Interest on reserves and monetary policy of targeting both interest rate and money supply

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Abstract

We build a dynamic model with currency, demand deposits and bank reserves. The monetary base is controlled by the central bank, while the money supply is determined by the interactions between the central bank, banks and public. In banking crises when banks cut loans, a Taylor rule is not efficient. Negative interest on reserves or forward guidance is effective, but deflation is still likely to be persistent. If the central bank simultaneously targets both the interest rate and the money supply by a Taylor rule and a Friedman's k-percent rule, inflation and output are stabilized.

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1 Introduction

In the Great Recession, the federal funds rate was near zero; however, the deflation pressure was still high as banks cut loans. That phenomenon raised many concerns among academics and policy makers on how the central bank's policy should be designed when the interest rate channel was weak. Possible solutions for pushing up inflation in this circumstance are forward guidance, helicopter money and quantitative easing (Bernanke, 2016*c*,*b*). The last two tools can change the money supply directly (rather than indirectly through a money creation process by banks); however they need to be integrated with standard monetary policy frameworks and cannot be a daily tool like an interest rate targeting policy. This paper argues that the central bank can utilize both instruments, interest rate and money supply, to do a better job at hitting the inflation target in a banking crisis.

The common traditional consensus among economists is that the central bank cannot target both interest rate and money supply at the same time. The central bank chooses either the monetary base as its main instrument (Meltzer, 1987; McCallum, 1988; Friedman, 1960) or the common interbank rate (Taylor, 1993). However, the introduction of a new monetary policy tool - interest on reserves (IOR) - and the transformation of the economy from making transactions with currency to with demand deposits will allow the central bank to use both above instruments. With IOR, the price of reserves might disconnect from the quantity of reserves in the banking system. With demand deposits, money can earn a positive nominal interest rate. Therefore, it is a possible scenario that that the central bank can increase the money supply and raise IOR at the same time.

This paper builds a dynamic model with bank reserves, currency, and demand deposits. The monetary base in our model is controlled by the central bank, while the money supply is determined by the interactions between the central bank, banks and the public. The interbank rate is controlled either by open market operations or by adjusting IOR, so a wide range of conventional and unconventional monetary policy can be assessed in this model.

We find that in normal times, an interest rate policy following a Taylor rule is a transparent and effective means of controlling the economy. When the central bank cuts rates, the amount of the money supply increases because banks create more loans. As the price is sticky in our model, the economy is stimulated due to the rise in the aggregate demand. The effect is identical to the standard New Keynesian model.

However, in banking crises, when banks cannot make loans due to the capital constraints, a policy following a Taylor rule is insufficient for pushing the real interest rate down. Even with the negative IOR or forward guidance, the outcome is only slightly better. The main reason is that the inflationary expectations also depend on the path of the money supply. In the case of banking crisis, the endogenous money supply declines. If the central bank does not inject liquidity directly into the market, the deflation pressure will be huge. Because of deflation and the wedge created by the capital constraint, the real prime rate will be high even though the interbank rate touches the zero lower bound.

Targeting both the money supply and the interest rate is very efficient in this situation. We find that the central bank only needs to follow a simple Taylor rule and a Friedman's k-percent rule so that both output and inflation will be stabilized. After the crisis time, the central bank can always come back to a simple Taylor rule.

Related Literature

Our model shares many similarities with the standard New Keynesian framework with the existence of the banking sector[1]. Banks play a role of intermediaries channeling funds from savers to borrower in these models; while ours focuses on the function of creating money in the banking sector (McLeay, Radia and Thomas, 2014). Banking crises create liquidity problem for agents in the private sector as the money supply declines. In this sense, our model is identical to Benigno and Nisticò (2017), where a shock worsening the liquidity of pseudo-safe assets can create a crisis with a persistent deflation.

The money supply in our model is endogenous. The interest rate channel and the credit channel are interdependent. In this aspect, our model is related to Bianchi and Bigio (2014) and Afonso and Lagos (2015), where these authors apply the search and matching theory to

[1]See Bernanke, Gertler and Gilchrist (1999), Christiano, Motto and Rostagno (2004), Gertler and Kiyotaki (2010), Curdia and Woodford (2011), Gertler and Karadi (2011).

study the banking sector. On the other hand, our banking sector is perfectly competitive and frictionless, so we can focus more on the impact of monetary policy on output and inflation. On the money demand side, we follow the cash-in-advance literature in (Lucas and Stokey, 1987; Belongia and Ireland, 2006, 2014), so households hold currency and bank deposits as they provide the liquidity. Cash goods and deposit goods are bundled together in a constant elasticity of substitution utility function.

To target both the money supply and interest rate, the central bank has to use IOR. This tool is already mentioned in the literature (Sargent and Wallace, 1985; Goodfriend et al., 2002; Kashyap and Stein, 2012; Ireland, 2014; Cochrane, 2014; Keister, 2016). Our model, different from this line of research, can connect IOR with banking reserves, money supply and the interbank rate in a micro-founded dynamic setup.

We also discuss two unconventional monetary policies: negative IOR and forward guidance. Analysis of monetary policy with a negative interest rate can be found in Rognlie (2015). Our model emphasizes the transmission through a negative IOR to the interbank rate and the deposit rate while the framework in Rognlie (2015) assumes directly that the central bank can impose a negative short term rate. In both papers, the negative rate might be an important tool when the interbank rate is near zero. We also examine the effect of the central bank's forward guidance policy[2] in a banking crisis context.

This paper extends the model in Ngotran (2017) to a more general environment, including both currency and electronic money. Banking crises are the main themes in both paper and the effectiveness of different hypothetical polices are assessed under this context. However, Ngotran (2017) studies the inflation dynamics after the Great Recession and the appropriate policy to get out of the low interest rate environment. This paper focuses on a new type of monetary policy when the central bank targets both the money supply and the interest rate by two common simple monetary rules.

The rest of the paper is divided into six parts. Section 2 and 3 describe the model. Section 4 and 5 study the equilibrium conditions as well as some theoretical results of this model. Section

[2]See (Eggertsson et al., 2003; Levin et al., 2009; Del Negro, Giannoni and Patterson, 2012; Campbell et al., 2012; Keen, Richter and Throckmorton, 2017*a*)

6 performs some experiments to assess the efficacy of different policies when banking crises happen. Section 7 gives the conclusion.

2 The Environment

2.1 Notation:

Let P_t be the price of the final good. We use lowercase letters to represent the real balance of a variable or its relative price. For example, the real reserves balance $n_t = N_t/P_t$, or the real price of intermediate goods $p_t^m = P_t^m/P_t$. The timing notation follows this rule: if a variable is determined or chosen at time t, it will have the subscript t. The gross inflation rate is $\pi_t = P_t/P_{t-1}$.

2.2 Goods and Production Technology

We follow closely the model description in Ngotran (2017). Our model extends Ngotran (2017) to the environment where currency and demand deposits coexist. There are four types of goods in the economy: cash-goods $y_{1,t}$ produced by c-retailers who only accept currency as the mean of payment, deposit-goods $y_{2,t}$ produced by d-retailers who only accept payment through banks, wholesale goods $y_t(j)$ produced by wholesale firm j and intermediate good y_t^m produced by households.

Each period households self-employ their labor l_t to produce the homogeneous intermediate good y_t^m under the production function:

$$y_t^m = l_t$$

Households sell y_t^m to wholesale firms in the competitive market with the price P_t^m.

There is a continuum of wholesale firms indexed by $j \in [0,1]$. Each wholesale firm purchases the homogeneous intermediate good y_t^m from households and differentiates it into a distinctive wholesale goods $y_t(j)$ under the following technology:

$$y_t(j) = y_t^m$$

Wholesale firms faces the Rotemberg adjustment cost when they change their prices.

Two types of retail firms both produce the final good $y_{i,t}$ $(i = 1,2)$ by aggregating a variety of differentiated wholesale goods $y_t(j)$:

$$y_{i,t} = \left(\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad i = 1,2$$

As the markets for cash-goods and deposit-goods are perfectly competitive and they have the same constant return to scale production function, they have the same price P_t.

2.3 Time, Demographics and Preferences

Time is discrete, indexed by t and continues forever. The model is in the deterministic setting and has six types of agents: bankers, households, wholesale firms, two types of retail firms, and the consolidated government.

There is a measure one of identical infinitely lived bankers in the economy. Bankers discount the future with the discount factor β. Each period, they gain utility from consuming a basket c_t that contains cash-goods $c_{1,t}$ and deposit-goods $c_{2,t}$. Their utility at the period t can be written as:

$$\sum_{s=0}^{\infty} \beta^s \log(c_{t+s}), \quad \text{with } c_t = \left[\sum_{i=1}^{2} \alpha_i^{\frac{1}{\sigma}} c_{i,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where α_i is the share of cash-goods or deposit-goods in the basket and σ is the elasticity of substitution between two goods in the basket.

There is also a measure one of identical infinitely lived households. Households discount the future with the discount factor $\widetilde{\beta}<\beta$, so they will borrow from bankers in the steady state. Similar to bankers, each period households gain utility from consuming the basket $\widetilde{c}_t$ and lose utility when providing labor l_t to their own production. Household's utility at the period t can

Banker	
Loans: $+S_t$	Deposits: + $q^l_t S_t$
	Net worth: + $(1-q^l_t)S_t$

Banker	
Loans: -$\delta_b B^h_{t-1}$	Deposits: -$\delta_b B^h_{t-1}$

Table 1: Banker issues loans (left) and collects on loans (right)

be written as:

$$\sum_{s=0}^{\infty} \widetilde{\beta}^s \left[\log(\tilde{c}_{t+s}) - \chi l_{t+s} \right], \quad \text{with } \tilde{c}_t = \left[\sum_{i=1}^{2} \alpha_i^{\frac{1}{\sigma}} \tilde{c}_{i,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

where χ is the weight of labor in the utility function.

Wholesale firms, retail firms are infinitely lived, owned by households.

The consolidated government includes both the government and the central bank, so for convenience, we assume there is no independence between the government and the central bank.

2.4 Assets

There are two types of financial assets: bank loans to households B^h_t and interbank loans B^f_t.

(a) **Bank loans to households (B^h_t)** : We follow Leland and Toft (1996) and Bianchi and Bigio (2014) to model the loan structure between bankers and households. The market for bank loan is perfectly competitive and the price of loan is q^l_t. When a household wants to borrow 1 dollar at time t, bankers will create an account for her and deposit q^l_t dollars to her account. In the exchange for that, this household promises to pay δ_b, $\delta_b(1-\delta_b)$, ..., $\delta_b(1-\delta_b)^{n-1}$, $\delta_b(1-\delta_b)^n$... dollars at time $t+1$, $t+2$,... $t+n$, $t+n+1$... where n runs to infinity and $0 < \delta_b \leq 1$ (Table 1). Loans are illiquid and bankers cannot sell loans.

Let B^h_t be the nominal balance of loan stock in the period t, let S_t be the nominal flow of new loan issuance, we have:

$$B^h_t = (1-\delta_b)B^h_{t-1} + S_t$$

(b) **Interbank loan (B^f_t)**: Bankers can borrow reserves from other bankers in the interbank market. The nominal gross interest rate in the interbank market is the interbank rate R^f_t.

2.5 Money and Payment System

There are three types of money in our economy: currency x_t, zero-maturity deposits m_t and reserves n_t.

(a) **Currency** (x_t): is issued by the central bank and held by households. If currency is held by bankers, it is automatically converted to reserve. Currency is used for transactions between households/bankers and c-retailers who sell cash-goods. Currency does not pay nominal interest. The amount of currency in circulation is endogenous in the equilibrium.

(b) **Zero maturity deposit (ZMD)** (m_t): is a type of e-money issued by bankers. ZMDs have the same unit of account as currency. When holding these deposits, households earn the gross nominal interest rate R_t^m which is determined by the perfectly competitive banking market. ZMDs are used for settling all transactions in the private sector, except for transactions between households/bankers and c-retailers. When the market between bankers and households open, household can convert ZMDs to currency or currency to ZMDs. They are insured by the central bank, so they are totally safe.

(c) **Reserve** (n_t): is a type of e-money issued by the central bank for only bankers. It has the same unit of account with currency. The central bank pays the gross interest rate R_t^n for these reserves. Interest on reserves R_t^n is a monetary policy tool of the central bank. At any moment, bankers can convert these reserves to currency and pay households. Reserves are used for settling transactions between bankers and bankers, bankers and the consolidated government.

Transactions with currency are simple. However, transactions with zero maturity deposits relates to many parties. We repeat the example in Ngotran (2017) to illustrate the connection between the flows of ZMDs and reserves. If wholesale firm A (whose account at bank A) pays 1 dollars to household B (whose account at bank B), the flows of payment will follow Table 2.

Wholesale firm A	
Deposit: -1	Payable:-1

Household B	
Deposit: +1	
Receivable:-1	

Bank A	
Reserve: -1	Deposit: -1

Bank B	
Reserve: +1	Deposit: +1

The Central Bank	
	Reserve (bank A): -1
	Reserve (bank B):+1

Table 2: Electronic Payment System

2.6 Central Bank and Monetary Policy

The central bank always uses the electronic payment system to conduct monetary policy. Each period, the central bank transfers τ_t dollars in checks to households[3]. For any transactions between the central bank and households, as the payments are conducted through the banking system, we should think that they contain two sub-transactions: one between the central bank-bankers is settled by reserves, one between bankers-households is settled by ZMDs.

The Central Bank		Banks		Public	
	Reserves:$+\tau_t$	Reserves:$+\tau_t$	Deposits: $+\tau_t$	Deposits:$+\tau_t$	Net worth: $+\tau_t$
	Net worth:$-\tau_t$				

Table 3: Helicopter Money / Lump-sum tax

2.7 Timing within one period

(i) Production takes place. Households sell goods to wholesalers, who, in turn, sell goods to retailers. All of the payments between households-wholesalers, wholesalers-retailer are delayed until the step (v).

[3] This can be seen as a shortcut of an open market operation process when the central bank purchases government bonds from the government. Then the government transfers the payoffs to households. When τ_t is negative, it is equivalent to a lump-sum tax

(ii) The cash-good market opens. Households need cash-in-advance to purchase from c-retailers. Bankers can convert reserves to cash to purchase from c-retailers.

(iii) The loan market between households and bankers opens. All the debt payments and loan issuance will be conducted electronically. The government transfers money to households. Households cannot exchange cash and deposits in this step.

(iv) The deposit-good market opens. Households need ZMD-in-advance to purchase goods from d-retailers. Bankers can create ZMDs to purchase d-goods.

(v) Payments in the non-bank private sector are settled. Profits from firms are transfered back to households under either form of cash or ZMDs. Households can go to banks and readjust their portfolio between cash and deposits.

(vi) The banking market opens. Bankers can adjust the level of reserves by borrowing in the interbank market, receiving new deposits.

3 Agents' Problems

3.1 Bankers

There is a measure one of identical bankers in the economy. These bankers have to follow the central bank's regulations. There are three types of assets on a banker's balance sheet: reserves (n_t), loans to households (b_t^h), loans to other bankers (b_t^f). His liability side contains the zero-maturity deposits that households deposit here (m_t).

Banker	
Reserves: n_t	Zero Maturity Deposits: m_t
Loans to households: b_t^h	Net worth
Loans to other bankers: b_t^f	

Cost: The banker faces a cost of managing loan, which is θb_t^h in terms of deposit-goods.

The banker can adjust the level of his deposits and reserves after households and firms pay each other or when households withdraw currency from bank account. When these happen, the banker can witness that the deposits and reserves outflow from or inflow to his bank. Let e_t be the net inflow of deposits and reserves go into his bank, he will treat e_t as exogenous. When the banking market opens, as the deposit market is perfectly competitive, he can choose any amount d_t of deposit inflows or outflows to his bank.

In each period, the banker treats all the prices as exogenous and chooses $\{c_t, c_{1,t}, c_{2,t}, n_t, b_t^h, s_t, m_t, b_t^f, d_t\}$ to maximize his utility over a stream of consumption:

$$\max \sum_{t=0}^{\infty} \beta^t \log(c_t)$$

subject to

$$\frac{R_{t-1}^n n_{t-1}}{\pi_t} + \frac{R_{t-1}^f b_{t-1}^f}{\pi_t} + d_t + e_t + \tau_t = n_t + b_t^f + c_{1,t} \quad \text{(Reserve Flows)} \tag{1}$$

$$m_t = \frac{R_{t-1}^m m_{t-1}}{\pi_t} + q_t^L s_t + \theta b_t^h - \delta_b \frac{b_{t-1}^h}{\pi_t} + c_{2,t} + d_t + e_t + \tau_t \quad \text{(Deposit Flows)} \tag{2}$$

$$b_t^h = (1-\delta_b)\frac{b_{t-1}^h}{\pi_t} + s_t \quad \text{(Loan Flows)} \tag{3}$$

$$c_t = \left[\sum_{i=1}^{2} \alpha_i^{\frac{1}{\sigma}} c_{i,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \quad \text{(Consumption)} \tag{4}$$

$$n_t \geq \varphi m_t \quad \text{(Reserves Requirement)} \tag{5}$$

$$n_t + b_t^f + b_t^h - m_t \geq \kappa_t b_t^h \quad \text{(Capital Requirement)} \tag{6}$$

The equation (1) shows the change in reserves in the banker's balance sheet. After receiving the IOR, the previous balance of reserves becomes $R_{t-1}^n n_{t-1}/\pi_t$. He also collects the payment from the interbank loans he lends out to other bankers in the previous period $R_{t-1}^f b_{t-1}^f/\pi_t$. He can also increase his reserves by taking more deposits d_t. When doing that, his reserves and his liability increase by the same amount d_t (Table 4). That is the reason we see d_t appear on both the equation (1) and (2). The similar effect can be found on τ_t- helicopter money and e_t. The banker treats τ_t and e_t exogenously. Then, he can leave reserves n_t at the central bank's account to earn interest rate, or lend reserves to another bankers b_t^f. To purchase the cash-goods $c_{1,t}$

Banker	
Reserves: $+d_t$	Deposits: $+d_t$

Banker	
Reserves: - b_t^f	
Interbank loan: + b_t^f	

Table 4: The banker takes more deposits (left) and makes interbank loan (right)

Central Bank	
	Reserves: $-c_{1,t}$
	Currency: $+c_{1,t}$

Banker	
Reserves: $-c_{1,t}$	Net worth: $-c_{1,t}$
(Vault Cash)	

c-Retailers	
Currency: $+c_{1,t}$	
Inventory: $-c_{1,t}$	

Table 5: Banker buys goods from c-retailers

Banker	
	Deposits: $+(c_{2,t}+\theta b_t^h)$
	Net worth: $-(c_{2,t}+\theta b_t^h)$

d-Retailers	
Deposits: $+(c_{2,t}+\theta b_t^h)$	
Inventory: $-(c_{2,t}+\theta b_t^h)$	

Table 6: Banker buys goods from d-retailers

from c-retailers, he converts reserves into currency (Table 5)[4].

The equation (2) shows the change in the banker's deposits. He makes loans to households by issuing deposits or creating ZMDs (Table 1)[5]. The banker also issues his own ZMDs to purchase the consumption good from d-retailers ($c_{2,t}$) and to pay for the cost (in terms of deposit-goods) related to lending activities (θb_t^h) (Table 6). It is noted that he cannot create infinite amount of money for himself to buy consumption goods as there exists the capital requirement. Even without the capital requirement, because deposits are bankers' debts, the No-Ponzi condition is enough to prevent that from happening.

The banker faces two constraints in every period. At the end of each period, he has to hold enough reserves as a fraction of total deposits, which is showed in the inequation (5).

The second constraint is the capital requirement constraint. The left hand side of (6) is the

[4]During one period, his reserves balance can go temporary negative. But in the end of every period, it must be positive and satisfies the regulation. Hence, the constraint in purchasing cash-goods implicitly lies in the reserve requirement

[5]It is assumed that households have to pay loans from the account at the bank they borrow. So if they want to use money from account at bank B to pay for loans from bank A, they need to transfer deposits from bank B to bank A first. In fact, this assumption does not matter in equilibrium.

banker's net worth (capital), which is equal to total assets minus total liabilities[6]. The constraint requires the banker to hold capital greater than a fraction of total loans in his balance sheet. We assume that κ_t is a constant κ in normal times. We later put the unexpected shock on this κ_t to reflect the shock in a banking crisis.

Let γ_t, μ_t^r and μ_t^c be respectively the Lagrangian multipliers attached to the reserves flows, reserves constraint and the capital constraint. Let r_t^h be defined as the real short-term lending rate. The first order conditions of the banker's problem can be written as:

$$\gamma_t = \left(\frac{\alpha_i c_t}{c_{i,t}}\right)^{1/\sigma} \frac{1}{c_t}, \quad i = 1,2 \tag{7}$$

$$\gamma_t = \frac{\beta R_t^f \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c \tag{8}$$

$$\gamma_t = \frac{\beta R_t^m \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c + \varphi \mu_t^r \tag{9}$$

$$\gamma_t = \frac{\beta R_t^n \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c + \mu_t^r \tag{10}$$

$$(q_t^l + \theta)\gamma_t = \frac{\beta[\delta_b + (1-\delta_b)q_{t+1}^l]\gamma_{t+1}}{\pi_{t+1}} + (1-\kappa)\mu_t^c \tag{11}$$

$$r_t^h = \frac{\delta_b + (1-\delta_b)q_{t+1}^l}{(q_t^l + \theta)\pi_{t+1}} \tag{12}$$

And two complimentary slackness conditions:

$$\mu_t^r \geq 0, \quad n_t - \varphi m_t \geq 0, \quad \mu_t^r (n_t - \varphi m_t) = 0 \tag{13}$$

$$\mu_t^c \geq 0, \quad n_t + b_t^f + (1-\kappa_t)b_t^h - m_t \geq 0, \quad \mu_t^c \left(n_t + b_t^f + (1-\kappa_t)b_t^h - m_t\right) = 0 \tag{14}$$

3.2 Households

There is a measure one of identical households. These self-employed households produce the homogeneous intermediate good y_t^m to sell to the wholesale firms at the price P_t^m. In each period, a household consumes the cash-goods ($\tilde{c}_{1,t}$) from c-retailers and the deposit-goods ($\tilde{c}_{2,t}$) from

[6]We use the book value B_t^h rather than the "market value" of loans $q_t^l B_t^h$ in the capital constraint. The reason is that illiquid bank loans should be treated differently from bonds. In reality, bank loans are often not revalued in the balance sheet when the interest rate changes.

d-retailers.

Let $\tilde{B}_t^h$ be the nominal debt stock that she borrows from bankers. The loan structure follows the description in Table (1). There is an exogenous borrowing constraint for households with the debt limit $\tilde{b}_t^h \leq \overline{b^h}$.

In each period, households choose $\{\widetilde{c}_t, l_t, a_t, x_t, \widetilde{b}_t^h, \widetilde{m}_t, \widetilde{s}_t, \tilde{c}_{1,t}, \tilde{c}_{2,t}\}$ to maximize their expected utility:

$$\max \sum_{t=0}^{\infty} \widetilde{\beta}^t \left(\log(\tilde{c}_t) - \chi l_t \right)$$

subject to

$$\text{CIA:} \qquad \tilde{c}_{1,t} \leq \frac{x_{t-1}}{\pi_t} \tag{15}$$

$$\text{Loan Market:} \qquad a_t + \delta_b \frac{\widetilde{b}_{t-1}^h}{\pi_t} = \frac{R_{t-1}^m m_{t-1}}{\pi_t} + q_t^l s_t + \tau_t \tag{16}$$

$$\text{DIA:} \qquad \tilde{c}_{2,t} \leq a_t \tag{17}$$

$$\text{Budget:} \qquad \tilde{m}_t + x_t + \tilde{c}_{1,t} + \tilde{c}_{2,t} = a_t + \frac{x_{t-1}}{\pi_t} + p_t^m y_t^m + \frac{\Pi_t}{P_t} \tag{18}$$

$$\text{Production:} \qquad y_t^m = l_t \tag{19}$$

$$\text{Loan flows:} \qquad \widetilde{b}_t^h = (1-\delta_b)\frac{\widetilde{b}_{t-1}^h}{\pi_t} + \tilde{s}_t \tag{20}$$

$$\text{Constraint:} \qquad \widetilde{b}_t^h \leq \overline{b^h} \tag{21}$$

$$\text{Consumption:} \qquad \tilde{c}_t = \left[\sum_{i=1}^{2} \alpha_i^{\frac{1}{\sigma}} \tilde{c}_{i,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{22}$$

When the cash-good market opens, the household brings (x_{t-1}/π_t) in cash to make transactions there. She faces the cash-in-advance (15) constraint when purchasing goods from c-retailers.

The loan market between bankers and households (16) only opens after that. Here the household pays a fraction of her old debts ($\delta_b b_{t-1}^h/\pi_t$) and borrows new loan ($q_t^l s_t$). All of the transactions are conducted electronically. We have assumed that she cannot readjust her portfolio between cash and deposits in this step. In the end, she brings a_t amount of ZMDs to purchase goods from d-retailers.

The equation (18) is the household's general budget constraint. After receiving the profits (Π_t/P_t) from wholesalers and revenue ($p_t^m y_t^m$) from selling the intermediate good, she can go to banks and readjust her portfolio between deposits (m_t) and currency (x_t).

Let $\eta_{1,t}$, $\eta_{2,t}$, η_t^b be the Lagrangian for the cash-in-advance, the deposit-in-advance and the borrowing constraint. Let λ_t be the Lagrangian for the budget constraint.

$$\lambda_t + \eta_{i,t} = \left(\frac{\alpha_i \tilde{c}_t}{\tilde{c}_{i,t}}\right)^{1/\sigma} \frac{1}{\tilde{c}_t}, \qquad i = 1,2 \tag{23}$$

$$p_t^m \lambda_t = \chi \tag{24}$$

$$\lambda_t = \frac{\widetilde{\beta}(\lambda_{t+1} + \eta_{1,t+1})}{\pi_{t+1}} \tag{25}$$

$$\lambda_t = \frac{\widetilde{\beta} R_t^m (\lambda_{t+1} + \eta_{2,t+1})}{\pi_{t+1}} \tag{26}$$

$$q_t^l(\lambda_t + \eta_{2,t}) = \frac{\widetilde{\beta}[\delta_b + (1-\delta_b)q_{t+1}^l](\lambda_{t+1} + \eta_{2,t+1})}{\pi_{t+1}} + \eta_t^b \tag{27}$$

And three complimentary slackness conditions:

$$\eta_{1,t} \geq 0, \quad \frac{x_{t-1}}{\pi_t} - \tilde{c}_{1,t} \geq 0, \quad \eta_{1,t}\left(\frac{x_{t-1}}{\pi_t} - \tilde{c}_{1,t}\right) = 0 \tag{28}$$

$$\eta_{2,t} \geq 0, \quad a_t - \tilde{c}_{2,t} \geq 0, \quad \eta_{2,t}(a_t - \tilde{c}_{2,t}) = 0 \tag{29}$$

$$\eta_t^b \geq 0, \quad \overline{b^h} - b_t^h \geq 0, \quad \eta_t^b\left(\overline{b^h} - b_t^h\right) = 0 \tag{30}$$

3.3 Retail Firms and Wholesale Firms

Following Rotemberg pricing, each wholesale firm j faces a cost of adjusting prices, which is measured in terms of final good and given by:

$$\frac{\iota}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi}\right)^2 y_t$$

where ι determines the degree of nominal price rigidity and $\overline{\pi}$ is the long-run inflation target. The wholesale firm j discounts the profit in the future with rate $\widetilde{\beta}^i \lambda_{t+i}/\lambda_t$. Her real marginal cost is p_t^m.

In a symmetric equilibrium, all firms will choose the same price and produce the same quantity $P_t(j) = P_t$ and $y_t(j) = y_t = y_t^m$. The optimal pricing rule then implies that:

$$1 - \iota\left(\pi_t - \overline{\pi}\right)\pi_t + \iota\widetilde{\beta}\left[\frac{\lambda_{t+1}}{\lambda_t}\left(\pi_{t+1} - \overline{\pi}\right)\pi_{t+1}\frac{y_{t+1}}{y_t}\right] = (1 - p_t^m)\varepsilon \tag{31}$$

3.4 The Central Bank and Government

The consolidated government uses the payoffs from tax or their assets to pay for the IOR, then injects (withdraws) $\hat{\tau}_t$ amount of money to (from) households to target the interbank rate. All transactions are conducted in the electronic system.

$$\tau_t = -\frac{(R_{t-1}^n - 1)n_{t-1}}{\pi_t} + \hat{\tau}_t \tag{32}$$

In the conventional monetary policy, we assume that the IOR R_t^n is fixed at a constant level $\overline{R^n}$. The interbank rate follows a common Taylor rule. To connect with the common New Keynesian literature, we assume that the central bank do not want to have excess reserves in the banking system so they never set R_t^f lower than $\overline{R^n} + \delta_f$ where $\delta_f > 0$[7]. Later, we relax the assumption and examine the situation when the banking system is awash of excess reserves and the central bank controls the interbank rate by adjusting R_t^n.
The conventional monetary policy rule can be described as:

$$R_t^n = \overline{R^n} \tag{33}$$

$$R_t^f = \max\left\{\frac{\bar{\pi}}{\beta}\left(\frac{\pi_{t+1}}{\bar{\pi}}\right)^{\phi_\pi}, \quad \overline{R^n} + \delta_f\right\} \tag{34}$$

4 Equilibrium

Definition: *A perfect foresight equilibrium is a sequence of bankers' decision choice* $\{c_t, c_{i,t}, n_t, b_t^h, s_t, m_t, b_t^f, d_t\}$*, household's choice* $\{\widetilde{c}_t, \tilde{c}_{i,t}, \widetilde{b}_t^h, \widetilde{s}_t, \widetilde{m}_t, x_t, l_t, y_t^m\}$*, the firms' choice* $\{y_t\}$*, the*

[7] When the reserve requirement is no longer binding, there are infinite levels of reserves that can satisfy the interbank rate at its lower bound. In this case, we need a rule governing the motion of reserves and change the standard Taylor Rule.

central bank' choice $\{\tau_t,\ R_t^n\}$*, and the market price* $\{q_t^l, R_t^f, \pi_t, p_t^m\}$ *such that:*

i Given the market price, the initial conditions and the central bank's choices, banker's choices solve the banker's problem, household's choices solve the household's problem, firm's choice solves the firm's problem.

ii All markets clear:

$$\text{Net inflows of deposits:} \qquad d_t + e_t = -(x_t - \frac{x_{t-1}}{\pi_t} - c_{1,t}) \tag{35}$$

$$\text{The interbank market:} \qquad b_t^f = 0 \tag{36}$$

$$\text{Total ZMDs:} \qquad m_t = \widetilde{m}_t$$

$$\text{Loan Market:} \qquad b_t^h = \widetilde{b}_t^h$$

$$\text{Good Market:} \qquad y_t = \sum_{i=1}^{2}(c_{i,t} + \tilde{c}_{i,t}) + \theta b_t^h + \frac{\iota}{2}(\pi_t - \overline{\pi})^2 y_t \tag{37}$$

Later, we set some different central bank's monetary polices subject to a set of equations in this perfect foresight equilibrium. In each case, we might also change the set of the central bank's monetary policy tools. For convenience, we define **(AD)** as the set of equations in the perfect foresight equilibrium, excluding the monetary policy and exogenous shocks.

Definition: *Let* **(AD)** *contain the set of equations and conditions in (C.1)-(C.23).*

5 Theoretical Results

We make the following assumption to ensure that in the steady state, households borrow from bankers.

Assumption 1. *The discount factors of bankers and households satisfy:*

$$\frac{\beta\delta_b - \theta\overline{\pi}}{\overline{\pi} - \beta(1-\delta_b)} > \frac{\widetilde{\beta}\delta_b}{\overline{\pi} - \widetilde{\beta}(1-\delta_b)}$$

The next assumption ensures that in the steady state, inflation is equal to the central bank's inflation target.

Assumption 2. *The monetary policy tools satisfy:*

$$\lim_{t\to\infty} \frac{\hat{\tau}_t}{n_t + x_t} = \frac{\overline{\pi} - 1}{\overline{\pi}}$$
$$\overline{R^n} + \delta_f < \frac{\overline{\pi}}{\beta}$$

We start with the first result showing the relationship between the interest on reserves, the deposit rate and the interbank rate.

Theorem 1. *In equilibrium:*

i The lower bound of the interbank rate and the deposit rate is the interest on reserves. In all cases, $R^n_t \leq R^m_t \leq R^f_t$

ii When the constraint of reserve requirement is not binding, $R^f_t = R^m_t = R^n_t$.

The benefits of holding reserves come from two sources. First, bankers earn the interest on reserves. Second, bankers satisfy the reserve requirement, showing in the shadow price of reserve constraint $\mu^r_t \geq 0$. When the banking system has a huge amount of excess reserves, second benefit is no longer there $\mu^r_t = 0$, and the interbank rate is equal to the interest on reserves.

Theorem 2. *In equilibrium, the level of the monetary base, as the sum of reserves and currency in circulation, is decided solely by the central bank:*

$$\frac{n_{t-1} + x_{t-1}}{\pi_t} + \hat{\tau}_t = n_t + x_t \tag{38}$$

When households withdraw currency from their bank accounts, it only changes the level of reserves but does not affect the level of monetary base. We assume that the central bank will target the interbank rate, so it implies that the central bank will never leave the banking system with the negative amount of reserves.

Theorem 3. *Under the Assumption (1)-(2) and if* κ *satisfies:*

$$\kappa < 1 - \frac{(1-\varphi)\overline{m}}{\overline{b^h}}$$

where $\overline{m}$ is the steady state value of m, then there exists a unique steady state. Moreover, in this steady state, the reserve requirement is binding while the capital constraint is not binding.

This unique steady state reflects well the banking system in the US before the Great Recession. There were no excess reserves and the federal funds rate was around 4 percent. The central bank's main tool was open market operations at that time, rather than the IOR. After the Great Recession, due to many rounds of quantitative easing (unconventional monetary policy), the excess reserves skyrocketed. However, in the long term, the central bank has a plan to scale down its balance sheet's size to the level before the Great Recession, so this unique steady state might reflect well the long-term position of the central bank.

6 Numerical Experiments

6.1 Calibration

The time period is one quarter. Data are calibrated to match the US economy before the Great Recession. The bankers' discount factor is set to the standard value 0.99. The reserves requirement is calibrated to reflect the ratio between the total level of reserves and the total ZMDs. In Dec 2007, before the financial crisis, the total level of reserves was around 9 billion dollars. The total MZM (Money Zero Maturity) was approximately 8130 billion, 75-80 percent of which are checkable deposits, saving deposits and money market deposit accounts. The level of ZMDs was therefore 6000 billion, and the ratio between reserves and ZMD is 0.0015, which we round up to 0.002. The monitoring cost and the loan amortization factor are set exogenously. The risk weight is calibrated so that it satisfies the condition in the Theorem 3 to ensure the unique steady state. For $\kappa \geq 0.2$, the capital constraint is binding in the steady state. Therefore, we set κ at 0.18.

The consumption basket is calibrated to match the ratio between currency and ZMDs in the economy. In Dec 2007, the total level of currency was 760 billion. Judson (2012) estimates that more than half of US dollar bills are held overseas, so we end up with around 330 billion in currency. At the same time, ZMDs was 6000 billion, so currency accounts for approximately 6

Table 7: Parameter values

Param.	Definition	Value
Bankers		
β	Banker's discount factor	0.99
φ	The reserves requirement	0.002
κ	The risk weight	0.18
θ	The monitoring cost	0.0005
δ_b	Loan amortization	0.5
Households		
$\widetilde{\beta}$	Household's discount factor	0.985
χ	Relative Utility Weight of Labor	0.586
$\overline{b^h}$	The borrowing limit	1.5
Consumption Basket		
α_1	Share of cash goods in the basket	0.06
α_2	Share of deposit goods	0.94
σ	Elasticity of substitution between two goods	10
Firms		
ε	Elasticity of substitution of wholesale goods	4
ι	Cost of changing price	80
Central bank		
$\overline{\pi}$	Inflation long-run target	1
ϕ_π	Policy responds to inflation	1.25
$\overline{R^n}$	The constant IOR	1+0.25/400
$\overline{R^n} + \delta_f$	The lower bound for FFR	1+0.251/400

percent of the total money supply (MZM). We calibrate $\alpha_1 = 0.06$ and $\alpha_2 = 0.94$ to match with this fact. The elasticity of substitution between cash goods and deposit goods is set exogenously 10.

For the central bank's parameters, the only unusual parameter is the interest on reserves. We set it at 25 basis points and consider it as the lower bound for IOR at most cases in our quantitative exercises. All other parameters are in the standard range in the New Keynesian literature.

6.2 Shock on the interbank rate

The first numerical experiment is to examine the response of the economy when the central bank cuts the interbank rate. The list of equations for monetary policy and exogenous shock is:

$$
\begin{aligned}
R_t^f &= \max\left\{\frac{\overline{\pi}}{\beta}\left(\frac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}\exp(u_t^f), \quad \overline{R^n}+\delta_f\right\} \\
R_t^n &= \overline{R^n} \\
u_t^f &= \rho_f u_{t-1}^f, \quad u_0^f \text{ is given}
\end{aligned}
\tag{M1}
$$

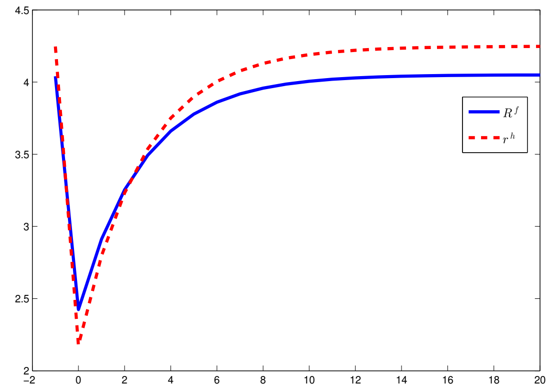
From the steady state, there is an unexpected shock on the interbank rate u_0^f, then agents know the shock will die slowly with the persistence ρ_f. These two parameters' values are in Table 8. We can summarize this problem as (**P1**) containing the set of conditions in **(AD)** and (**M1**). Figure 1 shows the response of the economy under this experiment.

When the central bank cuts the interbank rate by increasing the level of reserves, q^l increases and the real lending rate r^h is lower for households. As bankers lend out by creating money under the form of ZMDs, the money supply increases. It is noting that both currency and ZMD go up after this shock. The aggregate demand is stimulated and inflation goes up.

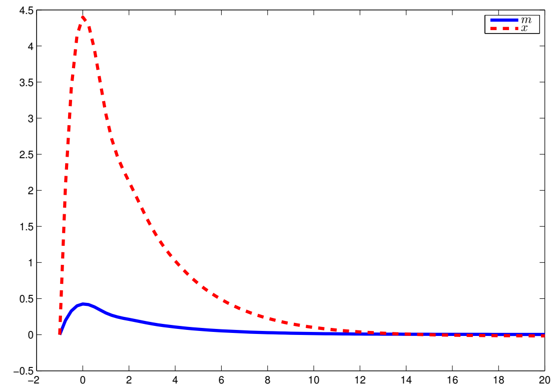
Basically, this is identical to the reaction in the standard New Keynesian model. The only key difference here is the role of commercial banks in creating money. Money supply is totally endogenous and depends on the interaction between the central bank, banks and the public. Another crucial point is that the effect of monetary policy, in this conventional setting, depends on the transmission from the interbank rate to the lending rate in the loan contract between bankers and households.

6.3 Financial Crisis - Taylor Rule

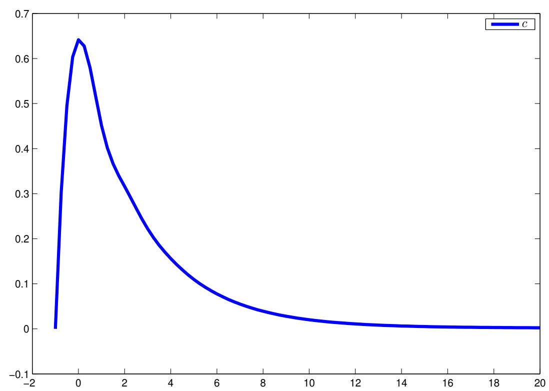
We examine a simple form of banking crisis by imposing an exogenous shock on κ_t, reflecting the increase in the bad loans that causes the capital constraint to bind. The central bank is



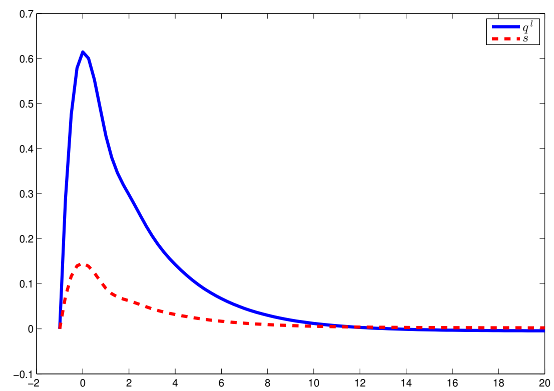
(a) Interbank rate and real lending rate



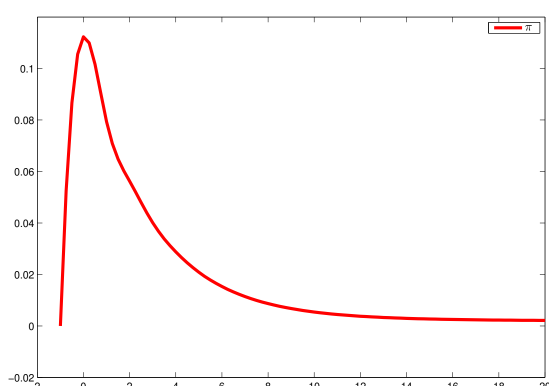
(b) Currency and Deposits



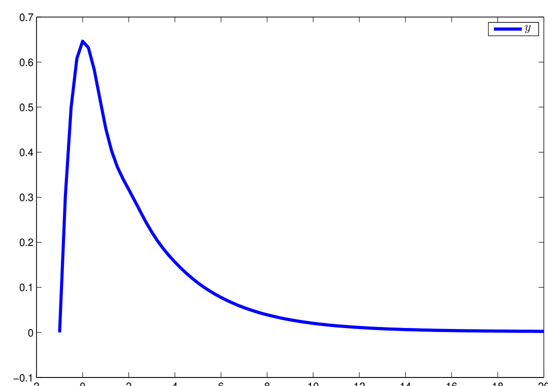
(c) Aggregate Consumption



(d) Loan price and New loan issuances



(e) Inflation



(f) Output

Figure 1: Impulse Response to Interest Rate Shock (**P1**)

assumed to respond to this crisis using a Taylor Rule.

$$
\begin{aligned}
R_t^f &= \max\left\{\frac{\overline{\pi}}{\beta}\left(\frac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}, \quad \overline{R^n}+\delta_f\right\} \\
R_t^n &= \overline{R^n} \\
\kappa_t &= \rho_\kappa \kappa_{t-1} + (1-\rho_\kappa)\overline{\kappa}, \quad \kappa_0 \text{ is given}
\end{aligned}
\tag{M2}
$$

From the steady state, there is an unexpected shock κ_0. After that, the shock dies with the persistence ρ_κ. These two parameters' values are reported in the Table 8. We can summarize this problem as (**P2**) containing the set of conditions in **(AD)** and (**M2**).

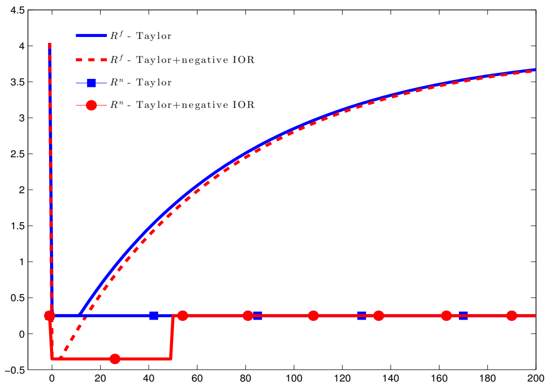
As the lower bound on the interbank rate is the IOR, the Taylor rule is constrained by the IOR. To see whether the negative IOR helps the central bank, we conduct another similar experiment but allow the interest on reserves is negative during T_e periods.

$$
\begin{aligned}
R_t^f &= \max\left\{\frac{\overline{\pi}}{\beta}\left(\frac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}, \quad \underline{R^n}+\delta_f\right\} \\
R_t^n &= \begin{cases} \underline{R^n}, & \text{if } t \le T_e \\ \overline{R^n}, & \text{if } t > T_e \end{cases} \\
\kappa_t &= \rho_\kappa \kappa_{t-1} + (1-\rho_\kappa)\overline{\kappa}, \quad \kappa_0 \text{ is given}
\end{aligned}
\tag{M3}
$$

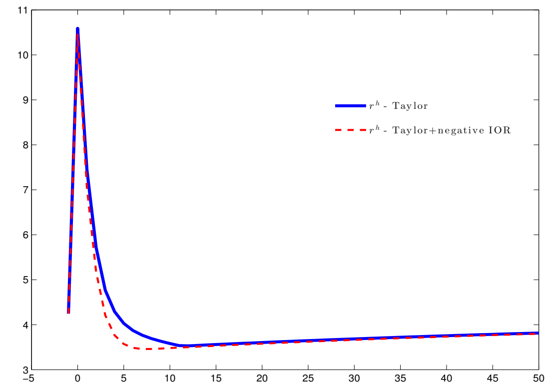
According to Bernanke (2016*a*), the Fed estimates that the interest rate paid on bank reserves in the U.S. could not practically be brought lower than about -0.35 percent to avoid the bank withdrawal. Hence, we set $\underline{R^n} = 1 - 0.35/400$ and $T_e = 50$. We can set the problem (**P3**) contains the equations in (**AD**) and (**M3**). Figure 2 shows the response of the economy under these two experiments. Here are some important remarks:

i. A Taylor rule with a negative IOR is more efficient than the one with zero lower bound. Rognlie (2015) gets the same result from a standard New Keynesian framework. However, the positive effect is very small at dealing with this type of financial crisis.

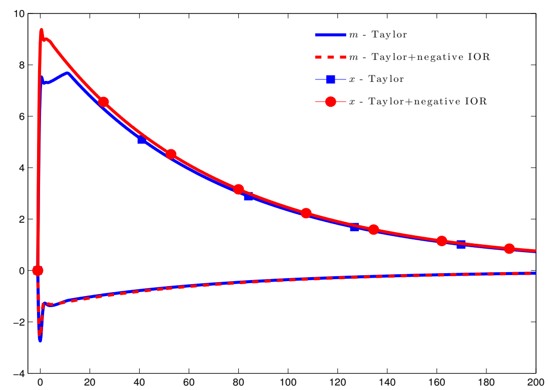
ii. The banking crisis is dangerous as the the central bank cannot rely on the pass-through from the interbank rate to the prime rate any more. In our simulation, the interbank rate is at its



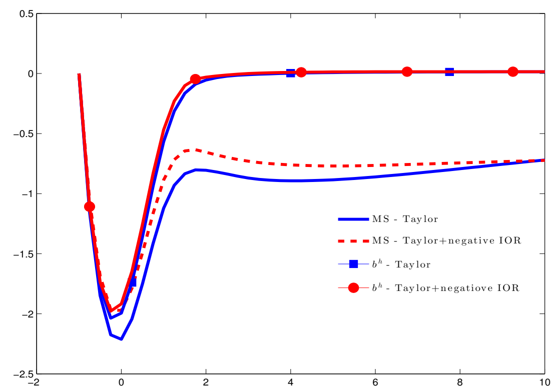
(a) Interbank rate and IOR



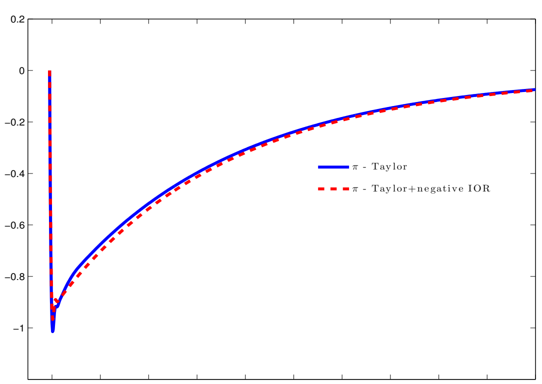
(b) Real lending rate- LR



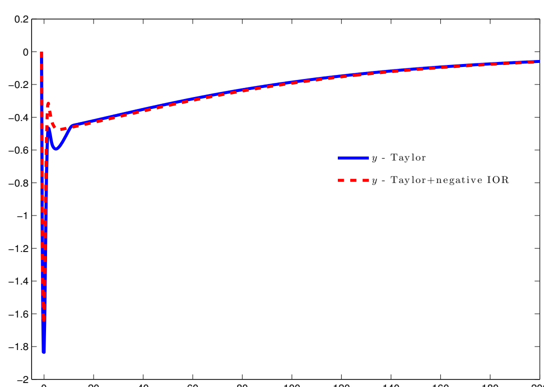
(c) Deposits and Currency



(d) Money Supply and Household Debt



(e) Inflation



(f) Output

Figure 2: Financial Crisis - Taylor Rule (**M2**) and (**M3**)

lower bound for 12 quarters with the IOR at 25 basis points and 4 quarters with the IOR at -35 basis points ; however, the real lending rate still goes up. When the capital constraint is binding $\mu_t^c > 0$, the wedge between the interbank rate and the prime rate must reflect this shadow price.

iii. The banking crisis is often accompanied by the deflation episode and the insufficient demand. Bankers cut loan; therefore, the total money supply plummets even though the monetary base increases. In our model, the level of currency goes up a little bit during the crisis. The deposit rate is near zero or even negative in our two experiments, making currency more favorable in households' eyes. However, this change does not affect much the total money supply because currency only accounts 6 percent of the total money supply in the steady state.

iv. Lacking liquidity, households cut their own consumption and output declines. A Taylor rule, even with a negative IOR, is not enough to stimulate the aggregate demand in this case. When the link between the interbank rate and the prime rate breaks, the conventional monetary policy is generally not effective.

6.4 Forward Guidance

The recent literature in monetary economics focuses on the forward guidance policy when interest policy is restricted by the zero lower bound. In this section, we do a simple experiment to see whether the forward guidance policy is useful in the banking crisis. There are two common ways to model how the central bank informs the public about the interest rate path in the future: (i) interest rate peg and (ii) news shock on the Taylor rule. We follow the latter in Keen, Richter and Throckmorton (2017*b*) to characterize the forward guidance as follows:

$$
\begin{aligned}
R_t^f &= \max\left\{\frac{\overline{\pi}}{\beta}\left(\frac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}\exp(\varepsilon_t), \quad \overline{R^n}+\delta_f\right\} \\
\varepsilon_t &= \begin{cases} \overline{\varepsilon}, & \text{if } t \leq T_{FG} \\ 0, & \text{if } t > T_{FG} \end{cases} \\
R_t^n &= \overline{R^n} \\
\kappa_t &= \rho_\kappa \kappa_{t-1} + (1-\rho_\kappa)\overline{\kappa}, \quad \kappa_0 \text{ is given}
\end{aligned}
\tag{\textbf{M4}}
$$

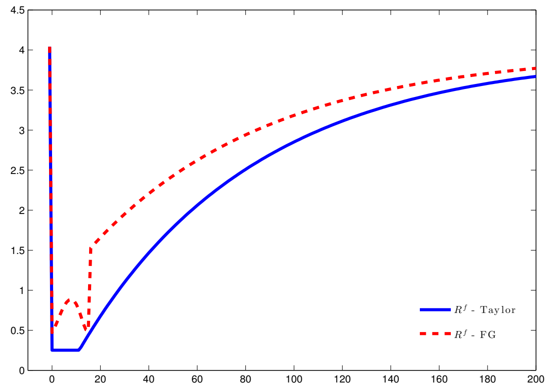
The central bank still follows the common Taylor rule. However, during the forward guidance period $0 \leq t \leq T_{FG}$, the central bank commits to lower the intercept of the Taylor rule by $\exp(\overline{\varepsilon})$. We set $\overline{\varepsilon} = -1/400$. In the previous experiment, when following the Taylor rule, the interbank rate is bounded by the IOR during the first 12 periods. Hence, we set the horizon for forward guidance as 4 years ($T_{FG} = 16$) to evaluate its efficacy in pushing up inflation.

The key channel that forward guidance affects the real economy is through increasing the expected inflation. Hence, it lowers the real short-term interbank rate, which in turn passes through to the real lending rate. In comparison to a common Taylor rule, forward guidance is much more effective at pushing up the expected inflation. Like all monetary models with the forward looking feature, path of inflation affects the current activities.

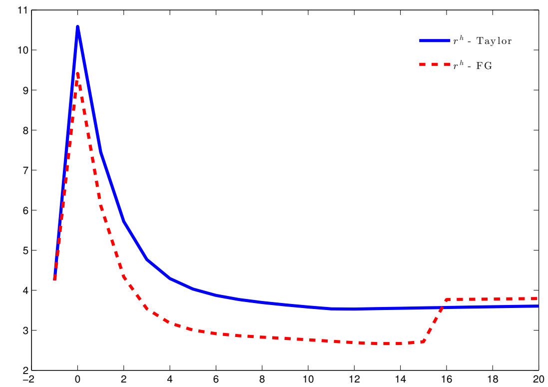
The effectiveness of forward guidance policy depends mostly on how far households look forward in the future. In our model, if the horizon of forward guidance is around 5 years, this policy cannot push up inflation. As banks cut loans due to the capital constraint, inflation should be also consistent with the decline in the money supply path.

6.5 Financial Crisis - Mixed Rule

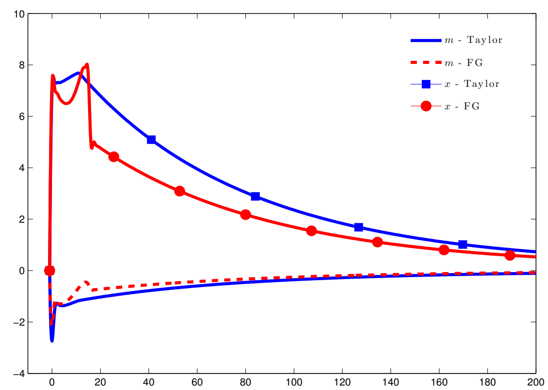
The previous section shows that a monetary policy targeting only the interbank rate is not efficient to deal with the banking crisis. What can the central bank do to improve the situation? If the problem is a lack of liquidity in the private sector when banks cut loans, a natural guess should be a policy of targeting the money supply directly. In this section, we examine a mod-



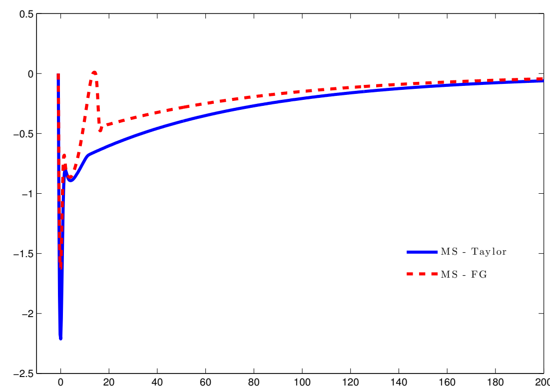
(a) Interbank rate



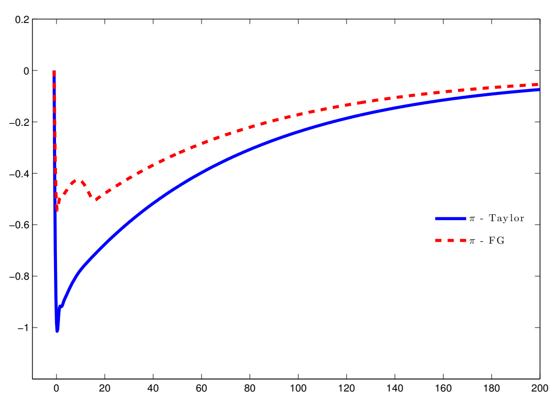
(b) Real lending rate



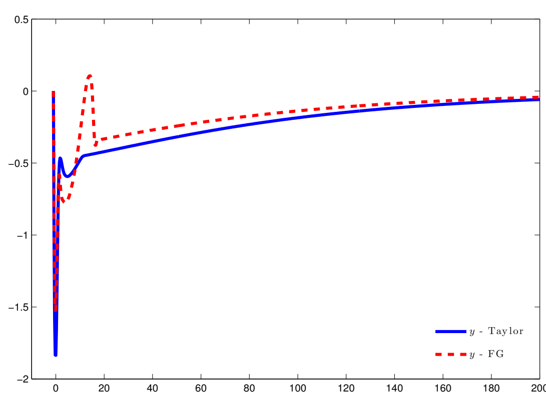
(c) Deposits and Currency



(d) Money Supply



(e) Inflation



(f) Output

Figure 3: Financial Crisis - Forward Guidance

ification of a Taylor rule. In normal times, the central bank still targets the interbank rate by a Taylor rule. However, in crises, when the interbank rate is pushed down to the level of IOR and the deflation is still severe, the central bank will switch to target the growth of money supply $m_t^s = m_t + x_t$. All the exogenous shocks and monetary policy can be described by the following system of equations:

$$\begin{aligned} R_t^f &= \frac{\overline{\pi}}{\beta}\left(\frac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}\left(\frac{m_t^s}{m_{t-1}^s}\right)^{\phi_m} \\ R_t^n &= \overline{R^n} \\ \kappa_t &= \rho_\kappa \kappa_{t-1} + (1-\rho_\kappa)\overline{\kappa}, \quad \kappa_0 \text{ is given} \end{aligned} \tag{M5}$$

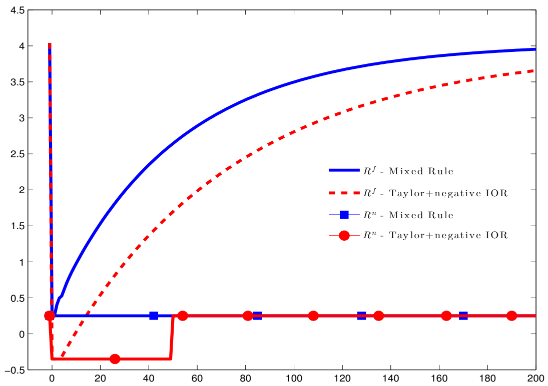
where $\phi_m = 0.2$ measures the reaction of the interbank rate to the growth of the money supply. The problem (**P5**) is defined to contain the equations in (**AD**) and (**M5**).

We set ϕ_m small relative to ϕ_π for two reasons. First, it means that in normal times, the interbank rate is not influenced much by the growth rate of the money supply. Second, it implies that, in crises, when the interbank rate is not enough for raising inflation, the central bank will respond aggressively by raising the money supply through helicopter money. Why? We can rewrite the modified Taylor rule as:

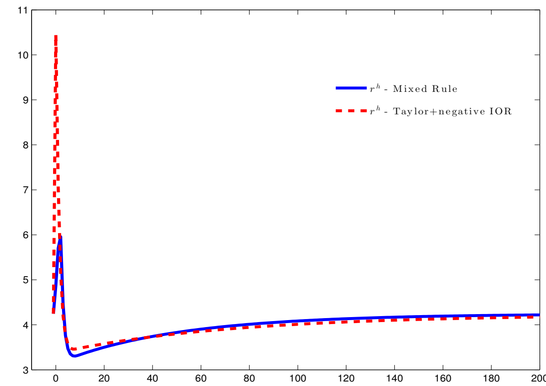
$$\frac{m_t^s}{m_{t-1}^s} = \left(\frac{R_t^f \beta}{\overline{\pi}}\right)^{1/\phi_m}\left(\frac{\overline{\pi}}{\pi_{t+1}}\right)^{\phi_\pi/\phi_m}$$

When the interbank rate is at its lower bound $R_t^f = R_t^n = \overline{R^n}$, it means that ϕ_π/ϕ_m shows how aggressively the central bank will raise the money supply to deal with inflation. Figure 4 shows the economy's response under this modified Taylor rule. Here are some important remarks:

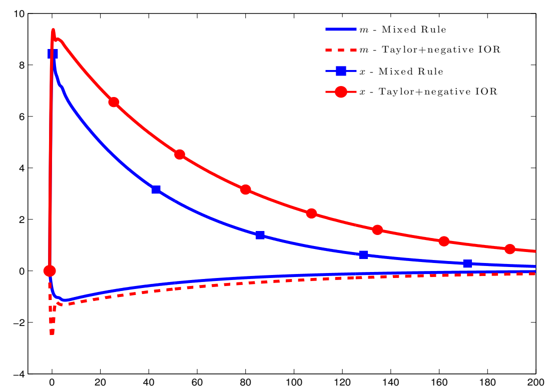
i. In this banking crisis, when targeting the interbank rate is not efficient anymore, switching to target the growth rate of the money supply is very effective. Both inflation and output paths in this experiment are smoother and less volatile than a Taylor rule with a negative IOR. A modified Taylor rule can anchor inflationary expectations better; thereby restricting the increase in the real lending rate. This result is similar to Christiano and Rostagno (2001). Their research also shows that when inflation is out of a bounded region, switching from a Taylor rule to target the growth rate of the money supply can reduce the volatility of the



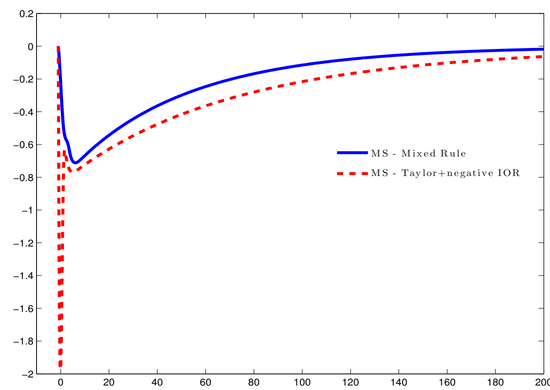
(a) Interbank rate and IOR



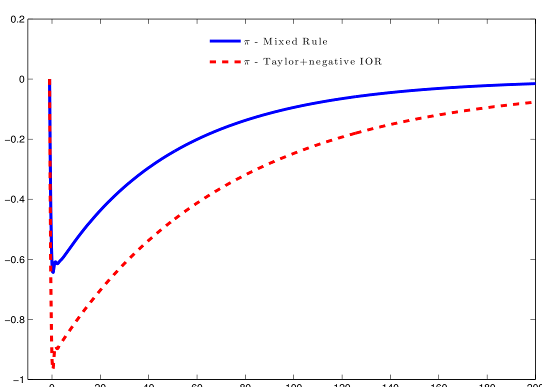
(b) Real lending rate



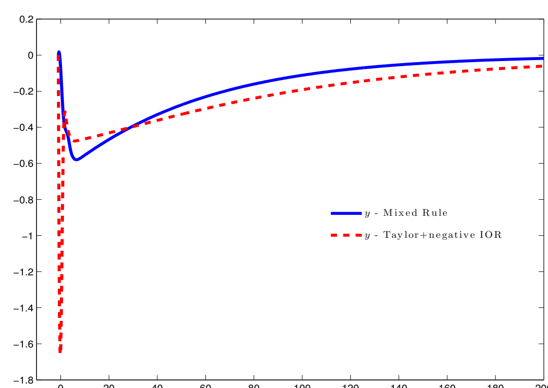
(c) Deposits and Currency



(d) Money Supply



(e) Inflation



(f) Output

Figure 4: Financial Crisis - Mixed Rule

economy.

ii. The interest rate path alone does not reflect the stance of the monetary policy in a banking crisis. If we only look at the paths of the interbank rate, a Taylor rule with a negative IOR keeps the interbank rate not only longer at the lower bound but also lower in every period in comparison to a mixed rule in this section (Figure 4a). If we follow the common New Keynesian logic, inflation should have been higher in the previous experiment. However, this is not the case here. When we model explicitly the microfoundation in the banking sector, the link between the money supply and the interest rate is not as tight as the one in the New Keynesian literature. Money supply is not determined solely by the central bank. Moreover, the central bank do not control the interest rate by directly changing the money supply here.

iii. inflationary expectation is anchored by both the money supply and the interest rate. Seemly, money supply is a more credible signal for the inflation path in banking crises.

6.6 Taylor Rule and Friedman's k-percent Rule

The new tool IOR allows the central bank to target both the interest rate and money supply at the same time. In this section, the central bank is assumed to follow the Friedman's k-percent rule during our crisis. The Friedman's k-percent rule indicates that the growth of the money supply is fixed at a constant level:

$$\frac{M_t}{M_{t-1}} = \overline{\pi}$$

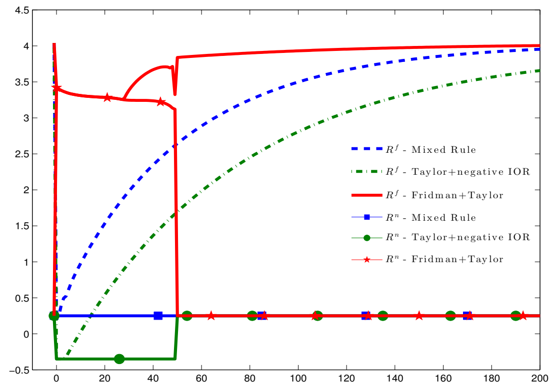
At the same time, IOR is adjusted following a Taylor rule. The set of monetary policies and the exogenous shock can be written as:

$$
\begin{aligned}
&\begin{cases}
\dfrac{m_t}{m_{t-1}} = \dfrac{\overline{\pi}}{\pi_t}, & \text{if } t \le T_e \\
R_t^f = \max\left[\dfrac{\overline{\pi}}{\beta}\left(\dfrac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}, \quad \overline{R^n} + \delta_f\right], & \text{if } t > T_e
\end{cases} \\
&R_t^n = \begin{cases}
(1-\alpha_n)\overline{R^n} + \alpha_n \dfrac{\overline{\pi}}{\beta}\left(\dfrac{\pi_{t+1}}{\overline{\pi}}\right)^{\phi_\pi}, & \text{if } t \le T_e \\
\overline{R^n}, & \text{if } t > T_e
\end{cases} \\
&\kappa_t = \rho_\kappa \kappa_{t-1} + (1-\rho_\kappa)\overline{\kappa}, \quad \kappa_0 \text{ is given}
\end{aligned}
\tag{M6}
$$

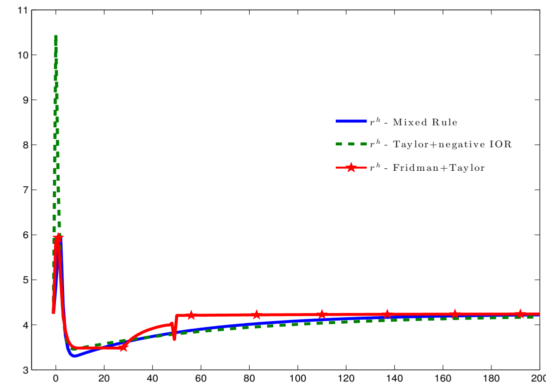
where $\alpha_n = 0.8$ and $T_e = 50$. Together with the equations in (**AD**), (**M6**) sets up the problem (**P6**). This experiment can be summarized as followings. Before time T_e, the central bank targets both the IOR and the money supply by, respectively, a Taylor rule and the Friedman's k-percent rule. After time T_e, the central bank comes back to its Taylor rule and only targets the interbank rate. Figure 5 compares the effect of monetary polices in (**P6**) with the previous experiment. Here are some important remarks:

i. Targeting both the money supply and the interest rate is extremely efficient. The inflation rate is nearly anchored at the target level for the whole time. As our model does not have any real rigidities, it implies the output is also at the steady state level.

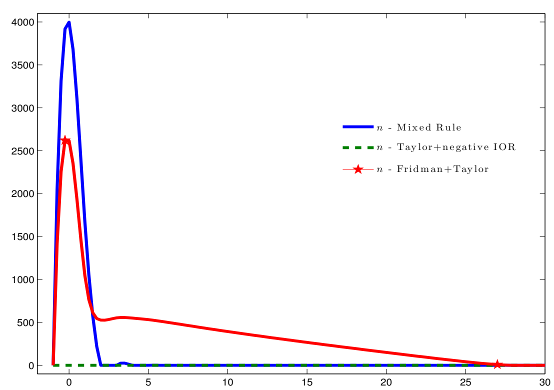
ii. The obvious byproduct of targeting the growth of money supply directly is clearly the sharp increase of reserves and excess reserves. Reserves increase by 25 times in our model and the reserves requirement is no longer binding for 25 periods. Many economists worry that a huge amount of excess reserves might prevent the effectiveness of the monetary policy or create hyperinflation. Our model shows these concerns have no foundations. By using the IOR, the central bank can control the interbank rate. The effect is very similar to the one when the central bank adjusts by using open market operations. There are also no reasons to believe the huge amount of reserves will create a huge amount of money supply. When the reserves requirement is no longer binding, we cannot use the logic in the money



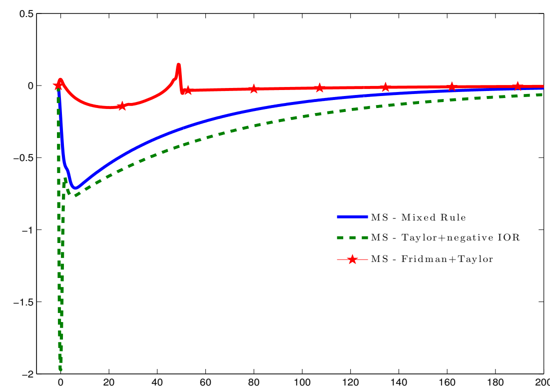
(a) Interbank rate and IOR



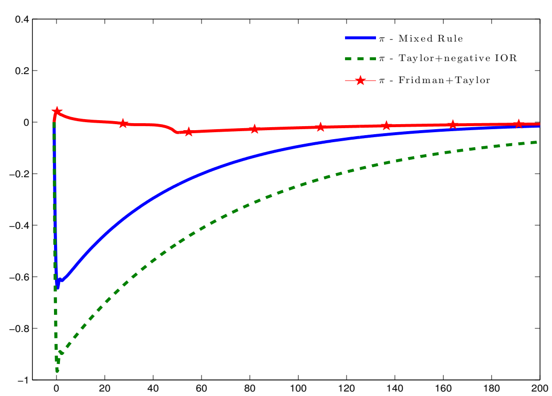
(b) Real lending rate



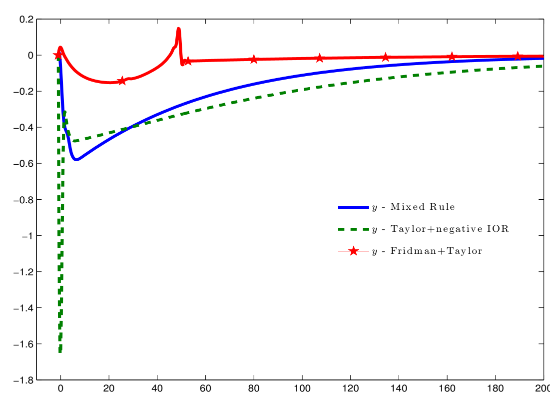
(c) Reserves



(d) Money Supply



(e) Inflation



(f) Output

Figure 5: Financial Crisis - a Taylor Rule and the Friedman's k-percent Rule

multiplier model to create the link between the monetary base and money supply anymore. Until there is still a borrowing constraint and capital requirement, the endogenous money supply is always bounded. Furthermore, the inflation, in the long run, is always determined by the central bank.

iii. Once again, we emphasize that the stance of monetary policy can only be judged when we observe both the nominal interbank rate, the money supply and the real short-term rate. The money supply and the interest rate can move in any directions. It can be the case that the central bank increases the money supply and the IOR at the same time. It might be a serious mistake to infer that it would cause a deflation.

iv. The above point raises an important issue about the central bank's communication. In reality, the stance of the monetary policy, when sending to the public, is often summarized by only one indicator: the interbank rate. Indeed, this is a common and good practice as the interbank rate is a unique short-term target that the central bank controls completely. In normal times, it is a good predicator of inflation path. However, the current situation is very tricky. The interbank rate in most developed countries has been near zero for a long time and the inflation is persistently lower than its target. The growth rate of the money supply should be included as part of the central bank's communication with the public.

7 Conclusion

With a huge amount of excess reserves in the banking systems, IOR is now the most crucial tool for the Fed. This tool opens a new opportunity for the conduct of monetary policy as the central bank can target both the interest rate and the growth rate of the money supply at the same time. Of course, in normal times, adjusting the interbank rate alone is always timely and much more transparent than targeting the money supply, which is not entirely controlled by the central bank. However, in banking crises, our research shows that the link between the interbank rate and the lending rate is very weak. If the central bank simultaneously targets both the interbank rate and the money supply, they can hit the inflation target.

References

Afonso, Gara, and Ricardo Lagos. 2015. “Trade Dynamics in the Market for Federal Funds.” *Econometrica*, 83: 263–313.

Belongia, Michael T., and Peter N. Ireland. 2006. “The Own-Price of Money and the Channels of Monetary Transmission.” *Journal of Money, Credit and Banking*, 38(2): 429–445.

Belongia, Michael T., and Peter N. Ireland. 2014. “The Barnett critique after three decades: A New Keynesian analysis.” *Journal of Econometrics*, 183(1): 5–21.

Benigno, Pierpaolo, and Salvatore Nisticò. 2017. “Safe Assets, Liquidity, and Monetary Policy.” *American Economic Journal: Macroeconomics*, 9(2): 182–227.

Bernanke, Ben. 2016*a*. “What tools does the fed have left? Part 1: Negative interest rates.” *Brookings Institution (blog)*.

Bernanke, Ben. 2016*b*. “What Tools Does the Fed Have Left? Part 2: Targeting Longer-Term Interest Rates.” *Brookings Institution (blog)*.

Bernanke, Ben. 2016*c*. “What tools does the Fed have left? Part 3: Helicopter money.” *Brookings Blogs*, 11–4.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. 1999. “The financial accelerator in a quantitative business cycle framework.” In *Handbook of Macroeconomics*. Vol. 1 of *Handbook of Macroeconomics*, , ed. J. B. Taylor and M. Woodford, Chapter 21, 1341–1393. Elsevier.

Bianchi, Javier, and Saki Bigio. 2014. “Banks, Liquidity Management and Monetary Policy.” National Bureau of Economic Research, Inc NBER Working Papers 20490.

Campbell, Jeffrey R, Charles L Evans, Jonas DM Fisher, and Alejandro Justiniano. 2012. “Macroeconomic effects of Federal Reserve forward guidance.” *Brookings Papers on Economic Activity*, 2012(1): 1–80.

Christiano, Lawrence J, and Massimo Rostagno. 2001. “Money growth monitoring and the Taylor rule.” National Bureau of Economic Research.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2004. “The Great Depression and the Friedman-Schwartz hypothesis.” Federal Reserve Bank of Cleveland Working Paper 0318.

Cochrane, John H. 2014. “Monetary policy with interest on reserves.” *Journal of Economic Dynamics and Control*, 49: 74–108.

Curdia, Vasco, and Michael Woodford. 2011. “The central-bank balance sheet as an instrument of monetarypolicy.” *Journal of Monetary Economics*, 58(1): 54–79.

Del Negro, Marco, Marc P Giannoni, and Christina Patterson. 2012. “The forward guidance puzzle.”

Eggertsson, Gauti B, et al. 2003. "Zero bound on interest rates and optimal monetary policy." *Brookings papers on economic activity*, 2003(1): 139–233.

Friedman, Milton. 1960. "A Program for Monetary Stability."

Gertler, Mark, and Nobuhiro Kiyotaki. 2010. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*. Vol. 3. 1 ed., , ed. Benjamin M. Friedman and Michael Woodford, Chapter 11, 547–599. Elsevier.

Gertler, Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics*, 58(1): 17–34.

Goodfriend, Marvin, et al. 2002. "Interest on reserves and monetary policy." *Federal Reserve Bank of New York Economic Policy Review*, 8(1): 77–84.

Ireland, Peter N. 2014. "The macroeconomic effects of interest on reserves." *Macroeconomic Dynamics*, 18(6): 1271–1312.

Judson, Ruth. 2012. "Crisis and calm: Demand for U.S. currency at home and abroad from the fall of the Berlin Wall to 2011." Board of Governors of the Federal Reserve System (U.S.) International Finance Discussion Papers 1058.

Kashyap, Anil K, and Jeremy C Stein. 2012. "The optimal conduct of monetary policy with interest on reserv." *American Economic Journal: Macroeconomics*, 4(1): 266–282.

Keen, Benjamin D, Alexander W Richter, and Nathaniel A Throckmorton. 2017*a*. "Forward Guidance and the State of the Economy." *Economic Inquiry*.

Keen, Benjamin D, Alexander W Richter, and Nathaniel A Throckmorton. 2017*b*. "Forward Guidance and the State of the Economy." *Economic Inquiry*.

Keister, Todd. 2016. "Interest on Reserves." In *Testimony before the Subcommittee on Monetary Policy and Trade*. Committe of Financial Services, United States House of Representatives, May 17.

Leland, Hayne, and Klaus Bjerre Toft. 1996. "Optimal Capital Structure, Endogenous Bankruptcy, and the Term Structure of Credit Spreads." *Journal of Finance*, 51(3): 987–1019.

Levin, Andrew T, David López-Salido, Edward Nelson, and Tack Yun. 2009. "Limitations on the effectiveness of forward guidance at the zero lower bound."

Lucas, Robert E, Jr, and Nancy L Stokey. 1987. "Money and Interest in a Cash-in-Advance Economy." *Econometrica*, 55(3): 491–513.

McCallum, Bennet T. 1988. "Robustness properties of a rule for monetary policy." Vol. 29, 173–203, North-Holland.

McGrattan, Ellen R. 1996. "Solving the stochastic growth model with a finite element method." *Journal of Economic Dynamics and Control*, 20(1-3): 19–42.

McLeay, Michael, Amar Radia, and Ryland Thomas. 2014. "Money creation in the modern economy."

Meltzer, Allan. 1987. "Limits of Short-run Stabilization Policy: Presidential Address to the Western Economic Association, July 3, 1986." *Economic Inquiry*, 25(1): 1–14.

Ngotran, Duong. 2017. "The E-Monetary Theory." University Library of Munich, Germany MPRA Paper.

Rognlie, Matthew. 2015. "What lower bound? monetary policy with negative interest rates." *Unpublished manuscript, Department of Economics, Harvard University (November 23).*

Sargent, Thomas, and Neil Wallace. 1985. "Interest on reserves." *Journal of Monetary Economics*, 15(3): 279–290.

Schmedders, Karl, Kenneth L Judd, and Felix Kubler. 2002. "A solution method for incomplete asset markets wih heterogeneous agents."

Taylor, John B. 1993. "Discretion versus policy rules in practice." Vol. 39, 195–214, Elsevier.

Wachter, Andreas, and T. Lorenz Biegler. 2006. "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming." *Mathematical Programming*, 106(1): 25–57.

Zangwill, Willard I, and CB Garcia. 1981. *Pathways to solutions, fixed points, and equilibria.* Prentice Hall.

A Parameter values in experiments

Table 8

Param.	Definition	Value
*Experiment (**M1**)*		
u_0^f	Initial interest shock	-2/400
ρ_f	The persistence of the interest shock	0.7
*Experiment (**M2**)*		
κ_0	Initial shock on the risk weight	0.224
ρ_κ	The persistence of shock	0.95
*Experiment (**M3**)*		
$\underline{R^n}$	The negative lower bound for IOR	1-0.35/400
T_e	Number of periods $R_t^n = \underline{R^n}$	50
*Experiment (**M4**)*		
$\overline{\varepsilon}$	The forward guidance signal	-1/400
T_{FG}	Number of periods in forward guidance	16
*Experiment (**M5**)*		
ϕ_m	Coefficient in mixed rule	0.2
*Experiment (**M6**)*		
α_n	Coefficient in Taylor Rule	0.8
T_e	Number of periods targeting both MS and IR	50

B Mathematical Appendix

Proof for Theorem 1:

We rewrite the equation (8), (9) and (10):

$$\gamma_t = \frac{\beta R_t^f \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c \tag{B.1}$$

$$\gamma_t = \frac{\beta R_t^m \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c + \varphi \mu_t^r \tag{B.2}$$

$$\gamma_t = \frac{\beta R_t^n \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c + \mu_t^r \tag{B.3}$$

As μ_t^c and μ_t^r are non-negative and $\gamma_t > 0$, we have $R_t^n \leq R_t^m \leq R_t^f$.

The $'' ='' $ happens when $\mu_t^r = 0$, or when the reserver requirement is no longer binding.

Proof for Theorem 2:

Substitute (32), (36) and (35) into the reserve flows equation (1), we have:

$$\frac{n_{t-1} + x_{t-1}}{\pi_t} + \hat{\tau}_t = n_t + x_t$$

Proof for Theorem 3:

We omit the subscript "t" to denote the steady state value of a variable. Under the Assumption (2) and the result of the Theorem (2) $\Rightarrow \pi = \overline{\pi}$. Under the Assumption (2) and the Taylor rule (34) $\Rightarrow R^f = \overline{\pi}/\beta$. Replace this value of R^f into (8) $\Rightarrow \mu^c = 0$ (the capital requirement is not binding). From (10) and (9) $\Rightarrow R^m$; from (11) $\Rightarrow q^l$.

$$\frac{\mu^r}{\gamma} = 1 - \frac{\beta R^n}{\pi}$$

$$R^m = \left(1 - \varphi \frac{\mu^r}{\gamma}\right) \frac{\pi}{\beta}$$

$$q^l = \frac{\beta \delta_b - \theta \pi}{\pi - \beta(1 - \delta_b)}$$

$$\hat{\tau} = \frac{\pi - 1}{\pi}$$

Under the steady state, (31) $\Rightarrow p^m = (\varepsilon - 1)/\varepsilon$. Next we move on the household's equation and can find the steady state of:

$$
\begin{aligned}
&(24) \Rightarrow && \lambda = \frac{\chi}{p^m} \\
&(25) \Rightarrow && \eta_1 = \left(\lambda - \frac{\widetilde{\beta}\lambda}{\pi}\right)\frac{\pi}{\widetilde{\beta}} \\
&(26) \Rightarrow && \eta_2 = \left(\lambda - \frac{\widetilde{\beta}R^m\lambda}{\pi}\right)\frac{\pi}{\widetilde{\beta}R^m} \\
&(27) \Rightarrow && \eta^b = q^l(\lambda + \eta_2) - \frac{\widetilde{\beta}[\delta_b + (1-\delta_b)q^l](\lambda + \eta_2)}{\pi} > 0 \\
&(21) \Rightarrow && b^h = \overline{b^h} \\
&(3) \Rightarrow && s = \left(1 - \frac{1-\delta_b}{\pi}\right)b^h
\end{aligned}
$$

From (23), we can perform $\tilde{c}_i$ as a function of $\tilde{c}$:

$$
\tilde{c}_i = \frac{\alpha_i(\tilde{c})^{1-\sigma}}{(\lambda + \eta_i)^{\sigma}} \tag{B.4}
$$

Substituting (B.4) into the aggregate consumptions (22), we can find the steady state value of $\tilde{c}$, then $\tilde{c}_1$ and $\tilde{c}_2$:

$$
\tilde{c} = \left(\sum_{i=1}^{2} \frac{\alpha_i}{(\lambda + \eta_i)^{\sigma-1}}\right)^{\frac{1}{\sigma-1}}
$$

All the constraints (15), (17) are binding:

$$
x = \pi\tilde{c}_1
$$

$$
a = \tilde{c}_2
$$

From (5), (32) and (16), we can find m and n from the following equations:

$$
\begin{aligned}
&\frac{\left[R^m - (R^n - 1)\varphi\right]m}{\pi} = a + \frac{\delta_b b^h}{\pi} - \hat{\tau} - q^l s \\
&n = \varphi m
\end{aligned}
$$

Replacing that into the deposit flows:

$$\sum_{i=1}^{2} c_i = m - \left(\frac{R^m m}{\pi} + q^l s + \theta b^h - \frac{\delta_b b^h}{\pi} - x + \frac{x}{\pi} - \frac{(R^n - 1)n}{\pi} + \tau\right) \tag{B.5}$$

From(7), (4) and use (B.5):

$$\begin{aligned}
c_i &= \frac{\alpha_i c^{1-\sigma}}{\gamma^\sigma} \\
\frac{c^{1-\sigma}}{\gamma^\sigma}(\alpha_1 + \alpha_2) &= c_1 + c_2 \\
c_i &= \frac{\alpha_i (c_1 + c_2)}{\alpha_1 + \alpha_2} \\
c &= \left[\sum_{i=1}^{2} \alpha_i^{1/\sigma} c_i^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}$$

We know that this steady state only exists if κ satisfy the condition such that capital constraint is not binding, so we need the condition:

$$\kappa < 1 - \frac{(1-\varphi)m}{\overline{b^h}}$$

C System of Equations in Equilibrium

Bankers:

$$\gamma_t = \left(\frac{\alpha_i c_t}{c_{i,t}}\right)^{1/\sigma} \frac{1}{c_t}, \quad i = 1, 2 \tag{C.1}$$

$$\gamma_t = \frac{\beta R_t^f \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c \tag{C.2}$$

$$\gamma_t = \frac{\beta R_t^m \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c + \varphi \mu_t^r \tag{C.3}$$

$$\gamma_t = \frac{\beta R_t^n \gamma_{t+1}}{\pi_{t+1}} + \mu_t^c + \mu_t^r \tag{C.4}$$

$$(q_t^l + \theta)\gamma_t = \frac{\beta[\delta_b + (1-\delta_b) q_{t+1}^l]\gamma_{t+1}}{\pi_{t+1}} + (1-\kappa)\mu_t^c \tag{C.5}$$

$$\frac{n_{t-1}}{\pi_t} + \frac{x_{t-1}}{\pi_t} + \hat{\tau}_t = n_t + x_t \tag{C.6}$$

$$m_t = \frac{R^m_{t-1} m_{t-1}}{\pi_t} + q^L_t s_t + \theta b^h_t - \delta_b \frac{b^h_{t-1}}{\pi_t} + c_{2,t} + c_{1,t} - x_t + \frac{x_{t-1}}{\pi_t} - \frac{(R^n_{t-1} - 1) n_{t-1}}{\pi_t} + \hat{\tau}_t \tag{C.7}$$

$$0 \le \mu^r_t \perp (n_t - \varphi m_t) \ge 0 \tag{C.8}$$

$$0 \le \mu^c_t \perp \left(n_t + (1 - \kappa_t) b^h_t - m_t \right) \ge 0 \tag{C.9}$$

$$b^h_t = (1 - \delta_b) \frac{b^h_{t-1}}{\pi} + s_t \tag{C.10}$$

Households:

$$\lambda_t + \eta_{i,t} = \left(\frac{\alpha_i \tilde{c}_t}{\tilde{c}_{i,t}} \right)^{1/\sigma} \frac{1}{\tilde{c}_t}, \qquad i = 1, 2 \tag{C.11}$$

$$p^m_t \lambda_t = \chi \tag{C.12}$$

$$\lambda_t = \frac{\widetilde{\beta}(\lambda_{t+1} + \eta_{1,t+1})}{\pi_{t+1}} \tag{C.13}$$

$$\lambda_t = \frac{\widetilde{\beta} R^m_t (\lambda_{t+1} + \eta_{2,t+1})}{\pi_{t+1}} \tag{C.14}$$

$$q^l_t (\lambda_t + \eta_{2,t}) = \frac{\widetilde{\beta}[\delta_b + (1 - \delta_b) q^l_{t+1}](\lambda_{t+1} + \eta_{2,t+1})}{\pi_{t+1}} + \eta^b_t \tag{C.15}$$

$$0 \le \eta_{1,t} \perp \left(\frac{x_{t-1}}{\pi_t} - \tilde{c}_{1,t} \right) \ge 0 \tag{C.16}$$

$$0 \le \eta_{2,t} \perp \left(\frac{R^m_{t-1} m_{t-1}}{\pi_t} + q^l_t s_t + \hat{\tau}_t - \frac{(R^n_{t-1} - 1) n_{t-1}}{\pi_t} - \delta_b \frac{b^h_{t-1}}{\pi_t} - \tilde{c}_{2,t} \right) \ge 0 \tag{C.17}$$

$$0 \le \eta^b_t \perp \left(\overline{b^h} - b^h_t \right) \ge 0 \tag{C.18}$$

Firms:

$$1 - \iota \left(\pi_t - \overline{\pi} \right) \pi_t + \iota \widetilde{\beta} \frac{\lambda_{t+1}}{\lambda_t} \left(\pi_{t+1} - \overline{\pi} \right) \pi_{t+1} \frac{y_{t+1}}{y_t} = (1 - p^m_t) \varepsilon \tag{C.19}$$

$$y_t = l_t \tag{C.20}$$

Market Clearing:

$$y_t = \sum_{i=1}^{2} (c_{i,t} + \tilde{c}_{i,t}) + \theta b^h_t + \frac{\iota}{2} \left(\pi_t - \overline{\pi} \right)^2 y_t \tag{C.21}$$

$$c_t = \left[\sum_{i=1}^{2} \alpha_i^{\frac{1}{\sigma}} c_{i,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{C.22}$$

$$\tilde{c}_t = \left[\sum_{i=1}^{2} \alpha_i^{\frac{1}{\sigma}} \tilde{c}_{i,t}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{C.23}$$

Central bank:

$$R_t^f = \max \left\{ \overline{R^f} \left(\frac{\pi_{t+1}}{\pi} \right)^{\phi_\pi}, \quad \overline{R^n} + \delta_f \right\} \tag{C.24}$$

$$R_t^n = \overline{R^n} \tag{C.25}$$

D Numerical Method

D.1 Inequality Constraints

There are 6 occasionally binding inequality constraints in our model: the reserve requirement, the capital requirement, the deposit-in-advance, the cash-in-advance, the household's borrowing constraint and the Taylor rule of the central bank. We deal with these occasionally binding constraints by three methods:

Method 1: We apply the method in Zangwill and Garcia (1981) and Schmedders, Judd and Kubler (2002) to transform the complementary conditions into the equality constraints. For example, if we have the following complementary condition:

$$\mu_t f(x_t) = 0, \quad \mu_t \geq 0, \quad f(x_t) \geq 0$$

We create a new variable $\underline{\mu}_t$ and transforms the above conditions into two following equations:

$$\mu_t = \max\{\underline{\mu}_t, 0\}^3$$
$$f(x_t) = \max\{-\underline{\mu}_t, 0\}^3$$

We apply this method for the reserves requirement, deposit-in-advance and cash-in-advance constraints.

Method 2: For the capital requirement and the household's borrowing constraint, we apply the penalty method in McGrattan (1996) to avoid the ill-conditioned of the system and deal with occasionally binding constraints. So the utility of banker and the capital constraint will be changed as:

$$U = \log c_t - \frac{\rho_e}{4} \max\{\underline{\mu}_t^c, 0\}^4$$
$$n_t + b_t^f + (1-\kappa_t) b_t^h - m_t = -\underline{\mu}_t^c$$

where $\rho_e = 1e6$ is the penalty coefficient. When the capital constraint is violated, banker will lose the utility. However, when they get positive net worth, they do not get reward for that. The household's utility also is changed to deal with the borrowing constraint.

Method 3: For the Taylor rule of the central bank, we use the soft max constraint to deal with the lower bound on $R^f_{min} = \overline{R^n} + \delta_f$ so we can still take derivative to solve the system of equations:

$$u_t = \overline{R^f} \left(\frac{\pi_{t+1}}{\bar{\pi}}\right)^{\phi_\pi}$$
$$R_t^f = \begin{cases} u_t + \frac{\log(1+\exp(s_{max}(R^f_{min}-u_t)))}{s_{max}}, & \text{if } u_t \geq R^f_{min} \\ R^f_{min} + \frac{\log(1+\exp(s_{max}(u_t-R^f_{min})))}{s_{max}}, & \text{if } u_t < R^f_{min} \end{cases}$$

When $s_{max} \to \infty$, the soft max constraint converges to the hard max constraint. We choose the coefficient $s_{max} = 1e4$.

D.2 Dynamics of Economy

We solve the perfect foresight equilibrium with the unexpected shock by assuming that after $T = 300$ quarters, the economy will converge back to the initial steady state. The initial position before the unexpected shocks is the steady state. Basically, we need to solve a large system of equations to determine the dynamic path of the economy. The transform of occasionally inequality constraints in the previous section ensures that every equation is continuous and differentiable.

For every application, we use homotopy method for solving this large system of equation, with the initial point starting from the steady state or the previous result. We use Ipopt written by Wachter and Biegler (2006) with the linear solver HSL[8] to conduct homotopy.

In all cases, we can solve successfully the perfect foresight path with the accuracy at least

[8]HSL. A collection of Fortran codes for large scale scientific computation. http://www.hsl.rl.ac.uk/

$1e-9$. In some results, only at the last period $T-1$, we observe that there is a big switch from deposits to currency while the total money supply does not change much. When we increase the length of periods from $T=300$ to $T=1000$ the path does not change. We omit the result in the last period to ensure the smoothness of the result.