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Competitive Pressure from Neighboring Markets and Optimal Privatization Policy*

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Abstract

We formulate a mixed oligopoly model in which one state-owned public enterprise competes with \( n \) private firms in the same market and \( m \) private firms in the neighboring market. We investigate how \( n \) and \( m \) affect the optimal degree of privatization. We find a nonmonotone (monotone) relationship between the optimal degree of privatization and the number of private competitors in the neighboring (same) market. The optimal degree of privatization is increasing in the number of private firms in the same market, and the relationship between the optimal degree of privatization and the number of private competitors in the neighboring market is an inverted U-shape. An increase in \( m \) more likely increases the optimal degree of privatization when the degree of product differentiation is low. Our results suggest that more competitive pressure from competitors supplying differentiated products can reduce the optimal degree of privatization.

JEL classification numbers: H44, L33, L44

Key words: market competitiveness, partial privatization, number of private firms.

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1 Introduction

In this study, we formulate the following two-product model. Product A and Product B are imperfectly substitutable products (differentiated products). In the Product A market, there is one state-owned public enterprise and \( n \) private enterprises, while in the Product B market, there are \( m \) private enterprises. All enterprises face Cournot competition. We investigate how \( n \) and \( m \) affect the optimal degree of privatization of the public enterprise. We find that the optimal degree of privatization is increasing with the number of private enterprises in the same market (Product A market). However, the relationship between the optimal degree of privatization and \( m \) are nonmonotone (inverted U-shaped). Our result suggests that stronger competitive pressure from the market in which the public enterprise exists increases the optimal degree of privatization, while stronger competitive pressure from the neighboring market might not.\(^1\)

In the literature on mixed oligopolies\(^2\), many studies already investigated the relationship between privatization policy and the number of private competitors. De Fraja and Delbono (1989) formulated a model of mixed oligopolies in which a welfare-maximizing public enterprise competes against \( n \) private enterprises in a homogeneous product market. The authors assumed that both public and private enterprises have an identical cost function and showed that full privatization more likely improves welfare when \( n \) is larger. Matsumura and Shimizu (2010) showed that this result holds even when multiple public enterprises exist and a cost difference between public and private enterprises is allowed. Lin and Matsumura (2012) adopted the partial privatization approach formulated by Matsumura (1998) and showed that the optimal degree of privatization is increasing with the number of private enterprises regardless of the nationality.

\(^1\)Mixed oligopolies in which state-owned public enterprises compete with private enterprises exist globally, and in many countries, privatization of these public enterprises is an important policy issue. The Japanese government partially privatized Japan Post, Postal Bank, and Kampo in 2015, sold a small share of Japan Post in 2017 again, and still holds the majority share in all three entities. The Brazilian government privatized major companies, such as Emnraer, and plans to privatize larger companies further, such as Centrais Eletricas Brasileiras S.A. The Vietnamese government recently changed its privatization policy from full to partial privatization (keeping at least a 35\% share) in 12 major national companies (Nikkei Newspaper, 2017/8/30, 2017/9/19).

\(^2\)For important examples of mixed oligopolies and recent development of the analysis of mixed oligopolies, see Heywood and Ye (2009), Bose et al. (2014), and Chen (2017).
of them. Matsumura and Okamura (2015) showed this is true even when private enterprises maximize relative profit rather than absolute profits. The implication of these studies is clear: the more competitive pressure public enterprises receive from private companies, the more the government should privatize the public enterprises.

All studies mentioned above assumed homogeneous product markets. However, many public enterprises are under competitive pressure from not only the same market but also from suppliers in highly differentiated but substitutable products for the public enterprises’ products. For example, the Japanese public broadcasting corporation, NHK, competes with several private companies, such as Nippon Television Network Corporation, in the TV broadcasting market, and both NHK and private companies also compete with internet TV and/or cable TV companies in neighboring markets. Postal Bank, a major public financial institution in Japan, directly competes with many commercial banks, such as Mizuho Bank, and indirectly competes with many other investment banks, such as Nomura Securities Co., Ltd. In Japan, several public gas companies, such as Business Administration of Otsu city, compete with private gas enterprises, such as Osaka Gas Co., Ltd., in the gas market and those gas companies also compete with other energy companies, such as Kansai Electric Power Co., Inc., which is a major player in the electric power market. NTT Docomo, in which the Japanese government indirectly owns the share, competes with KDDI and Softbank as mobile network operators, and those three companies compete with many small virtual mobile network operators. In such situations, the optimal degree of privatization must depend on the number of competitors in neighboring markets as well as that in the market in which the public enterprise exists.

In this study, we adopt Singh and Vives’ (1984) linear demand model, which is popular in the literature on mixed oligopolies. However, our model has one important deviation from

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3The relative profit maximization approach enables us to use the single quantity competition model to treat various competition structures, from collusive to perfectly competitive cases. Thus, this result implies that the optimal degree of privatization is increasing with the number of private enterprises under various competition structures. For a discussion on relative profit maximization, see Matsumura and Matsushima (2012) and Matsumura et al. (2013).

their model formulation. In their model, each differentiated product is produced by only one enterprise, while in our model, each differentiated product is supplied by multiple enterprises.\(^5\)

In other words, we consider a model in which competition within homogeneous product markets and competition across the two differentiated markets coexist. We find that the optimal degree of privatization is increasing with the number of private competitors in the same market of the public enterprise, whereas the relationship between the optimal degree of privatization and the number of private competitors in the neighboring market is nonmonotone (inverted U-shaped). Our result suggests that if public enterprises compete with a small number of private competitors in the same market and a large number of private competitors in neighboring markets, the optimal degree of privatization is small. This situation is typically observed in Japanese gas markets. The Japanese government concluded that gas companies face tough competition with electric power companies, although the competition in gas markets is not in fact tough (Report, Strategic Policy Committee, Advisory Committee for Natural Resources and Energy, 2015/1/13). Our result suggests possible welfare loss of the privatization of gas companies in Japan even if the public gas companies receive severe competitive pressure from the neighboring market.

2 The Model

We adopt a standard model with two differentiated products, Product A and Product B, and linear demand (Singh and Vives (1984)). Product A is supplied by one state-owned public firm, firm 0, and \( n \) private firms (firm 1, 2, ..., \( n \)). Product B is supplied by \( m \) private firms (firm \( n + 1, n + 2, ..., n + m \)). The quasi-linear utility function of the representative consumer is

\[
U(Q_A, Q_B, y) = a(Q_A + Q_B) - \frac{Q_A^2 + 2\delta Q_A Q_B + Q_B^2}{2} + y, \tag{1}
\]

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\(^5\)Yoshida (2017) formulated a private oligopoly model in which two differentiated products are supplied by multiple enterprises. He showed that an increase in the number of enterprises in a market harms consumer surplus when the number of enterprises is significantly larger than that in the other market.
where $Q_i$ is the consumption of Product $i$ ($i = A, B$) and $y$ is the consumption of an outside
good that is competitively provided (with a unitary price). Parameter $a$ is a positive constant
and $\delta \in (0, 1)$ represents a degree of product differentiation (the smaller $\delta$ is, the more Product
$A$ and Product $B$ are differentiated). The inverse demand functions for Product $i = A, B$ with
$i \neq j$ are
\begin{equation}
    p_i = a - Q_i - \delta Q_j,
\end{equation}
where $p_i$ is the price of product $i$.

The marginal cost of production is constant for all firms. We assume that common marginal
cost is zero for simplicity. Let $q_i$ be the output of firm $i$ ($i = 0, 1, ..., n + m$). Then, we obtain
$Q_A = \sum_{i=0}^{n} q_i$ and $Q_B = \sum_{i=n+1}^{n+m} q_i$. Let $\pi_i$ be the profit of firm $i$ ($i = 0, 1, ..., n + m$). $\pi_i = p_i q_i$
for $i = 0, 1, ..., n$ and $\pi_i = p_B q_i$ for $i = n + 1, n + 2, ..., n + m$. The objective of each private firm
is its own profit.

Following the standard approach formulated by Matsumura (1998), we assume that the
public firm’s objective function is a convex combination of social welfare (the sum of consumer
surplus and all firms’ profits) and its own profit. We denote this as
\begin{equation}
    \Omega = \alpha \pi_0 + (1 - \alpha) W,
\end{equation}
where $W$ is social welfare, given by
\begin{equation}
    W = a(Q_A + Q_B) - \frac{Q_A^2 + 2\delta Q_A Q_B + Q_B^2}{2} - p_A Q_A - p_B Q_B + \sum_{i=0}^{n} \pi_i + \sum_{i=n+1}^{n+m} \pi_i,
\end{equation}
and $\alpha \in [0, 1]$ represents the degree of privatization. In the case of full nationalization (i.e.,
$\alpha = 0$), firm 0 maximizes social welfare. In the case of full privatization (i.e., $\alpha = 1$), firm 0
maximizes its own profit.

The game runs as follows. In the first stage, the government chooses the degree of privatiza-
tion $\alpha$ to maximize the social welfare. In the second stage, each firm simultaneously chooses its
output to maximize its objective. We solve this game by backward induction and the equilibrium concept is the subgame perfect Nash equilibrium.

### 3 Results

First, we solve the second-stage game, given $\alpha$. The first-order conditions of public and private firms are

\[
\frac{\partial \Omega}{\partial q_0} = a - Q_A - \delta Q_B - \alpha q_0 = 0, \\
\frac{\partial \pi_i}{\partial q_i} = a - Q_A - \delta Q_B - q_i = 0 \quad (i = 1, ..., n), \\
\frac{\partial \pi_i}{\partial q_i} = a - Q_B - \delta Q_A - q_i = 0 \quad (i = n + 1, ..., n + m),
\]

respectively. The second-order conditions are satisfied. These first-order conditions yield the following reaction functions of public and private firms

\[
R_0(Q_B, Q_{A-0}) = \frac{a - \delta Q_B - Q_{A-0}}{1 + \alpha}, \\
R_i(Q_B, Q_{A-i}) = \frac{a - \delta Q_B - Q_{A-i}}{2} \quad (i = 1, ..., n), \\
R_i(Q_A, Q_{B-i}) = \frac{a - \delta Q_A - Q_{B-i}}{2} \quad (i = n + 1, ..., n + m),
\]

respectively, where $Q_{j-i}$ is total output of Product $j$ except for firm $i$ ($j = A, B, i = 0, 1, ..., m$).

We obtain the following equilibrium quantities of public and private firms

\[
q_0^*(\alpha) = \frac{(1 + m(1 - \delta))a}{(1 + \alpha)(1 + m) + \alpha(1 + m)n - (1 + n\alpha)m\delta^2}, \\
q_i^*(\alpha) = \frac{(1 + m(1 - \delta))aa}{(1 + \alpha)(1 + m) + \alpha(1 + m)n - (1 + n\alpha)m\delta^2} \quad (i = 1, ..., n), \\
q_i^*(\alpha) = \frac{((1 - \delta)(1 + n\alpha) + \alpha)a}{(1 + \alpha)(1 + m) + \alpha(1 + m)n - (1 + n\alpha)m\delta^2} \quad (i = n + 1, ..., n + m),
\]
respectively. We obtain the following equilibrium total output of Products A and B, and welfare

\[ Q_A^*(\alpha) = \frac{a(1+na)(1+m(1-\delta))}{(1+m)(1+\alpha(1+n)) - \delta^2m(\alpha n + 1)}, \]  

\[ Q_B^*(\alpha) = \frac{am(1-\delta)(1+na) + \alpha}{(1+m)(1+\alpha(1+n)) - \delta^2m(\alpha n + 1)}, \]  

\[ W^*(\alpha) = \frac{a^2X}{2((1+m)(1+\alpha(1+n)) - \delta^2m(\alpha n + 1))^2}, \]

respectively, where \( X := 1+2\alpha+(2m+m^2)(\alpha^2+4\alpha+2)+2\alpha(1+\alpha)(2m^2+4m+1)n+\alpha^2n^2(2m^2+4m+1) - 2m\delta(1+\alpha n)(\alpha mn + 2\alpha(m+n) + m + 3\alpha + 2) - 2m^2\delta^2(1+\alpha n)^2 + 2\delta^3m^2(1+\alpha n)^2. \)

We now present a result on the relationship between \( \alpha, Q_A, \) and \( Q_B \)

**Lemma 1** (i) \( Q_A^*(\alpha) \) is decreasing in \( \alpha \), (ii) \( Q_B^*(\alpha) \) is increasing in \( \alpha \), and (iii) \( Q_A^*(\alpha) + Q_B^*(\alpha) \) is decreasing in \( \alpha \).

**Proof**

From (3) and (4), we obtain

\[ \frac{\partial Q_A^*(\alpha)}{\partial \alpha} = -\frac{a(1+m)(1+m(1-\delta))}{((1+m)(1+\alpha(1+n)) - \delta^2m(\alpha n + 1))^2} < 0, \]

\[ \frac{\partial Q_B^*(\alpha)}{\partial \alpha} = \frac{am\delta(1+m(1-\delta))}{((1+m)(1+\alpha(1+n)) - \delta^2m(\alpha n + 1))^2} > 0, \]

\[ \frac{\partial Q_A^*(\alpha)}{\partial \alpha} + \frac{\partial Q_B^*(\alpha)}{\partial \alpha} = -\frac{a(1+m(1-\delta)^2}{((1+m)(1+\alpha(1+n)) - \delta^2m(\alpha n + 1))^2} < 0. \]

These results imply Lemma 1. ■

The larger \( \alpha \) is, the more the public firm is concerned with its own profit rather than consumer surplus. Therefore, an increase in \( \alpha \) makes the public firm less aggressive, and thus, directly reduces the output of the public firm. An increase in \( \alpha \) indirectly increases the output of each private firm in both markets through strategic interaction. Because the direct effect dominates the indirect strategic effect, an increase in \( \alpha \) reduces \( Q_A \). Because there is no direct effect in the Product B market, an increase in \( \alpha \) increases \( Q_B \).

Next, we discuss the government’s welfare maximization problem in the first stage. From
the first-order condition for interior solution \( \partial W^*/\partial \alpha = 0 \), we obtain
\[
\alpha^{**} = \frac{m\delta(1 - \delta)}{(m + 1)^2 - \delta m(n + m + 2) + \delta^2 mn}.
\] (6)

We present a result on the optimal degree of privatization \( \alpha^* \).

**Proposition 1** (i) \( \alpha^* > 0 \). (ii) \( \alpha^* = 1 \) if \( n \geq \tilde{n} := \frac{1 + m(1 - \delta)(2 - \delta + m)}{ma(1 - \delta)} \); otherwise \( \alpha^* = \alpha^{**} \).

**Proof**

(i) From (5), we obtain
\[
\frac{\partial W^*}{\partial \alpha} \bigg|_{\alpha = 0} = \frac{a^2 m \delta (1 - \delta)(1 + m(1 - \delta))}{(1 + m(1 - \delta^2))^3} > 0.
\]
This implies Proposition 1(i).

(ii) Because the second-order condition is satisfied, \( \alpha^* = 1 \) if and only if
\[
\frac{\partial W^*}{\partial \alpha} \bigg|_{\alpha = 1} \geq 0.
\]
From (5), we obtain
\[
\frac{\partial W^*}{\partial \alpha} \bigg|_{\alpha = 1} = \frac{a^2(1 + m(1 - \delta))(m(1 - \delta)(\delta(1 + n) - (2 + m)) - 1)}{(2 + n + (2 - \delta^2)m + (1 - \delta^2)mn)^3}.
\]
This is greater than or equal to zero if \( m(1 - \delta)(\delta(1 + n) - (2 + m)) - 1 \geq 0 \). Solving this inequality with respect to \( n \), we obtain \( n \geq \tilde{n} \). This implies Proposition 1(ii). ■

According to Proposition 1(i), full nationalization is never optimal, and similar results are repeatedly shown in different contexts (Matsumura, 1998; Lin and Matsumura, 2012; Matsumura and Okamura, 2015; Wu et al., 2016). A marginal increase in \( \alpha \) from zero reduces \( Q_A \), which reduces welfare. However, this effect is second order (the envelope theorem) because \( Q_A^*(0) \) is optimal given \( Q_B \). A marginal increase in \( \alpha \) from zero increase \( Q_B \), which improves welfare. This effect is first order because the price exceeds the marginal cost, even when \( \alpha = 0 \). Therefore, a marginal increase in \( \alpha \) from zero always improves welfare.

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6The second-order condition is satisfied.
According to Proposition 1(ii), full privatization is optimal if \( n \) is sufficiently large. When \( n \) is sufficiently large, the Product A market is competitive, and thus, there is a small welfare-reducing effect of a reduction of \( Q_A \) by an increase of \( \alpha \). Therefore, the maximal \( \alpha (\alpha = 1) \) is optimal.

We now present our main result, which shows the relationship between the optimal degree of privatization and the number of private firms.

**Proposition 2**

(i) \( \alpha^{**} \) is increasing in \( n \).

(ii) \( \alpha^{**} \) is increasing (decreasing) in \( m \) for \( m < \frac{\sqrt{1-\delta}}{1-\delta} \) \((m > \frac{\sqrt{1-\delta}}{1-\delta})\).

**Proof**

From (6), we obtain

\[
\frac{\partial \alpha^{**}}{\partial n} = \frac{m^2\delta^2(1-\delta)^2}{((m+1)^2 - \delta m(n + m + 2) + \delta^2 mn)^2} > 0.
\]

This implies Proposition 2(i).

From (6), we obtain

\[
\frac{\partial \alpha^{**}}{\partial m} = \frac{\delta(1 - \delta)(1 - m^2(1 - \delta))}{((m+1)^2 - \delta m(n + m + 2) + \delta^2 mn)^2}.
\]

This is positive (negative) if \( 1 - m^2(1 - \delta) > (\leq 0) \). Solving \( 1 - m^2(1 - \delta) > 0 \) with respect to \( m \), we obtain \( m < \frac{\sqrt{1-\delta}}{1-\delta} \). This implies Proposition 2(ii). ■

According to Proposition 1(ii), the maximal degree of privatization (full privatization) is more likely optimal when \( n \) is larger. According to Proposition 2(i), an increase in the number of private firms in the same market increases the optimal degree of privatization when the solution is interior. These results suggest that an increase in \( n \) accelerates privatization.

However, according to Proposition 2(ii), there is a nonmonotone relationship (inverted U-shaped relationship) between \( \alpha^{**} \) and \( m \). Moreover, \( \hat{n} \) in Proposition 1(ii) diverges to infinity when \( m \to 0 \) and \( m \to \infty \). This implies that the maximal degree of privatization (full privatization) is less likely optimal when \( m \) is sufficiently large or small. These results suggest a
nonmonotone relationship between the optimal degree of privatization and the number of private competitors in the neighboring market.

To understand the intuition of Proposition 2, we present a result on the composition of the two products.

**Proposition 3** \( Q_A(\alpha^{**})/Q_B(\alpha^{**}) > 1. \)

**Proof**

Substituting (6) into (3) and (4), we obtain

\[
Q_A(\alpha^{**}) = \frac{a((m+1)^2 - \delta m(m+2))}{(m+1)^2 - \delta^2 m(m+2)}
\]
\[
Q_B(\alpha^{**}) = \frac{am(1+m)(1+\delta)}{(m+1)^2 - \delta^2 m(m+2)}.
\]

Comparing these values, we obtain

\[
Q_A(\alpha^{**})/Q_B(\alpha^{**}) = 1 + \frac{1 + m(1-\delta)}{m(1+m)(1-\delta)} > 1.
\]

This implies Proposition 3. 

We explain the intuition behind Proposition 3. Welfare depends on both the total output level \( Q_A + Q_B \) and the output ratio \( Q_A/Q_B \). Because prices are strictly positive (exceed the marginal cost), an increase in \( Q_A + Q_B \) improves welfare given \( Q_A/Q_B \) (total output effect). Given \( Q_A + Q_B \), \( Q_A/Q_B = 1 \) is the best for welfare (composition effect).

An increase in \( \alpha \) decreases \( Q_A + Q_B \) (Lemma 1(iii)), and thus, reduces welfare (total output effect). An increase in \( \alpha \) reduces \( Q_A \) (Lemma 1(i)) and increases \( Q_B \) (Lemma 1(ii)), and thus, reduces \( Q_A/Q_B \). A reduction of \( Q_A/Q_B \) reduces welfare if \( Q_A/Q_B \leq 1 \) (composition effect). Therefore, if \( Q_A/Q_B \leq 1 \), a reduction of \( \alpha \) improves welfare. We show that \( \alpha^* > 0 \) (Proposition 1(i)). Under these conditions, \( Q_A/Q_B > 1 \) must hold in equilibrium.

We now explain the intuition behind our main result, Proposition 2. An increase in both \( n \) and \( m \) mitigates the welfare loss of the total output effect, because more firms increase their
output responding to an increase in $\alpha$, which partially compensates the reduction of $q_0$. An increase in $n$ increases $Q_A$ and reduces $Q_B$. Thus, an increase in $n$ strengthens the welfare-improving effect of the composition effect. Because an increase in $n$ reduces the welfare loss of the total output effect and increases the welfare gain of the composition effect, an increase in $n$ unambiguously increases the optimal degree of privatization.

By contrast, an increase in $m$ increases $Q_B$ and reduces $Q_A$. Thus, it weakens the welfare-improving effect due to the composition effect. Because the total output effect and composition effect move in the opposite directions, the effect becomes ambiguous. When $m$ is large (small), $Q_A/Q_B$ is small (large), and thus, a decrease in $Q_A/Q_B$ caused by an increase in the degree of privatization improves welfare less (more). Therefore, an increase in $m$ reduces (increases) the optimal degree of privatization when $m$ is large (small).

The larger $\delta$ is (i.e., the lower the degree of product differentiation is), the weaker the composition effect is. Therefore, an increase in $m$ more likely increases $\alpha^*$ when $\delta$ is larger.

4 Concluding Remarks

In this study, we investigate the relationship between the optimal privatization policy and the number of private competitors. We find that the optimal degree of privatization of the public firm is increasing with the number of private firms in the same market, and that the optimal degree of privatization is nonmonotone with the number of private firms in the neighboring market. This result implies that an increase of the competitive pressure does not always accelerate privatization.

Although we discuss a two-market model, we suppose that our principle can apply to a model with more than two markets. An increase in the number of firms that supply more differentiated products from those supplied by the public firm is less likely to increase the optimal degree of privatization.

In this study, we assume that both firms are domestic. However, the literature on mixed
oligopolies suggests that in the homogeneous market, an increase in the number of private competitors increases the optimal degree of privatization, regardless of the nationality of the private firms. Therefore, our result might hold even if private firms are owned by foreign investors. Extending our analysis to this direction remains for future research.

\[7\] Whether the private firm is domestic or foreign yields contrasting results in the literature on mixed oligopoly in other contexts. See Corneo and Jeanne (1994), Fjell and Pal (1996), Pal and White (1998), and Bárcena-Ruiz and Garzón (2005 a,b). The optimal degree of privatization is decreasing with the foreign ownership rate in private firms when the number of private firms is given exogenously (Lin and Matsumura, 2012), while it is increasing in free-entry markets (Cato and Matsumura, 2012).
References


