]). As ranking preference becomes stronger—that is, as the values of \( \frac{\partial u_{\text{Rank}}(\bar{q}_R, R)}{\partial R} \) become larger for any \( R \)—a household is willing to buy the goods for a higher price \( p_{\text{Rank}, R} \) than it did before. This means that, if a household’s ranking preference becomes stronger, the monopoly powers of the producers become stronger, and the producers of the higher-ranked goods can set even higher prices.

Inequalities (A9) and (A10) (or inequalities [A1] and [A4]) indicate that \( p_{\text{Rank}, i} > p_{\text{Rank}, j} \) if \( i < j \) where \( i, j \in R \). The producer of the goods with rank 1 therefore has the strongest monopoly power, and it can set the highest price \( (p_{\text{Rank}, 1}) \) relative to the other producers. If the ranking preference is extremely strong, the producer of the goods with rank 1 will monopolize almost all revenues in the industry.
A2.3.1.3 The strategy for non-rival goods and services

Even if ranking preference is very strong, however, producers of the goods may not necessarily set high prices for ranking value ($p_{\text{Rank,R}}$). Instead of setting $p_{\text{Rank,R}}$ high, they may plan to sell larger quantities by keeping $p_{\text{Rank,R}}$ relatively low if monopoly profits are maximized by doing so. Whether this strategy is adopted will depend on the degree of rivalry of the goods with respect to practical value. In the case of rival goods or services with respect to practical use, this low price strategy will not be adopted. However, in the case of non-rival goods or services (i.e., if very little cost is needed to produce and distribute additional units for practical use), then the strategy of setting relatively low prices for ranking value may be preferred.

In the case of non-rival goods or services, the marginal cost to produce not only ranking value but also practical value is almost zero. In this case, even if the price for the practical value is set relatively low, it is still above the marginal cost. That is, the supply curve of such goods (the marginal cost for practical value plus that for ranking value) will be situated at a very low price level and will be almost flat. On the other hand, lower prices will attract more consumers. If the demand curve is also almost flat in a low price range, the profits of the producer of the non-rival goods may be far larger when the strategy of setting a relatively low price for ranking value and thereby attracting a larger number of consumers is taken. That is, the maximum monopoly profits may be realized when the price is set relatively low. This low price strategy is closely related to the argument presented by Rosen (1981).

A2.3.2 The two-producer model

A2.3.2.1 The model

A two-producer version of the model is used in this section for simplicity to demonstrate the mechanism of superstardom. Suppose that there are only two producers: producers of goods with rank 1 and rank 2. Let them be producer 1 and producer 2, respectively. In addition, for simplicity, $p_{\text{Quant}}$ and $u_{\text{Quant}}$ are ignored. Hence, a household maximizes its utility

$$U = u_{\text{Rank}}(\tilde{q}_1,1) + u_{\text{Rank}}(\tilde{q}_2,2)$$

subject to its budget constraint

$$I = p_{\text{Rank},1}\tilde{q}_1 + p_{\text{Rank},2}\tilde{q}_2$$

where $I$, $p_{\text{Rank},1}$, and $p_{\text{Rank},2}$ are exogenously given. It is assumed that the ranking preference of the household is very strong—that is, $u_{\text{Rank}}(\tilde{q}_1,1) - u_{\text{Rank}}(\tilde{q}_1,2)$ is very large. Hence, the indifference curve is almost horizontal (Figs. A1 and A2).

Producers 1 and 2 set their prices to maximize their profits, such that

$$\Pi_r = p_{\text{Rank},r}\tilde{q}_r - c\tilde{q}_r$$

for $r = 1, 2$.

A2.3.2.2 Equilibrium prices and quantities

In the model with only two producers there is a duopoly. Suppose that producer 2 sets its price for ranking value $p_{\text{Rank},2}$, and then, considering $p_{\text{Rank},2}$, producer 1 sets its price for ranking value $p_{\text{Rank},1}$ to maximize its profits. The quantities $\tilde{q}_1$ and $\tilde{q}_2$ are determined at the point of contact between an indifference curve and the household budget constraint (Fig. A1).
As $P_{\text{Rank}, 1}$ is set higher, $\tilde{q}_2$ decreases; that is, $\frac{\partial P_{\text{Rank}, 1}}{\partial q_1} < 0$. Because producer 1 behaves to maximize its profits, it sets $P_{\text{Rank}, 1}$ so as to satisfy $\frac{d\Pi_1}{dq_1} = \frac{\partial P_{\text{Rank}, 1}}{\partial q_1} \tilde{q}_1 + P_{\text{Rank}, 1} - c = 0$; that is,

$$P_{\text{Rank}, 1} = c - \frac{\partial P_{\text{Rank}, 1}}{\partial q_1} \tilde{q}_1 > c.$$  

An important point is that, because of very strong ranking preference and almost horizontal indifference curves, there is a range of $P_{\text{Rank}, 1}$ where $\tilde{q}_1 > \tilde{q}_2$ even if $P_{\text{Rank}, 1} > P_{\text{Rank}, 2}$ (Fig. A1). That is, producer 1 can obtain far larger profits than producer 2.

However, because of the duopoly, game theoretic considerations apply. Producer 2 will change its price $P_{\text{Rank}, 2}$ after recognizing producer 1’s price $P_{\text{Rank}, 1}$ (Fig. A2). Each producer will adjust its prices for ranking value strategically by considering the other’s behavior. Most simply, $P_{\text{Rank}, 1}$, $P_{\text{Rank}, 2}$, $\tilde{q}_1$, and $\tilde{q}_2$ will be determined at a Cournot-Nash equilibrium. Each producer has its own response function, which indicates the producer’s set of best strategies when a strategy of the other producer is given (i.e. its best prices for ranking value when the other’s price for ranking value is given). Response functions are depicted as response curves on the $P_{\text{Rank}, 1}$ versus $P_{\text{Rank}, 2}$ plane in Figure A3. Equilibrium occurs at the point of intersection of the response curves. Note that neither producer sets its prices for ranking value below $c$ because it would suffer losses by doing so (Fig. A3).
Figure A2: The case of a given $P_{\text{Rank},1}$

The shapes of the response curves of producers 1 and 2 are very different because of the households’ very strong ranking preference—that is, because the indifference curve is almost horizontal. Because producer 1 can set significantly higher prices than producer 2, thanks to the very strong ranking preference, the response curve of producer 1 is situated at the upper side of the plane in Figure A3, whereas the response curve of producer 2 is situated at the left side of the plane. As a result, $p_{\text{Rank},1}$ is notably higher than $p_{\text{Rank},2}$ at the Cournot-Nash equilibrium. As the ranking preference increases, the response curve of producer 1 moves higher and that of producer 2 moves farther left, and the difference between $p_{\text{Rank},1}$ and $p_{\text{Rank},2}$ at the Cournot-Nash equilibrium also increases.

Figure A3 indicates that, if ranking preference is strong, the equilibrium quantity of goods with rank 1 will not decrease largely even if producer 2 sets its price at $p_{\text{Rank},2} = c$ (i.e., $\Pi_2 = 0$). This means that, if households’ ranking preference is strong enough, producer 2 must accept far smaller profits than producer 1 no matter which strategy producer 2 chooses. In other words, producer 1 can be a superstar. This is the mechanism of superstardom.

Note that producer 1 may set its price for ranking value ($p_{\text{Rank},1}$) very low to expel producer 2 out of the market and completely monopolize the profits. However, if producer 2 is expelled and only producer 1 remains in the market, the ranking becomes meaningless for households and thereby the ranking value of the goods with rank 1 will be zero. Therefore, producer 1 will set its price for ranking value ($p_{\text{Rank},1}$) sufficiently high so that producer 2 will not leave the market.
Figure A3: Cournot-Nash equilibrium under strong ranking preference
References