Freemium as Optimal Menu Pricing

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In online contents markets, content providers collect revenues from both consumers and advertisers by segmenting consumers who are willing to avoid advertisements and who are not. To analyze such situations, I construct a model of menu pricing by advertising platforms in two-sided markets. I find that, under certain condition, although a monopolistic platform can choose any menu of price-advertisement pairs, the optimal menu consists of only *two* services: ad-supported basic service and ad-free premium service. In addition, if the willingness to pay of advertisers is sufficiently high, the basic service is offered for free. This menu pricing is well known as *freemium*. Furthermore, this binary structure remains to hold an equilibrium menu pricing even under duopoly.
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1. Introduction

Freemium is a business model which is coined as a combination of the words *free* and *premium*. This word describes "a business model in which you give a core product away for free to a large group of users and sell premium products to a smaller fraction of this user base.[1]" The purpose of this paper is to show that this business model is optimal menu pricing for advertising platforms under certain conditions.

There are many instances of freemium in digital economy. As in Table 1, fair amount of major music- and video-streaming services adopt freemium business models. A prominent example of freemium business is Spotify, a music-streaming service with the largest

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[1]Freemium.org "What is Freemium?": http://www.freemium.org/what-is-freemium-2/

Service	Business Model
Spotify	Freemium
Apple Music	Subscription
Pandora	Freemium
Rhapsody	Subscription
Tidal	Subscription
Deezer	Freemium

Music-streaming services

Service	Business Model
YouTube	Freemium
Netflix	Subscription
Hulu	Subscription
Bing Videos	Ad-supported
Vimeo	Freemium
Daily Motion	Ad-supported

Video-streaming services

Table 1: Business models of major streaming services. Ad-supported business model offers free services to consumers and collect revenues from advertisers. Subscription business model offers paid services to consumers. The classification of business models is by my own.

market share in the world. Spotify offers two services, Free and Premium. In Free service, customers can shuffle several given playlists, with advertising audios interrupting in the time between songs. Customers who pay a monthly fee of $9.99 to subscribe Premium service can play any songs with better sound quality, create their original playlists, download musics, and listen offline, without being interrupted by advertisements. As another example of freemium, YouTube, a well-known ad-supported video-streaming platform, recently started to offer a paid and ad-free membership service, called YouTube Red. YouTube Red also has several additional functionalities such as saving videos on mobile devices or viewing original contents. Users of YouTube who want to avoid advertisements or get richer functionalities can upgrade their accounts to YouTube Red.

This type of business models can be seen as a class of second-degree price discrimination since the firm offers a menu of services (free and premium) and lets customers choose between them. A distinctive feature of this business model is that it uses the amount of advertisements as an instrument to screen customers.[2] Customers can choose ad-supported free service or ad-free premium service according to their nuisance from advertisements, and advertisers can show their advertisements only to free customers. Put differently, this is a price discrimination by a two-sided platform using the levels of interactions between agents on both sides as an instrument to price-discriminate.[3] This form of price discrimination is relatively new and thus have been subject to few research until recently.

There is a tradeoff when the platform uses the levels of interactions as instruments to price-discriminate. Consumers who want to enjoy contents without being annoyed by advertisements are willing to pay more to reduce the amount of advertisements. Thus, the platform can collect revenues from consumers by introducing a service with a fewer amount of advertisements and charging a higher fee. However, while offering a service with fewer advertisement may successfully collect revenues from consumers, this reduces

[2]There is another instrument of price discrimination and another form of freemium business model where free service and premium service differ in their intrinsic functionalities. Under this kind of business model, customers choose services according to their preference on the functionalities. This kind of freemium businesses include online applications (e.g., Evernote), publishers (IDES), massive open online courses (Coursera), and so on. As mentioned in Section 1.1, this kind of business models can be treated as versioning.

[3]For a brief review of the economics of two-sided markets, see Rysman (2009).

the revenue from advertisers as the total view of advertisements shrinks. Thus, the platform needs to take this tradeoff into account when they decide how to price-discriminate.

Treating freemium as a price discrimination by two-sided platforms, several questions arise; What is the optimal price discrimination for platforms? Is the freemium optimal price discrimination? Since platforms can potentially consider any nonlinear advertisement-price path to maximize its profit, and freemium is just one special class of such a price discrimination, it is natural to think that there would be better ways to collect revenues for the platforms. On the flip side of the coin, for freemium to be optimal, this must be superior to any other candidate menus platforms can design.

To answer these questions, I construct a model of menu pricing problem of advertising platforms in two-sided markets where consumers are annoyed by advertisements and advertisers benefit from listing advertisements. The platform potentially can offer any menu of services which specify the pairs of amount of advertisements and fixed fees.

My main result (Proposition 2.1) shows that *the optimal menu pricing should be binary* under certain conditions. More precisely, I show that, under the linear specification, which is commonly adopted in the literature of advertising platforms, the monopolistic platform optimally offers only *two* services: basic service with full advertisements and premium service with no advertisements. This menu pricing segments consumers into two groups: those who contribute to the platform's revenue by paying premium fees and those who contribute by viewing advertisements, leaving no intermediate segment of consumers. In fact, this simple segmentation is optimal when consumer nuisance from advertisements is linear, and the platform successfully collect revenues from both consumers and advertisers. Furthermore, if the advertisers' benefit from transaction is sufficiently high relative to the intrinsic value of the platform's service, then the basic service becomes free (Proposition 2.2). In this case, the optimal menu is literally freemium.

Then, I analyze several properties of optimal menu pricing. First, I examine welfare properties of binary menu pricing. I show that profit-maximizing price for advertisers is too high and the amount of consumers who view advertisements is too small in terms of social welfare (Proposition 2.3). As a result, the size of advertising network is too small relative to the social optimum. Next, I compare the binary menu with another business model called ad-supported business model. I find that, the platform provides more advertisers and less consumers who view advertisements under the binary menu pricing (Proposition 2.3). This difference stems from the difference in the appropriability of surplus from consumers who avoids advertisements. Under ad-supported business model, these consumers do not participate in the platform, and the platform collects no revenues . On the other hand, a platform which adopts binary menu can collect revenues from these consumers by providing ad-free services. This generates the incentive toward reducing (increasing) the amount of consumers who view (do not view) advertisements. This in turn reduces the average nuisance of consumers who view advertisements, and thus platform increases the amount of advertisers.

I also examine whether the binary structure remains to be valid under different situations. The property that platforms offer only two services remains to be valid under a duopoly situation (Proposition 3.1). In addition, if the advertisers' benefit from transaction is sufficiently high relative to the degree of product differentiation (or transportation cost) of the platform's service, then the basic services become free In this sense, my main result that freemium is an equilibrium, is robust to the competition.

In summary, the results in my paper provide economic foundations for the prevalence of freemium business models; once we accept the linear environment, offering only two free and premium services is actually the best strategy for platforms among a number of alternatives.

The rest of the paper proceeds as follows. In the next subsection, I review the related literature. Section 2 presents a model of menu pricing by a monopoly platform and main results. Section 3 presents a doupoly extension, and Section 4 concludes.

1.1. Related Literature

There are three groups of research related to my paper: literature on price discrimination, two-sided markets, and freemium.

Price Discrimination There is a huge literature on second-degree price discrimination (e.g., Mussa and Rosen (1978), Maskin and Riley (1984)). Papers related to my model are those which focus on the optimality of price discrimination, or versioning (e.g., Salant (1989), Deneckere and McAfee (1996), Varian (1997), Jing (2007), and Anderson and Dana (2009)).

Salant (1989) examines the condition under which second-degree price discrimination is suboptimal, and Deneckere and McAfee (1996) examine the profitability of price discrimination through the introduction of "damaged goods". These analyses are further developed by Varian (1997) and Anderson and Dana (2009). A common finding in these papers is that, if consumer preference and production cost are linear in quality, inducing self-selection through price discrimination will never be optimal. Intuition behind this result is that, when consumer preference and quality cost is linear in quality, marginal profit of increasing the quality for each consumer is constant and it is optimal to increase the quality as long as possible if the marginal profit is positive and not to provide the goods if the marginal profit is negative. Then, the monopolist optimally segments consumers into those who use the service and those who are excluded from the service. This segmentation is achieved by the uniform monopoly pricing on the good with highest quality.

Contrary to these research I show that even under the linear environment, price-discrimination is optimal for the platform. Intuition behind this result is the following. In two-sided markets, each customer has its "consumer value" that reflects the willingness to pay for the service, and "input value" that reflects the profit from procuring the consumer to list the advertisements. To exploit these values at the same time, the platform optimally offers two services.

One exception which obtains a similar result is Jing (2007). He examines a linear environment as specified in Salant (1989), except that there are direct network externalities. He shows that, when there are network externalities, then it is optimal for the firm to offer two products which consist of a good with lowest possible quality and with highest possible quality. One difference between Jing (2007) and this paper is that, while he considers the menu pricing with direct network externalities in one-sided markets, I analyze the properties of optimal menu pricing inherent to the two-sidedness of markets. This difference in environments leads to the different behavior of optimal menu pricing and derives different implications.[4]

[4]Another technical difference is that, while Jing (2007) requires some exogenous bounds on the possible

Two-sided markets and advertising platforms My model is based on the framework of two-sided markets (e.g., Rochet and Tirole (2003), Armstrong (2006), Weyl (2010)).

There is a burgeoning literature on nonlinear-pricing or price discrimination in two-sided markets (e.g., Bedre-Defolie and Calvano (2013), Choi et al. (2015), Jeon et al. (2016)). In the sense that a platform uses transaction as an instrument to price-discriminate, the model of Gomes and Pavan (2016) is the closest to mine. They consider a price-discrimination by a many-to-many matching platform where each agent is characterized by vertical types. They show the conditions under which the optimal mechanism will be a threshold rule and analyze the properties of optimal mechanisms. In this respect, they treat the broader range of environments than mine. On the other hand, by focusing on simpler environment, my model provides a tractable setting which enables more detailed analyses on the properties of optimal menu pricing, especially related to freemium. In addition, my result that freemium is adopted by platforms is robust to the competition, which is not easy to show in Gomes and Pavan's mechanism design framework.

There is also a literature on the behavior of advertising platforms in the framework of two-sided markets (e.g., Gabszewicz, Laussel, and Sonnac (2004), Anderson and Coate (2005), Anderson and Gabszewicz (2006), Peitz and Valletti (2008)). Anderson and Coate (2005) find that advertising platforms always underprovides advertisements in terms of social welfare when they can charge prices to consumers. My result is consistent with their result. The platform also underprovides advertisements under freemium. In addition, I show that the amount of advertisements is larger and the number of consumers who view advertisements is smaller under freemium than under ad-supported model. In this respect, I show that freemium alleviates the underprovision of advertisements to consumers, while it exacerbates the underprovision of consumers to advertisers.

Freemium The word freemium is disseminated by Anderson (2009). In the area of management, Eisenmann et al. (2011) examine the Dropbox's business model as a case study of freemium business.

In economics, there are a few studies on freemium which focus on the role of combating piracies and exploiting network externalities (e.g., Halmenschlager and Waelbroeck (2014), Nan et al. (2016)). There are few on freemium as a price discrimination by two-sided platforms. One exception is Zennyo (2016). Using a similar approach, he analyzes the behaviors of freemium pricing by duopoly advertising platforms. While his main focus is on the behavior of equilibrium pricing *given* that platforms adopt freemium, my focus is on the optimality of freemium in a broader class of selling procedures. My result shows that freemium is also optimal price discrimination and an equilibrium price discrimination. Thus, this paper contributes to the research on freemium pricing in a different way.

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qualities to derive the qualities of two goods, these bounds are endogenously determined by the platform in two-sided markets. First, the platform cannot assign the amount of advertisements less than zero, which gives the upper bound on the "quality" in my model. Second, the platform also cannot assign the amount of advertisements more than the amount of advertisers who actually participate the platform, which gives the lower bound on the quality. Finally, the amount of advertisers who participate is determined by the platform. These factors endogenize the bound on possible qualities.

2. Monopoly Menu Pricing

In this section, I present a model of monopoly menu pricing by an advertising platform. I show that freemium, providing only two basic and premium services, is optimal menu pricing for the platform. Then, I examine the behavior of the freemium pricing and its welfare properties. Finally, I compare freemium with ad-supported business model.

2.1. Model

The model consists of three groups of agents: a monopolistic *platform* which operates an advertising space, a unit mass of *consumers*, and a unit mass of *advertisers* who may potentially participate in the platform. Main features of this model are that (i) a platform can offer a menu of services which specify the intended amount of advertisements and fixed fees, (ii) consumers derive utilities from an intrinsic value of services, but incur nuisance cost from interactions with advertisers, and (iii) advertisers benefit from interactions with consumers.

Platform The platform can offer a menu of services and an advertising space to potential consumers and advertisers. A *menu* $M \equiv (m_k)_{k=0}^{K} \in \mathbb{R}_+^{2(K+1)}$ is a profile of $(K+1)$ services. For each $k = 0, 1, \ldots K$, the k-th *service* $m_k \equiv (\overline{a}_k, p_k)$ specifies a pair of an *intended amount of advertisers* $\overline{a}_k \in \mathbb{R}_+$ on that service and a *fixed fee* $p_k \in \mathbb{R}_+$ for using the service. The restriction that $p_k \in \mathbb{R}_+$ means that there is nonnegativity constraint in the consumer price. The platform also charges *per-transaction fees* $p_a \in \mathbb{R}$ to advertisers. Figure 1 shows the flow of transactions.

Let $a \in \mathbb{R}_+$ be the *total amount of advertisers* who participate the platform and a_k be the *actual amount of advertisers* on k-th service. I assume that

$$a_k = \min\{\overline{a}_k, a\} \text{ for each } k. \tag{1}$$

This assumption means that the actual amount of advertisers a_k who transact with consumers on each service cannot exceed the total amount of advertisers a who participate in the platform.[5] If $\overline{a}_k \leq a$, then the platform can assign the intended amount without problems.

Let $d_k \in \mathbb{R}_+$ denote the amount of consumers who choose m_k. The amount of transaction under k-th service is given by $d_k a_k$. Under this setting, the *total amount of transaction* T is given by

$$T = \sum_{k=0}^{K} d_k a_k.$$

The platform who offers M and p_a earns revenue Π from (i) fixed fees for each services from consumers, and (ii) transaction fees from advertisers, which can be expressed as

$$\Pi = \sum_{k=0}^{K} d_k p_k + T p_a = \sum_{k=0}^{K} d_k (p_k + a_k p_a).$$

[5] This assumption corresponds to the reciprocity condition in the model of Gomes and Pavan (2016).

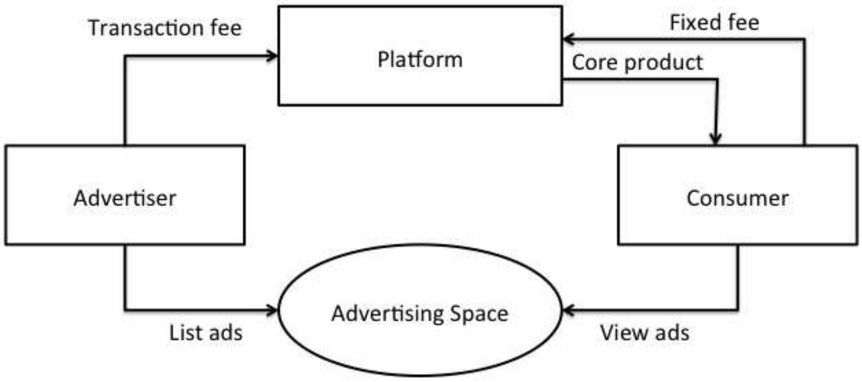

Figure 1: Flow of Transactions

Consumers Consumers obtain an intrinsic value $v \in \mathbb{R}_+$ from participating in the platform, which is independent of services chosen. For simplicity, I assume that v is the same among consumers.

I also assume that consumers are annoyed by the presence of advertisements, and these nuisance from advertisements are heterogeneous among consumers. In particular, each consumer incurs a *nuisance cost* $\tilde{c}$ per transaction with advertisers, which is privately known by the consumer and follows a strictly increasing, continuously differentiable distribution function F on $[0, C]$ with the density function f. I assume that $\frac{F(\tilde{c})}{f(\tilde{c})}$ is increasing in $\tilde{c}$. The specification that consumer nuisance cost is linear in the amount of advertisement is commonly adopted in the literature of advertising platforms (e.g., Anderson and Gabszewicz (2006)) and I follow this convention.

Imposing the quasi-linearity assumption, the utility of consumer with type $\tilde{c}$ who chooses k-th service can be expressed as $v - \tilde{c}a_k - p_k$.[6] Normalizing the value of outside option to zero, we can write the utility function of consumer with type $\tilde{c}$ as follows:

$$U(\tilde{c}) = \begin{cases} v - \tilde{c}a_k - p_k & \text{if } m_k \text{ is chosen,} \\ 0 & \text{if none is chosen.} \end{cases}$$

Finally, each consumer has a unit demand and chooses the alternative that gives the greatest utility.

[6]This specification implicitly assumes that each consumer correctly forms the expectation over the realization of a_k.

Advertisers Advertisers are heterogeneous in their per-transaction benefit b, which reflects an expected profit from consumers they transact with, and is privately known by the advertiser. I assume that b follows a strictly increasing, continuously differentiable distribution function G on $[0, B]$ with the density function g, and that $\frac{1-G(b)}{g(b)}$ is decreasing in b. For simplicity, I also assume that advertisers only differ in per-transaction benefits and that there is no benefit from just participating in the platform.

Each advertiser is equally assigned with the advertisement spaces, which means that each advertiser transacts with $\frac{T}{a}$ consumers on average. Thus, given the total amount of advertisers a, the total amount of transaction T, and the per-transaction fee p_a, the payoff of advertiser with type b is given by

$$(b - p_a)\frac{T}{a}.$$[7]

We can see that an advertiser with type b participates the platform if and only if $b \geq p_a$. Thus, the demand for the advertisement space is given by

$$a = 1 - G(p_a). \tag{2}$$

Note that, since G is strictly increasing, we can invert G and write p_a as $G^{-1}(1 - a)$.

Timing Timing is as follows.

1. Platform chooses M and p_a.
2. Observing M and p_a, advertisers decide whether to participate in the platform. At the same time, consumers decide which service to choose or not to participate in the platform, following the correct expectation on the amount of advertisers.
3. All outcomes realize.

2.2. Profit Maximization

Given the setting in the previous subsection, consider the profit maximization problem of the platform.

First, note that the choice variables $\overline{a}_k$ for $k = 0, \ldots, K$ and constraint (1) can be replaced by a_k for $k = 0, \ldots, K$ and the constraint

$$a_k \leq a \quad \text{for each } k = 0, \ldots, K, \tag{3}$$

[7] This is a natural extension of standard two-sided markets literature. For example, in the model of Rochet and Tirole (2003), the amount of transaction T between the amount d of consumers and the amount a of advertisers is given by

$$T = da.$$

In this case, the benefit function of advertisers in my model can be written as

$$(b - p_a)\frac{T}{a} = (b - p_a)d,$$

which is the same as the benefit function presented in Rochet and Tirole (2003).

since the platform can realize any $a_k \leq a$ by choosing the same value of $\overline{a}_k$. Hence, I consider the platform's problem as the choice of $(a_k, p_k)_{k=0}^K$ instead of $(\overline{a}_k, p_k)_{k=0}^K$.

Next, without loss of generality, assume that $a_0 \leq a_1 \leq \cdots \leq a_K$ and that $p_0 \geq p_1 \geq \cdots \geq p_K$.[8] Also, without loss of generality, assume that $p_0 \leq v$ so that m_0 is chosen by positive mass of consumers.[9] Then let c_0 be the type who is indifferent between m_0 and not buying, and let c_k be the type which is indifferent between m_k and m_{k-1} for each $k = 1, \ldots, K$. To break ties, I assume that each consumer with type c_k choose m_k rather than m_{k-1}. Then, we can see that

$$\begin{cases} c_0 = \dfrac{v - p_0}{a_0} \text{ and } c_k = \dfrac{p_{k-1} - p_k}{a_k - a_{k-1}} \text{ for } k = 1, \ldots, K & \text{if } a_0 > 0 \\ c_1 = \dfrac{p_0 - p_1}{a_1} \text{ and } c_k = \dfrac{p_{k-1} - p_k}{a_k - a_{k-1}} \text{ for } k = 2, \ldots, K & \text{if } a_0 = 0, \end{cases} \tag{4}$$

and that consumers with type $\tilde{c} \in (c_{k+1}, c_k]$ chooses m_k. In the case where $a_0 = 0$, all types $\tilde{c} \in [0, c_0]$ will choose m_0 as long as $p_0 \leq v$. Thus, the demand for each service is given by

$$\begin{cases} d_k = F(c_k) - F(c_{k+1}) \text{ for } k = 0, \ldots, K-1, \text{ and } d_K = F(c_K) & \text{if } a_0 > 0 \\ d_0 = 1 - F(c_1), d_k = F(c_k) - F(c_{k+1}) \text{ for } k = 1, \ldots K-1, \text{ and } d_K = F(c_K) & \text{if } a_0 = 0. \end{cases} \tag{5}$$

Putting these elements together, the profit maximization problem of the platform can be expressed as

$$\begin{aligned} \max_{(a_k, p_k)_{k=0}^K} \quad & \sum_{k=0}^{K} d_k (p_k + a_k p_a) \\ \text{s.t.} \quad & (2), (3), (4), \text{ and } (5). \end{aligned} \tag{6}$$

Then consider the solution to the maximization problem above. I say a menu M is *binary* if it consists of only two services. In this case, M can be expressed as $M = (m_B, m_P) \equiv ((a_B, p_B), (a_P, p_P))$ with $a_B > a_P$ and $p_B < p_P$. I call the service with lower price $m_B = (a_B, a_B)$ as *basic service* and the service with higher price $m_P = (a_P, p_P)$ as *premium service*. In addition, I say a binary menu (m_B, m_P) is *freemium* if $p_B = 0$, that is, the price of the basic service is zero. Let c be the type of consumer who is indifferent between basic service and premium service. The following proposition states the main result that the profit-maximizing menu is binary and satisfies certain properties.

Proposition 2.1. *Optimal menu is binary, that is, the platform offers only two services at the optimum. In particular, the profit-maximizing menu is of the form* $((a_B, p_B), (a_P, p_P)) = ((a, p), (0, v))$*, and in the case of interior solution,* c *and* p_a *are determined by the following equations:*

$$p_a = c + \frac{F(c)}{f(c)}, \tag{7}$$

$$c = p_a - \frac{1 - G(p_a)}{g(p_a)}. \tag{8}$$

[8] Further assuming that $p_0 \geq p_1 \geq \cdots \geq p_K$ is without loss of generality since if $p_k < p_{k+1}$ hold some k, then no consumer chooses m_{k+1} since m_k gives strictly greater utility for any consumer.

[9] If $p_0 > v$ and $p_1 \leq v$ we can induce the same demand by reintroducing the menu $(m'_k)_{i=0}^K$ such that $m'_k = m_{k-1}$ for $k = 1, \ldots, K$ and $m'_K = m_K$.

Proof. In Appendix. □

The intuition behind this result is the following. When consumer nuisance costs are linear in the amount of advertisements, it is always optimal for the platform to either increase or decrease the amount of advertisements to each consumer as long as possible. Thus, for any k-th intermediate service, the amount of advertisement will be equal to one of adjacent service. As a results, only two services with full advertisements and no advertisements remains. Moreover, once we accept that the platform only offer two services, it follows that the platform equates the marginal revenue and marginal cost of increasing an agents on one side who interacts with the agents on the other side. These incentives yield the equations (7) and (8). These equations are familiar in the literature of two-sided markets which states that platforms equate the sum of transaction prices and price semi-elasticity of demand of each side (e.g., Rochet and Tirole (2003)). In other words, provided that the freemium is optimal, its behavior is fairly standard two-sided pricing.

This intuition can be stated in another way. Consider the situation where the platform chooses the amount of advertisement $a(\tilde{c})$ for each consumer with type $\tilde{c}$. We can interpret $\tilde{c} + \frac{F(\tilde{c})}{f(\tilde{c})}$ as the *virtual marginal cost* of listing an advertisement to consumer with type $\tilde{c}$, in the sense that the platform need to pay that amount to induce the consumer to view that advertisement (see Myerson (1981)). On the other hand, marginal revenue from listing an advertisement to consumer is the per-transaction fee p_a from the advertiser. When nuisance costs are linear, these marginal cost and marginal revenue are constant in $a(\tilde{c})$ and thus it is optimal to increase (decrease) $a(\tilde{c})$ as much as possible if marginal revenue p_a exceeds (falls below) the virtual marginal cost $\tilde{c} + \frac{F(\tilde{c})}{f(\tilde{c})}$. This means that the optimal amount of advertisement $a(\tilde{c})$ for the consumer with type $\tilde{c}$ greater (smaller) than threshold $\tilde{c}$ given by the equation (7) will be 0 (a). In determining the threshold type p_a of advertisers, the platform equates the marginal revenue from increasing an amount of advertisers $p_a - \frac{1-G(p_a)}{g(p_a)}$ and the cost c of keeping the consumer demand unchanged. This yields the equation (8).

I have shown that the optimal menu pricing is binary. However, this menu pricing is not precisely the same as the freemium in the real world, since the basic goods might not be free. Then we can ask when does the optimal menu pricing corresponds with the freemium. In other words, the question is when $p = 0$ holds at the optimum. The answer is that, when the benefit of advertisers from transaction is sufficiently large relative to the intrinsic value of services, then $p = 0$ is optimal for the platform.

From the equation (8) and the nonnegative price constraint for p, we can see that if $v \leq (1 - G(p_a))\left(p_a - \frac{1-G(p_a)}{g(p_a)}\right)$, then $p = 0$, since

$$p = v - (1 - G(p_a))\left(p_a - \frac{1 - G(p_a)}{g(p_a)}\right)$$

must hold in the interior solution, which is negative and violates the nonnegative price constraint. That is, consumers who use the basic service need not to pay anything if the benefits of advertisers from listing advertisements are sufficiently large relative to the intrinsic value consumers derive from the platform. This is the common property which is observed in the models of two-sided markets. We can also see this inequality as the condition under which freemium in the literal sense ($p = 0$) is optimal. Then, because the platform cannot adjust the price p below zero under the nonnegative price constraint, its behavior slightly changes. The next result shows this property.

Proposition 2.2. *If $v \leq (1 - G(p_a))\left(p_a - \frac{1-G(p_a)}{g(p_a)}\right)$ holds at the optimum, the optimal menu is freemium. In addition, c and p_a are determined by the equation*

$$c = \frac{v}{1 - G(p_a)} \tag{9}$$

$$\eta_c(c)(p_a - c) = p_a - \frac{1 - G(p_a)}{g(p_a)}, \tag{10}$$

where $\eta_c(c) \equiv \frac{c}{F(c)} f(c)$ is the nuisance elasticity of demand for basic service.

This equation can be rewritten as follows:

$$\frac{\partial c}{\partial a}\frac{1}{c}\frac{f(c)c}{F(c)}(v - ap_a) = p_a - \frac{1 - G(p_a)}{g(p_a)} \tag{11}$$

The left-hand side is the cost of increasing the amount of advertisers: the product of the percentage change in threshold type due to the increase in the amount of advertisers, surplus elasticity of demand, and the per-consumer revenue net of opportunity cost of losing premium consumers. The right-hand side is the simple marginal revenue of increasing the amount of advertisements. These cost and benefit equate at the optimum.

This result is analogous to Gomes (2014). When the platform can use a side payment to adjust consumers' incentive, the platform just maximize the total virtual value from consumers and advertisers. On the other hand, when the platform cannot use a side payment due to the nonnegative constraint of fixed fees, the platform need to care about the demand elasticity of increasing the amount of advertisements.

Going back to the interior solution, the next result shows simple comparative statics of the behavior of freemium pricing. Roughly speaking, these results state that if either consumers' nuisance costs are more likely to be high, or advertisers' benefit is more likely to be high, then both of threshold types c and p_a will be higher.

Result 2.1. *The following comparative statics results hold.*

1. *If the distribution function F is replaced by a distribution function $\tilde{F}$ which dominates F according to reverse hazard rate,* [10] *both c and p_a increase.*
2. *Suppose the distribution function $G(b)$ has an inverse hazard rate function $\lambda(\theta, b)$ which is continuously differentiable, increasing in the first argument θ, and decreasing in the second argument b. Then p_a and c are increasing in θ.*

Proof. In Appendix. □

The first part of comparative statics is straightforward. The reverse hazard rate dominance implies that the consumer nuisance cost is more likely to be high. Then the platform decreases the amount of advertisements which is shown to the consumers, which leads to the increase in the threshold type c and p_a. Technically, the reverse hazard rate dominance

[10] A distribution function $\tilde{F}$ dominates another distribution function F according to reverse hazard rate if for any $\tilde{c} \in [0, C]$, $\frac{\tilde{f}(\tilde{c})}{\tilde{F}(\tilde{c})} \geq \frac{f(\tilde{c})}{F(\tilde{c})}$ holds.

implies that the lower virtual marginal cost of listing an advertisement to each consumer. Then by equation (7), for a given amount of advertisements, threshold type of consumer will be higher. Next, if the threshold type of consumer will be higher, then the virtual value of marginal advertiser will be higher by equation (8). These facts lead to the increase in c and p_a.

The effects of change in the distribution G of advertisers' types are not so clear. By equation (7) we can see that threshold types p_a and c moves in the same direction regardless of the type of exogenous shock, but the direction in which these thresholds move is unclear. However, parameterizing the distribution functions G by the inverse hazard rate function $\lambda(\theta, b)$, optimal values of p_a and c turn out to be increasing in θ.

2.3. Welfare Analysis

Consider the socially optimal menu pricing and the divergence between the social optimum and profit maximizing menu. I restrict the attention to the set of menus which contains two elements $(0, v)$ and (a, p). Actually, it can be shown that this class of menus are welfare-maximizing using the same logic as in the Proposition 2.1.

Result 2.2. *The socially optimal menu is binary.*

Proof. In Appendix. □

Thus, I focus on the binary menu.

First, consider the utility of consumers. Consumer who chooses $(0, v)$ obtains 0 utility. On the other hand, the consume who chooses (a, p) obtains the utility $v - \tilde{c}a - p$. Summing these up over consumers, the consumer surplus CS is obtained as

$$\begin{aligned} CS &= \int_0^c (v - \tilde{c}a - p) f(\tilde{c}) d\tilde{c} \\ &= F(c)(1 - G(p_a))(c - \hat{c}), \end{aligned} \tag{12}$$

where $\hat{c} = E[\tilde{c} | \tilde{c} \leq c]$ is the average disutility of consumers who choose the service with advertisements. On the other hand, the advertiser surplus AS is given by

$$\begin{aligned} AS &= F(c) \int_{p_a}^B (b - p_a) g(b) db \\ &= F(c)(1 - G(p_a))(\hat{b} - p_a), \end{aligned} \tag{13}$$

where $\hat{b} = E[b | b \geq p_a]$ is the average benefit of advertisers who participate the platform. Summing these and the platform's profit up, the total surplus TS is given by

$$\begin{aligned} TS = CS + AS + \Pi &= v + \int_0^c \left(\int_{p_a}^B bg(b) db - \tilde{c}a \right) f(\tilde{c}) d\tilde{c} \\ &= v + F(c)(1 - G(p_a))(\hat{b} - \hat{c}). \end{aligned} \tag{14}$$

We can see that the total surplus depends only on c and p_a. Thus, it suffices to consider the socially optimal values of c and p_a.[11] Taking derivatives with respect to c and p_a, we can

[11] Actually, any c can be chosen by the platform by choosing an appropriate value of p.

obtain the welfare-maximizing pricing, which is determined by the following equations:

$$c = \hat{b} \text{ and } p_a = \hat{c}. \tag{15}$$

This means that the threshold type of one side equals the average type of the other side who interacts. The next proposition is a natural consequence of Spence (1975) and Weyl (2010); Besides the market power, the profit-maximizing behaviors deviate from the social optimum to the extent that their effects on the marginal agent and average agent diverge. Following Weyl (2010), I call this divergence as *Spence distortion*.

Proposition 2.3. *Under the profit-maximizing pricing, the following equations for threshold types c and p_a hold:*

$$c = \underbrace{\hat{b}}_{\text{Social optimum}} + \underbrace{(p_a - \hat{b})}_{\text{Spence distortion}} - \underbrace{\frac{F(c)}{f(c)}}_{\text{market power distortion}} \tag{16}$$

$$p_a = \underbrace{\hat{c}}_{\text{Social optimum}} + \underbrace{(c - \hat{c})}_{\text{Spence distortion}} + \underbrace{\frac{1 - G(p_a)}{g(p_a)}}_{\text{market power distortion}} \tag{17}$$

Profit-maximizing price for advertisers is too high and the amount of consumers who view advertisements is too small in terms of social welfare. In total, the amount of transaction is insufficient.

In other words, profit-maximizing size of network is too small in terms of social welfare, because both the amount of consumers who choose the service with advertisement and the amount of advertisers who participate the platform is too small. In relation to the literature, I confirm that the result of Anderson and Coate (2005) that profit-maximizing amount of advertisements is too small in terms of social welfare remains to hold even under freemium.

2.4. Comparison between Business Models

It is interesting to compare properties of binary menu pricing with those of ad-supported business model since these business models are both prevalent in real world and their difference in revenue structures might lead to different behaviors. I say a menu is *ad-supported* if it consists of only one service (a, p). Then, the pricing problem of the platform which adopts ad-supported menu is given by

$$\max_{a,p} \quad F\left(\frac{v - p}{a}\right)(p + aG^{-1}(1 - a)). \tag{18}$$

Deriving the first-order conditions, we can see the following result.

Proposition 2.4. *Under the ad-supported business model, the optimal prices for consumers and advertisers are determined by the equations*

$$p_a = c + \frac{F(c)}{f(c)} - \frac{v}{a}, \tag{19}$$

$$c = p_a - \frac{1 - G(p_a)}{g(p_a)}. \tag{20}$$

In addition, p_a, and c are higher than under freemium.

Proof. In Appendix. □

This result implies that the amount of advertisers is lower and the amount of consumers who view advertisements are higher under ad-supported business than under freemium. Together with the Proposition 2.3., we can see that freemium alleviates the incentive of platforms to under-provide advertisements relative to ad-supported business but exacerbates the incentive to under-provide consumers who view advertisements. This result comes from the fact that the platform cannot collect revenues from consumers who do not want to view advertisements under the ad-supported business model. Platforms which adopt freemium can collect revenues from consumers who avoid advertisements. This changes incentives of platforms in the way that more consumers pay to avoid advertisements, exacerbating the underprovision of consumers who view advertisements. On the other hand, this underprovision of consumers decreases the type c of threshold consumer, which implies that the type p_a of threshold advertiser also decreases. This means that the amount of advertisements will be higher, alleviating the underprovision of advertisements.

2.5. Discussions

In this subsection, I discuss several aspects of monopoly menu pricing which are not treated in the model above.

Heterogeneous Intrinsic Value It seems restrictive that consumer intrinsic value from participating the platform is constant at v. Nevertheless, we can see that the qualitative result will not change even if this assumption is relaxed.

Suppose that v follows a strictly increasing, continuously differentiable distribution function H on $[0, V]$ with density h, and $\frac{1-H(v)}{h(v)}$ being decreasing in v. Setting the profit-maximization problem and deriving the first-order conditions for p_k, $k = 1, \ldots, K-1$ accordingly, we can obtain

$$\begin{aligned}&\int_{c_{k+1}}^{c_k} (1 - H(ca_k + p_k)) f(\tilde{c}) d\tilde{c} - \int_{c_{k+1}}^{c_k} h(\tilde{c}a_k + p_k) f(\tilde{c}) dc (p_k + a_k G^{-1}(1-a)) \\ &\quad + (1 - H(c_k a_k + p_k)) f(c_k)(c_k - G^{-1}(1-a)) \\ &\quad - (1 - H(c_{k+1} a_k + p_k)) f(c_{k+1})(c_{k+1} - G^{-1}(1-a)) = 0.\end{aligned} \tag{21}$$

This has a solution $c_k = c_{k+1}$. Thus, the similar result is observed.[12]

Linearity of Nuisance Costs Parts of my results depend crucially on the linearity of the environment. If the nuisance cost is convex in the amount of advertisements, it is optimal for the platform to offer a nonlinear schedule of advertisement-price pairs. The linearity assumption requires that the consumer nuisance from an additional advertisement is constant. Casually speaking, this assumption seems to hold in the cases where the amount of advertisements shown to consumers is not large. On the contrary, when the amount of advertisements is too large, these advertisements crowd out the original contents which the platform offers, generating the higher marginal nuisance from advertisements. However, if there is so large amount of advertisements that the advertisements crowd out

[12] The detail is available upon request.

the original contents, then the platform is likely to fail to attract consumers. Whether the reality can be approximated by the linear environment is rather an empirical issue.

Menu Pricing on Advertisers' Side So far, I have considered the situation where the platform engages in menu pricing only on the consumers' side. Even if we take the menu pricing on advertisers' side into account, the result does not change. Consider the situation where the platform's menu consists of $\overline{M} \equiv (M, \tilde{M})$, where the menu $\tilde{M} = (\tilde{m}_l)_{l=0}^{L}$ for advertisers consists of $L + 1$ services $m_l = (x_l, t_l)$. x_l is the amount of consumers the advertisers can reach and t_l is the fixed payment that the advertiser must make. Let y_l be the amount of advertisers who choose the service $\tilde{m}_l$. I impose the feasibility restriction that

$$\sum_{k=0}^{K} d_k a_k = \sum_{l=0}^{L} y_l x_l.$$

With this specification, it can be shown that the optimal menu pricing is the same as that presented in the previous subsections.[13]

In this respect, the assumption that the platform only charges transaction price on the advertisers' side is without loss of generality in this model.

Interdependence between Consumers' and Advertisers' Types It may be more plausible to allow for the interdependence between the nuisance cost of consumers and transaction benefit of advertisers. For example, if nuisance cost reflects the opportunity cost of time, and the consumer with higher income has the higher opportunity cost of time, then the transaction benefit of advertiser may be higher when it transacts with a consumer with higher nuisance cost. On the other hand, it may be the case that the consumer with little interest to the advertisers has a higher nuisance cost. In this case, there is the negative relation between nuisance costs and transaction benefits. The condition under which my result remains to hold in such situations is left for a future research.

3. Duopoly Menu Competition

In this section, I extend the model to duopoly competition. The qualitative result that the equilibrium menu pricing or optimal monopolistic menu pricing should be binary remains to be valid.

Consider a duopoly case of the previous section. I adopt a Hotelling specification. There are two platforms $i = 1, 2$ located on the edges of a unit interval, and consumers are uniformly distributed on that interval. Let $M_i = (m_{ki})_{k=0}^{K}$ be menu that platform i offers. I assume that advertisers multihome so that there is no competition between platforms for advertisers. Consumer's utility from participating the platform i's service is given by

$$U_i(\tilde{c}, x) = \begin{cases} v - \tilde{c}a_{ki} - p_{ki} - t|1_{i=2} - x| & \text{if } m_{ki} \text{ is chosen} \\ 0 & \text{if none is chosen,} \end{cases} \tag{22}$$

where x is the location on the unit interval $[0, 1]$, and $1_{i=2}$ is the indicator function which takes 1 if $i = 2$.

[13] The detail is available upon request.

Each consumer with type $(\tilde{c}, x)$ chooses the utility-maximizing services among the menus of two platforms, or chooses to buy nothing.

As in the previous section, suppose $a_{ki} \leq a_{k+1i}$ and $p_{ki} \geq p_{k+1i}$ for each $k = 0, \ldots, K-1$ and $i = 1, 2$.

In general, there might be numerous patterns of menus for a platform which will be a best response to a menu of the other platform. Although it is interesting, it is not easy to figure out these general patterns of Nash equilibrium pair of menus. Thus, I restrict an attention to the symmetric equilibrium, where $M_1 = M_2 = M = (m_k)_{k=0}^K$. In this simplified case, we can see that $c_{k1} = c_{k2} = c_k$ for each k, where c_{ki} is the type a consumer who is indifferent between m_{ki} and m_{k-1i}.

Suppose $(m_k)_{k=0}^K$ is the symmetric equilibrium menu. For simplicity, I assume that v is sufficiently large relative to C and t, so that in any equilibrium any consumer chooses some service. I also assume that the second-order condition for the equilibrium menu pricing is satisfied[14] Under the setting described above, I obtain the following result which shows that binary menu remains to be an equilibrium even if we take competition into account.

Proposition 3.1. *The symmetric equilibrium menu is binary. The symmetric equilibrium binary menu is of the form $(a, t - ap_a)$, $(0, t)$, and the price for advertisers are determined by the following equations:*

$$c = \underbrace{\hat{b}}_{\text{Social optimum}} + \underbrace{(p_a - \hat{b})}_{\text{Spence distortion}} \tag{23}$$

$$p_a = \underbrace{\hat{c}}_{\text{Social optimum}} + \underbrace{\frac{1 - G(p_a)}{g(p_a)}}_{\text{Market power distortion}} \tag{24}$$

where $\hat{c} \equiv E[\tilde{c}|\tilde{c} \leq c]$ is the average type of consumers who choose basic good.

Proof. In Appendix. □

Compared to the first-order condition in the monopoly case, we can see that there is no distortion on the consumer side. This effect is driven by the introduction of competition. As in Armstrong and Vickers (2010), when platforms compete in menu pricing, they offer the first-best contracts, and try to compete in fixed prices. These effects eliminate the distortion on the consumer side. By contrast, since each platform retains market power on advertisers' side, the distortion on that side still remains.

Through the derivation of the expressions in Proposition 3.1., I find the condition on which the freemium is the equilibrium menu. This condition is similar to that found in Result 2.1.

Corollary 3.1. *If the solution to the equations (22) and (23) satisfies the inequality $ap_a \geq t$, the equilibrium menu is freemium.*

[14] I do not derive the second-order condition but just assume that the second-order condition is satisfied since it is not the main focus of this paper. Zennyo (2016) derives the second-order condition under a similar specification to my model.

As in the monopoly case, the binary menu becomes freemium if the willingness to pay of advertisers is sufficiently high.

As another corollary of the proposition above, I find that the equilibrium profit is independent of any fundamentals of consumers or advertisers.

Corollary 3.2. *In the symmetric equilibrium, the equilibrium profit for each firm is* $\frac{t}{2}$.

This result is somewhat striking because all the benefits for platforms from adopting binary menu disappears once a competition is introduced. This property is known as *revenue neutrality property* that if all consumers are served, then any exogenous increase in revenues per consumers are competed away (Anderson and Gabszewicz 2006, Armstrong 2006).

As seen above, the main property that the platform segment consumers into those who view fewer advertisements and those who view all advertisements holds even under duopoly. This result indicates the robustness of binary pricing in different situations. In addition, as seen in the Result 2.1., I conjecture that the equilibrium menu would be freemium if the benefit of advertisers from interaction with consumers is sufficiently high.

4. Conclusion

Freemium with advertisement is so prevalent that any people who have ever used online applications have faced a choice between free service with a lot of advertisements and ad-free premium service. I examine the optimality of this business model, and show that under certain specifications which are naturally adopted in the literature of two-sided markets, freemium with advertisements is actually the best way to collect revenues from both consumer and advertisers. The property that at the optimal nonlinear pricing, there are two bunches of consumers at the top and at the bottom seems to be robust to several modifications of specifications.

One possible direction of future research is the analysis of a platform who uses quality and advertisements as instruments of price discrimination at the same time. In reality, platforms use not only the amount of advertisements but also the qualities of services as instruments of price discrimination. There might be an interesting interaction when we analyze these things together. In addition, there might be heterogeneity in the externality of each agent in one side on agents on another side, as in Gomes and Pavan (2016) and Jeon et al. (2016). Incorporating these elements may make other differences, such as complementarities or substitutabilities in quality and the amount of advertisements. Also, analyzing asymmetric equilibria under duopoly menu competition might be interesting since in the real world, different business models coexist and this cannot be explained by my simple model. However, just computing asymmetric equilibria using my model ends up being a tedious calculation without meaningful results. Thus, one have to invent more tractable framework to tackle with this problem.

A. Appendix: Proofs

A.1. Proof of Proposition 2.1

I introduce several lemmata and use them to prove the proposition.

Lemma A.1. *Optimal menu pricing satisfies* $a_0 = 0$*,* $p_0 = v$*.*

Proof. Let $(m_k)_{k=0}^K$ be an optimal menu and suppose that $a_0 > 0$. By the first-order condition for p_k, $k = 1, \ldots, K-1$, we have $c_k = c_{k+1}$ for $k = 1, \ldots, K-1$. In this case, the profit can be written as

$$d_0 p_0 + d_K p_K + (d_0 a_0 + d_K a_K) G^{-1}(1-a)).$$

Consider a menu $(m'_k)_{k=0}^K$ such that

$$\begin{aligned} a'_0 &= 0, \quad p'_0 = v \\ a'_1 &= a_0, \quad p'_1 = p_0 \\ a'_k &= a_k, \quad p'_k = p_k \text{ for } k = 2, \ldots, K. \end{aligned}$$

We can see that this menu obtains the profit

$$d_0 p_0 + d_K p_K + (d_0 a_0 + d_K a_K) G^{-1}(1-a) + (1 - F(c_0))v,$$

which is higher than the profit obtained by $(m_k)_{k=0}^K$, violating the optimality. Thus, we must have $a_0 = 0$.

Next, suppose that $(m_k)_{k=0}^K$ satisfies $a_0 = 0$ but $p_0 < v$. In this case, $c_1 = \frac{p_0 - p_1}{a_1}$. Then increasing p_k by small ε for all k does not change c_k for all k. Thus, this price change increases the profit by $\sum_{k=0}^K d_k \varepsilon > 0$. This contradicts the optimality of $(m_k)_{k=1}^K$. Thus, we must have $p_0 = v$. □

Lemma A.2. *Optimal menu pricing satisfies* $c_1 = \cdots = c_K$*.*

Proof. By the proof of Lemma A.1, we can see that $c_1 = \frac{v - p_1}{a_1}$. I next show that $c_2 = \cdots = c_K$. Consider the first-order condition for p_k for $k = 2, \ldots, K-1$:

$$\frac{\partial d_{k-1}}{\partial p_k}(p_{k-1} + a_{k-1} G^{-1}(1-a)) + \frac{\partial d_k}{\partial p_k}(p_k + a_k G^{-1}(1-a)) + \frac{\partial d_{k+1}}{\partial p_{k+1}}(p_{k+1} + a_{k+1} G^{-1}(1-a)) + d_k = 0.$$

This equation can be rewritten as

$$f(c_k)(c_k - G^{-1}(1-a)) + F(c_k) = f(c_{k+1})(c_{k+1} - G^{-1}(1-a)) + F(c_{k+1}),$$

which has the solution $c_k = c_{k+1}$.

Finally, I show that $c_1 = c_2$. The first-order condition for p_1 is given by

$$\frac{\partial d_0}{\partial p_1}(p_0 + a_0 G^{-1}(1-a)) + \frac{\partial d_1}{\partial p_1}(p_1 + a_1 G^{-1}(1-a)) + \frac{\partial d_2}{\partial p_1}(p_2 + a_2 G^{-1}(1-a)) + d_1 = 0.$$

This equation can be rewritten as

$$f(c_1)(c_1 - G^{-1}(1-a)) + F(c_1) = f(c_2)(c_2 - G^{-1}(1-a)) + F(c_2),$$

and $c_1 = c_2$ satisfies this condition. These imply that $c_1 = \ldots, c_K$. □

Lemma A.3. *Optimal menu pricing satisfies* $a_K = a$

Proof. By Lemma A.1 and Lemma A.2, the profit maximization problem is reduced to

$$\begin{aligned} \max_{a_K, a, p} \quad & \Pi = (1 - F(c))v + F(c)(p + a_K G^{-1}(1 - a)) \\ \text{s.t.} \quad & c = \frac{v - p}{a_K} \\ & a_K \leq a. \end{aligned}$$

In this case, as long as $a_K < a$, the platform can increase the profit by reducing a. Thus, $a_K = a$. □

Summarizing these lemmata, we obtain the first statement.

Next, the profit maximization problem (6) can be rewritten as

$$\begin{aligned} \max_{a, p} \quad & (1 - F(c))v + F(c)(p + aG^{-1}(1 - a)) \\ \text{s.t.} \quad & c = \frac{v - p}{a}. \end{aligned} \tag{25}$$

Proof. The first-order condition for p is given by

$$f(c)(c - G^{-1}(1 - a)) + F(c) = 0.$$

Rearranging this equation, we obtain

$$p_a - c = \frac{F(c)}{f(c)}.$$

The first-order condition for a is given by

$$cf(c)(c - G^{-1}(1 - a)) + F(c)(G^{-1}(1 - a) - aG^{-1'}(1 - a)) = 0.$$

Applying the inverse function theorem, substituting the first-order condition for p, and rearranging, we obtain

$$p_a - c = \frac{1 - G(p_a)}{g(p_a)}.$$

Finally, we can see that the second-order condition is satisfied when $\frac{F}{f}$ is increasing and $\frac{1-G}{g}$ is decreasing.

Putting these together, we obtain the Proposition 2.1. □

A.2. Proof of Result 2.2

Proof. First, consider the case where F is replaced by $\tilde{F}$ which dominates F according to reverse hazard rate. Then, we have $\frac{\tilde{F}(\tilde{c})}{\tilde{f}(\tilde{c})} \leq \frac{F(\tilde{c})}{f(\tilde{c})}$ for any $\tilde{c} \in [0, C]$. Let c' and p'_a be the threshold types under $\tilde{F}$ and c and p_a be the threshold types under F. I first show that $p'_a \geq p_a$. Suppose that $p_a > p'_a$. Then

$$c = p_a - \frac{1 - G(p_a)}{g(p_a)} > p'_a - \frac{1 - G(p'_a)}{g(p'_a)} = c'.$$

Combining equations (7) and (8), we obtain

$$\frac{1-G(p_a)}{g(p_a)} = \frac{F(c)}{f(c)}, \text{ and } \frac{1-G(p'_a)}{g(p'_a)} = \frac{\tilde{F}(c')}{\tilde{f}(c')}.$$

Putting these together, we obtain

$$\frac{1-G(p_a)}{g(p_a)} = \frac{F(c)}{f(c)} \geq \frac{F(c')}{f(c')} \geq \frac{\tilde{F}(c')}{\tilde{f}(c')} = \frac{1-G(p'_a)}{g(p'_a)},$$

which leads to $p_a \leq p'_a$ since $\frac{1-G}{g}$ is decreasing and derives contradiction. Thus, we have $p'_a \geq p_a$ and $c' \geq c$ follows.

Next, I show that p_a and c are increasing in θ in the second case. If the inverse hazard rate is characterized by $\lambda(\theta, b)$, then the first-order conditions can be written as

$$c + \frac{F(c)}{f(c)} - p_a = 0$$
$$p_a - \lambda(\theta, p_a) - c = 0.$$

Differentiating these equations by θ and rearranging, we can see that

$$\frac{dp_a}{d\theta} = \frac{-\frac{\partial \lambda(\theta,p_a)}{\partial \theta}\frac{\partial}{\partial c}\left(c + \frac{F(c)}{f(c)}\right)}{\left(1 - \frac{\partial}{\partial p_a}\left(p_a - \lambda(\theta, p_a)\right)\frac{\partial}{\partial c}\left(c + \frac{F(c)}{f(c)}\right)\right)} \geq 0$$

Thus, p_a is increasing in θ. Then the fact that c is increasing in θ is straightforward.

□

A.3. Proof of Result 2.3

Proof. First, note that the socially optimal menu must satisfy $a_0 = 0$ and $p_0 \leq v$. If $a_0 > 0$, then there is a positive mass of consumers who do not participate in the platform (i.e., $\tilde{c} > c_0$.). In this case, by introducing a new service m' with $a' = 0$ and $p' = v$, the welfare can be improved by the amount of such consumers times the intrinsic value of the content, $(1 - F(c_0))v$.

Given the menu M with $a_0 = 0$ and $p_0 \leq v$, the total surplus is given by

$$\begin{aligned} & v - \sum_{k=1}^{K} \int_{c_{k+1}}^{c_k} a_k \tilde{c} f(\tilde{c}) d\tilde{c} + \sum_{k=1}^{K} d_k \int_{p_a}^{B} \frac{b}{a} g(b) db \\ =& v + \sum_{k=1}^{K} \int_{c_{k+1}}^{c_k} \left(\hat{b} - \tilde{c}\right) f(\tilde{c}) d\tilde{c} \end{aligned} \tag{26}$$

where $\hat{b} = E[b|b \geq p_a]$. Then the first-order condition for p_k, $k = 2, \ldots, K-1$ can be writtens as

$$-(\hat{b} - c_k) f(c_k) + (\hat{b} - c_{k+1}) f(c_{k+1}) = 0, \tag{27}$$

which has a solution $c_k = c_{k+1}$ for $k = 2, \ldots, K-1$. Thus, $c_2 = \ldots, c_K$. It remains to show that $c_1 = c_2$. The first-order condition for p_1 can also be written as

$$(c_1 - \hat{b}) f(c_1) - (c_2 - \hat{b}) f(c_2) = 0,$$

which has the solution $c_1 = c_2$. Thus, we have $c_1 = \ldots, c_2$. This shows that the socially optimal menu pricing is binary. □

A.4. Proof of Proposition 2.3

Proof. Let c' and p'_a be the threshold types under ad-supported model and c and p_a be the threshold under binary menu. Suppose that $c' < c$. Then $p'_a < p_a$ is also follows from the equation (8) and the equation (19). Combining the first-order condition under each business model and assumption above, we obtain

$$\frac{1 - G(p'_a)}{g(p'_a)} < \frac{F(c')}{f(c')} \leq \frac{F(c)}{f(c)} = \frac{1 - G(p_a)}{g(p_a)}.$$

Thie inequality requires $p'_a > p_a$ to hold, which contradicts $p'_a < p_a$. Thus, we must have $c' \geq c$ and $p'_a \geq p_a$. □

A.5. Proof of Proposition 3.1

Proof. Suppose that platform 2 offers the binary menu described in Proposition 3.1. I argue that the best response of platform 1 is to adopt the same menu.[15]

Suppose that 1 can observe $\tilde{c}$ (but not x). I calculate 1's best response to 2 given $\tilde{c}$. Let

$$V_1 = \max_{m_{k1}} v - a_{k1}\tilde{c} - p_{k1}$$

be the type-$\tilde{c}$ consumer's gross utility when she buys from platform 1 and 2, respectively. The most profitable way to generate utility V_1 is

$$\begin{aligned} &\max_{p',a'} p' + a'G^{-1}(1 - a) \\ &\text{s.t.} \quad v - a'\tilde{c} - p' = V_1, \\ &\qquad\; a' \leq a. \end{aligned}$$

This problem has a solution

$$a(\tilde{c}) = \begin{cases} a & \text{if } \tilde{c} \leq G^{-1}(1 - a) \\ 0 & \text{otherwise.} \end{cases}$$

From this, we can see that (i) platform offers two services with $(0, p_P)$ and (a, p_B), and (ii) the threshold type c is determined by $c = G^{-1}(1 - a) = p_a$. These implies

$$c = \frac{p_P - p_B}{a} = p_a \Longrightarrow p_P = p_B + ap_a.$$

Putting these together, the equilibrium menu becomes (a, p_B), $(0, p_B + ap_a)$.

Next, I consider the remaining equilibrium values of a and p_B. Suppose that platform 1 slightly increases p_B by ϵ. Then, given type $\tilde{c}$ below p_a, the threshold location $\hat{x}(\tilde{c})$ is determined by

$$\hat{x}(\tilde{c}) = \frac{1}{2} - \frac{\epsilon}{2t}.$$

For type $\tilde{c}$ below, the threshold location is given by

$$\hat{x} = \frac{1}{2} - \frac{\epsilon}{2t}$$

[15] This proof is almost the copy of Armstrong and Vickers (2010).

Then the profit of platform 1 can be written as

$$\left(\frac{1}{2}-\frac{\epsilon}{2t}\right)(p_B+\epsilon+ap_a).$$

Then, the equilibrium condition is that the first-order condition is satisfied at $\epsilon = 0$:

$$p_B + ap_a = t.$$

Next, suppose that platform 1 slightly increases a by ε. First, the threshold nuisance cost changes from p_a to $G^{-1}(1-a-\varepsilon)$. Next, the threshold location for $\tilde{c}$ below $G^{-1}(1-a-\varepsilon)$ is given by

$$x'(\tilde{c}) = \frac{1}{2} - \frac{\tilde{c}\varepsilon}{2t}.$$

For $\tilde{c} \in [G^{-1}(1-a-\varepsilon), p_a]$, the threshold type is

$$x''(\tilde{c}) = \frac{1}{2} - \frac{(a+\varepsilon)G^{-1}(1-a-\varepsilon)-\tilde{c}a}{2t}.$$

For $\tilde{c} \geq p_a$, the threshold location becomes

$$x''' = \frac{1}{2} - \frac{(a+\varepsilon)G^{-1}(1-a-\varepsilon)-ap_a}{2t}.$$

Putting these together, the profit of platform 1 is

$$\left((1-F(p_a))x''' + \int_{G^{-1}(1-a-\varepsilon)}^{p_a} x''(c)f(\tilde{c})d\tilde{c} + \int_0^{G^{-1}(1-a-\varepsilon)} x'(\tilde{c})f(\tilde{c})d\tilde{c}\right)(p_B+(a+\varepsilon)G^{-1}(1-a-\varepsilon)).$$

The first-order condition is then

$$\left(-\frac{p_a-aG^{-1'}(1-a)}{2t}(1-F(p_a)) - \int_0^{p_a}\frac{\tilde{c}}{2t}f(\tilde{c})d\tilde{c}\right)(p_B+ap_a)+\frac{1}{2}\left(p_a-aG^{-1'}(1-a)\right)=0$$

$$\Longrightarrow p_a - \frac{1-G(p_a)}{g(p_a)} = \hat{c}.$$

Putting them together, I obtain equations (22) and (23). □

References

[1] Anderson, C., 2009, Free: The Future of a Radical Price, Hyperion: New York City.

[2] Anderson, E. T., Dana Jr, J. D., 2009, “When is Price Discrimination Profitable?" Management Science, 55, 980-989.

[3] Anderson, S. P., Coate, S., 2005, “Market Provision of Broadcasting: A Welfare Analysis," The Review of Economic Studies, 72, 947-972.

[4] Anderson, S. P., Gabszewicz, J. J., 2006, “The Media and Advertising: a Tale of Two-Sided Markets," Handbook of the Economics of Art and Culture, 1, 567-614.

[5] Armstrong, M., Vickers, J. (2010). Competitive non-linear pricing and bundling. The Review of Economic Studies, 77(1), 30-60.

[6] Armstrong, M., 2006, "Competition in Two-Sided Markets," The RAND Journal of Economics, 37, 668-691.

[7] Choi, J. P., Jeon, D. S., Kim, B. C., 2015, Net Neutrality, Business Models, and Internet Interconnection," American Economic Journal: Microeconomics, 7, 104-141.

[8] Bedre-Defolie, Ö., Calvano, E., 2013, "Pricing Payment Cards," American Economic Journal: Microeconomics, 5, 206-231.

[9] Deneckere, R. J., Preston McAfee, R., 1996, "Damaged Goods," Journal of Economics & Management Strategy, 5, 149-174.

[10] Eisenmann, T. R., Pao, M., Barley, L., 2011, "Dropbox: 'It Just Works'," Harvard Business School Case 811-065, January 2011.

[11] Gabszewicz, J. J., Laussel, D., Sonnac, N., 2004, "Programming and Advertising Competition in the Broadcasting Industry," Journal of Economics & Management Strategy, 13, 657-669.

[12] Gomes, R. (2014). Optimal auction design in two-sided markets. The RAND Journal of Economics, 45(2), 248-272.

[13] Gomes, R., Pavan, A., 2016, "Many-to-Many Matching and Price Discrimination," Theoretical Economics, 11, 1005-1052.

[14] Halmenschlager, C., Waelbroeck, P., 2014, "Fighting Free with Free: Feemium vs. Piracy," Working Paper, Available at SSRN: `https://ssrn.com/abstract=2475641`

[15] Jeon, D. Kim, B. Menicucci, D., 2016, "Second-Degree Price Discrimination by a Two-Sided Monopoly Platform," Working Paper, University of Toulouse

[16] Jing, B., 2007, "Network Externalities and Market Segmentation in a Monopoly," Economics Letters, 95, 7-13.

[17] Maskin, E., Riley, J., 1984, "Monopoly with Incomplete Information," The RAND Journal of Economics, 15, 171-196.

[18] Mussa, M., Rosen, S., 1978, "Monopoly and Product Quality," Journal of Economic Theory, 18, 301-317.

[19] Myerson, R. B., 1981, "Optimal Auction Design," Mathematics of Operations Research, 6, 58-73.

[20] Nan, G., Wu, D., Li, M., Tan, Y., 2016, "Optimal Pricing of Information Goods under the Freemium Strategy in the Presence of Piracy and Network Effects," Working Paper, Available at SSRN: `https://ssrn.com/abstract=2798421`.

[21] Peitz, M., Valletti, T. M., 2008, "Content and Advertising in the Media: Pay-TV versus Free-to-Air," International Journal of Industrial Organization, 26, 949-965.

[22] Rochet, J. C., Tirole, J., 2003, "Platform Competition in Two-Sided Markets," Journal of the European Economic Association, 1, 990-1029.

[23] Rysman, M., 2009, "The Economics of Two-Sided Markets," The Journal of Economic Perspectives, 23, 125-143.

[24] Salant, S. W., 1989, "When is Inducing Self-Selection Suboptimal for a Monopolist?" The Quarterly Journal of Economics, 104, 391-397.

[25] Spence, A. M., 1975, "Monopoly, Quality, and Regulation," The Bell Journal of Economics, 6, 417-429.

[26] Varian, H. R., 1997, "Versioning Information Goods," Working Paper University of California, Berkeley.

[27] Weyl, E. G., 2010, "A Price Theory of Multi-Sided Platforms," The American Economic Review, 100, 1642-1672.

[28] Zennyo, Y., 2016 , "Freemium on advertising platforms," Working Paper, Available at SSRN: `http://ssrn.com/abstract=2777918`.