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R&D Output Sharing in a Mixed Duopoly and Incentive Subsidy Policy

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Abstract

This study investigates the incentives for R&D output sharing in a mixed duopoly and shows that public firm chooses full sharing of their R&D output, whereas private firm enjoys free-riding. We then devise an agreement-based incentive R&D subsidy scheme, which can internalize R&D spillovers and induce both firms to earn higher payoffs through full sharing of their R&D output. We also show that an R&D subsidy policy is welfare-superior to a production subsidy policy.

JEL Classifications: L13; L32; H21

Keywords: Agreement-based R&D subsidy; Mixed duopoly; Production output subsidy; R&D output sharing

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1. Introduction

In the last generation, there has been considerable empirical and theoretical works on the R&D (research and development) incentives, and a significant number of studies conclude that R&D spillovers exist and their implications on innovation and competition policy are presently gaining importance in economies.[1] The rate of spillovers in the literature is usually

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[1] See, for example, d'Aspremont and Jacquemin (1988), Kamien et al. (1992) Poyago-Theotoky (1995), Beath, et al. (1998), Gil Molto, et al. (2011) and Kesavayuth, et al (2017) among others.

defined as institutional and/or technological factors and thus, most of the existing literature treats the R&D spillovers in innovation as exogenous.

However, some economists have investigated the extent to which R&D activities might allow firms to internalize R&D spillovers. They examined the role of endogenous spillovers whereby spillovers arise from information-sharing and/or research design through firm-specific strategies. From a theoretical viewpoint, for example, Katsoulacos and Ulph (1988), Poyago-Theotoky (1999) and Baranes and Tropeano (2003) construct a model for endogenous R&D spillovers in which firms choose the spillovers rate for R&D output sharing in the non-cooperative equilibrium within a RJV (research joint venture).[2] The main focus of the literature is on a comparison between non-cooperative R&D and the case in which firms coordinate their R&D activities to maximize their joint profits (through RJV or a R&D cartel).

However, all these research studies consider the case of ex-ante identical symmetric private firms and as a result, if they fail to account for RJVs, the effect of R&D spillovers is to reduce the amount of R&D undertaken by each firm. This occurs because the profit from investing in R&D cannot be fully appropriated. Thus, knowledge sharing between the competing firms can lead one of them to free-ride by benefiting from the knowledge spillovers in the innovating activities without exerting any effort.

Policy makers have recognized the importance of R&D activities and have thus enacted various policies to encourage them. Among the effective policy alternatives in the real world, governments are continuously increasing R&D subsidization toward public institutions and organizations, such that public firms are the key players in R&D-intensive industries in

[2] Further, many empirical works and case studies also report widespread voluntary exchange of information and knowledge sharing between firms in the industry. See, for example, Jaffe et al. (1993), Saxenian (1994) and Keller (2000).

contemporary economies, such as in healthcare, medicine, energy, and bio-agriculture.[3] For example, Katsoulacos (1993) and Conte et al (2009) reported that the majority of RJVs supported by European RTD (research, technological development and demonstration) framework programs give an R&D subsidy to the participating firms if they share information through integrated projects or coordination actions.[4]

Much of the recent study of the relationship between R&D activity and subsidies has been conducted in the context of mixed oligopolies, where public and private firms compete in R&D investments.[5] For example, Zikos (2007), Gil Molto, et al. (2011), Kesavayuth and Zikos (2013) and Haruna and Goel (2017) investigated the role of R&D subsidies as a policy instrument in mixed oligopolies. However, they also treated the R&D spillovers as exogenous in the model of cost-reducing R&D.

The policy consequences of R&D subsidies in mixed markets are somewhat in contrast to those in the analysis of private markets. Due to the asymmetry in the objective functions that private firms maximize their profits whereas public firms are controlled by the government, a symmetric equilibrium in information sharing does not occur in mixed oligopolies. In particular, if we assume a benevolent government that maximizes social welfare, public firms invest more as the rate of spillovers is higher, whereas private firms invest less in the absence of subsidies. As a result, public firms would like to internalize the externality of the

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[3] Aanestad, et al. (2003) and Godø, et al. (2003) provided attentional case studies in the medical and energy sectors in European and OECD countries, and reported that public firms are key players in R&D-intensive industries. See also other interesting examples in Gil-Moltó, et al. (2011).

[4] The 8th framework program of EU research funding is running for 2014-2020 under the name of Horizon 2020. For more detailed information, see http://ec.europa.eu/programmes/horizon2020/h2020-sections.

[5] The increasing interest in mixed markets stems from their importance in regulatory reforms in the economies of developed regions, such as Western Europe, Canada, and Japan, and transitionary economies, such as those of China and Eastern Europe. Regarding R&D competition without government subsidies, Delbono and Denicolo (1993), Poyago-Theotoky (1998), Ishibashi and Matsumura (2006), Heywood and Ye (2009) and Nie and Yang (2015) examined R&D competition in mixed oligopolies.

knowledge spillover but private firms prefer to be free-riders. Therefore, policy makers should consider the importance of R&D policies regarding private firms in order to encourage their R&D knowledge sharing.

In this study, we consider a mixed duopoly where a public firm competes with a private firm in both R&D investment and production output by endogeneizing the choice of the rate of R&D spillovers between the firms. We investigate the incentives of R&D output sharing and show that a public firm will choose full sharing whereas a private firm will enjoy free-riding. It shows that the internalization of externalities improves social welfare even though a private incentive does not incorporate externalities. We then devise an agreement-based incentive R&D subsidy scheme in which both firms earn higher payoffs by full sharing of their R&D output. This is because (i) the agreement can make each firm receive a beneficial cost spillover with a maximum rate from its rival firm, and (ii) the subsidy can increase the private firm's profits. Thus, it benefits both firms to internalize R&D spillovers by compensating for the loss of information sharing. Further, we show that an agreement-based R&D subsidy policy is welfare-superior to a production subsidy policy. This is because an incentive R&D subsidy can compensate for the wasteful cost asymmetry associated with the public firm's higher production output relative to that of the private firm.[6]

The organization of this paper is as follows: In Section 2, we present a mixed duopoly model, in which a public firm competes with a private firm in both R&D investment and production output. In Section 3, we show that in the absence of government subsidy, a public firm chooses full sharing of R&D output, whereas a private firm enjoys free-riding. In section 4, we devise an agreement-based incentive R&D subsidy scheme to internalize R&D spillovers

[6] It is well-known in the literature on mixed oligopolies with R&D investments that a public firm provides more production output than the private firm even though the public firm undertakes more R&D, and thus the distribution of production costs across firms is not efficient. On this point, see Gil Molto, et al. (2011), Kesavayuth and Zikos (2013), Lee and Tomaru (2017) and Lee, et al (2017).

in which both firms earn higher payoffs by full sharing of their R&D output. In Section 5, we compare the R&D subsidy with a production subsidy and show that an agreement-based R&D subsidy is welfare-superior to a production subsidy. Finally, we conclude in Section 6.

2. The Model

Consider a Cournot duopoly market, where two firms produce homogeneous goods. Let the inverse demand function be $P(Q) = a - Q$, where P is market price, $Q(= q_0 + q_1)$ is the total market production, and q_i is the production output of firm *i*, $(i = 0,1)$, respectively. Then, the consumer surplus can be computed as $\mathrm{CS} = Q^2/2$.

We consider that each firm has an ex-ante identical cost function with decreasing returns to scale in production and R&D investment. In particular, we assume the following specific cost functions, proposed by Gil Molto, et al. (2011) and Kesavayuth and Zikos (2013):

$$C(q_i, x_i) = (c - x_i - d_j x_j)q_i + q_i^2 \text{ and } \Gamma(x_i) = x_i^2, \quad i \neq j = 0,1 \tag{1}$$

The ex-ante cost c $(a > c > 0)$ is reduced by each firm's R&D output, x_i, and rival's R&D output, $d_j x_j$, where $d_j \in [0,1]$ denotes the R&D output sharing rate, which is determined by rival firm. It implies that R&D investment can reduce a firm's own cost by x_i and the rival firm's cost by $d_i x_i$ per unit of output, depending on the endogenous choice of R&D output sharing rate. For instance, $d_j = 0$ represents perfect protection of R&D output while $d_j = 1$ represents full sharing of R&D output. Further, the firm has to spend the amount of R&D investment, $\Gamma(x_i) = x_i^2$, to implement the cost-reducing R&D.

A few remarks are in order. First, we assume a quadratic production cost function, which is standard in mixed market literature, to rule out the uninteresting case of a public monopoly. Second, the production cost shows that both a firm's R&D output and the rival's R&D output sharing rate shift the firm's marginal production cost function downwards, as $\frac{\partial C}{\partial q_i} = c - x_i -$

$d_j x_j + 2q_i$, but do not alter its slope. Finally, we assume that the R&D spillovers rate is solely limited by the information that the providing firm allows (or protects). That is, the amount of knowledge spillovers that the recipient can receive depends on the provider.[7]

The profit of the firm and social welfare functions, respectively, are as follows:

$$\pi_i = \big(a - q_0 - q_1\big)q_i - (c - x_i - d_j x_j\big)q_i - q_i^2 - x_i^2, \quad i \neq j = 0,1. \tag{2}$$

$$W = CS + \pi_0 + \pi_1. \tag{3}$$

We consider a mixed duopoly market where a profit-maximizing private firm, firm 1, competes with a state-owned public firm, firm 0, which maximizes social welfare. In the followings, we will examine two different cases. In Section 3, we consider the case without a government subsidy and examine a three-stage simultaneous game: In the first stage, each firm chooses whether to share its R&D output with its rival. Then, observing the agreement on the R&D output sharing, each firm chooses its R&D investment level in the second stage and its production output level in the third stage. In Section 4, we extend the game structure and further consider the intervention of the government before the first stage of the game. Then, we construct an agreement-based R&D subsidy scheme. We analyze the subgame perfect Nash equilibrium by backward induction.

3. The Analysis without Government Intervention

In the third stage, the first-order conditions of the private firm and the public firm yields the following reaction functions for the firms:

$$q_0 = \frac{(a-c) - q_1 + x_0 + d_1 x_1}{3} \text{ and } q_1 = \frac{(a-c) - q_0 + x_1 + d_0 x_0}{4}. \tag{4}$$

[7] However, a recipient firm's capacity to benefit from the R&D conducted by other firms may also depend on the amount it spends on R&D activities. In this sense, the recipient of a spillover may be able to affect the maximum amount of the spillovers rate it receives through the actions it takes.

Noting that both firms' decisions on the final products are strategic substitutes, we have the equilibrium outputs of the second stage:

$$q_0^* = \frac{3(a-c)+4x_0-d_0x_0-x_1+4d_1x_1}{11} \text{ and } q_1^* = \frac{2(a-c)-x_0+3d_0x_0+3x_1-d_1x_1}{11}. \quad (5)$$

Then, we have $\frac{\partial q_0^*}{\partial x_i} > \frac{\partial q_1^*}{\partial x_i} > 0$: an increase in R&D by one firm increases the production of both firms, but that of the public firm is higher than that of the private firm. Thus, R&D investment will increase total industry productions, that is, $Q^* = q_0^* + q_1^*$ and $\frac{\partial Q^*}{\partial x_i} > 0$.

In the second stage, from the first-order conditions of public and private firms, using the envelope theorem and rearranging for the necessary calculations yield the following reaction functions:

$$x_0 = \frac{(a-c)(31+28d_0)-(14-14d_0(3-d_1)-45d_1)x_1}{197+14d_0(2-3d_0)} \text{ and } x_1 = \frac{2(3-d_1)(2(a-c)-(1-3d_0)x_0)}{103+2(6-d_1)d_1}. \quad (6)$$

It is noteworthy that the public firm's decision on R&D investment is strategic substitute (complement) to the private firm's decision when $14 - 14d_0(3 - d_1) - 45d_0 < (>)0$, while the private firm's decision is strategic substitute (complement) to the public firm's decision when $d_0 < (>)1/3$. Then, we have the equilibrium R&D investment of the first stage:

$$x_0^* = \frac{(a-c)(25+28d_0+2(4-d_1)d_1)}{167-42d_0^2+22d_1-4d_1^2+28d_0-6d_0d_1+2d_0d_1^2} \text{ and } x_1^* = \frac{2(a-c)(3+d_0)(3-d_1)}{167-42d_0^2+22d_1-4d_1^2+28d_0-6d_0d_1+2d_0d_1^2} \quad (7)$$

Thus, we have the following equilibrium production outputs:

$$q_0^* = \frac{(a-c)(53+d_0((15-14d_0))+4((4-d_1))d_1)}{167-42d_0^2+22d_1-4d_1^2+28d_0-6d_0d_1+2d_0d_1^2} \text{ and } q_1^* = \frac{11(a-c)(3+d_0)}{167-42d_0^2+22d_1-4d_1^2+28d_0-6d_0d_1+2d_0d_1^2} \quad (8)$$

The profit of the private firm and the social welfare at the equilibrium, respectively, can be rewritten as follows:

$$\pi_1^*(d_0, d_1) = \frac{2(a-c)^2(3+d_0)^2(103+2(6-d_1)d_1)}{(167-42d_0^2+22d_1-4d_1^2+28d_0-6d_0d_1+2d_0d_1^2)^2}, \tag{9}$$

$$W^*(d_0, d_1) = \frac{(a-c)^2[d_0\left(3662-d_0\left(2703+98d_0(8-3d_0)\right)\right)+8(74-81d_0)d_0d_1-4(2-d_0)(48+41d_0)d_1^2 +8(74-81d_0)d_0d_1-4(2-d_0)(48+41d_0)d_1^2-160d_1^3+20d_1^4+8(967+361d_1)]}{(167-42d_0^2+22d_1-4d_1^2+28d_0-6d_0d_1+2d_0d_1^2)^2} \tag{10}$$

In the first stage, each firm decides at which degree it will share its R&D output with its rival firm. Regarding the public firm's incentive, we can show that $\frac{\partial W^*}{\partial d_0} > 0$ for any $d_1 \in [0,1]$. This represents that the public firm's dominant strategy is to choose full sharing of its R&D output, i.e., $d_0^* = 1$. This is because the public firm can internalize the externalities in R&D output to improve welfare, which is the objective of a public firm. It also implies that the private firm's decision on R&D investment is always a strategic complement to the public firm's decision because $d_0^* > 1/3$ at equilibrium. Regarding the private firm's incentive, however, we can show that $\frac{\partial \pi_1}{\partial d_1} < 0$ when $d_0 = 1$. This represents that the private firm chooses perfect protection of its R&D output at equilibrium, i.e., $d_1^* = 0$. This is also because a profit-maximizing private firm is not willing to internalize the externalities to enjoy a free-riding effect. It is interesting to note that the public firm's decision on R&D investment is always a strategic substitute to that of the private firm at equilibrium.

From (7) and (8), we have:

$x_0^* = \frac{53(a-c)}{153} \cong 0.3464(a-c) > x_1^* = \frac{24(a-c)}{153} \cong 0.1569(a-c)$ and

$q_0^* = \frac{54(a-c)}{153} \cong 0.353(a-c) > q_1^* = \frac{44(a-c)}{153} \cong 0.2876(a-c)$.

Further, from (9) and (10), we have:

$\pi_1^*(1,1) \cong 0.130(a-c)^2 < \pi_1^*(1,0) \cong 0.141(a-c)^2$ and

$W^*(1,1) \cong 0.378(a-c)^2 > W^*(1,0) \cong 0.350(a-c)^2$.

These represent that there is a conflict of incentives between the public and private firms in deciding the agreement on the R&D output sharing.

Proposition 1. *In the equilibrium of R&D output sharing, the public firm chooses full sharing and undertakes a higher R&D investment, whereas the private firm chooses perfect protection and undertakes a lower R&D investment.*

4. Agreement-based Incentive R&D Subsidy Scheme

In this section, we consider the commitment of the government before the first stage of the previously discussed game, and provide an agreement-based incentive R&D subsidy scheme. In particular, we assume that the government can provide an R&D subsidy only when both the public and private firms make a mutual agreement on the R&D output sharing: if they fail to make this agreement, no subsidy will be provided.

We now examine the effect of an agreement-based incentive R&D subsidy when both firms make a mutual agreement regarding the rate of R&D spillovers, d_i. As shown in Proposition 1, welfare is maximized when $d_i = 1$ and thus, the public firm will prefer that the private firm chooses $d_1 = 1$. Further, due to the positive externalities, the private firm will always prefer that the public firm chooses $d_0 = 1$. Therefore, it is sufficient for a government to devise an incentive R&D subsidy scheme, which will induce the private firm to choose $d_1 = 1$ when $d_0 = 1$. This implies that a government's optimal choice is to provide an R&D subsidy only when the private firm chooses $d_1 = 1$, under which the subsidized profit is always higher than that with $d_1 < 1$. Thus, no subsidy policy is sufficient when $d_1 < 1$.

In the following analysis, we assume that the government provides a unit subsidy on the firm's R&D output. Then, the subsidized profit of the firm under the incentive R&D subsidy policy with full sharing, that is, when both $d_i = 1$, is as follows:

$$\pi_i = (a - q_0 - q_1)q_i - (c - x_i - x_j)q_i - q_i^2 - x_i^2 + s_x x_i, \;\; i \neq j = 0,1. \tag{11}$$

where s_x is the R&D subsidy rate. We assume that the subsidy is financed by the taxpayers in a lump-sum manner, and thus, it does not directly influence the welfare function. Note that the total R&D output, $\mathrm{X} = x_i + x_j$, affects the production cost of both firms.

In the below analysis, note that the R&D subsidy affects each firm's R&D decision in the second stage but does not explicitly affect their production output decision in the third stage. In the second stage, from the first-order conditions of the maximization problems of the public and private firm in terms of R&D investment, we have the following reaction functions:

$$x_0 = \frac{59(a-c)+59x_1}{183} \text{ and } x_1 = \frac{16(a-c)+16x_0+121s_x}{226}. \tag{12}$$

The reaction function of each firm increases with the rival's R&D investment. It is noteworthy that R&D investments under full sharing of R&D output are strategic complements for both firms. An increase in R&D investment by one firm leads to an increase in the production output by its rival firm, thereby increasing both firms' incentives to conduct R&D. Then, we have the following equilibrium R&D investments of the second stage:

$$x_0^R = \frac{1298(a-c)+649s_x}{3674} \gtrless x_1^R = \frac{352(a-c)+2013s_x}{3674} \text{ if } s_x \lessgtr \frac{946(a-c)}{1364} \cong 0.6935(\mathrm{a-c}) > 0. \tag{13}$$

These equations show that the relative R&D outputs depend on the R&D subsidy, which affects the asymmetric distribution of the production costs between the firms. In particular, the public firm undertakes more (less) R&D than the private firm as the R&D subsidy rate is lower (higher). Further, we have that $\frac{\partial x_1^R}{\partial s_x} > \frac{\partial x_0^R}{\partial s_x} > 0$, which shows that both firms' R&D investments increase with the R&D subsidy, but the increasing rate of the private firm's R&D is higher than that of the public firm. Hence, the total R&D output, $\mathrm{X^R} = x_0^R + x_1^R$, also increases with the R&D subsidy, that is, $\partial X^R/\partial s_x > 0$.

Thus, we have the following equilibrium production outputs:

$$q_0^R = \frac{66(a-c)+33s_x}{167} > q_1^R = \frac{44(a-c)+22s_x}{167}. \quad (14)$$

This shows that the public firm provides more production output than the private firm even though the public firm undertakes more or less R&D, depending on the R&D subsidy. Note that the R&D subsidy can induce both private and public firms to enlarge their production outputs and R&D investments as well. Therefore, the total industry outputs, $\mathrm{Q}^{\mathrm{R}} = q_0^R + q_1^R$, increases with the R&D subsidy, that is, $\partial Q^R/\partial s_x > 0$.

The government chooses an R&D subsidy rate to maximize the social welfare function:

$$W = \frac{(Q^R)^2}{2} + \pi_0(x_0^R, x_1^R, q_0^R, q_1^R) + \pi_1(x_0^R, x_1^R, q_0^R, q_1^R) - s_x(x_0^R + x_1^R). \quad (15)$$

Then, we have the following optimal R&D subsidy rate:

$$s_x = \frac{43(a-c)}{62} \cong 0.6935(\mathrm{a}-\mathrm{c}) > 0. \quad (16)$$

Finally, the equilibrium outcomes under the full sharing of R&D output are as follows:

$$x_0^R = x_1^R = \frac{59(a-c)}{124} \cong 0.476(a-c) \text{ and}$$

$$q_0^R = \frac{33(a-c)}{62} \cong 0.532(a-c) > q_1^R = \frac{11(a-c)}{31} \cong 0.333(a-c).$$

Note that $x_0^R = x_1^R$ and $q_0^R > q_1^R$ at equilibrium. It represents that the optimal R&D subsidy can induce both firms to undertake the same R&D investments, which will be fully internalized through the full sharing of R&D spillovers, but the public firm still provides more production output than the private firm, which results in a higher marginal production cost for the public firm than that of the private firm. It also provides the following profit and social welfare under the incentive R&D subsidy policy, respectively:

$$\pi_1^R = \frac{5465(a-c)^2}{15376} \cong 0.355(a-c)^2 \text{ and } W^R = \frac{59}{124}(a-c)^2 \cong 0.476(a-c)^2.$$

Proposition 2. *Under the incentive R&D subsidy, both firms agree on the full sharing of their R&D outputs and undertake the same R&D investments.*

Proof. It is sufficient to show that the private firm chooses the full sharing of its R&D output and can maximize its profit, that is, $\pi_1^R(1,1) \cong 0.355(a-c)^2 > \pi_1^*(1,0) \cong 0.141(a-c)^2$.

5. Comparison and Discussions

5.1 Comparison with efficient outcomes

Under the incentive R&D subsidy scheme, the public firm still provides more production output than that of the private firm even though both firms undertake the same R&D investments. Thus, the distribution of production costs across the two firms is not efficient. This implies that the obtained welfare in W^R is the second-best optimum. In this subsection, we will examine the efficient first-best allocations and investigate the welfare consequences of the incentive R&D subsidy scheme.

In the first-best allocations, the social welfare is maximized under the full sharing of R&D outputs and the marginal cost pricing rule in both R&D investments and production outputs. Then, we can directly obtain the following efficient outcomes:

$$x_0^F = x_1^F = \frac{a-c}{2},\ q_0^F = q_1^F = \frac{a-c}{2} \text{ and } W^F = (a-c)^2.$$

The efficient outcomes indicate that both firms undertake the same R&D investments and provide the same production output, which yields a symmetric distribution of production costs to two firms and the market price equals each firm's marginal production cost. Then, comparing the efficient outcomes with the second-best outcomes under the incentive R&D subsidy shows that both firms undertake less R&D investments, i.e., $x_i^* > x_i^R$, but the public firm over-produces, whereas the private firm under-produces, i.e., $q_0^R > q_i^F > q_1^R$. Thus, in addition to the R&D subsidy, other policy instrument is required, which results in the redistribution of the production output from the higher-marginal-cost public firm to the lower-marginal-cost private firm. For example, if the government is able to use the optimal policy mix of R&D and production output subsidies, it can be easily shown that the first-best outcome is obtained and thus social welfare is maximized.[8]

5.2 Comparison with production output subsidy

Instead of the incentive R&D subsidy scheme, we can consider an incentive production output subsidy scheme and examine the welfare effects.[9] We assume that government can provide a unit subsidy on the firm's production output level under a mutual R&D agreement with full sharing of the R&D output. Then, it can decide the optimal production output subsidy rate to maximize welfare. The subsidized profit of the private firm in (11) includes $s_q q_i$ as a production subsidy rate, instead of $s_x x_i$ in the R&D subsidy term. Using the same procedures taken in the previous section for the R&D subsidy, we have the following results under a production subsidy.

In the third stage, the production output subsidy decreases the production output of public

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[8] See Zikos (2007) and Lee and Tomaru (2017) on the optimal subsidization policy mix. On the other hand, Kesavayuth and Zikos (2013) and Lee, et al. (2017) compared the relative welfare effects of an R&D subsidy and a production output subsidy in a mixed duopoly.

[9] The use of a production subsidy has been proposed in the literature of mixed oligopolies, in which the so-called "irrelevance result" is argued, which states that privatization does not alter welfare as long as the regulator can subsidize the production output. For a recent discussion, see Matsumura and Tomaru (2013, 2015), Matsumura and Okumura (2013, 2017) and Lee, et al. (2017).

firm while it increases that of the private firm as follows:[10]

$$q_0^o = \frac{3(a-c+x_0+x_1)-s_q}{11} \text{ and } q_1^o = \frac{2(a-c+x_0+x_1)+3s_q}{11}. \quad (17)$$

In the second stage, we have the following relationship between the two firms' equilibrium R&D investments:

$$x_0^o = \frac{252s_q+1298(a-c)}{3674} \gtrless x_1^o = \frac{352(a-c)+408s_q}{3674} \text{ if } s_q \lessgtr \frac{946(a-c)}{156} \cong 6.064(\mathrm{a}-\mathrm{c}) > 0. \quad (18)$$

Note that the public firm undertakes more (less) R&D than the private firm as production output subsidy is lower (higher). Further, we have $\frac{\partial x_1^o}{\partial s_q} > \frac{\partial x_0^o}{\partial s_q} > 0$. This shows that both firms' R&D investments increases with the output subsidy, but the rate of increase for the private firm's R&D is higher than that of the public firm's R&D.

In the first stage, we have the following relationship between the two firms' equilibrium production outputs:

$$q_0^o = \frac{66(a-c)-7s_q}{167} > 0 \text{ if } s_q < \frac{66(a-c)}{7} \cong 9.4286(\mathrm{a}-\mathrm{c}). \quad (20)$$

$$q_0^o = \frac{66(a-c)-7s_q}{167} \gtrless q_1^o = \frac{44(a-c)+51s_q}{167} \text{ if } s_q \lessgtr \frac{11(a-c)}{29} \cong 0.3793(\mathrm{a}-\mathrm{c}) > 0. \quad (21)$$

It is noteworthy that the production output subsidy decreases the production output of the public firm while it increases that of the private firm. Thus, depending on the production output subsidy, the public firm might provide less production output than the private firm. This result comes from the fact that that both firms' production outputs are strategic

[10] Note that the subsidized profit of private firm is increasing in the subsidy rate, while social welfare, the objective of public firm, does not change. Since both firms' production outputs are strategic substitutes, the public firm decreases its production output as private firm increases its production output according to the increase of the subsidy rate.

substitutes and thus, the public firm provides less production output when the output subsidy rate is higher. Further, note that both firms can enjoy the full sharing of the R&D output and thus, their R&D choices do not matter with the R&D externalities in the cost-reducing effect. Thus, the production output choice of the public firm solely depends on the production output choice of the private firm under production output subsidy.[11]

Subsequently, the government chooses a production output subsidy rate to maximize the social welfare, which yields the following optimal production subsidy rate:

$$s_q = \frac{36289(a-c)}{55925} \cong 0.649(\mathrm{a}-\mathrm{c}) > 0. \quad (22)$$

The equilibrium outcomes under the full sharing of R&D output are as follows:

$$x_0^o = \frac{22247(a-c)}{55925} \cong 0.398(\mathrm{a}-\mathrm{c}) > x_1^o = \frac{9388(a-c)}{55925} \cong 0.168(\mathrm{a}-\mathrm{c}) \text{ and}$$

$$q_0^o = \frac{20581(a-c)}{55925} \cong 0.368(\mathrm{a}-\mathrm{c}) < q_1^o = \frac{25817(a-c)}{55925} \cong 0.46(\mathrm{a}-\mathrm{c}).$$

Note that $x_0^o > x_1^o$ but $q_0^o < q_1^o$ at equilibrium. This shows that the public firm undertakes more R&D but provides less production output than the private firm. This is contrast to the established result in the literature of mixed oligopoly, in which the public firm will undertake more R&D and provide more production output than the private firm even though the public firm can be inefficient in its production. However, under the full sharing of R&D output, this story can be reversed because the externality effect of the R&D spillovers can be fully internalized. Regarding the R&D investment, the welfare-maximizing public firm has a larger incentive to undertake more R&D than the profit-maximizing private firm, but both R&D investments are still lower than the first-best, that is, $x_i^F > x_0^o > x_1^o$. Regarding the

[11] In the Appendix, we analyze the optimal production output subsidy with the R&D spillovers rate and show that there exists a threshold of R&D spillovers, for which the public firm provides less (more) production output than the private firm with a higher (lower) R&D spillovers.

production output, however, the welfare-maximizing public firm provides less production output than the profit-maximizing private firm, but both production outputs are still lower than the first-best, that is, $q_i^F > q_1^o > q_0^o$. This is because a higher R&D cost discourages the public firm from increasing its production output, whereas a higher production output subsidy encourages the private firm to increase its production output. Due to the production output substitution effect between homogeneous products, this can result in a better balance of production costs between the firms, from the viewpoint of welfare. That is, each firm can receive beneficial R&D spillovers from its rival under an agreement-based incentive subsidy, but the cost savings under a production output subsidy can compensate for the wasteful cost asymmetry associated with the public firm's higher R&D investment relative to that of the private firm. Therefore, an incentive production output subsidy yields two production-related inefficiencies: the production output level is sub-optimal and the distribution of the production costs between the firms is not efficient.

Finally, we have the following profit and social welfare under the incentive production output subsidy policy, respectively::

$\pi_1^o = \frac{1244900434}{3127605625}(a-c)^2 \cong 0.398(a-c)^2$ and $W^o = \frac{23479}{55925}(a-c)^2 \cong 0.420(a-c)^2$.

Proposition 3. *Under a mutual agreement on the full sharing of R&D outputs, an incentive R&D subsidy provides higher welfare than an incentive production output subsidy.*

Proof. We can show that (i) the private firm will choose the full sharing of its R&D output under an incentive output subsidy, that is, $\pi_1^*(1,0) = 0.355(a-c)^2 < \pi_1^o(1,1) = 0.398(a-c)^2$, but (ii) the welfare under a production output subsidy is lower than that under an R&D subsidy, that is, $W^R(1,1) = 0.476(a-c)^2 > W^o(1,1) = 0.420(a-c)^2$.

This proposition implies that the efficiency gains from an R&D subsidy are relatively larger than those from a production subsidy.[12] It supports the findings in Kesavayuth and Zikos (2013), who assumed exogenous spillovers in the cost-reducing R&D and showed that if spillovers are high, then an increase in R&D incentivized by an R&D subsidy generates a larger overall cost reduction compared to a production output subsidy.[13]

6. Concluding Remarks

The role of the government in R&D sharing activities among innovative firms is significantly relevant in the current economics of the innovation system. We investigated the incentives of R&D output sharing in a mixed duopoly and showed that without the government's intervention, the public firm chooses full sharing whereas the private firm enjoys free-riding. We then devised an agreement-based incentive R&D subsidy scheme to internalize the R&D spillovers, in which both firms earn higher payoffs by full sharing of their R&D outputs. We also showed that an agreement-based R&D subsidy is welfare-superior to a production output subsidy policy.

There remain some topics for future research. We used the simplified Cournot duopoly model with homogeneous products in cost-reducing innovation under endogenous choice of R&D spillovers. However, the strategic choice of R&D output sharing crucially depends on the innovation of the products in different industries and regions. Thus, further examinations of the endogenous market structure, such as Cournot, Bertrand, and Stackelberg, should be analyzed under a differentiated products market.

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[12] It also shows that the irrelevance result does not hold in the presence of R&D. See, Lee and Tomaru (2017) and Lee, et al (2017).

[13] When spillovers are sufficiently low, however, Lee, et al. (2017) showed that a production output subsidy is welfare-superior to an R&D subsidy, but the government has a higher incentive to privatize the public firm under a production output subsidy than under the R&D subsidy.

Appendix: Optimal Production Output Subsidy with R&D Spillovers

We analyze the optimal production output subsidy under the symmetric R&D spillovers rates between the two firms, i.e., $d \in [0,1]$, where the profit of the firm is defined as follows:

$$\pi_i = \left(a - q_0 - q_1\right)q_i - \left(c - x_i - dx_j\right)q_i - q_i^2 - x_i^2 + s_q q_i, \quad i \neq j = 0,1.$$

In the third stage, solving the first-order conditions yields the equilibrium outputs:

$${q_0}^O = \frac{3(a-c)+x_0(4-d)-x_1(1-4d)-s_q}{11} \text{ and } .{q_1}^O = \frac{2(a-c)-x_0(1-3d)+x_1(3-d)+3s_q}{11}$$

In the second stage, using the previous results and taking output subsidy as given, solving the first-order conditions yields the equilibrium R&D investment:

$${x_0}^O = \frac{11(a-c)(25+36d-2d^2)-3(17-77d+20d^2-2d^3)s_q}{11(167+50d-52d^2+2d^3)} \text{ and } {x_1}^O = \frac{2(3-d)(11(a-c)(3+d)+(54+6d-9d^2)s_q)}{11(167+50d-52d^2+2d^3)}.$$

Note that output subsidy encourages the R&D investment of the private firm while its effect on that of the public firm depends on the R&D spillovers rate. In particular, it will discourage if the spillovers rate is low while it will encourages if the spillovers rate is high. This provides the equilibrium production outputs:

$${q_0}^O = \frac{11(a-c)(53+31d-18d^2)-(215-163d+11d^2+20d^3-6d^4)s_q}{11(167+50d-52d^2+2d^3)} \text{ and } {q_1}^O = \frac{11(a-c)(3+d)+(54+6d-9d^2)s_q}{167+50d-52d^2+2d^3}.$$

Note that output subsidy encourages the output of the private firm, but discourages that of the public firm. The profit of the private firm and social welfare are, respectively:

$$\pi_1^O = \frac{2(103+12d-2d^2)(11(A-c)(3+d)+(54+6d-9d^2)s_q)^2}{121(167+50d-52d^2+2d^3)^2} \text{ and}$$

$$W^O = \frac{\begin{array}{c}242(a-c)^2(7736+6550d-2495d^2-1728d^3+478d^4)\\+66(a-c)\left(6159+10255d+838d^2-4638d^3+90d^4+600d^5-108d^6\right)s_q\\-3(250025+136138d-168501d^2-21604d^3+34566d^4-7496d^5+368d^6+240d^7-36d^8)s_q^2\end{array}}{6749138+4041400d-3598056d^2-1096744d^3+702768d^4-50336d^5+968d^6}.$$

We turn to the first stage where the government determines the output subsidy to maximize welfare. Then, using previous results and differentiating welfare with respect to s_q yield the optimal production output subsidy rate:

$$s_q = \frac{11(a-c)(6159+10255d+838d^2-4638d^3+90d^4+600d^5-108d^6)}{250025+136138d-168501d^2-21604d^3+34566d^4-7496d^5+368d^6+240d^7-36d^8} > 0.$$

Note that the optimal production output subsidy rate is positive and increasing in the spillovers rate, i.e., $\frac{\partial s_q}{\partial d} > 0$. Finally, the equilibrium outcomes under the optimal production output subsidy are as follows:

$$x_0{}^O = \frac{2(a-c)(17774+34511d+1624d^2-10991d^3+1463d^4+233d^5-138d^6+18d^7)}{250025+136138d-168501d^2-21604d^3+34566d^4-7496d^5+368d^6+240d^7-36d^8} \text{ and}$$

$$x_1{}^O = \frac{2(a-c)(19449+10134d-10171d^2-2386d^3+1772d^4+98d^5-138d^6+18d^7)}{250025+136138d-168501d^2-21604d^3+34566d^4-7496d^5+368d^6+240d^7-36d^8}.$$

$$q_0{}^O = \frac{(a-c)(71420+61043d-42667d^2-16506d^3+10342d^4-1470d^5+162d^6)}{250025+136138d-168501d^2-21604d^3+34566d^4-7496d^5+368d^6+240d^7-36d^8} \text{ and}$$

$$q_1{}^O = \frac{11(a-c)(6483+5539d-1544d^2-1310d^3+154d^4+84d^5-18d^6)}{250025+136138d-168501d^2-21604d^3+34566d^4-7496d^5+368d^6+240d^7-36d^8}.$$

$$W^O = \frac{(a-c)^2(142787+121054d-58211d^2-28176d^3+11482d^4-1248d^5+144d^6)}{500050+272276d-337002d^2-43208d^3+69132d^4-14992d^5+736d^6+480d^7-72d^8}.$$

From the comparisons, we have $x_0{}^O \gtrless x_1{}^O$ and $q_0{}^O \lesseqgtr q_1{}^O$ if $d \gtrless 0.067$. Thus, we can show that there exists a threshold of R&D spillovers rate, for which the public firm provides less (more) production output and takes more (less) R&D investment than the private firm with a higher (lower) R&D spillovers. Note that the difference of $x_0{}^O - x_1{}^O$ is increasing in the spillovers rate while the difference of $q_0{}^O - q_1{}^O$ is decreasing in the spillovers rate. Finally, social welfare is also increasing in the spillovers rate.

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