Equilibrium co-existence of public and private firms and the plausibility of price competition

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EQUILIBRIUM CO-EXISTENCE OF PUBLIC AND PRIVATE FIRMS AND THE PLAUSIBILITY OF PRICE COMPETITION

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ABSTRACT. We consider a differentiated product duopoly where a regulated firm competes with a private firm. The instrument of regulation is the level of privatization. First, the regulator determines the level of privatization to maximize social welfare. Then both firms endogenously choose the mode of competition (that is, whether to compete in price or quantity). Finally, the two firms compete in the market. Under a very general demand specification, we show that when the products are imperfect substitutes (complements), there is co-existence of private and public (strictly partially privatized) firms. Moreover, in the second stage, the firms compete in prices.

JEL Classification: D4, L1, L2

Keywords: Partially private firm, price (Bertrand) competition, quantity (Cournot) competition

1. INTRODUCTION

What happens if, instead of two profit maximizing firms, we consider a regulated firm and a profit maximizing firm in the duoploy market with differentiated product? Singh and Vives [30] and Cheng [5] considered a two-stage game for a differentiated product duopoly market where both firms are profit maximizers. In the first stage, the firms credibly announce to play in either quantity or price strategies. If the goods are substitutes (complements), then it is shown that quantity or Cournot (price or Bertrand) competition is the SPNE outcome of this two stage game (see Singh and Vives [30] and Cheng [5]). In this paper we model the co-existence of a regulated firm and a profit maximizing firm and, in particular, we model the objective of the regulator and then (like Singh and Vives [30]) allow the firms to decide on the mode of competition before competing in the market. In a static scenario this calls for a three stage game which to the best of our knowledge has not been done in the differentiated product literature.[1] Moreover, there are many papers that provide important results by assuming quadratic utility function or CES utility function of the representative consumer. We want to come out of this limitation as well and allow for more general demand specifications to provide our results with the three stage game.

The primary reason for this three-stage game stems from the fact that when the goods are imperfect substitutes, it is not always the case that we find profit maximizing firms operating in a

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[1] All the models in the existing literature either endogenize the mode of competition or endogenize the objective of the non-profit maximizing firm but not both. Hence, we only have two stage (and not three-stage) models in the existing literature. See the related literature section for details.

market and competing in quantities (like the results in Singh and Vives [30] and Cheng [5] suggest). Objective different from profit maximization for imperfect substitutes is a special feature of many markets in many countries. Examples include the telecom sector, banking industry, airlines, postal services, health sector, and education sector (see for example Backx, Carney and Gedajlovic [3], Badertscher, Shroff and White [4], Doganis [11] and La Porta, Lopez-De-Silanes and Shleifer [21]). Even in developed countries we often find the co-existence of welfare maximizing public firm and profit maximizing private firms.[2] Therefore, one cannot deny the role of regulation in the differentiated products markets.[3]

Assuming a market where a private firm competes with a public firm, it was shown by Matsumura and Ogawa [24] that, with quadratic utility function of the representative consumer, price (Bertrand) competition is the SPNE of the two stage game regardless of whether goods are substitutes or complements. Therefore, one cannot unambiguously confirm that quantity competition will always follow in a differentiated product market when at least one firm is not a profit maximizer. However, what guarantees the co-existence of public firm and private firm in a differentiated product market? This requires a more careful modeling of the regulatory instrument and it is also for this reason that our contribution is important.

We first add an earlier (first) stage to the two-stage game of Singh and Vives [30] and Cheng [5]. In the first stage, a (regulator) government decides how much weight the partially privatized firm must attach to its own profit and social welfare assuming that the competing firm is a profit maximizer. We show that in such a set-up, when the goods are substitutes we uniquely end up in the co-existence of welfare maximizing public firm and profit maximizing private firms, that is, no privatization Bertrand equilibrium is the SPNE outcome of this game where the government sets zero (full) weight to profit (social welfare) of the partially privatized firm and both firms compete in prices (that is, Bertrand competition). When the goods are complements we uniquely end up in an SPNE outcome which we call strictly partial privatization Bertrand equilibrium where, in Stage 1, the government adds non-zero weights to both Firm 1's own profit and social welfare and, in Stage 2, firms play price strategies.

The first stage regulatory instrument of the government is the weight θ (lying in the closed interval $[0,1]$) attached to the profit of the partially privatized firm and the residual weight $(1-\theta)$ attached to the welfare of the society. According to Vives [33], when both firms are profit maximizers, then, with Cournot competition, there is less of a profit loss with price under-cutting than with Bertrand competition. However, when we have one partially privatized firm, then there exists a critical value of weight ($\theta \in (0,1)$) such that for each weight below this critical weight, there

[2]In case of China after early 1980 we have seen the coexistence of both public and private firms. For example, in the health sector in urban Chine we find such a co-existence. In case of USA and England, we find such a co-existence in both health and education sector.

[3]In case of the aviation sector in India, Air India is a government regulated enterprise competing with other private enterprise such as Kingfisher Airlines, Spice Jet etc. In the Indian banking sector there are nationalized (regulated) banks such as State Bank of India that competes with other private banks such as Axis Bank.

exists a critical price of Firm 2 below which Vives's [33] argument holds and, more importantly, above this critical price the reverse argument holds, that is, with Bertrand competition there is less of a profit loss with price under-cutting than with Cournot competition. It is precisely this feature that drives our main result when the goods are substitutes.

Our results hold under very general demand specifications. Moreover, our results are true even when the quantity reaction functions transformed in the price space are non-monotonic. In particular, for substitute goods, our result hold under the set of assumptions made by Cheng [5] and with an additional assumption on welfare which is general enough and was used in Ghosh and Mitra [16]. To prove our results we have at times made use of Cheng's [5] geometric approach and, to prove one lemma, we have also used the line integral techniques similar to the one used in Ghosh and Mitra [15], [16]. Specifically, to find the exact cut-off weight (θ) for the optimal choice of mode of competition for Firm 2 changes we use line integral techniques and then we apply Cheng's [5] geometric approach to sequentially eliminate possibilities other than the price competition.

The paper is organized as follows. We conclude this section with a brief discussion on the related literature. In Section 2, we introduce the basic framework, our assumptions with imperfect substitute goods and we explain the three stage game. In Section 3, we present our main theorem with imperfect substitutes. In Section 4, we present the result with complement goods. In Section 5, we address the robustness of our game with quadratic utility and we also address the issue of cost asymmetry. In Section 6 we provide our conclusions followed by an appendix section (Section 7) where we provide the proofs of all the results.

1.1. **Related literature.** The classic work by Singh and Vives [30] endogenize price and quantity strategies with profit maximizing firms in a differentiated product market. This was later generalized by Cheng [5] by providing an elegant geometric approach. There are papers that deal with Bertrand Cournot comparison with profit maximizing firm in a differentiated products market with general demand specifications (see for example Cheng [6], Häckner [18], Okuguchi [28] and Vives [33], [34]).

In this paper we apply two stages of endogenization. The first stage endogenization is the objective function of the partially privatized firm and, like Singh and Vives [30] and Cheng [5], the second stage endogenization is price and quantity strategies. The first stage endogenization of adding positive weights on welfare in a firm's objective function seems natural in the context of partially privatized firms (see, for example, the papers in the mixed-oligopoly literature by Anderson, De Palma, and Thisse [1], Ghosh and Mitra [15], [16], Matsumura [23] and Matsumura and Ogawa [24]). This literature focuses on mixed markets where both private and partially privatized (or public) firms coexist. In the early stages of industrialization of developing economies, there is often an upper bound on the extent of private ownership. When a foreign firm tries to enter a domestic market, the government can ask the foreign firm to pursue an objective different from profit maximization that includes Corporate Social Responsibility (for example, taking initiative to assess and take responsibility for the company's effects on the environment and impact on social welfare). If we assume that the government cares about social welfare and private firms' care about profit, then it seems plausible to assume that the partially privatized firm maximize a weighted combination of profit and welfare. Therefore, objectives different from profit is quite important and prevalent in the industrial organization literature. A paper with a very general objective function that allows for altruism and informational asymmetry is by Heifetz, Shannon and Spiegel [20]. However, Heifetz, Shannon and Spiegel [20] do not allow for either privatization based enodogeneity (like Stage 1 of our three stage game) or price-quantity based endogeneity (like Stage 2 of our three stage game). Even when we have fully privatized firms, we know from the managerial-delegation literature that managers maximize a weighted combination of profit and quantity/revenue/welfare and it is compatible with profit maximization (see Fershtman and Judd [13], Miller and Pazgal [25], Sklivas [31] and Vickers [32]).

With quadratic utility function there is a growing literature that studies the coexistence of partially privatized firm and a private firm in a differentiated product market. With quadratic utility, only Stage 1 endogeneity like ours was addressed by Fujiwara [14] and by Ohnishi [26]. In Fujiwara [14], it is argued that under Cournot competition it is optimal to choose a positive weight $\theta > 0$ for the partially privatized firm. In Ohnishi [26], it was argued that under Bertrand competition it is optimal to choose zero weight $\theta = 0$ for the partially privatized firm. Our analysis shows that, in general, if we also endogenize mode of competition along with θ, then Cournot competition (Fujiwara's [14] analysis) is never achieved as an equilibrium outcome. With quadratic utility, only Stage 2 endogeneity like ours was addressed by Matsumura and Ogawa [24] with an added assumption that one firm is fully public (that is, θ is exogenously fixed at 0). Matsumura and Ogawa [24] argued that Bertrand competition is the SPNE of the two stage game regardless of whether goods are substitutes or complements. We show that Bertrand competition is the SPNE of the three stage game, which allows for endogenous determination of the level of privatization θ. Moreover, our results hold for a very general demand specification.

De Fraja and Delbono [7] show that, in homogeneous goods Cournot oligopoly with decreasing returns to scale technology, coexistence of a fully public firm with one or more private firms results in lower social welfare compared to that in oligopoly with only private firms. However, full privatization of the public firm is not socially desirable either; instead partial privatization of the public firm is socially optimal (see Matsumura [23]). These results hold true in the case of differentiated products mixed oligopoly with constant returns to scale technology as well (see Fuziwara [14]). That is, when firms compete in quantities, it is inefficient to have a fully public firm in the industry and this inefficiency in mixed oligopoly can be mitigated by partially privatizing the public firm. On the other hand, when firms compete in prices, coexistence of a fully public firm with one or more private firms is socially desirable and, thus, privatization (partial or full) of the public firm looses its appeal under price competition (see Anderson, De Palma, and

Thisse [1]; Sanjo [29]; Ohnishi [26]), unless goods are complements (see Ohnishi [27]). This paper shows that, the level of privatization of the public firm has important consequences on the nature of product market competition and when firms can choose the mode of product market competition, coexistence of a fully public firm with one or more private firms is socially optimal, except in case of complementary goods. That is, optimality of partial privatization cannot be sustained when the nature of product market competition is endogenously determined when the goods are imperfect substitute. We further show (in Section 5) that this result can be valid even when the public firm is relatively inefficient (but not "too" inefficient) compared to its private counterparts.

2. PRELIMINARIES

We consider an economy with a competitive sector producing the numéraire good (money) y and with a imperfectly competitive sector where two firms operate. Each firm produces a differentiated good. For any firm $i \in \{1,2\}$, let p_i and q_i denote Firm i's price and quantity respectively. For convenience we define the following notations. Let $\Re_+$ represent the non-negative orthant of the real line $\Re$. For any $x = (x_1, x_2) \in \Re^2_+$ and any $y = (y_1, y_2) \in \Re^2_+$, $x \neq y$ means either $x_1 \neq y_1$ or $x_2 \neq y_2$, $x \geq y$ means $x_1 \geq y_1$ and $x_2 \geq y_2$, and, $x >> y$ means $x_1 > y_1$ and $x_2 > y_2$. We assume a representative consumer who maximizes $\mathcal{U}(q,y) := U(q) + y$ subject to $p_1q_1 + p_2q_2 + y \leq M$ where $q = (q_1, q_2) \geq (0,0)$, $p = (p_1, p_2) >> (0,0)$ and M denotes income of the representative consumer. For any function $G : \Re^2_+ \to \Re$, define for any $i \in \{1,2\}$, $G_i(x) := \frac{\partial G(x)}{\partial x_i}$, $G_{ii}(x) := \frac{\partial^2 G(x)}{\partial x_i^2}$ and for any $i,j \in \{1,2\}$ such that $i \neq j$, $G_{ij}(x) := \frac{\partial^2 G(x)}{\partial x_j \partial x_i}$ and $G_{ij}(x) = G_{ji}(x)$. Similarly, for any firm $i \in \{1,2\}$ and any firm specific function $H_i : \Re^2_+ \to \Re$, define for any $j,k \in \{1,2\}$, $H_{i,j}(x) := \frac{\partial H_i(x)}{\partial x_j}$, $H_{i,jk}(x) := \frac{\partial^2 H_i(x)}{\partial x_j \partial x_k}$.

Assumption 1. For $i,j = 1,2$ ($i \neq j$) and any $q >> (0,0)$, (i) $U_i(q) > 0$, (ii) $U_{ii}(q) < 0$, (iii) $U_{ij}(q) < 0$ and (iv) $|U_{ii}(q)| > |U_{ij}(q)|$.

Given $V(q,y)$ is quasi-linear, there is no income effect and hence the representative consumer's optimization is to select q to maximize $U(q) - p_1q_1 - p_2q_2$. Utility maximization yields the inverse demand function $p_i = U_i(q) := F_i^{QQ}(q)$ for all $q \geq (0,0)$ and for each $i \in \{1,2\}$. Using Assumption 1 it follows that $F_{i,i}^{QQ}(q) = U_{ii}(q) < 0$ and $F_{i,j}^{QQ}(q) = U_{ij}(q) < 0$ for $i \neq j$. From Assumption 1(iv) we know that the demand system is invertible. Therefore, given any price vector $p = (p_1, p_2) >> (0,0)$, we get the direct demand function $q_i = F_i^{PP}(p)$ for each $i \in \{1,2\}$. Let $|D| := U_{11}(q)U_{22}(q) - (U_{12}(q))^2 > 0$. Given Assumption 1, it also follows that $F_{i,i}^{PP}(p) = U_{jj}(q)/|D| < 0$ and $F_{i,j}^{PP}(p) = -U_{ij}(q)/|D| > 0$ for $i,j \in \{1,2\}$ with $i \neq j$. For any $i \in \{1,2\}$, any quantity $q_i \geq 0$, the level set $Q_i(q_i) = \{p \mid p >> (0,0), F_i^{QQ}(p) = q_i\}$ generates iso-quantity curve for Firm i in the price space. Due to Assumption 1, the slope of the iso-quantity curve at $q_i = \bar{q}_i$ is $\frac{dp_j}{dp_i} \mid_{\bar{q}_i} = -F_{i,i}^{PP}(p)/F_{i,j}^{PP}(p) > 0$. By Assumption 1, own effect dominates cross effect implying that Q_1 is steeper than Q_2 in the price space (see Cheng [5]). We assume symmetric total cost of both the firms and it is given by $C(y) = cy$ where $c > 0$ and $y \geq 0$. When both firms choose quantity as a strategic variable, profit of Firm i is given as $\pi_i^{QQ}(q) = (F_i^{QQ}(q) - c)q_i$ for $i, j = 1, 2$ with $i \neq j$. The profit function of Firm i when both chooses price as a strategic variable is given by $\pi_i^{PP}(p) = (p_i - c)F_i^{PP}(p)$ for all $i, j = 1, 2$ with $i \neq j$.

Assumption 2. For $i, j = 1, 2$ ($i \neq j$) and any $q >> (0, 0)$, (i) $\pi_{i,ij}^{QQ}(q) < 0$ and (ii) $\pi_{i,ii}^{QQ}(q) + |\pi_{i,ij}^{QQ}(q)| < 0$.

Assumption 3. For $i, j = 1, 2$ ($i \neq j$) and any $p >> (0, 0)$, (i) $\pi_{i,ij}^{PP}(p) > 0$ and (ii) $\pi_{i,ii}^{PP}(p) + |\pi_{i,ij}^{PP}(p)| < 0$.

Assumption 1, Assumption 2 and Assumption 3 are very standard and these are satisfied by any standard demand function when products are imperfect substitutes (see Cheng [5] and Vives [34]). Let $CS = U - p_1 q_1 - p_2 q_2$ denote the consumer surplus and $\pi = \pi_1 + \pi_2 = (p_1 - c)q_1 + (p_2 - c)q_2$ denote the aggregate profit with π_1 (π_2) representing profit of Firm 1 (Firm 2). The (social) welfare is given by $W = CS + \pi = U - c(q_1 + q_2)$. The welfare function when both firms choose quantity as a strategic variable is given by $W^{QQ}(q) = U(q) - c(q_1 + q_2)$ with $W_i^{QQ}(q) = F_i^{QQ}(q) - c$, $W_{ii}^{QQ}(q) = F_{i,i}^{QQ}(q) < 0$, and, $W_{ij}^{QQ}(q) = F_{i,j}^{QQ}(q) < 0$. The welfare function when both firms choose price as a strategic variable is given by $W^{PP}(p) = W^{QQ}(F_1^{PP}(p), F_2^{PP}(p)) = U(F_1^{PP}(p), F_2^{PP}(p)) - c(F_1^{PP}(p) + F_2^{PP}(p))$ with $W_i^{PP}(p) = (p_i - c)F_{i,i}^{PP}(p) + (p_j - c)F_{j.i}^{PP}(p)$.

Assumption 4. For $i, j = 1, 2$ and $(p_1, p_2) \geq (c, c)$, (i) $W_{ii}^{PP}(p) < 0$ and (ii) $W_{ii}^{PP}(p) + W_{ij}^{PP}(p) < 0$.

An assumption similar to Assumption 4 was used in Ghosh and Mitra [16]. Assumption 4 (i) is necessary to satisfy the second order condition of any welfare maximizing firm. We consider two very standard utility specifications. Suppose that the utility function of the representative consumer is given by

$$U(q) = a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2), \tag{1}$$

where a ($> c$) is a preference parameter, γ ($-1 < \gamma < 1$) is the product differentiation parameter (see Dixit [8] and Singh and Vives [30]). A positive (negative) value of γ indicates substitute (complement) goods. We first restrict attention to substitute goods case. One can show that the quadratic utility function given in (1) satisfies all our assumptions (that is, Assumption 1 to Assumption 4) when the goods are substitutes. Suppose that the utility function of the representative consumer is given by

$$U(q) = [q_1^s + q_2^s]^\gamma,\ s\gamma, \gamma, s \in (-\infty, 1), \tag{2}$$

where $\sigma = \frac{1}{1-s}$ measure the elasticity of substitution (see Dixit and Stiglitz [10] and Vives [34]). Goods are substitute if $\gamma, s \in [0, 1]$ and complement if $\gamma, s \in [-\infty, 0]$. We first restrict attention to substitute goods case. One can show that the CES utility function satisfies the first three assumptions (that is, Assumption 1 to Assumption 3). If $1 - 2s + \gamma s^2 > 0$, then Assumption 4 is satisfied by the CES utility functions given in (2).

Remark 1. It is important to note that we consider a weaker set assumptions than what is required for the stability of the equilibrium according to Dixit [9].

2.1. **The three stage game.** We assume that Firm 1 is partially privatized (maximizing a weighted sum of welfare and its own profit) and Firm 2 is a private firm (maximizing its own profit). Therefore, the payoff function of Firm 1 is $V_1 := \theta\pi_1 + (1-\theta)W$ (where θ is the privatization ratio) and that of Firm 2 is π_2. Specifically, if Firm 1 is a public (private) firm, then $\theta = 0$ ($\theta = 1$) and Firm 1 maximizes social welfare (its own profit). For any given weight $\theta \in (0,1)$, Firm 1 maximizes the weighted sum of its own profit and social welfare. We consider a three stage game Γ and the stages of the game are as follows.

- **Stage1:** The government decides the level of privatization ($\theta \in [0,1]$) in order to maximize social welfare.
- **Stage 2:** Each firm decides (simultaneously and independently) whether to adopt a price strategy (call it P) or a quantity strategy (call it Q). See Table 1.
- **Stage 3:** Firm 1 and Firm 2 compete in the market.

We solve the game using backward induction. Given the first stage choice of θ, let the optimal price and quantity of Firm i be $p_i^{XY}(\theta)$ and $q_i^{XY}(\theta)$ assuming Firm 1 adopts strategy X and Firm 2 adopts strategy Y where $X, Y \in \{P, Q\}$. We denote the consequent profit of Firm i at the optimal choice and contingent on XY by $\overline{\pi}_i^{XY}(\theta) = \pi_i^{QQ}(q_1^{XY}(\theta), q_2^{XY}(\theta)) = \pi_i^{PP}(p_1^{XY}(\theta), p_2^{XY}(\theta))$. Similarly, the consequent welfare at this optimal choice and contingent on XY is $\overline{W}^{XY}(\theta) = W^{QQ}(q_1^{XY}(\theta), q_2^{XY}(\theta)) = W^{PP}(p_1^{XY}(\theta), p_2^{XY}(\theta))$. So the optimal pay-off of Firm 1 and Firm 2 contingent on XY are $\overline{V}_1^{XY}(\theta) = \theta\overline{\pi}_i^{XY}(\theta) + (1-\theta)\overline{W}^{XY}(\theta)$ and $\overline{\pi}_2^{XY}(\theta)$ respectively. With this specification, in the second stage firms play the following stage game.

TABLE 1. **Stage 2 of Γ**

Firm 1Firm 2	Price	Quantity
Price	$\overline{V}_1^{PP}(\theta), \overline{\pi}^{PP}(\theta)$	$\overline{V}_1^{PQ}(\theta), \overline{\pi}_2^{PQ}(\theta)$
Quantity	$\overline{V}_1^{PQ}(\theta), \overline{\pi}_2^{PQ}(\theta)$	$\overline{V}_1^{QQ}(\theta), \overline{\pi}_2^{QQ}(\theta)$

Sub-game perfect equilibrium of Γ*:* For any $X, Y \in \{P, Q\}$, any $x_1 \in \{p_1, q_1\}$, any $y_2 \in \{p_2, q_2\}$, and, any $\theta^{XY} \in [0,1]$, a profile of strategies $(\theta^{XY}, (X, x_1^{XY}(\theta^{XY})), (Y, y_2^{XY}(\theta^{XY})))$ is a sub-game perfect Nash equilibrium (SPNE) of Γ if it induces a Nash equilibrium in every sub-game of Γ. First, in Stage 3, given θ^{XY} and given XY, $(x_1^{XY}(\theta^{XY}), y_2^{XY}(\theta^{XY}))$ is a Nash equilibrium choice vector (that is, $x_1^{XY}(\theta^{XY})$ and $y_2^{XY}(\theta^{XY})$ are respectively the optimum choice of X by Firm 1 given $y_2^{XY}(\theta^{XY})$ and the optimum choice of Y by Firm 2 given $x_1^{XY}(\theta^{XY})$). Second, in Stage 2, given θ^{XY},

X is a best response of Firm 1 against Y of Firm 2 and Y is a best response of Firm 2 against X of Firm 1. Finally, θ^{XY} induces XY in Stage 2 and maximizes $\overline{W}^{XY}(\theta)$ in Stage 1. Moreover, there does not exist θ that induces a mode of competition Z_1Z_2 (with $Z_i \in \{P, Q\}$ for $i = 1, 2$) and yields a higher welfare than $\overline{W}^{XY}(\theta^{XY})$.

We define four possible types of equilibria of Γ.

(i) Let $(\theta^{PP}, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$ be a Bertrand equilibrium with equilibrium weight θ^{PP}. If $\theta^{PP} = 0$, then we call it the *no privatization Bertrand equilibrium*. If $\theta^{PP} \in (0, 1)$, then we call it the *strictly partial privatization Bertrand equilibrium*.

(ii) Let $(\theta^{QQ}, (Q, q_1^{QQ}(\theta^{QQ})), (Q, q_2^{QQ}(\theta^{QQ})))$ be a Cournot equilibrium with equilibrium weight θ^{QQ}.

(iii) Let $(\theta^{PQ}, (P, p_1^{PQ}(\theta^{PQ})), (Q, q_2^{PQ}(\theta^{PQ})))$ be a Type 1 equilibrium with equilibrium weight θ^{PQ}.

(iv) Let $(\theta^{QP}, (Q, q_1^{QP}(\theta^{QP})), (P, p_2^{QP}(\theta^{QP})))$ be a Type 2 equilibrium with equilibrium weight θ^{QP}.

3. The main result

Theorem 1. Suppose Assumption 1, Assumption 2, Assumption 3 and Assumption 4 hold. The strategy combination $(\theta^{PP} = 0, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$, that is, no privatization Bertrand equilibrium, is the unique SPNE outcome of Γ.

Before going to the proof of Theorem 1 we illustrate the relevant reaction functions that will be helpful for our analysis. If both firms compete in prices, then for any $\theta \in [0, 1]$, let $\mathcal{SV}_1^{PP}(\theta) = \{p \mid p >> (0, 0), V_{1,1}^{PP}(p, \theta) = 0\}$ be the reaction function of Firm 1 in the price space. Given, Assumption 3 and Assumption 4, $\mathcal{SV}_1^{PP}(\theta)$ is invertible. Hence, we can represent it as $p_1 = SV_1^{PP}(p_2, \theta)$. In Figure 1, we represent $p_1 = SV_1^{PP}(p_2, 0)$ by the $S_1^{PP}S_1^{PP'}$ curve and we represent $p_1 = SV_1^{PP}(p_2, 1)$ by the $R_1^{PP}R_1^{PP'}$ curve and, for any $\theta \in (0, 1)$, the curve $p_1 = SV_1^{PP}(p_2, \theta)$ must lie between the curves $p_1 = SV_1^{PP}(p_2, 0)$ and $p_1 = SV_1^{PP}(p_2, 1)$ (since by Assumption 3 and Assumption 4 one can show that $V_{1,11}^{PP} < 0$). The reaction function of Firm 2 is the locus of all points in the set $\mathcal{R}_2^{PP} = \{p \mid p >> (0.0), \pi_{2,2}^{PP}(p) = 0\}$. By Assumption 3, we know that $\mathcal{R}_2^{PP}$ is a positively sloped curve with slope less than unity (see Cheng [5]) hence it is invertible. Therefore, we can represent it as $p_2 = R_2^{PP}(p_1)$. In Figure 1, we represent $p_2 = R_2^{PP}(p_1)$ by the $R_2^{PP}R_2^{PP'}$ curve.

Suppose that both firms are competing in quantities. For any $\theta \in [0, 1]$, the reaction function of Firm 1 is the locus of all points in the set $\mathcal{SV}_1^{QQ}(\theta) = \{q \mid q >> (0, 0), V_{1,1}^{QQ}(q, \theta) = 0\}$. By Assumption 1 and Assumption 3, it is possible to show that $V_{1,11}^{QQ}(q, \theta) < 0$, $V_{1,12}^{QQ}(q, \theta) < 0$ and $|V_{1,11}^{QQ}(q, \theta)| > |V_{1,12}^{QQ}(q, \theta)|$. Hence, in the (q_1, q_2) plane, the $\mathcal{SV}_1^{QQ}$ curve is negatively sloped and its slope is more than unity in absolute sense. Therefore, we can represent it as $q_1 = SV_1^{QQ}(q_2, \theta)$. The reaction function of Firm 2 is locus of all points in the set $\mathcal{R}_2^{QQ} = \{q \mid q >> (0, 0), \pi_{2,2}^{QQ}(q) =$

$0\}$. By Assumption 2 the reaction function $\mathcal{R}_2^{QQ}$ (in the (q_1, q_2) plane) is strictly decreasing with slope less than unity in absolute sense (see Cheng [5]) and hence is invertible. Therefore, we can represent it as $q_2 = R_2^{QQ}(q_1)$. The graphs of $\mathcal{R}_2^{QQ}$ and $\mathcal{SV}_1^{QQ}(\theta)$ in price space are respectively $\mathcal{P}(\mathcal{R}_2^{QQ}) = \{p \mid \pi_{2,2}^{QQ}(q) = 0 \text{ and } q_i = F_i^{PP}(p)\ \forall\ i = 1,2\}$ and $\mathcal{P}(\mathcal{SV}^{QQ}(\theta)) = \{p \mid V_{1,1}^{QQ}(q,\theta) = 0 \text{ and } q_i = F_i^{PP}(p)\ \forall\ i = 1,2\}$ and their respective equations in implicate form are $F_1^{PP}(p) - SV_1^{QQ}(F_2^{PP}(p), \theta) = 0$ and $F_2^{PP}(p) - R_2^{QQ}(F_1^{PP}(p)) = 0$. In Figure 1, the set of points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(0))$ is represented by the line $p_1 = c$. Like Cheng [5], one can show that the set of points $\mathcal{P}(\mathcal{R}_2^{QQ})$ must lie above the $R_2^{PP}R_2^{PP'}$. One such representation is the r_2r_2' curve in Figure 1.

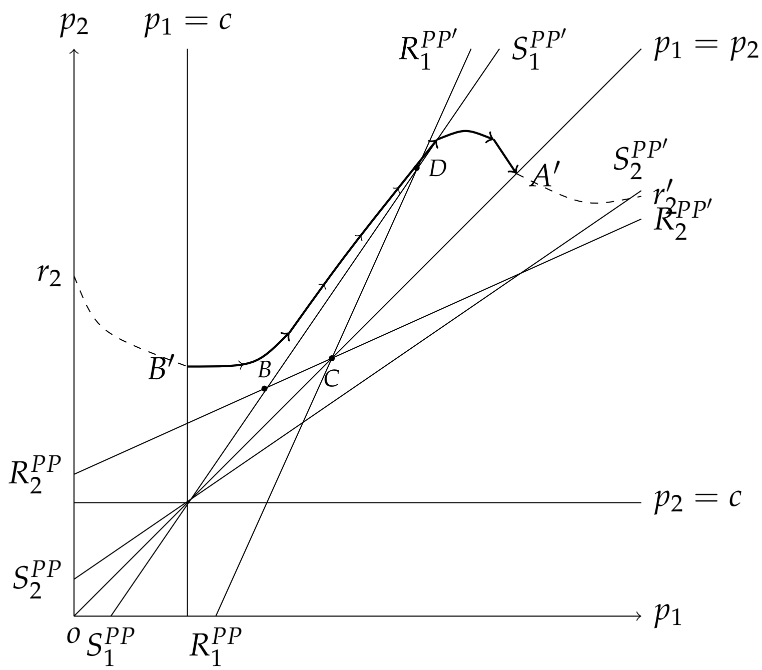

FIGURE 1. The case of imperfect substitutes

Lemma 1. For any weight $\theta \in (0,1)$, $\pi_{1,1}^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) > 0$ and for any Firm i with $i \in \{1,2\}$, $W_i^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) < 0$.

Lemma 1 states that with price competition and given any $\theta \in (0,1)$, at any equilibrium price vector $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ it is always optimal for Firm 1 to increase (decrease) price given Firm 2's price remains at $p_2^{PP}(\theta)$ when Firm 1 is a profit (welfare) maximizer.

Lemma 2. For any $\theta \in (0,1)$,

(i) $\frac{\partial q_1^{QQ}(\theta)}{\partial \theta} < 0$ and $\frac{\partial q_2^{QQ}(\theta)}{\partial \theta} > 0$.

(ii) $\frac{\partial p_1^{QQ}(\theta)}{\partial \theta} > 0$, and, for any $i = 1,2$, $\frac{\partial p_i^{PP}(\theta)}{\partial \theta} > 0$ and $\frac{\partial p_i^{PQ}(\theta)}{\partial \theta} > 0$.

Lemma 2 provides the standard comparative static results.

Lemma 3.

(i) There exists a unique $\theta_1 \in (0,1)$ such that $\overline{\pi}_2^{PP}(\theta) \gtreqless \overline{\pi}_2^{PQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_1$.

(ii) There exist $\underline{\theta}_3 \in (0,1)$ such that $\overline{V}_1^{PP}(\underline{\theta}_3) = \overline{V}_1^{QP}(\underline{\theta}_3)$ and, for any $\theta \in (0, \underline{\theta}_3)$, $\overline{V}_1^{PP}(\theta) > \overline{V}_1^{QP}(\theta)$.

(iii) There exist a unique $\theta_4 \in (0,1)$ such that $\overline{\pi}_2^{QP}(\theta) \gtreqless \overline{\pi}_2^{QQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_4$.

Assume that Firm 1 chooses price strategy. Lemma 3 (i) states that there exist a unique $\theta_1 \in (0,1)$ for which Firm 2 is indifferent between choosing price strategy and quantity strategy. Moreover, if $\theta < \theta_1$, then price strategy is optimal for Firm 2, and, if $\theta > \theta_1$, then quantity strategy is optimal for Firm 2. Next, assume that Firm 2 chooses price strategy. Lemma 3 (ii) states that there exist $\underline{\theta}_3 \in (0,1)$ for which Firm 1 is indifferent between choosing price or quantity strategy. Moreover, if $\theta < \underline{\theta}_3$, then Firm 1 chooses price strategy. Lemma 3 (iii) states that when Firm 1 chooses quantity strategy, there exist an unique $\theta_4 \in (0,1)$ for which Firm 2 is indifferent between choosing price strategy and quantity strategy. For any $\theta < \theta_4$, price strategy is optimal and, for any $\theta > \theta_4$, quantity strategy is optimal. The cut-off point θ_1 (θ_4) is associated with the case where Firm 1 chooses price (quantity) strategy. These cut-off points in Lemma 3 (i) and (iii) reflects the reversal in the cost of adopting price strategy for Firm 2 compared to quantity strategy. For the privatization weights below these cut-off points the reverse intuition of Vives [33] holds. To prove Lemma 3 (i) and Lemma 3 (iii) we use the line integral technique which is the two-variable asymmetric version of the one used in Ghosh and Mitra [15], [16].

Lemma 4. Under price competition in Stage 2, the resulting welfare $\overline{W}^{PP}(\theta)$ is maximized at $\theta = 0$. Moreover, at $\theta = 0$, the government can uniquely induce price strategy for both firms.

Lemma 4 indicates that no privatization Bertrand equilibrium is a possible SPNE outcome of Γ. Specifically, if we can rule out the other modes of competition (that is, if we can rule out both firms choosing quantity strategy and if we can rule out one firm choosing price strategy and the other firm choosing quantity strategy), then from Lemma 4 it will follow that the no privatization Bertrand equilibrium is the unique SPNE outcome of Γ. The remaining lemmas together rule out other modes of competition and completes the proof of Theorem 1. Lemma 5 and Lemma 6 that follows rule out the possibilities of Type 1 and Type 2 equilibria.

Lemma 5. There is no $\theta^{PQ} \in [0,1]$ such that $(\theta^{PQ}, (P, p_1^{PQ}(\theta^{PQ})), (Q, q_2^{PQ}(\theta^{PQ})))$ is an SPNE outcome of Γ.

Lemma 6. There is no $\theta^{QP} \in [0,1]$ such that $(\theta^{QP}, (Q, q_1^{QP}(\theta^{QP})), (P, p_2^{QP}(\theta^{QP})))$ is an SPNE outcome of Γ.

Finally, to rule out the possibility of quantity competition, let us first generate the *Cournot equilibrium path* in the (p_1, p_2) space by varying θ from 0 to 1 and plotting the corresponding price vector. See the path $B'A'$ in Figure 1 where B' corresponds to $(p_1^{QQ}(0), p_2^{QQ}(0))$ and A' corresponds to

$(p_1^{QQ}(1), p_2^{QQ}(1))$. The next lemma captures the exact behavior of the Cournot equilibrium path as we vary θ.

Lemma 7. Let $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ and $(p_1^{QQ}(\theta'), p_2^{QQ}(\theta'))$ be any two points on the Cournot equilibrium path. If $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ is closer to $(p_1^{QQ}(1), p_2^{QQ}(1))$ than $(p_1^{QQ}(\theta'), p_2^{QQ}(\theta'))$ in terms of arch length of the path, then $\theta > \theta'$.

Lemma 7 can be explained in terms of the $B'A'$ segment of the $r_2 r_2'$ in Figure 1. For each point in the segment $B'A'$, we can associate a $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ vector. Lemma 7 states that as we move along the $B'A'$ segment of the $r_2 r_2'$ curve (starting from B' and ending at A'), the underlying θ increases. Finally, to complete the proof of Theorem 1, we need to eliminate the possibility of quantity competition. Given Lemma 7 identifies the properties of the Cournot equilibrium path in terms of θ, we can use this path along with the cut-off point θ_4 (identified in Lemma 3 (iii)) to establish the impossibility of quantity competition. Hence, we have Lemma 8.

Lemma 8. There is no $\theta^{QQ} \in [0,1]$ such that $(\theta^{QQ}, (Q, q_1^{QQ}(\theta^{QQ})), (Q, q_2^{QQ}(\theta^{QQ})))$ is an SPNE outcome of Γ.

4. Complements

To obtain the equilibrium outcome when the goods are complement we use the following assumptions.

Assumption 5. For $i, j = 1, 2$ $(i \neq j)$ and any $q >> (0,0)$, (i) $U_i(q) > 0$, (ii) $U_{ii}(q) < 0$, (iii) $U_{ij}(q) > 0$ and (iv) $|U_{ii}(q)| > |U_{ij}(q)|$.

Assumption 6. For $i, j = 1, 2$ $(i \neq j)$ and any $q >> (0,0)$, (i) $\pi_{i,ij}^{QQ}(q) > 0$ and (ii) $\pi_{i,ii}^{QQ}(q) + |\pi_{i,ij}^{QQ}(q)| < 0$.

Assumption 7. For $i, j = 1, 2$ $(i \neq j)$ and any $p >> (0,0)$, (i) $\pi_{i,ij}^{PP}(p) < 0$ and (ii) $\pi_{i,ii}^{PP}(p) + |\pi_{i,ij}^{PP}(p)| < 0$.

Assumption 8. For $i, j = 1, 2$, $i \neq j$ and any $p >> (0,0)$ such that $p_i \leq c \leq p_j$, (i) $W_{ii}^{PP}(p) < 0$ and (ii) $W_{ii}^{PP}(p) - W_{ij}^{PP}(p) < 0$.

Assumption 5, Assumption 6 and Assumption 7 are very standard and these are satisfied by any standard demand function when the goods are complements (see Singh and Vives [30] and Vives [34]). Assumption 8 (i) is necessary to satisfy the second order condition of any welfare maximizing firm. With the quadratic (CES) utility function given by condition (1) (condition (2)), Assumption 5, Assumption 6, Assumption 7 and Assumption 8 are satisfied.

Before going to our result we explain the implications of Assumption 5, Assumption 6, Assumption 7 and Assumption 8 in terms of reactions functions in the price plane using Figure 2. In particular, we are interested in the function $p_2 = R_2^{PP}(p_1)$, the set $\mathcal{P}(\mathcal{R}_2^{QQ})$ for Firm 2 and, for

$\theta \in \{0,1\}$, we are interested in the function $p_1 = SV_1^{PP}(p_2, \theta)$ and the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ for Firm 1. In Figure 2, the curve R_2R_2' represents the reaction function of Firm 2 when Firm 1 chooses price strategy, that is, $p_2 = R_2^{PP}(p_1)$. By Assumption 7, it is decreasing in p_1 with an absolute slope less than unity. Given Assumption 7 (ii), $\pi_{2,22}^{PP}(p) < 0$ implying that in the region above the R_2R_2' curve $\pi_{2,2}^{PP}(p) < 0$ and in the region below the R_2R_2' curve we have $\pi_{2,2}^{PP}(p) < 0$. Therefore, given $\pi_{2,1}^{PP}(p) = (p_2 - c)F_{2,1}^{PP}(p) < 0$, in the region above the R_2R_2' curve, the iso-profit curve of Firm 2 is decreasing and in the region below the R_2R_2' curve, the iso-profit curve of Firm 2 is increasing. In each point in the set $\mathcal{P}(\mathcal{R}_2^{QQ})$, Firm 2 maximizes profit $\pi_2^{PP}(p)$ subject to $q_1 = F_1^{PP}(p)$. Hence, each point in the set $\mathcal{P}(\mathcal{R}_2^{QQ})$ is a point of tangency between the iso-profit curve of Firm 2 and the iso-quantity curve of Firm 1. By Assumption 5, the iso-quantity curve of Firm 1 is negatively sloped implying that the tangency of the iso-quantity curve of Firm 1 with the iso-profit curve of Firm 2 must lie above the R_2R_2' curve. Therefore the set of points in $\mathcal{P}(\mathcal{R}_2^{QQ})$ lie above the R_2R_2' curve. Finally, as we move along the R_2R_2' curve towards the p_2 axis, the profit of the Firm 2 increases since $\frac{d\pi_2^{PP}(p_1, R_2^{PP}(p_1))}{dp_1} = \frac{\partial \pi_2^{PP}(p_1, R_2^{PP}(p_1))}{\partial p_1} < 0$. In Figure 2, the R_1R_1' curve is the reaction function of Firm 1, that is, $p_1 = SV_1^{PP}(p_2, 1)$ for $\theta = 1$. By Assumption 7, it is decreasing and the slope is greater than unity. One can also show that each point in the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ lies to the right of the R_1R_1' curve. By definition, $p_1 = c$ represents the set of points in the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(0))$. In Figure 2, the S_1S_1' curve represents the function $p_1 = SV_1^{PP}(p_2, 0)$ and it satisfies the following condition.

$$(p_1 - c)F_{1,1}^{PP}(p) + (p_2 - c)F_{2,1}^{PP}(p) = 0. \tag{3}$$

By Assumption 5, $F_{1,1}^{PP}(p) < 0$, $F_{2,1}^{PP}(p) < 0$ and $|F_{1,1}^{PP}(p)| > |F_{2,1}^{PP}(p)|$ and hence using (3) it follows that the S_1S_1' curve must lie between the $p_1 = c$ line and the $p_1 + p_2 = 2c$ line (see line PP' in Figure 2). Similarly, the S_2S_2' curve represents the locus of points satisfying $W_2^{PP}(p) = 0$ and this curve lies between the $p_2 = c$ and the PP' lines. In Figure 2, point B is the intersection point between the R_1R_1' curve and the R_2R_2' curve representing the Bertrand equilibrium point for $\theta = 1$. Since both firm are facing symmetric demand and identical cost conditions, point B lies on the $p_1 = p_2$ line. Point A is the point of intersection between the S_1S_1' curve and the R_2R_2' curve representing the Bertrand equilibrium for $\theta = 0$. Given any $\theta \in [0,1]$, the function $p_1 = SV_1^{PP}(p_2, \theta)$ lies between the S_1S_1' curve and the R_1R_1' curve. Therefore, for any θ, the equilibrium price vector $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ must belong to the segment AB of the R_2R_2' curve.

Proposition 1. Suppose Assumption 5, Assumption 6, Assumption 7 and Assumption 8 hold. There exists $\theta^{PP} \in (0,1)$ such that the strictly partial privatization Bertrand equilibrium strategy combination $(\theta^{PP}, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$ is the unique SPNE outcome of Γ.

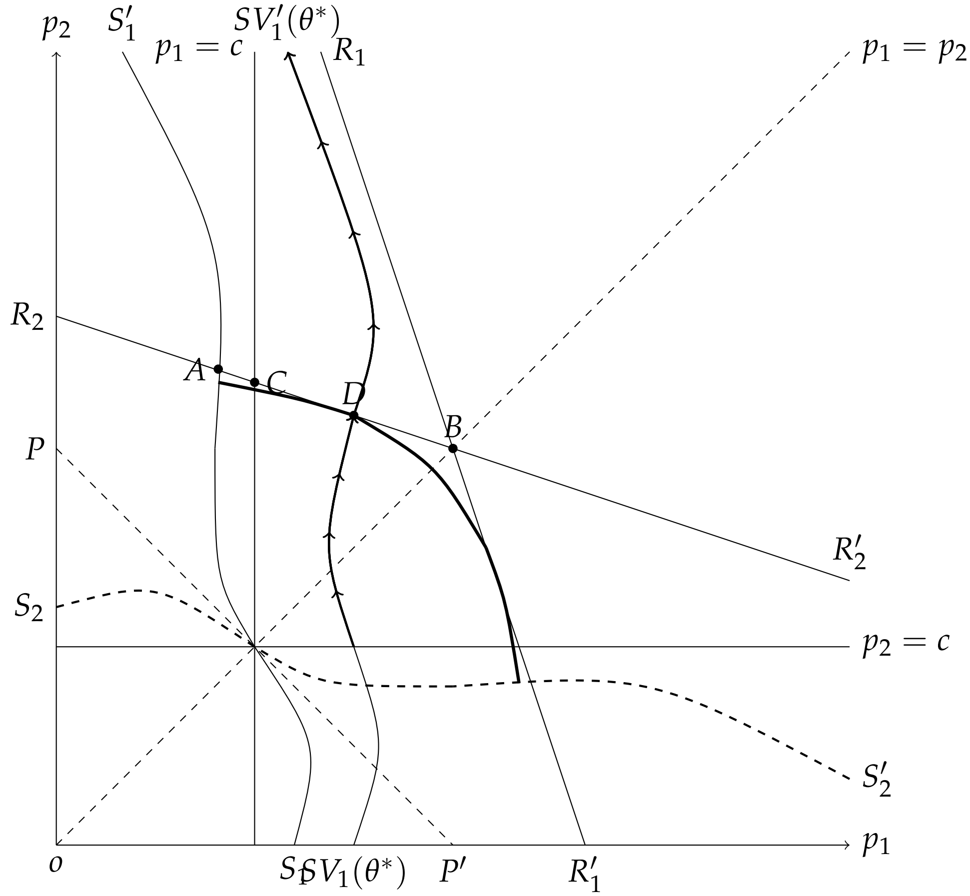

FIGURE 2. The case of complements

5. ROBUSTNESS

Following Kreps and Scheinkman's [22] argument on the importance of game form, we first check how important our three stage game Γ is in driving Theorem 1 and Proposition 1. We do this robustness check with quadratic utility function given by (1).

(a) Firstly, if we interchange Stage 1 and Stage 2 of Γ, then, in case of imperfect substitutes, we have no privatization Bertrand equilibrium as the unique SPNE outcome, and, in case of complements, we have strictly partial privatization Bertrand equilibrium with $\theta^{PP} = -\gamma(1+\gamma)/(4+3\gamma) \in (0,1)$ as the unique SPNE outcome.

(b) Keeping everything else unchanged, suppose in Stage 1 of Γ we replace the objective function of the government by $V_1 := \theta\pi + (1-\theta)W$ where $\theta \in [0,1]$. For both imperfect substitutes and complements, no privatization Bertrand equilibrium is the unique SPNE outcome.

(c) Suppose, ceteris paribus, in Stage 1 we replace the objective function of the government by $V_1 := \pi + (1-\theta)CS$ where $\theta \in [0,1]$. In case of substitute we have strictly partial privatization Bertrand equilibrium with $\theta^{PP} = \gamma/(4+\gamma-6\gamma^2-3\gamma^3)$ as the unique SPNE outcome provided the goods are 'sufficiently' differentiated. Specifically, this result holds for $\gamma \in (0, \hat{\gamma})$ where $\hat{\gamma} \approx 0.8$. In case of complements, we have no privatization Bertrand equilibrium as the unique SPNE outcome and it holds for all $\gamma \in (-1, 0)$.

(d) We check the importance of our symmetric cost assumption. Suppose $C_i(q) = c_i q_i$ is the total cost function of Firm i for $i = 1, 2$ and assume that $c_1 \neq c_2$. If the difference in the marginal costs of the two firms is 'large enough', then results can change for imperfect substitutes (see Zanchettin [35]). Keeping the game Γ unchanged, if we assume cost asymmetry, then, with quadratic utility function given by (1), we have the following results. In case of imperfect substitutes, if $\gamma(3 - \gamma^2)/2 < \alpha_1/\alpha_2 < 1/\gamma$, then we have no privatization Bertrand equilibrium as the unique SPNE outcome of Γ where $\alpha_i = a - c_i > 0$ for all $i = 1, 2$. In case of complements, for any $\gamma \in (-1, 0)$, we have the strictly partial privatization Bertrand equilibrium as the unique SPNE outcome of Γ with $\theta^{PP} = -\gamma(1 - \gamma^2)(\alpha_2 - \gamma\alpha_1) / \left[(4 - 3\gamma^2)(\alpha_1 - \gamma\alpha_2) - \gamma(\alpha_2 - \gamma\alpha_1)\right] \in (0, 1)$.

Therefore, with differentiated duopoly products, price competition is an inescapable equilibrium outcome when a regulated partially privatized firm competes with a private firm provided the cost difference between the two firms is not too much.

(e) Finally, if we have one regulated firm and more than one profit maximizing firms competing in a differentiated product market, then, by taking a general form of the quadratic utility function given by (1), we can show that in this three stage game it is optimal to select zero weight on profit of the regulated firm under price competition. However, in this scenario it was established by Haraguchi and Matsumura [19] that one cannot always induce price competition. Specifically, Haraguchi and Matsumura [19] show that for any given number of private firms greater than one, we always have a cut-off value of the substitution parameter γ below which one can induce price competition but above which one cannot.

Thus, even in an oligopoly framework, co-existence of a fully public firm and many profit maximizing firms is a possible equilibrium outcome under symmetric cost conditions and with sufficiently low values of the substitution parameter γ.

6. CONCLUSION

6.1. **Government ownership as a policy instrument.** Efficiency of a market crucially depends on the nature of strategic interaction among firms in the market. For example, unless firms are capacity constrained, price competition among firms results in higher social welfare than competition in terms of quantity. However, it is often difficult for a social planner to find appropriate policy instrument to influence the nature of firms strategic interaction. Analysis of this paper reveals that, when firms are free to choose the strategic variableprice contract vis-á-vis quantity contract, the equilibrium modes of competition depends on the level of privatization of the public firm.

It implies that the level of government ownership of one of the firms operating in a market is an effective policy instrument to influence the nature of strategic interaction among firms in that market in favor of the social planner.

6.2. **Implementation aspect of the policy instrument.** One can question the implementability aspect of regulating weight on profit of a partially privatized firm. The difficulty of implementability is a valid criticism if, as a policy, one has to sustain a weight on profit of the partially privatized firm which is neither zero nor one (like our SPNE outcome with complements). Specifically if, as a policy, the regulator has to maintain an exact weight $\theta \in (0, 1)$ on profit of the partially private firm, then it is difficult to implement it if the existing weight on profit of the partially private firm is $\theta' \neq \theta$ since the transition to θ calls for redistribution of private and public shares of the firm which may be difficult and costly. Moreover, there may be other legal difficulties in the form of upper bounds on private shares. However, by completely disallowing private stakeholders (that is, by retaining only government shares as a rule) in a partially privatized firm, the regulator can transform a partially privatized firm to a public firm. In that sense our result on imperfect substitute that prescribes the co-existence of a purely private firm and a purely public firm is easy to implement relative to our result on complement goods. However, the need for regulation to change the mode of competition is absent when the goods are complement.

6.3. **Regulating both firms.** If the government regulates both the firms in an otherwise three stage game like ours, then (due to marginal cost pricing) equilibrium social welfare is higher than that of our SPNE outcomes. Moreover, in that case, the mode of competition is also irrelevant. However, in reality we rarely see more than one regulated firm in a differentiated product duopoly (oligopoly) market. In that sense our approach to regulate only one firm is more realistic.

6.4. **On the adverse effect of transforming the objective of a public firm towards more profit orientation.** A public firm may choose to go private either for significant financial gain of the shareholders and CEOs' and/or to reduced regulatory requirements in order to focus on long-term goals. However, in developed countries (like the USA and the UK), the harmful effects of transition of a public firm towards private firm on the stakeholders was pointed out by Greenfield (see Greenfield [17]).[4] It is also argued that a public firm going private may induce more overall efficiency in the long-run. Specifically, the English government has radically restructured its school system under an assumption that school autonomy delivers benefits to schools and students. However, the paper by Eyles, Machin and McNally [12] shows that there is no evidence of improvement either in pupil performance or in teaching quality resulting from this conversion.

[4]According to Greenfield (see Greenfield [17]), "There may be somewhat more freedom for private firms to operate with a view toward stakeholder interests, but the impact is likely to be marginal. And that freedom could cut the other way, giving private firms the ability to insulate themselves from stakeholder interests and public oversight, making them even more profit-oriented and less concerned about the public interest".

The harmful short-run effects of more profit orientation in a differentiated product oligopoly market was pointed out by Anderson, de Palma and Thisse [1] (when only quantity competition is admissible and with CES utility function of the representative consumer). Our paper adds to this harmful effects argument of more profit orientation from the social welfare angle for the differentiated product market under symmetric cost conditions. From a policy perspective, our result suggests that if for some reason (other than welfare maximization) the regulator wants to change the orientation of the public firm (in a market with imperfect substitutes) towards more profit (by allocating non-zero weight on profit of the partially private firm), then we can have two types of welfare losses. Not only there is a certain welfare loss due to the increase in profit orientation of the partially private firm, there is a further chance of welfare loss due to a shift in the mode of competition from price to something else.[5] Since our results hold under very general demand specifications, when the goods are substitutes, the policy prescription is to try not to make a public firm more profit oriented.

7. Appendix

Proof of Lemma 1: We use two steps to prove the result.
Step 1: Given any weight $\theta \in [0,1]$ in Stage 1 and given that firms compete in prices in Stage 2, the Stage 3 optimum choice $(p_1^{PP}(\theta), p_2^{PP}(\theta))$ is unique.
Proof of Step 1: In Stage 3, given p_2, Firm 1 chooses p_1 to maximizing $V_1^{PP}(p,\theta) = \theta.\pi_1^{PP}(p) + (1-\theta)W^{PP}(p)$ and, given p_1, Firm 2 chooses p_2 to maximize $\pi_2^{PP}(p) = (p_2 - c)F_2^{PP}(p)$. The first order conditions are the following:

$$V_{1,1}^{PP}(p,\theta) = \theta F_1^{PP}(p) + (p_1 - c)F_{1,1}^{PP}(p) + (1-\theta)(p_2 - c)F_{2,1}^{PP}(p) = 0, \tag{4}$$

and

$$\pi_{2,2}^{PP}(p) = F_2^{PP}(p) + (p_2 - c)F_{2,2}^{PP}(p) = 0. \tag{5}$$

Using Assumption 3 (ii) and Assumption 4 (i) it follows that $V_{1,11}^{PP} < 0$ and $\pi_{2,22}^{PP} < 0$. Therefore, second order conditions for maximization are satisfied. Since $\pi_{2,12}^{PP} > 0$, Firm 2's reaction function is increasing in (p_1, p_2). Moreover, $|\pi_{2,22}^{PP}| > |\pi_{2,12}^{PP}|$ implies that the slope of the reaction function of the Firm 2 is less than unity. The sign of $V_{1,12}^{PP}$ can be anything. If for some $(p_1^{PP}(\theta), p_2^{PP}(\theta))$, $V_{1,12}^{PP} > 0$, then by Assumption 4 (ii), the slope of the reaction function of the Firm 1 must be greater than unity implying that the intersection of this reaction function with Firm 2's reaction function is unique since, along the $\frac{\partial V_1^{PP}}{\partial p_1} = 0$ curve, given any p_2 we have only one p_1, the locus of the function $\frac{\partial V_1^{PP}}{\partial p_1} = 0$ will never intersect Firm 2's reaction function twice. If for some $(p_1^{PP}(\theta), p_2^{PP}(\theta))$, $V_{1,12}^{PP} = 0$, then at that point Firm 1's reaction function has a slope of ∞. Given that the slope of the reaction function of Firm 2 is increasing (and is less than unity), we have a unique best response for Firm 1 given any p_2 implying uniqueness of the equilibrium point. Finally, if for some $(p_1^{PP}(\theta), p_2^{PP}(\theta))$, $V_{1,12}^{PP} < 0$, then it is obvious that we will have a unique intersection.
Step 2: $\pi_{1,1}^{PP}(p_1^{PP}(0), p_2^{PP}(0)) > 0$.
Proof of Step 2: At $\theta = 0$, the equilibrium price vector $(p_1^{PP}(0), p_2^{PP}(0))$ satisfy following first order conditions

$$(p_1^{PP}(0) - c)F_{1,1}^{PP}(p_1^{PP}(0), p_2^{PP}(0)) + (p_2^{PP}(0) - c)F_{2,1}^{PP}(p_1^{PP}(0), p_2^{PP}(0)) = 0, \tag{6}$$

[5] For complements, the first type of welfare loss is present but the second type of welfare loss is absent since price competition is a dominant strategy.

and

$$(p_2^{PP}(0)-c)F_{2,2}^{PP}(p_1^{PP}(0),p_2^{PP}(0))+F_2^{PP}(p_1^{PP}(0),p_2^{PP}(0))=0. \tag{7}$$

By definition $q_2^{PP}(0):=F_2^{PP}(p_1^{PP}(0),p_2^{PP}(0))>0$ and, by Assumption 1, $F_{2,2}^{PP}(p_1^{PP}(0),p_2^{PP}(0))<0$. Therefore, from (7) we have $p_2^{PP}(0)>c$. Given $p_2^{PP}(0)>c$, and, $F_{2,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))>0$ and $F_{1,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))<0$ (by Assumption 1), from (6) we get $p_1^{PP}(0)>c$. By Assumption 1 we also have $F_{2,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))=F_{1,2}^{PP}(p_1^{PP}(0),p_2^{PP}(0))<|F_{1,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))|$. Hence, from condition (6) we get $p_2^{PP}(0)>p_1^{PP}(0)>c$. Using $p_2^{PP}(0)>p_1^{PP}(0)>c$ and using the fact that the demands are symmetric with own effect dominant cross effect we have,

$$F_1^{PP}(p_1^{PP}(0),p_2^{PP}(0))>F_1^{PP}(p_1^{PP}(0),p_1^{PP}(0))=F_2^{PP}(p_1^{PP}(0),p_1^{PP}(0))>F_2^{PP}(p_1^{PP}(0),p_2^{PP}(0)). \tag{8}$$

Finally,

$$\begin{aligned}\pi_{1,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))&=(p_1^{PP}(0)-c)F_{1,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))+F_1^{PP}(p_1^{PP}(0),p_2^{PP}(0))\\&=-(p_2^{PP}(0)-c)F_{2,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))+F_1^{PP}(p_1^{PP}(0),p_2^{PP}(0))\\&>(p_2^{PP}(0)-c)F_{2,2}^{PP}(p_1^{PP}(0),p_2^{PP}(0))+F_1^{PP}(p_1^{PP}(0),p_2^{PP}(0))\\&>(p_2^{PP}(0)-c)F_{2,2}^{PP}(p_1^{PP}(0),p_2^{PP}(0))+F_2^{PP}(p_1^{PP}(0),p_2^{PP}(0))\\&=0.\end{aligned}$$

Here the first equality is by definition, the second equality is due to (6), the first inequality follows from the fact $-F_{2,1}^{PP}(p_1^{PP}(0),p_2^{PP}(0))>F_{2,2}^{PP}(p_1^{PP}(0),p_2^{PP}(0))$ and last inequality is due to (8). This proves Step 2.

To complete the proof we also use Figure 3. Given any θ, its (unique) corresponding equilibrium price vector $(p_1^{PP}(\theta),p_2^{PP}(\theta))$ is the intersection of the reaction function of Firm 1 $p_1=SV_1^{PP}(p_2,\theta)$, and the reaction function of the Firm 2 $p_2=R_2^{PP}(p_1)$. By condition (5), $R_2^{PP}(c)>c$ and $0<dR_2^{PP}(p_1)/dp_1<1$ implying that $p_2=R_2^{PP}(p_1)$ must intersect the $p_1=p_2$ line from above (see Figure 3). Thus, to the left of the $p_1=p_2$ line along Firm 2's reaction function $p_2=R_2^{PP}(p_1)$ we have $p_2>p_1$. Moreover, by symmetry of the firms, at $\theta=1$ we have $p_1^{PP}(1)=p_2^{PP}(1)$. Hence, the intersection point of the curve $p_2=R_2^{PP}(p_1)$ and the line $p_1=p_2$ is also the intersection point of the curves $p_2=R_2^{PP}(p_1)$ and $p_1=SV_1^{PP}(p_2,1)$. By Step 2, the intersection point of $p_2=R_2^{PP}(p_1)$ and $p_1=SV_1^{PP}(p_2,0)$ must lie to the left of $p_1=SV_1^{PP}(p_2,1)$ and, for any $\theta\in(0,1)$, $p_1=SV_1^{PP}(p_2,\theta)$ is bounded between $p_1=SV_1^{PP}(p_2,0)$ and $p_1=SV_1^{PP}(p_2,1)$ (given Assumption 3 and Assumption 4(i)). As a result, every equilibrium price vector $(p_1^{PP}(\theta),p_2^{PP}(\theta))$ must belongs to the segment of $p_2=R_2^{PP}(p_1)$ that lie between intersection of $p_1=SV_1^{PP}(p_2,0)$ and $p_1=SV_1^{PP}(p_2,1)$, that is, the over braced segment PP' in Figure 3.

The PP' segment in Figure 3 lies to the left of $p_1=SV_1^{PP}(p_2,1)$ implying $\pi_{1,1}^{PP}(p_1^{PP}(\theta),p_2^{PP}(\theta))>0$. Moreover, the PP' segment also lies to the right of $p_1=SV_1^{PP}(p_2,0)$ implying $W_1^{PP}(p_1^{PP}(\theta),p_2^{PP}(\theta))<0$. Finally, for $p_2>p_1>c$, the PP' segment in Figure 3 must lie completely above the $p_1=p_2$ line implying $W_2(p_1^{PP}(\theta),p_2^{PP}(\theta))<0$.

Proof of Lemma 2: To prove $\frac{\partial q_1^{QQ}}{\partial\theta}<0$ and $\frac{\partial q_2^{QQ}}{\partial\theta}>0$, we differentiate the conditions $V_{1,1}^{QQ}(q_1^{QQ}(\theta),q_2^{QQ}(\theta),\theta)=0$ and $\pi_{2,2}^{QQ}(q_1^{QQ}(\theta),q_2^{QQ}(\theta))=0$ with respect to θ and then solve for $\frac{\partial q_1^{QQ}}{\partial\theta}$ and $\frac{\partial q_2^{QQ}}{\partial\theta}$. This results in

$$\frac{\partial q_1^{QQ}}{\partial\theta}=-\frac{\pi_{2,22}^{QQ}(q^{QQ}(\theta))\frac{\partial V_{1,1}^{QQ}}{\partial\theta}}{|A^{QQ}|},$$

and

$$\frac{\partial q_2^{QQ}}{\partial\theta}=\frac{\pi_{2,12}^{QQ}(q^{QQ}(\theta))\frac{\partial V_{1,1}^{QQ}}{\partial\theta}}{|A^{QQ}|},$$

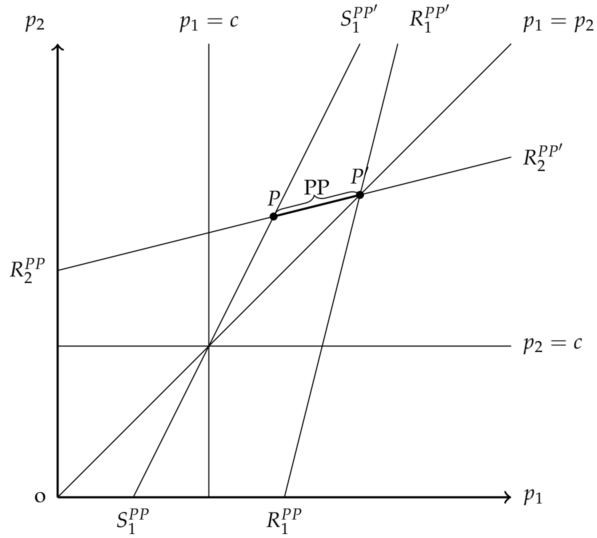

FIGURE 3. Region of potential Bertrand equilibria

where for any $\theta \in [0,1]$, $q^{QQ}(\theta) := (q_1^{QQ}(\theta), q_2^{QQ}(\theta))$, $\frac{\partial V_{1,1}^{QQ}}{\partial \theta} = \pi_{1,1}^{QQ} - W_1^{QQ} = q_1^{QQ}(\theta) F_{1,1}^{QQ} < 0$ and $|A^{QQ}| = V_{1,11}^{QQ} \pi_{2,22}^{QQ} - V_{1,12}^{QQ} \pi_{2,12}^{QQ} > 0$. Hence, we have $\frac{\partial q_1^{QQ}}{\partial \theta} < 0$ and $\frac{\partial q_2^{QQ}}{\partial \theta} > 0$.

Note that $\frac{\partial p_1^{QQ}}{\partial \theta} = F_{1,1}^{QQ} \frac{\partial q_1^{QQ}}{\partial \theta} + F_{1,2}^{QQ} \frac{\partial q_2^{QQ}}{\partial \theta}$ and that $\frac{\partial q_2^{QQ}}{\partial \theta} / \frac{\partial q_1^{QQ}}{\partial \theta} = -\pi_{2,12}^{QQ}(q)/\pi_{2,22}^{QQ}(q) = dR_2^{QQ}(q_1)/dq_1$. From Assumption 1 and Assumption 2 we have $F_{1,1}^{QQ} + F_{1,2}^{QQ}(dR_2^{QQ}(q_1)/dq_1) < 0$. Hence, using the earlier result $\frac{\partial q_1^{QQ}}{\partial \theta} < 0$, we get $\frac{\partial p_1^{QQ}}{\partial \theta} = \left(F_{1,1}^{QQ} + F_{1,2}^{QQ}(dR_2^{QQ}(q_1)/dq_1)\right) \frac{\partial q_1^{QQ}}{\partial \theta} > 0$.

For any $\theta \in [0,1]$, define $p^{PP}(\theta) := (p_1^{PP}(\theta), p_2^{PP}(\theta))$. To show $\frac{\partial p_i^{PP}(\theta)}{\partial \theta} > 0$ for $i = 1,2$, we first differentiate the functions $V_{1,1}^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta), \theta) = 0$ and $\pi_{2,2}^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) = 0$ with respect to θ. This gives

$$V_{1,11}^{PP}(p^{PP}(\theta)) \frac{\partial p_1^{PP}(\theta)}{\partial \theta} + V_{1,12}^{PP}(p^{PP}(\theta)) \frac{\partial p_2^{PP}(\theta)}{\partial \theta} = -(\pi_{1,1}^{PP}(p^{PP}(\theta)) - W_1^{PP}(p^{PP}(\theta))), \tag{9}$$

and

$$\pi_{2,12}^{PP}(p^{PP}(\theta)) \frac{\partial p_1^{PP}(\theta)}{\partial \theta} + \pi_{2,22}^{PP}(p^{PP}(\theta)) \frac{\partial p_2^{PP}(\theta)}{\partial \theta} = 0. \tag{10}$$

Solving for $\frac{\partial p_1^{PP}(\theta)}{\partial \theta}$ and $\frac{\partial p_2^{PP}(\theta)}{\partial \theta}$ from (9) and (10) we obtain

$$\frac{\partial p_1^{PP}(\theta)}{\partial \theta} = \frac{\pi_{2,22}^{PP}(p)(W_1^{PP}(p^{PP}(\theta)) - \pi_{1,1}^{PP}(p^{PP}(\theta)))}{|A^{PP}|},$$

and

$$\frac{\partial p_2^{PP}(\theta)}{\partial \theta} = \frac{\pi_{2,12}^{PP}(p)(\pi_{1,1}^{PP}(p^{PP}(\theta)) - W_1^{PP}(p^{PP}(\theta)))}{|A^{PP}|}.$$

The term $|A^{PP}| = V_{1,11}^{PP}(p^{PP}(\theta))\pi_{2,22}^{PP}(p^{PP}(\theta)) - V_{1,12}^{PP}(p^{PP}(\theta))\pi_{2,12}^{PP}(p^{PP}(\theta))$ is positive due to Assumption 3 and Assumption 4. Given Lemma 1, for every $\theta \in (0,1)$, $\pi_{1,1}^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) - W_1^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta)) > 0$. Hence, for each $\theta \in (0,1)$, $\frac{\partial p_1^{PP}}{\partial \theta} > 0$ and $\frac{\partial p_2^{PP}}{\partial \theta} > 0$.

Next, we prove that $\frac{\partial p_i^{PQ}}{\partial \theta} > 0$ for $i = 1,2$. Suppose, given any q_2, Firm 1 chooses p_1 to maximize $V_1^{PQ}(p_1, q_2) = \theta \pi_1^{PQ}(p_1, q_2) + (1-\theta) W^{PQ}(p_1, q_2)$ and, given any p_1, Firm 2 chooses q_2 to maximize $\pi_2^{PQ}(p_1, q_2) = (F_2^{PQ}(p_1, q_2) -$

$c)q_2$ where, for $i = 1, 2$, $F_i^{PQ}(p_1, q_2)$ is the demand function of Firm i. The first order condition of Firm 1 is

$$V_{1,1}^{PQ}(p_1, q_2) = \theta F_1^{PQ}(p_1, q_2) + (p_1 - c) F_{1,1}^{PQ}(p_1, q_2) = 0. \tag{11}$$

The first order condition of Firm 2 is

$$\pi_{2,2}^{PQ}(p_1, q_2) = (F_2^{PQ}(p_1, q_2) - c) + q_2 F_{2,2}^{PQ}(p_1, q_2) = 0. \tag{12}$$

Observe that the reaction function of Firm 1 is $p_1 = F_1^{QQ}(SV_1^{QQ}(q_2, \theta), q_2)$ and that of Firm 2 is $q_2 = F_2^{PP}(p_1, R_2^{PP}(p_1))$. For any $\theta \in [0, 1]$, $p_1^{PQ}(\theta) = F_1^{QQ}(SV_1^{QQ}(q_2^{PQ}(\theta), \theta), q_2^{PQ}(\theta))$ and $q_2^{PQ}(\theta) = F_2^{PP}(p_1^{PQ}(\theta), R_2^{PP}(p_1^{PQ}(\theta)))$. Differentiating $p_1^{PQ}(\theta)$ and $q_1^{PQ}(\theta)$ with respect to θ and then solving for $\frac{\partial p_1^{PQ}}{\partial \theta}$ and $\frac{\partial q_2^{PQ}}{\partial \theta}$ we get,

$$\frac{\partial p_1^{PQ}}{\partial \theta} = \frac{F_{1,1}^{QQ}(q) \frac{\partial SV_1^{QQ}}{\partial \theta}}{|A^{PQ}|},$$

and

$$\frac{\partial q_2^{PQ}}{\partial \theta} = \frac{F_{1,1}^{QQ}(q) \frac{\partial SV_1^{QQ}}{\partial \theta} \left(F_{2,1}^{PP}(p) + F_{2,2}^{PP}(p) \frac{dR_2^{PP}}{dp_1} \right)}{|A^{PQ}|},$$

where $|A^{PQ}| = 1 - \left(F_{1,1}^{QQ}(q) \frac{dSV_1^{QQ}}{dq_2} + F_{1,2}^{QQ}(q) \right) \left(F_{2,1}^{PP}(p) + F_{2,2}^{PP}(p) \frac{dR_2^{PP}}{dp_1} \right) > 0$ and $\frac{\partial SV_1^{QQ}}{\partial \theta} = -q_1 F_{1,1}^{QQ} / V_{1,11}^{QQ}(q) < 0$.[6] Hence, given $F_{1,1}^{QQ}(q) < 0$, we get $\frac{\partial p_1^{PQ}}{\partial \theta} > 0$. Finally, $\frac{\partial q_2^{PQ}}{\partial \theta} / \frac{\partial p_1^{PQ}}{\partial \theta} = F_{2,1}^{PP}(p) + F_{2,2}^{PP}(p)(dR_2^{PP}/dp_1)$ implies that

$$\begin{aligned} \frac{\partial p_2^{PQ}}{\partial \theta} &= F_{2,1}^{PQ}(p_1, q_2) \frac{\partial p_1^{PQ}}{\partial \theta} + F_{2,2}^{PQ}(p_1, q_2) \frac{\partial q_2^{PQ}}{\partial \theta} \\ &= \left[F_{2,1}^{PQ}(p_1, q_2) + F_{2,2}^{PQ}(p_1, q_2) \left(F_{2,1}^{PP}(p) + F_{2,2}^{PP}(p) \frac{dR_2^{PP}}{dp_1} \right) \right] \frac{\partial p_1^{PQ}}{\partial \theta} \\ &= \frac{dR_2^{PP}}{dp_1} \frac{\partial p_1^{PQ}}{\partial \theta} > 0. \end{aligned} \tag{13}$$

Proof of Lemma 3: To prove part (i) and part (iii) of this result we use an application of the Fundamental (Gradient) Theorem of Line Integrals that states the following: Consider any function $f : \Re_+^2 \to \Re$ which is twice differentiable. For any $a = (a_1, a_2) >> (0, 0), a' = (a_1', a_2') >> (0, 0)$ and for any scalar $t \in [0, 1]$ such that $a(t) = (a_1(t), a_2(t)) = (ta_1' + (1 - t)a_1, ta_2' + (1 - t)a_2) >> (0, 0)$,

$$f(a') - f(a) = (a_1' - a_1) \int_0^1 \frac{\partial f(a(t))}{\partial a_1(t)} dt + (a_2' - a_2) \int_0^1 \frac{\partial f(a(t))}{\partial a_2(t)} dt. \tag{14}$$

Condition (14) specifies that given any smooth path $a(t)$ connecting points a and a' in the domain of a function f, the line integral through the gradient of the function f equals the difference in its scalar at the endpoints (that is, $f(a') - f(a)$) (see Apostol [2] for a more detailed discussion on line integrals).

Proof of (i): In the price space, given any $\theta \in (0, 1)$, if Firm 2 chooses price strategy, then Firm 1's reaction function is $p_1 = SV_1^{PP}(p_2, \theta)$, and, if Firm 2 chooses quantity strategy, then Firm 1's reaction function is the set of points $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ and can be written in implicit form as $F_1^{PP}(p) - SV_1^{QQ}(F_2^{PP}(p), \theta) = 0$. Given Firm 1 chooses price strategy, Firm 2's reaction function is $p_2 = R_2^{PP}(p_1)$. Fix a $\theta \in [0, 1]$. Consider $p_1(t) = tp_1^{PP}(\theta) + (1 - t)p_1^{PQ}(\theta)$

[6]Specifically, $|A^{PQ}| = 1 - \left(F_{1,1}^{QQ}(q) \frac{dSV_1^{QQ}}{dq_2} + F_{1,2}^{QQ}(q) \right) \left(F_{2,1}^{PP}(p) + F_{2,2}^{PP}(p) \frac{dR_2^{PP}}{dp_1} \right)$
$= \frac{U_{11}\left(U_{22} + U_{11} |\frac{dR_2^{PP}}{dp_1}| |\frac{dSV_1^{QQ}}{dq_2}| - U_{12} |\frac{dSV_1^{QQ}}{dq_2}| - U_{12} \frac{R_2^{PP}}{dp_1} \right)}{|D|} > \frac{U_{11} U_{12} \left(1 - \frac{dR_2^{PP}}{dp_1} \right) \left(1 - |\frac{dSV_1^{QQ}}{dq_2}| \right)}{|D|} > 0.$

defined for each $t \in [0,1]$. Applying the condition (14) on the function $\pi_{2,2}^{PP}(p)$ with endpoints $(p_1^{PP}(\theta), p_2^{PQ}(\theta))$ and $(p_1^{PQ}(\theta), p_2^{PQ}(\theta))$ we get

$$\pi_{2,2}^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) - \pi_{2,2}^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) = (p_1^{PP}(\theta) - p_1^{PQ}(\theta)) \int_0^1 \pi_{2,12}^{PP}(p_1(t), p_2^{PQ}(\theta))dt.$$

The point $(p_1^{PQ}(\theta), p_2^{PQ}(\theta))$ is on $p_2 = R_2^{PP}(p_1)$ implying $\pi_{2,2}^{PP}(p_1^{PQ}(\theta), p_2^{PQ}(\theta)) = 0$. As a result we have

$$\pi_{2,2}^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) = (p_1^{PP}(\theta) - p_1^{PQ}(\theta)) \int_0^1 \pi_{2,12}^{PP}(p_1(t), p_2^{PQ}(\theta))dt. \tag{15}$$

From Assumption 3 (i) it follows that $\int_0^1 \pi_{2,12}^{PP}(p_1(t), p_2^{PQ}(\theta))dt > 0$. Therefore, $p_1^{PP}(\theta) \gtreqless p_1^{PQ}(\theta)$ if and only if $\pi_{2,2}^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) \gtreqless 0$. Observe first that

$$\lim_{\theta \to 0} \pi_{2,2}^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) = \pi_{2,2}^{PP}(p_1^{PP}(0), p_2^{PQ}(0)) > 0. \tag{16}$$

Condition (16) holds since from the first order condition of profit maximization and welfare maximization we have $c = p_1^{PQ}(0) < p_1^{PP}(0) < p_2^{PP}(0)$ and since R_2^{PP} is increasing, that is, $p_2^{PQ}(0) < p_2^{PP}(0)$ therefore $(p_1^{PP}(0), p_2^{PQ}(0))$ lie below the R_2^{PP} hence implies (16). Also observe that

$$\lim_{\theta \to 1} \pi_{2,2}^{PP}(p_1^{PP}(\theta), p_2^{PQ}(\theta)) = \pi_{2,2}^{PP}(p_1^{PP}(1), p_2^{PQ}(1)) < 0. \tag{17}$$

Condition (17) holds since $p_1^{PP}(1) = p_2^{PP}(1) < p_2^{PQ}(1)$ implies that the point $(p_1^{PP}(1), p_2^{PQ}(1))$ lies above the R_2^{PP}. Conditions (16) and (17) implies that there exist θ_R,θ_S with $\theta_R \leq \theta_S$ such that for any $\theta \in (0, \theta_R)$ and any $\theta \in (\theta_S, 1)$ we have $p_1^{PP}(\theta) > p_1^{PQ}(\theta)$ and $p_1^{PP}(\theta) < p_1^{PQ}(\theta)$ respectively. Thus, $\pi_{2,2}^{PP}(p_1^{PP}(\theta_S), p_2^{PP}(\theta_S)) - \pi_{2,2}^{PP}(p_1^{PP}(\theta_R), p_2^{PP}(\theta_R)) = 0$ and applying condition (14) to this equality with end points $(p_1^{PP}(\theta_S), p_2^{PP}(\theta_S))$ and $(p_1^{PP}(\theta_R), p_2^{PP}(\theta_R))$ we get

$$(p_1^{PP}(\theta_S) - p_1^{PP}(\theta_R)) \int_0^1 \pi_{2,12}^{PP}(p_1(t), p_2(t))dt + (p_2^{PP}(\theta_S) - p_2^{PP}(\theta_R)) \int_0^1 \pi_{2,22}^{PP}(p_1(t), p_2(t))dt = 0. \tag{18}$$

By Assumption 3 and Lemma 2 (ii) it follows that for condition (18) to hold we must have $p_1^{PP}(\theta_S) > p_2^{PP}(\theta_S) > p_2^{PP}(\theta_R) > p_1^{PP}(\theta_R)$ if $\theta_R < \theta_S$. But for each $\theta \in [0,1]$ we have $p_2^{PP}(\theta) \geq p_1^{PP}(\theta)$. Therefore, $p_1^{PP}(\theta_S) > p_2^{PP}(\theta_S)$ is a contradiction, hence we have $\theta_S = \theta_R = \theta_1$. Thus, there exists a unique $\theta_1 \in (0,1)$ such that $p_1^{PP}(\theta) \gtreqless p_1^{PQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_1$.

Along Firm 2's reaction function $p_2 = R_2^{PP}(p_1)$, $\hat{\pi}_2^{PP}(p_1) = \pi_2^{PP}(p_1, R_2^{PP}(p_1))$. Given $R_2^{PP}(p_1) - c > 0$ and $F_{2,1}^{PP}(p_1, R_2^{PP}(p_1)) > 0$, $d\hat{\pi}_2^{PP}(p_1)/dp_1 = \pi_{1,1}^{PP}(p_1, R_2^{PP}(p_1)) = (R_2^{PP}(p_1) - c)F_{2,1}^{PP}(p_1, R_2^{PP}(p_1)) > 0$. Therefore, along the reaction function $p_2 = R_2^{PP}(p_1)$, Firm 2's profit increases in p_1. For any $\theta \in [0, \theta_1)$, $p_1^{PP}(\theta) > p_1^{PQ}(\theta)$ holds. Hence, $\overline{\pi}_2^{PP}(\theta) = \pi_2^{PP}(p_1^{PP}(\theta), R_2^{PP}(p_1^{PP}(\theta))) > \pi_2^{PP}(p_1^{PQ}(\theta), R_2^{PP}(p_1^{PQ}(\theta))) = \overline{\pi}_2^{PQ}(\theta)$. Thus, if Firm 1 chooses price strategy, then Firm 2 optimally chooses price strategy. When $\theta = \theta_1$, if Firm 1 chooses price strategy, $p_1^{PP}(\theta_1) = p_1^{PQ}(\theta_1)$ implying $\overline{\pi}_2^{PP}(\theta_1) = \overline{\pi}_2^{PQ}(\theta_1)$ and Firm 2 is indifferent between price and quantity strategies. When $\theta \in (\theta_1, 1]$, if Firm 1 chooses price strategy, then $p_1^{PP}(\theta) < p_1^{PQ}(\theta)$ and by similar reasoning we can show that $\overline{\pi}_2^{PP}(\theta) < \overline{\pi}_2^{PQ}(\theta)$ so that it is always optimal for Firm 2 to choose quantity strategy.

Proof of (ii): Consider the difference $\overline{V}_1^{PP}(\theta) - \overline{V}_1^{QP}(\theta)$ evaluated at $\theta = 0$. It is quite easy to observe that $\overline{V}_1^{PP}(0) - \overline{V}_1^{QP}(0) = W^{PP}(p_1^{PP}(0), p_2^{PP}(0)) - W^{PP}(p_1^{QP}(0), p_2^{QP}(0)) > 0$. In particular, whatever be the shape of the locus of $W_1^{PP}(p) = 0$, starting from the point (c,c) as we move along that locus by increasing p_2, the welfare has to fall (see Figure 4) and, since the transformed reaction function ($\pi_{1,1}^{QQ}(q) = 0$) of Firm 1 in price space must lie above $p_1 = R_2^{PP}(p_2)$, we have $p_2^{PP}(0) > p_2^{QP}(0)$ and $(p_1^{PP}(0), p_2^{PP}(0))$ and $(p_1^{QP}(0), p_2^{QP}(0))$ lie on the locus of $W_1^{PP}(p) = 0$.

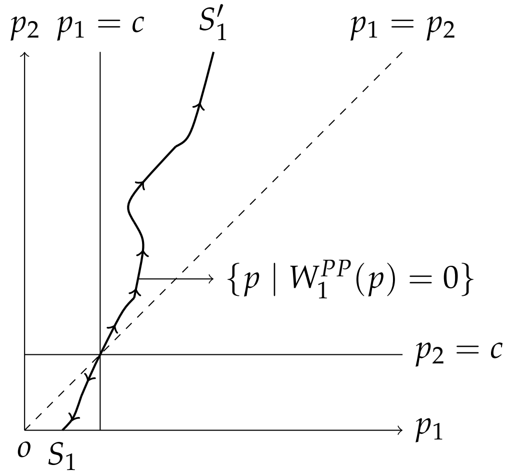

FIGURE 4. Welfare reaction function in price space

Consider $\overline{V}_1^{PP}(\theta) - \overline{V}_1^{QP}(\theta)$ at $\theta = 1$. We have, $\overline{V}_1^{PP}(1) - \overline{V}_1^{QP}(1) = \pi_1^{PP}(p_1^{PP}(1), p_2^{PP}(1)) - \pi_1^{QP}(p_1^{QP}(1), p_2^{QP}(1)) < 0$ since for a profit maximizing firm quantity strategy strictly dominates price strategy. Since $\overline{V}_1^{PP}(\theta) - \overline{V}_1^{QP}(\theta)$ is a continuous function of θ the result follows.

Proof of (iii): For this proof we restrict our attention to the quantity space (q_1, q_2). Given any $\theta \in (0,1)$, if Firm 2 chooses quantity strategy, then Firm 1's reaction function is $q_1 = SV_1^{QQ}(q_2, \theta)$. If Firm 2 chooses price strategy, then Firm 1's reaction function is $p_1 = SV_1^{PP}(p_2, \theta)$. If we transform $p_1 = SV_1^{PP}(p_2, \theta)$ to the quantity space, then we can be write it implicitly as $F_1^{QQ}(q) - SV_1^{QQ}(F_2^{QQ}(q), \theta) = 0$. Given Firm 1 chooses quantity strategy, Firm 2's reaction function is $q_2 = R_2^{QQ}(q_1)$.

Fix a $\theta \in [0,1]$. Consider $q(t) = tq_1^{QP}(\theta) + (1-t)q_1^{QQ}(\theta)$ defined for each $t \in [0,1]$. Applying the condition (14) on the function $\pi_{2,2}^{QQ}(q)$ with end points $(q_1^{QP}(\theta), q_2^{QQ}(\theta))$ and $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ we have

$$\pi_{2,2}^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) - \pi_{2,2}^{QQ}(q_1^{QQ}(\theta), q_2^{QQ}(\theta)) = (q_1^{QP}(\theta) - q_1^{QQ}(\theta)) \int_0^1 \pi_{2,12}^{QQ}(q_1(t), q_2^{QQ}(\theta))dt.$$

The point $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ is on $q_2 = R_2^{QQ}(q_1)$ implying $\pi_{2,2}^{QQ}(q_1^{QQ}(\theta), q_2^{QQ}(\theta)) = 0$. Hence

$$\pi_{2,2}^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) = (q_1^{QP}(\theta) - q_1^{QQ}(\theta)) \int_0^1 \pi_{2,12}^{QQ}(q_1(t), q_2^{QQ}(\theta))dt. \tag{19}$$

Using Assumption 2 (i) it follows that $\int_0^1 \pi_{2,12}^{QQ}(q_1(t), q_2^{QQ}(\theta))dt < 0$ and hence we have $q_1^{QP}(\theta) \lesseqqgtr q_1^{QQ}(\theta)$ if and only if $\pi_{2,2}^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) \gtreqqless 0$. Let $r_1^{PP}(q_1)$ be the transformed price reaction of Firm 1. Given $q_2^{QP}(1) < q_2^{QQ}(1)$, we have $r_1^{PP}(q_2^{QP}(1)) > R_1^{QQ}(q_2^{QP}(1)) > R_1^{QQ}(q_2^{QQ}(1))$ implying $q_1^{QP}(1) > q_1^{QQ}(1)$. Hence $(q_1^{QP}(1), q_2^{QQ}(1))$ must lie above the R_2^{QQ} curve. Thus, we have

$$\lim_{\theta \to 1} \pi_{2,2}^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) = \pi_{2,2}^{QQ}(q_1^{QP}(1), q_2^{QQ}(1)) < 0. \tag{20}$$

Also observe that

$$\lim_{\theta \to 0} \pi_{2,2}^{QQ}(q_1^{QP}(\theta), q_2^{QQ}(\theta)) = \pi_{2,2}^{QQ}(q_1^{QP}(0), q_2^{QQ}(0)) > 0 \tag{21}$$

Condition (21) holds since the price welfare reaction function of the Firm 1 in the quantity space must intersect the R_2^{QQ} curve to the left of $F_1^{QQ}(q) = c$. Therefore, $(q_1^{QP}(0), q_2^{QQ}(0))$ lies below the R_2^{QQ} curve. Condition (20) and (21) implies that there exist θ_R', θ_S' with $\theta_R' \leq \theta_S'$ such that for all $\theta \in (0, \theta_R')$ and $\theta \in (\theta_S', 1)$ we have $q_1^{QQ}(\theta) > q_1^{QP}(\theta)$ and

$q_1^{QQ}(\theta) < q_1^{QP}(\theta)$ respectively. Therefore, $\pi_{2,2}^{QQ}(q_1^{QQ}(\theta'_S), q_2^{QQ}(\theta'_S)) - \pi_{2,2}^{QQ}(q_1^{QQ}(\theta'_R), q_2^{QQ}(\theta'_R)) = 0$ and applying the condition (14) to this equality with end points $(q_1^{QQ}(\theta'_S), q_2^{QQ}(\theta'_S))$ and $(q_1^{QQ}(\theta'_R), q_2^{QQ}(\theta'_R))$ yields

$$(q_1^{QQ}(\theta'_S) - q_1^{QQ}(\theta'_R)) \int_0^1 \pi_{2,12}^{QQ}(q_1(t), q_2(t))dt + (q_2^{QQ}(\theta'_S) - q_2^{QQ}(\theta'_R)) \int_0^1 \pi_{2,22}^{QQ}(q_1(t), q_2(t))dt = 0. \tag{22}$$

By Assumption 2 and Lemma 2 (i), it follows that for condition (22) to hold with $\theta'_R < \theta'_S$, we must have $q_1^{QQ}(\theta'_R) > q_2^{QQ}(\theta'_S) > q_2^{QQ}(\theta'_R) > q_1^{QQ}(\theta'_S)$. But we know that for all $\theta \in [0,1]$, $q_2^{QQ}(\theta) < q_1^{QQ}(\theta)$ and we have a contradiction. As a result we must have $\theta'_S = \theta'_R = \theta_4$. Thus, there exists a unique $\theta_4 \in (0,1)$ such that $q_1^{QP}(\theta) \lesseqgtr q_1^{QQ}(\theta)$ if and only if $\theta \lesseqgtr \theta_4$.

Along Firm 2's reaction function $q_2 = R_2^{QQ}(q_1)$ we have $\hat{\pi}_2^{QQ}(q_1) = \pi_2^{QQ}(q_1, R_2^{QQ}(q_1))$. Given that $R_2^{QQ}(q_1) > 0$ and $F_{2,1}^{QQ}(q_1, R_2^{QQ}(q_1)) < 0$ (by Assumption 1), $d\hat{\pi}_2^{QQ}(q_1)/dq_1 = \pi_{2,1}^{QQ}(q_1, R_2^{QQ}(q_1)) = R_2^{QQ}(q_1)F_{2,1}^{QQ}(q_1, R_2^{QQ}(q_1)) < 0$. Therefore, along $q_2 = R_2^{QQ}(q_1)$, Firm 2's profit decreases in q_1. For any $\theta \in [0, \theta_4)$, $q_1^{QQ}(\theta) > q_1^{QP}(\theta)$ holds. Hence, we obtain $\overline{\pi}_2^{QQ}(\theta) = \pi_2^{QQ}(q_1^{QQ}(\theta), R_2^{QQ}(q_1^{QQ}(\theta))) < \pi_2^{QQ}(q_1^{QP}(\theta), R_2^{QQ}(p_1^{QP}(\theta))) = \overline{\pi}_2^{QP}(\theta)$. Thus, if Firm 1 chooses quantity strategy, then Firm 2 optimally chooses price strategy. When $\theta = \theta_4$, if Firm 1 chooses quantity strategy, $q_1^{QQ}(\theta_4) = q_1^{QP}(\theta_4)$ implying $\overline{\pi}_2^{QQ}(\theta_4) = \overline{\pi}_2^{QP}(\theta_4)$ and Firm 2 is indifferent between price and quantity strategies. When $\theta \in (\theta_4, 1]$, if Firm 1 chooses price strategy, then $q_1^{QQ}(\theta) < q_1^{QP}(\theta)$ and by similar reasoning we can show that $\overline{\pi}_2^{QQ}(\theta) > \overline{\pi}_2^{QP}(\theta)$ so that it is always optimal for Firm 2 to choose quantity strategy.

Proof of Lemma 4: If we assume price competition in Stage 2, then, in Stage 1, the government chooses $\theta \in [0,1]$ to maximize welfare. Given $\overline{W}^{PP}(\theta) = W^{PP}(p_1^{PP}(\theta), p_2^{PP}(\theta))$, differentiating $\overline{W}^{PP}(\theta)$ with respect to θ we get,

$$\frac{\partial \overline{W}^{PP}(\theta)}{\partial \theta} = W_1^{PP}(p^{PP}(\theta)) \frac{\partial p_1^{PP}(\theta)}{\partial \theta} + W_2^{PP}(p^{PP}(\theta)) \frac{\partial p_2^{PP}(\theta)}{\partial \theta}. \tag{23}$$

By Lemma 2 (ii), $\frac{\partial p_i^{PP}(\theta)}{\partial \theta} > 0$ and, by Lemma 1, $W_i^{PP} < 0$. Therefore, from equation (23), we get $\frac{\partial \overline{W}^{PP}(\theta)}{\partial \theta} < 0$ for all $\theta \in (0,1)$. Since $\overline{W}^{PP}(\theta = 0) > \overline{W}^{PP}(\theta = 1)$, the optimal choice of θ in Stage 1 under price competition is $\theta = 0$.

If $\theta = 0$ is the optimal choice of Stage 1, then, given $\theta = 0 < \theta_1$, it is optimal for Firm 2 to choose price strategy when Firm 1 chooses price strategy (Lemma 3 (i)). Moreover, since $\theta = 0 < \theta_4$, it is optimal for Firm 2 to choose price strategy even when Firm 1 chooses quantity strategy (Lemma 3 (iii)). Therefore, with $\theta = 0$, choosing price is the dominant strategy for Firm 2 in Stage 2. Moreover, since $\theta = 0 < \underline{\theta}_3$ and since choosing price is the dominant strategy for Firm 2, it is optimal for Firm 1 to choose price strategy (Lemma 3 (ii)). Hence, given $\theta = 0$, in Stage 2 it is optimal for both firms to choose price strategy and it is the unique Nash equilibrium of the sub-game of Γ starting from Stage 2.

Proof of Lemma 5: We prove Lemma 5 using the following figure.

In Figure 5, the curve $R_1^{PP}R_1^{PP'}$ represents the function $p_1 = SV_1^{PP}(p_2, 1)$.[7] By Assumption 3 the curve $R_1^{PP}R_1^{PP'}$ is increasing in the price plane with slope greater than unity and hence must lie to the right of the $p_1 = c$ line. Since $S_1^{PP}S_1^{PP'}$ represents the function $p_1 = SV_1^{PP}(p_2, 0)$, it must lie between the $p_1 = c$ and $p_1 = p_2$ lines. Similarly, $R_2^{PP}R_2^{PP'}$ represents the function $p_2 = R_2^{PP}(p_1)$ and, by Assumption 3, it is always increasing in the price plane with slope less than unity and hence must lie above the $p_2 = c$ line. Therefore, the intersection point of $R_1^{PP}R_1^{PP'}$ and $R_2^{PP}R_2^{PP'}$ is the Bertrand equilibrium point C for $\theta = 1$ and by Assumption 3 this point is unique. Since firms have identical cost and symmetric demand conditions, point C must lie on the $p_1 = p_2$ line. By Assumption 3 and Assumption 4, the intersection of $R_2^{PP}R_2^{PP'}$ and $S_1^{PP}S_1^{PP'}$ is the Bertrand equilibrium point (B) for $\theta = 0$ and, by Step-2 of the Lemma 1, the point B must lie to the left of point C on R_2^{PP}. We do not impose any restriction on the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ implying that it can take any shape and can intersect the curve $R_2^{PP}R_2^{PP'}$ more than ones. But the

[7]In this Figure 5, we draw all curves as straight line just for simplicity of exposition.

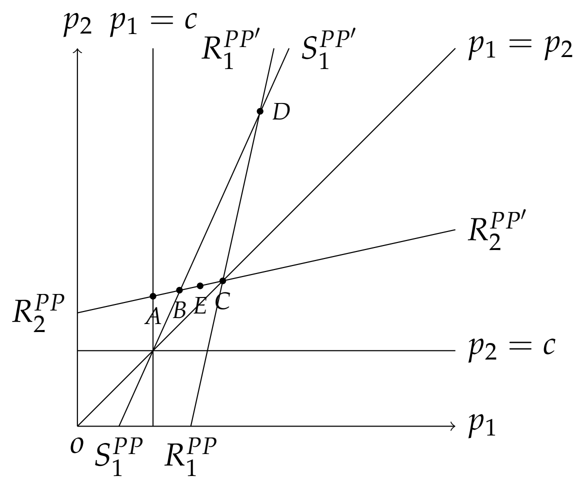

FIGURE 5. Impossibility of Type I equilibrium

locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ must lie to the left of the $R_1^{PP}R_1^{PP'}$ curve (see Cheng [5]). Hence, any intersection point between $R_2^{PP}R_2^{PP'}$ and the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$ must lie to the right of point C on the $R_2^{PP}R_2^{PP'}$ curve.

The line $p_1 = c$ is the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(0))$. If Firm 1 select price strategy, then Firm 2's optimal reaction is to react along the $R_2^{PP}R_2^{PP'}$ curve (see Singh and Vives [30]) in the price space. Again, given some $\theta \in [0,1]$, if Firm 2 select quantity strategy, then Firm 1 optimally reacts (in terms of prices) according to the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ in the price space. Since $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ must lie between the line $p_1 = c$ and the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$, for any given θ, when Firm 1 chooses price strategy and Firm 2 chooses quantity strategy, the equilibrium point must lie on the $R_2^{PP}R_2^{PP'}$ curve and it must also lie on or to the right of point A.

By Lemma 3 (i), when Firm 1 chooses price strategy, there exist a $\theta_1 \in (0,1)$ at which Firm 2 is indifferent between choosing price strategy and quantity strategy, and, for $\theta < (>)\theta_1$, it chooses price (quantity) strategy. Hence, at θ_1 the Bertrand equilibrium price vector $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$ and Type-1 equilibrium price vector $(p_1^{PQ}(\theta_1), p_2^{PQ}(\theta_1)$ induces same profit for Firm 2. The point $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$ is the intersection point of $R_2^{PP}R_2^{PP'}$ and the locus of $p_1 = SV_1^{PP}(p_2, \theta_1)$ and the point $(p_1^{PQ}(\theta_1), p_2^{PQ}(\theta_1))$ is intersection point of $R_2^{PP}R_2^{PP'}$ and locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta_1))$ in the price space. Since along the $R_2^{PP}R_2^{PP'}$ curve, any two distinct points generate distinct profits, we must have $p_i^{PP}(\theta_1) = p_i^{PQ}(\theta_1)$. Hence, at $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$, the locus of $p_1 = SV_1^{PP}(p_2, \theta_1)$ and the locus of $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta_1))$ intersect on the $R_2^{PP}R_2^{PP'}$ curve in the price space and the intersection point is unique by Lemma 3 (i). Since the locus of $p_1 = SV_1^{PP}(p_2, \theta_1)$ must lie between $S_1^{PP}S_1^{PP'}$ and $R_1^{PP}R_1^{PP'}$ and since $\theta_1 \in (0,1)$, the point $(p_1^{PP}(\theta_1), p_2^{PP}(\theta_1))$ must lie at the interior on the segment BC of the R_2^{PP} curve. Without loss of generality, let E be that point. By Lemma 2 (ii) $\frac{\partial p_i^{PQ}}{\partial \theta} > 0$, any point on the segment AE excepting point E corresponds to $\theta < \theta_1$. Hence, we cannot find any selection θ in Stage 1 for the government that can induce any (p_1, p_2) combination that lie in this segment of AE (except point E). Finally, the government won't induce any point on or to the right of E since each such point (on the $R_2^{PP}R_2^{PP'}$) generates less welfare than at point B. Since the point B can be induced by choosing $\theta = 0$ (by Lemma 4), the result follows.

Proof of Lemma 6: Consider Figure 6. In Figure 6 we introduce two new curves. The first one is the iso-welfare curve corresponding to welfare level of point B (that is, the welfare level $\overline{W}^{PP}(0)$). The second one is the $S_2^{PP}S_2^{PP'}$ curve which is the locus of $W_2^{PP}(p) = 0$. Point B is Bertrand equilibrium for $\theta = 0$ and, by Lemma 4, this point can be uniquely induced by choosing $\theta = 0$. If the resulting welfare from any strategy associated with Type II equilibrium

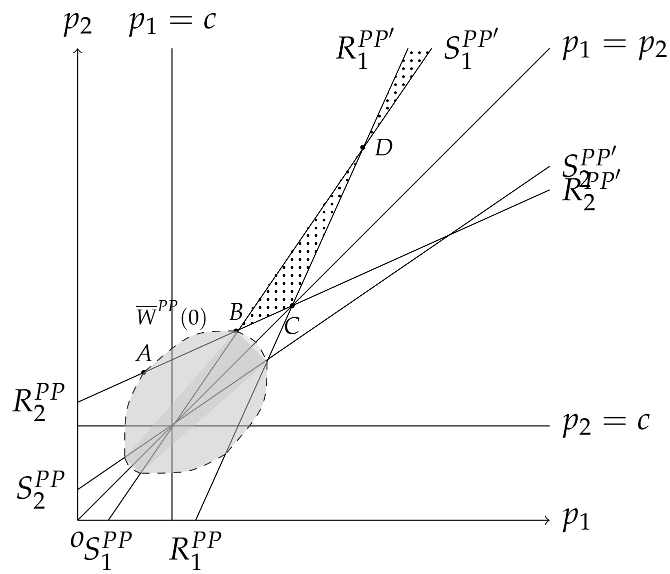

FIGURE 6. Impossibility of Type II equilibrium

yields a welfare less than the welfare level corresponding to point B, then the possibility of Type II equilibrium is ruled out. By Assumption 4 (i), in the regions above and below both $S_1^{PP}S_1^{PP'}$ and $S_2^{PP}S_2^{PP'}$ curves, the iso-welfare curve is upward sloping and in the region lying between these curves, the iso-welfare curve is downward sloping. The Bertrand equilibrium point at $\theta = 0$ (that is, point B) lies on the $S_1^{PP}S_1^{PP'}$ curve and is located above the $S_2^{PP}S_2^{PP'}$ curve. Therefore, to the left of point B the iso-welfare curve is increasing and to the right of point B it is decreasing. Since a consequence of welfare maximization in terms of quantity choice yields $(p_1 = c, p_2 = c)$ as the resulting price vector, it is the global maximum of $W^{PP}(p)$. Therefore, the upper contour set $\Omega_W^{PP} = \{p \mid W^{PP}(p) \geq \overline{W}^{PP}(0)\}$ of B is the region shaded in gray in Figure 6 that always includes point (c, c) as an interior point. When Firm 1 chooses quantity strategy and Firm 2 chooses price strategy, then the reaction function of Firm 1 is the locus of $p_1 = SV_1^{PP}(p_2, \theta)$ lying between the $R_1^{PP}R_1^{PP'}$ and the $S_1^{PP}S_1^{PP'}$ curves. The reaction function of Firm 2 is the locus of the set $\mathcal{P}(\mathcal{R}_2^{PP})$ that lies completely above the $R_2^{PP}R_2^{PP'}$. Therefore, any potential Type II equilibrium point must belong to the region lying between the $R_1^{PP}R_1^{PP'}$ curve and the $S_1^{PP}S_1^{PP'}$ curve and must also lie above the $R_2^{PP}R_2^{PP'}$ as shown in the Figure 6 by the dotted region (where the boundary is not included for the BC segment). Hence, the set in which the Type II equilibrium can occur is $E^{QP} = \{p \mid \pi_{2,2}^{PP}(p) > 0, W_1^{PP}(p).\pi_{1,1}^{PP}(p) \leq 0\}$. Since, due to Assumption 4, the $S_1^{PP}S_1^{PP'}$ curve can never bend back and since the only intersection of the closure of E^{QP} and the Ω_W^{PP} is point B and B is not in E^{QP}, the set Ω_W^{PP} and the set E^{QP} must be disjoint. Hence, for any price vector associated with Type II equilibrium, the resulting welfare is always less than the welfare corresponding to point B. Therefore, Type II equilibrium is ruled out.

Proof of Lemma 7: Consider Figure 9 where in Figure 7 we consider the quantity space and in Figure 8 we consider the price space. In Figure 7, the curves R_1R_1', RC and R_2R_2' corresponds respectively to the function $q_1 = SV_1^{QQ}(q_2, 1)$, $q_1 = SV_1^{QQ}(q_2, 0)$ and $q_2 = R_2^{QQ}(q_1)$. Each curve is negatively sloped and both R_1R_1' and R_1C curves have an absolute slope of more than unity and the R_2R_2' curve has an absolute slope of less than unity. If $\theta = 1$, then firms are symmetric and hence we have $q_1^{QQ}(1) = q_2^{QQ}(1)$. Hence, the intersection point of R_1R_1' and R_2R_2' must lie on the $q_1 = q_2$ line (see point A in Figure 7). For any point on the R_1C curve we have $p_1 = c$ and for any point on the R_1R_1' curve we have $p_1 > c$ excepting at point R_1 where we have $q_1 = 0$ and hence we also have $p_1 = c$. Since by Assumption 1 own effect on indirect demand is negative, the R_1C curve must lie to the right of the R_1R_1' curve . Consider point B (in Figure 7) which is the point of intersection between the R_1C and the R_2R_2' curves. Point B must lie to the right of point A and both A and B are on R_2R_2'. Point B is the Cournot equilibrium vector $(q_1^{QQ}(0), q_2^{QQ}(0))$. Firstly, by Lemma 2, $\frac{\partial q_1^{QQ}}{\partial\theta} < 0$ and $\frac{\partial q_2^{QQ}}{\partial\theta} > 0$. Secondly, one can show that $\frac{\partial q_2^{QQ}}{\partial\theta} = \frac{dR_2^{QQ}}{dq_1}\frac{\partial q_1^{QQ}}{\partial\theta}$ (see the proof of Lemma 2 (i)). Thirdly, for any $\theta \in [0,1]$, the equilibrium point $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ must lie on the R_2R_2' curve. Hence, for all $\theta \in [0,1]$, $(q_1 = q_1^{QQ}(\theta), q_2 = q_2^{QQ}(\theta))$ is the parametric representation of the AB segment of R_2R_2' with A (B) representing the quantity vector corresponding to $\theta = 1$ ($\theta = 0$). As θ varies from 0 to 1 we move from point B to point A along R_2R_2' as shown by the arrows in Figure 7.

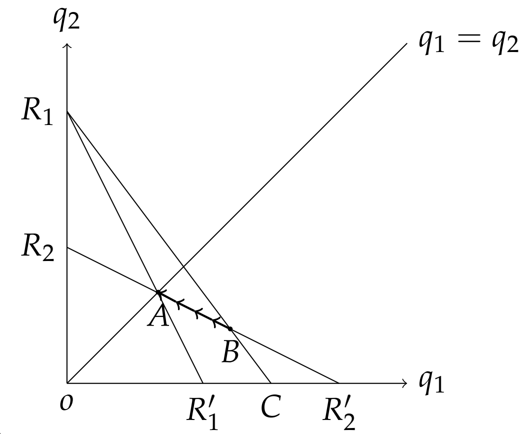

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FIGURE 7. Quantity Space

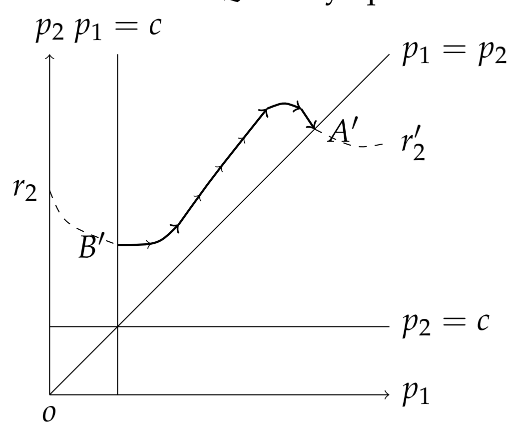

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FIGURE 8. The Price Space

FIGURE 9. Quantity reaction function in the quantity and price space

Consider Figure 8 and let the curve r_2r_2' represent the set $\mathcal{P}(\mathcal{R}_2^{QQ})$. Point A' and B' in Figure 8 correspond to the points A and B respectively of Figure 7. Since for any Cournot equilibrium the resulting price vector must satisfy $p_2 \geq p_1 \geq c$, the segment $B'A'$ must lie between the $p_1 = c$ line and the $p_1 = p_2$ line and above the $R_2^{PP}R_2^{PP'}$ curve (see Cheng [5]). By Assumption 1 and Assumption 2, the Cournot equilibrium quantity vector $(q_1^{QQ}(\theta), q_2^{QQ}(\theta))$ is unique for each θ implying that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ is also unique. Therefore, for the segment $B'A'$, given any p_1 we must get a single p_2 and this segment can be represented as a function $p_2 = r_2^{QQ}(p_1)$ defined for $p_1 \in [c, p_1^{QQ}(1)]$. For each p_1, $r_2^{QQ}(p_1)$ is always well-defined and given continuity of AB segment, the $B'A'$ segment is also continuous. Starting from B' if we move towards A' along the segment $B'A'$, the underlying θ increases since the $B'A'$ segment has a functional representation it cannot be backward bending. Hence, given $\frac{\partial p_1^{QQ}}{\partial \theta} > 0$, $p_1^{QQ}(\theta)$ increases along the segment $B'A'$ when we start from B'.

Proof of Lemma 8: Consider Figure 10. Given any $\theta \in [0,1]$, if $q^{QQ}(\theta)$ is Cournot equilibrium quantity vector, then $q_2^{QQ}(\theta) = R_2^{QQ}(q_1^{QQ}(\theta))$ and $q_1^{QQ}(\theta) = SV_1^{QQ}(q_2^{QQ}(\theta), \theta)$ and the resulting price of Firm i is $p_i^{QQ}(\theta) = F_i^{QQ}(q^{QQ}(\theta))$ implying that the price vector $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in \mathcal{P}(\mathcal{R}_2^{QQ}) \cap \mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$. The graph $\mathcal{P}(\mathcal{R}_2^{QQ})$ must lie above R_2^{PP} in the price space and $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$ is bounded between $p_1 = c$ and the graph $\mathcal{P}(\mathcal{SV}_1^{QQ}(1))$. Again, since the firms face identical demand and cost conditions, the Cournot equilibrium price vector must lie in $E^{QQ} = \{p \mid \pi_{1,1}^{PP}(p) > 0,\ p_1 > c \text{ and } p_2 \geq p_1\}$. Therefore, the region $\mathcal{A}$ in Figure 10 represents the set $E^{QQ} \cap \Omega_W^{PP}$. This region $\mathcal{A}$ represents the set of points where Cournot equilibrium can occur and resulting welfare is higher compared to point B. If $\mathcal{P}(\mathcal{R}_2^{QQ}) \cap \Omega_W^{PP} = \emptyset$, then, in Stage 1, the government's optimal choice of θ can never induce Cournot competition since, by choosing $\theta = 0$, the government can improve the level of welfare.

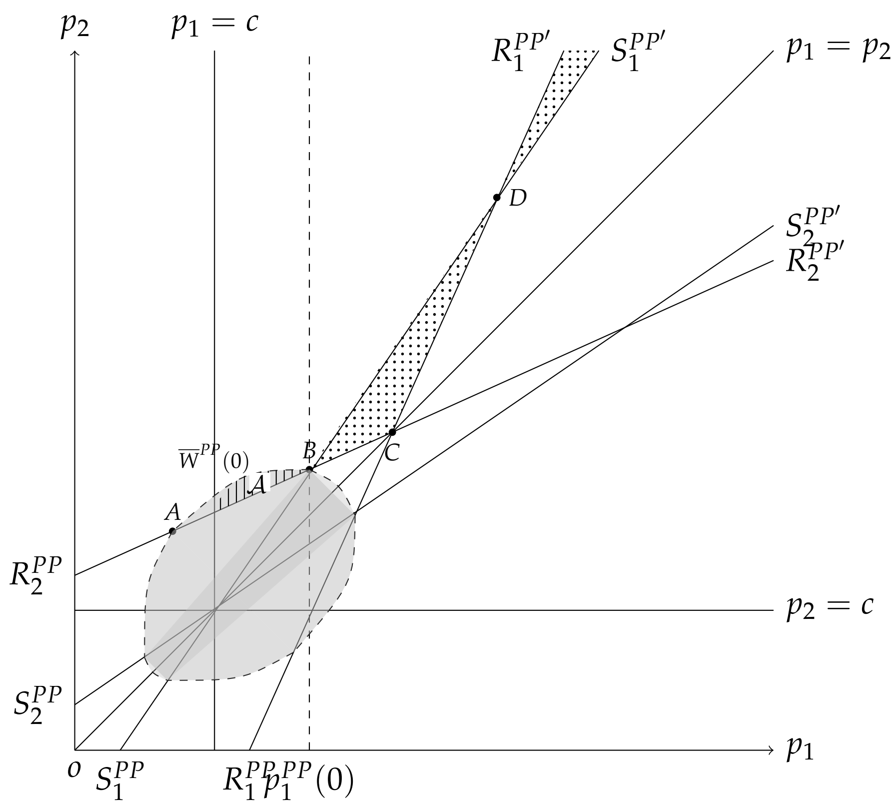

FIGURE 10. Case 1

If $\mathcal{P}(\mathcal{R}_2^{QQ}) \cap \Omega_W^{PP} \neq \emptyset$, then can the government induce quantity competition by choosing θ in such a way that the resulting price vector $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in E^{QQ} \cap \Omega_W^{PP}$? Consider the sets $E_{\geq}^{QQ} = \{p \mid p \in E^{QQ} \text{ and } p_1 \geq p_1^{PP}(0)\}$ and $E_{<}^{QQ} = \{p \mid p \in E^{QQ} \text{ and } p_1 < p_1^{PP}(0)\}$. Observe that $E_{\geq}^{QQ} \cap E_{<}^{QQ} = \emptyset$ and $E_{\geq}^{QQ} \cup E_{<}^{QQ} = E^{QQ}$. We consider two exhaustive cases.

Case 1: $E^{QP} \cap E_{<}^{QQ} = \emptyset$.

Case 2: $E^{QP} \cap E_{<}^{QQ} \neq \emptyset$

For Case 1, E^{QP} lies to the right of the vertical line $p_1 = p_1^{PP}(0)$ (see Figure 10). Since $E^{QP} \subset E^{QQ}$, we must have $E^{QP} \subset E^{QQ}_{\geq}$. Given $E^{QP} \cap \mathcal{P}(\mathcal{R}_2^{QQ}) \neq \emptyset$, $E^{QP} \cap \Omega_W^{PP} = \emptyset$ (by Lemma 6) and the continuity of the graph of $\mathcal{P}(\mathcal{R}_2^{QQ})$ in the price plane, there exists exactly one compact set $S_P(\subset \mathcal{P}(\mathcal{R}_2^{QQ}))$ such that (a) the interior of S_P is contained in the complement set of $E^{QP} \cap \Omega_W^{PP}$, (b) we can find (p_1, p_2) in the intersection of the boundaries of the sets S_P and Ω_W^{PP}, and, (c) we can find another (p_1, p_2) in the intersection of the boundaries of the sets S_P and E^{QP}. Using Lemma 7 we can now say that each θ for which $(p_1^{QQ}(\theta), p_2^{QQ}(\theta))$ in the interior of S_P is higher compared to every θ such that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in \Omega_W^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$ and is lower compared to every θ such that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in E^{QP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$. By Lemma 3 (iii), $(p_1^{PP}(\theta_4), p_2^{PP}(\theta_4)) \in E^{QP}$. Hence, for every θ such that $(p_1^{QQ}(\theta), p_2^{QQ}(\theta)) \in \Omega_W^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$, $\theta < \theta_4$. Thus, it is impossible for the government to induce Cournot competition by choosing θ such that resulting price vector belongs to $\Omega_W^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$.

For Case 2, the entire E^{QP} does not lie to the right of the vertical line $p_1 = p_1^{PP}(0)$ (see Figure 11). Consider the set $E^{QP}_{<} = \{(p_1, p_2) \mid (p_1, p_2) \in E^{QP}, p_1 < p_1^{PP}(0)\}$. If $E^{QP}_{<} \cap \mathcal{P}(\mathcal{R}_2^{QQ}) = \emptyset$, then the analysis is similar to Case 1 and Cournot competition cannot be sustained. Finally, if $E^{QP}_{<} \cap \mathcal{P}(\mathcal{R}_2^{QQ}) \neq \emptyset$, then given $\mathcal{P}(R_2^{QQ}) \cap \Omega_W^{PP} \neq \emptyset$, $\Omega_W^{PP} \cap E^{QP} = \emptyset$ and continuity of the graph of $\mathcal{P}(\mathcal{R}_2^{QQ})$ in the price plane, we can find at least one $S^P \subset \mathcal{P}(\mathcal{R}_2^{QQ})$ for which we have three mutually exclusive sets S^{Pa}, S^{Pb} and S^{Pc} such that $S^{Pa} \cup S^{Pb} \cup S^{Pc} = S^P$, $S^{Pa} \subset E^{QP}$, $S^{Pb} \subset \Re^2_{++} \setminus \{\Omega_W^{PP} \cup E^{QP}\}$ and $S^{Pc} \subset \Omega_W^{PP}$. Assume that there are M such S^Ps'. Denote a representative S^P as S_m^P where $m \in \{1, 2, ..., M\}$. Therefore, for each S_m^P we have a set $S_m^{P'} \subset E^{QP}$. Can we find $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$? The following argument shows that the answer is no. By Lemma 7, along the graph of the set S_m^{Pa} in the price plane, p_1 is increasing (along the segment $B'A'$ in Figure 8) and it must contain at least two points in the boundary of E^{QP} each of which corresponds to Type I equilibrium price vector for $\theta = 0$. Along the graph S_m^{Pa}, the behavior of $p_1^{QP}(\theta)$ is shown in Figure 12. Suppose $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$. By Lemma 3 (ii) θ_4 is unique and by Lemma 2, $p_1^{QQ}(\theta)$ is increasing in θ. Therefore $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4))$ is unique. Hence, if $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$, then there dose not exist any $k \in \{1, 2, ..., M\}$ with $k \neq m$ such that $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_k^{Pa}$.

Let $\overline{OT}$ denote the length of the OT segment in Figure 12. Given $(p_1^{QQ}(\theta_4), p_2^{QQ}(\theta_4)) \in S_m^{Pa}$, $\theta_4 > \overline{OT}$ is not possible. If $\theta_4 = \overline{OT}$, then for $p_1^{QQ}(\theta) = p_1^{QP}(\theta)$ at $\theta = \theta_4$ either $p_1^{QQ}(\theta)$ has slope of ∞ at θ_4 (which is impossible since $V_{1,11}^{QQ}\pi_{2,22}^{QQ} - V_{1,12}^{QQ}\pi_{2,12}^{QQ} \neq 0$) or $p_1^{QQ}(\theta)$ should intersect $p_1^{QP}(\theta)$ twice which is again a contradiction due to uniqueness of θ_4 (see Lemma 3 (iii)). If $\theta_4 < \overline{OT}$, then (given $p_1^{QQ}(\theta) = p_1^{QP}(\theta)$ holds for at most one θ) the only possibility is that $p_1^{QQ}(\theta)$ is tangent to the lower segment of $p_1^{QP}(\theta)$ at $\theta = \theta_4$ which is again a contradiction since, in that case, we can find at least one $\theta > \theta_4$ such that $p_1^{QP}(\theta) > p_1^{QQ}(\theta)$.

Proof of Proposition 1: We use four steps to prove the result.
Step (i): The value of θ that maximizes $\overline{W}^{PP}(\theta)$ must belongs to $(0, 1)$.
Proof of Step (i): The first order condition of Stage 1 under the assumption that firms select price strategy in Stage 2 is given by

$$\frac{\partial \overline{W}^{PP}}{\partial \theta} = W_1^{PP}(p^{PP}(\theta))\frac{\partial p_1^{PP}(\theta)}{\partial \theta} + W_2^{PP}(p^{PP}(\theta))\frac{\partial p_2^{PP}(\theta)}{\partial \theta}. \tag{24}$$

Like Lemma 2, when goods are complement one can show that $\frac{\partial p_1^{PP}(\theta)}{\partial \theta} > 0$, $\frac{\partial p_2^{PP}(\theta)}{\partial \theta} < 0$, and, $\frac{\partial p_2^{PP}(\theta)}{\partial \theta} = \frac{dR_2^{PP}(p_1)}{dp_1}\frac{\partial p_1^{PP}(\theta)}{\partial \theta}$. Therefore, from condition (24) we get,

$$\frac{\partial \overline{W}^{PP}}{\partial \theta} = \left(W_1^{PP}(p^{PP}(\theta)) + \frac{dR_2^{PP}}{dp_1}W_2^{PP}(p^{PP}(\theta))\right)\frac{\partial p_1^{PP}(\theta)}{\partial \theta}. \tag{25}$$

At $\theta = 0$, the price vector $(p_1^{PP}(0), p_2^{PP}(0))$ corresponds to point A in the Figure 2. At A $W_1^{PP}(p_1^{PP}(0), p_2^{PP}(0)) = 0$ (since, point A must lie on the S_1S_1' curve), $W_2^{PP}(p_1^{PP}(0), p_2^{PP}(0)) < 0$ (since, point A must lie above PP') and, by

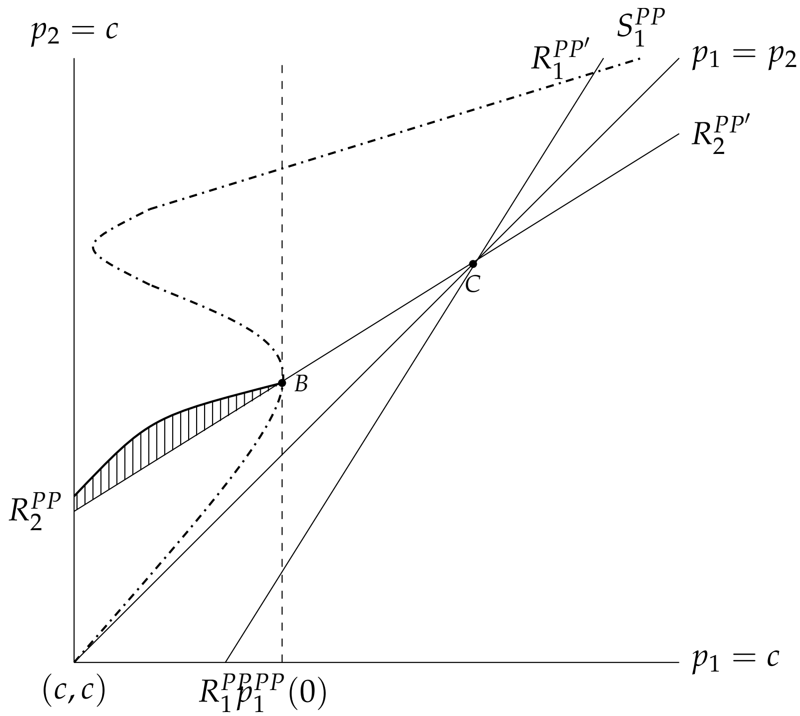

FIGURE 11. Case 2a

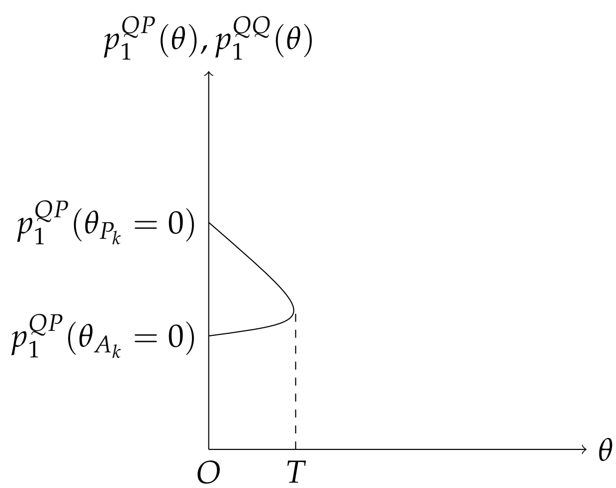

FIGURE 12. Case 2b

Assumption 7, we also have $(dR_2^{PP}(p_1)/dp_1) < 0$. Hence, at $\theta = 0$, $\frac{\partial \overline{W}^{PP}}{\partial \theta} = \frac{d\overline{W}^{PP}}{d\theta} > 0$. At $\theta = 1$, the price vector $(p_1^{PP}(1), p_2^{PP}(1))$ corresponds to point B in the Figure 2 where we have $W_1^{PP}(p_1^{PP}(1), p_2^{PP}(1)) < 0$ (since point B must lie to the right of S_1S_1'), $W_2^{PP}(p_1^{PP}(1), p_2^{PP}(1)) < 0$ (since point B must lie above S_2S_2'), $W_1^{PP}(p_1^{PP}(1), p_2^{PP}(1)) = W_2^{PP}(p_1^{PP}(1), p_2^{PP}(1))$ (since point B must lie on $p_1 = p_2$ line and the welfare function is symmetric) and (applying Assumption 7) we also have $-1 < (dR_2^{PP}(p_1)/dp_1) < 0$. Thus, at $\theta = 1$, $\frac{\partial \overline{W}^{PP}}{\partial \theta} = \frac{d\overline{W}^{PP}}{d\theta} < 0$. Given $\frac{d\overline{W}^{PP}}{d\theta} > 0$ at $\theta = 0$ and $\frac{d\overline{W}^{PP}}{d\theta} < 0$ at $\theta = 1$, and, given the second order condition $\frac{d^2\overline{W}^{PP}}{d\theta^2} < 0$, it follows that the optimal stage 1 choice of θ is some θ^* that lies in the open interval $(0, 1)$. Hence, at $\theta = \theta^*$ the equilibrium price vector $(p_1^{PP}(\theta^*), p_2^{PP}(\theta^*))$ must belong to the interior of the segment AB (say some point like D in Figure 2) where the iso-welfare curve is tangent to the R_2R_2' curve.

Step (ii): On the (p_1, p_2) plane, for any given $\theta \in [0,1]$, all points in $\mathcal{SV}_1^{PP}(\theta)$ must lie to the left all points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$.

Proof of Step (ii): In Stage 2, if Firm 2 chooses price strategy, then Firm 1's reaction function is given by

$$V_{1,1}^{PP}(p,\theta) = (p_1 - c)F_{1,1}^{PP}(p) + \theta F_1^{PP}(p) + (1-\theta)F_{2,1}^{PP}(p) = 0. \tag{26}$$

In Stage 2, if Firm 2 chooses quantity strategy, then Firm 1's reaction function is

$$V_{1,1}^{QQ}(q,\theta) = F_1^{QQ}(q) - c + \theta q_1 F_{1,1}^{QQ}(q) = 0. \tag{27}$$

If $(\hat{p}_1, \hat{p}_2)$ is a solution to (27), then $\hat{p}_1 - c + \theta F_1^{PP}(\hat{p}_1, \hat{p}_2) F_{1,1}^{QQ}(F_1^{PP}(\hat{p}_1, \hat{p}_2), F_2^{PP}(\hat{p}_1, \hat{p}_2)) = 0$ implying that $\hat{p}_1 - c = -\theta F_1^{PP}(\hat{p}_1, \hat{p}_2) F_{1,1}^{QQ}(F_1^{PP}(\hat{p}_1, \hat{p}_2), F_2^{PP}(\hat{p}_1, \hat{p}_2))$. At $(\hat{p}_1, \hat{p}_2)$, $V_{1,1}^{PP}(\hat{p}_1, \hat{p}_2, \theta) = (\hat{p}_1 - c)F_{1,1}^{PP} + \theta F_1^{PP} + (1-\theta)F_{2,1}^{PP} = \theta \left(1 - F_{1,1}^{PP} F_{1,1}^{QQ}\right) F_1^{PP} + (1-\theta)F_{2,1}^{PP} < 0$. Therefore, given any $\theta \in [0,1]$, we have $SV_1^{PP}(\hat{p}_2, \theta) < \hat{p}_1$, that is, all points satisfying $p_1 = SV_1^{PP}(p_2, \theta)$ must lie to the left of all points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$.

Step (iii): In Stage 2, choosing price is the dominant strategy for Firm 2.

Proof of Step (iii): When Firm 1 chooses price strategy, the curve R_2R_2' is the reaction function of Firm 2. Therefore, the singleton set $\mathcal{SV}_1^{PP} \cap \mathcal{R}_2^{PP}$ must lie to the left of all points in the set $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta)) \cap \mathcal{R}_2^{PP}$. Therefore, at any given θ, if Firm 1 chooses price strategy, then it is always optimal for Firm 2 to choose price strategy. When Firm 1 chooses quantity strategy, the set of points $\mathcal{P}(\mathcal{R}_2^{QQ})$ represent the reaction function of Firm 2 in terms of prices. Again, like Lemma 7, one can show that if we generate the Cournot equilibrium path in the price space by changing θ from 0 to 1 and plotting the corresponding price vector, then, along that Cournot equilibrium path, as we move from price vector $(p_1^{QQ}(0), p_2^{QQ}(0))$ to price vector $(p_1^{QQ}(1), p_2^{QQ}(1))$ the underlying θ increases. Like Lemma 2(ii), one can also show that $\frac{\partial p_1^{QQ}}{\partial \theta} > 0$. Hence, along that Cournot equilibrium path, p_1 also increases. By Assumption 5, $\frac{\partial \pi_2^{QQ}}{\partial q_1} > 0$ and $F_{1,1}^{PP} < 0$ implying $d\pi_2^{QQ}(F_1^{PP}(p), R_2^{QQ}(F_1^{PP}(p)))/dp_1 = \frac{\partial \pi_2^{QQ}}{\partial q_1} F_{1,1}^{PP} < 0$. Therefore, along the Cournot equilibrium path, the profit of the Firm 2 decreases as we move from $(p_1^{QQ}(0), p_2^{QQ}(0))$ to $(p_1^{QQ}(1), p_2^{QQ}(1))$. Since, by Step (ii) for any θ the set of point $\mathcal{SV}_1^{PP}(\theta)$ lie to the left of the set of points in $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta))$, the profit associated with the point in the singleton set $\mathcal{P}(\mathcal{SV}_1^{QQ}(\theta)) \cap \mathcal{P}(\mathcal{R}_2^{QQ})$ is less than profits from all point in the set $\mathcal{SV}_1^{PP}(\theta) \cap \mathcal{P}(\mathcal{R}_2^{QQ})$. Therefore, at any given θ, if Firm 1 chooses quantity strategy, then also it is optimal for Firm 2 to choose price strategy.

Step (iv): In Stage 2, if Firm 2 chooses price strategy, then Firm 1 also chooses price strategy.

Proof of Step (iv): On the region lying above the set $\mathcal{T} := \{p \mid p_2 \geq c, p_1 + p_2 \geq 2c\}$, $V_{1,2}^{PP}(p,\theta) = \theta \pi_{1,2}^{PP}(p) + (1-\theta)W_2^{PP}(p) < 0$. Therefore, $V_1^{PP}(p,\theta)$ is decreasing in p_2 for all points in the set $\mathcal{SV}_1^{PP}(\theta) \cap \mathcal{T}$. Again, the reaction function of Firm 2 given Firm 1 chooses price (that is, the R_2R_2' curve in Figure 2) lies above the line $p_2 = c$ and each point in this reaction function lies below all points in the set $\mathcal{P}(\mathcal{R}_2^{QQ})$. Moreover, all points on the AB segment in Figure 2 is contained in $\mathcal{T}$ and for all points on the AB segment we have $dV_1^{PP}(SV_1^{PP}(p_2,\theta), p_2)/dp_2 < 0$. Therefore, at the intersection point of the R_2R_2' curve and the $p_1 = SV_1^{PP}(p_2,\theta)$ curve, we value of $V_1^{PP}(p,\theta)$ is higher compared to all points in the set $\mathcal{SV}_1^{PP} \cap \mathcal{P}(\mathcal{R}_2^{QQ})$. Hence, given Firm 2 chooses price strategy, it is always optimal for Firm 1 to choose price strategy. Hence, Step (iv) follows.

Step (iii) and Step (iv) shows that given any $\theta \in [0,1]$, price competition is the only Nash equilibrium of the sub-game starting from Stage 2. By Step (i), at some $\theta^*(\in (0,1))$, the government maximizes welfare under price competition. Therefore, the strategy combination $(\theta^{PP} = \theta^*, (P, p_1^{PP}(\theta^{PP})), (P, p_2^{PP}(\theta^{PP})))$ is the unique SPNE of Γ.

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