Internal Rebellions and External Threats: A Model of Government Organizational Forms in Ancient China

Zhou, Haiwen

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Haiwen Zhou

Abstract  
In ancient China, a ruler needed to handle both internal rebellions and external threats. To decrease the possibility of internal rebellions, a ruler could organize the government to establish the balance of power among ministers. While effective in preventing internal rebellions, this approach could make the defense of the country against external threats less effective. The tradeoff between preventing internal rebellions and dealing with external threats in a ruler’s choice of government organizational form is affected by the size of the population, the level of coordination efficiency, and the degree of increasing returns in the military sector. If the magnitude of external threats increases, regardless of the type of equilibrium organizational form, the equilibrium level of the concentration of power among division heads increases.

Keywords: Government organizational form, feudalism, county system, Chinese history, balance of power

JEL Classification Numbers: N15, N45, P40

1. Introduction

In ancient China, to rule successfully, a ruler needed to handle both internal rebellions and external threats. Internal rebellions were frequently observed, as recorded in Sima (1988) and Sima et al. (1084). A general might rebel, a high rank civilian officer might usurp power, and peasants might rebel. External threats were also common in China’s history. For example, in the early stage of the Han Dynasty (206 BC-220), the threat from Xiongnu was severe. More significantly, the Song Dynasty (960-1279) was replaced by the invading Mongols and the Ming Dynasty (1368-1644) was replaced by the invading Manchu.

To prevent internal rebellions and to deal with external threats, a ruler may choose from three types of government organizational forms. First, the ruler might adopt the fengjian

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1 I thank Laura Razzolini and two anonymous referees for their very insightful suggestions. I am solely responsible for all remaining errors.

2 Internal rebellions could interact with external threats. For example, when Li Yuan was the leader of an army of the Sui Dynasty stationed in Taiyuan to handle external threats from the Turkish, he used the possibility of an imminent Turkish attack to justify his action to recruit a large number of soldiers. With a large number of soldiers, Li Yuan rebelled against the Sui government and became the founding emperor of the Tang Dynasty.

3 Huang (1997) provides a modern interpretation of China’s history. His book also contains examples of rebellions in China’s history.

4 For the dates of dynasties, if not explicitly specified, the date should be interpreted as AD.

5 One alternative way to decrease the possibility of rebellions was to put “good” individuals less likely to rebel into important positions, as emphasized by the Confucianism school. A famous example of the success of this approach was the Duke of Zhou in the Zhou Dynasty. The Duke of Zhou had the power to punish the emperor for the emperor’s wrong doing while he did not usurp the power of the emperor (Huang, 1997).
feudalism) form. Under this organizational form, important positions of the country would be inherited and would not be controlled by the central government and the power of the central government would be limited. Second, the ruler may adopt the jun-xian (commandery-county) organizational form. For simplicity of presentation, for the rest of this paper, this organizational form is referred to as the county form. Under the county form, local officials would be appointed by the central government and government power would be concentrated at the central government. While the county form would be useful to pool resources for the central government and might make the handling of external threats more effective, internal rebellions could be a threat to the ruler because a minister’s control of the central government would lead to the usurpation of power. Third, the ruler might adopt a mixed organizational form in which some parts of the country were organized as kingdoms while other parts of the country were organized as counties.

As discussed in more detail in Section 2, within a given government organizational form, a ruler could take various measures to decrease the possibility of internal rebellions. Those measures worked by decreasing the concentration of power among ministers. But the deconcentration of power means that organization efficiency could be harmed and the handling of external threats would be less effective. That is, while a policy could decrease the possibility of internal rebellions, at the same time it could make the defense against external threats less effective.

Scholars in China have debated on the choice of organizational form for a long time, going back to the Qin Dynasty (221 BC-206 BC) more than two thousand years ago. Each type of organizational form has its pros and cons. Under the county form, without kingdoms established, powerful ministers could usurp power. Under the feudal form, when kingdoms were established, the existence of kingdoms could deter powerful ministers from usurping power. However, kings might rebel and fight among themselves. While no organizational form dominated, historically a ruler’s choice of government organizational form was not random and could be affected by the size of the population, the magnitude of external threats, the level of coordination efficiency, and the degree of returns to scale in the military. For example, the choice of organizational form could

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6 The term “feudalism” used in the study of European history is frequently used to refer to dynasties in China after the Qin Dynasty. However, there were significant differences between the European feudalism and the Chinese dynasties (Huang, 1999, pp. 146-147).

7 In China’s history, a dynasty could be commonly viewed as organized as a county form even though remote regions might be organized as highly autonomous units.
be affected by the degree of returns in the military sector. If an organizational form leads to multiple small armies rather than one large army, the possibility of internal rebellions by a military leader would be smaller but the aggregate military strength of the country and hence the ability of this country to defend against external threats would be sacrificed when there are increasing returns in the military sector.

In the literature, there are numerous valuable researches on ancient China. For example, Huang (1974) addresses taxation and governmental revenue in the Ming Dynasty. Chao (1986) examines the evolution of land-labor ratio in ancient China. Pomeranz (2000) explores the reasons of ancient China’s lack of industrialization and challenges some received wisdoms on this issue. Pomeranz and Topik (2005) check the impact of trade on China and other cultures. Shiue (2002), Keller and Shiue (2007), and Shiue and Keller (2007) study the degree of market integration in the Qing Dynasty (1644-1911) and compare it with that of Europe. Wong (1997) and Rosenthal and Wong (2011) revisit different experiences between China and Europe over centuries and propose studying the impact of competition on a political regime’s long-run performance.

However, to our best knowledge, there is no formal model addressing the choice of government organizational form in ancient China. In this paper, we study a ruler’s choice of the organizational form in ancient China in a simple formal model to provide a framework to organize our thinking on this important issue. In the model, a ruler chooses the organizational form to maximize his expected benefit.\(^8\) Both the feudalism form and the county form have their advantages and disadvantages. While the feudalism form needs less coordination among different divisions of the government, it does a less good job in exploiting increasing returns in the military sector. While the county form may exploit increasing returns in the military sector better, with the division of power established endogenously within the county form, it could suffer from a lack of coordination among different divisions of the government. If the degree of increasing returns in the military sector is small or if the level of coordination efficiency is low, the ruler will choose the feudal organizational form. If the degree of increasing returns in the military sector is high or the level of coordination efficiency is high, the ruler will choose the county form.

\(^8\) In the literature, Davis (2003) studies organizations from a transaction cost perspective. Zhou (2005) studies how the relative performance of the unitary organizational form and the multidivisional organizational form is affected by market structure such as the number of firms competing in the same industry.
If the magnitude of external threats increases, we show that a robust result is that regardless of the type of equilibrium organizational form, the equilibrium number of divisions decreases and thus the degree of the concentration of power among division heads increases. Thus the observation in Zhang (2004) that division heads of the government would be given more power when the external threats increased was rational. With a higher degree of the concentration of power among division heads, the possibility of internal rebellions by a division head could increase.

This paper contributes to the literature by analyzing the impact of internal rebellions and external threats on a ruler’s choice of government organizational form in a unified model rather than treating the impact of internal rebellions and external threats separately. This unified framework is useful in explaining some historical puzzles. For example, one puzzle in China’s history is that compared with other dynasties such as the Tang Dynasty (618-907), the Song Dynasty was richer while militarily weaker (Elvin, 1973). Why the wealth of the Song Dynasty did not translate into a stronger military force? This puzzle can be explained by using our unified framework: to develop a strong army, the concentration of military resources would be needed. But this concentration of military resources would increase the possibility of a rebellion by a military leader. Because policies to develop a strong army were not adopted, the wealth of the Song Dynasty did not turn into military power. Our model is also useful in explaining why the rulers in ancient China tried to disarm the country after a unification of the country rather than to maintain a strong army: recognizing that a strong army may not only be used to deal with external threats but also be used by ministers to rebel, rulers frequently chose to disarm.

When a ruler designed the institutions for his dynasty, he could be mainly concerned with his and his family’s rule rather than economic efficiency. Actually many institutions designed by rulers harmed economic efficiency. For example, Huang (1999) argues that various institutions designed by Zhu Yuanzhang, the founding emperor of the Ming Dynasty, were not based on efficiency considerations. In this model, we provide one reason for the persistence of inefficient institutions: while an institution such as the separation of power within the government may be inefficient from economic efficiency perspective, it is useful to the ruler because it could decrease the possibility of internal rebellion. That is, the multiple roles played by an institution make an economically inefficient institution persistent.

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9 See Davis (2008) and Hall et al. (2010) for recent studies of the role of institutions on economic performance.
In this paper, while we focus on the impact of external wars on ancient China, various aspects of our study are also relevant to Europe. For example, we discuss how the preparation of war with Xiongnu led to a strengthening of the central government in the Han Dynasty under the rule of Liu Che. As discussed in Tilly (1992) and Rosenthal and Wong (2011), preparation of war also had significant impact in ancient Europe. The growth of military technology has played an important role in driving state formation in Europe. Military technology changed how war was conducted (Parker, 1996). Compared with national states, the small scale and fragmented sovereignty of city-states was a clear disadvantage. The city-state and city-empire lost out when mass armies recruited from state’s own population became crucial to successful warfare (Tilly, 1992). However, war threat was more significant in Europe (Rosenthal and Wong, 2011). This difference led to differences in the location of industries between ancient China and Europe. In ancient China, as the war threat was less significant, hand-craft industries frequently located at rural areas to be close to raw materials, to save labor costs, and to avoid the high mortality rate in cities. In Europe, as the war threat was significant, industries were concentrated in walled cities to get protection and cheap capital. To raise the huge amounts of money for wars, rulers in Europe encouraged the creation of credit market. This might have lead to the unintended consequence of European industrialization (Rosenthal and Wong, 2011).

In this model, a ruler’s choice of organizational form is affected by various parameters. One interesting question is to posit a reason for the change in parameters that produced the changing organizational forms. In the literature, there are various mechanisms inducing parameter changes. First, in Jones (2001) and Zhou (2009), population growth serves as the mechanism that induces changing behavior. In Galor and Weil (2000), population density forces changes. Second, in Tamura (1996a, 2006), human capital accumulation can induce changing costs of organization and hence changing organizational form. In this model, the size of the population is also a key parameter. Other parameters in this model such as the degree of returns in the military sector can be affected by population change and human capital accumulation and interact with them.

The rest of the paper is organized as follows. Section 2 illustrates the organizational responses to internal rebellions and external threats in ancient China to motivate the model. Section 3 specifies the model and establishes the equilibrium conditions. Section 4 studies the equilibrium organizational form. Section 5 presents numerical results on the impact of parameter changes on the optimal choice of the organizational form. Section 6 relates historical evidence of
the organization of governments to results in this model. Section 7 discusses some generalizations and extensions of the model and concludes.

2. Organizational Responses to Internal Rebellions and External Threats in Ancient China

Historically, a ruler’s choice of government organizational form in ancient China was complicated and was affected by various factors. For example, a ruler may establish some kingdoms to reward surrounded generals, such as the kingdoms established in the early period of the Qing Dynasty. Even though there were various factors affecting a ruler’s choice of government organizational form, his concern for internal rebellions and external threats was a very significant one. For example, the concern for internal rebellions and external threats was dominant when Zhu Yuanzhang designed the political system in the Ming Dynasty. In this section, we illustrate the organizational responses to internal rebellions and external threats in ancient China.

In China’s history, first, the organizational form adopted in the Zhou Dynasty (1045 BC-256 BC) was close to the feudalism form. Rulers of the Zhou Dynasty established many kingdoms to reward their relatives and subordinates and to place nobles from the replaced Shang Dynasty. The central government was weak under the feudal system and the Zhou ruling house was more like a nominal leader rather than actual leader of the kingdoms. Second, during the Spring Autumn Period (770 BC-476 BC) and the Warring States Period (475 BC-221 BC), kingdoms fought with each other for survival. A king’s direct control of resources was important for his success in fighting. To increase military strength, when a kingdom acquired a new piece of land, this piece of land could be organized as a county ruled directly by the king rather than as an autonomous unit awarded to a Dafu ruled indirectly by the king. In this sense, the arise of counties could be viewed as an organizational innovation. Examples of the county organizational form include the Qin Dynasty and the Tang Dynasty. For the county form, because the power was concentrated at the central government, if a powerful minister was able to control the central government, he could

10 The performance of a political system can be affected by the distribution of human capital in a society. See Tamura (1996b, 2002) for models on the role of human capital in the process of economic development.
11 Here kings in the Warring States period would either set up counties or autonomous units. In other parts of this paper, “kingdom” becomes synonymous with the autonomous unit. A reader may want to be aware of the following fact in China’s history to avoid being confused. Qi Shi-huang-di (259 BC-210 BC) at the end of the Warring States period was the first emperor in China. Before him, the highest ruler would be called the king. After him, the highest ruler would be called the emperor instead of the king.
12 One important part of the famous Shang Yang Reform at that time was the establishment of counties. The reform laid the foundation for Qin to unite China.
usurp the government. Third, examples of the mixed organizational form include the early stages of the Han Dynasty, the Western Jin Dynasty (265-316), and the Ming Dynasty.

No organizational form had a perfect record in preventing internal rebellions. For the feudalism form in the Zhou Dynasty, the authority of the Zhou ruling house declined gradually and kingdoms engaged in fighting among themselves. The Zhou ruling house was eventually eliminated by the kingdom Qin. For the county form, one example of internal rebellions was the usurpation of the central government by Wang Mang in the Han Dynasty, who was a relative of the royal family. Another example is that during the Three Kingdoms Period (220-280), when the ruler from the Cao family was young, Sima Yi usurped the power through a military coup. For the three dynasties adopted the mixed organizational form, all suffered from the rebellions of kingdoms. In the Han Dynasty, several kingdoms engaged in a rebellion in 154 BC; In the Western Jin Dynasty, eight kingdoms engaged in wars lasted between 291 to 306; In the Ming Dynasty, the second emperor Zhu Yunwen was overthrown by the rebelling king Zhu Di (Zhu Yunwen’s uncle) after a war lasted between 1399 to 1402. Zhu Di became the third emperor of the Ming Dynasty.

Within each type of organizational form, a ruler could take various measures to decrease the possibility of internal rebellions. Balance of power was attempted between the kingdoms and counties. For example, in the Han Dynasty, the domains of the kingdoms intersected with the domains of the counties so that they could monitor one another. To decrease the possibility of the rebellions of kingdoms, Liu Che, an emperor in the Han Dynasty, changed the law on the inheritance of the kingdoms. Before his change of the law, only one offspring (usually the oldest son) of a king could inherit the kingdom and the size of a kingdom may not change over time. After his change of the law, every son of a king was required by the emperor to be granted the right to inherit a part of the kingdom. Since a king in ancient China usually had multiple wives and thus multiple sons, a kingdom would be divided into several smaller kingdoms after the death of the incumbent king. The size of a kingdom would decrease and the threat of a kingdom to the central government would decrease over time.

Under the county form, to decrease the possibility of usurpation of power by civilian officers, the rulers established the division of power within the governments. For the organization of the central government, the three “Sheng” (department) of the central government of the Sui Dynasty (589-618) and the Tang Dynasty had balance of power among them in the following way. The Zhongshu Sheng was responsible for drafting an order, the Mengxia Sheng was responsible
for reviewing the order, and the Shangshu Sheng was responsible for implementing the order. The Shangshu Sheng was further divided into six “Bu” (ministry) specializing on personnel administration, finance, rites, military, justice, and public works respectively. There was also balance of power in the organization of local governments. For example, in the Qing Dynasty before the Taiping Tianguo movement (1851-1864), the official in a province supervising the army would be different from the official collecting government revenues. Without enough revenue to reward his subordinates, it would be difficult for the official supervising the army to recruit enough dedicated followers among his subordinates and thus to rebel. In China’s history such as the Han Dynasty, the central government also frequently sent out officials to local districts to monitor local government officials.

Under the county form, a prime minister occupied an important position and could be a challenge to the emperor. In China’s history, as the emperors tried to consolidate power, the power of a prime minister decreased over time. Early on, in the Qin Dynasty (the first dynasty established central government rule in ancient China) and the early stage of the Han Dynasty, a prime minister was responsible for running the government and had significant power. For example, the prime minister Li Si played an important though infamous role in deciding who would be the next emperor after the death of the emperor Qin Shi-huang-di. Meanwhile, the emperor Liu Che of the Han Dynasty adopted a strategy of using junior officials rather than well-established officials as prime ministers because junior officials would not have the authorities to challenge him. Liu Che also used his own office rather than the prime minister’s office to implement some government orders. In the Tang Dynasty, the heads of the three “Sheng” were prime ministers. Thus there were three or more prime ministers and the power of a prime minister in the Tang Dynasty was smaller than that in the Han Dynasty. Later on, in the Ming Dynasty, the founding emperor Zhu Yuanzhang used the rebellion of the prime minister Hu Weiyong as an excuse to take an extreme measure of eliminating the position of the prime minister altogether. Sacrificing coordination efficiency, Zhu Yuanzhang distributed the power of the premier to lower rank officials.

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13 During the war to put down the Taiping Tianguo movement, to improve coordinate efficiency, officials leading an army such as Zeng Guofan were frequently granted the right to collect tax or and to supervise officials collecting taxes. This increase of the power of local officials at the province level planted the seed of the collapse of the Qing Dynasty. Later, during the Boxer movement in 1900, officials from provinces in Southeastern China did not follow the central government’s order to declare war on foreign governments. During the revolution in 1911, when provinces disobeyed the order of the central government, the Qing Dynasty ended.

14 Qian (2001) provides a descriptive discussion of the evolution of political systems in ancient China.
officials (heads of the six “Bu”). The Qing Dynasty (the last dynasty of China) adopted many institutions of the Ming Dynasty and there was no prime minister in the Qing Dynasty.

Emperors took specific measures to decrease the possibility of rebellions by military officers. In various cases, the central government tried to support multiple military leaders so that they could deter one another from rebelling rather than to concentrate resources on one military leader. For example, in the Qing Dynasty, Zeng Guofan was leading an army to put down the rebellion of the Taiping Tianguo Movement. As the emperor at that time was a child, Empress Dowager Cixi was the actual ruler of the government. Cixi was worrying about a potential rebellion of Zeng because Zeng had the largest army in the country at that time. To decrease the risk of a rebellion of Zeng, Cixi tried to provide more resources to Zeng’s colleagues rather than Zeng even though this strategy could make the fighting with the Taiping army less effective.

A systematic approach to prevent rebellions of military leaders was pursued by Zhao Kuangyin, the founding emperor of the Song Dynasty. Before becoming the emperor, Zhao was a high rank military officer of Chai Yong, the incumbent emperor of the Later Zhou Dynasty (951-960). After Chai Yong’s death, his son Chai Zongxun became the new emperor who was only seven years old. During this transition of power, it was claimed that there was an invasion and the government sent Zhao to fight the enemies. On his way to fight the enemies, Zhao’s subordinates initiated a coup and Zhao was installed as the new emperor. Based on his own experience of becoming an emperor and the lessons learned from previous dynasties such as the rebellion of An Lushan in the Tang Dynasty, Zhao created a system of the balance of power within the army to decrease the possibility of rebellions by military leaders. Under Zhao’s system, first, there was balance of power between armies stationed in the capital and armies stationed at local governments: armies stationed in the capital were strong enough to put down any rebellion from an army stationed in a local government and armies stationed at local governments together could put down a rebellion from the armies stationed in the capital. Second, some important military positions such as Zhao’s former position in the military were eliminated. For each remaining important position, it could be split into several positions so that the role of one individual would not be too significant. Third, to prevent personal attachment between generals and soldiers, the generals training soldiers would be different from the generals leading soldiers in fighting. In a

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15 Qian (2001) provides a discussion of the military institutions in the Han, Tang, Song, and Ming Dynasties.
military campaign, it was common that generals were not familiar with their soldiers and soldiers were not familiar with the generals who were leading them.

Measures used to decrease the possibility of internal rebellions could make the handling of external threats less effective, and vice versa. The conflict between preventing internal rebellions and dealing with external threats was common in ancient China. On the one hand, measures to deal with external threats could increase the possibility of internal rebellions. The change of the military recruitment system in the Tang Dynasty provides an example of this type of conflict. Before the emperor Li Longji, the Tang Dynasty rotated soldiers who were also peasants for military services. To deal with external threats more effectively, Li Longji adopted a policy that changed the compulsory military recruitment system to a voluntary system in which soldiers became professional armies. While the handling of external threats was successful, this change led to the rise of powerful generals, such as An Lushan. The rebellion of An Lushan and Shi Siming led to a civil war that lasted for several years. Even though the rebellion was eventually put down, it led to the decline of the Tang Dynasty and finally the collapse of the Tang Dynasty.

On the other hand, measures to decrease the possibility of internal rebellions could make the handling of external threats less effective. To prevent the generals from rebelling, rulers frequently sent out their personal representatives (usually eunuchs) to monitor the generals. As those personal representatives normally did not know much about how to fight wars, their decisions frequently decreased the effectiveness of military officers and thus the handling of external threats. For the Song Dynasty specifically, while the institutions were effective in preventing internal rebellions from happening, the Song government was defeated and replaced by the Mongols even though the Song Dynasty was more developed than its enemies (Elvin, 1973, chap. 7). For example, the prevention of personal attachment between the generals and soldiers decreased the possibility of rebellions by generals, but it also decreased the military strength of the army. Also, to prevent internal rebellions, the Song government absorbed many peasants without

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16 Elvin (1973, chap. 5) has a discussion of the recruitment of soldiers in the Tang Dynasty. The rotary system depended on a relative equal distribution of land. As the distribution of land became more concentrated over time and many peasants lost their land, the rotary system was not functioning because not many peasants could afford the expenses any more.

17 Interestingly, the Song Dynasty lasted for more than three hundred years and was the longest dynasty in China since 221 BC. In this sense, the strategies adopted by the founding emperor Zhao Kuangyin to preserve the dynasty were successful.

18 In China’s history, strong military leaders such as Yue Fei in the Song Dynasty, Qi Jiguang in the Ming Dynasty (Huang, 1982, chap. 6), and Zeng Guofan in the Qing Dynasty depended on their personal attachment with soldiers. For example, Zeng recruited soldiers from the same region where he came from and the generals of Zeng’s army were
land and criminals into the army, and this type of practice decreased the fighting power of the army.\textsuperscript{19}

As the magnitude of external threats and the possibility of internal rebellions changed over time, an emperor’s policy could change over time, even dramatically. For example, in the Song Dynasty, when the degree of external threats from the Western Xia (1038-1227) was severe, an official named Fan Zhongyan initiated a reform in 1043 to deal with this external threat. This reform included measures such as the establishment of a professional army so that generals would be familiar with the soldiers, the elimination of redundant officials in the government, and an increase of the power of the prime minister. Fan’s reform was initially supported by the emperor Zhao Zhen. However, when a peace treaty was signed between the Song government and the Western Xia government and the degree of external threats decreased, the concern for internal rebellions increased. Some officials in the Song government suggested to the emperor Zhao Zhen that Fan’s policies were dangerous to the emperor’s rule and the emperor demoted Fan Zhongyan. As Fan was a highly regarded official in China’s history, in popular books the demotion of Fan was attributed to the criticisms of his “bad” colleagues to highlight the conflicts between good ministers and bad ministers. Our framework suggests that it was the change of the fundamentals affecting the relative importance of dealing with external threats and preventing internal rebellions that contributed to the rise and fall of Fan.

3. Model Specification

In this section, we specify the model and establish conditions for equilibrium. The population of the country is $L$, which is exogenously given. The ruler chooses the number of kingdoms and the number of counties to maximize his expected benefit, which is affected by the possibility of internal rebellions and the existence of external threats. First, if the ruler only establishes kingdoms in equilibrium, the organizational form is called the feudalism form. Second, if the ruler only establishes counties in equilibrium, the organizational form is called the county relatives, students, and friends of Zeng. In Zeng’s army, a general was responsible for recruiting his subordinates and if the general of a division was killed during the war, the whole division would be disbanded. As soldiers came from the same hometown and were well connected with one another, they were more likely to help one another in the battlefield rather than to escape from their wounded colleagues because of personal attachment and fleeing soldiers would carry stigmas not only in the army, but also in their hometowns.

\textsuperscript{19} To prevent peasants without enough food from rebelling, governments frequently organized relief activities. See Shiue (2004, 2005) for famine and disaster relief in the Qing Dynasty.

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form. Third, if the ruler establishes both kingdoms and counties, the organizational form is called the mixed form. The ruler engages in a tournament with a “foreign” ruler and the military strength of the foreign ruler measures the magnitude of external threats. If the ruler wins the tournament and there is no successful internal rebellion, the ruler gets an exogenously given reward. To simplify notation, this reward is normalized to one. If the ruler loses the tournament with the foreign ruler or if there is any successful internal rebellion, for simplicity, the ruler gets a reward of zero. When the level of military strength of the ruler is $\Psi$ and the level of military strength of the foreign ruler is $\Omega$, the ruler’s probability of winning the tournament is specified as $\Psi / (\Psi + \Omega)$.\(^{20}\)

The military strength of the foreign ruler is assumed to be exogenously given.\(^{21}\) The military strength of the ruler is affected by factors such as the level of domestic population and his choice of organizational form. Suppose in equilibrium the ruler establishes $m$ kingdoms, $m \geq 0$, and $n$ counties, $n \geq 0$.\(^{22}\) The sum of the number of kingdoms and the number of counties is referred to as the number of divisions in the government. Thus the total number of divisions in the government is $m + n$. All divisions are assumed to have the same population size $l$. The sum of the population in all divisions is equal to the total size of the population of the country:

$$(m + n)l = L. \quad (1)$$

When the ruler chooses the number of divisions, his decision is affected by the level of coordination efficiency and the degree of increasing returns in the military sector. If the ruler establishes counties, these counties need to be coordinated. For $0 \leq c \leq 1$, $c$ measures the level of coordination efficiency among the counties and a higher value of $c$ indicates that the level of coordination efficiency is higher. When there are $n$ counties, the aggregate level of coordination efficiency among all counties is specified as $c^n$. That is, other things equal, a higher number of counties decreases the aggregate level of coordination efficiency among the counties.

The degree of returns to scale in the military sector is $s$. When there are decreasing degrees of returns in the military sector, the feudalism form will always dominate. That would not

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\(^{20}\) This specification of the determination of the winner follows the rent-seeking literature, as studied in Tullock (1980). The determination of the winner could be different in R&D tournaments, as studied in Zhou (2006).

\(^{21}\) This assumption can be motivated as follows. In ancient China, the military strengths of nomadic groups were affected by various types of uncertainties, such as the amount of snow. The amount of snow could affect the quantities of living stock such as the number of horses and thus the military power of the nomadic groups.

\(^{22}\) To simplify presentation, the number of divisions is a real number, rather than restricted to be an integer number.
be very interesting. Thus, we focus on the interesting case that \( s \geq 1 \). Increasing returns in the military sector can be motivated as follows. A higher number of soldiers allows a better division of labor among soldiers and this can make military operations more effective. In ancient China, complicated technologies to attack cities and to defend cities were developed. A large army would be conducive in applying these military technologies.

With increasing returns in the military sector, will the ruler choose as few divisions as possible to exploit increasing returns? Not necessarily. When the number of divisions of the government is small, a division head has significant power and may rebel. When the total number of divisions increases, power becomes less concentrated among division heads and the probability of successful rebellion by a division head decreases. If a division head’s probability of a successful rebellion decreases at a rate faster than the rate of the increase of the total number of divisions, the total probability of a successful rebellion by any divisional head decreases with the total number of divisions. This claim can be motivated as follows. When there are \( m + n \) divisions, if a division head decides to rebel, the division members will consider the possibility of the success of the rebellion before following the division head’s decision to rebel. It is reasonable to assume that division members are less likely to follow their division head when their division head controls a smaller percentage of the military force. The percentage of the military force controlled by a division head is \( \frac{1}{m + n} \). For simplicity, suppose the possibility that division members follow their division head to rebel with probability \( \frac{1}{m + n} \). Even though the division members follow their division head to rebel, to rebel successfully, this division head still needs to win the fight with other divisions. Conditioning on his subordinates are willing to support him, suppose the probability that this rebelling division head wins the fight with other divisions is \( \frac{1}{m + n} \). Thus, the probability that a division head is able to rebel successfully is \( \frac{1}{(m + n)^2} \). Since there are \( m + n \) divisions, if the probabilities of successful rebellions of divisions are independent, the total probability of any successful rebellion by a division head is \( \frac{1}{m + n} \). That is, the probability of no successful rebellion is \( 1 - \frac{1}{m + n} \) and it increases if the total number of divisions increases.

Because the counties and the kingdoms are organized in different ways, increasing returns may not operate at the national level. Since the kingdoms were quite autonomous, increasing returns for a kingdom applied at the individual kingdom level. Each of the \( m \) kingdoms has a
military strength of $l^s$ and military strength from all kingdoms is $\sum_{i=1}^{m} l^s_i$. Increasing returns for the counties apply at the size of the population organized under the counties. The size of the population under the counties is $L - ml$, taking into account of the level of coordination efficiency, total military strength from counties is $c^n (L - ml)^s$. The total military strength of the ruler is the sum of the military strength from the kingdoms and the counties: $\Psi = \sum_{i=1}^{m} l^s_i + c^n (L - ml)^s$.

Taking into account of the probability of no successful rebellions $1 - \frac{1}{m+n}$ and the probability $\Psi / (\Psi + \Omega)$ of winning the tournament, the ruler’s expected payoff is $\left(1 - \frac{1}{m+n}\right) \frac{\Psi}{\Psi + \Omega}$.

Plugging the value of $\Psi$, the expected payoff of the ruler can be expressed as

$$
\left(1 - \frac{1}{m+n}\right) \frac{\sum_{i=1}^{m} l^s_i + c^n (L - ml)^s}{\sum_{i=1}^{m} l^s_i + c^n (L - ml)^s + \Omega} + \Omega
$$

$$
= \left(1 - \frac{1}{L}\right) \frac{c^n n^s + m + \Omega}{c^n n^s + m + \Omega}.
$$

When the ruler chooses the organizational form, there is a conflict between preventing internal rebellions and dealing with external threats. To deal with external threats, it is better to establish a small number of divisions to exploit increasing returns in the military sector. However, a small number of divisions will make internal rebellions more likely. By sacrificing some military power in dealing with the external threats, the probability of internal rebellions decreases. As a result, the expected payoff of the ruler could be higher.

In this model, we assume that it is impossible for the ruler to choose a pure county system with only one county. There were at least two reasons for this assumption. First, if an emperor led an army, the repercussion of a defeat could be much larger than a defeat led by someone else. Under normal circumstances, the bureaucracy would oppose the emperor from leading an army. Thus it was not strange that Li Shimin of the Tang Dynasty was directly involved in leading

---

23 Interestingly, in the Ming Dynasty, the emperor Zhu Houzhao did a very unusual thing by awarding himself a title of grand general so that he could circumvent the opposition of the bureaucracy to lead an army (Huang, 1982). Zhu’s behavior was a headache to the government bureaucracy.
military campaigns before he became the emperor while he was much less directly involved in leading military campaigns after he had become the emperor. Second, it may not be feasible for the emperor to lead an army. After the death of an emperor, the new emperor could be very young and would not be able to head an army. Since the founding emperors established institutions to help their offspring to rule as long as possible, it is better to have an institution that would work under different situations such as the situation of a young emperor or/and a not much competent emperor. With the above two justifications, we assume that the emperor would not establish only one division for the country and be the head of this only division.

By choosing the number of kingdoms, the number of counties, and the size of each division, the ruler tries to maximize Equation 2 subject to the constraint (1) and the constraints that the number of divisions should be nonnegative.

For \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) denoting the costate variables, the Lagrangian is

\[
Max: \quad \left(1 - \frac{l}{L}\right) \frac{(c^n n^s + m)}{c^n n^s + m + \Omega l^{-s}} + \lambda_1 [L - (m + n)l] + \lambda_2 l + \lambda_3 n .
\]

The first order condition with respect to \( m \) is

\[
\left(1 - \frac{l}{L}\right) \frac{\Omega l^{-s}}{(c^n n^s + m + \Omega l^{-s})^2} - \lambda_1 l + \lambda_2 \geq 0 . \tag{3}
\]

The first order condition with respect to \( n \) is

\[
\left(1 - \frac{l}{L}\right) \frac{(sn^{-1} c^n + n^s c^n \ln c)\Omega l^{-s}}{(c^n n^s + m + \Omega l^{-s})^2} - \lambda_1 l + \lambda_3 \geq 0 . \tag{4}
\]

The first order condition with respect to \( l \) is

\[
s \left(1 - \frac{l}{L}\right) \frac{(c^n n^s + m)\Omega l^{-s-1}}{(c^n n^s + m + \Omega l^{-s})^2} - \frac{1}{L} \frac{(c^n n^s + m)}{c^n n^s + m + \Omega l^{-s}} - \lambda_4 (m + n) = 0 . \tag{5}
\]

In addition to Equations 1, 3, 4, and 5, other necessary conditions for the ruler’s optimization include

\[
\lambda_2 m = 0 , \lambda_2 \geq 0 , \ m \geq 0 ,
\]

\[
\lambda_3 n = 0 , \lambda_3 \geq 0 , \ n \geq 0 .
\]

\[24 \text{ If the sum of the number of kingdoms and counties was smaller than one, the possibility of internal rebellions would be higher than one. Thus, the equilibrium number of kingdoms and counties should be higher than one. Whether this condition is satisfied or not can be checked once the values of parameters such as the size of the population are specified.} \]
We can arrive at the following results by applying the envelope theorem on Equation 2. First, the expected payoff of the ruler increases with the size of the population. Thus an increase of the size of the population is beneficial to the ruler. Second, the expected payoff of the ruler decreases with the magnitude of external threats. Thus an increase of the magnitude of external threats is harmful to the ruler. Third, the expected payoff of the ruler increases with the level of coordination efficiency. Thus an increase of the level of coordination efficiency is beneficial to the ruler. Fourth, the expected payoff of the ruler increases with the degree of returns in the military sector. Thus an increase of the degree of returns in the military sector is beneficial to the ruler.

4. Equilibrium Organizational Form

Depending on the values of parameters such as the level of coordination efficiency, we can have either a corner solution or an interior solution. There are two types of corner solutions: one is the pure feudalism form in which the ruler only establishes kingdoms and the other is the pure county form in which the ruler only establishes counties. There is one interior solution in which the ruler establishes both kingdoms and counties in equilibrium. In the following, we first study the feudalism form, then the mixed organizational form, and finally the county form.

4.1. The Feudalism Organizational Form

In this subsection, we study the corner solution in which the government is organized in the pure feudal form. Under the pure feudalism form, \( n = 0 \).

From Equations 3 and 5, the number of kingdoms under the feudalism form is defined by

\[
 s - 1 - \frac{s}{m} - \frac{L^s}{m^s \Omega} = 0. \tag{6}
\]

Equation 6 has three implications. When the government is organized as the feudalism form, first, an increase of the size of the population increases the optimal number of kingdoms. Second, an increase of the magnitude of external threats decreases the optimal number of kingdoms. Third, an increase of the degree of increasing returns in the military sector decreases the number of kingdoms.

4.2. The Mixed Organizational Form
In this subsection, we study the mixed organizational form in which the ruler establishes both kingdoms and counties in equilibrium. When there are both kingdoms and counties, we have $\lambda_2 = 0$, $\lambda_3 = 0$. From Equations 1, 3, 4, and 5, we can derive the following system of three equations defining three variables $m$, $n$, and $l$ as functions of exogenous parameters:\(^{25}\)

\[
\begin{align*}
V_1 &\equiv L - (m + n)l = 0, \quad (7a) \\
V_2 &\equiv c^n n^{-1} (s + n \ln c) - 1 = 0, \quad (7b) \\
V_3 &\equiv s(c^n n^s + m) - (m + n) - \frac{l}{\Omega L} \left( c^n n^s + m + \Omega l^{-1} \right) = 0. \quad (7c)
\end{align*}
\]

Partial differentiation of $V_1 - V_3$ with respect to $m$, $n$, $l$, $L$, $\Omega$, $c$, and $s$ leads to

\[
\begin{vmatrix}
\frac{\partial V_1}{\partial m} & \frac{\partial V_1}{\partial n} & \frac{\partial V_1}{\partial l} \\
0 & \frac{\partial V_2}{\partial n} & 0 \\
\frac{\partial V_3}{\partial m} & \frac{\partial V_3}{\partial n} & \frac{\partial V_3}{\partial l}
\end{vmatrix}
\begin{pmatrix}
dm \\
dn \\
dl
\end{pmatrix}
= - \begin{vmatrix}
\frac{\partial V_1}{\partial L} \\
0 \\
\frac{\partial V_3}{\partial L}
\end{vmatrix}
\begin{pmatrix}
dL \\
d\Omega \\
dc \\
ds
\end{pmatrix}
- \begin{vmatrix}
0 \\
0 \\
\frac{\partial V_2}{\partial s}
\end{vmatrix}
ds. \quad (8)
\]

The following proposition studies the impact of an increase of the size of the population on the ruler’s choice of organizational form.

Proposition 1: When the government is organized in the mixed form, an increase of the size of the population leaves the number of counties unchanged while increases the number of kingdoms.

Proof: Let $\Delta$ denote the determinant of the coefficient matrix of System 8. For stability, it is assumed that $\Delta < 0$.\(^{26}\) An application of the Cramer’s rule on the System 8 leads to $dn / dL = 0$ and

\[
\frac{dm}{dl} = \frac{\partial V_2}{dn} \left( \frac{\partial V_1}{dL} \frac{\partial V_3}{dL} - \frac{\partial V_1}{dL} \frac{\partial V_3}{dL} \right) / \Delta. \quad (9)
\]

Partial differentiation of Equations 7a and 7c leads to

---

\(^{25}\) Equations 7a, 7b, and 7c are derived as follows. First, Equation 7a is the same as Equation 1. Second, with $\lambda_2 = 0$ and $\lambda_3 = 0$, Equation 7b results from plugging the value of $\lambda_1$ from Equation 3 into Equation 4. Third, with $\lambda_2 = 0$, Equation 7c results from plugging the value of $\lambda_1$ from Equation 3 into Equation 5.

\(^{26}\) Samuelson (1983, chap. 8) has a detailed illustration of this type of assumptions.
If the marginal benefit of establishing one more county increases when the number of counties increases, the mixed form will not be stable. For the mixed form to be stable, we need that the marginal benefit of establishing one more county decreases when the number of counties increases. Since the marginal benefit of establishing one more county is $c^n c^{n-1} (s + n \ln c)$, the stability of the mixed form requires that $\partial V_2 / \partial n < 0$. With $\Delta < 0$, from Equation 9, it is clear that $dm / dL > 0$.

To understand Proposition 1, from Equation 7b, the number of counties under the mixed form is determined by the level of coordination efficiency and the degree of increasing returns in the military sector. Thus a change of the size of the population does not change the number of counties. The expected payoff of the ruler is like a quasilinear utility function if the numbers of the two types of divisions are interpreted as the consumption of two goods. Equation 7b shows the relative marginal utility of having one more county (which is $c^n c^{n-1} (s + n \ln c)$) with that of having one more kingdom (which is 1). An increase of the size of the population is similar to an increase of income and this additional income is spent on the consumption of the good with relative constant marginal utility (kingdom).

From Proposition 1, when the size of the population increases, since the total number of divisions increases, the possibility of internal rebellions decreases. Because both the number of divisions and the size of the population increase, whether the size of each division increases or not is unclear.

The following proposition studies the impact of an increase of the magnitude of external threats on the ruler’s choice of the organizational form.
Proposition 2: When the government is organized in the mixed form, an increase of the magnitude of the external threats does not change the number of counties. It decreases the number of kingdoms and increases the size of each division.

Proof: An application of Cramer’s rule on the System 8 leads to

\[
\frac{dn}{d\Omega} = 0, \\
\frac{dm}{d\Omega} = \left(\frac{\partial V_1}{\partial l} \frac{\partial V_2}{\partial n} \frac{\partial V_3}{\partial \Omega}\right) / \Delta < 0, \\
\frac{dl}{d\Omega} = -\left(\frac{\partial V_1}{\partial m} \frac{\partial V_2}{\partial n} \frac{\partial V_3}{\partial \Omega}\right) / \Delta > 0.
\]

To understand Proposition 2, from Equation 7b, the number of counties under the mixed form does not change the magnitude of external threats. When the magnitude of external threats increases, the marginal benefit of having a larger division increases. As a result, the size of each division increases. Since the size of each division increases and the size of the population is fixed, the total number of divisions decreases.

The following proposition studies the impact of an increase of the level of coordination efficiency on the ruler’s choice of the organizational form.

Proposition 3: When the government is organized in the mixed form, an increase of the level of coordination efficiency increases the number of counties.

Proof: An application of Cramer’s rule to the System 8 leads to

\[
\frac{dn}{dc} = \frac{\partial V_2}{\partial c} \left(\frac{\partial V_1}{\partial l} \frac{\partial V_3}{\partial m} - \frac{\partial V_1}{\partial m} \frac{\partial V_3}{\partial l}\right) / \Delta. 
\] (10)

Partial differentiation of Equations 7a and 7c leads to

\[
\frac{\partial V_1}{\partial l} \frac{\partial V_3}{\partial m} - \frac{\partial V_1}{\partial m} \frac{\partial V_3}{\partial l} < 0. 
\] (11)

From Equation 10 and Inequality 11, it is clear that \(dn / dc > 0\).

The following proposition studies the impact of an increase of the degree of returns in the military sector on the ruler’s choice of the organizational form.
Proposition 4: When the government is organized in the mixed form, an increase of the degree of returns increases the number of counties.

Proof: An application of the Cramer’s rule on the System 8 leads to

\[
\frac{dn}{ds} = \frac{\partial V_2}{\partial s} \left( \frac{\partial V_1}{\partial l} \frac{\partial V_3}{\partial m} - \frac{\partial V_1}{\partial m} \frac{\partial V_3}{\partial l} \right) / \Delta.
\]  

(12)

Partial differentiation of Equations 7a and 7c leads to

\[
\frac{\partial V_1}{\partial l} \frac{\partial V_3}{\partial m} - \frac{\partial V_1}{\partial m} \frac{\partial V_3}{\partial l} = -(s-1)(m+n) - \frac{l^{s+1}(c^n n^s + m)(c^n n^s + m + \Omega l^{-s})}{\Omega(L-l)^2}
\]

\[
= \frac{(c^n n^s + m)[l^{s+1}(c^n n^s + m)(s+1) + \Omega l]}{\Omega(L-l)} + \frac{l^{s+1}(m+n)(c^n n^s + m + c^n n^s + m + \Omega l^{-s})}{\Omega(L-l)}.
\]

Since

\[
- \frac{(c^n n^s + m)[l^{s+1}(c^n n^s + m)(s+1) + \Omega l]}{\Omega(L-l)} + \frac{l^{s+1}(m+n)(c^n n^s + m + c^n n^s + m + \Omega l^{-s})}{\Omega(L-l)} < 0,
\]

it is clear that

\[
\frac{\partial V_1}{\partial l} \frac{\partial V_3}{\partial m} - \frac{\partial V_1}{\partial m} \frac{\partial V_3}{\partial l} < 0.
\]  

(13)

From Equation 12 and Inequality 13, it is clear that \(dn / ds > 0\).

4.3. The County Form

In this subsection, we study the corner solution in which the ruler only establishes counties in equilibrium. Under this pure county form, \(m = 0\).

From Equations 4 and 5, the number of counties under the county system is defined by

\[
c^n L' + \Omega + (n^2 - n)\Omega \ln c = 0.
\]  

(14)

Equation 14 has four implications. When the government is organized in the county form, first, an increase of the size of the population increases the optimal number of counties.\(^\text{27}\) Second,

\(^{27}\) Fairbank and Goldman (1998, p.106) provide an account of the evolution of the number of counties in ancient China. The number of counties was 1180 in the Han Dynasty, 1255 in the Sui Dynasty, 1235 in the Tang Dynasty, 1230 in the Song Dynasty, 1115 in the Yuan Dynasty, 1385 in the Ming Dynasty, and 1360 in the Qing Dynasty. A county had a population of 50000 in later Han but a population of 300000 in late Qing. Skinner (1977) observes that the number of counties did not grow in two millennia despite long time population growth. This does not necessarily mean that theoretical results in this model are inconsistent with historical evidence on the number of counties over time. In this model, an increase of the number counties means an increase of the division of power among counties.
an increase of the magnitude of external threats decreases the optimal number of counties. Third, an increase of the level of coordination efficiency increases the optimal number of counties. Fourth, an increase of the degree of returns in the military sector increases the optimal number of counties.

4.4. Parameters Affecting Equilibrium Organizational Form

In this subsection, we discuss the impact of a change of the size of the population, the magnitude of external threats, the level of coordination efficiency, and the degree of returns in the military sector on the equilibrium choice of organizational form.

Since a kingdom has the same population size as a county, the choice of the organizational form depends on a comparison of the marginal benefit of establishing one more kingdom and the marginal benefit of establishing one more county. First, when the marginal benefit of establishing one more kingdom is higher than the marginal benefit of establishing one more county, the number of counties will be zero. From Equations 3 and 4, the number of counties is zero if

\[ e^{n_{s^{-1}}(s + n \ln c)} < 1. \]

Second, when the marginal benefit of establishing one more kingdom is equal to the marginal benefit of establishing one more county, or \[ e^{n_{s^{-1}}(s + n \ln c)} = 1, \] the mixed organizational form will be adopted. Third, when the marginal benefit of establishing one more kingdom is lower than the marginal benefit of establishing one more county, the number of kingdoms will be zero. From Equations 3 and 4, the number of kingdoms will be zero if

\[ e^{n_{s^{-1}}(s + n \ln c)} > 1, \]

where \( n \) is defined in Equation 14. In Inequality 15, because the value of \( n \) depends on the value of \( c \), whether Equation 14 is valid or not ultimately depends on the value of \( c \) and \( s \). From Equation 14 and Inequality 15, it is clear that when \( c \) is large or when \( s \) is large, Equation 14 is more likely to be valid. Thus, when the level of coordination efficiency is high or the degree of returns in the military sector is high, the ruler will choose the county system.

First, for an increase of the size of the population, from Proposition 1, if the current organizational form is the mixed form, it will not switch to a different organizational form. Second, for an increase of the magnitude of the level of external threats, from Proposition 2, if the current organizational form is the mixed form, it will not switch to a different organizational form. Third, for an increase of the level of coordination efficiency, from Proposition 3, if the current organizational form is the mixed form, it could switch to the county organizational form if the
increase of the level of coordination efficiency makes Inequality 15 to be satisfied. Finally, for an increase of the degree of returns in the military sector, from Proposition 4, if the current organizational form is the mixed form, it could switch to the county organizational form if the increase of the degree of returns makes Inequality 15 to be satisfied. Those results are supported by the numerical results in the next section.

5. Numerical Results

In this section, we present results of the impact of a change of the four exogenous parameters: the size of the population, the magnitude of external threats, the level of coordination efficiency, and degree of increasing returns on a ruler’s choice of organizational form by solving Equations 6, 7a, 7b, 7c, and 14 and then calculating the payoff in Equation 2 numerically. For Tables 1-4 below, the first row of a table is the parameter to be changed, the second row of a table is the payoff of the ruler if the feudalism organizational form is adopted, the third row of a table is the payoff of the ruler if the mixed organizational form is adopted, the fourth row of a table is the payoff of the ruler if the county organizational form is adopted, the fifth row of a table is the optimal organizational form (for some sets of parameter values, two organizational forms have the same payoff). With the specification of the expected payoff of the ruler in Equation 2, it is clear that the expected payoff is always between zero and one.

First, in Table 1, we calculate the impact of a change of the size of the population on the ruler’s optimal choice of the organizational form. For Table 1, \( \Omega = 30 \), \( c = 0.95 \), and \( s = 1.2 \), while the size of the population changes from 92 to 108. From Table 1, for this set of parameter values, the mixed organizational form is optimal regardless of the size of the population.

<table>
<thead>
<tr>
<th>L</th>
<th>92</th>
<th>94</th>
<th>96</th>
<th>98</th>
<th>100</th>
<th>102</th>
<th>104</th>
<th>106</th>
<th>108</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feudalism</td>
<td>0.7672</td>
<td>0.7712</td>
<td>0.7750</td>
<td>0.7788</td>
<td>0.7823</td>
<td>0.7858</td>
<td>0.7892</td>
<td>0.7925</td>
<td>0.7956</td>
</tr>
<tr>
<td>Mixed</td>
<td>0.7693</td>
<td>0.7732</td>
<td>0.7770</td>
<td>0.7806</td>
<td>0.7842</td>
<td>0.7876</td>
<td>0.7909</td>
<td>0.7941</td>
<td>0.7972</td>
</tr>
<tr>
<td>County</td>
<td>0.7379</td>
<td>0.7414</td>
<td>0.7448</td>
<td>0.7481</td>
<td>0.7513</td>
<td>0.7544</td>
<td>0.7574</td>
<td>0.7603</td>
<td>0.7631</td>
</tr>
</tbody>
</table>
Second, in Table 2, we calculate the impact of a change of the magnitude of external threats on the ruler’s optimal choice of the organizational form. For Table 2, \( L = 100, c = 0.95, \) and \( s = 1.2, \) while the magnitude of external threats changes from 22 to 38. From Table 2, for this set of parameter values, the mixed organizational form is optimal regardless of the size of the population.

\[
\begin{array}{c|cccccccc}
\Omega & 22 & 24 & 26 & 28 & 30 & 32 & 34 & 36 & 38 \\
\hline
\text{Feudalism} & 0.8244 & 0.8133 & 0.8026 & 0.7923 & 0.7823 & 0.7727 & 0.7633 & 0.7542 & 0.7454 \\
\text{Mixed} & 0.8257 & 0.8147 & 0.8042 & 0.7940 & 0.7842 & 0.7746 & 0.7654 & 0.7565 & 0.7478 \\
\text{County} & 0.7879 & 0.7785 & 0.7692 & 0.7602 & 0.7513 & 0.7427 & 0.7343 & 0.7260 & 0.7180 \\
\text{Optimum} & Mixed & Mixed & Mixed & Mixed & Mixed & Mixed & Mixed & Mixed & Mixed \\
\end{array}
\]

Third, in Table 3, we calculate the impact of a change of the level of coordination efficiency on the ruler’s optimal choice of the organizational form. For Table 3, \( L = 100, \Omega = 30, \) and \( s = 1.2, \) while the level of coordination efficiency changes from 0.91 to 0.99. From Table 3, when the level of coordination efficiency increases, first the feudalism organizational form is optimal, then the mixed organizational form is optimal, finally the county organizational form becomes optimal.

\[
\begin{array}{c|cccccccc}
C & 0.91 & 0.92 & 0.93 & 0.94 & 0.95 & 0.96 & 0.97 & 0.98 & 0.99 \\
\hline
\text{Feudalism} & 0.7823 & 0.7823 & 0.7823 & 0.7823 & 0.7823 & 0.7823 & 0.7823 & 0.7823 & 0.7823 \\
\text{Mixed} & 0.7819 & 0.7820 & 0.7820 & 0.7820 & 0.7842 & 0.7865 & 0.7918 & 0.8061 & 0.7820 \\
\text{County} & 0.6979 & 0.7101 & 0.7230 & 0.7367 & 0.7513 & 0.7674 & 0.7852 & 0.8061 & 0.8325 \\
\text{Optimum} & Feudalism & Feudalism & Mixed & Mixed & Mixed & Mixed & Mixed & Mixed, County \\
\end{array}
\]
Finally, in Table 4, we calculate the impact of a change of the degree of increasing returns in the military sector on the ruler’s optimal choice of the organizational form. For Table 4, $L=100$, $\Omega=30$, and $c=0.95$, while the degree of returns in the military sector changes from 1.1 to 1.9. From Table 4, when the degree of increasing returns in the military sector increases, first the feudalism organizational form is optimal, then the mixed organizational form is optimal, finally the county organizational form becomes optimal.

Table 4: Impact of a Change of the Degree of Increasing Returns in the Military Sector

<table>
<thead>
<tr>
<th>$s$</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feudalism</td>
<td>0.7657</td>
<td>0.7823</td>
<td>0.8018</td>
<td>0.8207</td>
<td>0.8378</td>
<td>0.8530</td>
<td>0.8662</td>
<td>0.8777</td>
<td>0.8876</td>
</tr>
<tr>
<td>Mixed</td>
<td>0.7655</td>
<td>0.7842</td>
<td>0.8131</td>
<td>0.8472</td>
<td>0.8782</td>
<td>0.8522</td>
<td>0.8654</td>
<td>0.8769</td>
<td>0.8869</td>
</tr>
<tr>
<td>County</td>
<td>0.6840</td>
<td>0.7513</td>
<td>0.8049</td>
<td>0.8465</td>
<td>0.8782</td>
<td>0.9024</td>
<td>0.9207</td>
<td>0.9348</td>
<td>0.9457</td>
</tr>
<tr>
<td>Optimum</td>
<td>Feudalism</td>
<td>Mixed</td>
<td>Mixed</td>
<td>Mixed</td>
<td>Mixed, County</td>
<td>County</td>
<td>County</td>
<td>County</td>
<td>County</td>
</tr>
</tbody>
</table>

6. Government Organizational Forms in Ancient China

A casual observation of China’s history leads to the following impression of the historical choice of government organizational form in ancient China. First, the feudalism form died with the Zhou Dynasty more than two thousand years ago. Second, the mixed form was used in different periods of China with different levels of population. The mixed organizational form could be used when the founding emperor of a dynasty hoped to combine the strengths of the county form and the feudalism in the following sense. By having counties, the power of the central government would be established. By having kingdoms ruled by relatives of the royal family, powerful ministers would be deterred from usurping power. For the Western Jin Dynasty that
adopted the mixed form, the central government became weak and the dynasty ended in 316 after the rebellion by the eight kings was put down in 306. For the Han Dynasty and the Ming Dynasty that adopted the mixed form in the early periods of the dynasties, after the rebellions by kings were put down, the governments moved to the county form. Overall, the mixed form did not seem to be stable. It had a tendency to degenerate to the county form.\textsuperscript{28} Third, the county form was the most common form to organize the government since the Qin Dynasty two thousand years ago.

How do the results in this model fit with historical evidence? It is not easy to answer this question and here we provide some rudimentary discussions of the impact of the four parameters such as the size of the population on the historical choice of organizational form. First, China’s population had a long-run trend of increase (Chao, 1986, p. 41). An increase of the size of the population might lead to the disappearance of the feudalism form, but through a channel not addressed in this model. Historically, the increase of population in the Zhou Dynasty meant land became scarcer and conflicts among the borders of the kingdoms increased significantly. The fight among kingdoms led to the demise of the Zhou Dynasty and the feudalism form. The impact of the size of the population on the choice between the mixed form and the county form was not clear-cut: the mixed organizational form appeared in different dynasties with significant different sizes of population.

Second, historically, an increase of the magnitude of external threats did not lead to the dominance of either the county form or the mixed organizational form. On the one hand, in the Han Dynasty, the emperor Liu Che eliminated the kingdoms established by his ancestors so that he could concentrate resources to deal with the external threats from Xiongnu. On the other hand, in the Ming Dynasty, because the emperor Zhu Yuanzhang did not trust his generals in leading the armies, he established kingdoms for his sons (Zhu Di was one of the sons of Zhu Yuanzhang) to deal with the external threats.

Third, factors such as the improvements of the transportation sector might have increased coordination efficiency historically. This might also have contributed to the demise of the feudalism form in China and the popularity of the county form.

\textsuperscript{28} For the Qing Dynasty that established kingdoms to reward surrounded generals from the Ming Dynasty, after the rebellions of those generals were put down, the land formally controlled by those generals became to be ruled by the central government directly.
Finally, the degree of increasing returns in the military sector might have increased historically, at least for some periods of time. Huang (1997, p. 22) argues that army sizes increased significantly in the Zhou Dynasty. Fairbank and Goldman (1998, p. 54) argue that the use of iron led to larger armies. Parker (1996) discusses factors leading to larger armies in Europe, and some factors he discussed such as the existence of walled cities were also relevant to ancient China. An increase of the degree of returns in the military sector might also have contributed to the demise of the feudalism form and the popularity of the county form.

7. Conclusion

In this paper, we have studied how the tradeoff between preventing internal rebellions and dealing with external threats in the choice of government organizational form in ancient China could be affected by the size of the population, the level of coordination efficiency, and the degree of increasing returns in the military sector in a simple model. We have established the following results. First, when the degree of increasing returns in the military sector is high or when the level of coordination efficiency is high, counties will be more likely to be established than kingdoms. Second, under a mixed organizational form, an increase of the size of the population leaves the number of counties unchanged while increases the number of kingdoms. An increase of the magnitude of the external threats makes the number of counties unchanged while decreases the number of kingdoms and increases the size of each division. Finally, a robust feature of the model is that when the magnitude of the external threats increases, regardless of the type of equilibrium organizational form, the equilibrium number of divisions decreases and thus the degree of the concentration of power among division heads increases.

Compared with her long and splendid history, the level of theoretical research on ancient China is quite limited. This paper shows that there are some important issues in China’s history that could be analyzed by using formal models. There are several interesting generalizations and extensions of the model. First, the feudalism form was associated with the Confucianism school, while the county form was associated with the Legalism school (Waley, 1982). In ancient China, starting from Liu Che in the Han Dynasty, to rule more effectively, governments used a combination of the ideas from the Confucianism school and the Legalism school. Why was it necessary to combine the Confucianism school and the Legalism school to rule effectively?
Second, in ancient China, not only powerful ministers such as generals may rebel, but also peasants may rebel. With the possibility of peasant rebels, there are some interesting questions to be addressed. For example, in the Ming Dynasty, the peasant rebellion of Li Zicheng exacerbated the threats from Manchu. The Ming Dynasty had to fight with Li and Manchu at the same time. As the Ming Dynasty struggled with fighting in two fronts, it eventually collapsed. Should the Ming government negotiate with one enemy so that it could concentrate on fighting with the other enemy? If so, which enemy should it negotiate with? What was the acceptable cost of truce?

Finally, the performance of an organizational form was affected by the abilities of the emperors. For example, while the military system introduced by Zhao Kuangyin in the Song Dynasty operated relatively well when he was the emperor, the operation of the system deteriorated in later generations of the Song Dynasty. Historically, whether there was an internal rebellion was affected by the relative abilities of the emperor and the most capable general. While a founding emperor could be extremely capable, the succeeding emperors might have much lower levels of abilities. If the ability of the emperor was low or if the emperor was young, either ambitious generals could engage in military coups or civilian officials may usurp the central government. The evolution of the abilities of emperors in a given dynasty can be modeled as a stochastic process. The model can be generalized to a dynamic setup in which the impact of the uncertainties of the abilities of emperors on the performance of different organizational forms can be explicitly addressed.
References


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