Non-binary preferences

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Abstract

We prove the existence of a Pareto optimal state of a finite society that has socially differentiated persons, each with non-binary personal preferences that quasi-order a finite set of alternatives. Everybody engages in a volitional act of choice by maximization of *non-binary* preferences. As a consequence of interpersonal interaction among social creatures, the *social interaction outcome* defined as belonging to a *nonempty social maximal* set exists, and thus is Pareto optimal. Injustices inflicted by one group of persons upon a socially distinct one, arising from social diversification, are, however, consistent with such a collective outcome. (95 words)

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Social Diversification, Injustices, and Pareto Optimality with Non-binary Preferences

1. Introduction

The formal mathematical expression of differentiated social identities of persons in a finite society is of vital importance to economics. Such an expression can determine whether a loose economic intuition such as social-identity based injustices by one group or persons against another has a coherent, logical meaning. This paper presents work on a *non-binary* preference based economic theory of a finite society of persons using formal mathematical reasoning, to breathe life into a society with persons who are differentiated by social identity, and simultaneously to recognize that the collective outcome is characterized by a great many injustices that arise from social differentiation itself.[1] Interpersonal interaction among persons socially differentiated in society culminates in the existence of a collective outcome in which everyone has maximized personal non-binary preferences (which are reflexive and acyclic). This outcome is also Pareto optimal. To prove this is the object of this paper. The work reported here is built on the choice theory of non-binary preference of a single individual, developed by the late Stig Kanger in the 1970's, but which was only brought to light by Amartya Sen (1994) in the 1990's.

It is interesting and it is important for economic theory to embrace the notion of a human being as a social creature, with multiple shared identities with others in society. And a specific social identity of a person derives exclusively from being affiliated with a distinguishable and distinct community (or subset) of persons. The scope and reach of such an economic theory is considerably greater in explaining phenomena such as racial and gender discrimination than existing economic theory, which is predicated on each person being represented exclusively by a *binary* ranking relation of weak preference, interpreted as 'at least as good as' defined on a set of mutually exclusive alternatives.

The binariness property strips off every person any semblance of social identity such as male or female, Black, White or Hispanic, or any other identity whatsoever. The only possible distinction between two persons with binary preferences is that one has a ranking of a pair of alternatives and the other's ranking of the same pair is its negation. No other distinction is admissible. The binariness property thus obliterates every *other* conceivable distinction between two persons, including differences in gender, race, ethnicity, religion and the like.

These marks of social identification in persons are rendered undetectable by the binariness property of each person's preferences because they are *tertiary* considerations that are *external to* an interpersonal distinction in a pairwise ranking of any given pair of alternatives by two persons.[2] It is, therefore, not a false alarm sounded by Amartya Sen and Bernard Williams (1982), that 'Persons do not count as individuals in this any more than

[1] Injustices that spring from the illusion of a much-valued social mark of distinction between two groups, which sometimes goes hand-in hand with violence also, has been observed based on tribal allegiance or religious intolerance, as well as discrimination against females in society, or racial discrimination against African Americans, among others such phenomena.

[2] There can be differences between the feasible sets of distinct persons, but such differences do nothing to generate social differentiation is society, except richer and poorer persons, the importance of which must not be underestimated, because by itself this information is valuable in its own right, particularly if income distributional features of a society during a given period and its dynamics over time are of interest.

individual petrol tanks do in the analysis of the national consumption of petroleum.'[3] They cannot be differentiated by race or gender nor indeed by a multitude of other possible social identification markers, rendering binary-preference-salient economic theory inadequate for the task of examining injustices such as racial or gender discrimination, or of any other injustice based on social differentiation.

Since mainstream economic theory, as in Arrow and Debreu (1954), is completely silent on the matter of recognition of, and thus removal of, remediable social injustices that arise from social differentiation of persons in society, our models fail to be conceptually up to snuff to recognizing or displaying social diversification phenomena in society. There is a gap between the existence of social differentiation of persons in society and of injustices based on this stratification, contrasted with the capability of economic theory to explain these facts, and it is a deficit of theory. Building on the work of Sen (1997), and taking guidance from the many warnings in Arrow (1998), we attempt in this paper to zero-out this deficit.

By contrast, the assignment of a finite array of non-binary ranking relations to each person in society defined on their respective feasible sets of outcomes produces an economic theory that permits an examination of social injustices that derive from *the illusion of destiny* identified by Sen (2006) at the base of *identity and violence*, to permit at some point an investigation of means of removal of the remediable social injustices, in Sen's (2009) sense of *the idea of justice*.

In particular, by assigning to each person multiple *socially-differentiated identities* in a finite society (*Axiom S*), *non-binary* preferences (*Axiom N*), that are reflexive and acyclic, and thus a *quasi-ordering* (*Axiom Q*), while every person engages in a volitional act of choice by *maximization* of one's own preferences defined on one's own feasible (budget) set, (*Axiom M*), and social interaction is really interpersonal *interaction* among social creatures (*Axiom I*), in this paper we generalize economic theory so that it can examine cures for social injustices that arise from social diversification of persons in a finite society. We also prove that in such an economy, which we call a non-binary economy, there exists a Pareto optimal social state.[4]

What is more, we do this for non-binary personal preferences defined on a finite set of social states, but our result is also true for the budget sets of '*consumption units*, typically families or individuals but including also institutional consumers,' in Arrow and Debreu (1954, p.268).[5] Thus we do *not* redefine the elements of the feasible set to investigate social-identity-based injustices such as gender or racial discrimination. This is in keeping with Arrow's (1998, p.95), warning to not 'risk… turning the explanation into tautology.'

In the main theorem and its proof, and in the concluding section, we do not restrict the feasible sets of individuals to be budget sets, but take on more general forms.[6] This provides a rich conceptual framework for investigating formally some socio economic phenomena that binary economic theory cannot.

[3] See Sen and Williams, p.4.
[4] Contrasted with points on the contract curve in an Edgeworth box, and contrasted with the core of a game, this is the weakest set of conditions under which the existence of a Pareto optimal state has been proven to date. That is, the core of a game with non-binary players' preferences is non-empty under *Axioms S*, *N*, *Q*, *M* and *I*. This is the content of our main *Theorem 3.1* below.
[5] Emphasis in original.
[6] Though for racial and gender discrimination, we do require the feasible set to be the Arrow-Debreu budget set.

Section 2 seeks to provide the motivation behind the notion of non-binary preferences. Section 3 contains some preliminary results, which are used in Section 4 to prove our main result. Section 5 contains some concluding remarks.

2. Motivating Non-binary Preferences

We prove the existence of a Pareto optimal state of a finite society with non-binary personal preferences. To our knowledge, our statement constitutes the weakest set of conditions under which the existence of a Pareto optimal state has been proven to date. This paper can be seen as extending to the case of a *finite society* the work of Amartya Sen (1997) on maximization as a *personal* act of volitional choice based on non-binary preferences, which is an enormous aid in our construction of a theory of a *social interactional outcome*, construed here as a generalization of the very concept of equilibrium in economic theory. Such an "equilibrium" social outcome is defined as the existence of a nonempty *social interactional maximal* set.[7]

How does our theory differ from mainstream economic theory? Arrow's (1998, p. 93-94), remarks clarify this matter,

> "The answer depends in part as to what we mean by economic theory. Certainly, "rational choice theory" is broader than "economic theory." Rational choice theory means that the individual actors act rationally (that is, by maximizing according to a complete ordering) within the constraints imposed by preferences, technology, and beliefs, and by the institutions which determine how individual actions interact to determine outcomes. Further, the beliefs are themselves formed by some kind of rational process. By economic theory, we mean that in some sense, markets are the central institution in which individual actions interact and that other institutions are of negligible importance."

Since a market supports *impersonal* exchange, in the competitive case it is irrelevant whether a seller sells to one demander or another, nor does it matter to a buyer as to from which seller the purchase is made. All inter-personal differences in *social or any other identities of distinct individuals* are obliterated in both rational choice theory and present-day economic theory.

The principal reason that a new formal theory needs construction is that in mainstream economic theory, a person in a society (1) is identified exclusively by a *binary* relation of (weak) preference, (2) some restrictions are placed on this relation that generates a complete (or sometimes incomplete) ordering of the set of feasible personal alternatives of choice, (3) the sole motivation of the person is to maximize personal preference so as to determine personal choice, and (4) a set of additional sufficient restrictions are imposed to precipitate the existence of a cohesive *social (un)interactional outcome*, which is typically referred to as a *general equilibrium*, as in Arrow and Debreu.

To be able to investigate social phenomena where gender or race or any other such differences in personal identities are operationally significant, a release from the present confining format of economic theory is needed, that is alluded to by Arrow. To that we now proceed.

3. Preliminaries

For a given set V_i^j, let $R_i(V_i^j)$ be person i's binary relation of weak preference that stands for "at least as good as", which is defined on a finite set S_i of alternatives social states, and V_i^j is a background set on which the binary relation R_i is dependent, with $i = 1, \dots, n$, and $j =$

[7] To be distinguished from a nonempty personal maximal set.

$1, \dots, k_i$ specifying the possible parametric variations, V_i^j, of person i's background set. Here, $n \geq 2$ is finite, $k_i \geq 2$ is finite, and S_i has at least three elements.

For $R_i(V_i^j)$, we can define the asymmetric part $P_i(V_i^j)$ that stands for "strict preference", and the symmetric part $I_i(V_i^j)$ that stands for "indifference" as follows.

DEFINITION 1: $(\forall i, \forall j \,\&\forall x, y \in S_i)$: $[[xR_i(V_i^j)y] \& \sim[yR_i(V_i^j)x]] \leftrightarrow [xP_i(V_i^j)y]$.

DEFINITION 2: $(\forall i, \forall j \,\&\forall x, y \in S_i)$: $[[xR_i(V_i^j)y] \& [yR_i(V_i^j)x]] \leftrightarrow [xI_i(V_i^j)y]$.

In this context, it is important to note that a variation in a *tertiary* consideration, viz., a parametric variation in the background set, can, in general, alter the order of personal preference insofar as $(\forall i, \exists x, y \in S_i \,\&\, \exists j = l, m, l \neq m)$: $[xR_i(V_i^l)y] \,\&\, \sim[xR_i(V_i^m)y]$, are both admissible, thereby rendering $R_i(V_i^j)$ a non-binary relation.[8]

Our purpose here, instead, is to examine if a *social interaction outcome* based on *non-binary* personal preferences exists, and whether it is Pareto optimal. We prove that, in fact, there exists such an outcome, and that it is Pareto optimal, under some extremely mild conditions, at least judging by the restrictions imposed in the literature to demonstrate the existence of alternative types of equilibria that are based on mainstream utility theory and choice theory. Clearly, such a social outcome is a generalization that goes considerably beyond all of the alternative concepts of equilibrium in general equilibrium theory or in game theory that are predicated on binary personal preferences. There are three significant implications.

First, notice that since $R_i(V_i^j)$ a non-binary relation insofar as $(\forall i, \exists x, y \in S_i \,\&\, \exists j = l, m, l \neq m)$: $[xR_i(V_i^l)y] \,\&\, \sim[xR_i(V_i^m)y]$ are both admissible, the person in question decisively declares x to be at least as good as y if the background of the person is V_i^l but finds y to be at least as good as x or is indecisive about preference over this pair if the person's background is V_i^m . However, the pair of alternatives x and y are still not rank comparable if background set variations are also accommodated in the conceptual framework in which an investigation of such tertiary considerations is undertaken.

Second, identities, prejudices and biases, and shared identities and biases, can all be operationally captured by the personal background sets, which can then, in a one-way direction, have influence over the form that personal preferences may take. This is a new route that is opened up, for every person is society, to have personal beliefs and values influenced by, though not entirely determined by, social norms and customs.

Third, non-binariness of personal preferences violates some standard behavioral assumptions of rational choice theory such as the *Weak Axiom of Revealed Preference*, according to which, if x is chosen from $S \subseteq X$, and $x \in T \subseteq S$, then x must not be rejected in choice from T in favor of a distinct $y \in T$. The theory of non-binary choice based on preference can, in fact, accommodate this violation of *WARP* as well, and thus its scope and reach extend well beyond the present-day theories of existence of equilibria – general or of a game – that rely entirely on standard binary choice theory for determining behavior of an agent or a player.

This has substantive implications for enriching the class of social phenomena that can be explained axiomatically. Moreover, a finite array of heterogeneous non-binary preferences

[8] Sen (1997) develops the theory of maximization as a *personal* act of non-binary choice based on non-binary personal preference. Our purpose is not to examine the issues that arise in the context of binariness or non-binariness *per se*; Sen has already done that in his groundbreaking work in the context of personal choice.

ranking relations defined on the respective feasible sets, each assigned to a person in society, constitutes a conceptual framework that is sufficiently rich to permit an examination of injustices such as racial or gender discrimination that arise from different persons bearing distinct marks of social identification, which in turn, arise from each person's affiliation with distinct, though not disjoint, sets of communities (subsets) of persons in society.

4. Existence of a Pareto Optimal State

To achieve our objective, we utilize three lemmas in Sen (1970) with relatively minor generalizations to prove an existence theorem.[9] First, however, some definitions are in order.

DEFINITION 3: *Reflexivity*: $R_i(V_i^j)$ is reflexive over S_i if and only if $(\forall i, \forall j \,\&\forall x \in S_i)$: $[xR_i(V_i^j)x]$.

DEFINITION 4: *Acyclicity*: holds if and only if $(\forall i, \forall j \,\&\, \forall x_1,\ x_2, \ldots, x_l\ \in S_i,)$: $[(x_1 P_i(V_i^j)x_2$ & $x_2 P_i(V_i^j)x_3, \ldots \& \, x_{l-1} P_1(V_i^j)x_l)]\&(l \geq 3) \rightarrow\ x_1 R(V_i^j)x_l$.

DEFINITION 5: A ranking relation that is reflexive and acyclic is called an *acyclic-incomplete-ordering*.

Let $J = \bigcup_{i=1}^{n}(\bigcup_{j=1}^{k_i} V_i^j)$, and $S = \bigcap_{i=1}^{n} S_i \neq \emptyset$, and assume that *S* has at least three elements.

DEFINITION 6: A social interaction outcome *rule* is a functional relation *f* that assigns exactly one social ranking $R(S,J)$ of *S* to an inter-personal non-binary preference profile, such that

$R(S,J) = f\left(R_1(V_1^j), \ldots, R_n(V_n^j)\right)$, where $\forall i, j$: $R_i(V_i^j)$ is an acyclic-ordering of S_i.[10]

By $P(S,J)$we denote the asymmetric part of $R(S,J)$.We next turn to unanimity over a pair of alternatives under all possible variations of the background set to define Pareto preference.

DEFINITION 7A: $\forall j \,\&\forall x, y \in S$: $[\forall i{:}\, x\ R_i(V_i^j)\ y] \leftrightarrow x\ \hat{R}(S,J)\ y$.

DEFINITION 7B: $\forall j \,\&\forall x, y \in S$: $[\forall i{:}\, x\ P_i(V_i^j)\ y] \leftrightarrow x\ \hat{P}(S,J) y$.

Remark: Definition 7A is a generalization of the Pareto 'preference or indifference' rule to non-binary personal preferences over the set *S* of alternative social states, denoted by $\hat{R}(S,J)$, and similarly, Definition 7B is a generalization of the Pareto 'strict preference' rule, denoted by $\hat{P}(S,J)$.

DEFINITION 8: A social state *x* in *S* is *Pareto optimal* if and only if it is not Pareto dominated by any state *y* in *S* in accordance with Definition 7B.

Finally, using Definitions 6, *7A*, and *7B*, and by requiring that $[x\ R(S,J)\ y] \leftrightarrow [x\ \hat{R}(S,J)y]$ and $[x\ P(S,J)\ y] \leftrightarrow [x\ \hat{P}(S,J)y]$, we can obtain a maximal social interaction outcome by using the following two lemmas.

[9] *Lemma 4.2* below is Sen's (1997) own generalization to non-binary preferences (under transitivity). *Lemma 3.1* below is our generalization to non-binary preferences, and *Theorem 4.1* is our generalization of a lemma in Sen (1970), also to the case of non-binary preferences.

[10] A social interaction outcome rule is a generalization of what Sen (1970) calls a collective choice rule.

LEMMA 4.1. *$\hat{R}(S,J)$ is a reflexive and acyclic ranking of S.*

Proof: (See Sen (1970, Lemma 2*a, p.29)).

$\forall j$ & $\forall x \in S$, since by Definition 6, $\forall i\colon x\ R_i\left(V_i^j\right)\ x$, it follows that $\hat{R}(S,J)$ is reflexive. Also,

$$(\forall x_1,\ x_2, \dots, x_l\ \in S)\colon [(x_1\hat{P}(S,J)x_2\ \&\ x_2\hat{P}(S,J)x_3, \dots \&\ x_{l-1}\hat{P}(S,J)x_l)] \rightarrow\ x_1R\left(V_i^j\right)x_l$$

$$\rightarrow [(x_1\hat{R}(S,J)x_2\ \&\ x_2\hat{R}(S,J)x_3, \dots \&\ x_{l-1}\hat{R}(S,J)x_l)]$$

$$\rightarrow \left[\forall j\ \&\ \left\{\forall i\colon x_1\ R_i\left(V_i^j\right)\ x_2\ \&\ x_2\ R_i\left(V_i^j\right)\ x_3 \dots\ x_{l-1}\ R_i\left(V_i^j\right)\ x_l\right\}\right]$$

$$\rightarrow \forall j\ [\forall i\colon x_1\ R_i\left(V_i^j\right)\ x_l]$$

$$\rightarrow x_1\ \hat{R}(S,J)x_l\ .\ \blacklozenge$$

Next, consider

DEFINITION 9: $M(\hat{R},S,J) = \{x | x \in S\ \&\ {\sim}[\exists y \in S\colon y\ \hat{P}(S,J)\ x]\}$.

Remark: The *social interaction maximal set* of socially un-dominated elements of S is fully captured by Definition 9 with respect to the Pareto rule $\hat{P}(S,J)$, which is the asymmetric part given in Definition 7B .

DEFINITION 10: A *social interaction outcome* exists if and only if $M(\hat{R},S,J) \neq \emptyset$.

Remark: Definition 10 defines the concept of general equilibrium in this society, and is referred to here as a social interaction outcome.

LEMMA 4.2. *The maximal set is non-empty for every finite set that is reflexive and acyclically ranked by a non-binary preference relation.*

Proof: (See Sen (1970, Lemma 1*b, p.11, and Sen (1997)). Let $S = \{x_1, \dots, x_m\}$. Assign the real number $a_1 = x_1$, and follow the recursive rule $x_{q+1}\ \hat{P}(S,J)x_q \rightarrow a_{q+1} = x_{q+1}$, and $a_{q+1} = a_q$ otherwise, so that by construction, x_m is a maximal element. ♦

Remark: Note that non-binariness of *personal* preferences, in the sense that $(\forall i, \exists x, y \in S_i\ \&\ \exists j = l, m, l \neq m)\colon \left[xR_i\left(V_i^l\right)y\right]\ \&\ {\sim}[xR_i(V_i^m)y]$ are both admissible, poses no problem for obtaining a nonempty *social* maximal set since the personal non-comparability of a pair of alternatives in S is rendered irrelevant for defining the maximal set. This, of course, is not true of the social *optimal* set of best elements that is defined as $C\left(\hat{R},S,J\right) = \{x | x \in S\ \&\ \forall y \in S\colon x\ \hat{R}(S,J)y\}$, which would necessarily be empty if $(\forall i, \exists x, y \in S_i\ \&\ \exists j = l, m, l \neq m)\colon \left[xR_i\left(V_i^l\right)y\right]\ \&\ {\sim}[xR_i(V_i^m)y]$ are both admissible.

Thus, requiring maximizing behavior as an act of volitional personal choice, instead of the more demanding optimization, does have an advantage in the case of non-comparability arising from non-binariness of personal preferences. In fact, it should not come as a surprise that once there is a social acyclic-ordering which ranks at least one pair of alternatives, though not necessarily all such pairs, if and only if these two alternatives are comparable over all individuals and over all background sets, there must be an element which is Pareto un-dominated and thus Pareto optimal. This would also follow from Zorn's lemma.

In the case of personal choice theory, Sen (1997) exploits precisely this combination of (A) non-binariness of preferences (and the entailed partially non-comparable ranking), *and* demanding (B) maximizing personal behavior, that precipitates the existence of a maximal element despite non-comparability. The existence of an optimal element is more demanding, in that it requires (a) optimizing personal behavior, which yields the existence of such an optimal element only if (b) all personal preferences are binary, and they are reflexive transitive and also complete – this is the formulation of standard rational choice theory – both in general equilibrium theory and in game theory. We merely take Sen's (1997) work one step further to obtain a social interactional, rather than a personal, nonempty maximal set, and hence have the following result.

THEOREM 4.1. *For every set of non-binary personal preferences* $\left(R_1\left(V_1^j\right), \dots, R_n\left(V_n^j\right)\right)$ *over a finite set S of alternative social states, where* $\forall i, j: R_i\left(V_i^j\right)$ *is an acyclic-ordering, there exists a nonempty maximal social interaction set* $M(\hat{R}, S, J)$ *that contains at least one Pareto-optimal state.*[11]

Proof: (See Sen (1970, Lemma 2*e, p.30)). By Lemma 1, the Pareto preference or indifference relation $\hat{R}(S, J)$ is an acyclic-ordering of the set S of alternative social states. And considering Definitions 8, 9 and 10, it follows that the Pareto-optimal subset of S is equivalent to the social maximal set $M(\hat{R}, S, J)$. Further, since S is finite, and $\hat{R}(S, J)$ acyclically-orders it, by Lemma 2, $M(\hat{R}, S, J)$ is nonempty. Hence a non-binary personal preferences based social interaction outcome exists, and it is Pareto optimal. ♦

This proves the main result of this paper.

5. Concluding Remarks

To appreciate the minimal contribution of this work, let us, first, stay within the confines of the only economic theory that we know for sure is economic theory, to wit, in Arrow and Debreu. In characterizing their model, Arrow and Debreu write, "we assume the existence of a number of consumption units, typically families or individuals but including also institutional consumers." They further go on to say, "The set of consumption vectors X_i available to individual i (=1, … , m) is a closed convex subset of $\mathbb{R}^{l}$, which is bounded from below; i.e., there is a vector $\xi_i \leq x_i$ for all $x_i \in X_i$." (p. 278).

We impose upon individual behavior the discipline that every person is engaged in a volitional act of choice from amongst mutually exclusive actions, and that this (i) behavior is motivated by the aim of maximization of one's own preferences, (ii) which are reflexive and acyclic, and (iii) defined on the budget sets described above by Arrow and Debreu for all persons in a finite society, without redefining any element of these individual feasible sets. In this framework we ask: how can we escape homogeneity of social identity?

Answer: by assigning non-binary preferences to each person, $R(V)$, which is a ranking relation defined on the personal feasible set, which in the present case is taken to be the budget set S, where R is a binary ranking relation defined on the set S, and V is a set of which R is a function. Thus, $R(V_i), i = 1, \dots\ k, k \geq 1,$ is defined on $\forall V_i$, but for a finite k. For $k = 1$, $R(V) = R$ is a binary relation, as a special case. However, in general, $\exists x, y \in S: xR(V_1)y$ & $\sim[xR(V_2)y], \exists\ V_1 \neq\ V_2$.

With such preferences, social differentiation of persons emerges in society. Each individual, in this social economic theory, is a social creature with multiple identities, with

[11] It can be verified that *Axioms S*, *N*, *Q*, *M* and *I* constitute the conditions under which this theorem is true.

each identity arising from the person's affiliation with a well-defined, distinguishable and distinct community (subset) or persons. Communities are non-disjoint sets of persons who share multiple identities by membership of several communities.

Arrow and Debreu are looking for a positive price vector (that belongs to a unit simplex) at which for every commodity the quantity demanded and supplied are equal in the aggregate. For the aggregate, it is not material whether the demand for a commodity is coming from a man or a woman or from a Black or a White person. Since they do not need social differentiation in society, they model every individual by identifying the individual with a binary ranking relation defined on the individual's budget set.[12]

What we do here is to replace in the Arrow-Debreu model their assumption about preferences by a slightly modified set of personal preferences, in particular, non-binary preferences in the sense of Kanger-Sen above, and the model then exhibits social differentiation without redefining the personal feasible sets in Arrow-Debreu. The model is capable also of exhibiting injustices based on social identity, such as gender or racial discrimination that are impossible to observe in the strict Arrow-Debreu model, due to the binariness property of preferences.

We take this approach to modeling and examining interpersonal interaction of social creatures, that gets expressed as social interaction, and the outcome is some forms of social cohesion and justice, but also some forms of social tension and social injustices.[13] This is the immediate purpose of this paper.

Arrow (1998) warns, that in developing a theory of the injustice of racial discrimination, some approaches are inadequate,

> Most analysts, following Becker (1957), add to the usual list of commodities some special disutility which whites attach to contact with blacks, taste-based discrimination. … The trouble with these explanations is that they contradict in a direct way the usual view of employers as simple profit-maximizers. While they do not contradict rational choice theory, they undermine it by introducing an additional variable.
>
> There are at least two objections to this line of analysis. One is that introducing new variables easily risks turning the "explanation" into a tautology. …and it certainly would be a parody of economics to multiply entities in this anti-Occamian fashion. Perhaps more serious is the neglect of Darwinian principles. (pp. 94-95)

In the contributions of Becker (1957) in the analysis of discrimination, and of Akerlof and Kranton (2000) in their examination of identity in economics, since Becker as well as Akerlof and Kranton ascribe utility functions to persons, and thereby implicitly impose the binariness restriction on personal or players' preferences, their formal, utility-function based frameworks cannot bear the burden of supporting their claims, simply because social identity differentiation is impossible under binariness. [14]

[12] Arrow and Debreu do this in the interest of parsimony. Indeed, it would be anti-Occamian for them adopt a more general personal preference structure.

[13] This approach also has the advantage that it is in agreement with the suggestions for a theory of racial discrimination that Arrow (1998) outlines in his searching examination of the matter.

[14] I refer strictly to the *utility indicator function* as developed from rational choice theory by Arrow and Debreu (1954). This function $u = f(x)$ is a real-valued numerical representation of individual preferences, and it exists if personal preferences are represented by (i) a binary ranking relation R that stands for "at least as good as", it completely orders the person's feasible set S of alternatives (is (ii) reflexive, (iii) transitive and (iv) complete), is (v) continuous, and if (vi) all elements of S are bounded from below, rendering $f(x)$ also continuous and

However, the conceptual framework of non-binariness of preferences in society permits the investigation of a great many issues in addition to those related to social differentiation and social injustices that, in some cases, arise from that stratification, with higher or lower social valuation attached to persons belonging to different strata. Issues relating to matters of intra-family interaction, for example, could be investigated, though for this it will be necessary to redefine the feasible set of each member who is capable of exerting influence in the creation of the intra-family wellbeing distribution.[15] This will permit us to make cross-country comparisons of two societies' distributions to ascertain, for example, in which society female children receive better treatment.

Another fruitful route to pursue is to formalize each person's background set as the collection of shared beliefs both about the description of social reality and the social evaluation of different sub-groups in society being more or less valued. Sacred and profane, social anthropologists tell us, are ideas that reside in the collective consciousness of every tribe and every society that has ever existed or exists. Male is good, female is bad in the culture of the Indian subcontinent, for example, with fatal consequences for literally millions of female fetuses, little girls, and women.[16]

Also, White is sacred, and Black profane in the shared beliefs of people in the United States of America. This culture is so powerful that when Asians come in as new immigrants, they develop disdain for African Americans; over-valuation of one group while devaluation of a distinct group is what the new immigrants imbibe from the cup of the new culture they enter, oblivious of the fact that neither they nor their forefathers had ever faced any issues of contention with African Americans.

By specifying in each personal background set both (a) community values and beliefs (such as giving one's seat to a senior citizen, or not grabbing the largest slice of cake, as morally good values to have for a person), and (b) several observable marks of identification (such as life expectancy, gender, race, and so on), our theory of a society can provide formal explanations for the existence of social outcomes that can exhibit racial or gender discrimination, not to mention many other social injustices, but it can also produce forces of cohesion among members within communities, a matter that is not always appreciated, just because it is commonplace.[17]

Moreover, moving from *form* to alternative *interpretations* of the background set, and by considering parametric variations of this set, many of the inadequacies in explanations of social and economic phenomena entailed by binariness are entirely jettisoned, and indeed replaced by a much more comprehensive conceptual structure that still exhibits the unanimity (over a pair of alternatives) property embodied in the Pareto rule.

Since these discriminatory social states also are Pareto optimal, advocacy of such optimality implies tolerating discrimination, which could be construed as indictment of Pareto optimality as an objective worthy of pursuit.[18]

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bounded from below. Such a real valued, continuous function bounded from below qualifies to be called a utility function if and only if $\forall x, y \in S: ((xRy) \leftrightarrow ([f(x) \geq f(y)])\ \&\ \big((xPy) \leftrightarrow [f(x) > f(y)]\big)$.

[15] See Sen (1989) for a discussion of issues relating to such matters of intra-family relationships that our non-binary preference model made up of distinct members can also formalize.

[16] See Sen (1992).

[17] Notice that, unlike the case of the existence of equilibrium in an Arrow-Debreu exchange economy, and unlike the case of existence of Nash equilibrium, for our existence result we do not impose any requirements other than reflexivity and acyclicity on non-binary personal preferences.

[18] This technical note can also be seen as serving to provide the formal basis of the claims made in Naqvi (2010).

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