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Abstract

This article proposes a differential-game model, in order to analyze markets in which regional regulation is operative and competition is based on quality. The case we have in mind is healthcare public service, where consumers (patients) choose the provider mainly basing on the providers' location and the quality of services, while prices play a more limited role. In most European countries, within the same

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State, regional (or local) providers compete on quality to attract demand. Market regulation is set at national and/or regional level. Our model highlights the features of equilibrium in such a framework, and specifically investigates how the differences in product quality evolve among regions, and how inter-regional demand flows behave. Differently from some available similar models (that do not take into account the regional dimension of the decision process), we find that quality differentials among regions may persist in equilibrium.

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1 Introduction

This article flows within the literature stream dealing with dynamic quality competition in markets under price regulation. A reference case is given by healthcare service, where prices are typically regulated, providers compete on quality, and consumers' demand choices are mainly driven by product quality and providers' location. In healthcare markets, the current relevance of quality competition is made clear by the fact that many reforms have implemented over the past years in several countries, with the final aim of increasing the product quality through harsher quality competition among providers. Generally speaking, such reforms allow the final consumers to choose the provider. Given that prices are regulated (and paid by a Public Authority, on a prospective payment system, or by insurance companies), the consumers' choices are based on the quality, along with the localization of the providers (Aiura [1]), rather than price. Thus, the providers have to compete to attract demand (and payments), by improving the quality of the offered services. However, our model can be appropriate for other sectors with similar characteristics, like education or long-term care.

A body of literature exists, focusing on the effect of different forms of competition and different regulation mechanisms upon the available quality level over time: Brekke et al. [4], [5], Siciliani et al. [16] are among the main references –a review is provided by Brekke et al. [6]. The specific problems studied in the mentioned articles range from the dynamic evolution of investment efforts made by providers and the corresponding available quality levels, to the relation between regulated prices and investment efforts to improve quality. Attention is paid to how different assumptions concerning the information sets used by players (and the corresponding game solution concepts) affect the features of the equilibrium path of control and state variables, and steady state allocation.

However, in this literature vein, the fact that different providers have a territorial / regional character, and regulation of relevant variables is made at the regional level, is usually overlooked. In Bisceglia et al. [3] spatial competition, both intra-regional and inter-regional, between providers is studied by means of a sequential game.

In the present article we aim to merge the mentioned research points concerning quality competition with a different research point, specifically concerning the territorial distribution of the service quality, the regional regulation, and the effect of decentralization decision processes upon available quality: the reference literature body –limiting our attention to markets with regulated prices and quality competition– includes, e.g., Balia et al. [2], Brekke et al. [7], [8], Levaggi et al. [14], [15].

While the former literature line employs dynamic (or, more specifically, differential) game tools to analyze the evolution of quality over time, the latter line typically resorts to static –or, at most, repeated or sequential– games. In the present article we propose a differential game model with the main aim of analyzing the implications of decentralized decision processes (specifically, regional regulation) upon the available quality levels. The model can be interpreted as a combination between models belonging to the mentioned literature lines; however, its conclusions are far from being a trivial sum of the points made by available articles. Our model shows that an equilibrium exists with different levels of quality across regions; differences in quality are permanent, but they may shrink or enlarge over time, also depending on the decision rules taken by the regulators. The model we present is analytically solvable thanks to its linear-quadratic structure; it provides some insights on the problems faced by providers and regional regulators in markets with quality competition and price regulation.

The structure of the article is as follows. Section 2 presents the model set-up. Section 3 presents, discusses and compares the open-loop and the feedback closed-loop solution of the game, assuming that regional regulators choose the investments able to move the quality. Section 4 presents a modification of the model, assuming that investments in quality are set by the providers, that are interested in profit rather than social welfare. Section 5 mentions further possible modifications, and concludes.

2 The model set-up

The basic set-up of the model is common to a number of available articles based on the Hotelling (1929) linear city model (see Calem and Rizzo [10], as a seminal contribution for the literature relevant to this present analysis; see also Brekke et al. [4]).

Consider a market with two providers located at either end of the unit line $S = [0, 1]$. On this line segment there is a uniform distribution of consumers, with total mass equal to 1. Since our reference example is healthcare provison, we will even refer to hospitals and patients for providers and consumers, respectively. Assuming that each consumer inelastically demands one unit of the considered service, the utility of a consumer located at $x \in S$ and buying from provider i, located at $z_i \in \{0, 1\}$, is given by

$$U(x, z_i) = v + kq_i - \tau |x - z_i|, \qquad (1)$$

where $v > 0$ is a parameter representing the gross valuation of consumption, q_i is the quality of the product (service) offered by provider i, $k > 0$ is a parameter measuring the marginal willingness to pay for quality, and $\tau > 0$ is the marginal transportation cost. In what follows we set $k = 1$ without loss of generality. Notice that in the case of competition among hospitals, it makes sense to assume that physical locations are fixed; the fact that locations are at the end points of the line is immaterial to our conclusions. Since the distance between providers is equal to one, the consumer who is indifferent between i and j is located at x_i^D, is characterised by

$$v - \tau x_i^D + q_i = v - \tau \left(1 - x_i^D\right) + q_j, \tag{2}$$

so that the demand for provider i is:

$$x_i^D(t) = \frac{1}{2} + \frac{q_i(t) - q_j(t)}{2\tau} \tag{3}$$

The demand is consistent with the assumptions of uniform consumer distribution (with mass 1), exogenous locations of providers and full market coverage. We call 1 and 2 the provider located at 0 and 1, respectively.

We propose here to introduce the assumption that the linear space S is administratively divided into two regions, called R_1 and R_2: consumers located between 0 and 1/2 belong to region R_1, and consumers located between 1/2 and 1 belong to region R_2. Again, the provider located at 0 and 1, will be indexed by 1 and 2, respectively, as they are under the administrative control of regulator of R_1 and R_2, respectively. Thus, each region has one regulator and one provider within its administrative space.

We can split the demand for each provider into two components, corresponding to "domestic" and "extra-regional" demand; formally: $x_i^D = x_i^i + x_i^j$, where x_i^i is the domestic demand (demand from residents in R_i met by the provider located in the same R_i) and x_i^j is the demand for provider i coming from residents in Region j. Specifically, we have:

$$\begin{cases} x_i^i = \frac{1}{2} - \frac{q_j - q_i}{2\tau},\ x_i^j = 0 & \text{if } q_i \leq q_j \\ x_i^i = \frac{1}{2},\ x_i^j = \frac{q_i - q_j}{2\tau} & \text{if } q_i > q_j \end{cases}$$

that is,

$$x_i^i = \min\left(\frac{1}{2}, \frac{1}{2} - \frac{q_j - q_i}{2\tau}\right), \qquad x_i^j = \max\left(0, \frac{q_i - q_j}{2\tau}\right).$$

Following available models, we assume that the cost function of each provider is linear in the quantity, and quadratic in the quality levels (q) and investment to improve quality levels (I); cost may also include a fixed cost:

$$C_i = c_i x_i + \frac{\beta}{2} q_i^2 + \frac{\gamma}{2} I_i^2 + F_i \tag{4}$$

where c_i, β, γ and F_i are positive parameters. Notice that we assume that constant marginal costs c_i and fixed cost F_i may differ across regions; this corresponds to the fact that institutional (organizational) aspects matter on the cost structure (and it is well known that differences in efficiency between hospitals in different regions exist). Parameters β and γ are assumed to be equal across regions, to easy the analytics. In what follows, γ is normalized to 1 without loss of generality.

Each hospital receives a price p_i (fixed by the domestic regional regulator) for each unit of produced service consumed by domestic patients, while the price for extra-regional treatment, p, is exogenously set by a central authority. This set of assumptions is consistent with what happens in the health system of several countries, like Italy or Spain, where regional regulators set the price for domestic treatment, while a system of centrally fixed prices hold for extra-regional treatments; moreover, hospitals may receive from the domestic regulator a possible lump-sum transfer to break-even, if the operative profit is negative.

Following the mentioned differential game literature, we assume that that demand is decided by each consumer at each instant of time $t \in [0, +\infty)$. The services' quality levels move over time, thanks to investment I aimed at improving quality. At the beginning, quality levels are $q_1(0) = q_{01} > q_2(0) = q_{02} > 0$, i.e., the quality level of the provider located in R_1 is higher than the quality level of provider of R_2. Then, the dynamics of quality is ruled by the following equation:

$$\dot{q}_i = I_i - \delta q_i \tag{5}$$

where $\delta > 0$ is a depreciation rate. Note that no externalities are at work across regions, and the service quality level of each provider only depends on his investment efforts (apart form the initial conditions).

Models in mentioned available literature differ as far as the objective functions concern. In some models, profit-oriented (or partially altruistic or motivated) providers aim at their own maximum result, while regulator(s) care(s) about social welfare; in other models, there is no distinction between provider and regulator, as far as the objective function concerns. In the main version of the present model (Section 3), we adopt the latter (and simpler) assumption that each regulator, aiming at maximizing the social welfare of his own region, sets the quality level; a modification, where providers aiming at maximum profit set the quality levels, is developed in Section 4. Different assumptions can be considered as well in different formulations of the model.

3 regional regulators as quality setters

Since we are mainly interested in studying the dynamics of regional differences in quality of offered services, and the dynamics of regional mobility of consumers, we start by keeping the model as simple as possible: we assume that the regulators set the quality (in fact, they set the investments able to move the quality);[1] moreover, each regional regulator takes into account the surplus of the citizens of its own region, and the profit of the hospital located within the region, along with the (public) expenditure borne by himself. Formally, the instantaneous objective function of Region i at tome t is:

$$\Pi_i(t) = \sigma_i(t) + \pi_i(t) - G_i$$

where σ_i is the surplus of residents in Region i; π_i is the profit of the hospital located in region i and G_i is the public expenditure for the service, that is, the payment from the regulator to the domestic (and possibly to the extra-regional) hospital; G_i may also include a lump-sum transfer to the provider of the same region i, to reach the break-even point, in the case of a negative operative profit. Since revenues for the hospital coming from

[1] See Cellini and Lamantia [11] as a model, sharing some characteristics with the present one, in which regulators set miminum quality standard instead of quality levels.

domestic treatment ($p_i x_i^i$) are paid by the same regulator, this sum will not appear in the objective function. In this set of assumptions we follow Siciliani et al. [16]. Differently, sums paid by a region to the other region enter the objective function of both regulators (of course, with opposite sign). Still following Siciliani et al. [16], an opportunity cost $\lambda > 0$ is associated to the public expenditure different from the transfer covering the payment for domestic treatment $p_i x_i^i$. Thus,

$$\sigma_1(t) = \int_0^{\frac{1}{2}} (v + q_1 - \tau x)\, dx \tag{6}$$

$$\sigma_2(t) = \int_{\frac{1}{2}}^{\frac{1}{2}+\frac{q_1-q_2}{2\tau}} (v + q_1 - \tau x)\, dx + \int_{\frac{1}{2}+\frac{q_1-q_2}{2\tau}}^{1} [v + q_2 - \tau(1-x)]\, dx \tag{7}$$

$$\pi_1(t) - G_1(t) = -(1+\lambda)[c_1(\frac{1}{2} + \frac{q_1 - q_2}{2\tau}) + \frac{\beta}{2}q_1^2 + \frac{1}{2}I_1^2] + p(\frac{q_1 - q_2}{2\tau}) \tag{8}$$

$$\pi_2(t) - G_2(t) = -(1+\lambda)[c_2(\frac{1}{2} - \frac{q_1 - q_2}{2\tau}) + \frac{\beta}{2}q_2^2 + \frac{1}{2}I_2^2] - (1+\lambda)p(\frac{q_1 - q_2}{2\tau}) \tag{9}$$

Hence, the dynamic problem of Region i is:

$$\begin{cases} \max\limits_{I_i \geq 0} \displaystyle\int_0^{\infty} e^{-\rho t}\Pi_i(t)dt \\ \dot{q}_i = I_i - \delta q_i, q_i(0) = q_{i0} \\ \dot{q}_j = I_j - \delta q_j, q_j(0) = q_{j0} \end{cases}$$

Notice that there is no difference, in this framework, between regulator and provider: the regulator sets the provider's investment aimed to improve the quality level of the produced service. The regulator also sets the unit price for the service delivered to domestic residents. However, the entailed public expenditure, $p_i x_i^i$, corresponds to revenue for the provider, so that its amount is immaterial to the objective function, and prices p_i $(i = 1, 2)$ do not enter the problems (this feature disappears from more complex version of the model, such as the formulation presented in Section 4).

Quality levels affect the demand and hence the inter-regional patients' mobility, along with the production costs. Clearly, interdependence between the two regulator's problems does exist, as long as the state variable of a player enters the problem of the other player. Hence, we are in front of a differential game.

We solve this differential game under two different assumptions concerning the information set used by the players, and correspondingly we depict the equilibrium under two different solution concepts. Firstly, we find the *open-loop* solution, where both players are assumed to be unable to observe the evolution of state variables over time, and they compute the optimal path of the choice variables at the beginning of time, and then stick to this solution forever. The open-loop solution is of type $I_i(t) = f(t; q_{i0}, q_{j0})$. Secondly, we will find the Markovian closed-loop feedback solution, where each player is assumed to be able to observe the dynamic evolution of state variables, and the optimal value of the choice variable depends on the current value of state variables, so that the solution is of type: $I_i(t) = f(t, q_i(t), q_j(t))$. Features, pros and cons of these solution concepts are widely discussed in the differential game literature (see, e.g., Dockner et al. [13]); feedback solutions are strongly time consistent, while open-loop solutions are, in general, only weakly time consistent.

3.1 Open Loop Solution

We are interested in studying the dynamics of the model in the presence of asymmetry between providers. The assumption of asymmetry can be seen as a novelty with respect to available models (like Brekke et al. [4], [5] or Siciliani et al. [16] where symmetry across providers is assumed). To fix the reference point, we introduce the following Assumptions, entailing that Region 1 is the more efficient, hence with the higher quality level, and attracting consumers from the other region.

Assumptions

$$\tau > \frac{1}{2\beta(\lambda+1)} \tag{10}$$

$$c_1 < c_2 - \frac{\lambda}{\lambda+1}p \tag{11}$$

Of course, situation (and conclusions) could be reversed, switching Region 1 with 2 and viceversa.

Proposition 1 *Under Assumptions (10)-(11), the pair of strategies* $(I_1(t), I_2(t))$, *solving the following ODE system in the state-control variables, is an Open*

Loop Nash Equilibrium:

$$\begin{cases} \dot{I}_1 = (\rho+\delta)I_1 + \frac{1}{1+\lambda}\left[\beta(\lambda+1)q_1 + \dfrac{c_1(\lambda+1)-p}{2\tau} - \dfrac{1}{2}\right] \\ \dot{I}_2 = (\rho+\delta)I_2 + \frac{1}{1+\lambda}\left[\beta(\lambda+1)q_2 + \dfrac{q_1-q_2}{2\tau} + \dfrac{(c_2-p)(\lambda+1)}{2\tau} - \dfrac{1}{2}\right] \\ \dot{q}_1 = I_1 - \delta q_1 \\ \dot{q}_2 = I_2 - \delta q_2 \end{cases} \tag{12}$$

With initial conditions: $q_1(0) = q_{01} > q_2(0) = q_{02} > 0.$

The equilibrium point of this linear system is given by:

$I_i^{OL} = \delta q_i^{OL}, \quad i = 1, 2$

where q_i^{OL} *are the open-loop steady state quality levels given by*

$$q_1^{OL} = \frac{p+\tau-c_1(\lambda+1)}{2\tau(\lambda+1)(\delta^2+\delta\rho+\beta)}$$

$$q_2^{OL} = \frac{2\tau(\delta^2+\delta\rho+\beta)(\lambda+1)[(p-c_2)(\lambda+1)+\tau]+c_1(\lambda+1)-p-\tau}{2\tau(\delta^2+\delta\rho+\beta)(\lambda+1)[2\tau(\delta^2+\delta\rho+\beta)(\lambda+1)-1]}$$

and it constitutes a saddle point.

Proof. *See Appendix A.*

It is interesting to note that the steady state level of the quality produced by the provider in R_1 (with the higher quality level) does not depend on the cost parameter c_2 pertaining to the provider of the other region, while the opposite does not hold: the steady-state quality produced in R_2 is affected by the cost structure of the provider with higher quality too. Technically, this is consistent with the fact that in system (12) only c_1 and q_1 appear in the dynamic equation pertaining to I_1, while both q_1 and q_2, along with

c_2, appear in the dynamics of I_2. From a substantial point of view, this is due to the assumption that the provider of Region 1 is (and remains) the more efficient one. Thus, since the provider of Region 1 is aware that he serves the whole domestic market and attracts consumers from the other region, the cost structure of Region 2 does not affect the optimal choice of the provider of Region 1. On the opposite, the quality level of provider 1 (influenced by his own cost structure) affects the amount of residents of Region 2 who decide to migrate, and hence affects the optimal investment decision of provider 2.

Consistently with the quality differential between the two providers in steady state, the steady-state inter-regional demand flow, constituted by residents of R_2 who demand the service from the provider of R_1, turns out to be equal to

$$(x_1^2)^{OL} = \frac{q_1^{OL} - q_2^{OL}}{2\tau} = \frac{(1+\lambda)(c_2 - c_1) - \lambda p}{2\tau[2\tau(1+\lambda)(\delta^2 + \delta\rho + \beta) - 1]}$$

As far as the dynamics of such inter-regional flow is concerned, we can simply notice that, since the steady state quality levels do not depend on the initial ones, we can have a higher or a lower equilibrium quality gap between the two regions with respect to the initial value, and this obviously results in a higher or lower inter-regional patients flow.

Simple comparative statics provide the following results concerning steady state allocation.

As to the effect of τ on quality levels and inter-regional migrational flow, it is easy to check that:

$$\frac{\partial q_1^{OL}}{\partial \tau} = \frac{c_1(1+\lambda) - p}{2\,\tau^2\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)}$$

$$\frac{\partial q_2^{OL}}{\partial \tau} = \frac{c_1(1+\lambda) - p}{2\,\tau^2\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)} + \frac{2\,(\lambda+1)\,(p-(\lambda+1)\,(c_1-c_2+p))\,\left(\delta^2+\rho\,\delta+\beta\right)}{\left(2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)-1\right)^2}$$

$$\frac{\partial (x_1^2)^{OL}}{\partial \tau} =$$

$$-\frac{p-(\lambda+1)\,(c_1-c_2+p)}{2\,\tau^2\,\left(2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)-1\right)} - \frac{(\lambda+1)\,(p-(\lambda+1)\,(c_1-c_2+p))\,\left(\delta^2+\rho\,\delta+\beta\right)}{\tau\left(2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)-1\right)^2}$$

It results $\frac{\partial q_1^{OL}}{\partial \tau} < 0$ if $p > c_1(1+\lambda)$. The economic meaning is immediate: lower travel cost entail harsher competition (as discussed in several contributions, including Siciliani et al. [16]); harsher competition leads to higher quality levels, if unit price is sufficiently high, and providers have a financial incentive to attract extra-regional demand. On the opposite, if the regulated unit price is "too low" (and financial losses are entailed by serving the market), harsher competition leads providers to exert lower efforts to increase quality, since higher demand would entail lower revenues. Our present result perfectly mimics the outcome in Siciliani et al. [16], where -in subsection 3.2.2- a brief discussion is provided on cases in which policy measures aimed at increasing competition have been based on travel cost reduction, for instance by reimbursing consumers/patients who choose to move to different regions. Clearly, such policy measures succeed in obtaining the expected increase of quality levels, only if they are associated to sufficiently high price for the service.

Similarly, note that $\frac{\partial q_2^{OL}}{\partial \tau} < 0$ if $p > \frac{(\lambda+1)}{\lambda}(c_2 - c_1)$, which can be interpreted in the same way as explained above for the quality of the provider of R_1. Under the same parametric condition, it holds that $\frac{\partial q_2^{OL}}{\partial \tau} > \frac{\partial q_1^{OL}}{\partial \tau}$. Moreover, condition $\tau > \frac{1}{2(\delta^2+\delta\rho+\beta)(\lambda+1)}$ and $p > \frac{(\lambda+1)}{\lambda}(c_2 - c_1)$ are sufficient to ensure that $(x_1^2)^{OL} > 0$, and $\frac{\partial (x_1^2)^{OL}}{\partial \tau} < 0$. This means that harsher competition (i.e., lower τ) leads to higher quality levels in both regions, but to a larger quality differential too: the region with lower marginal cost (and higher initial quality) increases the steady state quality by a larger amount, in front of lower consumers' travel cost, so that the amount of inter-regional migration increases. If we imagine that τ can be reduced through appropriate pro-competition policies, then such policies are beneficial for the available quality levels, but they entails larger quality differences and larger patient mobility.

As to the effect of p, the centrally regulated price for extra-regional treatment, we have:

$$\begin{aligned}
\frac{\partial q_1^{OL}}{\partial p} &= \frac{1}{2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)} > 0 \\
\frac{\partial q_2^{OL}}{\partial p} &= \frac{\lambda}{2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)-1} + \frac{1}{2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)} \\
\frac{\partial (x_1^2)^{OL}}{\partial p} &= -\frac{\lambda}{2\,\tau\,\left(2\,\tau\,(\lambda+1)\,\left(\delta^2+\rho\,\delta+\beta\right)-1\right)}
\end{aligned}$$

An increase of the price for extra-regional treatment leads to higher quality levels in the most efficient region, that attracts extra-regional demand. The same effect on the quality in the less efficient region occurs only if condition $\tau > \dfrac{1}{2(\delta^2 + \delta\rho + \beta)(\lambda + 1)}$ is met. Under the same condition it results: $\dfrac{\partial (x_1^2)^{OL}}{\partial p} < 0$. Verbally, under high transport cost, an increase of the price for extra-regional treatment leads the region with lower quality level to exert higher effort to increase its quality, in order to reduce the number of its citizens who decide to migrate to buy the service; on the opposite, if transportation costs are low, the less efficient region finds it convenient to reduce its quality level, inducing its citizens to buy the service from the provider of the other region.

3.2 Feedback Solution

Assumptions We must again assume that τ is greater than the threshold specified in (10). In addition to this, for $\rho \to 0$, it must also hold:

$$c_1 < c_2\sqrt{1 - \frac{1}{X}} + p\left[\frac{1}{\lambda + 1} - \sqrt{1 - \frac{1}{X}}\right] \tag{13}$$

where X is defined as:

$$X := 2\tau(\lambda + 1)(\beta + \delta^2) > 0 \tag{14}$$

and where $1 - \frac{1}{X} > 0$; and $\frac{1}{\lambda+1} - \sqrt{1 - \frac{1}{X}} > 0$ if $\tau < \frac{\lambda+1}{2(\beta+\delta^2)[(\lambda+1)^2-1]}$.

Under these Assumptions, we will explain later, the following Proposition provides a linear Feedback Nash Equilibrium of the considered differential game.

Proposition 2 *A linear FNE is given by the following couple of strategies* (I_1, I_2)*:*

$$I_1 = \frac{1}{1 + \lambda}(\alpha_1 + \alpha_3 q_1) \tag{15}$$

$$I_2 = \frac{1}{1+\lambda}(k_1 + k_3 q_2 + k_5 q_1) \tag{16}$$

where, defining[2]*:*

$$A := \tau(\lambda+1)[4\beta + (2\delta+\rho)^2] - 2 \tag{17}$$

we have:

$$\alpha_3 = \frac{\lambda+1}{2}[(2\delta+\rho) - \sqrt{4\beta+(2\delta+\rho)^2}] \tag{18}$$

$$k_3 = \sqrt{\frac{\lambda+1}{4\tau}}[\sqrt{\tau(\lambda+1)}(2\delta+\rho) - \sqrt{A}] \tag{19}$$

$$k_5 = \sqrt{\frac{\lambda+1}{4\tau}}[\sqrt{A} - \sqrt{A+2}] = -k_4 \tag{20}$$

$$\alpha_1 = \frac{N_{\alpha_1}}{D_{\alpha_1}}, \quad k_1 = \frac{N_{k_1}}{D_{k_1}} \tag{21}$$

with:

$$N_{\alpha_1} = \sqrt{\tau}(\sqrt{A} + \rho\sqrt{\tau}\sqrt{\lambda+1})\sqrt{4\beta+(2\delta^2+\rho)^2} - \rho\sqrt{\tau}\sqrt{A} +$$

$$+4\beta\sqrt{\lambda+1}(p - c_1(\lambda+1)) - \sqrt{\lambda+1}(4c_1\delta(\delta+\rho)(\lambda+1) - 4\delta^2 p - \rho(4\delta p - \rho\tau))$$

$$D_{\alpha_1} = 4\sqrt{\tau}(\beta + \delta(\delta+\rho))(\sqrt{A} + \rho\sqrt{\tau(\lambda+1)})$$

$$N_{k_1} = \sqrt{\tau(4\beta + 4\delta^2 + 4\delta\rho + \rho^2)}[\rho\sqrt{A\tau(\lambda+1)} + 2\beta(\lambda+1)(c_1(\lambda+1) - p + \tau) + 2c_1\delta(\delta+\rho)(\lambda+1)^2 +$$

$$+2\delta^2(\lambda+1)(\tau-p) + 2\delta\rho(\lambda+1)(\tau-p) + \rho^2\tau(\lambda+1) - 1] +$$

$$-[\sqrt{A(\lambda+1)}(2\beta(c_1(\lambda+1) + \lambda(c_2-p) - 2p + c_2) + 2c_1\delta(\delta+r)(\lambda+1) - 2\delta^2(\lambda(p-c_2) + 2p - c_2) +$$

[2]Note that, for $\rho \to 0 : A = 2(X-1)$.

$$-\rho(2\delta(\lambda(p-c_2)+2p-c_2)-\rho\tau))-\rho\sqrt{\tau}(2(\lambda+1)((\lambda+1)(p-c_2)-\tau))(\beta+\delta^2+\delta\rho)-\rho^2\tau(\lambda+1)+1)]$$

$$D_{k_1}=4\sqrt{\tau}(\rho\sqrt{\tau}(\beta+\delta(\delta+\rho))\sqrt{A(\lambda+1)}+2\beta^2\tau(\lambda+1)+\beta(\tau(\lambda+1)(2\delta+\rho)^2-1)+$$

$$+\delta(2\delta^3\tau(\lambda+1)+4\delta^2\rho\tau(\lambda+1)+\delta(3\rho^2\tau(\lambda+1)-1)+\rho(\rho^2\tau(\lambda+1)-1)))$$

Proof. *See Appendix B.*

Steady state qualities are thus given by:

$$q_1^F=\frac{\alpha_1}{\delta(\lambda+1)-\alpha_3}$$

$$q_2^F=\frac{k_1+k_5q_1^F}{\delta(\lambda+1)-k_3}$$

In this framework their analytic expressions are really cumbersome and intractable; hence in the remainder of this section we will focus on the particular case when the discount rate is negligible.

Remark 1 *Notice that Assumption (10) ensures $k_3<0$, which in turn implies that the first order condition provides a maximum point for the RHS of the HJB equation of the less efficient hospital (see Appendix B).*

From (15) and (16) it follows that

$$q_1^F>q_2^F \iff q_1^F>\frac{k_1}{\delta(\lambda+1)-(k_3+k_5)}$$

Therefore, recalling that $0>\alpha_3=k_3+k_5$, a sufficient condition for $q_1^F>q_2^F$ is:

$$\alpha_1>k_1$$

Furthermore, since the equilibrium point of the linear system in (q_1, q_2) *that we obtain from the FNE strategies and the quality levels dynamics is a stable node (see Appendix B), implying monotonic trajectories for the quality levels, the considered assumption is also a sufficient condition for* $q_1^F(t) > q_2^F(t)$, $\forall t \in [0, \infty)$. *Also in this case, since the steady-state qualities do not depend on the initial quality levels, there may be an increase or a decrease in the inter-regional migrational flow over time.*

Since, given the cumbersome expressions for α_1 *and* k_1 *in the general case with* $\rho > 0$*, it is really difficult to state the assumption* $\alpha_1 > k_1$ *in a convenient and meaningful form, we limit our attention to the particular case where* $\rho \to 0$*, obtaining Assumption (13).*

For $\rho \to 0$ we obtain the following steady-state qualities:

$$q_1^F = \frac{\sqrt{\tau(X-1)} + (p - c_1(\lambda+1))\sqrt{2(\beta+\delta^2)(\lambda+1)}}{2\sqrt{\tau}(\beta+\delta^2)(\lambda+1)\sqrt{X-1}}$$

$$q_2^F = \frac{N_{q_2^F}}{D_{q_2^F}}$$

where

$$N_{q_2^F} = \sqrt{2(\beta+\delta^2)^3(\lambda+1)(X-1)}(c_1(\lambda+1)-p) + 4B\beta^3\tau^{3/2}(\lambda+1)^2 +$$

$$+2\beta^2\sqrt{\tau}(\lambda+1)[6B\delta^2\tau(\lambda+1) - (B+\tau)] +$$

$$+\beta\sqrt{\tau}(12B\delta^4\tau(\lambda+1)^2 - 4\delta^2(\lambda+1)(B+\tau)+1) + \delta^2\sqrt{\tau}(2\delta^2\tau(\lambda+1)-1)(2B\delta^2(\lambda+1)-1)$$

with $B := (\lambda+1)(p-c_2)+\tau$, and

$$D_{q_2^F} = 2\sqrt{\tau}(\beta+\delta^2)(\lambda+1)(X-1)(2\beta^2\tau(\lambda+1)+\beta(4\delta^2\tau(\lambda+1)-1)+\delta^2(2\delta^2\tau(\lambda+1)-1))$$

For $\rho \to 0$ we also have[3]:

$$\frac{\partial q_1^F}{\partial p} = \frac{1}{\sqrt{X(X-1)}} > 0$$

$$\frac{\partial q_2^F}{\partial p} = \frac{1}{(1-X)\sqrt{X}[2\tau(\lambda+1)[(\beta+\delta^2)^2+2\beta\delta^2]-(\beta+\delta^2)]} \cdot \{(\beta+\delta^2)^{3/2}\sqrt{X-1}+$$
$$-2\beta^3\tau(\lambda+1)^2\sqrt{8\tau(\lambda+1)}+\beta^2(\lambda+1)^{3/2}\sqrt{2\tau}(1-6\delta^2\tau(\lambda+1))+$$
$$+2\sqrt{2\tau}\beta\delta^2(\lambda+1)^{3/2}(1-3\delta^2\tau(\lambda+1))-\sqrt{2\tau}\delta^4(\lambda+1)^{3/2}(2\delta^2\tau(\lambda+1)-1)\}$$

this quantity is positive if[4]:

$$X(X-1) > \frac{1}{(\lambda+1)^2} \iff \tau < \frac{\sqrt{(\lambda+1)^2+4}-(\lambda+1)}{4(\beta+\delta^2)(\lambda+1)^2} \text{ or } \tau > \frac{\sqrt{(\lambda+1)^2+4}+(\lambda+1)}{4(\beta+\delta^2)(\lambda+1)^2}$$

It is interesting to note that an increase of the exogenous price of extra-regional treatment has a clear (and positive) impact on the quality of the service provided in R_1, the region with the higher quality level (and hence attracting the extra-regional demand), while the effect can be positive or negative upon the quality of the service provided in R_2, the region with the lower quality level. The intuition runs as follows. The region with the higher quality has a clear incentive to increase the amount of extra-regional treatment, since the marginal gain is positive and increasing in p. The

[3] It is easy to prove that the derivative of the steady-state quality of the most efficient region w.r.t. the price p is positive also in the general case with $\rho > 0$.

[4] Both these thresholds are in general compatible with Assumption (10).

region with lower quality level, that has to pay for extra-regional treatments, could find it convenient to reduce the extra-regional patients' flow, or to increase it, depending on the relation between parameters connected with costs (transport cost, operative cost, opportunity cost). Consistently, the effect of an increase of p upon the size of inter-regional flow is

$$\frac{\partial x_1^2}{\partial p} = \frac{[\sqrt{X} - (\lambda+1)\sqrt{X-1}]}{2\tau(X-1)^{3/2}}$$

and this derivative is positive if:

$$\tau < \frac{\lambda+1}{2\lambda(\lambda+2)(\beta+\delta^2)}$$

Finally, one can check that

$$\frac{\partial q_1^F}{\partial \tau} = \frac{[c_1(\lambda+1) - p](2X-1)}{2\tau(X-1)^{3/2}\sqrt{X}} < 0 \iff p > c_1(\lambda+1)$$

exactly as it is for the Open Loop case.

3.3 A comparison between steady-state qualities for $\rho \to 0$

It is easy to check that

$$q_1^{OL} - q_1^F = [p - c_1(\lambda+1)]\frac{\sqrt{X-1} - \sqrt{X}}{X\sqrt{X-1}}$$

where X is given in (14); therefore:

$$q_1^F > q_1^{OL} \iff p > c_1(\lambda+1)$$

The result emerges, according to which the stronger competition entailed by the feedback behavior rule (as compared to open-loop rule) leads to a higher level of steady state quality, in the region with the higher quality level, at least for sufficiently high price level of extra-regional treatment. However, this conclusion does not apply to the provider with the lower quality level; indeed:

$$q_2^{OL} - q_2^F = (p - c_1(\lambda+1))\frac{(\beta+\delta^2)\sqrt{X(X-1)} - Y}{X(X-1)Y}$$

where $Y := 2\beta^2\tau(\lambda+1) + \beta[1 - 4\delta^2\tau(\lambda+1)] - \delta^2[2\delta^2\tau(\lambda+1) - 1]$.

Since we have: $\frac{(\beta+\delta^2)\sqrt{X(X-1)}-Y}{X(X-1)Y} > 0 \iff \frac{\tau[\sqrt{X-1}-\sqrt{X}]}{(X-1)^{3/2}} < 0$, it follows:

$$q_2^{OL} > q_2^F \iff p > c_1(\lambda+1)$$

Thus, under a sufficiently high price p, the region with the lower quality level arrives at a steady state in which quality is lower under the feedback behavior rule as compared to the open-loop rule. Hence, a harsher competition (as captured by closed-loop behaviour of players instead of open-loop behavior rule) results in a lower quality level in the less efficient region.

Hence, still limiting attention to steady state, in the presence of a sufficiently high price p, quality differential between regions is larger under feedback behaviors than under open-loop. Formally, for $p > c_1(\lambda+1)$, we have:

$$q_1^F - q_2^F > q_1^{OL} - q_2^{OL} \Longrightarrow (x_1^2)^F > (x_1^2)^{OL}$$

while the opposite holds for $p < c_1(\lambda+1)$. Verbally, stronger competition in the form of feedback behaviour of providers, instead of open-loop behavior, leads to a larger quality differential between regions and hence to larger inter-regional flow. This consequence is usually overlooked by current debates on pros and cons of competition in healthcare markets and new governance of national health systems, and regulation of markets where quality competition takes place. Our model makes clear that distance matters, and travel costs play a role in individual and social welfare consideration.

4 Extension: a model with providers as quality setters

In the version of the model above, prices p_1 and p_2 do not enter the problem of the regulators, as they have only redistributive effects. This assumption can be easily removed, to show that the levels of price set at the regional level may have relevant impact on quality (and hence demand). An easy way to show this point is to assume, as in Brekke et al. [4], that the providers choose the investment level at each instant in time, taking price (p_1 and p_2, in this case) as known; in such a case, objective function to maximize for provider H_i is:

$$J_i = \int_0^\infty \pi_i(t) e^{-\rho t}\, dt$$

where instantaneous profit is:

$$\pi_i = (p_i - c_i) x_i^D - \frac{\gamma}{2} I_i^2 - \frac{\beta}{2} q_i^2$$

Notice that provider i receives p_i for each case treated; however, Region i pays for each consumer belonging to R_i, while the same amount p_i is paid from region R_j for any patient belonging to x_i^j. Notice also that, for the sake of easiness, we assume that the objective function of each provider is simply its own profit, without taking into account the wide body of considerations supporting the occurrence of motivated (i.e., semi-altruistic) providers (see, e.g., Siciliani et al. [16] and references therein).

As in the model presented in the previous section, control variable is investment in quality, which affects the quality level, according to equation (5), again given an initial condition $q_1^0 > 0, q_2^0 > 0$. The following propositions provide the OLNE and the linear FNE of the model into consideration.

Proposition 3 *The ODE system in the state-control variables that an Open Loop Nash Equilibrium strategies pair $(I_1(t), I_2(t))$ solves is given by:*

$$\begin{cases} \dot{I}_i = (\rho + \delta)I_i + \beta q_i - \dfrac{p_i - c_i}{2\tau} \\ \dot{q}_i = I_i - \delta q_i \end{cases} \tag{22}$$

with $i = 1, 2$. The equilibrium point of this linear system is given by:

$$I_i^{OL} = \delta q_i^{OL}, \quad i = 1, 2$$

$$q_i^{OL} = \frac{p_i - c_i}{2\tau(\delta^2 + \delta\rho + \beta)}$$

and it constitutes a saddle point.

Proof. The proof is omitted since it is analogous to that provided in Brekke et al. [4] for the case of constant marginal treatment costs with $\varphi = 0$ and $\gamma = 1$ but allowing for different coefficients p_i and c_i.

Proposition 4 *A linear FNE strategy for provider $H_i, i \in \{1,2\}$, is given by:*

$$I_i(t) = \varphi_i + \omega q_i(t)$$

where:

$$\varphi_i = \frac{p_i - c_i}{\tau\sqrt{4\beta + (\rho + 2\delta)^2}}$$

$$\omega = \delta + \frac{\rho}{2} - \sqrt{\beta + \left(\delta + \frac{\rho}{2}\right)^2}$$

Proof. The proof is omitted since it employs the same technique shown in Appendix B and it yields a solution analogous to that obtained by Brekke et al. [4] for the considered case with different treatment marginal costs c_i and prices p_i across hospitals.

Note that our solution constitutes the only linear FNE in which the control chosen, in every time instant, by each hospital depends only on the current value of its own quality level (and on the price chosen by its Region).

From the FNE strategies, by using the state variable dynamic, we obtain the time-path of the quality level for H_i:

$$q_i(t) = \left[q_i^0 - \frac{p_i - c_i}{2\tau(\delta^2 + \beta + \rho\delta)}\right] e^{\left(\frac{\rho}{2} - \sqrt{\beta + \left(\frac{\rho}{2} + \delta\right)^2}\right)t} + \frac{p_i - c_i}{2\tau(\delta^2 + \beta + \rho\delta)} \tag{23}$$

Sufficient condition for $q_i(t) > 0$ is $p_i > c_i$. It holds:

$$\Delta q(t) = [\Delta q^0 - \frac{\Delta p - \Delta c}{2\tau(\delta^2 + \beta + \rho\delta)}] e^{\left(\frac{\rho}{2} - \sqrt{\beta + \left(\frac{\rho}{2} + \delta\right)^2}\right)t} + \frac{\Delta p - \Delta c}{2\tau(\delta^2 + \beta + \rho\delta)}$$

where:

$$\Delta q(t) := q_1(t) - q_2(t), \quad \Delta q^0 := q_1^0 - q_2^0, \quad \Delta p := p_1 - p_2, \quad \Delta c := c_1 - c_2.$$

Since we have: $\frac{\rho}{2} - \sqrt{\beta + \left(\frac{\rho}{2} + \delta\right)^2} < 0$, in steady-state it holds that: $\Delta q > 0 \iff \Delta p > \Delta c$, and furthermore the steady-state values coincide with those obtained in the open-loop case, as in Brekke et al. [4].

Let us consider, without loss of generality: $\Delta q^0 > 0$. Consequently it is realistic to consider also $\Delta c < 0$. It holds:

1. if $\Delta p > \Delta c$ then $\forall t \in [0, \infty) : \Delta q(t) > 0$;

2. if $\Delta p < \Delta c$ then $\Delta q(t) > 0$ for all $t < \bar{t}$, where:

$$\bar{t} = \frac{\ln \left(\frac{\Delta p - \Delta c}{(\Delta p - \Delta c) - \Delta q_0 [2\tau(\delta^2 + \beta + \rho\delta)]} \right)}{\left(\frac{\rho}{2} - \sqrt{\beta + \left(\frac{\rho}{2} + \delta\right)^2} \right)} \tag{24}$$

   while for all $t > \bar{t}$ we have: $\Delta q(t) < 0$. Not surprisingly, $\bar{t}$ is increasing in the initial quality difference and in the marginal disutility of traveling and it is decreasing in the term $\Delta p - \Delta c$.

Finally we notice that, as in the symmetric model of Brekke et al. [4], and differently from the model presented in the previous section, the open-loop and the feedback solutions lead to the same steady-state values.

The interest of the version of the model presented in this section rests in the fact that it shows that quality differential between regions may switch its algebraic sign at a point in time, in front of specific parameters' configuration: a "too low" regulated price level may induce the more efficient provider to select investment plans that lead to a lower quality as compared to the other region's provider. The further step left to future research, consists in considering regional prices as endogenous, and set by the regulators in order to maximize a social welfare function, different from the objective function of providers.

5 Concluding remarks

In this paper we have proposed a differential game model to highlight the role of local regulation in a market where prices are given and competition is based on quality. We have shown that quality differential across region can persist, under equilibrium conditions. We have studied the relations between regulated price levels, and quality levels and dynamics. Of course, quality levels affect demand and hence inter-regional mobility. Our theoretical model has a clear empirical counterpart in healthcare markets, as well as in other markets like childcare, long-term care, or even education (especially at the primary and secondary school levels), with very similar characteristics as healthcare: competition among providers bases on quality rather than price; consumers' choices are driven by location and quality of providers.

From a mathematical viewpoint, both the model of Section 3 and its variant in Section 4 are linear-quadratic differential games with one control variable for each player (I_i) and two state variables (q_1 and q_2), which have very simple dynamics. The setting is simple enough to let us to derive analytically the Open Loop and the linear Feedback Nash Equilibria of those games. However, it is interesting to notice that they are not linear-state games (see Dockner et al. [13]), since the instantaneous pay-off functions are not linear in the state variables: this implies that the Open loop Nash equilibria are not sub-game perfect. In particular we have a saddle-point equilibrium in both the models, implying that, given the initial quality levels, we can find initial values of the co-state variables (which in turn determine the initial optimal investment choices) such that the system converges to the steady state as time approaches infinity. Conversely, in the corresponding games with feedback information pattern, we considered the stationary linear FNE (which is, by definition, a sub-game perfect equilibrium) which stabilizes the states for every possible initial condition. Interestingly, these two different equilibrium paths lead to the same steady-state framework in the model of Section 4, while this is not the case for the model of Section 3. In both the models there is asymmetry between the players, but the *degree of asymmetry* is much bigger in the first one, in which the players ex-ante "know" that the *most efficient* one will always have higher quality levels (so they have different objective functionals form), and this leads to the feature that -for both the obtained equilibria- the equilibrium strategy of this player -differently from what happens for the *less efficient* one- does not explicitly depend on the state variable of the other one. Conversely, the model presented in Section 4 -which is a generalization of Brekke et al. [4]

work- is characterized by the fact that both in the open loop and in the linear feedback Nash equilibrium with asymptotically stable steady-state, the strategy of player i does not explicitly depend on the state variable of player j. In this simpler setting we have also investigated under which conditions an equilibrium trajectory with inversion of quality levels can occur. Specific assumptions concerning the choice variables of regulators lead to different properties of equilibria. From a simple comparison between the models of Section 3 and 4, one can see that the steady state under open-loop vs. feedback behavior rule may coincide or not, depending on whether the quality is chosen by the regulator or by the (profit-oriented) provider.

Our model can be extended along different routes. The most immediate one is to make the regulated prices endogenous, imagining that prices are set to maximize some social welfare functions in which regulated prices matter. In the present version of the model, local prices do not enter the objective functions of local regulators, and they have a mere redistributive role.

Though very simple, the present version of the model can provide some policy prescriptions. For instance, pro-competition policies are effective in fostering available quality of services only under specific conditions (as shown, by the way, by other available models with different focuses from regional distribution of quality levels and inter-regional consumers' mobility); more important, out present model shows that pro-competition policies may lead to increasing or decreasing quality differential across regions, depending on the parameter configuration, that is, the initial framework configuration.

Thus, our model suggests that the outcome of pro-competitive policy measures (which have been very popular over the past decades in several western countries, as applied to markets characterized by quality competition) are far from having obvious and similar results across different countries and institutional contexts.

A Proof of Proposition 1

In order to find an OLNE for the considered differential game, we apply the Pontryagin maximum principle. The current Hamiltonian function of player $i, i \in \{1,2\}$, is given by[5]:

$$H_i = \Pi_i + \mu_i(I_i - \delta q_i) + \phi_i(I_j - \delta q_j) \tag{25}$$

where μ_i and ϕ_i are the current co-state variables and the instantaneous profit functions Π_i are obtained by considering equations (6),(7),(8),(9). The adjoint equations are given by:

$$\dot{\mu}_1 = \rho\mu_1 - \frac{\partial H_1}{\partial q_1} \Longrightarrow \dot{\mu}_1 = (\rho+\delta)\mu_1 + \beta(\lambda+1)q_1 + \frac{c_1(\lambda+1)-p}{2\tau} - \frac{1}{2} \tag{26}$$

$$\dot{\phi}_1 = \rho\phi_1 - \frac{\partial H_1}{\partial q_2} \Longrightarrow \dot{\phi}_1 = (\rho+\delta)\phi_1 - \frac{c_1(1+\lambda)-p}{2\tau} \tag{27}$$

$$\dot{\mu}_2 = \rho\mu_2 - \frac{\partial H_2}{\partial q_2} \Longrightarrow \dot{\mu}_2 = (\rho+\delta)\mu_2 + \beta(\lambda+1)q_2 + \frac{q_1-q_2}{2\tau} + \frac{(c_2-p)(\lambda+1)}{2\tau} - \frac{1}{2} \tag{28}$$

$$\dot{\phi}_2 = \rho\phi_2 - \frac{\partial H_2}{\partial q_1} \Longrightarrow \dot{\phi}_2 = (\rho+\delta)\phi_2 - \frac{(c_2-p)(1+\lambda)+q_2-q_1}{2\tau} \tag{29}$$

[5] For notational simplicity we omit time dependencies.

Transversality conditions are: $\lim_{t \to \infty} e^{-\rho t} \mu_i q_i = 0$, $\lim_{t \to \infty} e^{-\rho t} \phi_i q_j = 0$. First order conditions for a maximum point of the Hamiltonian functions give:

$$\frac{\partial H_i}{\partial I_i} = 0 \iff \mu_i = (1 + \lambda) I_i \tag{30}$$

Please note that the second order conditions are satisfied if the Hamiltonian of each player is concave in its control and state variables. As well known, to this end, a necessary and sufficient condition is that their Hessian matrices $\nabla^2_{H_i}$ are negative semidefinite. Since these Hessian matrices are given by:

$$\nabla^2_{H_1} = \begin{pmatrix} -(1+\lambda) & 0 \\ 0 & -\beta(1+\lambda) \end{pmatrix}$$

$$\nabla^2_{H_2} = \begin{pmatrix} -(1+\lambda) & 0 \\ 0 & \frac{1}{2\tau} - \beta(1+\lambda) \end{pmatrix}$$

it is trivial to see that this condition is verified under Assumption (10).

Differentiating equation (30) w.r.t. time yields:

$$\dot{\mu}_i = (1 + \lambda) \dot{I}_i$$

which, substituted into (26) and (28), together with the quality stock dynamic equations, lead to the linear ODE system given in Proposition 1.

In this system, by imposing $\dot{I}_i = \dot{q}_i = 0$, we get the equilibrium point. In order to determine its *nature*, we compute the eigenvalues of the coefficient matrix of the system (12):

$$\lambda_1 = \frac{\rho}{2} + \sqrt{\delta^2 + \delta\,\rho + \frac{\rho^2}{4} + \beta}$$

$$\lambda_2 = \frac{\rho}{2} - \sqrt{\delta^2 + \delta\,\rho + \frac{\rho^2}{4} + \beta - \frac{1}{2\tau(1+\lambda)}}$$

$$\lambda_3 = \frac{\rho}{2} + \sqrt{\delta^2 + \delta\,\rho + \frac{\rho^2}{4} + \beta - \frac{1}{2\tau(1+\lambda)}}$$

$$\lambda_4 = \frac{\rho}{2} - \sqrt{\delta^2 + \delta\,\rho + \frac{\rho^2}{4} + \beta}$$

furthermore, under Assumption (10), it follows that all these eigenvalues are real and:

$$\lambda_4 < \lambda_2 < 0 < \lambda_3 < \lambda_1$$

therefore we get eigenvalues with positive values and others with negative values, hence the equilibrium point constitutes a saddle point.

B Proof of Proposition 2

In order $I_i(t, q_i, q_j)$, with $i, j \in \{1, 2\}, i \neq j$, to be a FNE strategy for player i, we look for a value function $V(t, q_i, q_j)$, continuously differentiable, which satisfies the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\rho V_i = \max_{I_i \geq 0} \left\{ \Pi_i + \frac{\partial V_i}{\partial q_i}(I_i - \delta q_i) + \frac{\partial V_i}{\partial q_j}(I_j - \delta q_j) \right\} \qquad (31)$$

The FOC for the maximum point of the RHS of this HJB equation leads to:

$$I_i = \frac{1}{\gamma(1+\lambda)} \frac{\partial V_i}{\partial q_i} \qquad (32)$$

Such condition is also sufficient for a maximum point since the expression to be maximized in the HJB equation is strictly concave in the control variable.

In order to find a pair of linear stationary feedback strategies that constitute a FNE of the differential game, we look for two value functions quadratic w.r.t. q_1 and q_2:

$$V_1(q_1, q_2) = \alpha_0 + \alpha_1 q_1 + \alpha_2 q_2 + \frac{\alpha_3}{2} q_1^2 + \frac{\alpha_4}{2} q_2^2 + \alpha_5 q_1 q_2 \qquad (33)$$

$$V_2(q_1, q_2) = k_0 + k_1 q_2 + k_2 q_1 + \frac{k_3}{2} q_2^2 + \frac{k_4}{2} q_1^2 + k_5 q_1 q_2 \qquad (34)$$

where $\alpha_0, \dots, \alpha_5, k_0, \dots, k_5$ are unknown coefficients to be determined.

By substituting in the FOC (32) we obtain:

$$I_1 = \frac{1}{1+\lambda}(\alpha_1 + \alpha_3 q_1 + \alpha_5 q_2) \qquad (35)$$

$$I_2 = \frac{1}{1+\lambda}(k_1 + k_3 q_2 + k_5 q_1) \qquad (36)$$

By substituting in the HJB equation of R_1 we obtain:

$$\tau[8\rho(\lambda+1)\alpha_0 - 4\alpha_1^2 + 4c_1(\lambda+1)^2 - 8\alpha_2 k_1 + (\lambda+1)(\tau - 4v)]+$$

$$+[2\tau\alpha_1[\delta(\lambda+1)-\alpha_3+\rho(\lambda+1)]+c_1(\lambda+1)^2-2\tau\alpha_2 k_5-(\lambda+1)(p+\tau)-2\tau\alpha_5 k_1]q_1+$$

$$+[-2\tau\alpha_1\alpha_5-c_1(\lambda+1)^2+2\tau\delta\alpha_2(\lambda+1)+2\tau\alpha_2[\rho(\lambda+1)-k_3]+p(\lambda+1)-2\tau k_1]q_2+$$

$$+[\beta(\lambda+1)^2 + 2\delta(\lambda+1)\alpha_3 - \alpha_3^2 + \rho(\lambda+1)\alpha_3 - 2\alpha_5 k_5]q_1^2+$$

$$+[2\delta(\lambda+1)\alpha_4 + \rho(\lambda+1)\alpha_4 - \alpha_5^2 - 2\alpha_4 k_3]q_2^2+$$

$$+[2\delta(\lambda+1)\alpha_5 - \alpha_3\alpha_5 + \rho(\lambda+1)\alpha_5 - \alpha_5 k_3 - \alpha_4 k_5]q_1 q_2 = 0$$

Similarly, by substituting in the HJB equation of R_2 we obtain:

$$\tau[8\alpha_1 k_2 - (\lambda+1)(8\rho k_0 + 4\lambda c_2 + \tau - 4(v - c_2)) + 4k_1^2]+$$

$$+[2\tau\alpha_1 k_4 - 2\delta\tau(\lambda+1)k_2 + 2\tau\alpha_3 k_2 - (\lambda+1)[\lambda(p-c_2) + 2\rho\tau k_2 + p - c_2] + 2\tau\alpha_5 k_1]q_1 +$$

$$+[-2\tau\alpha_1 k_5 + 2\delta\tau(\lambda+1)k_1 - (\lambda+1)[\lambda(p-c_2) - 2\rho\tau k_1 + p + \tau - c_2) - 2\tau(\alpha_5 k_2 + k_1 k_3)]]q_2 +$$

$$+[4\delta\tau(\lambda+1)k_4 - 4\tau\alpha_3 k_4 + (\lambda+1)(2\rho\tau k_4 - 1) - 2\tau k_5^2]q_1^2 +$$

$$+[2\beta\tau(\lambda+1)^2 + 4\delta\tau(\lambda+1)k_3 + (\lambda+1)(2\rho\tau k_3 - 1) - 2\tau(2\alpha_5 k_5 + k_3^2)]q_2^2 +$$

$$+[4\delta\tau(\lambda+1)k_5 - 2\tau\alpha_3 k_5 + [\lambda(2\rho\tau k_5 + 1) + 2\rho\tau k_5 + 1] - 2\tau\alpha_5 k_4 - 2\tau k_5 k_3]q_1 q_2 = 0$$

For the equality to hold, all the terms in brackets in the above equations have to be equal to zero.

We concentrate on the last three equations for both the players, which do not depend on $\alpha_0, \alpha_1, \alpha_2, k_0, k_1, k_2$, obtaining a non-linear system of 6 equations in 6 unknowns.

A solution of this system gives:

$$\alpha_4 = \alpha_5 = 0$$

$$k_4 = \sqrt{\frac{\lambda+1}{4\tau}}[\sqrt{\tau(\lambda+1)(4\beta + (2\delta+\rho)^2)} - \sqrt{A}]$$

with A defined by (17), and the values of α_3, k_3, k_5 in equations (18), (19), (20), respectively.

From this solution we obtain the values for α_1 and k_1 given in Proposition 2[6]. Finally, by substituting these obtained values into equations (35) and (36), we get the FNE given in Proposition 2.

[6] Obviously also the values for $\alpha_0, \alpha_2, k_0, k_2$ can be uniquely determined, but we can omit them since they do not enter the expression of the FNE strategies.

In order for the obtained solution to provide a maximum point for our problem which is also globally asymptotically stable we must impose (see Brekke et al. [4]):

$$\alpha_3 < 0$$

$$\alpha_3 + \alpha_5 < 0$$

$$k_3 < 0 \iff 4\beta\tau(\lambda + 1) - 2 > 0$$

which results in Assumption (10).

From the FNE strategies and the quality levels dynamic equations we obtain that the FNE qualities time paths are the solutions of the following linear ODE system:

$$\begin{cases} \dot{q}_1 = \frac{1}{1+\lambda}[\alpha_1 + (\alpha_3 - \delta)q_1] \\ \dot{q}_2 = \frac{1}{1+\lambda}[k_1 + k_5 q_1 + (k_3 - \delta)q_2] \end{cases}$$

The eigenvalues $\lambda_{1,2}$ of its coefficient matrix A are real since it holds:

$$(trA)^2 - 4\det A = \frac{(\alpha_3 + k_3 - 2\delta)^2}{(1+\lambda)^2} - 4\frac{(\alpha_3 - \delta)(k_3 - \delta)}{(1+\lambda)^2} = \frac{(\alpha_3 - k_3)^2}{(1+\lambda)^2} > 0$$

Since the equilibrium point is globally asymptotically stable, the conclusion $\lambda_{1,2} \in \mathbb{R}$ implies that these eigenvalues are negative, therefore the equilibrium point of the system is a stable node.

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