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Vorobyev, Oleg Yu.

Siberian Federal University, Institute of Mathematics and Computer  
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# The bet on a bald

**Oleg Yu. Vorobyev**

Institute of mathematics and computer science  
Siberian Federal University  
Krasnoyarsk  
<mailto:oleg.yu.vorobyev@gmail.com>  
<http://www.sfu-kras.academia.edu/OlegVorobyev>  
<http://olegvorobyev.academia.edu>

**Abstract.** *A fixed company of players observes a person selected from a fixed queue. After each observation, players are asked to bet the dollar secret from others, either on the fact that person is bald, or on what is not. A definite formula of the gain is suggested, such that every time **after bets** the gain of each player from a given company are completely determined by this formula. However, **before bets** player's gain is an uncertain value. Is it possible for a given company of players and a given queue of people **before bets** to build a correct mathematical model of uncertain gain of each player within the framework of Kolmogorov's probability theory? If not, what else do you need to add to the foundations of probability theory so that **before bets** to be able to use this model for decision making? The paper answers these questions within the framework of the new theory of experience and of chance (the certainty theory) [1] that consists of two dual halves: the believability theory and the probability theory, and that is intended for the mathematical description of experienced-random experiments, the uncertainty in outcomes of which is generated by the observer.*

**Keywords.** *Eventology, event, co~event, probability, believability, certainty, theory of experience and of chance, certainty theory, bet on bald.*

## 1 Formulation of the problem



A fixed company of players observes a person selected from a fixed queue.

After each observation, players are asked to bet the dollar secret from others, either on the fact that the person is bald, or on what is not. If, as a result of the bets made, the player's choice is in the minority, the player loses the bet. If, on the other hand, in the majority, the player is returned his bet plus an equal share of the loser players' bets. With a draw, the equality of the number of betting on the bald and the number of betting on the not bald, all players remain at their own: they are returned their bets<sup>1</sup>. Every time after the bets, the gain of each player from this company is fully certain. But before the bets, the gain is an uncertain value.

Is it possible for a given company of players and a given queue of persons *before bets* to build a correct mathematical model of uncertain gain of each player within the framework of Kolmogorov's probability theory?

If not, what else do you need to add to the foundations of probability theory so that *before bets* to be able to use the correct mathematical model for decision making under uncertainty?

<sup>1</sup>If, without loss of generality, consider that a company consists of an odd number of players, a draw situation can be ignored.

The negative answer to the first question and the meaningful answer to the second one follow from the new theory of experience and of chance, or *the certainty theory* [1], which is postulated as a mathematical theory for a describing the outcomes of the *experienced-random experiment*, defined as the Cartesian product of experienced and random experiments, and is a «product» of the dual halves: *the believability theory* and *the probability theory*. From the point of view of this «product» theories, the problem «*the bet on a bald*» describes an experienced-random experiment in which a fixed company of observer-players conducts observations of a person selected by one from the fixed queue until the queue is exhausted. What occurs in this experienced-random experiment, the new theory calls *co~event*  $\mathcal{R} \subseteq \langle \Omega | \Omega \rangle$ , which is defined as a measurable binary relation on the Cartesian product  $\langle \Omega | \Omega \rangle = \langle \Omega | \times | \Omega \rangle$  of *the set of elementary incomes (the bra-set)*  $\langle \Omega |$  and *the set of elementary outcomes (the ket-set)*  $| \Omega \rangle$  within the framework of the *certainty space (the bracket-space)*  $\langle \Omega, \mathcal{A}, \mathbf{B} | \Omega, \mathcal{A}, \mathbf{P} \rangle = \langle \Omega, \mathcal{A}, \mathbf{B} | \times | \Omega, \mathcal{A}, \mathbf{P} \rangle = (\langle \Omega | \Omega \rangle, \langle \mathcal{A} | \mathcal{A} \rangle, \Phi)$ , where  $\langle \mathcal{A} | \mathcal{A} \rangle$  is the sigma-algebra of subsets of  $\langle \Omega | \Omega \rangle$ , and  $\Phi$  is a certainty measure on  $\langle \mathcal{A} | \mathcal{A} \rangle$ . In the certainty theory each co~event  $\mathcal{R}$  generates its own *element-set labelling* of the set  $\langle \Omega | \Omega \rangle$  of elementary incomes-outcomes in the form  $\langle \mathfrak{X}_{\mathcal{R}} | \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \rangle = \langle \mathfrak{X}_{\mathcal{R}} | \times | \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \rangle$ , where  $\langle \mathfrak{X}_{\mathcal{R}} | \subseteq \langle \mathcal{A} |$  is the set of bra-events  $\langle x | \subseteq \langle \Omega |, x \in \mathfrak{X}_{\mathcal{R}}$ , and  $| \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \rangle \subseteq | \mathcal{A} \rangle$  is the set of terraced ket-events  $|\text{ter}(X // \mathfrak{X}_{\mathcal{R}}) \rangle \subseteq | \Omega \rangle, X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \subseteq \mathcal{P}(\mathfrak{X}_{\mathcal{R}})$ . All other theoretical details of the certainty model of the experienced-random experiment can be found in my work [1]. Here I will confine myself to an applied interpretation of new for the reader theoretical concepts within the framework of this experienced-random experiment «*the bet on bald*», in which the company of observer-players and the queue of observed persons participate.

## 2 Answers and solutions

We will assume that both the company of observers-players and the queue of observed persons are finite sets<sup>2</sup>. We associate with each of the  $M$  observers-players an elementary income  $\langle \omega^* | \in \langle \Omega |$ , that is, we assume that the bra-set  $\langle \Omega | = \{ \langle \omega_1^* |, \dots, \langle \omega_M^* | \}$  has a power  $|\langle \Omega | | = M$ . With each of the  $N$  observed persons, we associate an elementary outcome  $|\omega \rangle \in | \Omega \rangle$ , reasonably assuming that the ket-set  $| \Omega \rangle = \{ |\omega_1 \rangle, \dots, |\omega_N \rangle \}$  has a power  $|\Omega \rangle = N$ . Then any outcome of bets on a bald in this experienced-random experiment is defined by the co~event  $\mathcal{R} \subseteq \langle \Omega | \Omega \rangle = \{ \langle \omega^* | \omega \rangle : \langle \omega^* | \in \langle \Omega |; |\omega \rangle \in | \Omega \rangle$ , where

$$\mathcal{R} = \{ \langle \omega^* | \omega \rangle : \text{the observer-player } \langle \omega^* | \text{ bets on the bald } |\omega \rangle \} \subseteq \langle \Omega | \Omega \rangle, \quad (1)$$

and any outcome of bets on a non-bald is defined by the complementary co~event  $\mathcal{R}^c = \langle \Omega | \Omega \rangle - \mathcal{R}$ , where

$$\mathcal{R}^c = \{ \langle \omega^* | \omega \rangle : \text{the observer-player } \langle \omega^* | \text{ bets on the non-bald } |\omega \rangle \} \subseteq \langle \Omega | \Omega \rangle. \quad (2)$$

### 2.1 Probability means of a gain and a believability in a gain of a player

The believability  $b(X // \mathfrak{X}_{\mathcal{R}}) = \mathbf{B}(\langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}}} | \rangle)$  of the terraced bra-event  $\langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}}} | \subseteq \langle \Omega |$  dualistic to the given terraced ket-event  $|\text{ter}(X // \mathfrak{X}_{\mathcal{R}}) \rangle \subseteq | \Omega \rangle$ , is a believability measure of those observers-players form the labelling subset  $X \subseteq \mathfrak{X}_{\mathcal{R}}$  that «betting on a bald». By the condition of the bet an observer-player gets a gain when his/her choice is in majority. That is, when  $b(X // \mathfrak{X}_{\mathcal{R}}) > 1/2$  then winning observers-players form the subset  $X \subseteq \mathfrak{X}_{\mathcal{R}}$ ; and when  $b(X^c // \mathfrak{X}_{\mathcal{R}}) > 1/2$  then winning observers-players form the complementary subset  $X^c = \mathfrak{X}_{\mathcal{R}} - X$ ; and at last when  $b(X // \mathfrak{X}_{\mathcal{R}}) = b(X^c // \mathfrak{X}_{\mathcal{R}}) = 1/2$  then all observers-players remain at their own bets. Note that observers-players form the subset  $X \subseteq \mathfrak{X}_{\mathcal{R}}$  with probability  $p(X // \mathfrak{X}_{\mathcal{R}}) = \mathbf{P}(|\text{ter}(X // \mathfrak{X}_{\mathcal{R}}) \rangle)$  that is the probability of terraced ket-event  $|\text{ter}(X // \mathfrak{X}_{\mathcal{R}}) \rangle \subseteq | \Omega \rangle$ ; and, when  $b(X // \mathfrak{X}_{\mathcal{R}}) > 1/2$  each of them gets the gain  $1 + (1 - b(X // \mathfrak{X}_{\mathcal{R}})) / b(X // \mathfrak{X}_{\mathcal{R}}) = 1 / b(X // \mathfrak{X}_{\mathcal{R}})$ . When  $b(X^c // \mathfrak{X}_{\mathcal{R}}) > 1/2$  observers-players who have formed a complementary subset  $X^c \subseteq \mathfrak{X}_{\mathcal{R}}$  receive a gain  $1 + (1 - b(X^c // \mathfrak{X}_{\mathcal{R}})) / b(X^c // \mathfrak{X}_{\mathcal{R}}) = 1 / b(X^c // \mathfrak{X}_{\mathcal{R}})$ . And at last when  $b(X // \mathfrak{X}_{\mathcal{R}}) = b(X^c // \mathfrak{X}_{\mathcal{R}}) = 1/2$  the gain of all observers-players is one.

Consider an observer-player  $\langle \omega | \in \langle x | \subseteq \langle \Omega |$  that «bets on a bald» on an observed person  $|\omega \rangle \in |x \rangle$  from ket-event  $|x \rangle \subseteq | \Omega \rangle$ , and «bets on a non-bald» on an observed person  $|\omega \rangle \in |x \rangle^c$  from complementary ket-event  $|x \rangle^c = | \Omega \rangle - |x \rangle$ . Such observer-player is one-to-one connected with the ket-event  $|x \rangle \subseteq | \Omega \rangle$ .

<sup>2</sup>This is a weak restriction, which, frankly, I do not use anywhere, and I enter only to avoid intimidating the unprepared readers.

### 2.1.1 Probability mean of player's gain

The observer-player's gain  $\langle \omega | \in \langle x |$  is a random variable (r.v.)  $G_{\mathcal{R}}^{|\langle x |}$  that takes on  $|\Omega\rangle$  the values:

$$G_{\mathcal{R}}^{|\langle x |}(\omega) = \begin{cases} 1/b(X//\mathfrak{X}_{\mathcal{R}}), & \text{if } |\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle \subseteq |x\rangle \text{ и } b(X//\mathfrak{X}_{\mathcal{R}}) > 1/2, \\ 1/b(X^c//\mathfrak{X}_{\mathcal{R}}), & \text{if } |\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle \subseteq |x\rangle^c \text{ и } b(X^c//\mathfrak{X}_{\mathcal{R}}) > 1/2, \\ 1, & \text{if } |\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle \text{ и } b(X//\mathfrak{X}_{\mathcal{R}}) = b(X^c//\mathfrak{X}_{\mathcal{R}}) = 1/2. \end{cases} \quad (3)$$

The probability mean of a gain of observer-player  $\langle \omega | \in \langle x |$  for every  $x \in \mathfrak{X}_{\mathcal{R}}$  is an expectation of this r.v. by probability measure  $\mathbf{P}$ :

$$\mathbf{E}_{\mathbf{P}} \left( G_{\mathcal{R}}^{|\langle x |} \right) = \sum_{\substack{x \in X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \\ b(X) > 1/2}} p(X//\mathfrak{X}_{\mathcal{R}})/b(X//\mathfrak{X}_{\mathcal{R}}) + \sum_{\substack{x \notin X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \\ b(X^c) > 1/2}} p(X//\mathfrak{X}_{\mathcal{R}})/b(X^c//\mathfrak{X}_{\mathcal{R}}) + \sum_{\substack{X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \\ b(X) = 1/2}} p(X//\mathfrak{X}_{\mathcal{R}}). \quad (4)$$

### 2.1.2 Probability mean of player's believability in a gain

A believability of the observer-player  $\langle \omega | \in \langle x |$  in a gain is a random variable (r.v.)  $H_{\mathcal{R}}^{|\langle x |}$  that, if  $b(X//\mathfrak{X}_{\mathcal{R}}) > 1/2$ , takes on  $|\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle \subseteq |x\rangle$  the values of believabilities  $b(X//\mathfrak{X}_{\mathcal{R}}) = \mathbf{B}(\langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} |)$  of terraced bra-events; and that, if  $b(X//\mathfrak{X}_{\mathcal{R}}) < 1/2$ , takes on  $|\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle \subseteq |x\rangle^c$  the values of believabilities  $b(X^c//\mathfrak{X}_{\mathcal{R}}) = \mathbf{B}(\langle \text{Ter}_{X^c//\mathfrak{X}_{\mathcal{R}}} |)$  of terraced bra-events:

$$H_{\mathcal{R}}^{|\langle x |}(\omega) = \begin{cases} b(X//\mathfrak{X}_{\mathcal{R}}), & \text{if } b(X//\mathfrak{X}_{\mathcal{R}}) > 1/2 \text{ и } x \in X, \\ b(X^c//\mathfrak{X}_{\mathcal{R}}), & \text{if } b(X^c//\mathfrak{X}_{\mathcal{R}}) > 1/2 \text{ и } x \in \mathfrak{X} - X, \\ 0, & \text{if } b(X//\mathfrak{X}_{\mathcal{R}}) = b(X^c//\mathfrak{X}_{\mathcal{R}}) = 1/2. \end{cases} \quad (5)$$

The probability mean of believability in gain of observer-player  $\langle \omega | \in \langle x |$  for each  $x \in \mathfrak{X}_{\mathcal{R}}$  is an expectation of r.v.  $H_{\mathcal{R}}^{|\langle x |}$  by the probability measure  $\mathbf{P}$ :

$$\mathbf{E}_{\mathbf{P}} \left( H_{\mathcal{R}}^{|\langle x |} \right) = \sum_{\substack{x \in X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \\ b(X) > 1/2}} p(X//\mathfrak{X}_{\mathcal{R}})b(X//\mathfrak{X}_{\mathcal{R}}) + \sum_{\substack{x \notin X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}} \\ b(X^c) > 1/2}} p(X//\mathfrak{X}_{\mathcal{R}})(1 - b(X//\mathfrak{X}_{\mathcal{R}})). \quad (6)$$

## 2.2 Believability means of a gain and a probability of a gain of players in a bet

Consider the next observed person  $|\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle$  on which observers-players  $\langle \omega | \in \langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} | \subseteq \langle \Omega |$  «bet on a bald», and observers-players  $\langle \omega | \in \langle \text{Ter}_{X^c//\mathfrak{X}_{\mathcal{R}}} |^c = \langle \Omega | - \langle \text{Ter}_{X^c//\mathfrak{X}_{\mathcal{R}}} |$  «bet on a non-bald». Such observed person is one-to-one connected with the terraced bra-event  $\langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} |$ .

### 2.2.1 Believability mean of player's gain in a bet

A gain of observers-players  $\langle \omega | \in \langle \Omega |$  in a bet on an observed person  $|\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle$  is an experienced value (e.v.)  $G_{\mathcal{R}}^{*\langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} |}$  that takes on  $\langle \Omega |$  for every  $X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}}$  the values:

$$G_{\mathcal{R}}^{*\langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} |}(\omega) = \begin{cases} 1/b(X//\mathfrak{X}_{\mathcal{R}}), & \text{if } \langle \omega | \in \langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} | \text{ и } b(X//\mathfrak{X}_{\mathcal{R}}) > 1/2, \\ 1/b(X^c//\mathfrak{X}_{\mathcal{R}}), & \text{if } \langle \omega | \in \langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} |^c \text{ и } b(X^c//\mathfrak{X}_{\mathcal{R}}) > 1/2, \\ 1, & \text{if } b(X//\mathfrak{X}_{\mathcal{R}}) = b(X^c//\mathfrak{X}_{\mathcal{R}}) = 1/2. \end{cases} \quad (7)$$

A believability mean of a gain of observers-players  $\langle \omega | \in \langle \Omega |$  in a bet on an observed person  $|\omega\rangle \in |\text{ter}(X//\mathfrak{X}_{\mathcal{R}})\rangle$  for every  $X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}}$  is an expectation of this e.v. by believability measure  $\mathbf{B}$ :

$$\mathbf{E}_{\mathbf{B}} \left( G_{\mathcal{R}}^{*\langle \text{Ter}_{X//\mathfrak{X}_{\mathcal{R}}} |} \right) = 1 = \begin{cases} b(X//\mathfrak{X}_{\mathcal{R}})/b(X//\mathfrak{X}_{\mathcal{R}}), & b(X//\mathfrak{X}_{\mathcal{R}}) > 1/2, \\ b(X^c//\mathfrak{X}_{\mathcal{R}})/b(X^c//\mathfrak{X}_{\mathcal{R}}), & b(X^c//\mathfrak{X}_{\mathcal{R}}) > 1/2, \\ 1, & b(X//\mathfrak{X}_{\mathcal{R}}) = 1/2. \end{cases} \quad (8)$$

### 2.2.2 Believability mean of a probability of a player's gain in a bet

A *believability of an observer-player*  $\langle \omega | \in \langle \Omega |$  in a gain in a bet on an observed person  $|\omega \rangle \in |\text{ter}(X // \mathfrak{X}_{\mathcal{R}})\rangle$  is an *experienced value (e.v.)*  $H_{\mathcal{R}}^{*\langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |}$  that for each  $X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}}$  takes on  $\langle \Omega |$  the values:

$$H_{\mathcal{R}}^{*\langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |}(\langle \omega |) = \begin{cases} p_x, & \text{if } \langle \omega | \in \langle x | \subseteq \langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |} \text{ и } b(X // \mathfrak{X}_{\mathcal{R}}) \geq 1/2, \\ 0, & \text{if } \langle \omega | \in \langle x | \subseteq \langle \text{Ter}_{X^c // \mathfrak{X}_{\mathcal{R}} |} \text{ и } b(X // \mathfrak{X}_{\mathcal{R}}) > 1/2, \\ 0, & \text{if } \langle \omega | \in \langle x | \subseteq \langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |} \text{ и } b(X // \mathfrak{X}_{\mathcal{R}}) < 1/2, \\ 1 - p_x, & \text{if } \langle \omega | \in \langle x | \subseteq \langle \text{Ter}_{X^c // \mathfrak{X}_{\mathcal{R}} |} \text{ и } b(X // \mathfrak{X}_{\mathcal{R}}) \leq 1/2. \end{cases} \quad (9)$$

A *believability mean* of a probability of a gain of observers-players  $\langle \omega | \in \langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |}$  in a bet on an observed person  $|\omega \rangle \in |\text{ter}(X // \mathfrak{X}_{\mathcal{R}})\rangle$  for each  $X \in \mathfrak{Z}^{\mathfrak{X}_{\mathcal{R}}}$  is an expectation of e.v.  $H_{\mathcal{R}}^{*\langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |}$  by the believability measure  $\mathbf{B}$ :

$$\begin{aligned} \mathbf{E}_{\mathbf{B}} \left( H_{\mathcal{R}}^{*\langle \text{Ter}_{X // \mathfrak{X}_{\mathcal{R}} |} \right) &= \sum_{\substack{x \in X \\ b(X // \mathfrak{X}_{\mathcal{R}}) \geq 1/2}} b_x p_x + \sum_{\substack{x \in \mathfrak{X} - X \\ b(X // \mathfrak{X}_{\mathcal{R}}) \leq 1/2}} b_x (1 - p_x) \\ &= \begin{cases} \sum_{x \in X} b_x p_x, & b(X // \mathfrak{X}_{\mathcal{R}}) > 1/2, \\ \sum_{x \in \mathfrak{X} - X} b_x (1 - p_x), & b(X // \mathfrak{X}_{\mathcal{R}}) < 1/2, \\ \sum_{x \in X} b_x p_x + \sum_{x \in \mathfrak{X} - X} b_x (1 - p_x), & b(X // \mathfrak{X}_{\mathcal{R}}) = 1/2. \end{cases} \quad (10) \end{aligned}$$

## 3 Instead of the results: a bald versus a basketball match

I dare say, at the risk of being considered rash: to make a «bet on winning a basketball match» is the same as making a «bet on a bald». You can, of course, continue to argue as usual: «a bald *can not* change his baldness, and the basketball players-participants *can* change its outcome», reasonably believing that the statement I made is unfounded. However, from the point of view of the theory of experience and of chance, the analogy between the bald and the players-participants in the match on which this conclusion is based is a common and frustrating misconception. In accordance with this new theory and the players-participants in the match, and the audience should be likened to players who bet on the match bookmaker. So analogy to bald here is more correct to consider not the players-participants of the match, but the *match, as such: its course and outcome*. Participants in the match, basketball players, significantly affect its outcome by its game. But their game contribution is just «their bets» for the match. Spectator-players also influence, perhaps to a lesser extent, the outcome of the game with their support. But their support is also just «their bet» for the match<sup>3</sup>. Players making bets on the match, of course, influence its outcome with their monetized betting odds, which are commonly known to everyone, including basketball players and spectator-players, until the end of the match. By the way, the bookmaker also influences the outcome of the match, setting his own odds, which serve as «his bets» for the match. So the bookmaker should also be likened to the player betting on the match.

### References

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<sup>3</sup>Of course, the «bets» for the match by basketball players and spectator-players are usually not measured in coins, but this can not be the reason for excluding them from the number of players who bet on the match, but only complicates the mathematical description of the match betting model within the framework of the theory of experience and of chance.