Intermediate Inputs and External Economies

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Abstract

Is the degree of external economies (at the industry level) higher than the degree of internal increasing returns (at the firm level)? If so, what is the exact source of this difference? In this general equilibrium model in which firms producing final goods choose the degree of specialization of their technologies, external economies arise from the usage of intermediate inputs and the existence of internal increasing returns. It is frequently assumed that increasing returns are absent at the firm level while are present at the industry level. In this model, the existence of increasing returns at the firm level is necessary for the existence of external economies at the industry level. We show that the degree of external economies increases with the level of linkage effects. However, a higher linkage effect does not always lead firms to choose more specialized technologies.

Keywords: External economies, internal increasing returns, linkage effects, choice of technology, oligopolistic competition

JEL Classification Numbers: F10, L10, R10

1. Introduction

Internal increasing returns to scale refer to the case that a firm's average cost of production decreases with its own level of output, rather than with the output of the entire industry. One common source of internal increasing returns is the existence of fixed costs of production. External increasing returns to scale or external economies refer to the case that a firm's average cost of production decreases with the output of the entire industry, rather than its own level of output.[1] External economies may result from the existence of forward and backward linkages. Forward linkage refers to the use by one firm or industry of produced inputs from another firm or industry. Backward linkage refers to the provision by one firm or industry of produced inputs to another firm or industry. The level of linkage effects generated by an industry is the sum of the levels of forward linkages and backward linkages generated by this industry. With the existence of linkage effects, the expansion of an industry may lead input suppliers to choose more specialized technologies. More specialized technologies lead to lower average costs and thus lower prices of inputs. As the prices of inputs are lower, other things equal, the average cost of producing a final good decreases and external increasing returns result.

[1] Marshall (1920, Book IV, Chapter 10) discusses various factors leading a region to maintain an industry, such as knowledge spillovers, labor market pooling, and the usage of machines in the production of inputs.

The existence of external economies has been invoked to explain various issues. First, it has been used to explain the agglomeration of firms producing similar goods. Second, the existence of external economies could make coordinated investment in multiple sectors necessary. Because the private sector in a developing country may not be able to engage in investment in multiple sectors, the existence of external economies has been used to explain the failure of industrialization in developing countries. Third, with the existence of external economies, a large country will have a lower cost of production of a product than a small country. The smaller country may lose from the opening of international trade because some strategic industries would not survive foreign competition. Under this kind of scenario, the existence of external economies has been used to justify protection in international trade.

With the general relevance of external increasing returns, a microfoundation of external economies can be useful in illustrating several interesting issues. First, in prescribing practical strategies for economic development, Hirschman (1958) has estimated the levels of backward and forward linkages for various industries. An industry with a higher total linkage effects may receive priority in terms of investment. However, why an industry with a higher level of linkage effects should receive priority in terms of investment has not been demonstrated in a formal model. We know that an industry with a higher level of linkage effects will generate more demand for other industries. But if the production function has constant returns to scale, a higher demand does not lead to a lower average cost and the desirability of a higher linkage effects is not clear. Second, linkage effects and a firm's choice of its degree of specialization are elements packaged into the "black box" of external economies. However, the interaction between the linkage effects and a firm's choice of its degree of specialization has not been studied. In reality, this interaction is important. For example, it is used by Porter (1990) to explain the concentration of industries in given locations. Does a higher level of linkage effects always lead firms to choose more specialized technologies? Third and finally, what is the relationship between external economies and internal increasing returns? Is the degree of external economies (at the industry level) always higher than the degree of internal increasing returns (at the firm level)? If so, what is the exact source of this difference?

In this paper, starting with internal increasing returns at the firm level, we show that external economies at the industry level arise from the usage of intermediate inputs and the existence of internal increasing returns in a general equilibrium model incorporating the interaction between the linkage effects and technological choices of firms.[2] In this model, there is a continuum of final goods. Because final goods are produced by using the composite input and the composite input uses both labor and final goods as inputs, there exist linkage effects similar to the input-output matrix. Firms producing final goods are assumed to engage in oligopolistic competition.[3] A firm producing a final good chooses its production technology to maximize its profit. A more specialized technology has a higher fixed but a lower marginal cost of production. The existence of fixed costs of production for a firm indicates the existence of internal increasing returns.

In this model, the combination of internal increasing returns and linkage effects leads to external economies. We show that the degree of external economies is not smaller than the degree of internal increasing returns. The difference between internal and external increasing returns lies in the existence of linkage effects. The degree of external economies (if $Y = L^{\xi}$, where Y is industry output and L is industry input, the degree of external economies is ξ) increases with the level of linkage effects. Actually, without linkage effects, the degrees of internal and external increasing returns would be the same.

Empirical evidence such as Roberts and Tybout (1996) has shown that increasing returns at the firm level could be small. Since the main aspect of industrialization is the adoption of increasing returns technologies, increasing returns at the firm level may not be advanced as an explanation for the large effect of industrialization on aggregate income. Consequently, in any model where external economies cannot increase at a rate faster than the internal increasing returns it cannot help to explain large effect of industrialization on the aggregate income and productivity. In this model, with the existence of linkage effects, we show that the degree of external increasing

[2] In this general equilibrium model, firms engage in oligopolistic competition. Potential issues associated with incorporating oligopolistic competition into a general equilibrium model are discussed in Neary (2003).

[3] The relevance of oligopolistic competition in the United States is discussed in detail in Chandler (1990). With increasing returns in manufacturing, distribution, and management, a firm may monopolize an industry. As monopoly is not allowed with the introduction of antitrust laws, many industries have been characterized by a few dominant firms engaging in oligopolistic competition since the Second Industrial Revolution.

returns can be much higher than the degree of internal increasing returns.[4] Thus our model can be used to explain the large increase of aggregate income from industrialization.[5]

We show that if the degree of internal increasing returns is zero, the degree of external economies will also be zero. This result puts doubts on some assumptions frequently made in the literature. It is frequently assumed that there are constant returns at the firm level while there are increasing returns at the industry level. The reasons for making the above assumption are the following. On the one hand, constant returns at the firm level leads to perfect competition as the market structure and powerful tools developed for perfect competition can be applied and the difficulties in modeling associated with imperfect competition can be avoided. On the other hand, increasing returns at the industry level are assumed because increasing returns are essential to the questions to be addressed.

Interestingly, a higher degree of linkage effects does not always make a firm producing a final good choose a more specialized technology. An increase in the level of linkage effects makes a higher output for a firm producing a final good self-fulfilling. This increase in the level of output for a firm producing a final good leads to the choice of more advanced technologies. However, there is one additional effect when the level of linkage effect changes: a change of the level of linkage effects also changes the marginal product of labor and the marginal product of a final good. For the latter effect, the demand for final goods increases only when compared with labor, final goods become relatively more productive in the production of the composite input. Overall, a sufficient condition for the degree of specialization of a firm producing a final product to increase with the level of linkage effects is that compared with labor, the relative productivity of final goods increases.

[4] This magnification effect through intermediate inputs is similar to Jones (2011) who presents a neoclassical model with constant returns to scale. In his model, the degree of substitution among intermediate inputs is used to formalize the concept of weak links in economic development. He demonstrates that tax distortion can be magnified by the usage of intermediate inputs in production. However, there are some differences between his model and this one. In his model, the production of each final good uses capital, labor, and a fraction of the final good produced in the previous period. In this model, the production of each final good uses the intermediate good only. This intermediate input is produced by combining labor and all final goods with a Cobb-Douglas technology.

[5] The existence of linkage effects is related to the usage of intermediate inputs in production. Ciccone (2002) studies the implications of the usage of intermediate inputs in the process of industrialization. He shows that the introduction of input chains is able to explain large differences of income among countries. In Ciccone, even if a downstream firm is unprofitable, adoption of increasing returns to scale technology is socially profitable because it can increase the profit of upstream firm.

Emphasizing complementarity of different industries, Rosenstein-Rodan (1943) has argued that a large-scale coordinated investment may be profitable even though an individual project is not profitable. Murphy et al. (1989) have shown that demand spillovers alone do not lead to multiple equilibria. In the baseline version of their model, a firm generates positive externalities if and only if it earns a positive profit. In this paper, multiple equilibria may exist. However, the mechanism leading to multiple equilibria here is different from that in Murphy et al. (1989). In Murphy et al. (1989), firms face a binary choice in the choice of technologies: either a constant returns technology or an increasing returns technology. The condition for the existence of multiple equilibria based on the comparison of the change of the ratios of marginal and fixed costs in this model is absent in a binary technology choice model.

In terms of relating increasing returns at the micro level to increasing returns at the macro level, this model is related to Ethier (1982) and Markusen (1990). Ethier (1982) has studied international increasing returns with an increasing number of varieties. Markusen (1990) has explored the circumstances under which the implied industry production functions are consistent with ad hoc specifications used in international trade and economic growth. This paper contributes to the literature by incorporating technological choice into the study of external economies in a general equilibrium model and by clarifying the relationship between internal and external increasing returns.

In terms of the choice of technology, this paper is related to Zhou (2004, 2007). Zhou (2004) has shown that the choice of technology can be used to formalize Adam Smith's insight on the mutual dependence between the division of labor and the extent of the market in a general equilibrium model. More relevantly, Zhou (2007) has demonstrated that the choice of technology can be used to provide a link between internal and external increasing returns. By incorporating linkage effects into the choice of technology, this paper is able to address some interesting issues not addressed in Zhou (2007). First, this paper is able to address the interaction between the degree of specialization of a firm and the level of linkage effects. Second, this paper is able to show that the existence of external increasing returns depends on the existence of increasing returns and the difference between internal and external increasing returns lies in the existence of linkage effects.

The plan of the paper is as follows. Section 2 sets up the model. Section 3 examines the number of equilibria. Section 4 conducts comparative statics to address issues such as the impact of a higher linkage effects on a firm's choice of its technologies. Section 5 derives the degrees of internal and external increasing returns and establishes the relationship between internal and external increasing returns. Section 6 discusses some generalizations and extensions of the model and concludes.

2. The model

In this section, we specify the model and establish equilibrium conditions for a general equilibrium. First, we establish conditions for a consumer's utility maximization. Second, we establish conditions for the profit maximization of firms, including a firm producing the intermediate input and a firm producing a final good. Third, we establish conditions for markets to clear, including the market for final goods and the market for the intermediate input.

We study a closed economy with a population size of L. In this economy, there is a continuum of final goods indexed by a number $\varpi \in [0,1]$ with a total measure of one.[6] Final goods are symmetric in the sense that all the final goods have the same costs of production and they enter a consumer's utility function in the same way. The production of each final good requires a composite intermediate input. This composite input is produced by using both labor and all final goods as inputs.

First, we establish conditions for a consumer's utility maximization. A representative consumer's consumption of the final good ϖ is $c(\varpi)$. This consumer's utility function is specified as

$$U = \int_0^1 u[c(\varpi)]d\varpi . \tag{1}$$

We assume that $u' > 0$ and $u'' < 0$ and define the elasticity of demand for a final good as $\varepsilon = -u''(c)c/u'(c)$. Because an individual is assumed to have no preferences for leisure, each individual supplies one unit of labor inelastically. As firms earn zero profits in equilibrium, wage income is the only source of income for a consumer. A consumer's budget constraint states that her total spending on all final goods equals her wage income:

$$\int_0^1 p(\varpi)c(\varpi)d\varpi = w . \tag{2}$$

[6] The motivation of having a continuum of final goods instead of one final good is to eliminate a final good producer's market power in the market for intermediate inputs (Neary, 2003). Even though a final good is produced by a small number of firms and a final good producer has market power in the market for this final good, with a continuum of final goods, there is an infinite number of firms demanding intermediate inputs and a final good producer will be a price taker in the market for intermediate inputs.

A consumer takes the wage rate w and the price of all final goods as given and chooses quantities of consumption to maximize her utility (1) subject to the budget constraint (2). In a symmetric equilibrium, equal amount of income is spent on each final good:

$$p(\varpi)c(\varpi) = w \, . \tag{3}$$

Second, we establish conditions for the profit maximization of firms. The amount of final good ϖ used in the production of the composite input is $z(\varpi)$. For μ denoting a positive constant between zero and one, output of the composite input is specified as

$$Q = L^{1-\mu}[\int_0^1 z(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{\sigma-1}} \, . \tag{4}$$

In equation (4), μ captures the contribution of the final goods in the production of the composite input and in this model it measures the level of linkage effects. A higher value of μ indicates a higher level of linkage effects. As will be shown in later sections, this parameter plays an important role in this paper. In Section 4, we will study how this parameter interacts with the parameter measuring a firm's level of technology. In Section 5, we will study how this parameter will affect the degree of external economies.

The price of one unit of the composite intermediate input is p_I, I for intermediate. For a firm producing the composite input, its total revenue is $p_I L^{1-\mu}[\int_0^1 z(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{1-\sigma}}$.[7] Its cost from hiring labor is wL and its cost from purchasing final goods is $\int_0^1 p(\varpi)z(\varpi)d\varpi$. Thus the profit for a firm producing the composite input is $p_I L^{1-\mu}[\int_0^1 z(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{\sigma-1}} - wL - \int_0^1 p(\varpi)z(\varpi)d\varpi$. We assume that firms producing the composite input engage in perfect competition. Given the wage rate and the prices of final goods, a firm producing the composite input chooses the quantity of labor and quantities of final goods to maximize its profit. Optimal choices of labor and final goods for a firm producing the composite input require that

$$(1-\mu)p_I L^{-\mu}[\int_0^1 z(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{\sigma-1}} - w = 0 \, , \tag{5a}$$

[7] Since the sector producing the composite input exhibits constant returns to scale, the level of output and thus the amount of labor hired by a firm producing the composite input are undetermined. But this indeterminacy is inessential.

$$\mu p_I L^{1-\mu}[\int_0^1 z(\varpi)^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{\sigma-1}-1} z^{-1/\sigma} - p(\varpi)=0\,. \tag{5b}$$

With free entry and exit in the production of the composite input, a firm producing the composite input earns a profit of zero:

$$p_I L^{1-\mu}[\int_0^1 z^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{\sigma-1}} - w\,L - \int_0^1 p(\varpi)\,z(\varpi)d\varpi = 0\,. \tag{6}$$

The production of a final good uses the composite input only. To produce a final good, we assume that there is a continuum of technologies indexed by a positive number n. Technologies differ in their degrees of specialization. A technology with a higher index indicates a more specialized or a more advanced technology. For technology n, the marginal cost in terms of the units of the composite input needed is $\beta(n)$ and the fixed cost in terms of units of the composite input needed is $f(n)$. To capture the substitution between marginal and fixed costs of production, we assume that the fixed cost increases while the marginal cost decreases with the level of technology: $f'(n) > 0$ and $\beta'(n) < 0$. That is, a more specialized or advanced technology has a higher fixed cost but a lower marginal cost of production, as in Zhou (2004, 2007, 2011).

In this model, a technology is defined as "more specialized" or "more advanced" when it has relatively higher fixed costs. This definition may not fit technological advances in some industries. In those industries, technological advance is about getting rid of high fixed cost technologies and using flexible small-scale technologies. However, the following example of the adoption of containers in the transportation sector illustrates that the definition used here captures technology advances in some sectors. Before the adoption of containers in the 1950s, cargos were handled by longshoremen and were labor intensive. The adoption of containers led to a sharp increase in the level of fixed costs in the transportation sector because containerships and container ports are costly (Levinson, 2006). With containerization, the marginal cost of loading and unloading goods decreased sharply. If the volume of transportation is high, adoption of costly containerships and container ports will be profitable because the high fixed costs of can be spread to a high level of output and the average cost decreases.

For a firm producing a final good with an output level x, its total revenue is px and its cost from purchasing the composite input is $[f(n)+\beta(n)x]p_I$. Thus this firm's profit is

$px-[f(n)+\beta(n)x]p_I$. The number of firms producing the same final good is m.[8] Firms producing the same final good are identical and they are assumed to engage in Cournot competition.[9] A firm producing a final good takes the wage rate and the price of the composite input as given and chooses its level of output and the degree of specialization of its technology to maximize its profit. The optimal choice of output for a firm producing a final good requires that marginal revenue equals marginal cost:

$$p+x\frac{\partial p}{\partial x}=\beta\, p_I. \tag{7}$$

The optimal choice of technology for a firm producing a final good leads to the following equation which can be used together with the second order condition to show that a firm producing a final good with a higher level of output will choose a more specialized technology:

$$f'(n)+\beta'(n)x=0. \tag{8}$$

We assume that the following second order condition is valid: $f''(n)+\beta''(n)x>0$. Plugging the value of x from equation (8) into this second order condition leads to

$$f''(n)\beta'(n)-\beta''(n)f'(n)<0. \tag{9}$$

Free entry and exit leads to zero profit for a firm producing a final good:

$$px-[f(n)+\beta(n)x]p_I=0.^{10} \tag{10}$$

Third, we study the market clearing conditions. For a final good, demand has two components: the amount used for consumption Lc, and the amount used in the production of the composite input z. Thus the total demand for a final good is $Lc+z$. Each of the $m(\varpi)$ firms supplies $x(\varpi)$ units of output and the total supply of a final good is $m(\varpi)x(\varpi)$. The clearance of the market for a final good requires that

$$Lc+z=m(\varpi)x(\varpi). \tag{11}$$

Each firm produces a final good demands $f(n)+\beta(n)x$ units of the composite input and each of the sector producing a final good demands $m[f(n)+\beta(n)\,x]$ units of the composite input. Thus the total demand for the composite input is $\int_0^1 m[f(n)+\beta(n)x]d\varpi$. Total supply of the

[8] To facilitate presentation, the number of firms is allowed as a real number rather than restricted to be an integer number.
[9] For an example of a model of oligopolistic competition with differentiated goods, see Wang and Zhao (2007).
[10] For examples of oligopolistic competition with free entry, see Chao and Yu (1997), Zhang (2007), and Chen and Shieh (2011).

composite input is Q. The clearance of the market for the composite input requires that $\int_0^1 m[f(n)+\beta(n)x]d\varpi = Q$. By combining this equation with equation (4), the clearance of the market for the composite input requires that

$$\int_0^1 m[f(n)+\beta(n)x]d\varpi = L^{1-\mu}[\int_0^1 z^{\frac{\sigma-1}{\sigma}} d\varpi]^{\frac{\mu\sigma}{\sigma-1}}. \tag{12}$$

In a Cournot equilibrium, when a firm chooses its level of output, it takes output of other firms as given. With this in mind, partial differentiation of equation (11) leads to

$$\frac{\partial x}{\partial p} = \frac{\partial(Lc+z)}{\partial p} = L\frac{\partial c}{\partial p} + \frac{\partial z}{\partial p}$$

$$= \frac{Lc}{p}[\frac{\partial c}{\partial p}\frac{c}{p}] + \frac{z}{p}[\frac{\partial z}{\partial p}\frac{p}{z}] = -\frac{\varepsilon Lc + \sigma z}{p}.$$

Plugging the above value of $\partial x/\partial p$ into equation (7) leads to

$$p - \frac{px}{\varepsilon Lc + \sigma z} - \beta(n)\, p_I = 0. \tag{13}$$

In the following, we study the equilibrium that there is symmetry in the sectors producing final goods: the number of firms producing each final good m, price p, consumption c, output x, and technology n are the same for all final goods. With the symmetry among all goods, $z(\varpi)$ is simplified as z. Also, the integral can be dropped because the total measure is one. As a result, equations (5a) and (5b) lead to

$$\frac{(1-\mu)z}{\mu L} = \frac{w}{p}. \tag{5}$$

Equations (3), (5), (6), (8), and (10)-(13) form a system of eight equations defining eight variables p, p_I, x, n, m, w, c, and z as functions of exogenous parameters An equilibrium is a tuple (p, p_I, x, n, m, w, c, z) satisfying those eight equations. For the rest of this paper, the wage rate is used as the numeraire: $w \equiv 1$.

3. The number of equilibria

In this section, we study the number of equilibria. We proceed by simplifying the system of eight equations to a manageable number of equations by keeping variables of direct interest while eliminating other variables.

From equations (5) and (6), the price of the composite input can be expressed as a function of the wage rate and the prices of final goods:

$$p_I = \frac{1}{\mu^{\mu}(1-\mu)^{1-\mu}} p^{\mu} . \tag{14}$$

Plugging the value of p_I from the above equation into equation (10) yields

$$\mu^{\mu}(1-\mu)^{1-\mu} x\, p^{1-\mu} - [f(n) + \beta(n)x] = 0 . \tag{15}$$

Plugging the value of p_I from equation (14) into equation (13) yields

$$p^{1-\mu} - \frac{x\, p^{2-\mu}}{\left(\varepsilon + \frac{\sigma\mu}{1-\mu}\right)L} - \frac{\beta(n)}{\mu^{\mu}(1-\mu)^{1-\mu}} = 0 . \tag{16}$$

With the above manipulation, the system of eight equilibrium conditions for this economy reduces to the following system of three equations:[11]

$$V_1 \equiv -\beta'(n)x - f'(n) = 0 , \tag{17a}$$

$$V_2 \equiv [f(n) + \beta(n)\, x] - \mu^{\mu}(1-\mu)^{1-\mu} x\, p^{1-\mu} = 0 , \tag{17b}$$

$$V_3 \equiv \left(\varepsilon + \frac{\sigma\,\mu}{1-\mu}\right) f(n)\, L - p\, x[f(n) + \beta(n)x] = 0 . \tag{17c}$$

In equation (17c), since ε is a function of c, through equation (3), it is a function of p. Thus equations (17a)-(17c) form a system of three equations defining three variables n, x, and p as functions of exogenous parameters. To focus on the production side of the economy, in the remaining of this paper, a consumer's elasticity of demand ε is treated as a constant. For example, the utility function $u(c) = c^{\alpha}$ exhibits constant elasticity of demand.

Plugging the value of p from equation (17c) and the value of x from equation (17a) into equation (17b), as p can be expressed as a function of n, we can derive the following nonlinear equation which defines n implicitly:

$$\mu^{\mu}[\varepsilon(1-\mu) + \sigma\mu]^{1-\mu} f(n)^{1-\mu} L^{1-\mu}\left(-\frac{f'(n)}{\beta'(n)}\right)^{\mu} - \left(f(n) - \frac{\beta(n)\, f'(n)}{\beta'(n)}\right)^{2-\mu} = 0 . \tag{18}$$

[11] Equations (17a)-(17c) are derived as follows. First, equation (17a) is the same as equation (8). Second, equation (17b) is the same as equation (15). Third, equation (17c) is derived by plugging the value of p from equation (17b) into equation (16).

To simplify presentation, we define $\Gamma \equiv \mu^{\mu}[\varepsilon(1-\mu)+\sigma\mu]^{1-\mu} L^{1-\mu}$. With this notation, equation (18) reduces to $\Gamma f(n)^{1-\mu}\left(-\frac{f'(n)}{\beta'(n)}\right)^{\mu} - \left(f(n)-\frac{\beta(n) f'(n)}{\beta'(n)}\right)^{2-\mu} = 0$. The derivative of the left-hand side of equation (18) with respect to n is not always positive or negative. Thus the left-hand side of equation (18) is not a monotonic function of n. Depending on the specifications of marginal and fixed costs, the number of equilibria may be unique or multiple. When the marginal and fixed costs are specified, the number of equilibria can be checked.[12] Here we provide an example of the existence of multiple equilibria. Let $f(n) = e^{n}$, $\beta(n) = 1/n$, and $\mu = 0.9$. Choose parameters so that $\Gamma = 3$. From equation (18), the level of technology is defined by $3n^{1.8} - (1+n)^{1.2} e^{0.1n} = 0$. Solving this equation numerically leads to two solutions: 0.8628 and 31.2629.

We now provide a sufficient condition for a unique equilibrium to obtain. Differentiation of the left-hand side of equation (18) with respect to n yields

$$\Gamma(1-\mu) f^{-\mu}\left(-\frac{f'}{\beta'}\right)^{\mu} f' + \mu\Gamma f^{1-\mu}\left(-\frac{f'}{\beta'}\right)^{\mu-1} \frac{(f'\beta''-f''\beta')}{(\beta')^2}$$
$$-(2-\mu)\left(f-\frac{\beta f'}{\beta'}\right)^{1-\mu}\left(f'-\frac{(\beta')^2 f'+\beta\beta' f''-\beta f'\beta''}{(\beta')^2}\right).$$

By using equation (18), the above expression is equal to

$$\left(f-\frac{\beta f'}{\beta'}\right)^{2-\mu}\left(\frac{(1-\mu)f'}{f} - \beta'\left(\frac{f'\beta''-f''\beta'}{(\beta')^2}\right)\left(\frac{\mu}{f'} + \frac{(2-\mu)\beta}{f\beta'-\beta f'}\right)\right). \tag{19}$$

If (19) is always positive, a unique equilibrium will obtain. From (9), $f''\beta'-\beta''f' < 0$. Since $(1-\mu)f'/f > 0$, a sufficient condition for (19) to be positive and thus the left-hand side of equation (18) to be a monotonic function of the level of technology is that $\frac{\mu}{f'} + \frac{(2-\mu)\beta}{f\beta'-\beta f'} > 0$. Simplification of this inequality leads to the following inequality as a sufficient condition for a unique equilibrium to obtain:

$$\frac{2(1-\mu)f'}{\mu f} < \frac{-\beta'}{\beta}.$$

[12] Saaty (1981) contains a discussion of the existence and uniqueness of solutions for nonlinear equations.

A strict positive linkage effect is not necessary for the existence of multiple equilibria. As it will be shown in the next section, a higher linkage effect does not always make a firm producing a final good chooses a more specialized technology, thus it is not clear whether a higher level of linkage effects makes the existence of multiple equilibria more likely.

In this model, when the sufficient condition for a unique equilibrium to obtain is not satisfied, if all firms choose a higher level of output, the cost saving may justify the choice of a more advanced technology. That is, multiple equilibria may result when the sufficient condition for a unique equilibrium is not satisfied. If the sufficient condition is satisfied, the rate of the decrease in the level of marginal cost is higher than the rate of the increase in the level of the fixed cost. As a result, a firm's own output can justify the choice of a more advanced technology and equilibrium will be unique.

While multiple equilibria may exist in this model and Murphy et al. (1989), the mechanisms leading to multiple equilibria are different. In their model, firms face a binary choice in the choice of technologies: either a constant returns technology or an increasing returns technology. The comparison of the change of the ratios of marginal and fixed costs is absent in a binary technology choice model.

When there are multiple equilibria, the following proposition shows that an equilibrium when firms producing final goods choose less specialized technologies is Pareto dominated by an equilibrium when firms producing final goods choose more specialized technologies.

Proposition 1: When there are multiple equilibria, an equilibrium with more specialized technologies leads to a higher level of utility for a consumer.

Proof: A consumer's utility is determined by the prices of final goods and the wage rate. Since the wage rate is normalized to one, a consumer's utility increases when the prices of final goods decrease. Plugging the value of x from equation (17a) into equation (17b) yields the following equation showing the relationship between the level of technology and the prices of final goods:

$$\Omega \equiv -\frac{f'}{\beta'}\mu^{\mu}(1-\mu)^{1-\mu}p^{1-\mu} - (f - \frac{\beta f'}{\beta'}) = 0\,.$$

Partial differentiation of Ω leads to

$$\frac{\partial\Omega}{\partial n} = \mu^{\mu}(1-\mu)^{1-\mu} p^{1-\mu} \frac{f''\beta' - f'\beta''}{(\beta')^2} - \left(f' - \frac{(\beta' f' + \beta f'')\beta' - \beta f'\beta''}{(\beta')^2} \right)$$

$$= [\mu^{\mu}(1-\mu)^{1-\mu} p^{1-\mu} - \beta] \left(\frac{f'\beta'' - f''\beta'}{(\beta')^2} \right).$$

From (9), $f'\beta'' - f''\beta > 0$. From equation (15), $\mu^{\mu}(1-\mu)^{1-\mu} p^{1-\mu} - \beta > 0$. Thus, $\partial\Omega / \partial n > 0$. Also, $\partial\Omega / \partial p > 0$, thus $dp / dn < 0$. If there are two equilibria with different levels of technologies, then the equilibrium with more specialized technologies has lower prices of final goods and a consumer has a higher level of utility. ■

4. Properties of an equilibrium

In this section, we conduct comparative statics to explore the properties of an equilibrium. This type of inquiry is appropriate when there is a unique equilibrium.[13] We are especially interested in the interaction between the level of linkage effects and the level of technology for a firm producing a final good.

Partial differentiation of equations (17a)-(17c) with respect to n, x, p, L, and μ leads to

$$\begin{pmatrix} \frac{\partial V_1}{\partial n} & 0 & \frac{\partial V_1}{\partial x} \\ 0 & \frac{\partial V_2}{\partial p} & \frac{\partial V_2}{\partial x} \\ \frac{\partial V_3}{\partial n} & \frac{\partial V_3}{\partial p} & \frac{\partial V_3}{\partial x} \end{pmatrix} \begin{pmatrix} dn \\ dp \\ dx \end{pmatrix} = - \begin{pmatrix} 0 \\ 0 \\ \partial V_3 / dL \end{pmatrix} dL - \begin{pmatrix} 0 \\ \partial V_2 / \partial\mu \\ \partial V_3 / \partial\mu \end{pmatrix} d\mu . \tag{20}$$

Let Δ denote the determinant of the above coefficient matrix. From Appendix A, the stability of the system of equations (17a)-(17c) requires that $\Delta < 0$.

When μ increases, the contribution of final goods in the production of the composite input increases. What is the impact of a change of μ on the level of technology for a firm producing a final good? This impact is interesting because it measures the interaction between the level of linkage effects and a firm's choice of technology. Does a higher level of linkage effects always

[13] When multiple stable equilibria exist, Milgrom and Roberts (1994) show that comparative statics can be based on the comparison of highest equilibrium or the lowest equilibrium.

lead a firm to choose a more specialized technology? The following proposition provides a sufficient condition for the positive interaction between linkage effects and the level of technology for a firm producing a final good.

Proposition 2: A sufficient condition for $\frac{dn}{d\mu} > 0$ is that $\frac{d(p/p_I)}{d\mu} > 0$.

Proof: An application of Cramer's rule to the system (20) leads to

$$\frac{dn}{d\mu} = \frac{\partial V_1}{\partial x}\left(\frac{\partial V_2}{\partial p}\frac{\partial V_3}{\partial \mu} - \frac{\partial V_2}{\partial \mu}\frac{\partial V_3}{\partial p}\right) / \Delta.$$

Since $\Delta < 0$, $\partial V_1 / \partial x > 0$, $\partial V_2 / \partial p < 0$, $\partial V_3 / \partial p < 0$, and $\partial V_3 / \partial \mu > 0$, a sufficient condition for $dn / d\mu > 0$ is that $\partial V_2 / \partial \mu < 0$. From equations (10) and (17b), it can be shown that $\frac{p}{p_I} = \mu^{\mu}(1-\mu)^{1-\mu} p^{1-\mu}$. Thus $\partial V_2 / \partial \mu < 0$ is equivalent to $d(p/p_I)/d\mu > 0$. ■

The sufficient condition requires that the ratio of the price of the composite input to the price of a final good to decrease with μ. To interpret this sufficient condition, for the ratio of the price of the composite input to the price of a final good to decrease, from equation (14), compared with the price of a final good, the wage rate needs to decrease. From equation (4), because the production of the composite input exhibits constant returns, each factor used in the production of the composite input is paid by its marginal value product. That is, the wage rate is the marginal value product of labor and the price of a final good is the marginal value product of a final good. For the ratio of the price of a final good to the wage rate to increase, the relative marginal productivity of a final good should increase.

The intuition behind Proposition 2 is as follows. When μ increases, there are two effects affecting the level of technology for a firm producing a final good. First, as μ measures the contribution of final goods in the production of the intermediate input and the intermediate input is used to produce final goods, an increase in the value of μ makes a higher output for a firm producing a final good self-fulfilling. This increase in the level of output for a firm producing a final good leads to the choice of more advanced technologies. Second, when μ changes, both the marginal product of labor and the marginal product of a final good change. On the one hand, if the sufficient condition is satisfied, compared with labor, final goods become relatively more productive. This leads to an increase in the level of demand for final goods and firms producing final goods choose more specialized technologies. Thus when the sufficient condition is satisfied, the two effects work on the same direction and a firm producing a final good chooses a more specialized technology. On the other hand, when the sufficient condition is not satisfied, compared with labor, final goods become less productive. This decreases the demand for final goods. The two effects work on opposite directions. Whether the degree of specialization increases or not depends on which of the two effects is stronger.

Some additional results are available from the system (20). First, other things equal, a larger population size means a higher level of purchasing power. Thus population size can be used to measure the size of the market. By applying Cramer's rule to the system (20), it can be shown that the level of output for a firm producing a final good increases with the size of the market. Also, a firm produces a final good chooses a more specialized technology when the size of the market increases. Thus a larger market size leads input suppliers to choose more specialized technologies. Second, it can be shown that the price of a final good decreases with the size of the market. That is, a representative consumer's utility increases with the size of the market. Third, for a firm producing the intermediate input, its elasticity of demand for a final good measures the degree of substitution among various final goods. It can be shown that an increase of the elasticity of demand by a firm producing the composite input leads to a higher level of output, a more advanced technology for a firm producing a final good, and a lower price of a final good.

5. Relationship between internal and external increasing returns

In this section, we derive the degrees of internal and external increasing returns. Then we establish the relationship between internal and external increasing returns.

In the literature it is frequently assumed that increasing returns are absent at the firm level while are present at the industry level. The following proposition shows that the existence of internal increasing returns is necessary for the existence of external increasing returns.[14]

[14] The menu of technologies available to a firm could be dependent on industry size. When an industry becomes bigger, technology breakthroughs may lead to better technologies with lower marginal costs. If this is true, the existence of external economies will not depend on the existence of internal economies.

Proposition 3: If the degree of internal increasing returns is zero, the degree of external increasing returns will also be zero.

Proof: When there is no fixed cost of production ($f = 0$), there are no internal increasing returns. For a final good, mx is the industry output and z is the amount used in the production of the intermediate input, thus $mx - z$ is the net industry output of a final good. From equations (3), (5), (6), (10), and (11), it can be shown that the relationship between net industry output $mx - z$ and input L is given by

$$mx - z = \frac{(1-\mu)\mu^{\mu/(1-\mu)}}{\beta^{1/(1-\mu)}} L . \tag{21}$$

Without internal increasing returns, β is a constant. From equation (21), there are constant returns to scale at the industry level. ■

Suppose there are increasing returns at the firm level. Let ρ and τ denote positive constants. In this section, the fixed and marginal costs of producing final goods are specified as

$$f(n) = n^{\rho} , \tag{22a}$$

$$\beta(n) = n^{-\tau} . \tag{22b}$$

The above cost functions are simple and thus easy to handle. Also, by plugging the above cost functions into equation (18), it can be verified that there exists a unique equilibrium.

From (A6) in the Appendix B, the degree of increasing returns for an individual firm is $(\rho + \tau)/\rho$. From (A11) in the Appendix B, for "$\propto$" denoting "be proportional to", net industry output $mx - z$ is related to labor input L in the following way:

$$mx - z \propto L^{1+\frac{\tau}{\rho-\mu\rho-\mu\tau}} . \tag{23}$$

As labor L is the primary input, formula (23) relates net final good at the industry level as a function of the primary input. As output of a final good is expressed as a function of the primary input, equation (23) provides a microfoundation for external increasing returns. From (23), the degree of external increasing returns at the industry level is $1 + \dfrac{\tau}{\rho - \mu\rho - \mu\tau}$.

The following proposition studies how the degree of external returns is affected by various parameters.

Proposition 4: The degree of external increasing returns to scale increases with μ and τ, and decreases with ρ.

Proof: It can be shown that $1+\frac{\tau}{\rho-\mu\rho-\mu\tau}$ increases with μ and τ, and decreases with ρ. ■

Proposition 4 establishes formally the desirability of a higher linkage effect as argued in Hirschman (1958): for the same level of input, output increases with the level of linkage effects. To understand Proposition 4, first, a higher linkage effect increases the degree of income spillovers. As a result, the degree of external increasing returns is higher. Second, a higher value of τ means that marginal cost decreases at a faster rate. As a result, the degree of external increasing returns is higher. Finally, a higher value of ρ means that fixed costs increase at a faster rate. As a result, the degree of external increasing returns is lower.

The following proposition compares the magnitudes of the degrees of returns.

Proposition 5: With the existence of linkage effects, the degree of external increasing returns is higher than the degree of internal increasing returns.

Proof: The degree of external increasing returns at the industry level is $1+\frac{\tau}{\rho-\mu\rho-\mu\tau}$. The degree of increasing return for an individual firm is $(\rho+\tau)/\rho$. It can be shown that $1+\frac{\tau}{\rho-\mu\rho-\mu\tau}>1+\frac{\tau}{\rho}$ if and only if $\mu>0$. ■

The intuition behind Proposition 5 is as follows. The source of the difference between the degrees of external and internal increasing returns is the existence of linkage effects. If linkage effect is absent ($\mu=0$), the degrees of returns will be the same. For a firm, it takes the output of other firms as given. With the existence of linkage effects, an increase in the level of industry output will lead to multiple rounds of output expansion, similar to the working of the multiplier effect. Thus the degree of increasing returns at the industry level is higher than that at the firm level.

Increasing returns at the firm level are empirically found to be small and consequently might not be advanced as an explanation for the large effect of industrialization on aggregate income (Roberts and Tybout, 1996). With the existence of linkage effects, the degree of external increasing returns can be much higher than the degree of internal increasing returns. Thus our model can be used to explain the large increase in the level of aggregate income from industrialization.

6. Conclusion

In this paper, we have demonstrated that external economies arise from the usage of intermediate inputs and the existence of internal increasing returns in a general equilibrium model incorporating the interaction between the linkage effects and the choices of technologies. The model is tractable and results are derived analytically. First, multiple equilibria may exist and can be ranked. Second, if the degree of internal increasing returns is zero, the degree of external increasing returns will also be zero. The degree of external economies increases with the level of linkage effects. Third, an increase in the size of the market leads firms producing final goods to choose more specialized technologies. The price of a final good decreases with the size of the market. Finally, a sufficient condition for the degree of specialization to increase with the level of linkage effects is that compared with labor, the relative productivity of final goods in the production of the composite input increases.

There are some interesting extensions and generalizations of the model. First, it can be easier for one firm to learn from other firms in the same region than from firms in a different region. Incorporation of local knowledge spillovers may be an interesting avenue for future research. Second, the model can be used to study the impact of international trade. Suppose there is one additional agricultural sector with constant returns to scale and the manufacturing sector is specified as in this paper. With homothetic preferences, it can be shown that a country with a larger population has a comparative advantage in the production of manufactured goods. The extended model may be used to address the core-periphery pattern in international trade. Finally, in this model, industries have the same level of linkage effects. To address the role of a leading sector in economic development, a model incorporating different levels of linkage effects among industries could be an interesting avenue for future research.

Appendix A: Stability of the system of equations (17a)-(17c)

The purpose of this appendix is to show that the stability of the system of equations (17a)-(17c) requires that $\Delta < 0$. As discussed in Metzler (1945) and Samuelson (1983, chap. 9), to derive comparative statics, we may need to introduce explicit dynamics into the model. We now specify the dynamics for the system of equations (17a)-(17c). First, we assume that a firm producing a final good adopts a more specialized technology if the marginal benefit of adopting a more specialized technology is higher than its marginal cost. The marginal benefit of adopting a more specialized technology is the saving of marginal cost $-\beta' x$ and the marginal cost is the increased fixed cost f'. A more specialized technology will be adopted if and only if $-\beta' x - f' > 0$. Let a dot over a variable denote its time derivative. Thus, the evolution of the level of technology is given by

$$\dot{n} = -\beta' x - f' . \tag{A1}$$

Second, we assume that the price of a firm producing a final good is going to rise if the difference between a firm's total cost and its total revenue is increasing. It can be shown that the difference between a firm's cost and revenue is positively related to $(f + \beta\, x) - \mu^{\mu} (1-\mu)^{1-\mu} x\, p^{1-\mu}$. Thus, the evolution of the price of a firm producing a final good is given by

$$\dot{p} = (f + \beta\, x) - \mu^{\mu} (1-\mu)^{1-\mu} x\, p^{1-\mu} . \tag{A2}$$

Third, we assume that the level of output for a firm producing a final good increases when its marginal revenue is higher than its marginal cost. It can be shown that the difference between marginal revenue and marginal cost is positively related with $\left(\varepsilon + \frac{\sigma \mu}{1-\mu}\right) f\ L - p\, x (f + \beta x)$.

Thus, the evolution of the output for a firm producing a final good is given by

$$\dot{x} = \left(\varepsilon + \frac{\sigma \mu}{1-\mu}\right) f\ L - p\, x (f + \beta x) . \tag{A3}$$

For the system of equations (A1)-(A3), as discussed in Metzler (1945) and Takayama (1985, chap. 3), a necessary condition for the system to be stable is that the principal minors of Δ alternative in sign. With $\partial V_1 / \partial n < 0$, $\Delta < 0$.

Appendix B: Derivation of the degrees of internal and external increasing returns

First, we derive the degree of internal increasing returns. Plugging equations (22a) and (22b) into equation (18), the level of technology can be expressed as a function of exogenous parameters:

$$n = \left(\frac{\mu^{\mu} (\varepsilon - \varepsilon \mu + \sigma \mu)^{1-\mu} \rho^{\mu} \tau^{2-2\mu}}{(\rho + \tau)^{2-\mu}} \right)^{\frac{1}{\rho - \mu \rho - \mu \tau}} L^{\frac{1-\mu}{\rho - \mu \rho - \mu \tau}}. \tag{A4}$$

From equations (8), (22a), and (22b), the relationship between the level of output for a firm producing a final good and its technology can be expressed as

$$x = \frac{\rho}{\tau} n^{\rho + \tau}. \tag{A5}$$

Equation (A5) can be used to express f and β as functions of x. As a result, the relationship between the level of output x and the amount of the intermediate input $f + \beta x$ is

$$x = \left[\left(\frac{\tau}{\rho} \right)^{\frac{\rho}{\rho + \tau}} + \left(\frac{\rho}{\tau} \right)^{\frac{\tau}{\rho + \tau}} \right]^{-\frac{\rho + \tau}{\rho}} (f + \beta x)^{\frac{\rho + \tau}{\rho}}. \tag{A6}$$

From equation (A6), the degree of increasing returns for an individual firm is $(\rho + \tau) / \rho$.

Second, we derive the degree of external increasing returns. From equation (12), the number of firms producing a final good can be expressed as

$$m = \frac{L^{1-\mu} Z^{\mu}}{f + \beta x}. \tag{A7}$$

From equations (3), (5), (11), and (A7), the level of intermediate input can be expressed as

$$z = \mu^{\frac{1}{1-\mu}} \left(\frac{x}{f + \beta x} \right)^{\frac{1}{1-\mu}} L. \tag{A8}$$

From equations (A5)-(A7), it can be shown that z differs from $L^{1 + \frac{\tau}{\rho - \mu \rho - \mu \tau}}$ by a positive constant. The relationship between z and L can be expressed as

$$z \propto L^{1+\frac{\tau}{\rho-\mu\,\rho-\mu\,\tau}}. \tag{A9}$$

From equation (A7), the industry level of output $m\,x$ can be expressed as

$$m\,x = L^{1-\mu} Z^{\mu} \frac{x}{f+\beta x}. \tag{A10}$$

From equations (A7) and (A8)-(A10), it can be shown that $m\,x$ differs from $L^{1+\frac{\tau}{\rho-\mu\,\rho-\mu\,\tau}}$ by a positive constant. Thus the relationship between industry output level and labor input can be expressed as $mx \propto L^{1+\frac{\tau}{\rho-\mu\,\rho-\mu\,\tau}}$. With $z \propto L^{1+\frac{\tau}{\rho-\mu\,\rho-\mu\,\tau}}$ and $mx \propto L^{1+\frac{\tau}{\rho-\mu\,\rho-\mu\,\tau}}$, net industry output $m\,x - z$ is related to labor input L in the following way:

$$m\,x - z \propto L^{1+\frac{\tau}{\rho-\mu\,\rho-\mu\,\tau}}. \tag{A11}$$

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