Time-Varying Price Discovery and Autoregressive Loading Factors: Evidence from SP 500 Cash and E-Mini Futures Markets

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Abstract

The error correction coefficients, known as the loading factors, are a key component for price discovery measurement. To date, only constant loading factors have been considered for the price discovery measurement. This paper attempts to consider the autoregressive loading factors and their implications for the price discovery measurement. Based on the minute-by-minute data from the S&P 500 cash and E-mini futures markets, this paper reveals that the loading factors are indeed autoregressive. Furthermore, we propose three AR(1) processes for the loading factors and assess their performance in price discovery measurement compared to the constant loading factor model. Overall, this research provides supporting empirical evidence for using autoregressive loading factors for the price discovery measurement.

Key words: Price Discovery, Information Share, S&P 500 E-mini Futures, AGDCC GARCH, Loading Factor, Error Correction Coefficient

JEL code: G13; G14; G15

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1. Introduction

Price discovery measurement has been studied in the literature for decades. So far, there have been two groups of measures quantifying how security prices in a cointegrated system assimilate new information driving their fundamental values. One group focuses on the static price discovery measurement (e.g. Hasbrouck 1995, Gonzalo and Granger 1995, Harris et al. 2002, Yan and Zivot 2010, Lien and Shrestha 2009 & 2014, Wang and Yang 2011 & 2015, Gramming and Peter 2013). The other group focuses on the time-varying price discovery measurement (e.g. Ates and Wang 2005, Chen and Gau 2009 & 2010, Xu and Wan 2015, Taylor 2011, Avino et al. 2015, Bell et al. 2016). However, neither of the two groups allows the error correction coefficients (i.e. the loading factors) in the vector error correction model (VECM) to be autoregressive for the estimation of the price discovery measures.

According to the literature, the effects of informed trading on prices may change over time (Hasbrouck 1991, Easley and O'Hara 1992, Dufour and Engle, 2000). Hence one may expect that the error correction coefficients that capture the impacts of informed trading on the formation of efficient prices via the arbitraging activities change over time as well. In other words, the long-run impacts of new information on price series may well be time-dependent and reflects how informed trading behaves at a specific point in time while the cointegration relationship is violated. Thus it is important to investigate the behaviour of non-static loading factors.

To our knowledge, there have been few studies in the literature investigating whether the error correction coefficients follow an autoregressive process. Further, the question whether taking into account the autoregressive loading factors benefits the analysis of price discovery remains open.

This paper aims to fill these research gaps in the literature. First, this study tests whether the error correction coefficients are autoregressive. Specifically, this study considers stationary autoregressive processes of order 1 (AR(1)) for the loading factors in the VECM. Three AR(1) processes are proposed: the conventional AR(1), the sine-function-based AR(1) and the cosine-function-based AR(1)[1]. Empirical tests are carried out for these models based on the minute-by-minute data of the S&P 500 cash index and the E-mini futures contacts.

[1] It should be noted that we only focus on the basic triangular functions to construct AR(1) models for the loading factors. Any other combination of these triangular functions will be left to future studies.

Second, this paper investigates the benefits of taking into account the autoregressive loading factors for price discovery measurement. We computes the Hasbrouck (1995)'s information share (IS), Lien and Shrestha (2009)'s modified information share (MIS) and Lien and Shrestha (2014)'s generalised information share (GIS) measures by using both the proposed AR(1) loading factors and the conditional covariance of innovations of the VECM[2]. The latter is predicted by an asymmetric generalised dynamic-conditional-correlation (AG DCC) GARCH model that is estimated under the assumption that returns follow a more generalised skewed Student's *t* distribution. The same measures calculated by the constant loading factors are derived for comparison purpose[3]. In particular, we compare the AR(1) loading factors with the constant ones in terms of model evaluation, information share measure estimates and their standard deviations. Moreover, a comparison across the autoregressive loading factors is conducted using the same criteria. The best AR(1) model on the loading factors is then identified.

This paper contributes to the literature as follows. It is the first one in the literature to investigate the autoregressive error correction coefficients in the context of price discovery measurement. This paper confirms that taking into account the autoregressive loading factors in the measurement of price discovery produces better results than using the constant loading factors. In particular, it is found that the S&P 500 E-mini futures market plays a leading role in price discovery compared to its cash counterpart. The result aligns with the established consensus in the literature on price discovery (e.g. Hasbrouck 2003, Kurov and Lasser 2004, Ates and Wang 2005). However, the constant loading factors fail to achieve the above result.

Overall, our research reveals that the autoregressive loading factors can enhance the understanding on impacts of informed trading on prices of the cointegrated financial markets.

The remainder of the paper is organised as follows. Section 2 briefly reviews the relevant literature on price discovery measure. Section 3 develops the methodology adopted for this paper. Data and some preliminary analyses are described in Section 4. Section 5 presents the empirical results. Concluding remarks are given in Section 6.

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[2] We also calculate the PT/GG measure based on the constant and autoregressive loading factors for the comparison purpose.

[3] Price discovery measures generated by the constant loading factors include: (i) the constant IS, MIS, GIS and PT/GG measures from estimates of the conditional mean of the VECM-AGDCC GARCH model; (ii) the same measures from estimates of both the conditional mean and the conditional covariance matrix of the VECM-AGDCC GARCH model.

2. Price discovery measures

2.1. Static price discovery measures

Based on a vector error correction representation proposed by Engle and Granger (1987) for cointegrated price series, Hasbrouck (1995) transforms such representation into a vector moving average model from which information share measure is derived. The Hasbrouck information share calculates the contribution of one market to the total variance of the long-run impacts of new information on prices, as that market's contribution to price discovery. The information share measure is attractive due to the fact that it incorporates both the error-correction coefficients and the innovation covariance matrix (Lien and Shrestha 2009). The measure has been widely applied to study price discovery in the literature (see, e.g. Tse 1999, Hasbrouck 2003, So and Tse 2004, Tse et al. 2006, Tao and Song 2010).

Another well-known static price discovery measure is the permanent-temporary measure (PT/GG) (Gonzalo and Granger 1995) in which the price series is decomposed into a permanent component and a transitory component. The permanent component is assumed to be a linear function of the original series. In particular, it is considered to be the common factor driving the prices in all the markets. The normalised coefficient of the weight is used as the measure of price discovery for the market (Booth et al. 1999 & 2002, Chu et al. 1999, Harris et al. 2002, Covrig et al. 2004). The advantage of the PT/GG measure is that it delivers a unique value for evaluating price discovery performance, which facilitates the hypothesis testing on a market's contribution to price discovery. However, this measure ignores the innovation covariance matrix, i.e., information generation process.

Note that the Hasbrouck information share and the PT/GG measure are closely related to each other (De Jong 2002, Ates and Wang 2005). Neither of them is superior to the other (Hasbrouck 2002). The major difference between them is that the PT/GG measure takes into account the price reaction to new information while the IS incorporates both the long-run impacts of news on prices and the nature of information generation process (Lien and Shrestha 2014). These two measures deliver different results only when the markets are substantively correlated (Baillie et al. 2002). Compared to information share, the efficient price defined in the PT/GG measure is more volatile and autocorrelated. Hence, on average, the information share measure provides a more meaningful inference and has more economic relevance (Chen and Gau 2009)[4].

However, the information share approach has one drawback, i.e., it is not able to yield a unique measure on price discovery. Since this approach depends on the ordering of the series, one ends up with upper and lower bounds as indicators for price discovery performance on the underlying market. It is not problematic when the two bounds are close to each other. However, the difference between the two bounds increases as the correlation between the cointegrated markets increases. As many markets are highly correlated, it is inevitable to observe that the bounds are far apart (Lien and Shrestha 2009). A large gap between the two bounds of the IS measure makes it hard to draw a conclusion on price discovery performance of the underlying market. This problem is not resolved until the modified information share (MIS) is proposed by Lien and Shrestha (2009)[5].

One advantage of MIS over IS is that it provides a unique measure for a market's contribution to price discovery. This is achieved by proposing a different factorization structure of the covariance matrix of innovations. The way to factorise the correlation matrix instead of the covariance matrix of innovations in MIS gets rid of the ordering dependence due to the Cholesky factorisation that the IS measure uses. Consequently, the result of MIS for one market is unique and independent of the location of the price series of a market in a price vector. Alternatively, Gramming and Peter (2013) suggest a method to yield a unique market information share which is based on the different correlations of price innovations in the tails and in the centre of the distributions[6].

Although the non-uniqueness problem can be resolved, all of the IS, MIS and Gramming and Peter's information share measures require the assumption that the cointegrating vector is restricted to (1,-1). Consequently, they are applicable only when the markets are substantially correlated. To relax this restriction, Lien and Shrestha (2014) propose a generalised information share (GIS) measure which can appropriately deal with the situation where the

[4] Yan and Zivot (2010) also compare the IS measure with the PT/GG measure using a structural cointegration model.

[5] Baillie et al. (2002) suggest that a mid-point of the upper and lower bounds of the IS approach is a reasonable measure of a market's price discovery performance. It has been used in Hasbrouck (2003), Chakravarty et al. (2004), Ates and Wang (2005), and Chen and Gau (2009, 2010), among others. However, Lien and Shrestha (2009) point out that the average seems to be arbitrary since it cannot be shown to be related to any particular factorization structure. In addition, for the IS averages of more than two markets, the sum of them would not necessarily be 100%.

[6] Lien and Wang (2016) conclude that MIS is superior to Gramming and Peter's information share. Given that MIS is a special case of GIS, GIS may be superior to Gramming and Peter's information share as well. Hence we focus on MIS and GIS only for analysis in this paper.

cointegrating relationship is not one-to-one, that is, the markets correlate loosely with each other[7].

2.2. Time-varying price discovery measures

The measurements on price discovery illustrated so far assume that the contribution to the formation of efficient prices by a market is time-invariant. However, the validity of this assumption is in doubt according to some recent studies which find that price discovery may vary over time. The underpinning theoretical models suggest that the variation in price discovery can be traced back to the fundamental variables such as the number of agents collecting information and the intensity with which agents trade on information, assuming private information is long-lived (Admati and Pfleiderer 1988, Back and Pedersen 1998). The drivers for the time dependence of price discovery include volatility, trading activities variables such as trading volume and the number of trades, and other market microstructure variables such as investor structure, bid-ask spread and market share (Chakravarty et al. 2004, Capelle-Blancard 2001, Ates and Wang 2005, Chen and Gau 2009 & 2010, Xu and Wan 2015). In addition, Taylor (2011) finds that price discovery co-varies with a set of information asymmetry and liquidity variables in the market given that the parameters used for calculating information share co-move with those variables over time. More recently, Avino et al. (2015) propose a multivariate generalised autoregressive conditional heteroscedasticity (MGARCH) model to estimate the conditional covariance matrix of innovations, allowing the information share measure to be time-varying.

Note that the derivation of the time-varying price discovery measures is so far limited to four methods. The first method uses high-frequency tick data to calculate information share that varies at low frequencies. This method is adopted in Ates and Wang (2005), Chen and Gau (2009, 2010), Xu and Wan (2015), among others. The second one employs a rolling window approach to obtain the time-varying parameters in the vector error correction model (VECM) that ultimately leads to the time-varying price discovery measures (Bell et al. 2016). With respect to the third method, Taylor (2011) attaches several scaling factors that are based on the dynamic measures of information asymmetry and liquidity to the parameters in the VECM[8]. The time-varying price discovery is thus achieved. The last method of computing

[7] The weighted price contribution (WPC) to measure price discovery of non-overlapping financial markets is well documented by Wang and Yang (2011, 2015).

[8] The dynamic measures include intraday periodicity, time to maturity of futures contracts, liquidity variables, and the time period before and after the announcement of key macroeconomic data.

the time-varying price discovery measures is motivated by Avino et al. (2015). As mentioned above, they employ a MGARCH model to gauge the conditional covariance matrix of innovations that is a key component of the information share measure. Then the time-varying lower and upper bounds of IS are obtained.

3. Methodology

3.1. Price Discovery Measurement

Let Y_t be an $n \times 1$ vector of price series integrated of order 1. There are at most n - 1 cointegrating vectors that are stationary such that Y_t contains one single common stochastic trend (Stock and Watson 1988)[9]. Thus Y_t can be specified in the following vector error correction model (VECM) (Engle and Granger, 1987):

$$\Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^{k} A_i \Delta Y_{t-i} + \varepsilon_t. \tag{1}$$

where $\Pi = \alpha\beta^T$. α and β are $n \times (n-1)$ matrices where $\alpha\beta^T$ has *n-1* non-zero eigenvalues. $\beta^T Y_{t-1}$ consists of $(n - 1)$ cointegrating equations. Each column of α is comprised of adjustment coefficients which are known as the loading factors. The covariance matrix of the error term is given by $\Omega = \mathrm{E}[\varepsilon_t \varepsilon_t^T]$ where $\mathrm{E}[.]$is the expectation operator.

According to Stock and Watson (1988) and Hasbrouck (1995), the VECM can be rearranged as the following vector moving average (VMA) model:

$$\Delta Y_t = \Psi(L)\varepsilon_t, \tag{2}$$

or,

$$Y_t = Y_0 + \Psi(1)\sum_{s=1}^{t} \varepsilon_s + \Psi^*(L)\varepsilon_t. \tag{3}$$

The Engle-Granger representation theorem (Engle and Granger, 1987) suggests $\Psi(1)$ has the following important properties due to the cointegration (De Jong, 2002; Lehmann, 2002):

$$\beta^T \Psi(1) = 0 \text{ and } \Psi(1)\alpha = 0. \tag{4}$$

$\Psi(1)\varepsilon_t$ in Equation (3) represents the long run impact of innovations on the price series (Hasbrouck, 1995). This term is the major focus of information share measure. According to Baillie et al. (2002), $\Psi(1)$ can be represented by

$$\Psi(1) = \beta_\perp \mathcal{K} \alpha_\perp^T. \tag{5}$$

[9] *n* equals to 2 in this paper.

where $\alpha_{\perp}$ and $\beta_{\perp}$ are orthogonal matrices to α and β, respectively. $\mathcal{K}$ is a scalar since there is only one common stochastic trend in the system. The common efficient price is represented by $\alpha_{\perp}^{T}\sum_{s=1}^{t}\varepsilon_{s}$.

3.1.1. Hasbrouck Information Share

Assuming each of the pairwise cointegrating coefficients in β is (1, -1), Equation (4) implies that $\Psi(1)$ has identical rows. Let $\psi = (\psi_1, \psi_2, \ldots, \psi_n)$ be each row of $\Psi(1)$. $\psi\varepsilon_t$ represents the long-run impacts of innovations on each price series. Assuming that the covariance matrix Ω is diagonal, that is, the innovations are independent, the IS of market *j* is defined as

$$S_j = \frac{\psi_j^2\Omega_{jj}}{\psi\Omega\psi^T} \tag{6}$$

where ψ_j is the *j*th element of the vector ψ. $\psi\Omega\psi^T$ is the variance of $\psi\varepsilon_t$.

Note that since $\psi\varepsilon_t$ represents the long-run impacts of innovations on unit-root series, the IS of market *j* is the proportion of the variance of the long-run impacts that is attributable to the innovations of market *j* (Baillie et al. 2002). In other words, the IS of market *j* is the contribution of market *j* to the total variance of the common efficient price or permanent impact (Lien and Shrestha 2014). Thus, IS measures each market's capacity to assimilate new information.

When Ω is not diagonal, the IS of market *j* is given by (Hasbrouck 1995)

$$S_j = \frac{([\psi F]_j)^2}{\psi\Omega\psi^T} \tag{7}$$

where F is the Cholesky factorization of Ω such that $\Omega = FF^T$. $[\psi F]_j$ is the *j*th element of the vector ψF. Due to the Cholesky factorization, the upper (lower) bound of series *j*'s information share arises if series *j* is the first (last) variable in Y_t. This is known as the ordering problem where the calculation of IS of a particular series depends on its ordering in the price vector Y_t. Hence the IS measure of one market is not unique.

Let f_{ij} $(i = 1, \ldots, n, j = 1, \ldots, n)$ be an element of the triangular matrix F and γ_i be an element of the row vector of $\alpha_{\perp}^{T}$. According to Baillie et al. (2002), the upper and lower bounds of the IS of market *j* with $1 \le j \le n$ are given by

$$IS(UB)_j = \frac{[\sum_{i=1}^{n}\gamma_i f_{i1}]^2}{[\sum_{i=1}^{n}\gamma_i f_{i1}]^2 + [\sum_{i=2}^{n}\gamma_i f_{i2}]^2 + \cdots + [\gamma_n f_{nn}]^2}, \tag{8}$$

$$IS(LB)_j = \frac{[\gamma_n f_{nn}]^2}{[\sum_{i=1}^{n} \gamma_i f_{i1}]^2 + [\sum_{i=2}^{n} \gamma_i f_{i2}]^2 + \cdots + [\gamma_n f_{nn}]^2} \,. \tag{9}$$

Note that the upper bound incorporates the market's own contribution represented by f_{11} and its correlation with the other series as indicated by $f_{i1}(i = 2, \dots, n)$. The lower bound only takes the series' 'pure' contribution into account and it does not correlate with the other series as represented by f_{nn}. It is observed that the higher the correlation, the greater (smaller) the upper (lower) bound (Baillie et al. 2002).

3.1.2. Modified Information Share

A solution to resolve the ordering problem of Hasbrouck information share is proposed by Lien and Shrestha (2009). The new measurement without the ordering problem is called modified information share (MIS). The MIS employs a different factor structure that is based upon the correlation matrix of innovations instead of the covariance matrix. Let Φ represent the innovation correlation matrix and Λ be the diagonal matrix consisting of the eigenvalues of the correlation matrix. The corresponding eigenvectors are the column vectors of matrix *G*. Further, let *V* be a matrix containing the standard deviations of innovations on the diagonal. Then we have the following relationship:

$$\varepsilon_t = \hat{F} z_t. \tag{10}$$

where $\hat{F} = [G\Lambda^{-1/2} G^T V^{-1}]^{-1}$ and $\Omega = \hat{F}\hat{F}^T$. z_t are the innovations with mean zero and identity covariance matrix; i.e., $E[z_t] = 0$ and $E[z_t {z_t}^T] = I$. The MIS of market *j* is given by

$$S_j^M = \frac{[\psi_j^M]^2}{\psi \Omega \psi^T}. \tag{11}$$

where $\psi^M = \psi\hat{F}$ and ψ_j^M is the *j*th element of ψ^M. It is noteworthy that the factor structure $\hat{F}$ leads to the IS measure being independent of ordering[10]. Thus the upper and lower bounds of the IS are waived[11].

3.1.3. Generalised Information Share

Both IS and MIS measures are under the assumption that each pair of the cointegrating coefficients in β is (1,-1). This restriction implies that each row of $\Psi(1)$ is the same. However, the one-to-one cointegrating relationship does not necessarily hold in reality. Lien

[10] See Lien and Shrestha (2009, p. 392) for the proof.

[11] Although the MIS computes a single measure instead of the upper and lower bounds by the IS, the calculation process cannot guarantee that the measure can get rid of uniqueness due to the use of square-root matrix (Lien and Shrestha 2009).

and Shrestha (2014) propose a new measure that is independent of the restriction on β. Therefore, such measure can apply to series that do not have the one-to-one cointegrating relationships.

Suppose that the cointegrating matrix β contains a diagonal matrix $\Gamma_{(n-1)}$ and an (n-1) column vector $\iota_{(n-1)}$. $\Gamma_{(n-1)} = Diag(\beta_1, \beta_2, \dots, \beta_{(n-1)})$ and $\iota_{(n-1)} = [1, \dots, 1]^T$. Then β can be represented by

$$\underset{(n-1)\times n}{\beta^T} = [\iota_{(n-1)} \colon -\Gamma_{(n-1)}] \quad (12)$$

Equation (12) shows that the cointegrating matrix β is less restrictive than the one used to obtain the IS and MIS in terms of values on cointegrating coefficients. Further, it implies that the rows of $\Psi(1)$ are not identical. Let ψ_i^g be the ith row of $\Psi(1)$. Then the following relationship holds:

$$\psi_1^g = \beta_{i-1}\psi_i^g, \quad i = 2, \dots, n \quad (13)$$

Thus the long-run impact of innovations on the ith series is

$$\psi_i^g \varepsilon_t = \psi_1^g \beta_{i-1}^{-1} \varepsilon_t, \; i = 1, \dots, n. \quad (14)$$

where $\theta_0 = 1$ and ψ_1^g is the first row of $\Psi(1)$.

When the innovations are independent, the variance of long-run impact on the ith series is

$$Var(\psi_i^g \varepsilon_t) = \psi_i^g \Omega \psi_i^{gT} = \sum_{j=1}^{n} \psi_{ij}^2 \Omega_{jj} = \beta_{i-1}^{-2} \sum_{j=1}^{n} \psi_{1j}^2 \Omega_{jj}. \quad (15)$$

where ψ_{ij} is the jth element of the row vector ψ_i^g and ψ_{1j} is the jth element of the row vector ψ_1^g. The contribution of series j to the total variance of the common factor corresponding to series i is then represented by

$$S_{j,i}^G = \frac{\psi_{1j}^2 \Omega_{jj}}{\psi_1^g \Omega \psi_1^{gT}}. \quad (16)$$

and

$$S_{j,1}^G = S_{j,2}^G = \cdots = S_{j,n}^G, j = 1, 2, \dots, n. \quad (17)$$

$S_{j,i}^G$ is generalised information share (GIS) of series j which is independent of row order i. When the innovations are not independent, the GIS of series j can be calculated as

$$S_j^G = \frac{(\psi_j^G)^2}{\psi_1^g \Omega \psi_1^{gT}}. \quad (18)$$

where $\psi^G = \psi_1^g F^g, F^g = \hat{F} = [G\Lambda^{-1/2}G^T V^{-1}]^{-1}$. ψ_j^G is the jth element of ψ^G. The GIS measure uses the same factor structure as the MIS; thus it is also independent of the ordering problem[12].

3.1.4. Gonzalo-Granger Permanent-Temporary (PT/GG) Measure

We employ the permanent-temporary decomposition proposed by Gonzalo and Granger (1995) for comparison purposes. According to Gonzalo and Granger (1995), the vector of unit-root series Y_t is decomposed into permanent (common factor) component f_t and transitory component $\tilde{Y}_t$. f_t is an $I(1)$ series while $\tilde{Y}_t$ is stationary. f_t has a dimension of 1 when Y_t has one common stochastic trend. The following two assumptions are made to achieve the identification of the two components: (i) f_t is a linear function of Y_t; (ii) $\tilde{Y}_t$ does not Granger cause f_t in the long run. Then under the linear condition f_t can be represented by

$$f_t = \omega^T Y_t. \tag{19}$$

where ω is the n×1 permanent component coefficient vector and ω is orthogonal to the adjustment coefficient matrix α in Equation (1), i.e., $\omega = \alpha_\perp$. Equation (19) implies that the original unit-root series potentially contribute to the common factor (Lien and Shrestha, 2009). Gonzalo and Granger (1995) propose the ith element of ω, i.e. ω_i, as the contribution of market i to the price discovery process. This approach is discussed in Booth et al. (1999, 2002) and Harris et al. (2002). Harris et al. (2002) normalise elements of ω to measure price discovery where the sum of the elements equals to 1. When Y_t has two unit-root series, the normalised ω is given by

$$\omega = (\omega_1, \omega_2)^T = \left(\frac{\alpha_2}{\alpha_2 - \alpha_1}, \frac{\alpha_1}{\alpha_1 - \alpha_2}\right)^T. \tag{20}$$

where α_1 and α_2 are the elements of α.

The difference between the PT/GG measure and the IS type measure is that the PT/GG approach uses information on $\Psi(1)$ whereas the IS type method uses information on both $\Psi(1)$ and innovation covariance matrix Ω. Hence, the PT/GG measure only considers the error correction process that prices respond to new information whereas the IS type method takes both such process and the nature of information generation process into account (Lien and Shrestha, 2014).

[12] Lien and Shrestha (2014) suggest that the Hasbrouck IS measure can be computed by replacing ψ with ψ_1^g in Equation (7) where the cointegrating relationship is not one-to-one.

3.2. Asymmetric Generalised DCC GARCH Model

We employ a bivariate asymmetric generalised (AG) DCC GARCH model proposed by Cappiello et al. (2006) to specify the time-varying covariance matrix of the error terms of Equation (1). Typically, the AG-DCC GARCH model estimates both the individual heteroscedastic processes and the conditional correlations between innovations simultaneously. The error term of Equation (1) is specified as

$$\varepsilon_t|\Xi_{t-1} \sim F(0, H_t) \tag{21}$$

where $\varepsilon_t = [\varepsilon_{1,t}, \varepsilon_{2,t}]^T$ is a 2×1 vector. Ξ_{t-1} represents all the available information at time *t*-1. F denotes a bivariate distribution. H_t is a conditional covariance matrix, which is decomposed as

$$H_t = D_t R_t D_t, \tag{22}$$

with

$$D_t = diag\{h_{11,t}^{\frac{1}{2}}, h_{22,t}^{\frac{1}{2}}\}, \tag{23}$$

and

$$R_t = diag\{Q_t\}^{-1/2} Q_t diag\{Q_t\}^{-1/2}. \tag{24}$$

where D_t is a 2×2 diagonal matrix containing the square roots of the individual conditional variances $h_{ii,t}$ $(i = 1,2)$ on the diagonal; R_t is a conditional correlation matrix of innovations comprised of the conditional covariance matrix of standardized innovations (Q_t) where standardized innovations $\epsilon_{i,t} = \frac{\varepsilon_{i,t}}{\sqrt{h_{ii,t}}} (i = 1,2)$.

Adopting exponential GARCH (EGARCH) model (Nelson, 1991) to specify $h_{ii,t}$, we have

$$\log(h_{ii,t}) = \lambda_{1i} + \lambda_{2i} \left|\frac{\varepsilon_{i,t-1}}{h_{ii,t-1}}\right| + \lambda_{3i} \frac{\varepsilon_{i,t-1}}{h_{ii,t-1}} + \lambda_{4i} \log(h_{ii,t-1}). \tag{25}$$

where *i* =1,2. In Equation (25), the positivity of the conditional variance $h_{ii,t}$ is warranted in the estimation process. Parameter λ_{2i} measures the size effect and should be positive theoretically, as a shock with a higher absolute value should have a stronger effect on volatility. Moreover, the model estimates the asymmetric effects of positive and negative lagged values of $\varepsilon_{i,t}$ on the conditional variances simultaneously. The effects are captured by the parameter λ_{3i}. λ_{3i} is expected to be negative since a negative shock has a stronger effect on volatility than an equally positive shock. Note that for $h_{ii,t}$ to be stationary, λ_{4i} should be less than 1.

A diagonal version of the AG DCC model is adopted to specify Q_t. Then we have

$$Q_t = (\bar{R} - A^T\bar{R}A - B^T\bar{R}B - G^T\bar{S}G) + A^T\epsilon_{t-1}\epsilon_{t-1}^T A + B^T Q_{t-1} B + G^T s_{t-1} s_{t-1}^T G. \tag{26}$$

where A, B and G are 2×2 diagonal matrices with coefficients a_{ii}, b_{ii} and g_{ii} (i = 1,2) on the diagonal. ϵ_t is a 2×1 vector of standardized innovations where $\epsilon_t = [\epsilon_{1,t}, \epsilon_{2,t}]^T$. $s_t = I_t \odot \epsilon_t$ where I_t is a 2×1 indicator function which equals to 1 if $\epsilon_t < 0$ and 0 otherwise. $\odot$ is the element-by-element multiplication operator. $\bar{R} = E[\,\epsilon_t\, \epsilon_t^T]$ represents the unconditional covariance of ϵ_t. $\bar{S} = E[s_t\, s_t^T]$ is the unconditional covariance of s_t.

In Equation (26), asymmetry in correlation of innovations is captured by the term $G^T s_{t-1} s_{t-1}^T G$. For Q_t to be positive definite, a sufficient condition requires that the intercept, $\bar{R} - A^T\bar{R}A - B^T\bar{R}B - G^T\bar{S}G$, is positive semi-definite and the initial covariance matrix Q_0 is positive definite. Note that the asymmetric DCC (A-DCC) is a special case of the AG-DCC where A, B and G are scalars. A diagonal version of the AG-DCC is used since it can sufficiently reduce the number of parameters that convey little information and thus alleviate the computation burden of estimation process. In addition, the diagonal model is preferred by applications to a small number of assets (Cappiello et al., 2006).

3.3. Autoregressive Loading Factors

The regular VECM model as in Equation (1) specifies the error correction coefficients of price series to be constant over time. However, the error correction coefficients may not be constant since the long run impacts of news on prices can change with time (Dufour and Engle 2000). Thus an autoregressive loading factor may better reflect a market's ability to assimilate new information.

To this end, α in Π of Eq.(1) is specified to follow an AR(1) process[13] as

$$\alpha_t = \delta + \theta \odot \alpha_{t-1}. \tag{27}$$

where δ and θ are both $n \times (n\text{-}1)$ matrices. $\odot$ is the element-by-element multiplication operator. In a bivariate model system, Equation (27) can be alternatively represented as

[13] The Ljung-Box test shows that the loading factors in α are significantly autocorrelated at order 1. Note that a rolling window method is used to generate the series of loading factors for the test. In addition, the unit root tests suggest the loading factor series are stationary. These results support the proposed stationary AR(1) process.

$$\alpha_{i,t} = \delta_i + \theta_i \alpha_{i,t-1}. \tag{28}$$

where i =1, 2. $\alpha_{i,t}$ is the element of α_t where $\alpha_{1,t}$ corresponds to the error correction coefficient of the first series in ΔY_t and $\alpha_{2,t}$ corresponds to the error correction coefficient of the second series in ΔY_t. δ_i and θ_i are the elements of matrix δ and θ, respectively. For $\alpha_{i,t}$ to be stationary, a sufficient condition is $-1 < \theta_i < 1$. Thus, a restriction is imposed on the estimation of θ_i . We henceforth refer to the VECM incorporating Equation (28) as Specification (II) for $\alpha_{i,t}$ in this study.

Furthermore, two alternative specifications are proposed to relax the restriction on parameter θ_i. To this end, two trigonometric functions, sine and cosine, are used. The reason why these two functions are employed is that using them can reduce the constraints on the autoregressive coefficient θ_i in the estimation process that ensure the stationarity of $\alpha_{i,t}$. Superior results from the sine and cosine functions are thus expected compared to Equation (28) given that the estimation efficiency is improved. The model using the sine function is

$$\alpha_{i,t} = \delta_i + sine(\theta_i)\alpha_{i,t-1}. \tag{29}$$

where $sine(.)$ denotes the sine function. When $\theta_i \neq \tau(\frac{\pi}{2})$ where τ is any non-zero integer, $-1 < sine(\theta_i) < 1$. Therefore $\alpha_{i,t}$ is a stationary process. It is clear that there are less restrictions on values of θ_i to secure stationarity of $\alpha_{i,t}$ in Equation (29) than Specification (II). The VECM incorporating Equation (29) is henceforth referred to as Specification (III) for $\alpha_{i,t}$.

The model using the cosine function is represented as

$$\alpha_{i,t} = \delta_i + cosine(\theta_i)\alpha_{i,t-1}. \tag{30}$$

where $cosine(.)$ denotes the cosine function. When $\theta_i \neq \tau\pi$ where τ is any integer including zero, $-1 < cosine(\theta_i) < 1$; then $\alpha_{i,t}$ is stationary. Equation (30) has less restrictions on parameter θ_i than Specification (II) for the stationarity of $\alpha_{i,t}$. It has the similar restrictions as Specification (III). The VECM incorporating such equation is referred to as Specification (IV) for $\alpha_{i,t}$.

In this study, we refer to the regular VECM as in Equation (1) as Specification (I) that estimates the constant loading factors. The AG-DCC GARCH model is employed to estimate the conditional covariance matrix of the error terms for each specification. In doing this, we derive the conditional IS, MIS, and GIS measures with both constant loading factors and conditional covariance matrix and those with both autoregressive loading factors and conditional covariance matrix[14]. The Specifications (II), (III), and (IV) are then compared with Specification (I)[15].

3.4. Model Estimation

Parameter estimates are obtained through maximum likelihood estimation (MLE) based upon the probability density function (PDF) of innovations ε_t. ε_t is assumed to follow a bivariate skewed Student's *t* distribution where both excess kurtosis and large skewness are taken into account. Excess kurtosis, which is reflected by fat tails of distribution, is widely observed in financial time series (Bollerslev 1987, Baillie and Bollerslev 1989). In addition, financial returns are often skewed, thus capturing the skewness for the conditional distribution is necessary (Park and Jei 2010). Relying on the conditional normality may lose efficiency in the estimation process of the multivariate GARCH models (Engle and Gonzalez-Rivera 1991, Park and Jei 2010). Thus a more generalised conditional distribution that captures both excess kurtosis and large skewness can yield more reliable results than the normality (Susmel and Engle 1994, Tse 1999, Bauwens and Laurents 2005).

We employ Bauwens and Laurents (2005)'s multivariate skewed-t density for the standardized innovations ϵ_t, which applies Fernandez and Steel (1998)'s skew filter to a multivariate Student's *t* distribution. The contribution of each observation at time *t* to the log-likelihood of a standardized bivariate skewed-t distribution can be expressed in general term as

$$l_t(\Theta) = \log(\tfrac{4}{\pi}) + \textstyle\sum_{i=1}^{2} \log(\tfrac{\xi_i s_i}{1+\xi_i^2}) + \log\left\{\Gamma\left(\tfrac{v+2}{2}\right)/(\Gamma\left(\tfrac{v}{2}\right)(v-2))\right\} - (1/2)\,(v+2)\log[1+(\kappa_t^T\kappa_t)/(v-2)]. \quad (31)$$

where

$$\kappa_t = (\kappa_{1t}, \kappa_{2t})^T$$

$$\kappa_{it} = (s_i\epsilon_{it}^* + m_i)\xi_i^{-I_i}$$

$$m_i = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\Gamma\left(\frac{v}{2}\right)}(\xi_i - \frac{1}{\xi_i})$$

[14] We also compute PT/GG measure with the constant and dynamic loading factors.
[15] It should be noted that we focus only on an AR(1) process of loading factors in this study. Besides, only sine and cosine functions are used to transform the regular AR(1) process for the ease of estimation.

$$s_i^2 = \left(\xi_i^2 + \frac{1}{\xi_i^2} - 1\right) - m_i^2$$

$$I_i = \begin{cases} 1 & \text{if } \epsilon_{it}^* \geq -\frac{m_i}{s_i} \\ -1 & \text{if } \epsilon_{it}^* < -\frac{m_i}{s_i} \end{cases}.$$

Note that $\Gamma(.)$ is the gamma function and v is the degree of freedom for the bivariate Student's t distribution. Note that v is restricted to be more than 2 to ensure the covariance matrix exists. v governs the thickness of tails of the distribution, that is, the kurtosis. The covariance matrix of ${\epsilon^*}_{it}$ $(i = 1,2)$ is an identity matrix. $m_i(\xi_i, v)$ and $s_i(\xi_i, v)$ are the mean and standard deviation of the non-standardized marginal skewed-t of Fernandez and Steel (1998). ξ_i is the skewness parameter where the sign of the logarithmic ξ_i indicates the sign of the skewness. When $ln\xi_i > 0$ (< 0), the skewness is positive (negative) and density is skewed to the right (left).

Θ is a parameter vector with all of the coefficients of the model specifications. Estimates of the parameter vector Θ can be obtained by the following equation

$$L(\Theta) = \sum_{t=1}^{T} l_t(\Theta). \tag{32}$$

where T is the sample size.

4. Data and Descriptive Statistics

In this study, transaction prices recorded at 1-minute intervals of the Standard & Poor's (S&P) 500 index and the S&P 500 E-mini futures contracts are collected for our analysis. The 1-minute time frequency is chosen since it is perceived to be a better choice than other frequencies given the trade-off between staleness of data and loss of information (Wu et al. 2005, Taylor 2011). The sample period is from October 1st, 2015 to December 31st, 2015. All data are obtained from Thomson Reuters Tick History (TRTH).

The E-mini S&P 500 futures contracts trade around the clock on the electronic GLOBEX trading system which was introduced by Chicago Mercantile Exchange (CME) in 1997. The size of S&P 500 E-mini contracts is one-fifth of the regular S&P 500 futures contracts and it has a notional value of $50 times the index. This makes E-mini index futures trading affordable to traders with small margin accounts and capital constraints (Kurov and Lasser 2004). In addition, the E-mini index futures contracts are traded electronically, in contrast to the floor-traded regular index futures. That is, for E-mini index futures trading, customer orders are placed, routed, and executed without human intermediation through the GLOBEX electronic trading system (Hasbrouck 2003, Kurov and Lasser 2004, Ates and Wang 2005).

We choose the E-mini S&P 500 index futures contracts for this study over the floor-traded counterparts for two reasons. First, previous studies utilise the static price discovery measures and reach a consensus that E-mini S&P 500 index futures makes a dominant contribution to the price discovery process over the cash index (see, e.g. Hasbrouck 2003, Kurov and Lasser 2004, Ates and Wang 2005, Taylor 2011). This finding aligns with the advantages of E-mini trading such as increased speed of execution, timely and accurate reporting of fills, improved pricing transparency, high liquidity and trader anonymity (Kurov and Lasser 2004). Hence, using the data of E-mini futures and S&P 500 cash index allows the comparison with the findings of price discovery measures in the literature. Second, the S&P 500 E-mini futures market is one of the most actively traded index futures markets in the world. Thus it is important to understand the S&P 500 E-mini futures market and the results on this market can also have important implications for others.

The E-mini S&P 500 index futures contracts are cash settled at 8.30 am on the third Friday of March, June, September, and December. The futures contracts are traded daily for almost 24 hours on GLOBEX, with two trading halts from 3.15 pm to 3.30 pm and from 4.30 pm to 5.00 pm Central Standard Time (CST). As trading hours of the underlying stock market extend from 8.30 am to 3.00 pm CST, our sample of observations consists of index cash and futures prices matched at time points between 8.40 am and 3.00 pm CST. Any observations outside of this time period are eliminated. Transaction data in the first 10 minutes of each trading day are also omitted following Stoll and Whaley (1990). Exclusion of such data can also help to get rid of the staleness of the reported index levels at the beginning of the day (Ates and Wang 2005). For our analysis, we use the futures contracts with the highest liquidity on each trading day. To this end, the most nearby contracts are selected since they are the most active ones in terms of trading volume. Furthermore, a most nearby contract is switched to the next available one during the second week in its maturity month.

[Insert Table 1 about here]

Table 1 gives descriptive statistics for intraday minute-by-minute data from October to December 2015 for the S&P 500 cash index and E-mini futures contracts. Original price series are taken in the form of natural logarithm. Returns are calculated by taking the first-order difference of the log prices. Note that the statistical properties of the logarithmic prices and returns are similar for both the index cash and futures contracts. This indicates that the two markets may be cointegrated. The JB test statistics strongly reject the normality of return series where large skewness and excess kurtosis are detected. The non-normality will be addressed in the model estimation process.

[Insert Tables 2 & 3 about here]

Table 2 suggests that prices of the S&P 500 cash index and E-mini futures contracts are both integrated at order 1. Cointegration between these two price series with the same integration order is tested in Table 3[16]. The Johansen (1991) trace and max-eigenvalue test statistics indicate that prices of the cash index and E-mini futures are cointegrated. There is a long-run equilibrium relationship between the two time series. Moreover, the likelihood ratio test statistic strongly rejects the null that the cointegrating vector equals to (1,-1). Thus it is reasonable to take into account the unrestricted cointegrating vector when calculating information share measures.

5. Empirical Results

5.1. Model estimates

The estimation results on the constant and autoregressive error correction coefficients corresponding to the VEC models with four specifications are presented in Table 4. Residual diagnoses of the estimated models are provided. As can be seen from Panel B of the table, the Ljung-Box test suggests that the residuals of all the specifications have no autocorrelations. The heteroscedasticity exists in the residuals across the four specifications and will be addressed later by the AG-DCC GARCH model.

[Insert Table 4 about here]

In Panel A of Table 4, α_1 and α_2 in Specification (I) are significant. This implies that both the S&P 500 cash index and the E-mini futures respond to the past deviations from the long-run equilibrium. There may be a bidirectional lead-lag relationship between the cash and futures markets in the long term. The fact that α_1 is positive suggests that the short-term momentum may exist in the cash market (Zhong et al. 2004).

[16] In this test, the cointegrating equation has mean zero and the underlying VAR has no intercept. AIC is used to choose the optimal lags of the VAR. The null hypothesis that an intercept in the cointegrating equation equals to zero is not rejected.

With respect to the dynamic loading factors, θ_1 in Specification (II) is significant at the 1% level. This indicates that $\alpha_{1,t}$ follows an autoregressive process of order 1. The null hypothesis that $|\theta_1| = 1$ is rejected at the 1% level[17]. Thus the autoregressive process of $\alpha_{1,t}$ is stationary. The θ_2 estimate is insignificant and it thus suggests that $\alpha_{2,t}$ does not follow an AR(1) process as defined in Specification (II). The conditional $\alpha_{1,t}$ series confirms the autoregressive nature of the long-run response of the cash market to the deviations in the past from its cointegrating relationship with the E-mini futures. Overall, the result suggests that the long-run impacts of news on cointegrated prices can be dependent on the past.

The autoregressive behaviour of the loading factors is confirmed by the significant estimates of $\theta_i (i = 1,2)$ in Specifications (III) and (IV). Thus both the sine and cosine functions can be used to describe the AR(1) processes of the loading factors $\alpha_{1,t}$ and $\alpha_{2,t}$. Significance of θ_1 and θ_2 may be attributed to less parameter constraints on θ_i than Specification (II). Moreover, the null hypotheses $|sine(\theta_i)| = 1$ $(i = 1,2)$ and $|cosine(\theta_i)| = 1$ $(i = 1,2)$ are all rejected at the 1% level[18]. Hence, the AR(1) processes of the derived loading factor series $\alpha_{1,t}$ and $\alpha_{2,t}$ by Specifications (III) and (IV) are all stationary. Overall, the conditional $\alpha_{1,t}$ and $\alpha_{2,t}$ estimated by the triangular specifications for the AR(1) process reconfirms that loading factors are affected by its past as in Specification (I).

In Panel B of Table 4, Specifications (II), (III), and (IV) are compared with Specification (I) in terms of penalized model fit. The log-likelihood values for Specifications (II)-(IV) are higher than that of Specification (I). Meanwhile, AIC, SIC, and HQIC for Specifications (II)-(IV) are lower than for Specification (I). Thus, the proposed specifications of the VECM that account for the autoregressive error correction coefficients provide better representativeness of data than the regular VECM. The results obtained by Specifications (II)-(IV) should be more accurate and reliable.

[Insert Table 5 about here]

Panel A of Table 5 presents the descriptive statistics of the estimated loading factor series. Both the means of $\alpha_{1,t}$ and $\alpha_{2,t}$ are positive for Specifications (II) and (III) whereas they are negative for Specification (IV). A positive mean of $\alpha_{1,t}$ aligns with the constant estimates of loading factors where the short-term momentum may exist in the cash market. A negative mean of $\alpha_{2,t}$ suggests that the short-term momentum may exist in the futures market. That is,

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[17] Test statistic is available upon request.

[18] Test statistics are available upon request.

when the E-mini futures contracts are under-priced (over-priced), investors sell (buy) the contracts. It makes prices of the futures contracts deviate more from the efficient ones in the short run. The scale of the mean for $\alpha_{1,t}$ ($\alpha_{2,t}$) increases from Specifications (II) to (IV). The same results hold for the medians. Table 5 reveals that Specification (IV) may capture more effects of news impacts on prices than the other two. The volatilities of $\alpha_{1,t}$ and $\alpha_{2,t}$ for rise Specification (IV) are much higher than those for Specifications (II) and (III). Thus the cosine specification yields more volatile loading factors than the other two. It is interesting to note that the higher the variability of the error correction coefficients is, the higher are the effects of news.

We test whether the means of dynamic loading factors statistically equal to zero in Panel B of Table 5. This is equivalent to testing the weak exogeneity of either markets. The zero means of $\alpha_{1,t}$ and $\alpha_{2,t}$ are rejected at the 1% level for all the specifications. The result suggests that the proposed specifications that yield the autoregressive error correction coefficients agree with a bidirectional lead-lag relationship between cash and futures markets in the long term. Neither market can completely dominate the other.

[Insert Table 6 about here]

The estimation results of the AG-DCC GARCH model for the four specifications for the conditional mean are presented in Table 6. The model fits the data well across all specifications as suggested by the Ljung-Box test as there are no autocorrelation and heteroscedasticity in the standardised residuals. In addition, the excess kurtosis and large skewness are accounted for given significant estimates of v and ξ_i $(i = 1,2)$. Note that given estimates of ξ_i, $\ln(\xi_i) > 0$, which is evident for all specifications. This confirms positive skewness of both return series, consistent with the result of Table 1.

The GARCH effects are captured for all specifications where volatility is not only affected by new shocks but old news in cash and futures markets. Asymmetry of volatility is evident in cash and futures markets according to Specifications (I), (II) and (III) given significant estimates of $\lambda_{3i}(i = 1,2)$. The exception is Specification (IV) for which such result does not hold. Moreover, the correlation between cash and futures market is conditioned on its past for all the specifications. Further, asymmetry of correlation where correlation gets higher when both cash and futures markets face price downturns is found for all specifications. Overall, Table 6 suggests that accounting for the autoregressive loading factors in the conditional mean of return series has little effect on the result of the conditional covariance matrix.

5.2. Time varying price discovery

The estimates of constant and time-varying price discovery measures are shown in Table 7. According to Panel A, the static information share and PT/GG measures indicate the cash market is superior to the E-mini future market in terms of price discovery performance. Price discovery is more likely to occur in the underlying stock market. The result apparently contradicts with the transaction cost hypothesis, proposed by Garbade and Silber (1983) and Flemming et al. (1996), that price discovery should take place in the futures market due to its lower trading costs. It is also inconsistent with the empirical work on the hypothesis suggesting it is the E-mini futures that leads the cash index in the long run (e.g. Hasbrouck 2003, Kurov and Lasser 2004, Ates and Wang 2005)[19].

[Insert Table 7 about here]

The result of the conditional price discovery performance presented in Panel B of Table 7 reveals a different scenario. The means and medians of the IS, MIS, GIS and PT/GG measures on the cash index decrease from Specifications (I) to (IV). Meanwhile, the means and medians of those measures on the E-mini futures increase from Specifications (I) to (IV)[20]. Moreover, for Specification (IV), the means and medians of the IS, MIS, and GIS of the E-mini futures are higher than those of the cash index while the contrary holds for the PT/GG measure. Overall, the result supports the transaction cost hypothesis and agrees with the finding in the literature that the E-mini futures market plays a leading role in price discovery with the cash market. It is in sharp contrast to the result suggested by Specification (I). The result implies that specifying an AR(1) process of the loading factors when the price discovery measures are calculated can help to unveil more about the information content of the E-mini futures prices than using the constant loading factors. It is revealed that taking into account the autoregressive error correction coefficients can improve the results obtained by using the constant coefficients. This may be attributed to the variability of loading factors that reflects the impacts of informed trading that may change across time. Utilising a cosine function to model the AR(1) process may have more benefits than a pure AR specification and a sine function for capturing such behaviour. The reason is that the cosine function may

[19] Note that the static price discovery measures in Table 7 are all derived from estimates of the conditional mean of Specification (I), that is, the regular VECM AGDCC GARCH model. We do not calculate the same measures that consider autoregressive loading factors and unconditional covariance matrix of innovations since heteroscedasticity exists in the innovations of all proposed specifications as revealed in Table 4.

[20] The changes of the IS for the spot (futures) market across all specifications are the total changes of the means and medians of the upper and lower bounds of the IS.

estimate more variability of the loading factors. A detailed comparison between the pure AR, sine and cosine specifications will be discussed later in this paper.

[Insert Figure 1 about here]

In addition, from Specifications (I) to (IV), the volatilities of MIS and GIS decrease while MIS and GIS of the E-mini futures increase[21]. The result can be visualised in Figure 1[22]. Thus, the cosine specification provides the highest and most stable MIS and GIS measures on the futures market. It can be also observed from Figure 1 that the estimated series of IS, MIS, and GIS of the cash and futures markets by the four specifications follow a mean-reverting process. The stationarity tests suggest that all the series are stationary[23]. Besides, it is evident that both GIS and MIS lie between the upper and lower bounds of IS for all specifications.

5.3. A comparison across autoregressive loading factors

The three specifications on VECM proposed in this paper that generate autoregressive loading factors are compared with each other in terms of log-likelihood of model estimation, penalised model-fit, and volatility of estimated conditional GIS series[24]. The log-likelihood and information criteria that measure model fit for each specification are shown in Table 4. From Specifications (II) to (IV), the log-likelihood values increases and values of the information criteria (AIC, SIC and HQIC) decrease. The VECM with a cosine-autoregressive process for loading factors possesses the highest log-likelihood and the lowest AIC, SIC and HQIC among the three specifications that yield dynamic loading factors. Thus Specifications (IV) fits the data the best.

[Insert Table 8 about here]

The standard deviations of the conditional GIS of the cash index and the E-mini futures with respect to each specification are reported in Table 8[25]. As can be seen from the table, the standard deviations of the futures' GIS decrease from Specifications (II) to (IV) and Specification (IV) yields the least volatile conditional GIS measure Hence, the AR(1) process

[21] Standard deviations of the IS lower and upper bounds are not monotonic across all specifications. This paper relies on changes of standard deviations of MIS and GIS since these two measures are more advanced than the other two.
[22] Note that MIS equals to GIS in magnitude. Thus MIS is not shown in Figure 1.
[23] Test results are available upon request.
[24] It should be noted that we rely on the standard deviations of GIS as a benchmark for comparison in this subsection. We do not use those of IS since the estimates of IS convey relatively inferior information given that the lower and upper bounds are far apart. The standard deviations of MIS are not used since they are equivalent to GIS.
[25] The standard deviation of the conditional GIS of the cash index with respect to each specification is equal to that of the E-mini futures and thus not reported in the table.

of loading factors specified by a cosine function in Specification (IV) outperforms the other two specifications in terms of stability of estimated GIS series.

The performance of Specification (IV) may be due to its capability to capture the variability of loading factors. The standard deviations of loading factors estimated by Specification (IV) are highest in Table 8. This implies that Specification (IV) can better capture the autoregressive nature of the error correction coefficients than either Specification (II) or (III). That is, Specification (IV) is superior in capturing dynamics of the impacts of informed trading. The more the dynamics is reflected, the higher is the accuracy in estimating its impacts on information content on prices. This is supported by an observation that an increase in the standard deviations of dynamic loading factors from Specifications (II) to (IV), especially for the loading factors of the E-mini futures market, is followed by a decrease in the information criteria for model fit, a decrease in the volatility of GIS and an increase in estimation log-likelihood value.

Furthermore, it is observed from Table 8 that the average GIS of the E-mini futures steadily increases with changes in the standard deviations of loading factors from Specifications (I) to (IV). This observation promotes one to consider the question whether changes in variability of loading factors relate to the improved price discovery performance of the futures market. To this end, the following regression equations are considered:

$$\left\{\ln\left(\frac{GIS_{f,t}^{spec(II)}}{GIS_{s,t}^{spec(II)}}\right) - \ln\left(\frac{GIS_{f,t}^{spec(I)}}{GIS_{s,t}^{spec(I)}}\right)\right\} = \Lambda_0^{spec(II)} + \Lambda_1^{spec(II)}(\alpha_{1,t}^{spec(II)})^2 + \Lambda_2^{spec(II)}(\alpha_{2,t}^{spec(II)})^2 + \omega_t^{spec(II)}, \tag{33}$$

$$\left\{\ln\left(\frac{GIS_{f,t}^{spec(III)}}{GIS_{s,t}^{spec(III)}}\right) - \ln\left(\frac{GIS_{f,t}^{spec(II)}}{GIS_{s,t}^{spec(II)}}\right)\right\} = \Lambda_0^{spec(III)} + \Lambda_1^{spec(III)}\{\left(\alpha_{1,t}^{spec(III)}\right)^2 - \left(\alpha_{1,t}^{spec(II)}\right)^2\} + \Lambda_2^{spec(III)}\{\left(\alpha_{2,t}^{spec(III)}\right)^2 - \left(\alpha_{2,t}^{spec(II)}\right)^2\} + \omega_t^{spec(III)}, \tag{34}$$

$$\left\{\ln\left(\frac{GIS_{f,t}^{spec(IV)}}{GIS_{s,t}^{spec(IV)}}\right) - \ln\left(\frac{GIS_{f,t}^{spec(III)}}{GIS_{s,t}^{spec(III)}}\right)\right\} = \Lambda_0^{spec(IV)} + \Lambda_1^{spec(IV)}\{\left(\alpha_{1,t}^{spec(IV)}\right)^2 - \left(\alpha_{1,t}^{spec(III)}\right)^2\} + \Lambda_2^{spec(IV)}\{\left(\alpha_{2,t}^{spec(IV)}\right)^2 - \left(\alpha_{2,t}^{spec(III)}\right)^2\} + \omega_t^{spec(IV)}. \tag{35}$$

where $spec(II)$, $spec(III)$, and $spec(IV)$ denote Specifications (II), (III) and (IV), respectively. We use the squared loading factors as a proxy for variability in the series[26]. In the equations above, the difference in the relative GIS of futures prices across specifications is specified as a function of the difference in variability of loading factors[27]. Positive $\Lambda_1^{spec(i)}$ and $\Lambda_2^{spec(i)}$ (i = II, III, IV) indicate that an increase in the variability of the loading factors leads to a better price discovery performance of the E-mini futures market.

[Insert Table 9 about here]

The estimation results for Equations (33), (34), and (35) are shown in Table 9. All the estimates of $\Lambda_1^{spec(i)}$ and $\Lambda_2^{spec(i)}$ (i = II, III, IV) are positive and statistically significant at the 1% level. Therefore, volatility of loading factors contributes to price discovery performance of the E-mini futures market. The more volatile the loading factor that a specification produces, the higher is the GIS of the futures market. The reason why Specification (IV) has the highest GIS of the futures market is that it generates the most volatile dynamic loading factors. The result confirms that more volatile loading factors capture more impacts of informed trading and thus gauge information content of futures prices more accurately. Hence, utilising a cosine function to specify an autoregressive process of loading factors in VECM benefits the calculation of GIS since such specification significantly improves the estimation results by the constant loading factors. The result for Specification (IV) also supports the fact that the E-mini futures market is a focal pit for information.

6. Concluding Remarks

Although price discovery of cointegrated markets has been extensively explored in the literature, the question whether the autoregressive error correction coefficients of the VECM should be taken into account remains open. This paper shed lights on this issue. Three new specifications on the VECM are proposed to estimate autoregressive loading factors where the loading factors are specified to follow a pure AR(1), a sine-function AR(1) and a cosine-function AR(1). The conditional covariance matrix of the error structure of each specification is specified by a bivariate AG-DCC GARCH model. The models are estimated with the assumption that security prices follow a skewed Student's *t* distribution.

[26] We also calculate time series of standard deviations of loading factors as a proxy by using a rolling window method. The window size is set to 100 observations and step size is 1 observation. The estimation results are similar to Table 9.
[27] Note that variability of loading factors in Specification (I) is zero.

Using minute-by-minute data of the S&P 500 cash index and the E-mini futures markets in a 3-month period, static and conditional Hasbrouck information share (IS), modified information share (MIS), generalised information share (GIS) and Gonzalo-Granger permanent-temporary (PT/GG) measure are estimated. The regular VECM and new specifications are compared in terms of model fit, estimates of price discovery measures and their standard deviations. The best specification is determined and its benefits on price discovery measurement are assessed.

This paper reveals that the loading factors of the cointegrated cash and futures markets are dependent on their past. They follow a stationary AR(1) process, which is evidenced by all of the three new specifications. The result implies that the long-term impacts of news on cointegrated price series are conditioned on their past.

It is also found that the AR(1) loading factors benefit price discovery measurement. Moving from constant to autoregressive loading factors, a significant improvement in the price discovery performance of the E-mini futures is confirmed. The IS, MIS, and GIS measures suggest that price discovery primarily takes place in the S&P 500 cash market under the regular, pure AR(1), and sine AR(1) specifications on the loading factors of the VECM. In stark contrast, the same information share measures derived from the specification with cosine AR(1) loading factors strongly suggest that it is the E-mini futures market that leads the cash counterpart in the long run. Such finding aligns with the established consensus on price discovery of the E-mini futures market in the literature. The result from the cosine specification is the most reliable because that specification has the best fit with the data. . It is concluded that a cosine-function-based AR(1) process of the loading factors improves the results of information share measures on the E-mini futures market compared to the ones obtained by the constant loading factors.

Moreover, we find that the volatility of the dynamic loading factors increases with GIS of the E-mini futures market. The more volatile are the loading factors, the higher is the GIS of the futures market. This result implies that the highest GIS of the futures market that the cosine AR(1) specification generates can be attributed to the most volatile loading factors that specification yields. The result suggests a critical role the variability of the loading factors plays in the modelling price discovery of futures prices. That is, the autoregressive loading factors can yield a better price discovery measure and reveal more insight on the impacts of informed trading than the constant loading factors.

Finally, some future research directions can be briefly mentioned. A future study can be devoted to exploring a variety of specifications for the loading factors that may capture more time-series behaviour. The benefits of these specifications can be evaluated in terms of to what extent they help to explain information content of asset prices. In addition, the three specifications that estimate the autoregressive loading factors proposed in this paper may apply to other price discovery measures. One may wish to examine if the benefits of the specifications revealed in this study still are still valid with those measures.

References

1. Admati, A. and Pfleiderer. P, A theory of intraday patterns: Volume and price variability. Review of Financial Studies, 1988, 1, 1–40.
2. Ates, A., and Wang, G.H.K., Information transmission in electronic versus open-outcry trading systems: An analysis of U.S. equity index futures markets. The Journal of Futures Markets, 2005, 25(7), 679-715.
3. Avino, D., Lazar, E., and Varotto, S., Time varying price discovery. Economic Letters, 2015, 126, 18-21.
4. Back, K. and Pederson, H., Long-lived information and intraday patterns. Journal of Financial Markets, 1998, 1, 385-402.
5. Baillie, R.T. and Bollerslev.T., The message of daily exchange rates. Journal of Business & Economic Statistics, 1989, 7, 287-305.
6. Baillie, R.T., Booth, G.G., Tse, Y., and Zabotina, T., Price discovery and common factor models. Journal of Financial Markets, 2002, 5, 309-321.
7. Bauwens, L., and Laurents, S., A new class of multivariate skew densities with application to GARCH models. Journal of Business & Economic Statistics, 2005, 23, 346–354.
8. Bell, A.R., Brooks, C., and Taylor, N., Time-varying price discovery in the eighteenth century: Empirical evidence from the London and Amsterdam stock markets. Cliometrica, 2016, 10 (1), 5-30.
9. Bollerslev, T. A conditionally heteroskedastic time series model for speculative prices and rates of return. Review of Economics and Statistics, 1987, 69,542-547.
10. Booth, G., So, R., and Tse, Y., Price discovery in German equity index derivatives market. Journal of Futures Markets, 1999, 19, 619–643.
11. Booth, G., Lin, J., Martikainen, T., and Tse, Y., Trading and pricing in upstairs and downstairs stock markets. Review of Financial Studies, 2002, 15, 1111–1135.
12. Capelle-Blancard, G., Volatility trading in the option market: how does it affect where informed traders trade? Working Paper, University of Paris, 2001.
13. Cappiello, L., Engle, R.F., and Sheppard, K., Asymmetric dynamics in the correlations of global equity and bond returns. Journal of Financial Econometrics, 2006, 4(4), 537-572.
14. Chakravarty, S., Gulen, H., and Mayhew, S., Informed trading in stock and option markets. Journal of Finance, 2004, 59, 1235–1257.
15. Chen Y.-L. and Gau, Y.-F., Tick sizes and relative rates of price discovery in stock, futures, and options markets: Evidence from the Taiwan stock exchange. The Journal of Futures Markets, 2009, 29(1), 74-93.
16. Chen, Y.-L., and Gau, Y.-F., News announcements and price discovery in foreign exchange spot and futures markets. Journal of Banking & Finance, 2010, 34, 1628-1636.
17. Chu, Q.C., Hsieh, W. G., and Tse, Y., Price discovery on the S&P 500 index markets: An analysis of spot index, index futures, and SPDRs. International Review of Financial Analysis 1999, 8(l), 21-34.

18. Chung, H., Sheu, G.-J., and Hsu, S., Trading platform, market volatility and pricing efficiency in the floor-traded and E-mini index futures markets. International Review of Economics and Finance, 2010, 9, 742-754.
19. Covrig, V., Ding, D. K., and Low, B. S., The contribution of a satellite market to price discovery: Evidence from the Singapore exchange. Journal of Futures Markets, 2004, 24, 981–1004.
20. De Jong, F. Measure of contributions to price discovery: A comparison. Journal of Financial Markets, 2002, 5, 323-327.
21. Dufour, A., and Engle, R., Time and the Price Impact of a Trade. Journal of Finance, 2000, 55 (6), 2467-2498.
22. Easley, D., and O'Hara, M, Time and the process of security price adjustment. Journal of Finance, 1992, 47, 577-605.
23. Engle, R., and Gonzalez-Rivera, G., Semiparametric ARCH model. Journal of Business & Economic Statistics, 1991, 9, 345-360.
24. Engle, R. F., and Granger, C. W. J., Co-integration and error correction: Representation, estimation, and testing. Econometrica, 1987, 55, 251–276.
25. Fernandez, C., and Steel, M., On Bayesian modelling of fat tails and skewness. Journal of the American Statistical Association, 1998, 93, 359–371.
26. Fleming, J., Ostdiek, B., and Whaley, R. E., Trading costs and the relative rates of price discovery in stock, futures, and options markets. Journal of Futures Markets, 1996, 16, 353–387.
27. Garbade, K. D., and Silber, W. L., Price movements and price discovery in futures and cash markets. Review of Economics and Statistics, 1983, 65, 289–297.
28. Gonzalo, J. and Granger, C., Estimation of common long-memory components in cointegrated systems. Journal of Business and Economic Statistics, 1995, 13, 27–35.
29. Grammig, J., and Peter, F.J., Tell-tale tails: A new approach to estimating unique market information shares. Journal of Financial and Quantitative Analysis, 2013, 48, 459-488.
30. Harris, F., McInish, T., and Wood, R., Security price adjustment across exchanges: In investigation of common factor components for Dow stocks. Journal of Financial Markets, 2002, 5, 277–308.
31. Hasbrouck, J. Measuring the information content of stock trades. Journal of Finance, 1991, 46, 179-207.
32. Hasbrouck, J. One security, many markets: Determining the contributions to price discovery. Journal of Finance, 1995, 50, 1175–1199.
33. Hasbrouck, J. Stalking the 'efficient price' in market microstructure specifications: An overview. Journal of Financial Markets, 2002, 5, 329-339.
34. Hasbrouck, J. Intraday price formation in U.S. equity index markets. The Journal of Finance, 2003, LVIII (6), 2375-2399.
35. Kaul, A., and Sapp, S., Trading activity, dealer concentration and foreign exchange market quality. Journal of Banking and Finance, 2009, 33, 2122–2131.
36. Kurov, A., and Lasser, D.J., Price dynamics in the regular and E-mini futures markets. Journal of Financial and Quantitative Analysis, 2004, 39(2), 365-384.
37. Lehmann, B. Some desiderata for the measurement of price discovery across markets. Journal of Financial Markets, 2002, 5, 259–276.
38. Lien, D., and Shrestha, K. A New Information Share Measure. The Journal of Futures Markets, 2009, 29(4), 377-395.
39. Lien, D. and Shrestha, K., Price discovery in interrelated markets. The Journal of Futures Markets, 2014, 34 (3), 203-219.
40. Lien, D., and Wang, Z.J., Estimation of Market Information Shares: A Comparison. The Journal of Futures Markets, 2016, 36(11), 1108–1124.
41. Nelson, D.B., Conditional heteroscedasticity in asset returns: A new approach. Econometrica, 1991, 59(2), 347-370.
42. Newey, W. K., and West, K. D., A simple, positive semi-definite, heteroskedasticity and autocorrelation consistent covariance matrix. Econometrica, 1987, 55(3), 703–708.

43. Park, S.Y., and Jei, S.Y., Estimation and hedging effectiveness of time-varying hedge ratio: Flexible bivariate GARCH approaches. Journal of Futures Markets, 2010, 30(1), 71-99.
44. So, R.W. and Tse, Y., Price discovery in the Hang Seng index markets: Index, futures, and the tracker fund. The Journal of Futures Markets, 2004, 24(9), 887-907.
45. Stock, J. H., and Watson, M. W., Testing for common trends. Journal of the American Statistical Association, 1988, 83, 1097–1107.
46. Susmel, R., and Engle, R.F., Hourly volatility spillovers between international equity markets. Journal of International Money and Finance, 1994, 13, 3–25.
47. Tao, L.B. and Song, F.M., Do small trades contribute to price discovery? Evidence from the Hong Kong Hang Seng index markets. The Journal of Futures Markets, 2010, 30 (2), 156-174.
48. Taylor, N. Time-varying price discovery in fragmented markets. Applied Financial Economics, 2011, 21, 717-734.
49. Tse, Y. K. Price discovery and volatility spillovers in the DJIA index and futures markets. The Journal of Futures Markets, 1999, 19(8), 911-930.
50. Tse, Y., Bandyopadhyay, P., and Shen, Y.-P., Intraday Price Discovery in the DJIA Index Markets. Journal of Business Finance & Accounting, 2006, 33, 1572–1585.
51. Wang, J., and Yang, M., Housewives of Tokyo versus the Gnomes of Zurich: measuring price discovery in sequential markets. Journal of Financial Markets, 2011, 14, 82–108.
52. Wang, J., and Yang, M., How well does the weighted price contribution measure price discovery? Journal of Economic Dynamics & Control, 2011, 55, 113–129.
53. Wu, C., Li, J., and Zhang, W. Intradaily periodicity and volatility spillovers between international stock index futures markets. Journal of Futures Markets, 2005, 25 (6), 553–585.
54. Xu, F., and Wan, D., The impacts of institutional and individual investors on the price discovery in stock index futures market: Evidence from China. Financial Research Letters, 2015, 15, 221-231.
55. Yan, B., and Zivot, E., A structural analysis of price discovery measures. Journal of Financial Markets, 2010, 13,1–19.
56. Zhong, M., Darrat, A. F., and Otero, R., Price discovery and volatility spillovers in index futures markets: Some evidence from Mexico. Journal of Banking and Finance, 2004, 28, 3037–3054.

Table 1. Descriptive Statistics of the S&P 500 cash index and E-mini index futures

	Nobs	*Mean*	*Median*	*Std*	*Skew*	*Kurt*	*JB statistics*
S&P 500 cash index							
Prices	24054	7.626	7.630	0.020	-1.257	4.946	$1.013\times10^{4***}$
Returns	24053	2.647×10^{-6}	0.000	3.960×10^{-4}	0.255	206.122	$4.135\times10^{7***}$
S&P 500 E-mini index futures							
Prices	24054	7.623	7.628	0.021	-1.233	4.802	$9.351\times10^{3***}$
Returns	24053	2.617×10^{-6}	0.000	4.440×10^{-4}	0.350	146.783	$2.072\times10^{7***}$

Notes: This table reports the descriptive statistics of the S&P 500 cash index and E-mini index futures. Prices are taken in the form of natural logarithms. Returns are calculated as the first differences of the logarithmic price series. *Nobs* denotes the number of observations; *Mean* denotes mean of sample; *Median* denotes median of sample; *Std* denotes standard deviation; *Skew* denotes skewness; *Kurt* denotes kurtosis; *JB statistics* denotes statistics of the Jarque-Bera test for normality. ***, **, and * indicate significance at the 1%, 5% and 10% level, respectively.

Table 2. Unit-root tests

	Logarithms of Prices			
	Level		*First Difference*	
	ADF	*PP*	*ADF*	*PP*
S&P 500 Cash Index	-3.348	-3.312	-37.759***	-149.105***
S&P 500 E-Mini Futures	-3.313	-3.259	-38.146***	-163.178***

Notes: This table shows the results of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit-root Tests on the natural logarithm of prices of the S&P 500 cash index and E-mini index futures. The results on the original level and the first difference of price series are reported. The first difference equals to the price at time *t* minus the price at time *t-1*. *ADF* denotes the Augmented Dickey-Fuller test statistic and *PP* denotes the Phillips-Perron test statistic. ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively.

Table 3. Johansen cointegration tests

Cointegrating Vector		*Zero Cointegrating Vector (r = 0)*		*One Cointegrating Vector (r = 1)*		*Restrictions on Cointegrating Vector*
Coeff. of S_t	*Coeff. of F_t*	λ_{max}	*Trace*	λ_{max}	*Trace*	*Likelihood –ratio test statistic*
1	-1.0004	12.390**	12.988**	0.598	0.598	7.758***

Notes: This table shows the results of the Johansen cointegration tests on prices of the S&P 500 cash index and E-mini index futures. *Coeff.* stands for cointegrating coefficient. S_t denotes cash index prices while F_t denotes E-mini futures prices. λ_{max} is the max-eigenvalue test statistic. *Trace* denotes the trace test statistic. *r* refers to the number of cointegrating vector. The likelihood ratio test is conducted on restricting the cointegrating vector to be (1,-1). ***, **, and * indicate significance at the 1, 5, and 10% levels, respectively.

Table 4. Constant and autoregressive loading factors

Panel A: Model estimates				
	Specification (I)	*Specification (II)*	*Specification (III)*	*Specification (IV)*
α_1	0.004^{**} (1.79×10^{-3})	-	-	-
α_2	0.006^{***} (2.01×10^{-3})	-	-	-
δ_1	-	0.022^{***} (1.25×10^{-3})	0.024 (2.76×10^{-2})	-5.858^{***} (9.32×10^{-3})
δ_2	-	0.025^{*} (1.34×10^{-2})	0.045^{***} (1.05×10^{-2})	-6.913^{***} (2.83×10^{-3})
θ_1	-	-0.995^{***} (1.14×10^{-3})	-2.982^{**} (1.37)	3.181^{***} (1.06×10^{-3})
θ_2	-	-0.632 (0.88)	-2.327^{***} (0.59)	3.102^{***} (1.06×10^{-3})

Panel B: Residual Diagnosis				
	Specification (I)	*Specification (II)*	*Specification (III)*	*Specification (IV)*
Log-likelihood	-37274.438	41996.490	43239.912	76991.051
AIC	3.102	-3.491	-3.594	-6.401
SIC	3.110	-3.482	-3.585	-6.392
HQIC	3.104	-3.488	-3.591	-6.398
LB(k)	12.824	0.070	0.539	1.021
	7.721	0.080	0.485	0.985
$LB^2(k)$	44.524^{***}	9.950	45.429^{***}	60.433^{***}
	62.804^{***}	21.572^{***}	68.187^{***}	87.185^{***}

Notes: This table reports the estimation results of the constant and dynamic error correction coefficients of the VECM. *Specification (I)* refers to the VECM with the constant error correction coefficients as in Equation (1); *Specification (II)* refers to the VECM with the dynamic error correction coefficients defined by Equation (28); *Specification (III)* refers to the VECM with the dynamic error correction coefficients defined by Equation (29); *Specification (IV)* refers to the VECM with the dynamic error correction coefficients defined by Equation (30). *AIC*, *SIC*, and *HQIC* refer to Akaike information criteria, Schwarz information criteria, Hannan-Quinn information criteria, respectively. The lag orders of the underlying VAR are chosen by AIC. *LB(k)* and *$LB^2(k)$* are Ljung-Box Q statistics for residuals and its squares up to *k* lags. *k* equals to 8. Figures in parenthesis are standard errors. ***, **, and * denotes significance at the 1%, 5%, and 10% levels, respectively.

Table 5. Descriptive statistics and hypothesis testing on autoregressive loading factors

Panel A: Descriptive statistics			
	Mean	*Median*	*Std*
Specification (II)			
$\alpha_{1,t}$	0.011	0.011	7.206×10^{-4}
$\alpha_{2,t}$	0.015	0.015	1.253×10^{-4}
Specification (III)			
$\alpha_{1,t}$	0.020	0.020	1.338×10^{-4}
$\alpha_{2,t}$	0.026	0.026	2.432×10^{-4}
Specification (IV)			
$\alpha_{1,t}$	-2.930	-2.930	0.483
$\alpha_{2,t}$	-3.458	-3.458	0.570
Panel B : Hypothesis testing			
Specification (II)			
H_0: $u_1 = 0$		$2.406\times10^{3***}$	
H_0: $u_2 = 0$		$1.863\times10^{4***}$	
Specification (III)			
H_0: $u_1 = 0$		$2.374\times10^{4***}$	
H_0: $u_2 = 0$		$1.650\times10^{4***}$	
Specification (IV)			
H_0: $u_1 = 0$		-940.356^{***}	
H_0: $u_2 = 0$		-940.732^{***}	

Notes: This table reports the descriptive statistics of the estimated loading factor series. Test results on the hypotheses that the series' means equal to zero are reported. *Specification (II)* refers to the VECM with the dynamic error correction coefficients defined by Equation (28); *Specification (III)* refers to the VECM with the dynamic error correction coefficients defined by Equation (29); *Specification (IV)* refers to the VECM with the dynamic error correction coefficients defined by Equation (30). *Std* denotes standard deviation. u_1 denotes the mean of $\alpha_{1,t}$; u_2 denotes the mean of $\alpha_{2,t}$. *t*-statistics for hypothesis testing are reported. *** denotes significance at the 1% level.

Table 6. AG-DCC GARCH model

	Specification (I)		*Specification (II)*		*Specification (III)*		*Specification (IV)*	
Coeff.	(i = 1)	(i = 2)	(i = 1)	(i = 2)	(i = 1)	(i = 2)	(i = 1)	(i = 2)
λ_{1i}	-0.150***	-0.154***	-0.163***	-0.174***	-0.226***	-0.226***	-0.171***	-0.159***
	(4.99×10^{-7})	(1.54×10^{-6})	(3.05×10^{-4})	(1.88×10^{-5})	(2.73×10^{-4})	(1.34×10^{-5})	(2.20×10^{-3})	(1.65×10^{-3})
λ_{2i}	0.118***	0.120***	0.076***	0.077***	0.084***	0.086***	0.142***	0.137***
	(9.18×10^{-6})	(2.23×10^{-5})	(7.81×10^{-4})	(1.22×10^{-3})	(9.54×10^{-4})	(1.13×10^{-3})	(3.21×10^{-3})	(3.99×10^{-3})
λ_{3i}	-0.123***	-0.106***	-0.022***	-0.024***	-0.017***	-0.019***	0.001	0.001
	(4.02×10^{-4})	(9.67×10^{-4})	(1.28×10^{-3})	(1.47×10^{-3})	(1.41×10^{-3})	(1.61×10^{-3})	(1.08×10^{-3})	(1.71×10^{-3})
λ_{4i}	0.984***	0.986***	0.992***	0.991***	0.988***	0.988***	0.993***	0.993***
	(1.14×10^{-6})	(1.69×10^{-6})	(8.97×10^{-6})	(3.57×10^{-5})	(2.31×10^{-5})	(3.63×10^{-5})	(1.29×10^{-4})	(1.46×10^{-4})
a_{ii}	0.100***	0.074***	0.100***	0.023***	0.100***	0.056*	0.060***	0.199***
	(7.67×10^{-7})	(1.19×10^{-5})	(6.91×10^{-4})	(8.25×10^{-3})	(1.58×10^{-4})	(2.88×10^{-2})	(1.67×10^{-3})	(4.38×10^{-2})
b_{ii}	0.930***	0.274***	0.990***	0.072***	0.991***	0.311***	0.991***	-0.162***
	(2.36×10^{-6})	(8.47×10^{-5})	(6.20×10^{-6})	(1.34×10^{-2})	(2.66×10^{-5})	(6.63×10^{-2})	(4.23×10^{-5})	(6.04×10^{-2})
g_{ii}	0.104***	0.016***	0.085***	0.034	0.076***	0.024	0.107***	-0.268***
	(1.68×10^{-6})	(1.26×10^{-5})	(1.74×10^{-3})	(2.99×10^{-2})	(1.35×10^{-3})	(3.92×10^{-2})	(5.02×10^{-3})	(9.92×10^{-2})
ξ_i	1.910***	1.914***	1.849***	1.897***	1.736***	1.768***	2.593***	2.513***
	(3.83×10^{-2})	(3.85×10^{-2})	(7.06×10^{-3})	(7.71×10^{-3})	(7.04×10^{-3})	(7.38×10^{-3})	(3.20×10^{-2})	(4.41×10^{-2})
v	4.613***		2.567***		2.585***		2.733***	
	(2.48×10^{-2})		(5.14×10^{-3})		(6.12×10^{-3})		(1.98×10^{-2})	
LB(k)	12.743	7.732	0.003	0.002	0.539	0.485	0.876	0.831
LB^2*(k)*	3.345	4.669	0.318	0.795	1.784	5.800	1.464	5.624

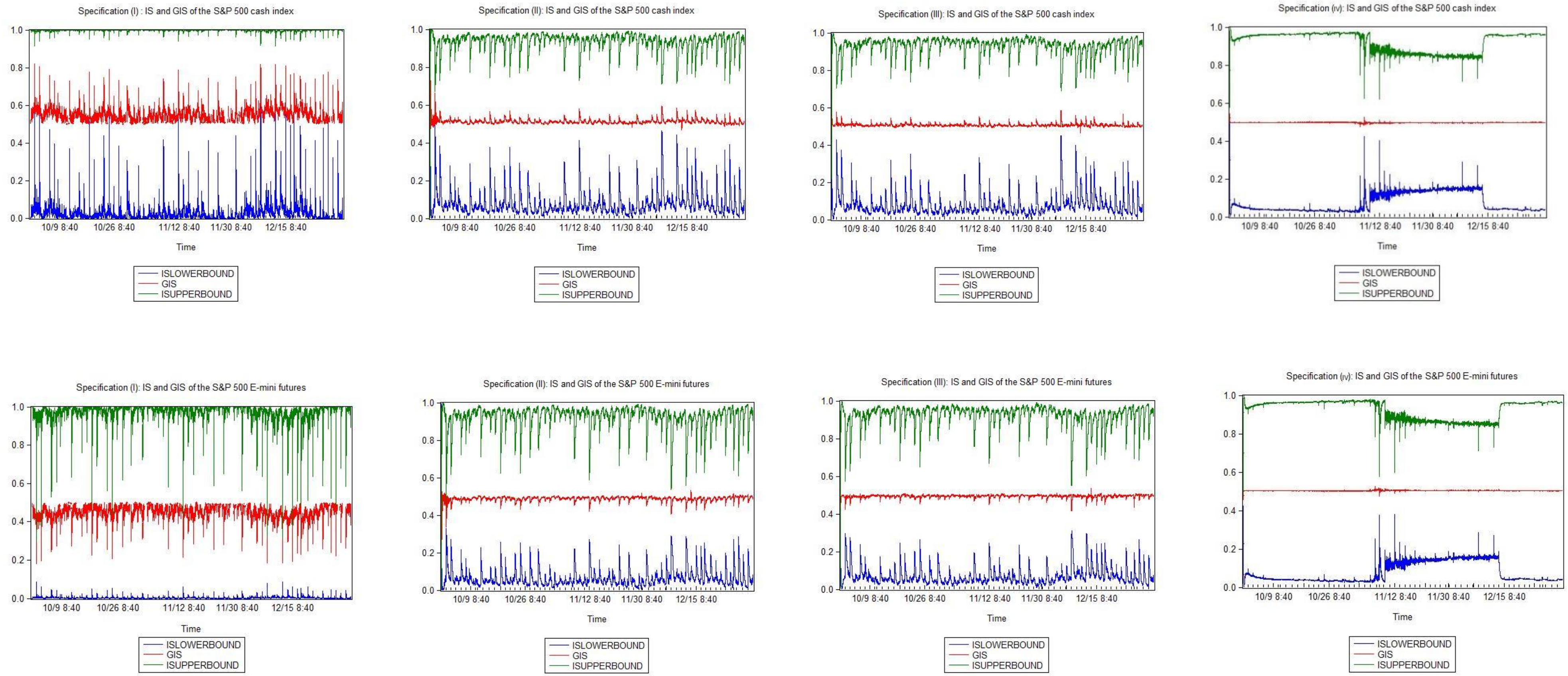
Notes: This table reports the estimation results of the bivariate AG-DCC GARCH model. Specifically, the coefficients of Equations (25), (26) & (31) are reported. *Specification (I)* refers to the VECM with the constant error correction coefficients as in Equation (1); *Specification (II)* refers to the VECM with the dynamic error correction coefficients defined by Equation (28); *Specification (III)* refers to the VECM with the dynamic error correction coefficients defined by Equation (29); *Specification (IV)* refers to the VECM with the dynamic error correction coefficients defined by Equation (30). *LB(k)* and *LB*^{2}*(k)* are the Ljung-Box Q statistics at order k for the standardized residuals and their squares, respectively. *k equals* to 8. *Coeff.* stands for coefficients. Figures in the parentheses are standard errors. ***, **, and * indicate significance at the 1, 5, and 10%, respectively.

Table 7. Static and conditional price discovery measures

	IS				MIS		GIS		PT/GG	
	Upper Bound	*Lower Bound*	*Upper Bound*	*Lower Bound*						
	Cash Index		*E-mini Futures*		*Cash Index*	*E-mini Futures*	*Cash Index*	*E-mini Futures*	*Cash Index*	*E-mini Futures*
Panel A : Static price discovery measure										
Estimates	0.985	0.025	0.975	0.015	0.518	0.482	0.518	0.482	0.593	0.407
Panel B: Conditional price discovery measure										
Specification (I)										
Mean	0.997	0.030	0.970	0.003	0.558	0.442	0.558	0.442	0.593	0.407
Median	0.999	0.015	0.985	0.001	0.551	0.449	0.551	0.449	0.593	0.407
Std	7.396×10^{-3}	0.060	0.060	7.396×10^{-3}	0.039	0.039	0.039	0.039	0	0
Specification (II)										
Mean	0.935	0.080	0.920	0.065	0.513	0.487	0.513	0.487	0.574	0.426
Median	0.949	0.059	0.941	0.051	0.510	0.490	0.510	0.490	0.574	0.426
Std	0.051	0.070	0.070	0.051	0.018	0.018	0.018	0.018	0.018	0.018
Specification (III)										
Mean	0.930	0.078	0.922	0.070	0.507	0.493	0.507	0.493	0.558	0.442
Median	0.945	0.059	0.941	0.055	0.505	0.495	0.505	0.495	0.558	0.442
Std	0.049	0.062	0.062	0.049	0.011	0.011	0.011	0.011	1.564×10^{-3}	1.564×10^{-3}
Specification (IV)										
Mean	0.923	0.074	0.926	0.077	0.497	0.503	0.497	0.503	0.541	0.459
Median	0.957	0.041	0.959	0.043	0.497	0.503	0.497	0.503	0.541	0.459
Std	0.052	0.051	0.051	0.052	0.005	0.005	0.005	0.005	3.317×10^{-5}	3.317×10^{-5}

Notes: This table reports the estimation result of the static and conditional IS, MIS, GIS, and PT/GG measures. Mean, median and standard deviation of the conditional measures are reported. *IS, MIS, GIS and PTGG* refer to information share, modified information share, generalised information share Gonzalo-Granger permanent-temporary measure, respectively. *Cash Index* denotes the S&P 500 cash index; *E-mini Futures* denotes the S&P 500 E-mini index futures. The AG-DCC GARCH model is used to predict the conditional covariance matrix of innovations for IS, MIS and GIS measures. *Specification (I)* refers to the VECM with the constant error correction coefficients as in Equation (1); *Specification (II)* refers to the VECM with the dynamic error correction coefficients defined by Equation (28); *Specification (III)* refers to the VECM with the dynamic error correction coefficients defined by Equation (29); *Specification (IV)* refers to the VECM with the dynamic error correction coefficients defined by Equation (30). *Std* denotes standard deviation.

Figure 1. Time-varying IS and GIS



Notes: Specification (I) is the VECM with the constant error correction coefficients as in Equation (1); Specification (II) is the VECM with the dynamic error correction coefficients defined by Equation (28); Specification (III) is the VECM with the dynamic error correction coefficients defined by Equation (29); Specification (IV) is the VECM with the dynamic error correction coefficients defined by Equation (30). The AG-DCC GARCH model is used to compute the conditional covariance matrix of innovations for IS, MIS and GIS measures. IS, information share; GIS, generalised information share. The estimates of MIS equal to GIS and thus are not showed in the figure.

Table 8. Volatility of autoregressive loading factors and GIS

	Specification(I)	*Specification(II)*	*Specification(III)*	*Specification(IV)*
Panel A: Variability of dynamic loading factors				
Std. of $\alpha_{1,t}$	0	7.206×10^{-4}	1.338×10^{-4}	0.483
Std. of $\alpha_{2,t}$	0	1.253×10^{-4}	2.432×10^{-4}	0.570
Panel B: Mean of conditional GIS				
$GIS_{s,t}$	0.558	0.513	0.507	0.497
$GIS_{f,t}$	0.442	0.487	0.493	0.503
Panel C: Volatility of conditional GIS				
Std. of $GIS_{f,t}$	0.039	0.018	0.011	0.005

Notes: This table reports the variability of estimated dynamic loading factors as well as the mean and volatility of estimated conditional GIS series. Note that the standard deviation of the conditional GIS of the cash index with respect to each specification is equal to that of the E-mini futures and thus not reported in this table. $\alpha_{1,t}$ refers to the conditional error correction coefficients for the S&P 500 cash index while $\alpha_{2,t}$ refers to the conditional error correction coefficients for the S&P E-mini futures. $GIS_{s,t}$ denotes the conditional GIS of the S&P 500 cash index; $GIS_{f,t}$ denotes the conditional GIS of the S&P E-mini futures. *Specification (I)* refers to the VECM with the constant error correction coefficients as in Equation (1); *Specification (II)* refers to the VECM with the dynamic error correction coefficients defined by Equation (28); *Specification (III)* refers to the VECM with the dynamic error correction coefficients defined by Equation (29); *Specification (IV)* refers to the VECM with the dynamic error correction coefficients defined by Equation (30). *Std.*, standard deviation; *GIS*, generalised information share.

Table 9. Model estimation of variability of autoregressive loading factors and GIS

	Eq.(33)		*Eq.(34)*		*Eq.(35)*
$\Lambda_0^{spec(II)}$	0.177^{***}	$\Lambda_0^{spec(III)}$	0.025^{***}	$\Lambda_0^{spec(IV)}$	0.039^{***}
	(5.66×10^{-2})		(9.32×10^{-4})		(9.84×10^{-4})
$\Lambda_1^{spec(II)}$	913.091^{***}	$\Lambda_1^{spec(III)}$	93.468^{***}	$\Lambda_1^{spec(IV)}$	0.004^{***}
	(126.58)		(3.33)		(9.21×10^{-5})
$\Lambda_2^{spec(II)}$	274.192^{***}	$\Lambda_2^{spec(III)}$	57.060^{***}	$\Lambda_2^{spec(IV)}$	0.003^{***}
	(71.35)		(2.24)		(6.61×10^{-5})
AIC	-1.146		-1.503		-2.988
SIC	-1.145		-1.503		-2.988
HQIC	-1.146		-1.503		-2.988

Notes: This table reports the estimation results of Equations (33), (34), and (35). $spec(II)$ denotes specification (II) that refers to the VECM with the dynamic error correction coefficients defined by Equation (28); $spec(III)$ denotes specification (III) that refers to the VECM with the dynamic error correction coefficients defined by Equation (29); $spec(IV)$ denotes specification (IV) that refers to the VECM with the dynamic error correction coefficients defined by Equation (30). AIC, SIC, and $HQIC$ refer to Akaike information criteria, Schwarz information criteria, Hannan-Quinn information criteria, respectively. Figures in parentheses are the Newey-West (1987) standard errors. ***, **, and * denotes significance at the 1%, 5% and 10% levels, respectively.