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From Solow to Romer: Teaching Endogenous Technological Change in Undergraduate Economics

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Abstract

Undergraduate students learn economic growth theory through the seminal Solow model, which takes the growth rate of technology as given. To understand the origin of technological progress, we need a model of endogenous technological change. The Romer model fills this important gap in the literature. However, given its complexity, undergraduate students often find the Romer model difficult. This paper proposes a simple method of teaching the Romer model. We add three layers of structure (one at a time) to extend the familiar Solow model into the less familiar Romer model. First, we incorporate a competitive market structure into the Solow model. Then, we modify the competitive market structure into a monopolistic market structure. Finally, we introduce an R&D sector that creates inventions.

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1 Introduction

Economic growth is an important topic in economics. As Acemoglu (2013) argues, "economics instructors should spend more time teaching about economic growth and development at the undergraduate level because the topic is of interest to students, is less abstract than other macroeconomic topics, and is the focus of exciting research in economics." Undergraduate students learn the theory of economic growth through the seminal Solow model originated from Solow (1956). This elegant model provides the following important insight: in the long run, economic growth must come from technological progress instead of capital accumulation. However, the Solow model takes the growth rate of technology as given, and hence, it does not provide insight on the determinants of technological progress. The Romer model, originated from Romer (1990), fills this important gap in the literature and enhances economists' understanding of endogenous technological change. Taylor (2010) writes that "teaching beginning students the Solow model, augmented with endogenous technology, is the first step toward teaching them modern macroeconomics."

Unfortunately, the Romer model is relatively complicated, and undergraduate students often find it difficult. In particular, although it is not difficult to demonstrate the key assumption in the Romer model that technological change is driven by research and development (R&D), it is much more difficult to demonstrate how the level of R&D is determined in the market equilibrium within the Romer model. Therefore, macroeconomic textbooks at the intermediate level, such as Jones (2016) and Barro, Chu and Cozzi (2017), often assume a given level of R&D when presenting the Romer model.¹

This paper proposes a simple method of teaching the Romer model by adding three layers of structure (one at a time) to extend the familiar Solow model into the less familiar Romer model. First, we incorporate into the Solow model a competitive market structure in which final goods are produced by competitive firms that employ labor and rent capital from households. Then, we modify the competitive market structure into a monopolistic market structure in which differentiated intermediate goods are produced by monopolistic firms. Finally, we introduce to the monopolistic Solow model an R&D sector, which develops new varieties of intermediate goods and gives rise to endogenous technological progress. Once we derive the endogenous growth rate of technology, we can then perform experiments in this mathematical laboratory by using comparative statics to explore the determinants of technological progress. All mathematical derivations are based on simple calculus and algebra at the level of intermediate microeconomics. We hope that by presenting it as a step-by-step extension of the Solow model, we have made the Romer model more accessible, at least to advanced undergraduate students in economics.

The rest of this paper is organized as follows. Section 2 presents the step-by-step transformation of the Solow model into the Romer model. Section 3 offers concluding thoughts.

¹See Aghion and Howitt (2009) and Jones and Vollrath (2013) for an excellent and complete treatment of the Romer model at the advanced undergraduate level.

2 From Solow to Romer

Section 2.1 reviews a basic version of the Solow model with *exogenous* technological progress. Section 2.2 incorporates a competitive market structure into the Solow model.² Section 2.3 modifies the competitive market structure into a monopolistic market structure. Section 2.4 introduces an R&D sector to the monopolistic Solow model, which becomes the Romer model with *endogenous* technological progress.

2.1 The Solow model

In this subsection, we consider a basic version of the Solow model with exogenous technological progress. Output Y is produced by an aggregate production function $Y = K^\alpha(AL)^{1-\alpha}$, where A is the level of technology that grows at an exogenous rate $g > 0$, K is the stock of capital, and L is the size of a constant labor force. The parameter $\alpha \in (0, 1)$ determines capital intensity α and labor intensity $1 - \alpha$ in the production process. The key equation in the Solow model is the capital-accumulation equation given by $\Delta K = I - \delta K$, where the parameter $\delta > 0$ is the depreciation rate of capital. Investment I is assumed to be a constant share $s \in (0, 1)$ of output Y . Substituting the investment function $I = sY$ and the production function $Y = K^\alpha(AL)^{1-\alpha}$ into the capital-accumulation equation yields

$$\frac{\Delta K}{K} = \frac{sY}{K} - \delta = s \left(\frac{AL}{K} \right)^{1-\alpha} - \delta. \quad (1)$$

Equation (1) can then be used to explore the transition dynamics of an economy from an initial state to the steady state, which is a common analysis in macroeconomic textbooks at the intermediate level. In the long run, the economy is on a balanced growth path, along which capital K grows at a constant rate implying that Y/K and A/K are constant in the long run. This in turn implies that in the long run, output Y and capital K grow at the same rate as technology A ; i.e.,

$$\frac{\Delta Y}{Y} = \frac{\Delta K}{K} = \frac{\Delta A}{A} \equiv g.$$

This is an important insight of the Solow model, which shows that in the long run, economic growth comes from technological progress (i.e., $g > 0$), without which the growth rate of the economy would converge to zero due to decreasing returns to scale of capital in production.

2.2 The Solow model with a competitive market structure

The basic Solow model above does not feature any market structure. Here we embed a market economy into the model in which competitive firms produce goods Y by employing labor L and renting capital K from households, which devote a constant share s of income to accumulate capital. The capital-accumulation equation is given by

$$\Delta K = s(WL + RK) - \delta K, \quad (2)$$

²Solow (1956) also discusses the implications of his model in a competitive market.

where W is the real wage rate and R is the real rental price of capital. Competitive firms produce goods Y to maximize real profit Π . The production function $Y = K^\alpha(AL)^{1-\alpha}$ is the same as before, whereas the profit function is given by $\Pi = Y - WL - RK$. From profit maximization, the first-order conditions that equate the real wage rate to the marginal product of labor and the real rental price to the marginal product of capital are given by

$$W = (1 - \alpha) \frac{Y}{L}, \quad (3)$$

$$R = \alpha \frac{Y}{K}. \quad (4)$$

Substituting (3) and (4) into (2) yields the same capital-accumulation equation as in (1) because wage income and capital income add up to the level of output. Therefore, dynamics and long-run growth in the two versions of the Solow model are the same.

2.3 The Solow model with a monopolistic market structure

In this subsection, we further introduce a monopolistic sector of differentiated intermediate goods into the Solow model. The production function of final goods Y is now given by

$$Y = L^{1-\alpha} \sum_{i=1}^N X_i^\alpha, \quad (5)$$

where X_i denotes intermediate goods $i \in [1, N]$ and the number of intermediate goods N increases at the rate g . The profit function of competitive firms that produce final goods is

$$\Pi = Y - WL - \sum_{i=1}^N P_i X_i.$$

From profit maximization, the first-order conditions are given by (3) and

$$P_i = \alpha L^{1-\alpha} X_i^{\alpha-1}, \quad (6)$$

for $i \in [1, N]$.

The monopolistic firm in industry i produces X_i units of intermediate goods by renting X_i units of capital from households. The profit function of the firm in industry i is

$$\pi_i = P_i X_i - R X_i. \quad (7)$$

The monopolistic firm chooses X_i to maximize π_i subject to the conditional demand function in (6). The profit-maximizing price of X_i is $P_i = R/\alpha$. For a more general treatment, we assume that firms may not be able to charge this profit-maximizing price due to price regulation as in Evans *et al.* (2003). In this case, the price of X_i is given by $P_i = \mu R$, where $\mu \in (1, 1/\alpha)$. Substituting $P_i = \mu R$ into (6) shows that $X_i = X$ for $i \in [1, N]$. Then the resource constraint on capital requires that $NX = K$; i.e., the usage of capital by all intermediate goods firms equals the total supply of capital.

Imposing symmetry $X_i = X$ and then substituting $X = K/N$ into (5) yield

$$Y = L^{1-\alpha}NX^\alpha = K^\alpha(NL)^{1-\alpha}, \quad (8)$$

which is the same production function as in the basic Solow model when $A = N$. Therefore, the level of technology A in the basic Solow model is interpreted as the number of differentiated intermediate goods N in this monopolistic Solow model. The growth rate of technology is determined by the growth rate of N , which is also g . As before, households devote a constant share s of income to accumulate capital. The capital-accumulation equation is³

$$\Delta K = s(WL + RK + N\pi) - \delta K. \quad (9)$$

From (3), we have $WL = (1-\alpha)Y$ whereas we can derive $N\pi = (\mu-1)RK$ from (7). Finally, using (6), one can show that $RK = \alpha Y/\mu$. Therefore, $WL + RK + N\pi = Y$, which together with (8) implies that (9) is the same capital-accumulation equation as in (1). Intuitively, wage income, capital income and monopolistic profit add up to the level of output, whereas investment is assumed to be a constant share of output. Once again, dynamics and long-run growth in the three versions of the Solow model are the same.

2.4 The Romer model

In the above models, the technology growth rate g is assumed to be exogenous. To endogenize technological progress, we introduce an R&D sector to the monopolistic Solow model, which then becomes a version of the Romer model with an exogenous saving rate s . Aside from the R&D sector, the Romer model is the same as the monopolistic Solow model except that labor is now allocated between R&D and the production of final goods. Equation (5) becomes

$$Y = L_Y^{1-\alpha} \sum_{i=1}^N X_i^\alpha,$$

where L_Y denotes production labor. From the profit maximization of competitive firms that produce final goods, the first-order conditions are given by (6) and

$$W = (1-\alpha) \frac{Y}{L_Y}. \quad (10)$$

We follow Romer (1990) to specify the following innovation equation for the creation of new differentiated intermediate goods:

$$\Delta N = \theta N L_R, \quad (11)$$

where the parameter $\theta > 0$ captures R&D productivity. L_R is R&D labor, and the resource constraint on labor is $L_R + L_Y = L$. Given that the R&D sector is perfectly competitive, zero profit implies that R&D revenue $\Delta N v$ equals R&D cost $W L_R$; i.e.,

$$\Delta N v = W L_R \Leftrightarrow \theta N v = W, \quad (12)$$

³Alternatively, one can assume that monopolistic profit does not contribute to capital investment; i.e., $I = s(WL + RK) = s(1-\alpha + \alpha/\mu)Y$, where $\mu > 1$ captures the monopolistic distortion that leads to a lower level of investment and capital. In this case, output and capital would still grow at g in the long run.

where v is the value of an invention. The value of an invention is the present value of future monopolistic profits. Let's use r to denote the exogenous real interest rate.⁴ In the steady state, the value v of an invention is given by

$$v = \frac{\pi}{r} = \frac{1}{r} \left(\frac{\mu - 1}{\mu} \right) \frac{\alpha Y}{N}, \quad (13)$$

where the second equality uses $N\pi = (\mu - 1)RK$ and $RK = \alpha Y/\mu$ from the previous subsection. Rewriting (10) yields $W/Y = (1 - \alpha)/L_Y$ whereas rewriting (12) yields $W/Y = \theta Nv/Y = \frac{\theta}{r} \left(\frac{\mu - 1}{\mu} \right) \alpha$. Equating these two equations yields the steady-state equilibrium level of production labor L_Y^* as shown in Figure 1.

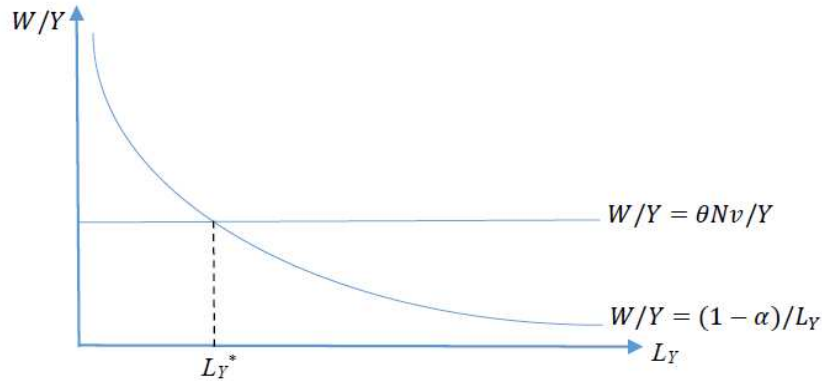


Figure 1: Equilibrium production labor

Substituting L_Y^* into the resource constraint on labor yields the steady-state equilibrium level of R&D labor L_R^* given by⁵

$$L_R^* = L - L_Y^* = L - \frac{1 - \alpha}{\alpha} \left(\frac{\mu}{\mu - 1} \right) \frac{r}{\theta}.$$

Therefore, the long-run growth rate of technology in the Romer model is

$$g^* \equiv \frac{\Delta N}{N} = \theta L_R^* = \theta L - \frac{1 - \alpha}{\alpha} \left(\frac{\mu}{\mu - 1} \right) r. \quad (14)$$

This endogenous technology growth rate g^* is also the long-run growth rate of output and capital. To see this, we use the following capital-accumulation equation:⁶

$$\frac{\Delta K}{K} = \frac{sY}{K} - \delta = s \left(\frac{NL_Y}{K} \right)^{1-\alpha} - \delta,$$

⁴In the original Romer model, the interest rate is endogenous and determined by the household's optimal consumption path. To avoid using dynamic optimization, we assume an *exogenous* interest rate. Under this assumption, the no-arbitrage condition $r = R - \delta$ may not hold given the investment rate s is also exogenous.

⁵Labor force L is assumed to be sufficiently large such that $L_R^* > 0$.

⁶To derive this equation, we assume households devote $s(WL_Y + RK + N\pi)$ to capital investment I , and R&D wage income WL_R is invested in intangible capital (i.e., the value of new inventions ΔNv). As in footnote 4, one could assume $I = s(WL_Y + RK) = s(1 - \alpha + \alpha/\mu)Y$ to allow for monopolistic distortion on capital accumulation, in which case the long-run growth rate of output and capital is still g^* in (14).

where production labor L_Y is stationary in the long run. Therefore, a constant growth rate of capital K on the balanced growth path implies that Y/K and N/K are stationary in the long run.

Given the expression for the endogenous growth rate in (14), we can now perform experiments in this mathematical laboratory by using comparative statics to explore the determinants of economic growth in the Romer model. Equation (14) shows that the equilibrium growth rate g^* is increasing in $\{\theta, \alpha, \mu, L\}$ and decreasing in r . The intuition of these comparative statics results can be explained as follows.

2.4.1 Experiment 1: changing R&D productivity

An improvement in R&D productivity θ increases the growth rate of technology for a given level of R&D labor and also makes R&D more attractive, which in turn increases R&D labor in the economy. Therefore, g^* is increasing in θ . This parameter θ captures the importance of human capital on the innovation capacity of an economy.

2.4.2 Experiment 2: changing capital and labor intensity in production

An increase in α increases capital intensity and reduces labor intensity in the production process, allowing more labor to be devoted to R&D. Therefore, g^* is increasing in α . This parameter α captures the effects of structural transformation of an economy from a labor-intensive production process to a capital-intensive production process.

2.4.3 Experiment 3: changing the monopolistic price

A larger μ enables monopolistic firms to raise their price and earn more profits, which in turn provide more incentives for R&D. Therefore, g^* is increasing in μ . This parameter μ captures the effects of the underlying economic institutions, such as antitrust policies, on R&D and economic growth.

2.4.4 Experiment 4: changing the interest rate

A higher interest rate r reduces the present value of future monopolistic profits and the value of inventions, which in turn decreases R&D in the economy. Therefore, g^* is decreasing in r . This parameter r captures the effects of financial frictions on innovation.

2.4.5 Experiment 5: changing the size of the labor force

Finally, a larger labor force L increases the supply of labor in the economy, which in turn increases R&D labor and the growth rate. Therefore, g^* is increasing in L . This is known as the scale effect in the literature.⁷

⁷This scale effect is often viewed as a counterfactual implication of the Romer model. To remove this scale effect, one can follow Jones (1995) to modify (11) into $\Delta N = \theta N^\phi L_R$, where the parameter $\phi < 1$ captures the degree of intertemporal knowledge spillovers.

3 Conclusion

Since the seminal work of Solow (1956), there has been much progress in the research of economic growth. However, the teaching of economic growth in undergraduate macroeconomic courses is still mostly based on the Solow model. Although this seminal model provides important insights, it takes the growth rate of technology as given. To understand the origin of technological progress, we need a model of endogenous technological change. The Romer model provides a useful framework for this purpose. However, given its complexity, undergraduate students often find the Romer model difficult. In this paper, we have proposed a method that serves as a bridge between the Solow model and the Romer model in three simple steps. Furthermore, the mathematical derivations involve only basic calculus and algebra. We hope that by providing a bridge with the Solow model, we have made the Romer model more accessible to undergraduate students in economics.

References

- [1] Acemoglu, D., 2013. Economic growth and development in the undergraduate curriculum. *Journal of Economic Education*, 44, 169-177.
- [2] Aghion, P., and Howitt, P., 2009. *The Economics of Growth*. Cambridge MA: The MIT Press.
- [3] Barro, R., Chu, A., and Cozzi, G., 2017. *Intermediate Macroeconomics*. UK: Cengage Learning.
- [4] Evans, L., Quigley, N., and Zhang, J., 2003. Optimal price regulation in a growth model with monopolistic suppliers of intermediate goods. *Canadian Journal of Economics*, 36, 463-474.
- [5] Jones, C., 1995. R&D-based models of economic growth. *Journal of Political Economy*, 103, 759-784.
- [6] Jones, C., 2016. *Macroeconomics* (Fourth Edition). New York: W. W. Norton & Company, Inc.
- [7] Jones, C., and Vollrath, D., 2013. *Introduction to Economic Growth* (Third Edition). New York: W. W. Norton & Company, Inc.
- [8] Romer, P., 1990. Endogenous technological change. *Journal of Political Economy* 98, S71-S102.
- [9] Solow, R., 1956. A contribution to the theory of economic growth. *Quarterly Journal of Economics*, 70, 65-94.
- [10] Taylor, J., 2000. Teaching macroeconomics at the principles level. *American Economic Review*, 90, 90-94.