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Vorobyev, Oleg Yu.

Siberian Federal University, Institute of Mathematics and Computer Science

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Oleg Yu. Vorobyev
Institute of mathematics and computer science
Siberian Federal University
Krasnoyarsk
mailto:oleg.yu.vorobyev@gmail.com
http://www.sfu-kras.academia.edu/OlegVorobyev
http://olegvorobyev.academia.edu

Abstract. *The pairwise independence of events does not entail their setwise independence (Bernstein's example, 1910-1917). The probability distributions of all pairs of events do not determine the probability distribution of the whole set of events (the triangular room paradox of negative probabilities of events [8, 9, 2001]). The pairwise preferences of events do not determine their setwise preferences (Blyth's paradox, 1972). The eventological theory of setwise event preferences, proposed in [8, 2007], gives an event justification and extension of the classical theory of preferences and explains Blyth's paradox «of three pies»[1] (that was already well-known to Yule[2]) by human ability to use triplewise and morewise preferences.*

Keywords. *Eventology, event, probability, preference, pairwise event preferences, setwise event preferences, theory of setwise event preferences.*

1 Introduction

The fact that pairwise independence of events does not imply setwise independence of events was mentioned for the first time in the correspondence in the years 1910 to 1917 between Chuprov[3] and Markov[4]. The bright example of the fact is attributted usually to Bernstein[5] with reference to [1, 1946, page 48].

In [9, 2016] an improved generalization of Feynman's[6] paradox of negative probabilities [3, 4] for observing three events which is directly related to the theory of quantum computing is presented. This generalization, first proposed in [7, 2001] and called the «paradox of the triangular room of negative probabilities of events», clearly demonstrates the fact that three probability distributions of pairs of events from a given triplet are insufficient to determine the probabilistic distribution of the whole triplet of events. In other words, three pairwise (partial) probability distributions of events do not determine the triplewise (joint) probability distribution of the whole triplet of events.

In this paper, I intend to briefly show the main advantages of the new *theory of setwise event preferences* developed in [8], and at the same time to demonstrate once again the failure of the pairwise to describe the whole. This time, using the example of preferences, when each of us is forced to make a comparison every time, hitting the next situation of choice, which requires a decision. The theory of setwise event preferences shows that the familiar pairwise comparisons do not provide a complete description of the

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[1]*Colin R. Blyth* was a Canadian mathematician.

[2]Yule, George Udny (1871–1951) was a Scotland statistician. An important contribution to the theory and practice of correlation, regression, time series analysis. Yule's distribution, discrete power law, and many other statistical concepts are named in his honor.

[3]*Alexander Alexandrovich Chuprov (1974–1926)* was a Russian statistician who worked on mathematical statistics, sample survey theory and demography.

[4]*Andrey Andreyevich Markov (1856–1922)* was a Russian mathematician. He is best known for his work on stochastic processes. A primary subject of his research later became known as Markov chains and Markov processes.

[5]*Sergei Natanovich Bernstein (1880–1968)* was a Russian and Soviet mathematician of Jewish origin known for contributions to partial differential equations, differential geometry, probability theory, and approximation theory.

[6]*Richard Phillips Feynman (1918–1988)* was an American scientist. The main achievements relate to the field of theoretical physics. One of the creators of quantum electrodynamics. Nobel Prise in Physics in 1965 for his contributions to the development of quantum electrodynamics.

whole set of preferences that can be trapped at every step. A vivid example of this failure of the pairwise to describe the whole is the explanation of the eventological theory of setwise event preferences of the famous Blyth's paradox «of three pies» [2, 1972].

2 Make decisions on the base of setwise event preferences

How does a person make a decision? Clarify: «How does the person make an event decision?», because from the point of view of the eventology theory [8] any decision accepted by a person is always an event decision. Let $(\Omega, \mathcal{F}, \mathbf{P})$ be a probability space in which the eventological theory of setwise event preferences answers this question.

Once in the situation of choice, first the person

1) *perceives the event circumstances of the environment*, that is, *perceives a subset* $F \subseteq \mathfrak{F} \subseteq \mathcal{A}$ *of happened events-circumstances*, in other words, *perceives the happened terraced event* $\text{ter}(F/\!\!/\mathfrak{F}) = \bigcap_{f \in F} f \bigcap_{f \in \mathfrak{F}-F} (\Omega - f) \subseteq \Omega$ generated by the set of events-circumstances $\mathfrak{F} \subseteq \mathcal{A}$. Then the person

2) *is conscious of possible own event decisions*, that is, *realizes the subset of possible events-decisions defined by* $F \subseteq \mathfrak{F} \subseteq \mathcal{A}$ *the subset of possible events-decisions* $D_F \subseteq \mathfrak{D} \subseteq \mathcal{A}$, where the set of his own events-decisions $\mathfrak{D}$ has the *probability distribution* $\boldsymbol{p}(D_F) = \{p(D/\!\!/\mathfrak{D}), D \subseteq D_F\}$. After that the person

3) *probabilistically chooses an event decision*, that is, *under the probability distribution* $\boldsymbol{p}(D_F)$ *the person chooses a subset of events-decisions* $D \subseteq D_F \subseteq \mathcal{A}$, in other words, *under* $\boldsymbol{p}(D_F)$ *the person chooses the terraced event-decision* $\text{ter}(D/\!\!/D_F) = \bigcap_{d \in D} d \bigcap_{d \in D_F - D} (\Omega - d) \subseteq \Omega$ from all terraced events-decisions generated by the set D_F. And at last the person

4) *creates an event decision*, that is, *the person creates events-decisions* from the choosen subset $D \subseteq D_F \subseteq \mathcal{A}$ (and does not create events-decisions from $D_F - D$), in other words, *the person creates the one terraced event-decision* $\text{ter}(D/\!\!/D_F) \subseteq \Omega$ from all terraced events-decisions generated by D_F.

Now we are interested in the second and third stages of decision-making by person. These two stages have long attracted the attention of both theorists and practitioners in decision-making and usually refer to what is called *theory of preferences* or *theory of choice*. We will offer the *theory of setwise event-based preferences* or *theory of setwise event-based choice*, which includes the formulation and decision of the common *problem of setwise event preferences* and in passing explains the Blyth's paradox «of three pies» [2, 5].

The Blyth Paradox (Simpson's paradox, the Yule-Simpson effect) is a statistical paradox in which the preferences of several groups of people change to the opposite, after the groups unite. With this seemingly impossible result, one encounters surprisingly often in sociology and medical statistics. The paradox was described by Simpson[7] in 1951 [6] and by Yule in 1903 [10]. The name of the «Simpson paradox» was given by Blyth in 1972 [2]. Since Simpson did not discover this paradox, a number of authors use impersonal names instead, such as the «reversal paradox» or the «amalgamation paradox». Since Blyth popularized this paradox with his vivid example of a choice of three pies, we call it

The Blyth paradox «of three pies». The restaurant owner, whose menu on different days contains a different subset of the set of three pies *«Apple, Cherry, Blueberry»*, noticed that when only two pies are on the menu, one regular visitor prefers an *apple* pie before *cherry* pie and never orders *blueberry* pie. However, when the menu contains all three pies, the visitor suddenly begins to prefer *Cherry* before *Apple*.

It would seem that we have a paradox: «the presence or absence of a third pie in the menu, which is never preferred by the visitor, changes his preference between the other two for the opposite». However, everything is much simpler: «from two pies, *Apple* and *Cherry*, the visitor prefers *Apple*, and from all — *Cherry*». The point is that the choosing person does not have to always carry out only pairwise comparisons and have only pairwise preferences, he is able to compare not only pairs of events, but also

[7] *Simpson, Edward Hugh (b. 1922)* is a British statistician best known for describing Simpson's paradox [6].

three, and consequently, an arbitrary set of events. In other words, the choosing person to be able to have not only pairwise, but also triplewise and setwise preferences. Moreover, the choosing person is capable *to prefer an arbitrary subset of events from the given set of events (to have a setwise event preference)*. These obvious eventological conclusions are the basis for the proposed by me *theory of setwise event preferences*, which, in a sense, generalizes the existing non-eventological theories of preferences and, in particular, gives an eventological explanation of the Blyth paradox «of three pies».

3 Eventological explanation of Blyth's paradox «of three pies»

Consider the detailed eventological explanation of Blyth's paradox «of three pies» [2, 5], before introducing the terminology of the theory of setwise event preferences, hoping that this simple example will help to master a new theory more quickly.

Let $\mathfrak{X} = \{x, y, z\} = \{$«Apple», «Cherry», «Blueberry»$\}$ be a set of names of pies, $\mathfrak{F} = \{f_x, f_y, f_z\} = \{$«Apple in menu», «Cherry in menu», «Blueberry in menu»$\} \subseteq \mathcal{A}$ be a set of events-circumstances. Then terraced events

$$\mathsf{ter}(F_X /\!\!/ \mathfrak{F}) = \bigcap_{x\in X} f_x \bigcap_{x\in\mathfrak{X}-X} (\Omega - f_x) = \text{«on menu there are name of pies from } X = \{x : f_x \in F\} \subseteq \mathfrak{X}\text{»} \subseteq \Omega$$

correspond to eight possible coincidences of events-circumstances $F_X \subseteq \mathfrak{F}$, $X \subseteq \mathfrak{X}$. Let

$$\mathfrak{D} = \{d_x, d_y, d_z\} = \{\text{«choice of Apple», «choice of Cherry», «choice of Blueberry»}\} \subseteq \mathcal{A}$$

be a set of possible events-decisions of choosing person, and let $D_{F_X} = \{d_x,\ x \in X\} \subseteq \mathfrak{D}$ be a subset of events-decisions, «imposed on» the person by a subset of events-circumstances $F_X \subseteq \mathfrak{F}$, $X \subseteq \mathfrak{X}$.

My eventological explanation of Blyth's paradox suggests the following probability distribution of the set of possible events-decisions of $\mathfrak{D}$ (See Venn diagrams in Fig. 1):

$$\begin{aligned}\boldsymbol{q}(\mathfrak{D}) &= \Big\{q(\emptyset),\ q(\{d_x\}),\ q(\{d_y\}),\ q(\{d_z\}), q(\{d_x, d_y\}),\ q(\{d_x, d_z\}),\ q(\{d_y, d_z\}),\ q(\{d_x, d_y, d_z\})\Big\} \\ &= \{0,\ 2/9, 4/9, 0,\ 0, 3/9, 0,\ 0\},\end{aligned} \tag{1}$$

where the probabilities are arranged in accordance with the events-decisions of choosing from the menu the corresponding combinations of names of pies: «Nothing, Apple, *Cherry*, Blueberry, Apple-Cherry, Apple-Blueberry, Cherry-Blueberry, Apple-Cherry-Blueberry».

In the theory of setwise event preferences, the probability distribution $\boldsymbol{q}(\mathfrak{D})$ is called the *triplewise event preference* of the choosing person, since it defines a probability of *set*-choice by this person from the *triplet* of events-decisions $\mathfrak{D} \subseteq \mathcal{A}$, in other words, the probabilistic choice by this person of any subset of the events-decisions $D \subseteq \mathfrak{D}$.

In accordance with the *triplewise setwise event preference* (1), when the menu offers all three pies, i.e., at the confluence of all three events-circumstances: $\mathsf{ter}(\mathfrak{F} /\!\!/ \mathfrak{F}) = f_x \cap f_y \cap f_z$, the *setwise choosing person*[8] prefers to choose only one cherry pie (y) because $q(\{d_y\}) = \max\limits_{D\subseteq\mathfrak{D}}\{q(D)\}$.

The *triplewise event mono-preference* is determined by the probabilities of the *mono*-choice[9] from the *triplet* of events-decisions. In the eventological explanation of the paradox in the case of monoplet choice from a triplet, the person prefers a cherry pie, the monoplet $\{d_y\}$, i.e. the person has the following *mono-preference*: «Apple, *Cherry*, Blueberry» $\sim \{2/9, 4/9, 0\} = \{q(\{d_x\}), q(\{d_y\}), q(\{d_z\})\}$, becuase $q(\{d_y\})\ \max\limits_{\{d\}\subseteq\mathfrak{D}}\{q(\{d\})\}$.

The *pairwise event mono-preferences* are defined by probabilities of *monoplet* choice from *doublets* of events-decisions, with probability distributions:

$$\boldsymbol{q}_{xy} = \Big\{q_{xy}(\emptyset),\ q_{xy}(\{d_x\}),\ q_{xy}(\{d_x\}),\ q_{xy}(\{d_x, d_y\})\Big\},$$

[8] *Setwise choosing person* is a person who is capable to choose any subset of events-decisions $D \subseteq \mathfrak{D}$ under the given probability distribution $\boldsymbol{q}(\mathfrak{D})$.

[9] *Monoplet choice* is a choice of *monoplets* of events-decisions $\{d\} \subseteq \mathfrak{D}$.

$$\boldsymbol{q}_{xz} = \Big\{ q_{xz}(\emptyset),\ q_{xz}(\{d_x\}),\ q_{xz}(\{d_z\}),\ q_{xz}(\{d_x, d_z\}) \Big\},$$

$$\boldsymbol{q}_{yz} = \Big\{ q_{yz}(\emptyset),\ q_{yz}(\{d_y\}),\ q_{yz}(\{d_z\}),\ q_{yz}(\{d_y, d_z\}) \Big\}$$

and are defined by probability distribution of the triplet $\mathfrak{D} \subseteq \mathcal{A}$. For example, the doublet probability distribution $\boldsymbol{q}_{xy}$ is defined via the triplet probability distribution $\boldsymbol{q}(\mathfrak{D})$ by formulas:

$$q_{xy}(\emptyset) = q(\emptyset) + q(\{d_z\}),\ q_{xy}(\{d_x\}) = q(\{d_x\}) + q(\{d_x, d_z\}),$$

$$q_{xy}(\{d_y\}) = q(\{d_y\}) + q(\{d_y, d_z\}),$$

$$q_{xy}(\{d_x, d_y\}) = q(\{d_x, d_y\}) + q(\{d_x, d_y, d_z\}).$$

In the eventological explanation of the paradox under monoplet choice from doublets the person has the following *event mono-preferences* between corresponding doublets of events-decisions:

«*Apple*, Cherry» $\sim \{5/9, 4/9\} = \{q_{xy}(\{d_x\}), q_{xy}(\{d_y\})\}$,

«*Apple*, Blueberry» $\sim \{2/9, 0\} = \{q_{xz}(\{d_x\}), q_{xz}(\{d_z\})\}$,

«*Cherry*, Blueberry» $\sim \{4/9, 3/9\} = \{q_{xy}(\{d_x\}), q_{xy}(\{d_y\})\}$

for the same reasons.

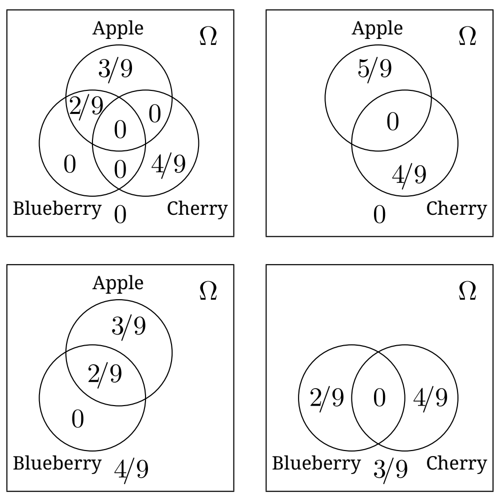

Figure 1: Venn diagrams of events-decisions illustrating the explanation of Blyth'paradox «of three pies» based on the theory of setwise event preferences. Events-decisions «pie choice» — apple pie (x), cherry pie (y) and blueberry pie (z) — form the triplet $\mathfrak{D} = \{d_x, d_y, d_z\}$ with probability distribution «Nothing, Apple, Cherry, Blueberry, Apple-Cherry, Apple-Blueberry, Cherry-Blueberry, Apple-Cherry-Blueberry» $\sim$ «0, 3/9, 4/9, 0, 0, 2/9, 0, 0», which is called a *triplewise event preference* of the choosing person. The *triplewise event mono-preference* «Apple, Cherry, Blueberry» $\sim$ «3/9, 4/9, 0» and *pairwise event mono-preferences*: «Apple, Cherry» = « 5/9, 4/9 », «Apple, Blueberry» $\sim$ «3/9, 0», «Cherry,Blueberry» $\sim$ «4/9, 2/» are defined by the theory of setwise event preferences.

4 Definition of setwise event preferences

Let $\mathfrak{D} \subseteq \mathcal{A}$ be a set of events-decisions of the person, who is capable to make a setwise choice of subsets of events-decisions in accordance with the probability distribution $\boldsymbol{q}(\mathfrak{D}) = \Big\{ q(D /\!\!/ \mathfrak{D}) = \mathbf{P}(\mathrm{ter}(D /\!\!/ \mathfrak{D})), D \subseteq \mathfrak{D} \Big\}$. The set $\mathfrak{D}$ is generated by the *choosing person* and is associated with a set of events-circumstance $\mathfrak{F} \subseteq \mathcal{A}$ by a *choosing map* $\varphi : 2^{\mathfrak{F}} \to 2^{\mathfrak{D}}$, that assigns to each subset of events-circumstances $F \subseteq \mathfrak{F}$ the only set of possible events-decisions $D_F = \varphi(F) \subseteq \mathfrak{D}$ from which the person chooses subsets making own *probabilistic set-wise choice*. In other words, at the confluence of events-circumstances $F \subseteq \mathfrak{F}$ the person *probabilistically setwise chooses* a subset of events-decisions $D \subseteq D_F$ from tha set D_F in accordance with partial probability distribution of D_F:

$$\boldsymbol{q}(D_F) = \Big\{ q(D /\!\!/ D_F),\ D \subseteq D_F \Big\}$$

where $q(D/\!\!/D_F) = \mathbf{P}(\mathrm{ter}(D/\!\!/D_F))$ are probabilities of terraced events

$$\mathrm{ter}(D/\!\!/D_F) = \bigcap_{d\in D} d \bigcap_{d\in D_F - D} (\Omega - d), \quad D \subseteq D_F,$$

generated by $D_F \subseteq \mathfrak{D}$.

The probability distribution $\boldsymbol{q}(\mathfrak{D})$ of the set of events-decisions $\mathfrak{D} \subseteq \mathcal{A}$ is associated with a probability distribution $\boldsymbol{p}(\mathfrak{F})$ of the set of events-circumstances $\mathfrak{F} \subseteq \mathcal{A}$ by conditional scheme formulas:

$$q(D/\!\!/D_F) = \sum_{F\subseteq\mathfrak{F}} q(D/\!\!/D_F|F)p(F), \quad D \subseteq \mathfrak{D},$$

where $\boldsymbol{p}(\mathfrak{F}) = \left\{p(F/\!\!/\mathfrak{F}),\ F \subseteq \mathfrak{F}\right\}$ is the probability distribution of the set of events-circumstances $\mathfrak{F} \subseteq \mathcal{A}$, and probabilities

$$q(D/\!\!/D_F|F) = \frac{\mathbf{P}(\mathrm{ter}(D/\!\!/D_F) \cap \mathrm{ter}(F))}{p(F)}$$

taking all together for $D \subseteq D_F$ forms

$$\boldsymbol{q}(D/\!\!/D_F|F) = \left\{q(D/\!\!/D_F|F),\ D \subseteq D_F\right\},$$

the conditional probability distribution of the set of events-decisions $\mathfrak{D} \subseteq \mathcal{A}$ under conditions of happened subsets of events-circumstances $F \subseteq \mathfrak{F}$. In other words, under happened terraced events-circumstances $\mathrm{ter}(F/\!\!/\mathfrak{F}),\ F \subseteq \mathfrak{F}$.

Definition (setwise event preference). The *setwise event preference* on the set of events-decisions $\mathfrak{D}$ is the probability distribution of the given set: $\boldsymbol{q}(\mathfrak{D}) = \left\{q(D/\!\!/\mathfrak{D}),\ D \subseteq \mathfrak{D}\right\}$. The *event n-preference* on the set of events-decisions $\mathfrak{D}$ is a set of probabilities of n-plets from the probability distribution of the given set: $\boldsymbol{q}_n(\mathfrak{D}) = \left\{q(D_n/\!\!/\mathfrak{D}),\ D_n \subseteq \mathfrak{D}\right\}$ where $D_n \subseteq \mathfrak{D}$ is an n-plet of events-decisions from $\mathfrak{D}$ (a subset of power $n = |D_n|$). In particular, the *event empty-preference* on the set of events-decisions $\mathfrak{D}$ is a set that consists of one probability $q(\emptyset) \in \boldsymbol{q}(\mathfrak{D})$ taking from the probability distribution of the given set: $\boldsymbol{q}_0(\mathfrak{D}) = \left\{q(\emptyset)\right\}$. The *event mono-preference* on the set of events-decisions $\mathfrak{D}$ is a set of probabilities of monoplets from the probability distribution of the given set: $\boldsymbol{q}_1(\mathfrak{D}) = \left\{q(\{d\}/\!\!/\mathfrak{D}),\ d \in \mathfrak{D}\right\}$. The *event doublet-preference* on the set of events-decisions $\mathfrak{D}$ is a set of probabilities of doublets from the probability distribution of the given set: $\boldsymbol{q}_2(\mathfrak{D}) = \left\{q(\{d,e\}/\!\!/\mathfrak{D}),\ \{d,e\} \subseteq \mathfrak{D}\right\}$. Obviously, that $\boldsymbol{q}(\mathfrak{D}) = \sum_{n=0}^{|\mathfrak{D}|} \boldsymbol{q}_n(\mathfrak{D})$.

A *setwise event-prefernce* defines a *probabilistic setwise choice* any subsets of events-decisions D from the set $\mathfrak{D}$ by the person. An *event empty-preference* defines a *probabilistic setwise choice* of *empty set* of events-decisions from the set $\mathfrak{D}$ by the person, in other words, it defines a *probability of person's inactivity* within the set of events-decisions $\mathfrak{D}$. An *event mono-preference* and an *event doublet-preference* define a *probabilistic setwise choice* of *monoplets and doublets* of events-decisions from the set $\mathfrak{D}$ by the person correspondingly.

5 Basic eventological assumption of the theory of setwise event preferences

The theory of setwise event preferences is based on the *non-trivial* eventological assumption that the mono-preferences of a person choosing only one event from the set of events $\mathfrak{X} \subseteq \mathcal{A}$ are defined not by the probabilities of the «mono events» $x \in \mathfrak{X}$, in other words, not a set of probabilities[10] $\left\{\mathbf{P}(x),\ x \in \mathfrak{X}\right\}$, but of probabilities of «terraced mono-events»

$$\mathrm{ter}(\{x\}/\!\!/\mathfrak{X}) = x \cap \left(\bigcap_{y\in\mathfrak{X}-\{x\}} (\Omega - y)\right), \quad x \in \mathfrak{X},$$

[10]As this is usually assumed in the different theories of preferences.

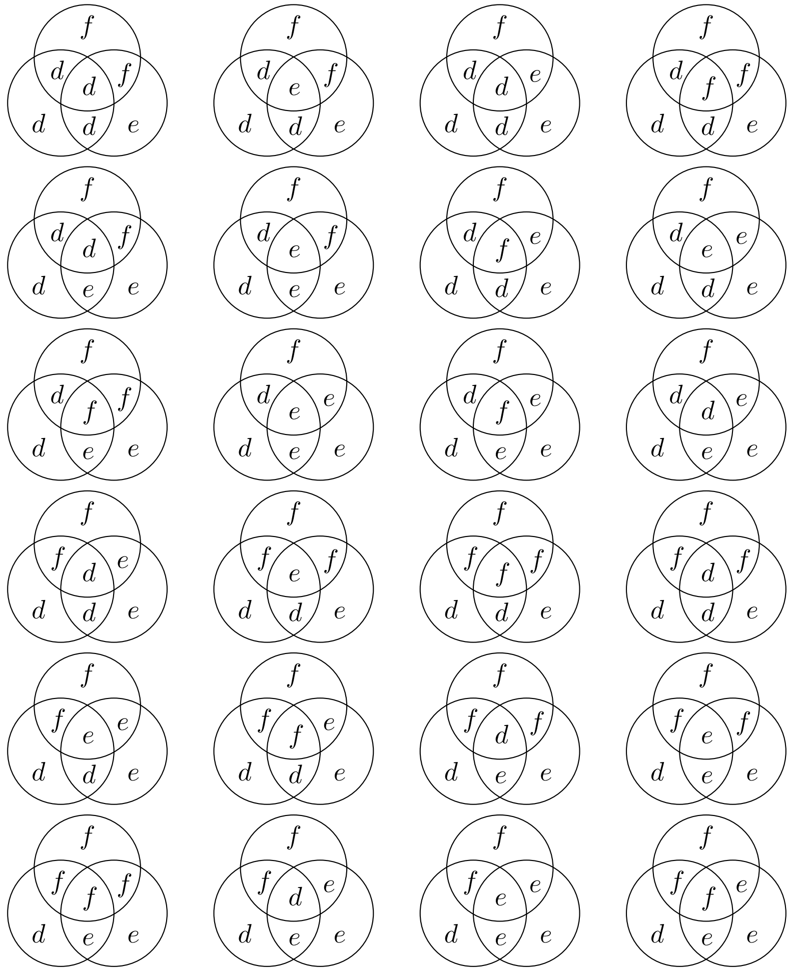

Figure 2: All twenty four setwise preferences of a one event-decision from the set of events-decisions $D \subseteq \mathfrak{D} \subseteq \mathcal{A}$, that can be defined on the triplet of events-decisions $\mathfrak{D} = \{d, e, f\}$. The last column on the right is made up of six setwise event preferences, corresponding to the structure of the setwise preferences in the Blyth paradox «of three pies». These six setwise preferences correspond to six possible options for renaming three events from $\mathfrak{D}$.

enumerated by monoplets $\{x\} \subseteq \mathfrak{X}$, i.e. by a set of probabilities

$$\Big\{\mathbf{P}\big(\mathsf{ter}(\{x\}/\!\!/\mathfrak{X})\big),\ x \in \mathfrak{X}\Big\}.$$

These two sets of probabilities begin to coincide, as soon as the events $x \in \mathfrak{X}$ do not pairwise intersect, since then $x = \mathsf{ter}(\{x\}/\!\!/\mathfrak{X}), \quad x \in \mathfrak{X}$. However, in the situation of an arbitrary set of events $\mathfrak{X}$ these two sets of probabilities can be completely different:

$$\Big\{\mathbf{P}(x),\ x \in \mathfrak{X}\Big\} \neq \Big\{\mathbf{P}\big(\mathsf{ter}(\{x\}/\!\!/\mathfrak{X})\big),\ x \in \mathfrak{X}\Big\}.$$

For example, for any set of probabilities of events $x \in \mathfrak{X}$ there exists a structure of event dependencies such that $\mathbf{P}\big(\mathsf{ter}(\{x\}/\!\!/\mathfrak{X})\big) = 0, \quad x \in \mathfrak{X}$. In the theory of setwise event preferences, this means the complete absence of a mono-choice, i.e. a making such a probabilistic setwise choice when the person never chooses events $x \in \mathfrak{X}$ separately, but always — together with other events from $\mathfrak{X}$.

The main eventological assumption of the theory of setwise event preferences in the situation of a probabilistic setwise choice is that the choice of n events forming an arbitrary subset $X_n \subseteq \mathfrak{X}$ of power $n = |X_n|$ is determined not by the probabilities of the following terraced events

$$p_{X_n/\!\!/\mathfrak{X}} = \mathbf{P}\Big(\mathsf{ter}_{X_n/\!\!/\mathfrak{X}}\Big) = \mathbf{P}\left(\bigcap_{x \in X_n} x\right),$$

i.e. of the intersections of all events from X_n, but by probabilities of other terraced events:

$$p(X_n /\!\!/ \mathfrak{X}) = \mathbf{P}\Big(\mathsf{ter}(X_n /\!\!/ \mathfrak{X})\Big) = \mathbf{P}\left(\bigcap_{x \in X_n} x \bigcap_{x \in \mathfrak{X} - X_n} (\Omega - x)\right),$$

which happens when only events from the subset $X_n \subseteq \mathfrak{X}$ happens. Thus, in the general situation, the main eventological assumption replaces the set of probabilities $\left\{p_{X_n /\!\!/ \mathfrak{X}}, X_n \subseteq \mathfrak{X}\right\}$ by a set of other probabilities: $\left\{p(X_n /\!\!/ \mathfrak{X}), X_n \subseteq \mathfrak{X}\right\}$.

Of course, the meaning and significance of the main eventological assumption of my *theory of setwise event preferences* do not reduce merely to a formal substitution one set of probabilities by another one. This eventological assumption allows us to clearly explain the Blyth paradox. This fact can only mean the one thing: the eventological assumption underlying the theory of setwise event-based preferences seems to reflect more preferably the mechanism of human implementation of the probabilistic setwise choice than the assumptions that are postulated by the different theories of preferences on the basis of only pairwise comparisons.

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