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Abstract

In this paper, we develop a new unit root testing procedure which considers jointly for structural breaks and nonlinear adjustment. The structural breaks are modeled by means of a logistic smooth transition function and nonlinear adjustment is modeled by means of an ESTAR model. The empirical size of test is quite close to the nominal one and in terms of power, the new unit root test is generally superior to the alternative test.

Keywords: Smooth Transition, nonlinearity, unit root, ESTAR

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1. Introduction

In the last forty years, the time series analysis of models with unit roots has increasingly become one of the major topics for the investigators and practitioners to understand the response of economic systems to shocks. The first tests for unit root were proposed by Fuller (1976) and Dickey and Fuller (1979). However, it is well known that the presence of structural breaks and nonlinearities in time series might affect the power of the traditional unit root tests. Accordingly, the Dickey-Fuller test fails to reject the null hypothesis of unit root and these types of tests would be powerless to separate the behaviour of a unit process from the behaviour of a stationary process with structural breaks.

Perron (1989) proposed a unit root test which takes into account structural breaks exogenously in the deterministic components and displayed that the traditional unit roots tests detect incorrectly that the series have a unit root when in fact they are stationary with structural breaks. Apart from Perron (1989), many authors have developed unit root tests in order to take into account structural breaks (Zivot and Andrews (1992); Lumsdaine and Papell (1997); Lee and Strazicich (2003). The main feature of these unit root tests is that the deterministic structural changes are assumed to occur instantaneously, only in certain points of time.

Nonetheless, individual agents can react simultaneously to a given economic stimulus; while some may be able to react instantaneously and so will adjust with different time lags. Thus, when considering aggregate behavior, the time path of structural changes in economic series is likely to be better captured by a model whose deterministic component permits gradual rather than instantaneous adjustment between different values (Leybourne et. al., 1998). From this point of view, some authors proposed different unit root tests that consider smooth rather than a sudden change. The main idea behind of these tests is that nonlinearities can be present in time series as an asymmetric speed of mean reversion and autoregressive parameter varies depending upon the values of a variable. This nonlinear behavior implies that there is a central regime where the series behave as a unit root whereas for values outside the central regime, the variable tends to revert to the equilibrium (Cuestas and Ordóñez, 2014).

The nonlinear dynamics for unit root testing procedures and the joint analysis of nonlinearity and nonstationarity have been popularised since about the last twenty years. Kapetanios et. al. (2003) proposed a unit root test within an exponential smooth transition autoregressive (ESTAR) model. Apart from Kapetanios et. al. (2003), Sollis (2009), Kruse (2011) present invaluable contributions to the testing of unit roots considering nonlinearity. Although these studies consider asymmetric speed of mean reversion, they do not take into account nonlinearities in the deterministic components.

On the other hand, Christopoulos and León-Ledesma (2010) developed tests for unit roots that account jointly for structural breaks and nonlinear adjustment. The prominent contribution of unit root test of Christopoulos and León-Ledesma (2010) is that this test takes into account asymmetric speed of mean reversion, as well as structural changes in the intercept, approximated by means of a Fourier function. Cuestas and Ordóñez (2014) also proposed a unit root test which extends the unit root test of Leybourne et. al. (1998) and takes into account both sources of nonlinearities, i.e. in the deterministic components, approximated by a logistic smooth transition function not only in the intercept, but also in the slope, an asymmetric adjustment of mean reversion.

In this paper, we develop a new unit root testing procedure which considers jointly for structural breaks and nonlinear adjustment. In our proposed test, structural breaks are modeled by means of a logistic smooth transition function that allows in the intercept, in the intercept under a fixed slope and in the intercept and slope terms. Nonlinear adjustment is modeled by means of an ESTAR model as suggested by Kruse (2011).

The rest of the paper is organized as follows: Section 2 describes the proposed test statistics and provides asymptotic critical values. Section 3 presents the results of power and size of our proposed test via Monte Carlo simulation experiments. The last section concludes the paper.

2. The Unit Root Test

In this section, we propose a unit root test which accounts jointly for structural breaks and nonlinear adjustment. The test, which is considered as an alternative to Leybourne et. al. (1998) and Kruse (2011), attempts to model structural change as a smooth transition between different regimes over time and also model the nonlinearities by means of ESTAR model.

In order to develop the new unit root testing strategy, we consider the following three logistic smooth transition models by following Leybourne et. al. (1998):

Model A: $y_t = \alpha_1 + \alpha_2 S_t\left(\lambda, \tau\right) + v_t$ (1)

Model B: $y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t\left(\lambda, \tau\right) + v_t$ (2)

Model C: $y_t = \alpha_1 + \beta_1 t + \alpha_2 S_t\left(\lambda, \tau\right) + \beta_2 t S_t\left(\lambda, \tau\right) + v_t$ (3)

where v_t is error term which is normally distributed with zero mean and unit variance and $S_t(\lambda,\tau)$ is the logistic smooth transition function, based on a sample of size T:

$$S_t(\lambda,\tau)=\left[1+\exp\{-\lambda(t-\tau T)\}\right]^{-1} \quad \lambda>0 \tag{4}$$

The parameter τ determines the timing of the transition midpoint and the speed of transition is determined by the parameter λ. If we assume v_t is a zero-mean $I(0)$ process, the in model A y_t is stationary around a mean which changes from the initial value α_1 to the final value $\alpha_1+\alpha_2$. Model B is similar to Model A, with the intercept changing from α_1 to $\alpha_1+\alpha_2$, but it allows for a fixed slope term. Finally, in Model C, in addition to the change in intercept from α_1 to $\alpha_1+\alpha_2$, the slope also changes contemporaneously, and with the same speed of transition β_1 to $\beta_1+\beta_2$. The null of unit root hypothesis may be stated as follows:

$$H_0: y_t=\mu_t,\ \mu_t=\mu_{t-1}+\varepsilon_t \tag{5}$$

where ε_t is assumed to be an $I(0)$ process with zero mean. The test statistics are calculated via a two step procedure. In the first step, we use nonlinear least squares (NLS) algorithm for estimating only deterministic component in model A, B and C, then we compute the NLS residuals,

Model A: $\hat{v}_t=y_t-\hat{\alpha}_1-\hat{\alpha}_2 S_t\left(\hat{\lambda},\hat{\tau}\right)$ (6)

Model B: $\hat{v}_t=y_t-\hat{\alpha}_1-\hat{\beta}_1 t-\hat{\alpha}_2 S_t\left(\hat{\lambda},\hat{\tau}\right)$ (7)

Model C: $\hat{v}_t = y_t - \hat{\alpha}_1 - \hat{\beta}_1 t - \hat{\alpha}_2 S_t\left(\hat{\lambda}, \hat{\tau}\right) - \hat{\beta}_2 t S_t\left(\hat{\lambda}, \hat{\tau}\right)$ (8)

In the second step, we apply the unit root test of Kruse (2011) to the residuals obtained in the first step:

$$\Delta \hat{v}_t = \delta_1 \hat{v}_{t-1}^3 + \delta_2 \hat{v}_{t-1}^2 + \sum_{i=1}^{p} \psi_i \Delta \hat{v}_{t-i} + \varepsilon_t \tag{9}$$

Kruse (2011) tests the null of unit root against the alternative of globally stationary ESTAR process, i.e.

$$\Delta \hat{v}_t = \gamma \hat{v}_{t-1}\left(1 - \exp\left\{-\theta\left(\hat{v}_{t-1} - c\right)^2\right\}\right) + \varepsilon_t \tag{10}$$

In order to test of the null of unit root, Kruse (2011) propose a first order Taylor approximation for equation (10) and obtain the auxiliary regression shown at equation (9). The test statistics of our new procedure are computed as a modified Wald type test statistic by following Kruse (2011) (For details see Kruse (2011)). We denote the value of test statistics as $\tau_{SNL\alpha}$ if Model A is used to construct the $\hat{v}_t$, $\tau_{SNL\alpha(\beta)}$ if Model B is used and $\tau_{SNL\alpha\beta}$ if Model C is used. Thus, the critical values of $\tau_{SNL\alpha}$, $\tau_{SNL\alpha(\beta)}$ and $\tau_{SNL\alpha\beta}$ test statistics have been obtained via stochastic simulations at 1%, 5% and 10% significance levels based on 50,000 replications for $T = 50, 100, 250, 500$. The critical values are reported in Table 1.

Table 1: Critical Values

	$\tau_{SNL\alpha}$			$\tau_{SNL\alpha(\beta)}$			$\tau_{SNL\alpha\beta}$		
T	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**	**1%**	**5%**	**10%**
50	13.390	9.662	8.014	27.252	15.619	10.897	17.315	12.404	10.409
100	13.567	9.839	8.925	16.895	12.621	10.749	16.897	12.621	10.749
250	13.454	9.613	7.958	16.579	12.730	10.928	16.586	12.728	10.925
500	13.247	9.525	7.846	17.107	12.895	11.053	13.656	9.830	8.050

3. Monte Carlo Study

This section involves the Monte Carlo investigation of the size properties and power performance of our new unit root test and also the power comparison of the new test with Kruse (2011) test.

First, we study the empirical size of test for different sample sizes i.e. $T = 50,100$ with a nominal size of 0.05. We generate the DGP as follows:

$$y_t = \mu_t, \quad \mu_t = \mu_{t-1} + \varepsilon_t, \quad \mu_0 = 0 \quad \varepsilon_t \sim NIID(0,1) \tag{11}$$

The results of empirical size of test, based on 5000 replications, are presented in Table 2. In general, we could conclude that the empirical size of test is quite close to the nominal one, 5%. A significant size distortion is only determined for $T = 50$ for $\tau_{SNL\alpha(\beta)}$ test. Nonetheless, the size distortion disappears for $T = 100$.

Table 2: Size Properties of Test

T	$\tau_{SNL\alpha}$	$\tau_{SNL\alpha(\beta)}$	$\tau_{SNL\alpha\beta}$
50	0.059	0.017	0.046
100	0.054	0.050	0.049

Next, we investigate the power of $\tau_{SNL\alpha}$, $\tau_{SNL\alpha(\beta)}$, $\tau_{SNL\alpha\beta}$ tests based on the following models, respectively:

$$y_t = 1 + \frac{10}{\left[1 + \exp\left\{-\lambda\left(t - \tau T\right)\right\}\right]} + v_t \tag{12}$$

$$y_t = 1 + 10t + \frac{10}{\left[1 + \exp\left\{-\lambda\left(t - \tau T\right)\right\}\right]} + v_t \tag{13}$$

$$y_t = 1 + 10t + \frac{10}{\left[1 + \exp\left\{-\lambda\left(t - \tau T\right)\right\}\right]} + \frac{10t}{\left[1 + \exp\left\{-\lambda\left(t - \tau T\right)\right\}\right]} + v_t \tag{14}$$

$$\Delta v_t = \gamma v_{t-1}\left\{1 - \exp\left(-\theta\left(v_{t-1} - c\right)^2\right)\right\} + \varepsilon_t \tag{15}$$

with $\lambda = 1.0$, $\tau = 0.5$ and $\gamma = -1.5$. The location parameter c is allowed by drawing from a uniform distribution with lower and upper bound of $-5(-10)$ and $5(10)$, respectively. Analogously, the parameter θ is allowed by drawing from a uniform distribution with lower and upper bound of $(0.001, 0.01)$ with slow transition between regimes (θl) and $(0.01, 0.1)$ with fast transition between regimes (θh), respectively. The nominal size of the tests are determined at 0.05, the number of replications is 5000 and the sample size is considered for $T = 50,100$. The results of power experiments and power comparison with Kruse (2011) test are displayed in Table 3.

Table 3: Power Experiments and Comparison

Model A

	$c_{\pm5},\theta l$		$c_{\pm5},\theta h$		$c_{\pm10},\theta l$		$c_{\pm10},\theta h$	
	$\tau_{SNL\alpha}$	τ	$\tau_{SNL\alpha}$	τ	$\tau_{SNL\alpha}$	τ	$\tau_{SNL\alpha}$	τ
T=50	0.068	**0.168**	0.795	**0.835**	**0.260**	0.138	**0.938**	0.815
T=100	**0.333**	0.304	0.964	**0.986**	**0.448**	0.265	**1.000**	0.983

Model B

	$c_{\pm5},\theta l$		$c_{\pm5},\theta h$		$c_{\pm10},\theta l$		$c_{\pm10},\theta h$	
	$\tau_{SNL\alpha(\beta)}$	τ	$\tau_{SNL\alpha(\beta)}$	τ	$\tau_{SNL\alpha(\beta)}$	τ	$\tau_{SNL\alpha(\beta)}$	τ
T=50	0.042	**0.166**	0.396	**0.642**	0.050	**0.138**	**0.998**	0.854
T=100	0.116	**0.134**	0.514	**0.692**	**0.846**	0.286	0.704	**1.000**

Model C

	$c_{\pm5},\theta l$		$c_{\pm5},\theta h$		$c_{\pm10},\theta l$		$c_{\pm10},\theta h$	
	$\tau_{SNL\alpha\beta}$	τ	$\tau_{SNL\alpha\beta}$	τ	$\tau_{SNL\alpha\beta}$	τ	$\tau_{SNL\alpha\beta}$	τ
T=50	0.077	**0.141**	**0.338**	0.227	**0.209**	0.026	**1.000**	0.760
T=100	**0.404**	0.170	**0.796**	0.507	**0.319**	0.043	0.773	**0.998**

Notes: The values are rejection rates of Kruse test (τ) and $\tau_{SNL\alpha}$, $\tau_{SNL\alpha(\beta)}$ and $\tau_{SNL\alpha\beta}$ tests and bold values display the cases where each test performs better.

The results of the power experiments and comparison show that the new unit root test is generally superior to the Kruse test. Only in some cases where the unit root test is applied for Model B, the Kruse test performs better than $\tau_{SNL\alpha(\beta)}$ test.

4. Conclusions

In this paper, we develop a new unit root testing procedure which considers jointly for structural breaks and nonlinear adjustment. The empirical size of test is quite close to the nominal one and in terms of power, the new unit root test is generally superior to the Kruse test.

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