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3 January 2017

Online at <https://mpra.ub.uni-muenchen.de/81753/>

MPRA Paper No. 81753, posted 03 Oct 2017 14:52 UTC

# Measuring the size of the shadow economy using a dynamic general equilibrium model with trends\*

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October 2, 2017

## Abstract

We propose a methodology for measuring the size and properties of the shadow economy. We use a two-sector dynamic deterministic general equilibrium model with four different trends: hours worked, investment-specific productivity, formal productivity, and shadow productivity. We find that the shadow productivity trend is endogenous, in the sense that it is an exact function of model parameters and the other three trends. We also document that, in order to be consistent with observed (real-world) trend growths, the shadow sector needs to exhibit increasing returns to scale, which is contrary to the standard procedure of imposing decreasing returns to this sector. We apply our methodology to a set of seven Latin American and Asian countries and document several empirical regularities that emerge from our analysis, the most important one being that the volatility of shadow sector output is considerably larger than the one in formal sector output.

**JEL codes:** E26, E32, O17.

**Keywords:** shadow economy, business cycles, DSGE models.

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\* We thank Jesús Rodríguez-López and Gary Krueger for valuable comments on earlier versions of the paper, as well as seminar participants at the 2016 Workshop on Macroeconomics Research at Liberal Arts Colleges, the 2016 Southern Economic Association meetings, and the University of Minnesota's Freeman Center for International Economic Policy's Global Policy Seminar. Finally, we gratefully acknowledge support from the Allianz Life Insurance Company Student Summer Research Fund.

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# 1 Introduction

This paper proposes a methodology for measuring the size and properties of the shadow economy based on a dynamic deterministic general equilibrium (DGE) model. While other authors have used DGE models as well, nearly all of them disregard the trend component found in most economic time series. In our methodology, we exploit the dynamics of observed trends to account for both the size and the cyclical nature of the shadow economy, as they impose a set of equilibrium restrictions over the growth rates of the model variables (including shadow sector output). Ignoring these restrictions—or imposing ad-hoc growth rates to quantify the dynamics of the shadow economy—may produce biased results.<sup>1</sup>

We show that explicitly incorporating the trends of the model variables into the analysis offers a very different picture of the shadow economy relative to earlier studies. We start from a two-sector model where the representative household has access to formal and shadow technologies; the former uses physical capital and labor for production while the latter is labor-exclusive. We go one step beyond by including trends on formal productivity, shadow productivity, investment-specific technological progress, and hours worked.<sup>2</sup>

While the first three trends are considered standard in the literature, the last one deserves some justification. Our main motivation is the fact that the number of hours worked (per capita), especially in developing countries, has been increasing steadily over time. As an example, consider the evolution of hours worked per capita for Brazil and Mexico (normalized to 100 in 1980), as shown in [Figure 1](#) below:

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<sup>1</sup> As an example, [Elgin and Öztunalı \(2012\)](#) calculate a biased estimate of the size of the shadow economy because the imputed growth rates of the shadow economy do not follow equilibrium conditions. In particular, they assume that the growth rate of the shadow sector productivity is the average of the growth rates of the formal sector productivity and the capital stock. See [Remark 4.2](#) for a discussion of this potential bias.

<sup>2</sup> Our modeling strategy and choice of trends is inspired by the work of [Lafourcade and de Wind \(2012\)](#).

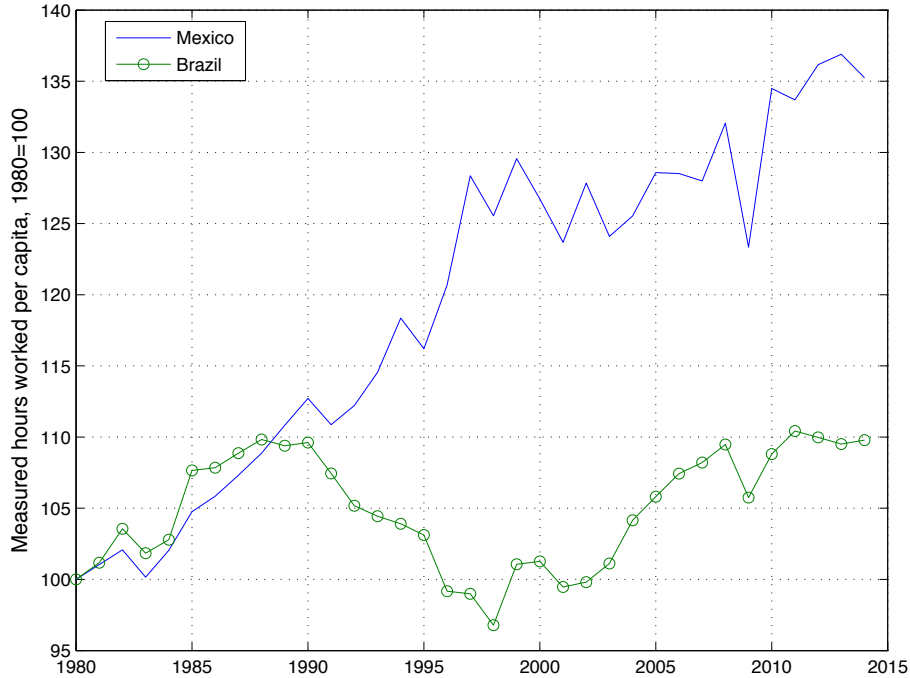


Figure 1: Hours worked per capita, Brazil and Mexico, 1980-2014.

The index for Brazil hovers around a mean of 105, yet the one for Mexico has been increasing throughout the sample. This growth can be accounted for by changes in the labor market (e.g., female workers entering the labor force) or the recurrent economic crises that have forced workers to take additional jobs or pushed additional household members to enter the labor force (this is commonplace in developing economies, where a social safety net is close to nonexistent). While our methodology remains silent about the underlying cause of this steady increase, we believe that omitting this trend can overlook relevant dynamics of the formal and shadow sectors.

We summarize our findings as follows. First, the model-imposed restrictions require the shadow output trend to be an endogenous function of the exogenous trends (formal output, investment-specific technological progress, and hours); hence, we can use real-world data to back out the dynamics of the shadow sector. Second, we show that in order to be consistent with the remaining trends, the shadow production technology necessarily exhibits increasing returns to scale. Third, we apply our methodology to a set of Latin American and Asian countries and find that the

evolution of the shadow sector responds to aggregate shocks in a way that is consistent with the economic history of each country. Finally, we document that the volatility of shadow sector output is considerably larger than the one in formal sector output.

The rest of the paper is structured as follows. [Section 2](#) presents a brief survey of the earlier literature on measuring the size of the shadow economy. [Section 3](#) presents the model economy in detail while [Section 4](#) shows how we deal with the model’s trends and lists the main results that allow us to derive the size of the shadow economy. [Section 6](#) provides the main time series and their statistical properties, while [Section 7](#) concludes.

## 2 Characterizing and measuring the shadow economy

The terminology regarding the shadow economy is often loosely used and exchanged for other concepts that are not necessarily equivalent. In this section, we aim to provide a clear map between the currently accepted notions and our working definition of the shadow economy. We also present and discuss how other researchers have developed their estimates for the size of the shadow economy.<sup>3</sup>

### 2.1 Defining the shadow economy

The Organisation for Economic Co-operation and Development (see [OECD 2002](#)) uses the term “non-observed economy” as a catch-all concept that stands in for the following categories of production:

1. Underground production: goods and services that are kept off the market in order to avoid taxes or regulations.
2. Illegal production: goods and services that are prohibited by law.

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<sup>3</sup> The material below borrows heavily from [Solis-Garcia and Xie \(2016\)](#).

3. Informal sector production: goods and services that are produced by firms that are either unregistered or below a threshold of employment.
4. Production of households for own-final use: goods and services produced within the household for self-consumption.
5. Statistical underground: goods and services that should be accounted for but are not because they are overlooked by statistical agencies.

In this paper, our concept of the shadow economy includes both the underground and informal sector production; the remaining three categories are left out of our unit of analysis. (For example, [Schneider, Buehn, and Montenegro 2010](#) rule out illegal production. The statistical underground is hard to quantify and the production of households for own-final use includes a different set of productive activities that are not meant to be traded in the market.)

## **2.2 Measurement of the shadow economy: early approaches**

Measuring the size of the shadow economy is not easy. In their survey paper, [Schneider and Enste \(2000\)](#) discuss three methodologies to calculate this value; here we examine them briefly and refer to their paper for additional details.

[Schneider and Enste](#) use the term “direct approaches” to refer to (direct) surveys and samples that attempt to quantify the number of productive entities that belong in the shadow economy. However, these methods often provide biased estimates as respondents may lie about their formal/shadow status. Also, the cost of implementing surveys of this kind makes it unlikely to be used in a frequent basis and, therefore, to provide a consistent time series over a long period of time.

The second methodology relies on macroeconomic indicators to infer the size of the shadow economy over time—[Schneider and Enste](#) denote this broadly as “indirect approaches.” First, they

consider the *discrepancy between the expenditure and income measures in national accounts*: since these (by construction) need to be the same, any difference between expenditures and income values of GDP could provide a measure of the shadow economy. Second, they look at the *discrepancy between the official and actual labor force*: a fall in the participation rate could point to an active shadow economy. The *transactions approach* conjectures a stable relation between the total volume of transactions and GDP and uses this as a base to quantify the size of the shadow economy. The *currency demand approach* assumes that all of the shadow economy transaction are carried out in cash, so that an increase in shadow economy activity will result in an increase in the demand for currency. Finally, the *physical input method* uses the (near unit) elasticity between electricity and GDP as well as the growth of electricity consumption to infer the growth of the shadow economy.

In the “model approach,” the researcher uses econometric models to back out the size of the shadow economy. The main idea behind this class of models is that the shadow economy does not have a single cause and does not exhibit a single effect when it operates over time. Hence, an econometric framework can be used to infer the size of the shadow sector by looking (simultaneously) both at the hypothesized causes of the shadow economy (e.g., tax rates and regulation) as well as the hypothesized effects (e.g., participation rates and currency demand). This methodology is called the multiple-indicators multiple-causes model (MIMIC) and is the technique used by [Schneider et al. \(2010\)](#) to derive the values that we use in this study. For comparison purposes, we include their estimates when assessing the performance of our model.

### 2.3 Measurement of the shadow economy: general equilibrium

Besides the work of [Schneider et al. \(2010\)](#) mentioned above, we want to point to several contributions that are related to this paper as they make full use of general equilibrium models.<sup>4</sup>

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<sup>4</sup> Even though it is a full-fledged dynamic stochastic general equilibrium (DSGE) model, we don’t include the work of [Gomis-Porqueras, Peralta-Alva, and Waller \(2014\)](#) as it is built on top of a monetary search model; in this sense, we believe it is closer in spirit to the currency demand approach than to the modeling strategy we follow below.

We first consider the work of [Elgin and Öztunalı \(2012\)](#). They use a DGE model (based on the work of [Ihrig and Moe 2004](#)) to build time series of the size of the shadow economy for a large panel—60 years and over 150 countries, compared to the 9 observations found in [Schneider et al.](#) While their methodology uses dynamic methods (as we do), we find that their estimates are potentially biased since the growth rates of the shadow economy are arbitrarily imposed and do not follow the equilibrium conditions of the model (see [Remark 4.2](#)). Nonetheless, we also include their estimates when assessing the performance of our model.

[Orsi, Raggi, and Turino \(2014\)](#) is perhaps the closest work to ours in terms of modeling approach. They build a two-sector DSGE model to infer the size of the shadow economy as a latent variable, while taking trend growth into account. That said, our paper is substantially different from theirs given that they only use a deterministic labor-augmenting technological progress trend and that the trend is the same in the formal and the underground economy.<sup>5</sup> In our paper, we allow for deterministic trends across different variables that differentiate the labor-augmenting technological progress across the two sectors. By virtue of the different trends, we believe our model may better capture the dynamics of the shadow economy. Though they use Bayesian methods to estimate their model parameters and derive the size and properties of the shadow economy from the model’s own forecasts, their focus is on Italy only. Our model is more flexible in terms of the data inputs required to estimate our results, and below we test it for a set of Latin American and Asian countries.

### 3 Model

The model consists of a representative household-producer and a government; both agents are described below. In what follows, uppercase letters denote trending variables while lowercase letters denote stationary variables.

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<sup>5</sup> In this case, all the endogenous variables in the model grow at the same rate, which at least for the case of the United States has not been supported by the data; see [Whelan \(2003\)](#).



### 3.1 Household-producer

The household-producer chooses sequences of consumption  $C_t$ , hours worked  $N_t$ , and investment  $X_t$  to maximize

$$\sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\phi \Gamma_{Ht} N_t^{1+\chi}}{1+\chi} \right),$$

where  $\beta \in (0, 1)$  is the discount factor,  $\sigma \in \mathbb{R}$  is the intertemporal elasticity of substitution,  $\phi > 0$  is a parameter quantifying the disutility of labor,  $\chi \geq 0$  is the inverse of the Frisch elasticity of labor supply, and  $\Gamma_{Ht}$  is a permanent shock that affects the household's choice of hours. Maximization is subject to a set of constraints: first, a law of motion for capital

$$K_{t+1} = (1 - \delta)K_t + X_t, \tag{3.1}$$

where  $\delta \in (0, 1)$  is the depreciation rate and  $K_t$  denotes the stock of physical capital. Second, a household time constraint

$$N_t = N_{Ft} + N_{St}, \tag{3.2}$$

where  $N_{Ft}$  denotes hours worked in the formal sector and  $N_{St}$  does so for the shadow sector. Finally, a budget constraint

$$C_t + \Gamma_{At} X_t = (1 - \tau_t) K_t^\alpha (\Gamma_{Ft} N_{Ft})^{1-\alpha} + (1 - \rho \hat{s} \tau_t) (\Gamma_{St} N_{St})^\eta, \tag{3.3}$$

where  $\alpha \in (0, 1)$  is the capital income share in formal output and  $\eta > 0$  is the labor share in shadow output,  $\Gamma_{At}$  is a permanent shock to the production of investment goods, and  $\tau_t \in (0, 1)$  is a tax on formal sector output  $K_t^\alpha (\Gamma_{Ft} N_{Ft})^{1-\alpha}$ . We let  $(\Gamma_{St} N_{St})^\eta$  denote shadow sector output; we assume

that the government cannot tax shadow sector output unless it is exposed by a tax audit.<sup>6</sup> In this sense,  $\rho \in [0, 1)$  represents the probability of a tax audit and  $\hat{s} > 1$  is a tax surcharge, as in [Ihrig and Moe \(2004\)](#) and [Orsi et al. \(2014\)](#). Finally, formal and shadow production technologies are subject to the permanent productivity shocks  $\Gamma_{Ft}$  and  $\Gamma_{St}$ .

### 3.2 Government

We include a government sector to account for the effect that tax rates have over the size of the shadow sector (e.g., [Ihrig and Moe 2004](#)). We assume that the government uses tax revenue to fund a stream of non-productive expenditure  $G_t$  and that it satisfies its period-by-period budget constraint

$$G_t = \tau_t K_t^\alpha (\Gamma_{Ft} N_{Ft})^{1-\alpha} + \rho \hat{s} \tau_t (\Gamma_{St} N_{St})^\eta. \quad (3.4)$$

Given equation (3.4), government expenditure is an endogenous variable in our model.

### 3.3 Exogenous processes

The set of permanent exogenous shocks is given by  $\Gamma_{Ht}$ ,  $\Gamma_{At}$ , and  $\Gamma_{Ft}$ . ([Proposition 4.1](#) shows that  $\Gamma_{St}$  is an endogenous variable.) In particular, for  $i = \{H, A, F, S\}$ ,

$$g_{it} = \frac{\Gamma_{it}}{\Gamma_{i,t-1}},$$

where  $g_{it}$  denotes the (gross) growth rate of variable  $i$ .<sup>7</sup>

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<sup>6</sup> We can think of the representative household-producer as having access to a formal production function and a “backyard technology” that produces the same good as the formal technology but with a lower productivity.

<sup>7</sup> Conversely,  $\Gamma_{it} = \prod_{s=1}^t g_{is}$ .

### 3.4 Equilibrium

The equilibrium conditions of the model are given by<sup>8</sup>

$$C_t + \Gamma_{At}X_t + G_t = K_t^\alpha(\Gamma_{Ft}N_{Ft})^{1-\alpha} + (\Gamma_{St}N_{St})^\eta \quad (3.5)$$

$$K_{t+1} = (1 - \delta)K_t + X_t \quad (3.6)$$

$$N_t = N_{Ft} + N_{St} \quad (3.7)$$

$$\begin{aligned} \Gamma_{At}C_t^{-\sigma} &= \alpha\beta C_{t+1}^{-\sigma}(1 - \tau_{t+1})K_{t+1}^{\alpha-1}(\Gamma_{F,t+1}N_{F,t+1})^{1-\alpha} \\ &\quad + \beta(1 - \delta)\Gamma_{A,t+1}C_{t+1}^{-\sigma} \end{aligned} \quad (3.8)$$

$$\phi\Gamma_{Ht}N_t^\chi = (1 - \alpha)C_t^{-\sigma}(1 - \tau_t)K_t^\alpha\Gamma_{Ft}^{1-\alpha}N_{Ft}^{-\alpha} \quad (3.9)$$

$$\eta(1 - \rho\hat{s}\tau_t)\Gamma_{St}^\eta N_{St}^{\eta-1} = (1 - \alpha)(1 - \tau_t)K_t^\alpha\Gamma_{Ft}^{1-\alpha}N_{Ft}^{-\alpha} \quad (3.10)$$

$$G_t = \tau_t K_t^\alpha(\Gamma_{Ft}N_{Ft})^{1-\alpha} + \rho\hat{s}\tau_t(\Gamma_{St}N_{St})^\eta. \quad (3.11)$$

We add expressions for formal, shadow, and total output

$$Y_{Ft} = K_t^\alpha(\Gamma_{Ft}N_{Ft})^{1-\alpha} \quad (3.12)$$

$$Y_{St} = (\Gamma_{St}N_{St})^\eta \quad (3.13)$$

$$Y_t = Y_{Ft} + Y_{St}, \quad (3.14)$$

and derive an expression for the decentralized price of investment goods; from equation (3.8),

$$P_{Xt} = \Gamma_{At}. \quad (3.15)$$

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<sup>8</sup> See the technical appendix for the derivation of these results.

## 4 Dealing with the trends

The model includes trends in the household's choice of hours worked ( $\Gamma_t^H$ ), the production of investment goods ( $\Gamma_t^A$ ), and formal and shadow technology productivity ( $\Gamma_t^F$  and  $\Gamma_t^S$ ). Hence, we need to find the relevant growth rates (as a function of the exogenous rates  $\gamma_t^H$ ,  $\gamma_t^A$ , and  $\gamma_t^F$ ) for each of the model's variables.

### 4.1 Deterministic growth rates

We use (3.5)-(3.15) to calculate the balanced growth path growth rates imposed by the model. (We will use “equilibrium” and “balanced growth path” interchangeably. Also, growth rates without a time subscript indicate equilibrium values.) From the aggregate resource constraint, law of motion for capital, and household time constraint we get

$$g_Y = g_C = g_A g_X = g_G \quad (4.1)$$

$$g_K = g_X \quad (4.2)$$

$$g_N = g_{N_F} = g_{N_S}. \quad (4.3)$$

The inter- and intratemporal conditions require

$$g_A = g_{Y_F} g_K^{-1} \quad (4.4)$$

$$g_H g_N^\chi = g_C^{-\sigma} g_{Y_F} g_{N_F}^{-1} \quad (4.5)$$

$$g_{Y_S} g_{N_S}^{-1} = g_{Y_F} g_{N_F}^{-1}, \quad (4.6)$$

where we set  $g_\tau = 1$  in (4.4) since we assume tax rates are stationary. From the production side

$$g_{Y_F} = g_K^\alpha (g_F g_{N_F})^{1-\alpha} \quad (4.7)$$

$$g_{Y_S} = (g_S g_{N_S})^\eta, \quad (4.8)$$

and the final restrictions are

$$g_Y = g_{Y_F} = g_{Y_S} \quad (4.9)$$

$$g_{P_X} = g_A. \quad (4.10)$$

We now present the following result (all proofs are contained in [Appendix B](#)):

PROPOSITION 4.1

*The equilibrium growth rates of the capital stock,  $g_K$ ; (formal and shadow) hours worked,  $g_N$ ; (formal, shadow, and total) output,  $g_Y$ ; and the permanent shock to the shadow production function,  $g_S$ , are given by*

$$g_K = g_H^{-1/(\sigma+\chi)} g_A^{-[\alpha(1-\sigma)+\sigma+\chi]/[(1-\alpha)(\sigma+\chi)]} g_F^{(1+\chi)/(\sigma+\chi)} \quad (4.11)$$

$$g_N = g_H^{-1/(\sigma+\chi)} g_A^{-\alpha(1-\sigma)/[(1-\alpha)(\sigma+\chi)]} g_F^{(1-\sigma)/(\sigma+\chi)} \quad (4.12)$$

$$g_Y = g_H^{-1/(\sigma+\chi)} g_A^{-\alpha(1+\chi)/[(1-\alpha)(\sigma+\chi)]} g_F^{(1+\chi)/(\sigma+\chi)} \quad (4.13)$$

$$g_S = g_H^{-(1+\eta)/[(\sigma+\chi)\eta]} g_A^{-\alpha[1+\chi+(1-\sigma)\eta]/[(1-\alpha)(\sigma+\chi)\eta]} g_F^{[1+\chi+(1-\sigma)\eta]/[(\sigma+\chi)\eta]}. \quad (4.14)$$

Equation (4.14) is central to our paper: it ties the evolution of the shadow sector to the exogenous growth rates of the model  $\{g_H, g_A, g_F\}$  along the equilibrium path and helps us quantify the size of the shadow economy over time.

*Remark 4.2.* Equation (4.14) also shows why imposing ad-hoc growth rates for the shadow economy can provide biased results. For example, [Elgin and Öztunalı \(2012\)](#) assume that shadow sector productivity grows at the average of the growth rates of the formal sector productivity and the capital stock. From (4.11) and (4.14), it's clear that  $\frac{1}{2}(g_K + g_F) \neq g_S$ .

## 4.2 Observable rates

Let  $\{\hat{g}_K, \hat{g}_{N_F}, \hat{g}_{Y_F}\}$  denote the long-run averages of the (observed) growth rates of physical capital, formal hours worked, and formal output. We now show how to link these real-world rates with the exogenous rates  $\{g_H, g_A, g_F\}$ .

### PROPOSITION 4.3

*The map between the exogenous growth rates  $\{g_H, g_A, g_F\}$  and the observed growth rates  $\{\hat{g}_K, \hat{g}_{N_F}, \hat{g}_{Y_F}\}$  is given by:*

$$g_H = \hat{g}_{Y_F}^{1-\sigma} \hat{g}_{N_F}^{-(1+\chi)} \quad (4.15)$$

$$g_A = \hat{g}_{Y_F} \hat{g}_K^{-1} \quad (4.16)$$

$$g_F = \left( \frac{\hat{g}_{Y_F}}{\hat{g}_K^\alpha \hat{g}_{N_F}^{1-\alpha}} \right)^{1/(1-\alpha)}. \quad (4.17)$$

## 5 Parametrization

As we show below, we only require values for a subset of the deep parameters of the model, namely,  $\{\alpha, \sigma, \chi\}$ ; in addition, we use real-world data to calibrate the values of  $\{\rho, \hat{s}, \eta\}$ .

For the first set of parameters, we set the capital income share in formal output  $\alpha$  to 1/3, the intertemporal elasticity of substitution  $\sigma$  to 1, and the inverse of the Frisch elasticity of labor supply  $\chi$  to 1. All of these values are standard in the literature—but see the technical appendix

for a sensitivity analysis over the values of  $\sigma$  and  $\chi$ . Perhaps more controversially, we set the probability of a tax audit  $\rho$  to zero; we offer some considerations in [Remark 5.1](#). Do note that this choice makes the tax surcharge rate  $\hat{s}$  irrelevant for the analysis below.

*Remark 5.1* (Probability of a tax audit). Our choice of  $\rho = 0$  may be puzzling to some readers. After all, why introduce a parameter when we will make it irrelevant as we take the model to the data? We do this not because we disregard the role of tax evasion, but from a purely empirical perspective: it is difficult to find accurate values for the probability of tax audits in *developed* economies, and more so in developing ones. That said, we perform a sensitivity analysis in the technical appendix, where we combine our methodology with the estimates of [Orsi et al. \(2014\)](#) to quantify the impact of our parametrization strategy with respect to  $\rho$ .<sup>9</sup> We find that the results with  $\rho > 0$  are qualitatively and quantitatively similar to our benchmark specification, unless we are willing to assume unrealistic values for this parameter.

In what follows, we explain how we calibrate the value of the shadow sector labor input parameter  $\eta$ . To start, we need the following assumption:<sup>10</sup>

ASSUMPTION 5.2. The observed (real-world) value of real GDP corresponds to formal output  $Y_{Ft}$ .

To obtain the value of  $\eta$ , we first take the shadow-formal output ratio from [Schneider et al. \(2010\)](#) for a base year  $t_0$ ; call this value  $Y_{[S/F],t_0}$ .

*Remark 5.3* (Relevance of the shadow-formal output ratio in the base year). We are aware of the criticisms that [Gyomai and van de Ven \(2014\)](#) make over the estimates of [Schneider et al. \(2010\)](#); we decide to use the values calculated by [Schneider et al.](#) for three reasons. First, they are among

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<sup>9</sup> As a reference, [Orsi et al. \(2014\)](#) set  $\hat{s} = 1.3$  and estimate  $\rho \in [0.015, 0.034]$ .

<sup>10</sup> [Fernández and Meza \(2015\)](#) also make this assumption in their study of the Mexican shadow employment.

the most widely used estimates of the size of the shadow economy so it's easy to compare our results to theirs. Second, our methodology only requires the value of the shadow-formal output ratio for a base year; this sets the level of the ratio but the dynamics are not connected to this particular number. Finally, changing the value of the base year to any other value is straightforward.

By construction,

$$Y_{[S/F],t_0} = \frac{Y_{S,t_0}}{Y_{F,t_0}}. \quad (5.1)$$

Given data for formal output, shadow output for  $t_0$  equals

$$Y_{S,t_0} = Y_{[S/F],t_0} Y_{F,t_0}. \quad (5.2)$$

Second, we use (5.1) again but now substitute the definition of  $Y_{S,t_0}$  in the numerator:

$$Y_{[S/F],t_0} = \frac{(\Gamma_{S,t_0} N_{S,t_0})^\eta}{Y_{F,t_0}}.$$

Solving for  $N_{S,t_0}$ ,

$$N_{S,t_0} = \frac{(Y_{[S/F],t_0} Y_{F,t_0})^{1/\eta}}{\Gamma_{S,t_0}}. \quad (5.3)$$

We can deduce the following result from equation (5.3).

*Remark 5.4* (Increasing returns to scale in the shadow economy). In equation (5.3), the term inside the parenthesis in the numerator is a fraction of formal output and is significantly larger than unity. In order to obtain reasonable values for  $N_{S_t}$  it must be the case that  $\eta \geq 1$  (otherwise shadow labor explodes); hence, in our methodology, the shadow sector necessarily exhibits increasing returns to scale.



Third, we take the intratemporal condition (3.10) and solve for  $\eta$ :

$$\eta = \frac{(1 - \alpha)(1 - \tau_{t_0})Y_{F,t_0}N_{S,t_0}}{(1 - \rho\hat{s}\tau_{t_0})N_{F,t_0}Y_{S,t_0}}.$$

Plugging from (5.2) and (5.3) and simplifying:<sup>11</sup>

$$\eta = \frac{(1 - \alpha)(1 - \tau_{t_0})Y_{F,t_0}^{1/\eta}Y_{[S/F],t_0}^{(1-\eta)/\eta}}{(1 - \rho\hat{s}\tau_{t_0})N_{F,t_0}\Gamma_{S,t_0}}. \quad (5.4)$$

Equation (5.4) is a nonlinear function of  $\eta$ , the (known) parameters  $\{\alpha, \rho, \hat{s}\}$ , and real-world values as  $\Gamma_{S,t_0}$  is itself a function of  $\eta$  and observed trend values  $\{\Gamma_{H,t_0}, \Gamma_{A,t_0}, \Gamma_{F,t_0}\}$ . We use a fixed point procedure to find the value of  $\eta$  such that (5.4) is satisfied. The algorithm behind the procedure is described next.

ALGORITHM 5.5.

1. Obtain data for the capital stock, formal hours worked, and formal output  $\{K_t, N_{Ft}, Y_{Ft}\}_{t=1}^T$ ; use these to calculate the long-run growth rate averages  $\{\hat{g}_{K_t}, \hat{g}_{N_{Ft}}, \hat{g}_{Y_{Ft}}\}_{t=1}^T$  and the triple  $\{g_{Ht}, g_{At}, g_{Ft}\}_{t=1}^T$  following (4.15)-(4.17).
2. Pick a base year  $t_0$  and fix values for  $\{\alpha, \sigma, \chi, \rho, \hat{s}\}$ . Pick a value  $M \gg 0$  and build a grid with  $M$  elements over the interval  $[\eta_L, \eta_H]$ , where  $\eta_L \geq 1$  (see Remark 5.4); call this set  $\mathcal{N}$ .
3. For every  $\eta_m \in \mathcal{N}$ ,
  - (a) Calculate  $\{\Gamma_{St}\}_{t=1}^T$  following (4.14); without loss of generality set  $\Gamma_{S,t_0} = 1$ .
  - (b) Calculate  $N_{S,t_0}$  following (5.3).

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<sup>11</sup> From (3.4) and period-by-period government budget balance we get that

$$\tau_{t_0} = \frac{G_{t_0}}{K_{t_0}^\alpha (\Gamma_{F,t_0} N_{F,t_0})^{1-\alpha} + \rho\hat{s}(\Gamma_{S,t_0} N_{S,t_0})^\eta} = \frac{G_{t_0}}{Y_{F,t_0} + \rho\hat{s}Y_{S,t_0}}.$$

These terms can be easily backed out from real-world data (recall that  $G_t$  is endogenous in our model).

- (c) Calculate  $\eta$  following (5.4).
4. Find the entry in  $\mathcal{N}$  where  $\|\eta_m - \eta\|$  is minimized. Set this value as the solution to (5.4) and use it to re-calculate  $N_{S,t_0}$  following (5.3).
  5. Use  $\{\hat{g}_{Nt}\}_{t=1}^T$  to back out  $\{\Gamma_{Nt}\}_{t=1}^T$  (see Footnote 7) and use this with  $N_{S,t_0}$  to back out  $\{N_{St}\}_{t=1}^T$ .
  6. Use  $\{\Gamma_{Nt}, N_{St}\}_{t=1}^T$  and  $\eta$  to back out  $\{Y_{St}\}_{t=0}^T$ . Use this series along  $\{Y_{Ft}\}_{t=1}^T$  to calculate  $\{Y_{[S/F],t}\}_{t=1}^T$ .

## 6 Application: size of the shadow economy

To show the usefulness of our methodology, we first use it to infer the size and dynamics of the shadow sector in Mexico. We also provide some historical context that links the behavior of the time series to real-world events. We then present results for a set of Latin American (Argentina, Brazil, and Venezuela) and Asian countries (Indonesia, Turkey, and Vietnam). We close the section by presenting standard business cycle statistics that compare formal, shadow, and total output.

For comparison purposes, we include the series calculated by [Schneider et al. \(2010\)](#) and [Elgin and Öztunalı \(2012\)](#). In what follows, we let the grid  $\mathcal{N}$  have  $M = 1,000,000$  elements (refer to [Algorithm 5.5](#)) and pick 2007 as the base year in order to ease the comparison with the other two series. (See [Appendix A](#) for a detailed description of the data used in the analysis.)

### 6.1 Mexico

Mexico makes an interesting case study for our methodology for several reasons. First, being a developing country, the influence of the shadow economy matters a lot. Second, the recurrent economic crises and the dismal economic growth experienced by the country since 1980 provide

a good opportunity to test the predictions of our methodology in the midst of a large economic downturn. The evolution of the shadow-to-formal output ratio is shown in [Figure 2](#).

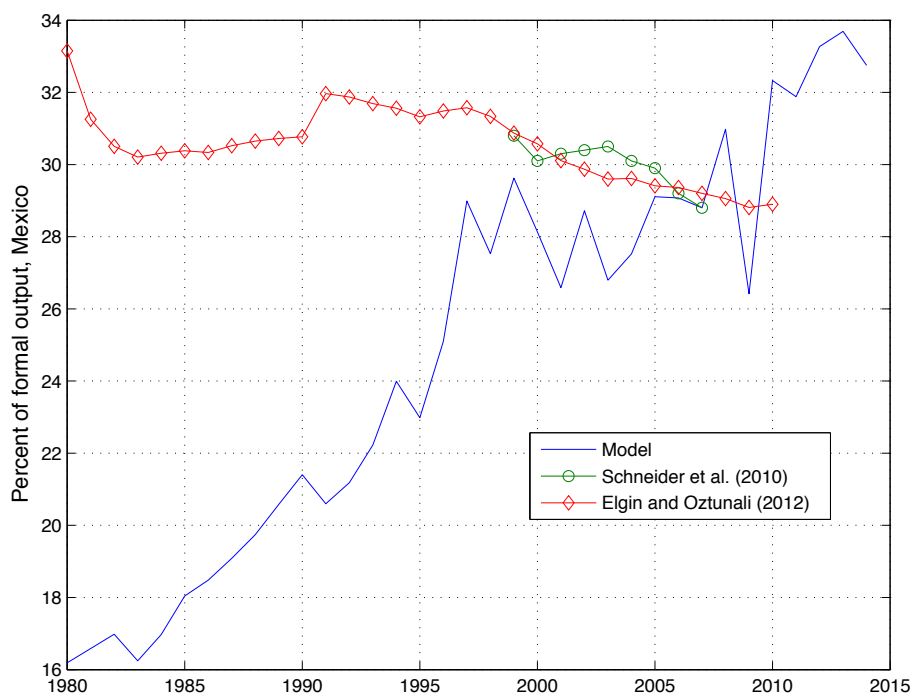


Figure 2: Mexico, shadow-to-formal output ratio, 1980-2014.

Our estimates show that the size of the shadow sector increased throughout Mexico’s “lost decade” (1980-1990) and up to the aftermath of the Tequila crisis: between 1980 and 1997, the average shadow sector size climbed from about 16 to 28%. The value remained stable until the Great Recession, when it increased to a peak of 34%. [Figure 2](#) also displays a sizable increase in the shadow sector size after the major economic downturns in Mexico: 1983 (a large depreciation of the Peso and banking sector nationalization; both happened in September 1982), 1995 (the Tequila crisis, which started in December 1994), and 2009 (the Great Recession).

[Figure 2](#) also compares our results to the estimates of [Schneider et al.](#): they somewhat agree with our calculations as they present a stationary shadow sector size (though it is slightly above our estimates). [Elgin and Öztunali](#) present a different picture for 1980-1997: their results show a slowly declining shadow sector size that’s inconsistent with the economic crises of 1983 and 1995.

Figure 3 aids in understanding the dynamics of the shadow-to-formal output ratio movements by showing the evolution of both shadow and formal outputs; for both series, the value for 1980 is normalized to 100. Note that the values for formal output come directly from the data while the values for shadow output are generated by our methodology. We find that formal output was 30% higher in 2014 relative to 1980, while shadow output was about 2.6 times larger. While the shadow output series is considerably more volatile than its formal counterpart, Figure 3 is consistent with a thriving shadow sector and a stagnant formal economy.

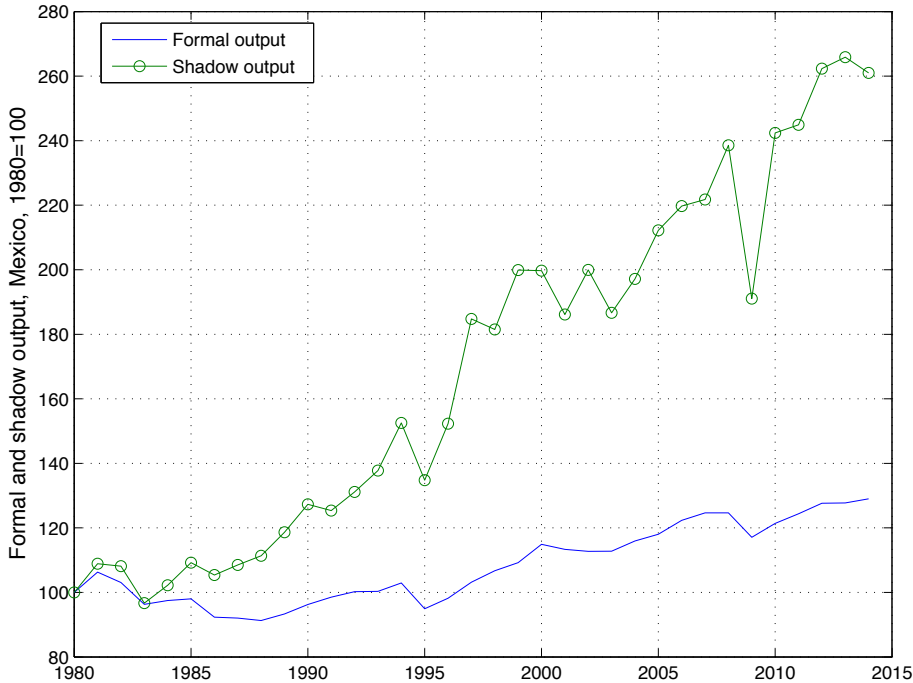


Figure 3: Mexico, formal and shadow output, 1980=100.

### 6.2 Latin American countries: Argentina, Brazil, and Venezuela

We now show the results of applying our methodology to a set of Latin American countries.

## Argentina

Figure 4 shows the evolution of the Argentinean shadow sector output. Our results suggest that the size of the shadow economy hovered around 17% from 1980 to 1994, when the influence of the “Tequila crisis” shows up: the size of the shadow economy falls to a bit to 14% in 1996 and starts increasing up to 1998. The recession of 1998-2002 has clear effects over the formal and shadow economies. As shown in Figure 4, our methodology implies that the shadow economy falls from 18 to 11% of formal output in these years. The estimates of Schneider et al. are diametrically opposite to our findings: according to their calculations, the shadow economy increased in size relative to the formal output between 1998 through 2002, then the ratio drops considerably. This means that shadow output increased far more than formal output, which is hard to believe in a recession of the magnitude experienced in Argentina. On the other hand, the estimates of Elgin and Öztunalı suggest a constant size of the shadow economy throughout the sample: the Tequila crisis and the 1998 recession didn’t have any effect over the size of the shadow economy.

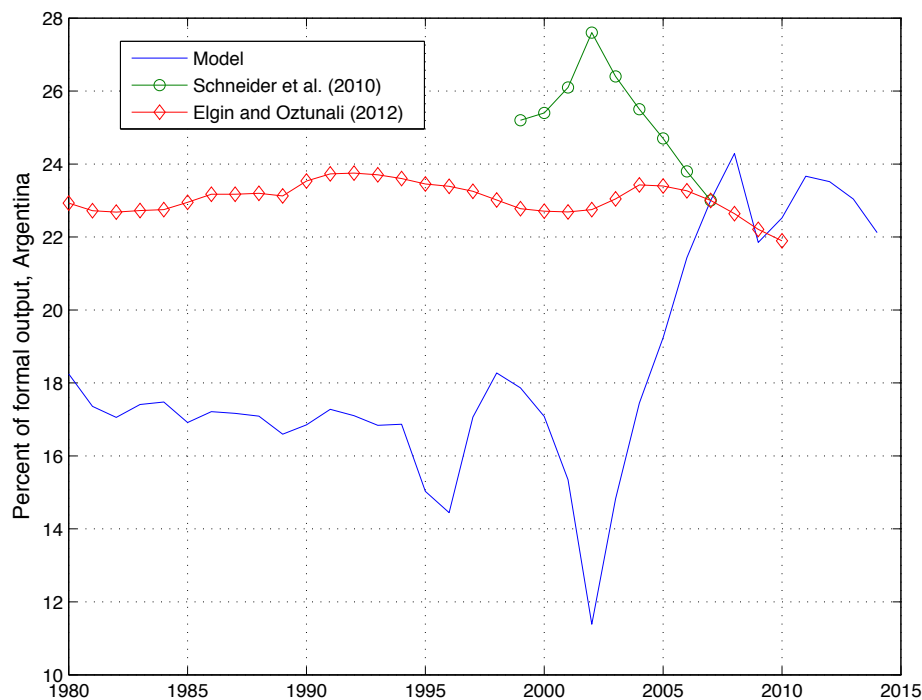


Figure 4: Argentina, shadow-to-formal output ratio, 1980-2014.

Figure 5 shows the evolution of the formal and shadow output for Argentina. Both formal and shadow output fall but the former does so less than the latter, which accounts for the fall in the relative size of the shadow economy. The fall in the shadow-to-formal output ratio during the recession of 1998-2002 follows the sizable contraction of the economy, as shown in Figure 5. Both formal and shadow output decrease: for the formal sector, the fall is about 22% (relative to 1998); for the shadow sector, the fall is nearly 51%. After the recession the Argentinean economy started to catch up; our results suggest that the formal sector recovered quickly yet the shadow sector did so at a faster rate.

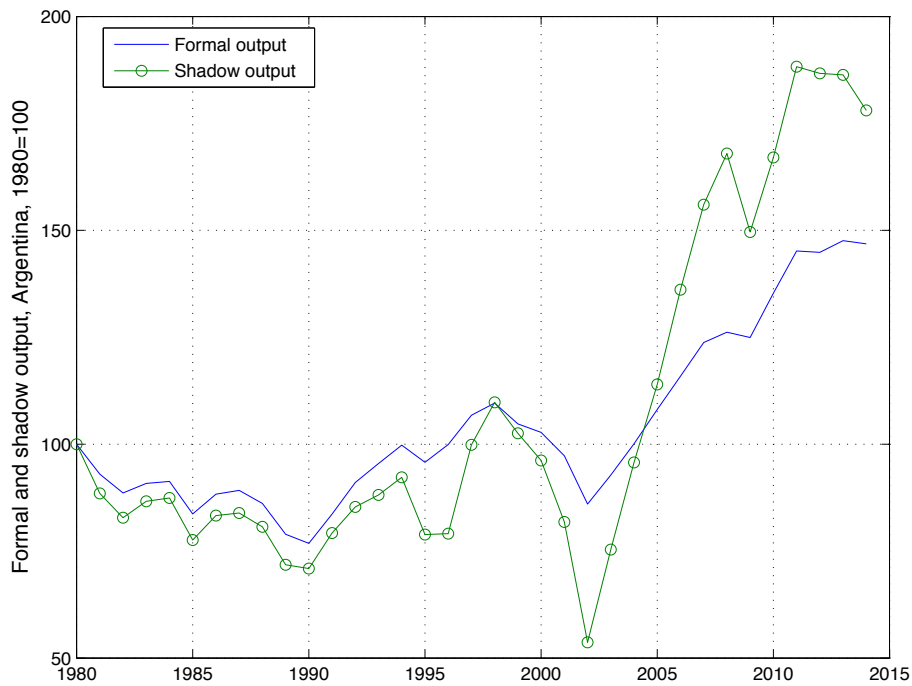


Figure 5: Argentina, formal and shadow output, 1980=100.

## Brazil

Figure 6 shows the evolution of the shadow economy for Brazil. As is well known, Brazil experienced an episode of stagnation in the 1980s: starting from a value of about 31% in 1980, the value increased to 38% by the late 1980s. The ratio then fell by almost 10 percentage points in a short time: by

1998 the shadow economy represented a bit over 28% of formal output; this seems consistent with the positive effects of the economic reforms of the early 1990s. Since then, there has been an upward trend in the ratio and currently it is back to the same value as in the late 1980s, around 38%.

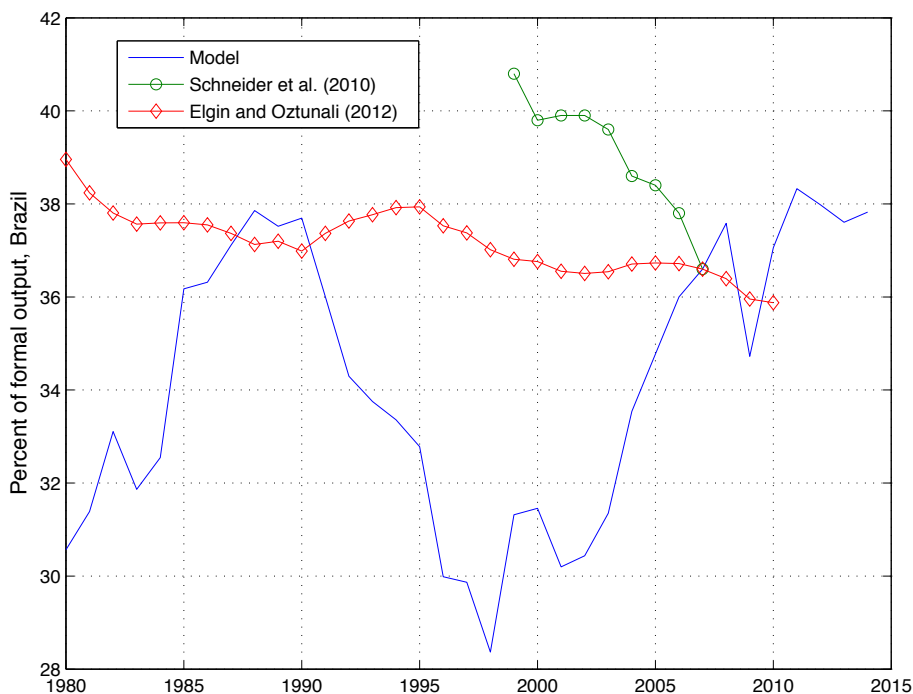


Figure 6: Brazil, shadow-to-formal output ratio, 1980-2014.

The results from [Schneider et al.](#) are completely different from ours. Their estimates suggest a falling trend from 1999 to 2007, opposite to our upward trend. Also, the results from [Elgin and Öztunali](#) do not capture [Schneider et al.](#)'s downward trend nor the large movements identified by our methodology. In addition, we can better understand the behavior of the shadow economy by inspecting [Figure 7](#), which shows the evolution of the formal and shadow output for Brazil. Consistent with the stagnation of the 1980s, the shadow economy grows faster than the formal economy in the late 1980s and then begins to shrink, remaining relatively stable until throughout the 1990s. Both the formal and shadow economies have grown since 2001 but the shadow economy has done so at a faster rate.

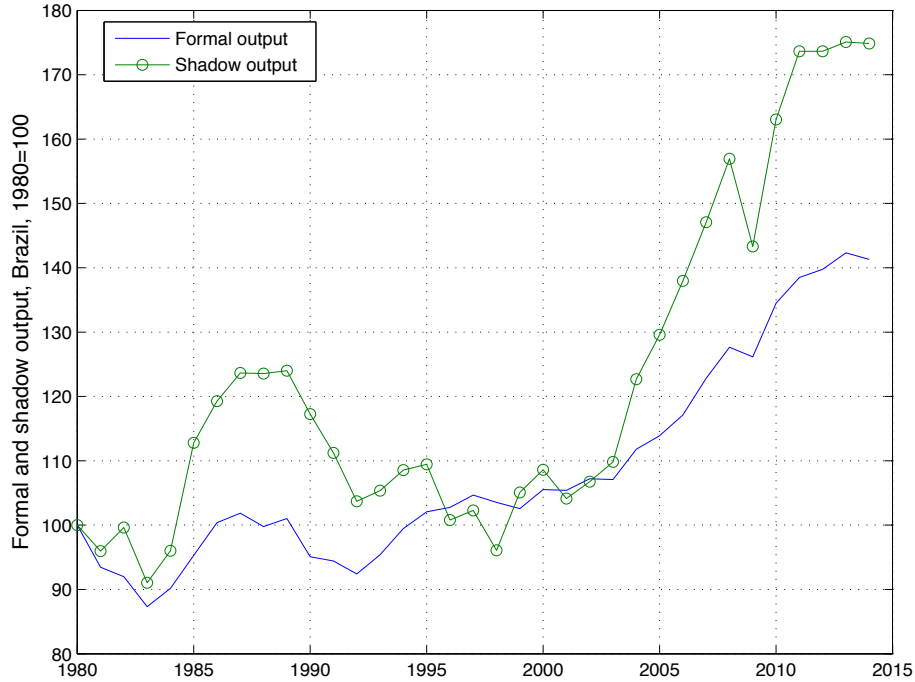


Figure 7: Brazil, formal and shadow output, 1980=100.

## Venezuela

We now analyze the shadow and formal sector performance in Venezuela; see [Figure 8](#). Our model suggests that the shadow economy was relatively stable between 1980 and 2000, at around 21% of formal output. The effect of the recession of 2001 can be seen in the figure, when the size of the shadow sector falls marginally. As the recession ends, the shadow economy takes a larger role in the Venezuelan economy, increasing nearly 15 percentage points in 5 years. The estimates of [Schneider et al.](#) paint a very different picture: the shadow sector increases through 2001-3 and then it starts to fall. [Elgin and Öztunalı](#) show a shadow economy that grows between 1980 and 2005 afterwards. Neither series captures the sharp increase in shadow sector output that our methodology detects between 2003 and 2008.

[Figure 9](#) presents formal and shadow output over time. Formal output suffered from a series of negative shocks starting from 1980—note that the index remains below 100 until 2007. The shadow economy also takes a dive but manages to return to 100 by the early 1990s. The figure



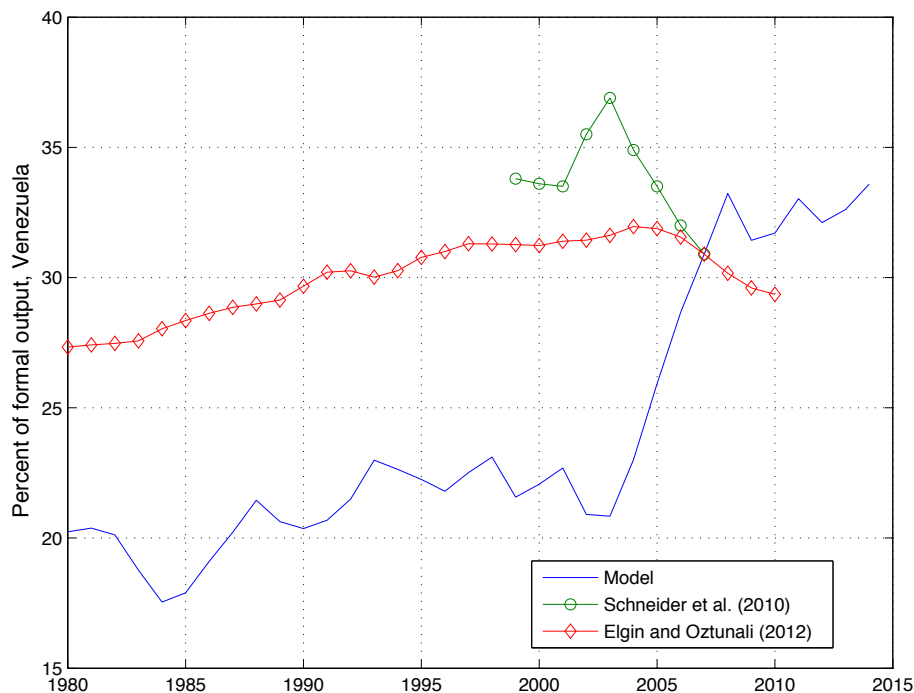


Figure 8: Venezuela, shadow-to-formal output ratio, 1980-2014.

shows that as the 2001-3 recession ends, both formal and shadow output recover, yet the recovery of the shadow sector is far stronger than the one experienced by the formal sector. This suggests that the economic policies taken by the government have affected the formal economy and that the shadow sector is acting like an escape valve.

### 6.3 Asian countries: Indonesia, Turkey, and Vietnam

We now show the results of applying our methodology to a set of Asian countries.

#### Indonesia

The evolution of the shadow-to-formal output ratio for Indonesia is shown in [Figure 10](#). After a roughly 50% increase in the size of the shadow sector—jumping from about 9 to 15%—throughout the 1980s, the ratio remained relatively constant up to 2006, increasing ever since. Note that the estimates of [Schneider et al.](#) and [Elgin and Öztunali](#) do not coincide with ours. The former show a

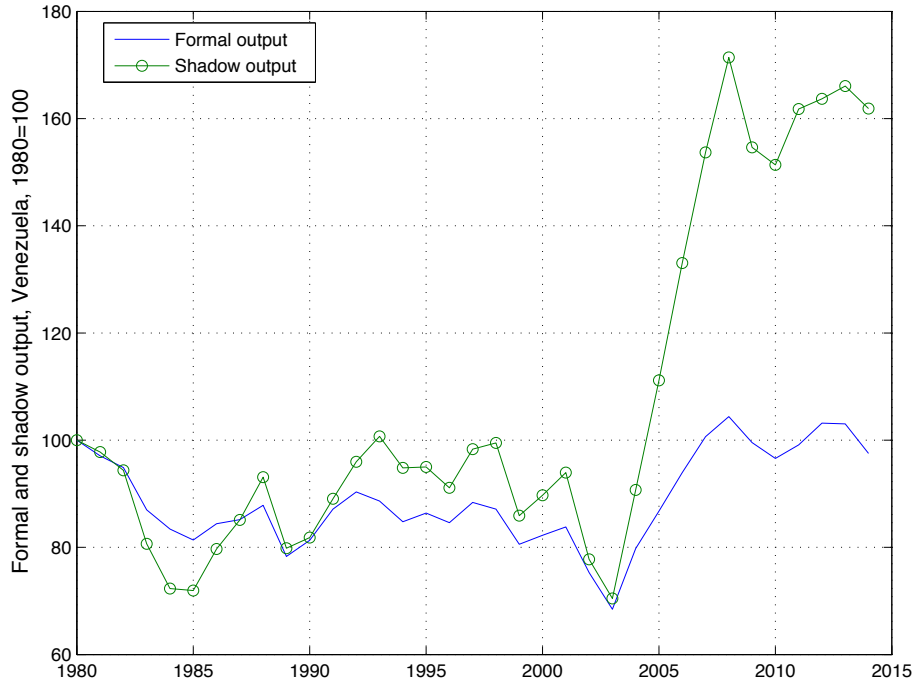


Figure 9: Venezuela, formal and shadow output, 1980=100.

downward trend while the latter suggest a steep decline up to 1997 and a stable ratio afterwards.

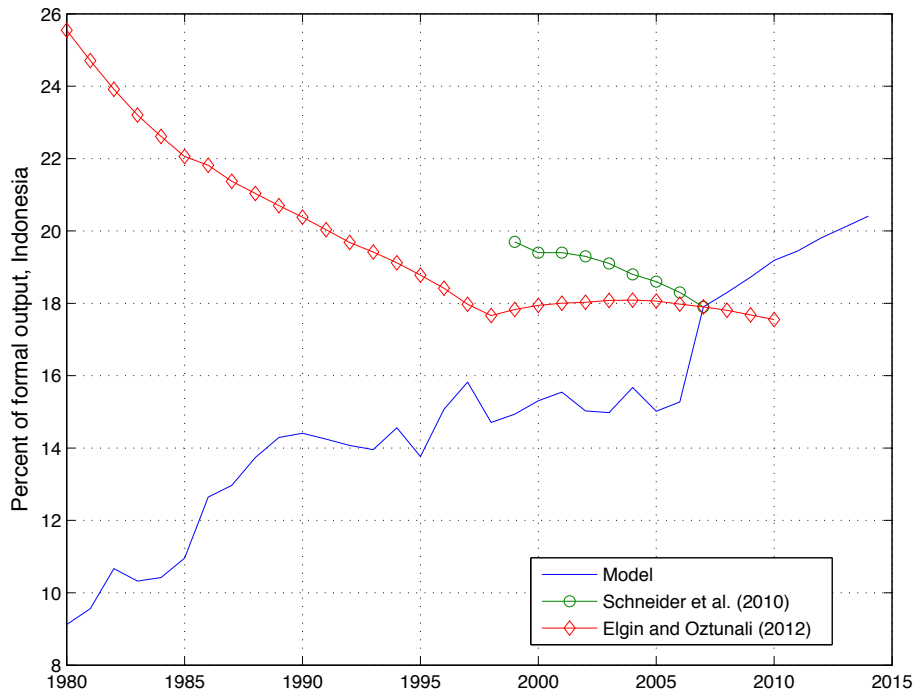


Figure 10: Indonesia, shadow-to-formal output ratio, 1980-2014.

Figure 11 shows the paths of shadow and formal output. Both series have increased dramatically: formal output is more than 3 times its size relative to 1980 while shadow output has had an almost 8-fold increase. The figure shows why our methodology predicts a counterintuitive decrease in the shadow-to-formal output ratio during the Asian financial crisis of 1997: formal output declined about 15% but shadow output fell by almost 21%.

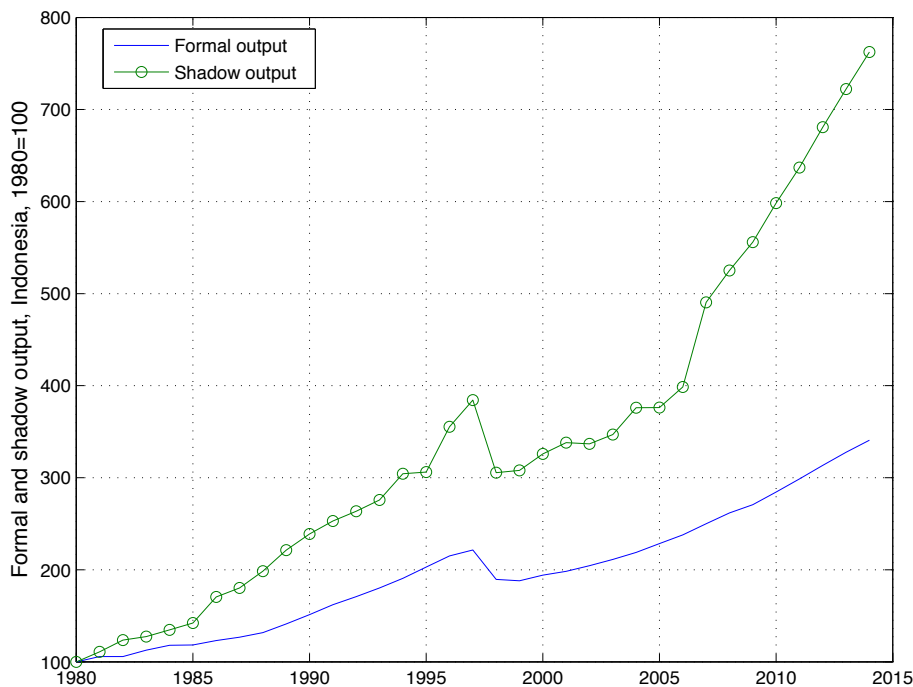


Figure 11: Indonesia, formal and shadow output, 1980=100.

## Turkey

We show our estimates for the case of Turkey in Figure 12. Our results show a drastic decrease in the size of the shadow economy (nearly 12 percentage points) from 1980 to 1988; the ratio remains relatively stable at around 30% and jumps again in 2009 to levels comparable to those in the 1980s. When comparing our results to those of Schneider et al., we see that both series share the downward trend in shadow sector size observed between 1999 and 2007; this trend is also captured by Elgin and Öztunalı yet their results are not consistent (both in trend and magnitude) for 1980-1995.

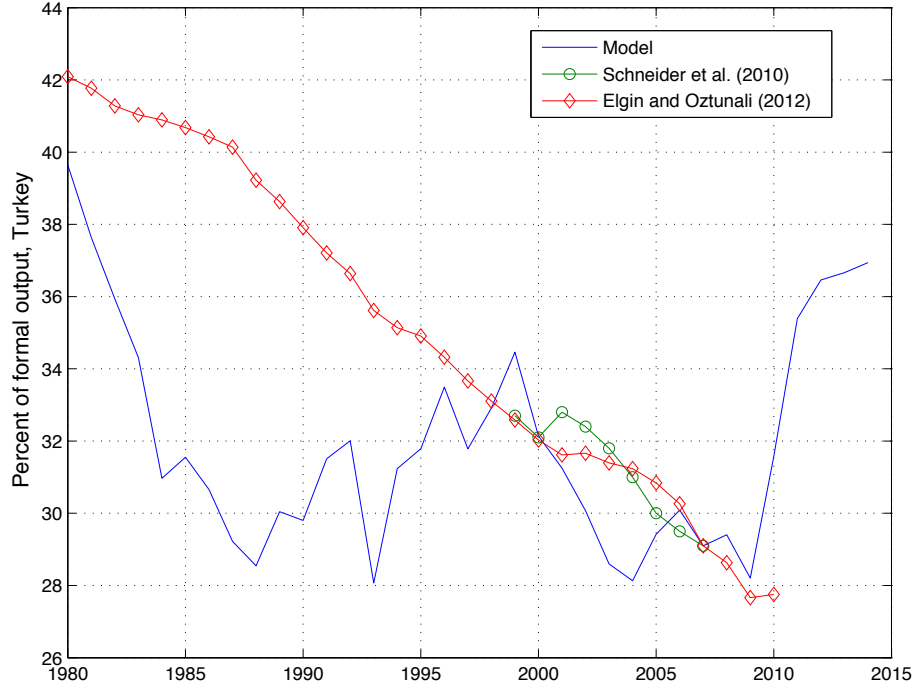


Figure 12: Turkey, shadow-to-formal output ratio, 1980-2014.

We show the performance of formal and shadow output in [Figure 13](#). Unlike the previous examples, Turkish formal output outperforms shadow output throughout the sample.

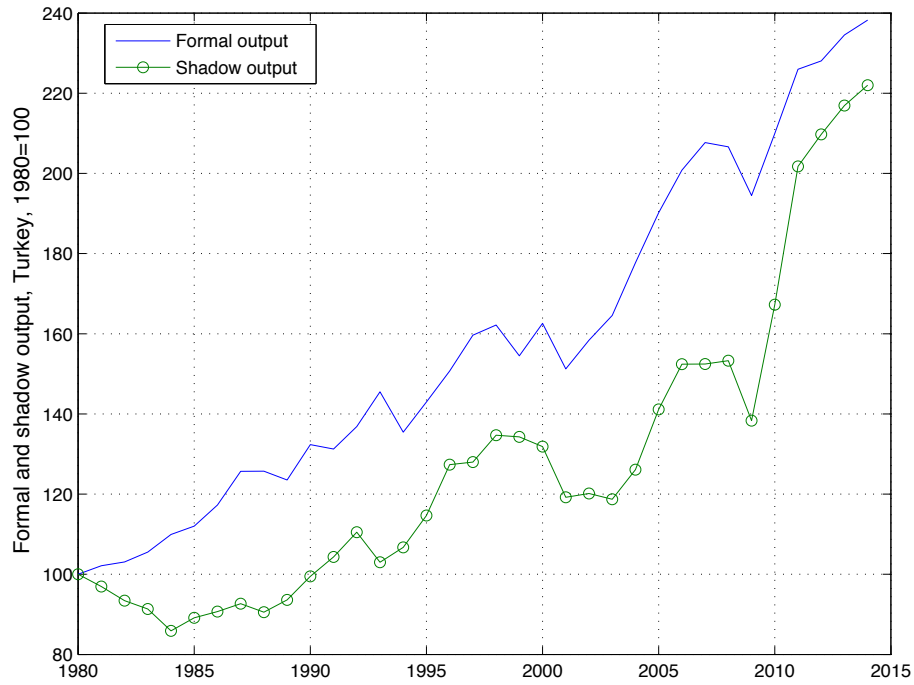


Figure 13: Turkey, formal and shadow output, 1980=100.

## Vietnam

Figure 14 presents the shadow-to-formal output ratio of Vietnam. Our results suggest that the size of the shadow economy has remained small: it has increased from 8 to 16% of formal output over the last 35 years (the ratio remained stable from 1985 to 2005). We also see that Schneider et al. show a downward-trending ratio that is not too far off from our results; the estimates of Elgin and Öztunalı suggest instead a dramatic decline in the ratio—a slow fall from 1980 to 1992 and then a steep decline up to 2010.

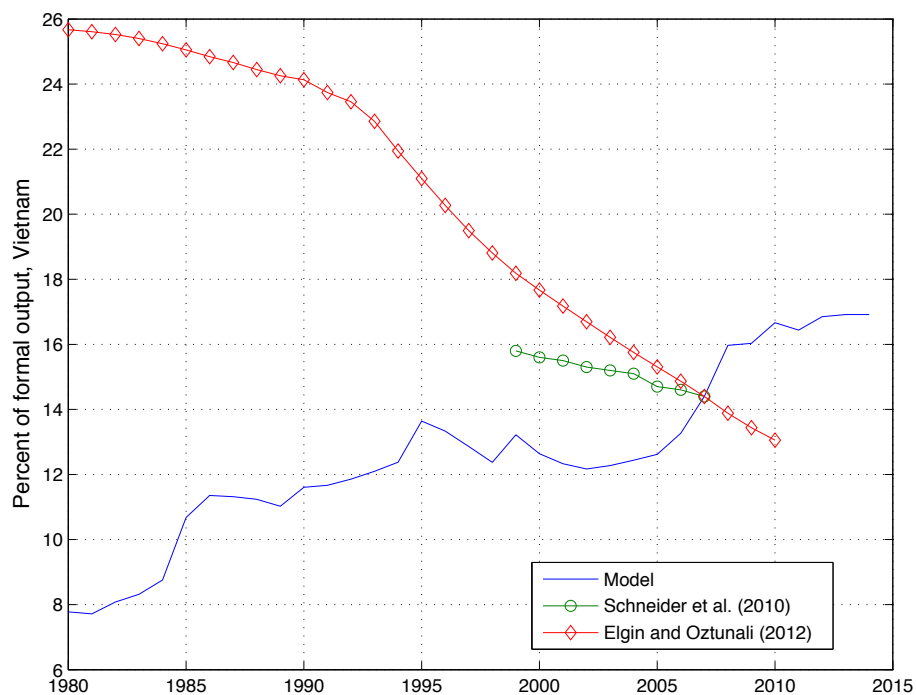


Figure 14: Vietnam, shadow-to-formal output ratio, 1980-2014.

The evolution of shadow and formal output in Vietnam is similar to that in Indonesia. Figure 15 shows a steady increase in the output of both sectors: formal output has increased by a factor of 5 while shadow output has done so by a factor of 11.

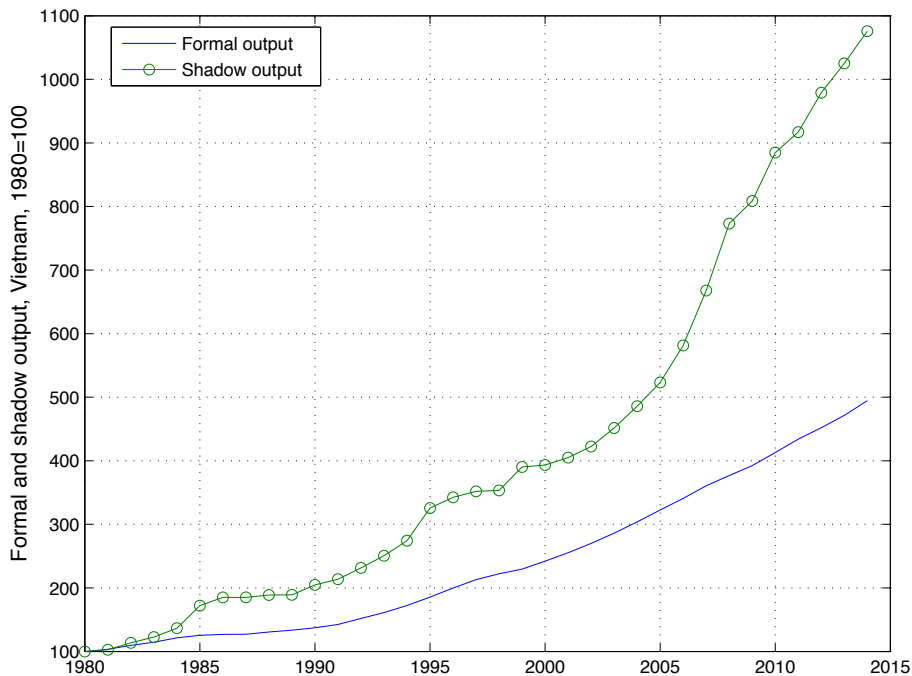


Figure 15: Vietnam, formal and shadow output, 1980=100.

#### 6.4 Business cycle statistics

We now present some basic business cycle statistics for all the countries analyzed; in particular, we document the volatilities of shadow, formal, and total output for our sample. We also present some relevant values that are by-products of our methodology: the value for  $\eta$ , which quantifies the returns to scale in the shadow production function; formal and shadow hours worked in 2007; as well as formal and shadow output per capita for the same year.

We can point to three main features derived from [Table 1](#). First, the standard deviation of the shadow sector output is higher than that of formal sector output for all countries in our sample. Second, the value of  $\eta$  averages 1.3, significantly above the constant returns to scale value of 1. Finally, hours worked in the shadow sector are smaller than those worked in the formal sector, though for some countries the difference is not that large (see Brazil).

Table 1: Business cycle statistics and other relevant values.

Country	$SD(Y_S)$	$SD(Y_F)$	$SD(Y_{TOT})$	$\eta$	$N_S$	$N_F$	$Y_S$	$Y_F$
Mexico	0.0511	0.0220	0.0264	1.33	553	825	\$4,553	\$15,808
Argentina	0.0963	0.0368	0.0439	1.39	392	746	\$3,911	\$17,006
Brazil	0.0377	0.0202	0.0234	1.28	746	874	\$4,829	\$13,193
Venezuela	0.0681	0.0389	0.0438	1.39	524	719	\$5,929	\$19,188
Indonesia	0.0408	0.0233	0.0243	1.24	336	895	\$1,337	\$7,470
Turkey	0.0438	0.0275	0.0290	1.40	410	549	\$4,666	\$16,035
Vietnam	0.0327	0.0085	0.0093	1.09	328	1,223	\$565	\$3,923
Average	0.0529	0.0255	0.0286	1.30	470	833	\$3,684	\$13,232

## 7 Conclusion

This paper has developed a methodology to measure the size and properties of the shadow economy. Our methodology is an improvement over other alternatives since (1) our procedure uses the restrictions imposed by a DGE model to estimate the size and dynamics of the shadow economy and (2) it requires minimal data and is relatively easy to implement. We believe that our methodology can prove useful to policymakers and academics interested in the shadow economy.

The work we present in the paper can easily be extended along several dimensions. First, the length of the shadow economy size time series can be used to determine whether developing economies are more volatile than developed ones, after taking the output of the shadow economy into consideration (see [Solis-Garcia and Xie 2016](#)). Second, the data can also be used to elaborate on the determinants of the shadow economy (e.g., [Friedman, Johnson, Kaufmann, and Zoido-Lobaton 2000](#)).

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## A Data sources

From the Penn World Table 9.0 (Feenstra, Inklaar, and Timmer 2015) we obtain the following variables:

1. Real GDP at constant 2011 national prices (`rgdpna`).
2. Capital stock at constant 2011 national prices (`rkna`).
3. Share of government consumption at current PPPs (`cs_h_g`).
4. Price level of capital formation (`pl_i`).

From The Conference Board<sup>12</sup> we obtain the following variables:

5. Midyear population.
6. Total annual hours worked.

From the items above, our model variables are obtained as follows:

7. Formal GDP per capita: 1/5.
8. Capital stock per capita: 2/5.
9. Price of investment: 4.
10. Hours worked per capita: 6/5.

The shadow economy size (relative to formal RGDP) is obtained from Elgin and Öztunalı (2012) and from Schneider, Buehn, and Montenegro (2010). The value of the tax rate follows the formulation found in Footnote 11.

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<sup>12</sup> The source is The Conference Board *Total Economy Database*<sup>TM</sup>, May 2016, <http://www.conference-board.org/data/economydatabase/>.

## B Proofs

**Proof of Proposition 4.1** First combine (4.1)-(4.10) to get the  $4 \times 4$  system

$$g_Y = g_A g_K \tag{B.1}$$

$$g_H (g_N)^{1+\chi} = g_Y^{1-\sigma} \tag{B.2}$$

$$g_Y = g_K^\alpha g_F^{1-\alpha} g_N^{1-\alpha} \tag{B.3}$$

$$g_Y = g_S^\eta g_N^\eta. \tag{B.4}$$

Substitute (B.1) in (B.2) and (B.3):

$$g_H g_N^{1+\chi} = g_A^{1-\sigma} g_K^{1-\sigma} \tag{B.5}$$

$$g_A g_K^{1-\alpha} = g_F^{1-\alpha} g_N^{1-\alpha}. \tag{B.6}$$

Now solve for  $g_N$  from (B.6)

$$g_N = g_A^{1/(1-\alpha)} g_K g_F^{-1}. \tag{B.7}$$

and plug in (B.5). Solving for  $g_K$  we get

$$g_K = g_H^{-1/(\sigma+\chi)} g_A^{-[\alpha(1-\sigma)+\sigma+\chi]/[(1-\alpha)(\sigma+\chi)]} g_F^{(1+\chi)/(\sigma+\chi)}, \tag{B.8}$$

which is equation (4.11). Now use (B.8) in (B.7) and solve for  $g_N$  to get

$$g_N = g_H^{-1/(\sigma+\chi)} g_A^{-\alpha(1-\sigma)/[(1-\alpha)(\sigma+\chi)]} g_F^{(1-\sigma)/(\sigma+\chi)}, \tag{B.9}$$

which is equation (4.12). Next, use (B.8) in (B.1); collecting common terms yields

$$g_Y = g_H^{-1/(\sigma+\chi)} g_A^{-\alpha(1+\chi)/[(1-\alpha)(\sigma+\chi)]} g_F^{(1+\chi)/(\sigma+\chi)}, \quad (\text{B.10})$$

which is equation (4.13). Finally, take (B.4) and substitute (B.9) and (B.10); after some algebra, we find that

$$g_S = g_H^{-(1+\eta)/[(\sigma+\chi)\eta]} g_A^{-\alpha[1+\chi+(1-\sigma)\eta]/[(1-\alpha)(\sigma+\chi)\eta]} g_F^{[1+\chi+(1-\sigma)\eta]/[(\sigma+\chi)\eta]},$$

as needed. □

**Proof of Proposition 4.3** Take (B.2) and solve for  $g_H$  to get (4.15). Take (B.1) and solve for  $g_A$  to get (4.16). Finally, take (B.3) and solve for  $g_F$  to get (4.17). □