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Mattesini, Fabrizio and Rossi, Lorenza

Universita Cattolica del Sacro Cuore, Milano

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Fabrizio Mattesini and Lorenza Rossi
Università di Roma "Tor Vergata"
Via Columbia 2, 00133 Rome, Italy

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Abstract

In this paper we analyze a general equilibrium DSNK model characterized by labor indivisibilities, unemployment and a unionized labor market. The presence of monopoly unions introduces real wage rigidities in the model. We show that as in Blanchard Galì (2005) the so called "divine coincidence" does not hold and a trade-off between inflation stabilization and the output stabilization arises. In particular, a productivity shock has a negative effect on inflation, while a reservation-wage shock has an effect of the same size but with the opposite sign. We derive a welfare-based objective function for the Central Bank as a second order Taylor approximation of the expected utility of the economy's representative household, and we analyze optimal monetary policy under discretion and under commitment. Under discretion a negative productivity shock and a positive exogenous wage shock will require an increase in the nominal interest rate. An operational instrument rule, in this case, will satisfy the Taylor principle, but will also require that the nominal interest rate does not necessarily respond one to one to an increase in the natural rate of interest. The results of the model are consistent with a well known empirical regularity in macroeconomics, i.e. that employment volatility is relatively larger than real wage volatility.

JEL codes: E24, E32, E50, J23, J51

1 Introduction

In the last ten years, the Dynamic Stochastic New Keynesian (DSNK) model has emerged as an important paradigm in macroeconomics and as a useful framework for the study of monetary policy. Most of the models proposed so far along this line of research, however, are based on the standard competitive model and completely ignore the role that trade unions play in determining wages and employment conditions in many countries. If this is probably an acceptable (although very strong) simplification for countries, like the U.S., where in the year 2002, only about 15% of workers were covered by collective contract agreements, it becomes instead problematic for other countries such as France, Italy or Sweden where the percentage of workers covered by collective contracts is above 84%.[1] Given that wage bargaining may introduce significant distortions in the functioning of a modern economy and have an impact on its behavior at the aggregate level, the study of unionized labor markets and of the consequences of these markets for monetary policy becomes of crucial importance if one wants to understand the functioning of many important economies around the world.

The purpose of this paper is to propose a model where wages are the result of a contractual process between unions and firms and where, at the same time, the movements of the rate of unemployment are explicitly accounted for. In order to evaluate movements of labor along the extensive margin, we assume, as in Hansen [22] and Rogerson and Wright [30], that labor supply is indivisible and that workers face a positive probability to remain unemployed. As in Maffezzoli [25] and Zanetti [38], we assume that wages are set by unions according to the popular monopoly-union model introduced by Dunlop [12] and Oswald [27]. This paper, therefore, contributes to a literature which has recently tried to improve on the "standard" DNK model by focusing on the behavior of the labor market. Models characterized by labor market frictions and price staggering, where labor is allowed to move not only along the intensive margin but also the extensive margin, have been proposed, among others, by Chéron and Langot [7], Walsh [34] [35], Trigari [32], [33], Moyen and Sahuc [26] and Andres et al. [2]. More recently Christoffel and Linzert [9] and Blanchard and Galì [4] [5] have proposed models characterized not only by labor market frictions and staggered prices, but also by real wage rigidities. Blanchard and Galì [4] show that, if real wages are assumed to adjust slowly, what they define as the "divine coincidence" does not hold any more: for a

[1] More precisely, the number of persons covered by collective agreeements over total employment was 94.5% in France in 2003, 84.1% in Italy in the year 2000 and 85.1% in Sweden in the year 2000. For a complete set of data on union coverage on the various countries see Lawrence and Ishikawa [24].

central bank pursuing, as a policy objective, the level of output that would prevail under flexible prices is not equivalent to pursuing the efficient level of output, in which case a trade-off between inflation stabilization and output gap stabilization arises. Blanchard and Galì [5] analyze a model where labor market frictions are not simply assumed but explicitly modelled and show that a policy trade-off does not only pertain to the output gap, but also to the rate of unemployment.

Also in the model we propose here, as in Blanchard and Galì, [4] [5] the "divine coincidence" does not hold, and changes in unemployment are explicitly modeled. Differently from the existing literature, however, instead of basing the analysis on the Pissarides [28] model of search and matching, we introduce unemployment in an alternative simple and tractable way which allows us to establish an inverse relationship between unemployment and the output gap and we focus on the consequences of union behavior for the response of the economy to exogenous shocks. The model is capable of producing a series of interesting results.

First, it shows that productivity shocks give rise to a significant policy trade-off between stabilizing inflation and stabilizing unemployment, and in this respect it provides a way to overcome an important shortcoming of the DNK model, i.e. its inability to account for the significant challenges that exogenous changes in technology represent for monetary policy in the real world. According to the "standard" DNK model, in fact, an optimal monetary policy that stabilizes output around its flexible price equilibrium, also produces zero inflation,[2] so that stabilizing inflation implies automatically an optimal response to a productivity shocks. This, however, not only is at odds with the historical accounts[3] and the widespread perception of financial markets, but there is also some recent empirical evidence indicating that, in most countries, central banks have actively responded to technology shocks, increasing or decreasing the nominal interest rate.[4] What is interest-

[2] This is shown quite clearly, for example, by Galì, Lopez Salido and Valles [16].

[3] In this respect the debate on the Fed's monetary policy during governor Greenspan tenure is quite instructive. There is a lot of anectodical evidence that the Fed has spent large efforts in understanding the increase in productivity growth that has characterized the American economy since the mid 1990s. The success of monetary policy in this period has been attributed by important commentators (Woodward 2000) to the ability of the Fed to respond to exogenous technological progress.

[4] Galì et al. [15], analyzing a 4 variable SVAR model where thechnology shocks were identified by assuming a unit root in labor productivity, found that the Fed, in the post-Volcker period, di not change the nominal interest rate in response to productivity shocks, but in a recent paper, Francis, Owyang and Theodoru [14] used the same methodology to analyze monetary policy for the G7 countries and found a wide range of variation in the behavior of different countries: while France, Japan, the United Kingdom (after

ing, in our model, is that this result is not the consequence of some kind of exogenous real wage inertia, as in Blanchard and Galì [4], but is simply the consequence of unions' monopoly power in the labor market. In our economy, in fact, a productivity slowdown, i.e. a negative productivity shock tends to lower efficient output but, since unions will keep real wages constant, the level of output that would prevail under price flexibility, that we define as "natural" output, decreases even more, so that the difference between efficient output and "natural" output increases. Since in sticky price models inflation depends on marginal costs and, in turn, marginal costs depend on the difference between "natural" output and actual output, then a Phillips curve, correctly defined as depending on the gap between efficient output and actual output, will depend on productivity shocks, and a trade-off between inflation stabilization and output gap stabilization arises.

Second, we show that a policy trade-off for the central bank arises not only in response to technology shocks, but also in response to exogenous wage push shocks. If the unions' reservation wage is subject to exogenous changes, and these changes tend to be persistent over time, then a welfare maximizing central bank must again face the problem of whether to accommodate these shocks with a easier monetary policy. Our model therefore provides a convenient framework to address important normative issues such as, for example, the optimal behavior of central banks in periods characterized by labor market turmoil and exogenous wage shocks.

Third, we derive the objective function of the central bank as a second order Taylor approximation of the expected utility of the representative household and we show that, when the economy is hit by technology and exogenous wage shocks, monetary policy presents some interesting peculiarities relative to the standard case. We first consider the problem of a central bank that cannot commit to future policy actions. In this case optimal monetary policy requires a decrease (increase) in the interest rate following a positive (negative) productivity shock and an increase in the interest rate following a reservation wage shock. An optimal instrument rule that implements such policy can be expressed as an interest rate reacting to the expected rate of inflation and to the natural rate of interest. In this model monetary policy satisfies the Taylor principle, i.e. the nominal interest rate must be raised more than proportionally with respect to the expected rate of inflation. Dif-

the break in monetary regime) seem to have reacted to a technology shock by increasing nominal interest rates, the U.S. (before and after Vocker), Canada, Germany, the U.K (pre-break) and Italy, seem to have reacted instead by lowering interest rates. A policy of no-reaction to a productivity shock consistent with the prescriptions of the standard DNK model, therefore, does not seem to have been widely adopted by the major industrialized countries.

ferently from the standard model, however, the nominal interest rate must not increase one to one with the natural rate of interest. If the persistence of the technological shock is greater than the persistence of the reservation wage shock the nominal interest will increase less than proportionately to an increase in the natural rate of interest.

A fourth, important result is that the model is able to account for a well known stylized fact in macroeconomics, i.e. the relatively smooth behavior of wages and the relatively volatile behavior of unemployment over the business cycle. When the level of unemployment that the economy achieves under an optimal discretionary policy is written as a function of the relevant shocks, an exogenous wage shock will in general induce a movement both in the real wage and in the rate of unemployment; a productivity shock, instead, will induce a movement in the rate of unemployment, but not in the real wage. An economy frequently hit by exogenous changes in technology will show, therefore, a strong variability in the rate of unemployment without experiencing, at the same time, significant movements in the real wage.[5]

The model is calibrated not only under the optimal rule, but also under other simpler operational rules. We show that an optimal discretionary monetary policy requires a reaction to inflation which is less aggressive than the one required by strict inflation targeting, but more aggressive than the one required by a policy of full employment stabilization. We also show that an optimal monetary policy may be replicated by a simple Taylor rule. A rule that is capable to deliver impulse response functions similar to the ones implied by the optimal rule, however, implies that the reaction to inflation is quite high relative to the most popular estimates, and a smaller reaction to the output gap.

The paper is organized as follows. In Section 2 we start by introducing indivisible labor in a standard DSNK model with Walrasian labor markets. In Section 3 we develop the unionized labor market model. In Section 4 we study optimal monetary policy and, finally, in Section 4 we calibrate the model under the optimal rule and some simpler policy rules.

[5] Also Gertler and Trigari [19] propose a model where wages and unemployment move consistently with the observed data. They achieve this result, however, by introducing exogenous multiperiod wage contracts.

2 A model with Indivisible Labor and Walrasian Labor Market

2.1 The Representative Household

We consider an economy populated by many identical, infinitely lived worker-households each of measure zero. Households demand a Dixit, Stiglitz [11] composite consumption bundle produced by a continuum of monopolistically competitive firms. In each period households sell labor services to the firms. As in Hansen [22], Rogerson [30] and Rogerson and Wright [31], for each household the alternative is between working a fixed number of hours and not working at all. We assume that agents enter employment lotteries, i.e. sign, with a firm, a contract that commits them to work a fixed number of hours, that we normalize to one, with probability N_t. The contract itself is being traded, so a household gets paid whether it works or not which implies that the firm is providing complete unemployment insurance to the workers. Since all households are identical, all will choose the same contract, i.e. the same N_t. However, although households are ex-ante identical, they will differ ex-post depending on the outcome of the lottery: a fraction N_t of the continuum of households will work and the rest $1 - N_t$ remains unemployed. The allocation of individuals to work or leisure is determined completely at random by a lottery, and lottery outcomes are independent over time.

Before the lottery draw, the expected intratemporal utility function is:

$$\frac{1}{1-\sigma} N_t \left[C_{0,t} \upsilon(0)\right]^{1-\sigma} + \frac{1}{1-\sigma} (1 - N_t) \left[C_{1,t} \upsilon(1)\right]^{1-\sigma} \tag{1}$$

where $C_{0,t}$ is the consumption level of employed individuals, $C_{1,t}$ is the consumption of unemployed individuals, N_t is the ex-ante probability of being employed and $\upsilon(\cdot)$ is the utility of leisure. Since the utility of leisure of employed individuals $\upsilon(0)$ and the utility of leisure of unemployed individuals $\upsilon(1)$ are positive constants, we assume $\upsilon(0) = \upsilon_0$ and $\upsilon(1) = \upsilon_1$. As in King and Rebelo [21], we assume $\upsilon_0 < \upsilon_1$.

If asset market are complete, households can insure themselves against the risk of being unemployed. Under perfect risk sharing we have:

$$C_{0,t}^{-\sigma} \upsilon_0^{1-\sigma} = C_{1,t}^{-\sigma} \upsilon_1^{1-\sigma} \tag{2}$$

which implies that the marginal utilities of consumption are equal for employed and unemployed individuals. Defining the average consumption level as:

$$C_t = N_t C_{0,t} + (1 - N_t) C_{1,t} \tag{3}$$

and given (2), equation (1) can be rewritten as:

$$\frac{1}{1-\sigma}C_t^{1-\sigma}\left[N_t\upsilon_0^{\frac{1-\sigma}{\sigma}}+(1-N_t)\,\upsilon_1^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}. \tag{4}$$

This allows us to write the life-time expected intertemporal utility function of a representative household as:

$$U_t=E_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\frac{1}{1-\sigma}\left[C_t\phi\left(N_t\right)\right]^{1-\sigma}, \tag{5}$$

where $\sigma>1$, and $0<\beta<1$ is the subjective discount rate and where the function

$$\phi\left(N_t\right)=\left[N_t\upsilon_0^{\frac{1-\sigma}{\sigma}}+(1-N_t)\,\upsilon_1^{\frac{1-\sigma}{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \tag{6}$$

can be interpreted as the disutility of employment for the representative household. The elasticity of $\phi\left(N_t\right)$ with respect to its argument is given by $\xi_\phi=\frac{\phi_N(N_t)}{\phi(N_t)}N<0$. The flow budget constraint of the representative household is given by:

$$P_tC_t+P_t^{-1}B_{t+1}\leq W_tN_t+B_t+\Pi_t-T_t \tag{7}$$

where P_t is the corresponding consumption price index (CPI) and W_t is the wage rate. Notice that here a worker is paid according to the probability that it works, not according to the work it does; in other words, the firm is automatically providing full employment insurance to the households. The purchase of consumption goods, C_t, is financed by labor income, profit income Π_t, and a lump-sum transfers T_t from the Government. Households have access to a financial market, where nominal bonds are exchanged. We denote by B_t the quantity of nominally riskless one period bonds carried over from period $t-1$, and paying one unit of the numéraire in period t. The maximization of (5) subject to (7) gives the following::

$$1=\beta R_tE_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\frac{\phi\left(N_t\right)}{\phi\left(N_{t+1}\right)}\right)^{1-\sigma}\frac{P_t}{P_{t+1}}\right] \tag{8}$$

$$\frac{W_t}{P_t}=-C_t\frac{\phi_N\left(N_t\right)}{\phi\left(N_t\right)} \tag{9}$$

where equation (8) is the standard consumption Euler equation and (9) gives us the supply of labor of the representative household.

2.2 The Representative Finished Goods-Producing Firm

The representative finished goods-producing firm uses $Y_t(j)$ units of each intermediate good $j \in [0,1]$ purchased at a nominal price $P_t(j)$ to produce Y_t units of the finished good with the constant returns to scale technology:

$$Y_t = \left[\int_0^1 Y_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \tag{10}$$

where θ is the elasticity of substitution across intermediate goods. Profit maximization yields the following set of demands for intermediate goods:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \tag{11}$$

Perfect competition and free entry drives the finished good-producing firms' profits to zero, so that from the zero profit condition we obtain:

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{12}$$

which defines the aggregate price index of our economy.

2.3 The Representative intermediate Goods-Producing Firm

We abstract from capital accumulation and assume that there is a continuum of intermediate good-producing firms $j \in (0,1)$ which hire N_t units of labor from the representative household and produce $Y_t(j)$ units of the intermediate good using the following technology:

$$Y_t(j) = A_t N_t(j)^{\alpha} \tag{13}$$

where A_t is an exogenous productivity shock. We assume that the $\ln A_t \equiv a_t$ follows follows the autoregressive process

$$a_t = \rho_a a_{t-1} + \hat{a}_t \tag{14}$$

where $\rho_a < 1$ and $\hat{a}_t$ is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_a.

Before choosing the price of its goods, a firm chooses the level of $N_t(j)$ which minimizes its costs, solving the following costs minimization problem:

$$\min_{\{N_t\}} TC_t = (1-\tau) W_t N_t(j)$$

subject to (13), where τ represents an employment subsidy to the firm[6]. The first order condition with respect to $N_t(j)$ is given by:

$$(1-\tau)\frac{W_t}{P_t} = MC_t(j)\,\alpha\frac{Y_t(j)}{N_t(j)}, \tag{15}$$

where $MC_t(j)$ represents firm's j marginal costs. Defining aggregate marginal costs as:

$$MC_t = \frac{(1-\tau)}{\alpha}\frac{W_t}{P_t}\frac{N_t}{Y_t}, \tag{16}$$

equation (15) implies,

$$MC_t = MC_t(j)\,\frac{Y_t(j)}{N_t(j)}\frac{N_t}{Y_t}. \tag{17}$$

2.4 Market clearing

Equilibrium in the goods market of sector j requires that the production of the final good be allocated to expenditure, as follows:

$$Y_t(j) = C_t(j) \tag{18}$$

considering (10) then

$$Y_t = C_t \tag{19}$$

which represents the economy resource constraint. Defining as X the steady state value of a generic variable X_t and by $x_t = \ln X_t - \ln X$ the log-deviation of the variable from its steady state value, then a linear first order approximation of the resource constrained around the steady state is given by:

$$y_t = c_t \tag{20}$$

Since the net supply of bonds, in equilibrium is zero, equilibrium in the bonds market, instead, implies

$$B_t = 0. \tag{21}$$

Labor market clearing implies

$$N_t = \int_0^1 N_t(j)\,dj \tag{22}$$

[6] We assume that the subsidy is covered by a lump sum tax in that the Government runs always a balanced budget.

given equation (11), (13) and (22) the aggregate production function can be expressed as

$$D_t Y_t = A_t N_t^{\alpha} \tag{23}$$

where

$$D_t = \left[\int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\frac{\theta}{\alpha}} dj \right]^{\alpha} \tag{24}$$

is a measure of price dispersion. Given that in a neighborhood of a symmetric equilibrium and up to a first order approximation $D_t \simeq 1$, log-linearizing equation (23) we obtain,

$$y_t = a_t + \alpha n_t. \tag{25}$$

2.5 The First Best Level of Output

The efficient level of output can be obtained by solving the problem of a benevolent planner that maximizes the intertemporal utility of the representative household, subject to the resource constraint and the production function. This problem is analyzed in the Appendix A1, where we show that the efficient supply of labor, in our economy, is given by:

$$\frac{\phi_N(N_t)}{\phi(N_t)} N_t = -\alpha. \tag{26}$$

Log-linearizing (26), and considering (25), we obtain[7]

$$y_t^{Eff} = a_t. \tag{27}$$

2.6 The Flexible Price Equilibrium and the Natural Output

Equilibrium in the labor market is obtained by equating (9) and (16). Substituting (19), this implies

$$-Y_t \frac{\phi_N(N_t)}{\phi(N_t)} = \frac{1}{(1-\tau)} \alpha M C_t \frac{Y_t}{N_t} \tag{28}$$

Under flexible prices, all firms set their prices equal to a constant markup over marginal cost. Assuming that firms mark-up, μ_t^P is constant, under the flexible price-equilibrium firms real marginal costs are costant at their steady state level and therefore given by:

$$MC_t = \frac{1}{1+\mu^P}. \tag{29}$$

[7] See appendix A2.

Considering now the log-linearization of (28) we obtain[8],

$$mc_t = \left[1 + \frac{\phi_N(N)}{\phi(N)}N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right]n_t \tag{30}$$

Considering that $mc_t = 0$,then $n_t = 0$ and, from the aggregate production function, we have that under the flexible price equilibrium:

$$y_t^f = a_t \tag{31}$$

Taking the difference between the log-linearized flexible and efficient output we obtain:

$$y_t^{Eff} - y_t^f = 0 \tag{32}$$

As in the standard DSNK model, when labor market is frictionless the difference between the efficient output (its first best) coincides with its flexible price equilibrium level (its second best) that we have defined as the natural level of output.[9] In other words, what Blanchard and Galì [4] call "the divine coincidence" will hold, since any policy that stabilizies output around its natural level, will stabilize it also around its efficient level. Notice that, in this model we have left the subsidy τ as parametric in order to show that the devine coincidence holds for any possible value of the subsidy. As in the standard case, also in this model an optimal subsidy could be set in order to eliminate the constant distortion induced by monopolistic competition.

2.7 The Phillips Curve

Firms choose $P_t(j)$ in a staggered price setting à la Calvo-Yun [6]. In the appendix A4 we show that, in our decreasing return to scale economy, the solution of the firm's problem is given by:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha mc_t \tag{33}$$

where $\lambda_\alpha = \frac{(1-\psi)(1-\beta\psi)}{\psi}\frac{\alpha}{\alpha+\theta(1-\alpha)}$ and ψ is the probability with which firms reset prices.
Given (25), (30) and (31), marginal costs can be rewritten in terms of the gap between actual and natural output,

$$mc_t = \left(1 + \frac{\phi_N(N)}{\phi(N)}N - \frac{\phi_{NN}(N)N^2}{\phi(N)\alpha}\right)\left(y_t - y_t^f\right) \tag{34}$$

[8] See appendix A3.

[9] Our result is equivalent to the one of Blanchard Galì, where they consider (log) real marginal costs instead of log-deviation from the steady state, and therefore the difference between the efficient and the flexible price output is constant and not zero.

so that, equation (33) can be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \left(1 + \frac{\phi_N (N)}{\phi (N)} N - \frac{\phi_{NN} (N) N^2}{\phi (N) \alpha} \right) x_t \tag{35}$$

where $\lambda_a = \frac{(1-\varphi)(1-\varphi\beta)}{\varphi} \frac{\alpha}{\alpha+\theta(1-\alpha)}$ and

$$x_t = y_t - y_t^f \tag{36}$$

is the output gap with respect to the natural rate of output. As in the standard case there is no trade-off between output stabilization and inflation stabilization, since a central bank that sets inflation to zero will immediately stabilize output.

3 A model with Indivisible Labor and a Unionized Labor Market

3.1 The Monopoly Union

As in the previous subsection the individual labor supply is indivisible. Each firm is endowed with a pool of household from which it can hire. In fact, as in Maffezzoli [25] and Zanetti [38], firms hire workers from a pool composed of infinitely many households so that the individual household member is again of measure zero. Since each household supplies its labor to only one firm, which can be clearly identified, workers try to extract some producer surplus by organizing themselves into a firm-specific trade union. The economy is populated by decentralized trade unions, so that each intermediate goods-producing firm negotiate with a single union $i \in (0, 1)$, which is too small to influence the outcome of the market. Unions negotiate the wage on behalf of their members.

The definition of the a union's objective function has been, in the past, the object of much controversy in labor economics and, as thoroughly discussed by Farber [13], there is no consensus on the appropriate microfoundation of this function. In this paper we choose to follow the approach proposed by Maffezzoli [25], who has recently analyzed the consequences of unionized wage setting in the RBC model. We assume that, consistently with the well-known monopoly union model introduced by Dunlop [12] and Oswald [27], the union chooses the nominal wage rate that maximizes the following welfare function[10]:

$$N_t(i)\frac{W_t}{P_t} + (1 - N_t(i))\frac{W_t^r}{P_t} \tag{37}$$

subject to firms' labor demand (15), where, W_t^r is the reservation wage. The reservation wage, here, is not a direct aggregation of workers reservation wages, but rather reflects the subjective evaluation, by union leaders of workers' disutility of labor.[11] With equation (37) we assume that unions are risk neutral and maximize members average wage. We assume that the reservation wage follows a stochastic process. Denoting by w_t^r the logarithm of W_t^r we assume that:

$$w_t^r = \rho_w w_{t-1}^r + \hat{w}_t^r \tag{38}$$

where $\rho_w < 1$ and w_t^r is a normally distributed serially uncorrelated innovation with zero mean and standard deviation σ_w.

The employment rate and the wage rate are determined in a non-cooperative dynamic game between the unions and the firms. We restrict the attention to Markov strategies, so that in each period union and firm solve a sequence of independent static games. Each union behaves as a Stackelberg leader and each firm as a Stackelberg follower. Once the wage has been chosen, each firm decides the employment rate along its labor demand function. Even if unions are large at the firm level, they are small at the economy level, and therefore they take the aggregate wage as given. The ex-ante probability of being employed is equal to the aggregate employment rate and the allocation of union members to work or leisure is completely random and independent over time.

From the first order conditions of the union's maximization problem wih respect to $W_t(i)$ we have:

$$\frac{W_t(i)}{P_t} = \frac{1}{\alpha}\frac{W_t^r}{P_t}. \tag{39}$$

Since $\frac{1}{\alpha} > 1$, this implies that the real wage rate is always set above the reservation wage.

[10] The utility function above correspondes to the risk neutral analogue of the utilitarian utility function of Oswald [27]. Anderson and Devereux [1] and Pissarides [29] use a similar utility function.

[11] In principle the reservation wage could also represent any unemployment subsidy provided by the government. In this model however, workers insure themselves against unemployment, and therefore these subsidies do not have any reason to exist.

3.2 Households

Similarly to what happens in the previous model, also in this case households enter employment lotteries i.e. sign, with a firm, a contract that commits them to work a fixed number of hours with probability N_t. By entering this contract, which is being traded, a firm provides complete unemployment insurance to the workers.[12] As in the previous model, also in this model the marginal utility of consumption will be equalized across employed and unemployed workers, so that households can be aggregated in a representative household. Therefore, also in this model the life-time expected intertemporal utility function of a representative household can be written again as the problem of maximizing (5) subject to (6) and the flow budget constraint (7).

The model is quite similar to the previous model with walrasian labor markets, except for the fact that now households, in solving their problem, take N_t as given, since the supply of labor is determined by the maximization problem of the monopoly union. The maximization of utility function subject to budget constraint gives the same Euler equation as in the Walrasian model, which is given by equation (8).

3.3 The Flexible Price Equilibrium and the Natural Level of Output

Given that both intermediate goods and finished goods producing firm problem are the same as in the previous problem, the aggregate labor demand function is again given by equation (16). Equating (16) and (39), we aobtain:

$$\frac{1}{\alpha}\frac{W_t^r}{P_t} = \frac{1}{(1-\tau)}\alpha MC_t \frac{Y_t}{N_t}. \tag{40}$$

Since under flexible prices all firms set their prices as a constant markup over marginal costs, which is given by equation (29), we can rewrite equation (40) as:

$$\frac{1}{\alpha}\frac{W_t^r}{P_t} = \frac{1}{(1-\tau)}\alpha \frac{1}{1+\mu^P}\frac{Y_t}{N_t} \tag{41}$$

Considering now the log-linearization of (40) we obtain the following expression for real marginal costs

$$mc_t = n_t - y_t + w_t^r \tag{42}$$

[12] Alternatively, we could consider an institutional arrangement where the union offers actuarally fair insurance contracts, redistributing income between employed and unemployed workers.

where w_t^r is the logarithm of the real reservation wage. Solving (25) for n_t and substituting in (42), we get:

$$mc_t = w_t^r + \frac{1-\alpha}{\alpha} y_t - \frac{1}{\alpha} a_t \tag{43}$$

Considering that $mc_t = 0$, substituting in (43) and solving for y_t we find an expression for the flexible-price level of output, which we define as the natural rate of output for our unionized economy:

$$y_t^f = \frac{1}{1-\alpha} a_t - \frac{\alpha}{1-\alpha} w_t^r \tag{44}$$

Recalling now that the efficient level of output, for our economy with indivisible labor, is given by equation (27) we immediately see that the difference between natural output and efficient output of the unionized economy is given by

$$\bar{y}_t^{Eff} - y_t^f = -\frac{\alpha}{1-\alpha} a_t + \frac{\alpha}{1-\alpha} w_t^r. \tag{45}$$

Unlike what happens in the walrasian model, this difference is not constant, but is a function of the relevant shocks that hit the economy. In this model therefore, as in Blanchard and Galì [4] stabilizing the output gap - the difference between actual and natural output - is not equivalent to stabilizing the *welfare relevant* output gap - the gap between actual and efficient output. In other words, what Blanchard and Galì call "the divine coincidence" will not hold, since any policy that brings the economy to its natural level is not necessarily an optimal policy.

3.4 The Long Run Labor Market Equilibrium: Optimal Subsidy

In this economy, when firms can revise their price at each time, beside the distortion created by monopolistic competition and firms' markup we have a distortion created by the monopoly union wage setting. We assume that, at the steady state, the government uses the employment subsidy to the firms τ, to bring steady state output to its efficient level, i.e. to the level at which

$$\frac{\phi_N(N)}{\phi(N)} N = -\alpha. \tag{46}$$

Since in the unionized economy labor market equilibrium is given by:

$$\frac{1}{\alpha}\frac{W^r}{P} = \frac{1}{(1-\tau)} \alpha \frac{1}{1+\mu^P} \frac{Y}{N}, \tag{47}$$

if the government sets a subsidy such that

$$-\frac{W^r}{P}(1-\tau)(1+\mu^P)\frac{N}{Y}=\frac{\phi_N(N)}{\phi(N)}N\alpha \tag{48}$$

which implies

$$\tau^*=\left[1+\frac{1}{1+\mu^P}\alpha\frac{\phi_N(N)}{\phi(N)}N\frac{Y}{N}\frac{P}{W^r}\right]. \tag{49}$$

then conditions (46) and (47) coincide.

3.5 The IS-Curve

In order to obtain the IS curve we start by log-linearizing[13] around the steady state the Euler equation (8) as:

$$c_t=E_t\{c_{t+1}\}-\frac{1-\sigma}{\sigma}\frac{\phi_N(N)\,N}{\phi(N)}E_t\{\Delta n_{t+1}\}-\frac{1}{\sigma}(\hat{r}_t-E_t\{\pi_{t+1}\}) \tag{50}$$

with $\hat{r}_t=r_t-\varrho$, where $r_t=\ln R_t$ and $\varrho=-\ln\beta$ which is the steady state interest rate all the variables without a subscript are taken at their steady state levels. Given that optimal subsidy setting implies $\frac{\phi_N(N)N}{\phi(N)}=-\alpha$, we can then rewrite equation (50) as

$$c_t=E_t\{c_{t+1}\}+\frac{\alpha(1-\sigma)}{\sigma}E_t\{\Delta n_{t+1}\}-\frac{1}{\sigma}(r_t-E_t\{\pi_{t+1}\}). \tag{51}$$

Given the economy resource constraint (20) and the production function (25), the Euler equation (51) can be written as:

$$y_t=E_t\{y_{t+1}\}-(1-\sigma)E_t\{\Delta a_{t+1}\}-(r_t-E_t\{\pi_{t+1}\}) \tag{52}$$

which represents the IS equation of our simple economy. Given (27) and (36) the IS equation can be rewritten in terms of the output gap as

$$x_t=E_tx_{t+1}+\sigma E_t\{\Delta a_{t+1}\}-(r_t-E_t\{\pi_{t+1}\}).$$

The natural rate of interest, instead, can be expressed as:

$$\hat{r}_t^n=\sigma E_t\{\Delta a_{t+1}\}=\sigma E_t\left\{\Delta y_{t+1}^{Eff}\right\}=-\sigma(1-\rho_a)\,a_t. \tag{53}$$

[13] In order to loglinearize $\phi(N_t)^{1-\sigma}$ we first log-linearize the term N_t obtaining $\phi[N(1+n_t)]^{1-\sigma}$.Applying a first order Taylor expansion, we obtain

$$\phi[N(1+n_t)]^{1-\sigma}=\phi(N)^{1-\sigma}+(1-\sigma)\phi(N)^{-\sigma}\phi_N(N)\,Nn_t$$

Notice that the natural rate of interest depends only on the productivity parameter that characterizes the economy's production function. Given (58), the definition of the efficient equilibrium output (27) and that for the natural interest rate (53) the IS relation can be rewritten as

$$x_t = E_t\{x_{t+1}\} - (r_t - E_t\{\pi_{t+1}\} - r_t^n). \tag{54}$$

3.6 The Phillips Curve

As in the Walrasian case, firms choose $P_t(j)$ in a staggered price setting à la Calvo-Yun [6] and the Phillips curve is again given by (33).
Given (42) and (44), marginal costs can be rewritten in terms of the gap between actual output and its natural level,

$$mc_t = \frac{1-\alpha}{\alpha}\left(y_t - y_t^f\right) \tag{55}$$

so that, equation (33) can now be rewritten as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha}\left(y_t - y_t^f\right) \tag{56}$$

Given the relationship between efficient and natural output, (see eq. (45)), equation (56) can finally be expressed as:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha} x_t - \lambda a_t + \lambda w_t^r \tag{57}$$

where

$$x_t = y_t - y_t^{Eff} \tag{58}$$

is the output gap with respect to the efficient natural rate of output. We can now state:

Result 1. *In a unionized labor market economy the "divine coincidence" does not hold, i.e., stabilizing inflation is not equivalent to stabilizing the output gap defined as the deviation of output from the efficient output. A positive (negative) productivity shock has a negative (positive) effect on inflation, while a cost push shock has an effect of the same size but with the opposite sign on inflation.*

This result depends on the existence of a real distortion in the economy, beside the one induced by monopolistic competition, and the nominal distortion caused by firms' staggered price setting. When a productivity shock hits the economy, efficient output, given by equation (27), increases by the same amount. Natural output instead (i.e., the level of output that would prevail in a flexible price equilibrium) increases more than proportionally so that the difference between efficient output and natural output decreases. This is due to the fact that in a unionized economy, following a productivity shock, real wages remain constant and therefore do not offset the effects of the shock on real marginal cost (see equation (55)).

Because of staggered price adjustment we know that inflation is proportional to real marginal costs which, in turn, because of monopolistic competition (see equation (56)) are proportional to the difference between actual and natural output. As we will see in the following paragraphs, a Central Bank pursuing an optimal monetary policy will decide to stabilize the distance between output and its efficient level. If the difference between efficient and flexible output were constant, as in the standard model with Walrasian labor markets, stabilizing the gap between actual and natural output would be equivalent to stabilizing the gap between actual and efficient output. In this case stabilizing the output gap with respect to the natural output would be sufficient to stabilize inflation. In our unionized economy, instead, the natural level of output differs from the efficient level because of productivity and cost-push shocks. As it is evident from equation (57), if the Central Bank stabilizes output around the efficient level, inflation will be completely vulnerable to productivity and cost-push shocks; in other words the output gap is no longer a sufficient statistics for the effect of real activity on inflation.

One interesting aspect of this model is that we are able to express the Phillips curve in its more traditional form, i.e. in terms of unemployment. From equations (25), (27) and (58) we obtain in fact that

$$n_t = \frac{x_t}{\alpha} \tag{59}$$

Expressing the rate of unemployment as $U_t = 1 - N_t$ and log linearizing around the steady state we obtain

$$u_t = -\frac{\eta}{\alpha} x_t \tag{60}$$

where $\eta = \frac{N}{1-N}$. We can therefore rewrite the Philllips curve as

$$\pi_t = \beta E_t \pi_{t+1} - \frac{\lambda_a (1-\alpha)}{\eta} u_t - \lambda_a a_t + \lambda_a w_t^r. \tag{61}$$

The relationship between unemployment and the output gap that we find in this model, therefore, allows us to consider, indifferently, the output gap and the unemployment rate as policy objectives for the central bank.

4 Optimal Monetary Policy

In the appendix A5 we show that also for the non-separable preferences assumed in our framework, consumers' utility can be approximated up to the second order by a quadratic equation of the kind:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} E_t \sum_{t=0}^{\infty} \left[\pi_{H,t+k}^2 + \frac{\lambda_a}{\theta\sigma} x_{t+k}^2 \right] + \bigcirc\left(\|\alpha\|^3\right) \tag{62}$$

where $\tilde{U}_{t+k} = U_{t+k} - \bar{U}_{t+k}$ is the deviation of consumers' utility from the level achievable in the frictionless equilibrium, and θ is the elasticity of substitution between intermediate goods, which are used as input in the final good sector. Notice that, the relative weights assigned to inflation and to the output gap are linked to the structural parameters reflecting preferences and technology.

4.1 Discretion

If the Central Bank cannot credibly commit in advance to a future policy action or a sequence of future policy actions, then the optimal monetary policy is discretionary, in the sense that the policy makers choose in each period the value to assign to the policy instrument, that here we assume to be the short-term nominal interest rate $\hat{r}_t$. In order to do so, the Central Bank maximizes the welfare-based loss function (62), subject to the Phillips curve (57), taking all expectations as given. Therefore, the Central Bank chooses the level of inflation and output gap that maximize:

$$\tilde{W}_t = -\frac{U_{Y,t}}{2}\left(\pi_t^2 + \frac{\lambda_a}{\theta\sigma} x_t^2\right) + H_t \tag{63}$$

subject to

$$\pi_t = \lambda_a \frac{1-\alpha}{\alpha} x_t - \lambda_a a_t + \lambda_a w_t^r + h_t \tag{64}$$

where $H_t = -\frac{U_{Y,t}}{2} E_t \sum_{t=1}^{\infty} \left[\frac{\theta}{\lambda_a}\pi_{t+k}^2 + \frac{1}{\sigma} x_{t+k}^2\right]$, $h_t = \beta E_t \pi_{t+1}$.

The first order conditions imply:

$$x_t = -\frac{1-\alpha}{\alpha}\theta\sigma\pi_t. \tag{65}$$

Substituting into (64) we obtain :

$$\pi_t = \frac{1}{\Omega}\left(\beta E_t \pi_{t+1} - \lambda_a a_t + \lambda_a w_t^r\right) \tag{66}$$

where $\Omega = 1 + \lambda \left(\frac{1-\alpha}{\alpha}\right)^2 \theta\sigma$.

Iterating forward (66),

$$\pi_t = -\frac{\lambda_a}{\Omega} E_t \sum_{i=0}^{\infty} \left(\frac{\beta}{\Omega}\right)^i \left(a_{t+i} - w_{t+i}^r\right) \tag{67}$$

and

$$E_t \pi_{t+1} = -\frac{\lambda_a}{\Omega} E_t \sum_{i=0}^{\infty} \left(\frac{\beta}{\Omega}\right)^i \left(a_{t+i+1} - w_{t+i+1}^r\right). \tag{68}$$

Given that,

$$E_t \left\{a_{t+i+1}\right\} = \rho_a^i a_t \tag{69}$$

$$E_t \left\{w_{t+i+1}^r\right\} = \rho_w^i w_t^r \tag{70}$$

(67) and (68) can be rewritten as,

$$\pi_t = -\frac{\lambda_a}{\Omega - \beta\rho_a} a_t + \frac{\lambda_a}{\Omega - \beta\rho_w} w_t^r \tag{71}$$

$$E_t \pi_{t+1} = -\frac{\lambda_a \rho_a}{\Omega - \beta\rho_a} a_t + \frac{\lambda_a \rho_w}{\Omega - \beta\rho_w} w_t^r \tag{72}$$

Notice that we can express current inflation as a function of the relevant shocks a_t and w_t^r. A positive productivity shock requires a decrease in inflation and a positive cost push shock requires an increase in inflation. Because of rational expectations we have a similar result for expected inflation.

Using (71), (72) and the definition of the natural interest rate (53) we can also rewrite expected inflation as:

$$E_t \pi_{t+1} = \rho_w \pi_t + \frac{(\rho_a - \rho_w)}{\sigma(1-\rho_a)} \frac{\lambda_a}{\Omega - \beta\rho_a} \hat{r}_t^n \tag{73}$$

The optimal level of inflation can be implemented by the Central Bank by setting the nominal interest rate. The interest rate rule can be obtained by substituting (65), (71) and (72) into the IS curve (54), in which case we obtain:

$$\begin{aligned} \hat{r}_t^* = &-\left[\left[1 + \left(\frac{1-\rho_a}{\rho_a}\right)\left(\frac{1-\alpha}{\alpha}\right)\theta\sigma\right]\left(\frac{\lambda_a \rho_a}{\Omega - \beta\rho_a}\right) + \sigma(1-\rho_a)\right] a_t + \\ &+ \left[1 + \left(\frac{1-\rho_w}{\rho_w}\right)\left(\frac{1-\alpha}{\alpha}\right)\theta\sigma\right]\left(\frac{\lambda_a \rho_w}{\Omega - \beta\rho_w}\right) w_t^r \end{aligned} \tag{74}$$

We can therefore state

Result 2. *Under discretion an optimal monetary policy requires a decrease in the nominal interest rate following a positive productivity shock and an increase in the nominal interest rate following a positive reservation wage shock.*

An interest rate rule that implements such optimal policy, can be found using (73) and (72). In this case we obtain:

$$\hat{r}_t^* = \left[1 + \left(\frac{1-\rho_w}{\rho_w}\right)\frac{1-\alpha}{\alpha}\theta\sigma\right] E_t\pi_{t+1} + \left[1 + \frac{(\rho_w - \rho_a)}{\rho_w (1-\rho_a)}\frac{\lambda_a\theta}{\Omega - \beta\rho_a}\frac{1-\alpha}{\alpha}\right]\hat{r}_t^n. \tag{75}$$

In Appendix A6 we show that under rule (75) equilibrium is determinate. Assuming, as a particular case $\rho_a = \rho_w = \rho$, equation (75) becomes

$$\hat{r}_t^* = \left[1 + \left(\frac{1-\rho}{\rho}\right)\frac{1-\alpha}{\alpha}\theta\sigma\right] E_t\pi_{t+1} + r_t^n \tag{76}$$

We can now state:

Result 3. *Optimal monetary policy under discretion requires a more than proportional increase in the nominal interest rate following an increase in the expected rate of inflation. However, an increase in the natural rate of interest implies a proportional increase in the nominal interest rate if and only if* $\rho_a = \rho_w = 1$. *Otherwise an increase in the natural rate implies a more than proportional increase in the nominal interest rate if* $\rho_w > \rho_a$ *and a less than proportional increase if* $\rho_w < \rho_a$.

Result 3 is quite interesting. As in the standard DSNK model, optimality requires that the Central Bank respond to increasing inflationary expectations by raising more than proportionally nominal interest rates. In other words, also for our unionized economy, the Taylor principle applies. The optimal response of the nominal interest rate to an increase in the natural rate of interest, instead, is different from the one that is usually obtained in the "standard" DSNK model. When labor markets are perfectly competitive, in fact, an increase in the natural rate of interest affects only the IS curve, and this implies that when the natural rate of interest increases, the nominal interest rate must be raised by the same amount. When wages are set by monopoly unions, instead, the natural rate of interest, which is basically determined by the productivity parameter that characterizes the economy, affects also the Phillips curve. This extra effect on the Phillips curve requires a further response of the central bank.

Notice that (64) and (60) together imply

$$u_t = -\frac{\eta(1-\alpha)\theta\sigma\lambda_a}{\alpha^2(\Omega-\beta\rho_a)}a_t + \frac{\eta(1-\alpha)\theta\sigma\lambda_a}{\alpha^2(\Omega-\beta\rho_w)}w_t^r. \tag{77}$$

Given the log-linearization of equation (39), we can now state

Result 4. *Under an optimal discretionary monetary policy a productivity shock will induce a change in the rate of unemployment without affecting the real wage rate.*

This result is quite important since it is consistent with a well known fact in macroeconomics, i.e. the relatively smooth behavior of wages along the business cycle together with the relatively volatile behavior of unemployment. In this simple model, wages move only when there is a shock in the reservation wages of households. Productivity shocks imply some degree of volatility in unemployment while real wages remain constant. Wages, in the simple set up we consider in this paper, are probably too rigid, as we assume that all markets are unionized. Neverthess, the model makes an interesting point, i.e. that the behavior of monopoly unions, in itself, is able to generate a dynamics of wages and unemployment that is roughly consistent with the one typically observed in the real economy

4.2 Constrained Commitment

Let us assume that the Central Bank follows a rule for the target variable x_t which depends on the fundamental shocks w_t^r and r_t^n. In order to obtaine an analytical solution we assume the following feedback rule equation

$$x_t^c = \omega\left(a_t - w_t^r\right) \qquad \forall t \tag{78}$$

and we also assume

$$\rho_a = \rho_w = \rho \tag{79}$$

where $\omega > 0$ is the coefficient of the feedback rule and the variable x_t^c is the value of x_t conditional on commitment to the policy.

Before solving the Central Bank problem under constrained commitment, we iterate forward the Phillips curve (57) and we obtain:

$$\pi_t^c = \left(1 - \frac{1-\alpha}{\alpha}\omega\right)\frac{\lambda_a}{1-\beta\rho}\left(w_t^r - a_t\right) \tag{80}$$

which, considering equation (78), can be rewritten as:

$$\pi_t^c = \lambda_a \frac{1-\alpha}{\alpha}\frac{1}{1-\beta\rho}x_t^c - \frac{\lambda_a}{1-\beta\rho}\left(a_t - w_t^r\right) \tag{81}$$

Notice that, in this case, a one percent contraction of x_t^c reduces π_t^c by the amount $\lambda_a \frac{1-\alpha}{\alpha} \frac{1}{1-\beta\rho_w}$, while under discretion, reducing x_t by one percent only produces a fall in π_t of $\lambda_a \frac{1-\alpha}{\alpha} < \lambda_a \frac{1-\alpha}{\alpha} \frac{1}{1-\beta\rho_w}$. As in the case analyzed by Clarida, Galì and Gertler [8], the Central bank will enjoy an improved trade off, due to the fact that commitment to a policy rule affects expectations on the future course of the output gap.

Given (78) and (80) we can now write the problem of the Central Bank under constrained commitment as follows:

$$W_t = E_t \sum_{t=0}^{\infty} \beta^t \tilde{U}_{t+k} = -\frac{U_{Y,t}}{2} \left[\left(\pi_t^c\right)^2 + \frac{\lambda_a}{\theta\sigma} \left(x_t^c\right)^2 \right] E_t \sum_{i=0}^{\infty} \left(\frac{w_{t+i} - a_{t+i}}{w_t^r - a_t} \right)^2 \tag{82}$$

subject to equation (81). The first order conditions imply:

$$x_t^c = -\frac{1-\alpha}{\alpha} \theta\sigma \frac{1}{1-\beta\rho} \pi_t^c \tag{83}$$

Since $\frac{1-\alpha}{\alpha}\theta\sigma\frac{1}{1-\beta\rho} < \frac{1-\alpha}{\alpha}\theta\sigma$ this implies that commitment to a rule makes it optimal, for the central bank, to induce a greater contraction of output in response to an increase in inflation. Substituting (83) into the Phillips curve and iterating forward we obtain:

$$\pi_t = -\frac{\lambda_a}{\Omega^c \left(1-\beta\rho\right)} \left(a_t - w_t^r\right) \tag{84}$$

and

$$E_t \pi_{t+1} = -\frac{\lambda_a \rho}{\Omega^c \left(1-\beta\rho_a\right)} \left(a_t - w_t^r\right) \tag{85}$$

where $\Omega^c = 1 + \lambda \left(\frac{1-\alpha}{\alpha} \frac{1}{1-\beta\rho} \right)^2 \theta\sigma > \Omega$. The interest rate rule can be obtained by substituting (83), (84) and (85) into the IS curve (54), in which case we obtain:

$$\begin{aligned} r_t^{c*} = & -\left[\left[1 + \left(\frac{1-\rho}{\rho} \right) \left(\frac{1-\alpha}{\alpha} \right) \frac{\theta\sigma}{1-\beta\rho} \right] \left(\frac{\lambda_a \rho}{\Omega^c \left(1-\beta\rho\right)} \right) + \sigma\left(1-\rho\right) \right] a_t + \\ & + \left[1 + \left(\frac{1-\rho}{\rho} \right) \left(\frac{1-\alpha}{\alpha} \right) \frac{\theta\sigma}{1-\beta\rho} \right] \left(\frac{\lambda_a \rho}{\Omega^c \left(1-\beta\rho\right)} \right) w_t^r \end{aligned} \tag{86}$$

Using equation (83), the one of the Phillips curve and the one of the IS-curve we find the following optimal instrument rule:

$$\hat{r}_t^{c*} = \left[1 + \left(\frac{1-\rho}{\rho} \right) \frac{1-\alpha}{\alpha} \frac{\theta\sigma}{1-\beta\rho} \right] E_t \pi_{t+1} + \hat{r}_t^n. \tag{87}$$

Since $\frac{1}{1-\beta\rho} > 1$, we have the following

Result 4. *Under commitment to a simple feedback rule, when $\rho_a = \rho_w = \rho$, an optimal interest rule requires that, in reacting to an increase in expected inflation, the nominal interest rate must be increased more than in the case of discretion .*

5 Calibration

In order to illustrate the qualitative properties of our model we analyze the responses of the output gap, inflation and the nominal interest rate to a productivity shock and a reservation wage shock. We start by the discussing our calibration of the model parameters, summarized in table 1.

We take each period to correspond to a quarter. For the parameters describing preferences, we set, as in Maffezzoli [25] the elasticity of intertemporal substitution $\sigma = 2$. The ouput elasticity of labor, $\alpha = 0.72$, is based on the estimate of Christoffel et al. [10]. The discount factor β, the Calvo parameter φ, and the elasticity of substitution among intermediate goods θ are set at values commonly found in the literature (for example in Galì [15]). In particular we set $\beta = 0.99$, $\varphi = 0.75$, which implies an average price duration of one year, and finally $\theta = 11$, which is consistent with a 10 percent markup in the steady state. The persistence of the technology shock ρ_a and its standard deviation σ_a are set as in Amato and Laubach [3], i.e. $\rho_a = .93$ and $\sigma_a = 0.687$. The persistence of the wage shock and its standard deviation are assumed to be as the persistence of a cost-push shock and its standard deviation, i.e., equal to 0.7 and 0.07, as estimated by Ireland [23]. As in Maffezzoli [25] $N = 0.62$.

In figure (1), (2), (3) and (4) we consider the effect of a one standard deviation, negative productivity shock on the nominal interest rate, inflation, the output gap and the rate of unemployment under different types of monetary policy. Under a policy aimed at stabilizing output, which we obtain by setting $x_t = 0$, a negative productivity shock will imply an increase in the nominal interest rate by more that 0.2 percentage points and will have a very large effect on the rate of inflation, that will initially increase by almost 0.15 per cent. Under a policy aimed at stabilizing inflation, i.e. a policy achieved by setting $\pi_t = 0$, a productivity shock will imply a much smaller response of the interest rate, which will initially increase by 0.22 percentage points. The output gap will increase by 1.7 percentage points and the rate of unemployment will fall by almost 4 percentage points.

As we could expect, the optimal policy under discretion, which is described in figure (3), stirs an intermediate course between these two extreme policies. A negative productivity shock will require an increase in the nominal interest rate of 0.2% and an increase in inflation of almost 0.1%. Initially output will fall by 0.7% and the rate of unemployment will have an initial increase of about 1.5 percentage points. An optimal monetary policy, therefore, will take into account the trade-off that exists between inflation stabilization and output stabilization: as a response to a productivity shock output will decrease less than in the extreme inflation targeting case and inflation will also increase less than in the policy aimed at fully stabilizing output and unemployment.

In figure (4) we report the results of an exercise aimed at replicating the optimal policy through a simple Taylor rule. We found that a rule that approximates quite well the optimal monetary policy (i.e. that achieves a response of the major variables quite close to the one achieved by our economy under the optimal discretional monetary policy) is given by

$$i_t = 2.5\pi_t + 0.05x_t \tag{88}$$

Notice that this rule implies a stronger response to inflation and a weaker response to the output gap than the ones found in the literature. Clarida Galì and Gertler [8] for the U.S. economy in the Volcker-Greenspan era, for example, found a response to inflation equal to 2.15 and a response to unemployment equal to 0.93 . Smets and Wouters [37] for the European economy found a response to inflation equal to 1.658 and a response to output of 0.145.

In figure (5)-(8) we show the responses of the interest rate, output and unemployment to a one standard deviation shock to the reservation wage. The responses are quite similar to those obtained for the negative productivity shock although, given the smaller persistence of the wage shock, the effect lasts for fewer quarters.

6 Conclusions

We have considered in this paper a DSNK model where labor is indivisible and where wages are set by monopoly unions. We found that, with respect to the standard DSNK framework, our model gives a more satisfactory description of the reality of modern industrialized economy, especially of those where collective bargaining dominates the labor market. In a unionized economy, significant trade-offs between stabilizing inflation and stabilizing unemployment arise, in response to technology and exogenous wage shocks. Because of real wage rigidity, an optimizing central bank must respond to negative (positive) technology shocks by increasing (decreasing) the interest rate and, similarly, must respond to exogenous increases in unions' reservation wage with an interest rate increase. Interestingly, if we consider an optimal instrument rule, an optimizing central bank not only will increase the interest rate more than proportionately in response to an increase in future expected inflation, but will also react to increases in the natural rate that are not necessarily one to one. The model is also capable of accounting for the greater volatility of unemployment relative to the wage volatility that is usually found in the data.

Even though we think that the model represents a step forward in the analysis of optimal monetary policy of contemporary economies, we are aware that it gives a representation of the working of the labor market which is still quite crude. For the sake of simplicity, many other market imperfections, like search and matching costs and firing costs are absent. Moreover, the model assumes thatl the whole labor market is unionized. A more realistic representation of the challenges provided to monetary policy by different institutional settings in the labor market would imply considering, for example, a two sector model where only a fraction of workers belong to unions and are covered by collective agreements. We leave however these challenges to future research.

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A Technical Appendix

A.1 The Ramsey Problem

We consider a social planner which maximizes the representative household utility subject to the economy resource constraint and production function as follows:

$$\max_{N_t} U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} \phi(N_t)^{1-\sigma} \tag{A1}$$

$$s.t. \tag{A2}$$

$$C_t = Y_t \tag{A3}$$

$$Y_t = A_t N_t^{\alpha}$$

Substituting the constraint into the utility function the problem is:

$$\max_{N} \frac{1}{1-\sigma} \left(A_t N_t^{\alpha}\right)^{1-\sigma} \phi(N_t)^{1-\sigma} \tag{A4}$$

the first order condition requires

$$\left(A_t N_t^{\alpha}\right)^{-\sigma} \alpha \frac{Y_t}{N_t} \phi(N_t)^{1-\sigma} = -\left(A_t N_t^{\alpha}\right)^{1-\sigma} \phi(N_t)^{-\sigma} \phi_N(N_t) \tag{A5}$$

simplifying

$$Y_t \frac{\phi_N(N_t)}{\phi(N_t)} = -\alpha \frac{Y_t}{N_t} \tag{A6}$$

Multiplying both sides of equation for $\frac{N_t}{Y_t}$ we find

$$\frac{\phi_N(N_t)}{\phi(N_t)} N_t = -\alpha \tag{A7}$$

and

$$\frac{U_N}{U_C} \frac{N_t}{Y_t} = \frac{\phi_N(N_t)}{\phi(N_t)} N_t = -\alpha \tag{A8}$$

A.2 Derivation of the Efficient Output

We consider the Ramsey solution (A17)

$$\phi_N(N_t) N_t = -\alpha \phi(N_t) \tag{A9}$$

in order to find an equation for the efficient output we first log-linearizing equation (A9) around the steady state, which implies

$$\left[\phi_N(N) + \phi_{NN}(N) N n_t\right] N (1 + n_t) = -\alpha\left(\phi(N) + \phi_N(N) N n_t\right) \tag{A10}$$

which can be rewritten as

$$\phi_N(N) N + \phi_N(N) N n_t + \phi_{NN}(N) N^2 n_t = -\alpha\left(\phi(N) + \phi_N(N) N n_t\right) \tag{A11}$$

Considering the steady state equation

$$\phi_N(N) N_t = -\alpha\phi(N) \tag{A12}$$

and collecting terms in n_t we obtain

$$\left(1 + \frac{\phi_N(N) N_t}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N)}\left(\frac{\phi_N(N) N_t}{\phi(N)}\right)^{-1}\right) n_t = 0 \tag{A13}$$

given that $1 + \frac{\phi_N(N)N_t}{\phi(N)} + \frac{\phi_{NN}(N)N^2}{\alpha\phi(N)}\left(\frac{\phi_N(N)N_t}{\phi(N)}\right)^{-1} \neq 0$ we require,

$$n_t = 0 \tag{A14}$$

and then from the aggregate production function we obtain equation (27) in the text.

A.3 Derivation of the Flexible Price Equilibrium Output in the Walrasian Model

Let us rewrite equation (28) as:

$$\phi_N(N_t) N_t = -\frac{\alpha}{(1-\tau)} MC_t \phi(N_t) \tag{A15}$$

at the steady state becomes,

$$\phi_N(N) N = -\frac{1}{(1-\tau)}\alpha MC\phi(N) \tag{A16}$$

Then log-linearizing,

$$\left[\phi_N(N) + \phi_{NN}(N) N n_t\right] N (1 + n_t) = -\frac{\alpha MC}{(1-\tau)}(1 + mc_t)\left[\phi(N) + \phi_N(N) N n_t\right] \tag{A17}$$

considering the steady state equation (A16) we have,

$$mc_t = \left(1 + \frac{\phi_N(N) N}{\phi(N)} + \frac{\phi_{NN}(N) N^2}{\phi(N)\alpha}\right) n_t \tag{A18}$$

given equation (A14) and considering the aggregate production function we obtain equation (31) in the text.

A.4 Derivation of the Phillips Curve

Following Calvo [6] we assume that each firm may reset its price with probability $1-\varphi$ each period, indipendently from the time elapsed since the last adjustment. This means that each period a measure $1-\varphi$ of firms reset their price, while a fraction φ of them keep their price unchanged. The law of motion of the aggregate price is given by:

$$\ln P_t = \varphi \ln P_{t-1} + (1-\varphi) \ln P_t^* \tag{A19}$$

which implies

$$\pi_t = (1-\varphi) \ln \left(\frac{p_t^*}{P_{t-1}} \right) \tag{A20}$$

where $\ln P_t^*$ denotes the (log) price set by a firm i adjusting its price in period t. Under Calvo [6] price-setting structure $p_{t+k}(i) = p_t^*$ with probability φ^k for $k = 0, 1, 2, ...$, hence firms have to be forward-looking.

Given that the individual firm technology is characterized by decreasing return to scale, the optimal price setting rule should take into account that marginal cost is no longer common across firms. In particular, in the neighborhood of the zero inflation steady state, we have the following price-setting rule:

$$\ln P_t^*(i) = \mu^P + (1-\beta\varphi) \sum_{k=0}^{\infty} (\beta\varphi)^k E_t \left\{ mc_{t,t+k}^n \right\} \tag{A21}$$

where $mc_{t,t+k}^n$ is the log-linearized nominal marginal cost in period $t+k$ of a firm which last set its price in period t. Considering the equation of real marginal cost and the one of the aggregate production function,

$$\begin{aligned} MC_{t,t+k} &= (1-\tau) \frac{(W_{t+k}/P_{t+k})}{\alpha (Y_{t,t+k}/N_{t,t+k})} \\ &= MC_{t+k} \frac{(Y_{t+k}/N_{t+k})}{(Y_{t,t+k}/N_{t,t+k})} \\ &= MC_{t+k} \left(\frac{Y_{t+k}}{Y_{t,t+k}} \right)^{\frac{1-\alpha}{\alpha}} \\ &= MC_{t+k} \left(\frac{P_t^*}{P_{t+k}} \right)^{\theta \frac{1-\alpha}{\alpha}} \end{aligned} \tag{A22}$$

taking the logs

$$\ln MC_{t,t+k} = \ln MC_{t+k} - \theta \frac{1-\alpha}{\alpha} \ln \left(\frac{P_t^*}{P_{t+k}} \right) \tag{A23}$$

Considering that all firms resetting prices in period t will choose the same price P_t^+ we can rewrite equation (A21) as,

$$\begin{aligned}\ln\left(\frac{P_t^*(i)}{P_{t-1}}\right) &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^n - \ln P_{t-1}\right\} \\ &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^n\right\} + \\ &\quad + \sum_{k=0}^{\infty}(\beta\varphi)^k\left\{\pi_{t+k}\right\} \qquad \text{(A24)}\end{aligned}$$

substituting equation (A5) which can be rewritten as

$$\begin{aligned}\ln\left(\frac{P_t^*(i)}{P_{t-1}}\right) &= \mu^P + (1-\beta\varphi)\sum_{k=0}^{\infty}(\beta\varphi)^k E_t\left\{\ln MC_{t,t+k}^n - \theta\frac{1-\alpha}{\alpha}\ln\left(\frac{P_t^*}{P_{t+k}}\right)\right\} \\ &\quad + \sum_{k=0}^{\infty}(\beta\varphi)^k\left\{\pi_{t+k}\right\} \qquad \text{(A25)}\end{aligned}$$

then

$$\ln P_t^*(i) - \ln P_{t-1} = \mu^P + \beta\varphi E_t\left\{\ln P_{t+1}^* - \ln P_t\right\} + (1-\beta\varphi)\ln MC_t \qquad \text{(A26)}$$

Combining (A26) with (A19) we obtain

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_\alpha mc_t \qquad \text{(A27)}$$

as in the text.

A.5 The Welfare-Based Loss Function

A second-order Taylor expansion of the period utility around the efficient equilibrium yields,

$$\begin{aligned}U_t = \bar{U}_t + \bar{U}_{\bar{C},t}\bar{C}_t\tilde{C}_t + \frac{1}{2}\bar{U}_{\bar{C}\bar{C},t}\bar{C}_t^2\tilde{C}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 + \\ + \bar{U}_{\bar{C}\bar{N},t}\bar{C}_t\bar{N}_t\tilde{C}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \qquad \text{(A28)}\end{aligned}$$

where the generic $\tilde{X} = \ln\left(X/\bar{X}_t\right)$ denotes log-deviations from the efficient equilibrium and $\bar{X}_t$ denotes the value of the variable under efficient equilibrium. Moreover, we denote as $\bar{x}_t = \ln\left(\frac{\bar{X}_t}{X}\right)$.

Considering the flexible prices economy resource constraint,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t\tilde{Y}_t + \frac{1}{2}\bar{U}_{\bar{Y}\bar{Y},t}\bar{Y}_t^2\tilde{Y}_t^2 + \bar{U}_{\bar{N},t}\bar{N}_t\tilde{N}_t + \frac{1}{2}\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2\tilde{N}_t^2 + + \bar{U}_{\bar{Y}\bar{N},t}\bar{Y}_t\bar{N}_t\tilde{Y}_t\tilde{N}_t + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A29)}$$

Collecting terms yields

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \begin{bmatrix} \tilde{Y}_t + \frac{\bar{U}_{\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t + \frac{1}{2}\frac{\bar{U}_{\bar{Y}\bar{Y},t}}{\bar{U}_{\bar{Y},t}}\bar{Y}_t\tilde{Y}_t^2 + \\ +\frac{1}{2}\frac{\bar{U}_{\bar{N}\bar{N},t}\bar{N}_t^2}{\bar{U}_{\bar{Y},t}\bar{Y}_t}\tilde{N}_t^2 + \frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}}\tilde{Y}_t\tilde{N}_t \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A30)}$$

Considering that, $\frac{\bar{U}_{\bar{Y}\bar{N},t}\bar{N}_t}{\bar{U}_{\bar{Y},t}} = \frac{\phi_{\bar{N},t}(\bar{N}_t)\bar{N}_t}{\phi(\bar{N}_t)} = -(1-\sigma)\alpha$, we have,

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left[\frac{\phi_{NN}(\bar{N}_t)}{\phi(\bar{N}_t)} - \sigma\left(\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A31)}$$

It can be shown that $\frac{\phi_{NN,t}(\bar{N}_t)}{\phi(\bar{N}_t)} = \frac{2\sigma-1}{\sigma}\left(\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\right)^2$, hence

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 + (1-\sigma)\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\bar{N}_t\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left[\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\left(\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\right)^2\right]\bar{N}_t^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A32)}$$

$$U_t = \bar{U}_t + \bar{U}_{\bar{Y},t}\bar{Y}_t \begin{bmatrix} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\alpha\tilde{Y}_t\tilde{N}_t + \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{bmatrix} + \bigcirc\left(\|\alpha\|^3\right) \quad \text{(A33)}$$

We now take a first-order expansion of the term $\bar{U}_{\bar{Y},t}\bar{Y}_t$ around the steady state.

$$\begin{aligned} \bar{U}_{\bar{Y},t}\bar{Y}_t &= U_Y\left(1 + (1-\sigma)\bar{y}_t + (1-\sigma)\frac{\phi_N(N)}{\phi(N)}N\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \\ &= U_Y\left(1 + (1-\sigma)\bar{y}_t - (1-\sigma)\alpha\bar{n}_t\right) + \bigcirc\left(\|\alpha\|^2\right) \end{aligned} \quad \text{(A34)}$$

$$\frac{\phi_N(\bar{N}_t)}{\phi(\bar{N}_t)}\bar{N}_t = \frac{\phi_N(N)}{\phi(N)}N + \Gamma_n\bar{n}_t + + \bigcirc\left(\|\alpha\|^2\right) \quad \text{(A35)}$$

where $\Gamma_n = \left(\frac{\phi_N(N)N}{\phi(N)} + \frac{\phi_{NN}(N)N^2}{\phi(N)} - \frac{\phi_N(N)^2N^2}{\phi(N)^2}\right)$

$$\left(\frac{\phi_N(\bar{N}_t)\bar{N}_t}{\phi(\bar{N}_t)}\right)^2 = \left(\frac{\phi_N(N)N^2}{\phi(N)}\right)^2 + \Lambda_n\bar{n}_t + \bigcirc\left(\|\alpha\|^2\right) \quad \text{(A36)}$$

where $\Lambda_n = 2\left(\frac{\phi_N(N)\phi_{NN}(N)N}{\phi(N)^2} + \left(\frac{\phi_N(N)N}{\phi(N)}\right)^2 - \left(\frac{\phi_N(N)}{\phi(N)}\right)^3 N\right)$
given that $\bar{n}_t = 0$, and that $\frac{\phi_N(N)N}{\phi(N)} = -\alpha$, substituting (A35) and (A36) into the Welfare function,

$$U_t = \bar{U}_t + U_Y\left(1 + (1-\sigma)\,\bar{y}_t\right)\left[\begin{array}{c} \tilde{Y}_t - \alpha\tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - \alpha(1-\sigma)\,\tilde{Y}_t\tilde{N}_t \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\alpha^2\tilde{N}_t^2 \end{array}\right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A37}$$

Given the aggregate production function and that the log-deviations of the price dispersion index $-d_t = \tilde{Y}_t - \alpha\tilde{N}_t$ are of second-order, and that:

$$\tilde{Y}_t^2 = \alpha^2\tilde{N}_t^2 \qquad n_t\alpha\tilde{N}_t = n_t\tilde{Y}_t \qquad y_t\alpha\tilde{N}_t = y_t\tilde{Y}_t \qquad \tilde{Y}_t\alpha\tilde{N}_t = \tilde{Y}_t^2$$

considering only terms up to the second-order we have:

$$U_t = \bar{U}_t + U_Y\left[\begin{array}{c} \tilde{Y}_t - \tilde{N}_t - \frac{\sigma}{2}\tilde{Y}_t^2 - (1-\sigma)\,\tilde{Y}_t^2 \\ +\frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - \sigma\right)\tilde{Y}_t^2 \end{array}\right] + \bigcirc\left(\|\alpha\|^3\right) \tag{A38}$$

$$\begin{aligned} \tilde{U}_t &\equiv U_t - \bar{U}_t = -U_Y\left\{d_t + \frac{1}{2}\left(\frac{2\sigma-1}{\sigma} - 2\right)\tilde{Y}_t^2\right\} + \bigcirc\left(\|\alpha\|^3\right) \\ &= U_t - \bar{U}_t = -U_Y\left\{d_t - \frac{1}{2\sigma}\tilde{Y}_t^2\right\} + \bigcirc\left(\|\alpha\|^3\right) \end{aligned} \tag{A39}$$

As proven by Galì and Monacelli [18], the log-index of the relative-price distortion is of second-order and proportional to the variance of prices across firms, which implies that:

$$d_t = \ln\left(\int_0^1 \left(\frac{P_t(i)}{P_t}\right) di\right) = \frac{\theta}{2} var_i\left\{p_t(i) + \bigcirc\left(\|\alpha\|^3\right)\right\} \tag{A40}$$

proof Galì and Monacelli [18].

As shown in Woodford [36], this means that

$$\sum_{t=0}^{\infty}\beta^t var_i\left\{p_t(i)\right\} = \frac{1}{\lambda_a}\sum_{t=0}^{\infty}\beta^t\pi_t^2 \tag{A41}$$

where $\lambda = (1-\psi)(1-\psi\beta)/\psi$.

Finally, denoting the output gap $\tilde{Y}_t$ as in the standard way x_t, the Welfare-Based loss-function can be written as,

$$\begin{aligned} W_t &= E_t\sum_{t=0}^{\infty}\beta^t\tilde{U}_{t+k} = -\frac{U_Y}{2}E_t\sum_{t=0}^{\infty}\left[\frac{\theta}{\lambda_a}\pi_{t+k}^2 + \frac{1}{\sigma}x_{t+k}\right] + \bigcirc\left(\|\alpha\|^3\right) \\ &= E_t\sum_{t=0}^{\infty}\beta^t\tilde{U}_{t+k} = -\frac{U_Y}{2}E_t\sum_{t=0}^{\infty}\left[\frac{\theta}{\lambda_a}\pi_{t+k}^2 + \frac{1}{\sigma}x_{t+k}\right] + \bigcirc\left(\|\alpha\|^3\right) \end{aligned} \tag{A42}$$

A.6 Stability and Determinacy in the Reduced Form Dynamical System

Our model can be expressed in the following reduced form:

$$x_t = E_t x_{t+1} - [r_t - E_t \pi_{t+1} - \hat{r}_t^n] \tag{A43}$$

$$\pi_t = \beta E_t \pi_{t+1} + \lambda_a \frac{1-\alpha}{\alpha} x_t + \frac{\lambda_a}{\sigma(1-\rho_a)} \hat{r}_t^n + \lambda_a w_t^r \tag{A44}$$

the model is completed adding the optimal instrument rule interest rate which, under discretion is given by:

$$\hat{r}_t^* = \Phi_\pi E_t \pi_{t+1} + \Phi_r \hat{r}_t^n \tag{A45}$$

where $\Phi_\pi = 1 + \left(\frac{1-\rho_w}{\rho_w}\right) \frac{1-\alpha}{\alpha} \theta\sigma$ and $\Phi_r = 1 + \frac{(\rho_a - \rho_w)}{\sigma(1-\rho_a)} \frac{\lambda\theta}{\Omega - \beta\rho_a}$.

In order to verify if the optimal policy can guarantee the uniqueness of the equilibrium we combine the IS the AS and the optimal interest rate with the natural interest rate equation, so that we can write the following reduced dynamical system:

$$x_t = E_t x_{t+1} + (1-\Phi_\pi) E_t \pi_{t+1} + (1-\Phi_r) r_t^n \tag{A46}$$

$$\begin{aligned} \pi_t = {} & \lambda_a \frac{1-\alpha}{\alpha} E_t x_{t+1} + \left[\lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi) + \beta\right] E_t \pi_{t+1} + \\ & + \left(1 + \frac{\lambda_a}{\sigma(1-\rho_a)} - \Phi_r\right) r_t^n + \lambda_a w_t^r \end{aligned} \tag{A47}$$

which can be rewritten as

$$\varpi_t = A_1 E_t \varpi_{t+1} + A_2 u_t \tag{A48}$$

where $\varpi_t = E_t [x_t; \pi_t]^T$ and $A_2^T = \begin{bmatrix} 1-\Phi_r & 0 \\ 1 + \frac{\lambda}{\sigma(1-\rho_a)} - \Phi_r & \lambda_a \end{bmatrix}$, $u_t = [r_t^n, w_t^r]^T$ while the transition matrix is given by:

$$A_1 = \begin{bmatrix} 1 & 1-\Phi_\pi \\ \lambda_a \frac{1-\alpha}{\alpha} & \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi) + \beta \end{bmatrix} \tag{A49}$$

Given that ϖ_t is a 2-vector of non-predetermined endogenous variable, rational expectation equilibrium is determinate if and only if the matrix A_1 has both eigen values outside the unit circle, which occurs if and only if[14]:

$$\det A_1 < 1, \tag{A50}$$

$$|-trA_1| < 1 + \det A_1. \tag{A51}$$

[14] See Proposition 1 in the Appendix of Woodford [36].

Notice that, in our case:

$$\det A_1 = \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi) + \beta - \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi) = \beta < 1 \tag{A52}$$

$$|-trA_1| = 1 + \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi) + \beta < 1 + \beta \quad \text{given that} \quad \Phi_\pi > 1 \tag{A53}$$

which implies that the rational expectations equilibrium of our model under an optimal discretionary policy is determinate.

The optimal instrument rule commitment is given by:

$$\hat{r}_t^{c*} = \Phi_\pi^c E_t \pi_{t+1} + \Phi_r^c \hat{r}_t^n \tag{A54}$$

where $\Phi_\pi^c = 1 + \left(\frac{1-\rho}{\rho}\right) \frac{\alpha}{1-\alpha} \frac{\theta}{1-\beta\rho}$ and $\Phi_r^c = 1$.

Under commitment matrix A_1 becomes:

$$A_1 = \begin{bmatrix} 1 & 1-\Phi_\pi^c \\ \lambda_a \frac{1-\alpha}{\alpha} & \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi^c) + \beta \end{bmatrix} \tag{A55}$$

Notice that

$$\det A_1 = \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi^c) + \beta - \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi^c) = \beta < 1 \tag{A56}$$

$$|-trA_1| = 1 + \lambda_a \frac{1-\alpha}{\alpha}(1-\Phi_\pi^c) + \beta < 1 + \beta \quad \text{given that} \quad \Phi_\pi^c > 1 \tag{A57}$$

which implies that also under constrained commitment the optimal interest rule can guarantee the uniqueness of the equilibrium.

Tables and Figures

Table 1.

Parameters	value	Description
β	0.99	Discount factor
σ	2	Coefficient of relative risk adversion
θ	11	Elasticity of substitution across intermediate goods
α	0.72	Output elasticity to labor input
φ	0.75	Probability of goods price non-adjustment
ρ_w	0.7	Persistence of the reservation wage shock
σ_w	0.07	Standard deviation of the reservation wage shock
ρ_a	0.93	Persistence of the reservation productivity shock
σ_a	0.867	Standard deviation of the reservation productivity shock
N	0.61	Steady state employment

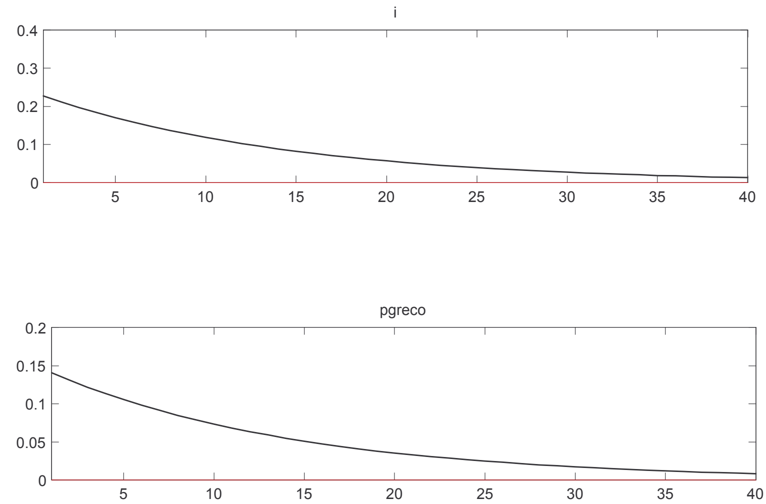

Figure 1: IRFs to a negative technology shock under constant output

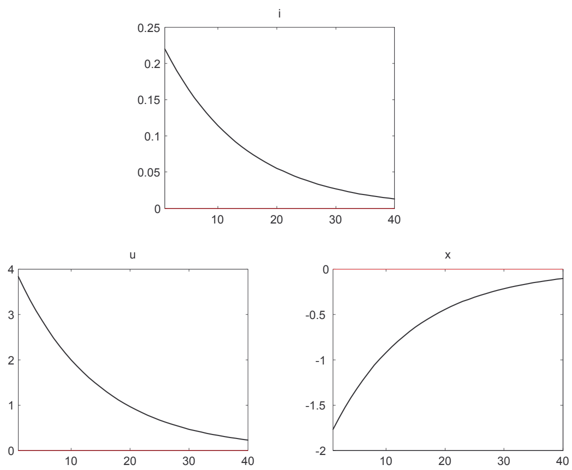

Figure 2: IRFs to a negative tecnology shock under constant inflation

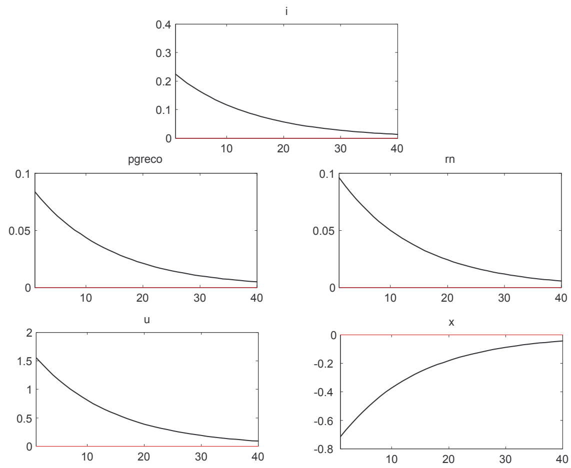

Figure 3: IRFs to a negative tecnology shock under the discretionary optimal rule

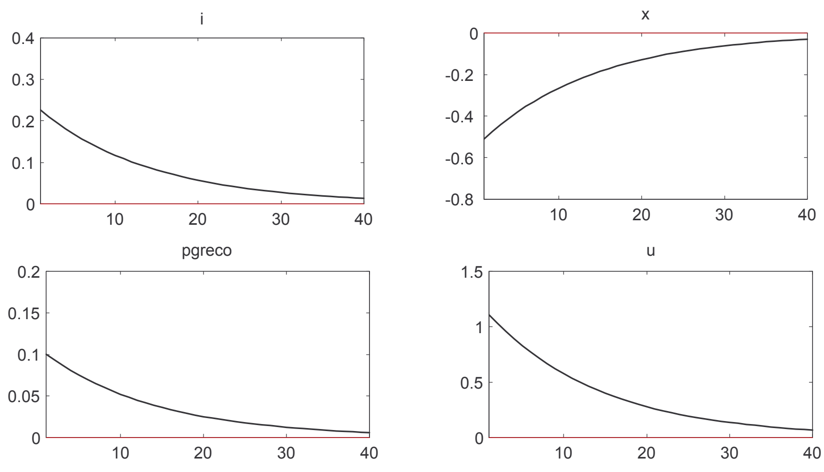

Figure 4: IRFs to a negative tecnology shock under a simple taylor rule

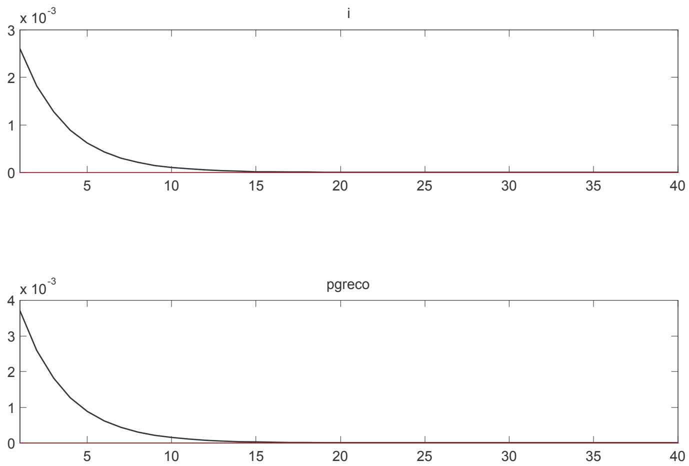

Figure 5: IRFs to a reservation wage shock under constant output

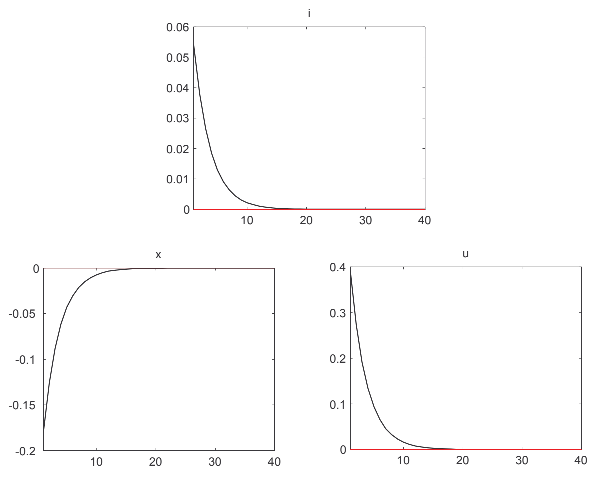

Figure 6: IRFs to a reservation wage shock under constant inflation

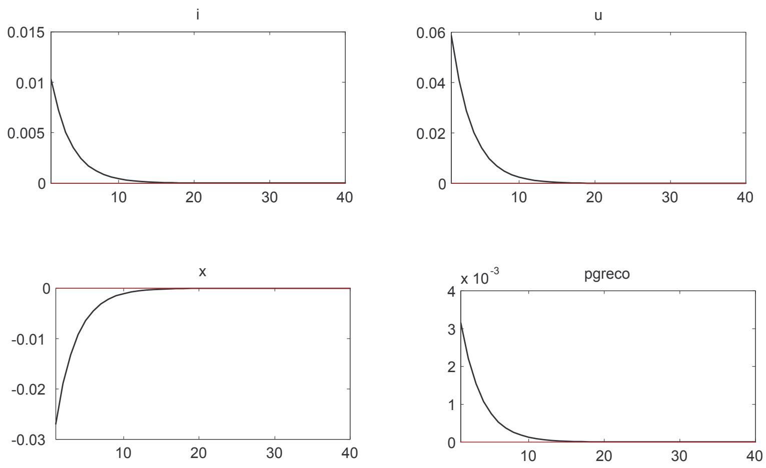

Figure 7: IRFs to a reservation wage shock under the discretionary optimal rule

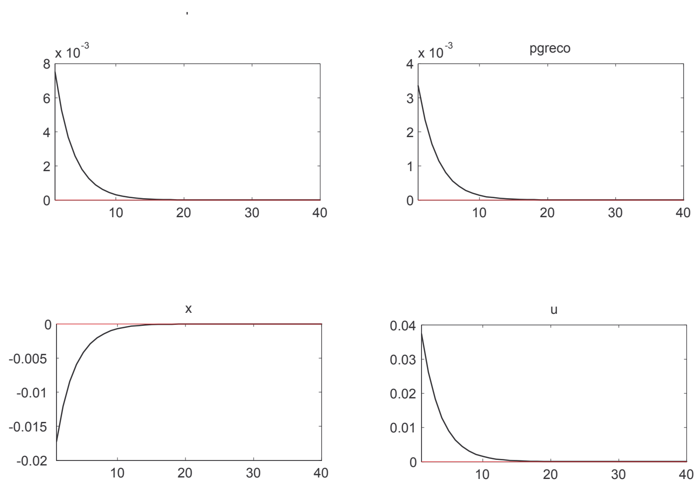

Figure 8: IRFs to a reservation wage shock under a simple Taylor rule