Triangle room paradox of negative probabilities of events

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Abstract. *Here an improved generalization of Feynman's paradox of negative probabilities [1, 2] for observing three events is considered. This version of the paradox is directly related to the theory of quantum computing. Imagine a triangular room with three windows (see Fig.* **??***), where there are three chairs, on each of which a person can seat [4]. In any of the windows, an observer can see only the corresponding pair of chairs. It is known that if the observer looks at a window (to make a pair observation), the picture will be in the probabilistic sense the same for all windows: only one chair from the observed pair is occupied with a probability of 1/2, and there are never busy or free both chairs at once. Paradoxically, existing theories based on Kolmogorov's probability theory do not answer the question that naturally arises after such pairs of observations of three events: «What is really happening in a triangular room, how many people are there and with what is the probability distribution they are sitting on three chairs?».*

Keywords. *Eventology, event, probability, triangle room paradox of negative probabilities, quantum computing, event as a superposition of two states.*

The formulation of the triangular room paradox of negative probabilities (see Fig. 1). Let $(\Omega, \mathcal{A}, \mathbf{P})$ be a probability space. The triplet of events $\mathfrak{X} = \{x, y, z\} \subseteq \mathcal{A}$ unobserved completely, but having three observable «pair projections», three doublets of events $\{x, y\}, \{x, z\}$, and $\{y, z\}$ obeying the probability distributions from the table 1, can have only a *phantom (containing negative probabilities)* probability distribution the general form of which is indicated in the table 2 and in the diagram 2.

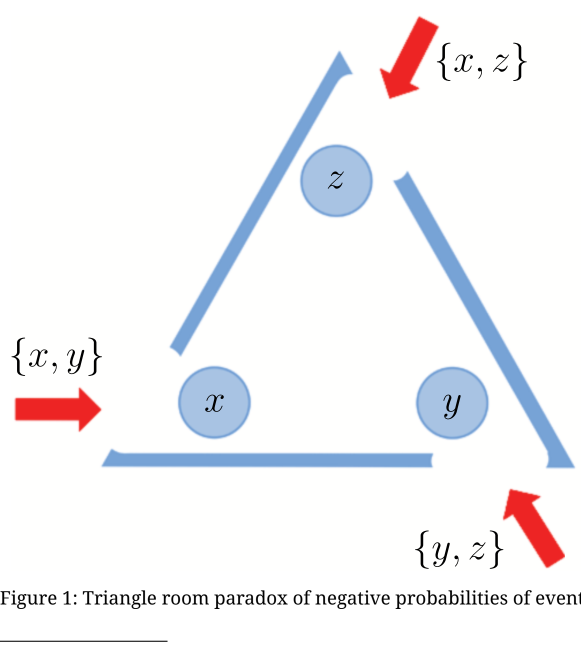

Figure 1: Triangle room paradox of negative probabilities of events.

Oleg Vorobyev (ed.), Proc. XV FAMEMS'2016, Krasnoyarsk: SFU

Proof of the phantomcity of events in a triangular room. Since from the form of the probability distributions of the observed doublets it follows that, firstly,

$$p(\emptyset /\!\!/ \mathfrak{X}) + p(\{z\} /\!\!/ \mathfrak{X}) = p(\emptyset /\!\!/ \{x, y\}) = 0, \tag{1}$$

$$p(\emptyset /\!\!/ \mathfrak{X}) + p(\{y\} /\!\!/ \mathfrak{X}) = p(\emptyset /\!\!/ \{x, z\}) = 0, \tag{2}$$

$$p(\emptyset /\!\!/ \mathfrak{X}) + p(\{x\} /\!\!/ \mathfrak{X}) = p(\emptyset /\!\!/ \{y, z\}) = 0, \tag{3}$$

secondly,

$$p(\mathfrak{X} /\!\!/ \mathfrak{X}) + p(\{x, y\} /\!\!/ \mathfrak{X}) = p(\{x, y\} /\!\!/ \{x, y\}) = 0, \tag{4}$$

$$p(\mathfrak{X} /\!\!/ \mathfrak{X}) + p(\{x, z\} /\!\!/ \mathfrak{X}) = p(\{x, z\} /\!\!/ \{x, z\}) = 0, \tag{5}$$

$$p(\mathfrak{X} /\!\!/ \mathfrak{X}) + p(\{y, z\} /\!\!/ \mathfrak{X}) = p(\{x, z\} /\!\!/ \{y, z\}) = 0, \tag{6}$$

then if one does not resolve the probabilities from the triplet distribution to be less than zero, then all eight probabilities must be zero, which contradicts the normalization of the probability[1].

X	$\mathbf{1}_x$	$\mathbf{1}_y$	$p(X /\!\!/ \{x, y\})$	$p(X /\!\!/ \{x, y\})$
$\emptyset$	0	0	$p(\emptyset /\!\!/ \{x, y\})$	0
$\{x\}$	1	0	$p(\{x\} /\!\!/ \{x, y\})$	1/2
$\{y\}$	0	1	$p(\{y\} /\!\!/ \{x, y\})$	1/2
$\{x, y\}$	1	1	$p(\{x, y\} /\!\!/ \{x, y\})$	0
			$\sum_{X \subseteq \{x, y\}} p(X /\!\!/ \{x, y\}) = 1$	1

X	$\mathbf{1}_x$	$\mathbf{1}_z$	$p(X /\!\!/ \{x, z\})$	$p(X /\!\!/ \{x, z\})$
$\emptyset$	0	0	$p(\emptyset /\!\!/ \{x, z\})$	0
$\{x\}$	1	0	$p(\{x\} /\!\!/ \{x, z\})$	1/2
$\{z\}$	0	1	$p(\{z\} /\!\!/ \{x, z\})$	1/2
$\{x, z\}$	1	1	$p(\{x, z\} /\!\!/ \{x, z\})$	0
			$\sum_{X \subseteq \{x, z\}} p(X /\!\!/ \{x, y\}) = 1$	1

X	$\mathbf{1}_y$	$\mathbf{1}_z$	$p(X /\!\!/ \{y, z\})$	$p(X /\!\!/ \{y, z\})$
$\emptyset$	0	0	$p(\emptyset /\!\!/ \{y, z\})$	0
$\{y\}$	1	0	$p(\{y\} /\!\!/ \{y, z\})$	1/2
$\{z\}$	0	1	$p(\{z\} /\!\!/ \{y, z\})$	1/2
$\{y, z\}$	1	1	$p(\{y, z\} /\!\!/ \{y, z\})$	0
			$\sum_{X \subseteq \{y, z\}} p(X /\!\!/ \{y, z\}) = 1$	1

Table 1: Venn tables of probability distributions of three ***observed*** doublets of events $\{x, y\}, \{x, z\}$ и $\{y, z\}$ (from top to down) from the triangle room paradox of negative probabilities.

X	$\mathbf{1}_x$	$\mathbf{1}_y$	$\mathbf{1}_z$	$p(X /\!\!/ \mathfrak{X})$	$f(X /\!\!/ \mathfrak{X})$
$\emptyset$	0	0	0	$p(\emptyset /\!\!/ \mathfrak{X})$	$-p$
$\{x\}$	1	0	0	$p(\{x\} /\!\!/ \mathfrak{X})$	p
$\{y\}$	0	1	0	$p(\{y\} /\!\!/ \mathfrak{X})$	p
$\{z\}$	0	0	1	$p(\{z\} /\!\!/ \mathfrak{X})$	p
$\{x, y\}$	1	1	0	$p(\{x, y\} /\!\!/ \mathfrak{X})$	q
$\{x, z\}$	1	0	1	$p(\{x, z\} /\!\!/ \mathfrak{X})$	q
$\{y, z\}$	0	1	1	$p(\{y, z\} /\!\!/ \mathfrak{X})$	q
$\mathfrak{X}$	1	1	1	$p(\mathfrak{X} /\!\!/ \mathfrak{X})$	$-q$
				$\sum_{X \subseteq \mathfrak{X}} p(X /\!\!/ \mathfrak{X}) = 1$	$2(p + q) = 1$

Table 2: Venn table of ***unobserved*** phantom probability distributions of triplet of events $\mathfrak{X} = \{x, y, z\}$ from the triangle room paradox with two negative probabilities.

[1]Here we use the following abbreviation: $p(X /\!\!/ \mathfrak{X}) = \mathbf{P}\Big(\mathrm{ter}(X /\!\!/ \mathfrak{X})\Big) = \mathbf{P}\left(\bigcap_{x \in X} x \bigcap_{x \in \mathfrak{X} - X} x^c\right)$, where $x^c = \Omega - x$.

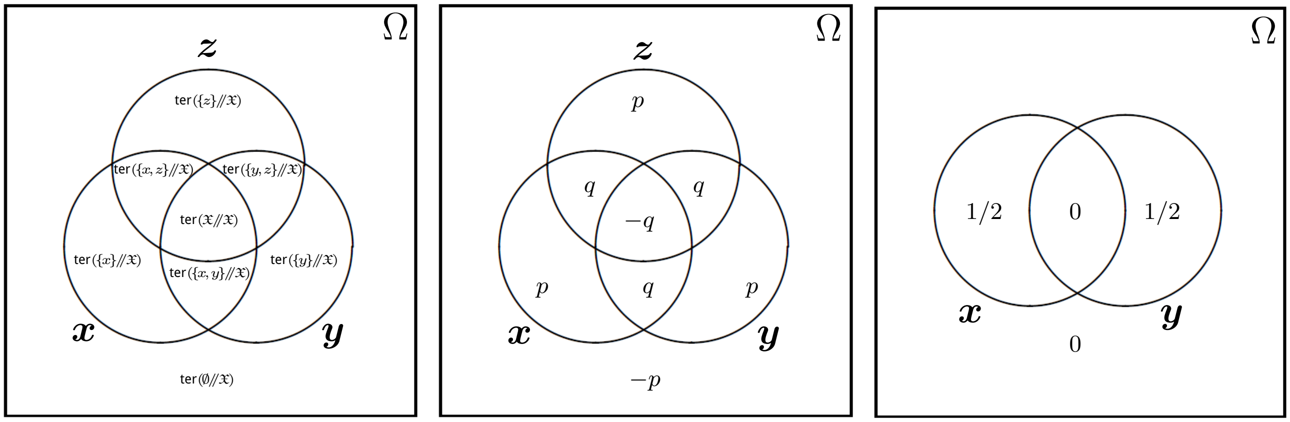

Figure 2: Venn diagram of the ***unobserved*** phantom triplet events $\mathfrak{X} = \{x, y, z\}$ (center); one of three its pair projections: the ***observed*** doublet of events $\{x, y\}$ (right); and a terraced labelling the partition of the space Ω by the triplet of events $\mathfrak{X} = \{x, y, z\}$ (left).

If we allow negative probabilities, i.e. for $p > 0, q > 0$ we set

$$p(\emptyset /\!\!/ \mathfrak{X}) = -p,$$

$$p(\mathfrak{X} /\!\!/ \mathfrak{X}) = -q,$$

then the solutions of the paradox will be the phantom probability distributions of the triplet $\mathfrak{X} = \{x, y, z\}$ of the general form indicated in the table 2, where

$$\mathbf{P}(x) = \mathbf{P}(y) = \mathbf{P}(z) = p + q = 1/2$$

are probabilities of events of the triplet, which follows from the probability normalization:

$$\sum_{X \subseteq \mathfrak{X}} p(X /\!\!/ \mathfrak{X}) = 2(p + q) = 1.$$

Note that this phantom probability distribution of general form satisfies all 12 equations that connect the triplet $\mathfrak{X} = \{x, y, z\}$ with its three pair projections $\{x, y\}, \{x, z\}$ and $\{y, z\}$, that is, the phantom distribution satisfies six more equations:

$$p(\{x\} /\!\!/ \{x, y\}) = p(\{x\} /\!\!/ \mathfrak{X}) + p(\{x, z\} /\!\!/ \mathfrak{X}) = p + q = 1/2, \tag{7}$$

$$p(\{z\} /\!\!/ \{x, z\}) = p(\{z\} /\!\!/ \mathfrak{X}) + p(\{y, z\} /\!\!/ \mathfrak{X}) = p + q = 1/2, \tag{8}$$

$$p(\{y\} /\!\!/ \{y, z\}) = p(\{y\} /\!\!/ \mathfrak{X}) + p(\{x, y\} /\!\!/ \mathfrak{X}) = p + q = 1/2, \tag{9}$$

$$p(\{y\} /\!\!/ \{x, y\}) = p(\{y\} /\!\!/ \mathfrak{X}) + p(\{y, z\} /\!\!/ \mathfrak{X}) = p + q = 1/2, \tag{10}$$

$$p(\{x\} /\!\!/ \{x, z\}) = p(\{x\} /\!\!/ \mathfrak{X}) + p(\{x, y\} /\!\!/ \mathfrak{X}) = p + q = 1/2, \tag{11}$$

$$p(\{z\} /\!\!/ \{y, z\}) = p(\{z\} /\!\!/ \mathfrak{X}) + p(\{x, z\} /\!\!/ \mathfrak{X}) = p + q = 1/2. \tag{12}$$

Conclusions and warnings. The paradox of negative probabilities of events in a triangular room, which serves as an irrefutable generalization of the Feynman paradox of negative probabilities [1, 2, 1982,1987] and is directly related to the theory of quantum computing[2], generates the next far-reaching warning for all observers. If we agree with Wittgenstein [8, 1921] and Russell [3, 1946], it is not difficult to understand that we live not in the world of material things, but in the world of events that happen, or do not happen with certain probabilities. All we were born as observers. Any observation is the observation of some set of events. From the paradox of the negative probabilities of events in a triangular room, it follows that even the observation of all without an exception pairs of events from the observed set of events can not give the observer a complete picture of what happens when the observed set of events happens. It can not give because the desired complete picture simply does not have a consistent mathematical language description (probability distribution) within the framework of existing theories of observing the set of events, but has every chance of obtaining such a description within the framework of my new *theory of experience and of chance* [7, 6, 5].

[2]If we consider the bits $1 = \mathbf{1}_x(\omega)$ and $0 = \mathbf{1}_{x^c}(\omega)$ as the results of observations of the event $x \subseteq \Omega$, and the qubits — as the event $x \subseteq \Omega$ itself with a probability distribution $(p_x, 1 - p_x)$, where $p_x = \mathbf{P}(x), 1 - p_x = \mathbf{P}(x^c)$, i.e. as the superposition of two possibilities: x and x^c.

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