Balanced per capita contributions and levels structure of cooperation

Gómez-Rúa, María and Vidal-Puga, Juan

Universidade de Vigo, Universidade de Vigo

9 April 2008

Balanced per capita contributions and levels structure of cooperation*

María Gómez-Rúa
Departamento de Estatística e Investigación Operativa
Universidade de Vigo

Juan J. Vidal-Puga†
Research Group in Economic Analysis and
Departamento de Estatística e Investigación Operativa
Universidade de Vigo

April 9, 2008

Abstract

We define a new value for games with levels structure. We introduce a new property in this class of games, *balanced per capita contributions*, which is related with others in the literature. We provide an axiomatic characterization of this value using this new property.

Keywords: levels structure, value, balanced *per capita* contributions.

1 Introduction

In many real situations the agents cooperate in order to get a benefit. This situation can be modelled as a transferable utility (TU, for short) game in

*Financial support from the Spanish Ministerio de Ciencia y Tecnología and FEDER through grant SEJ2005-07637-C02-01/ECON and the Xunta de Galicia through grants PGIDIT06PXIC300184PN and PGIDIT06PXIB362390PR is gratefully acknowledged.

†Corresponding author. E-mail: **vidalpuga@uvigo.es**

which the players partition themselves into groups for the purpose of bargaining. These situations are modelled as TU games with coalition structure.

However, in many situations a coalition structure does not provide a complete description of the cooperation structure.

For instance, consider the members of the European Union Parliament. In this situation, even though all of them have the same rights, they do not act independently. The natural cooperation structure will be form by the parties. However, on a higher level, parties may associate according to their ideology in larger groups, such like the European People's Party (EPP), the European Democrats (ED), the Party of European Socialist (PES), etc. In an even higher level, the EPP and the ED form a larger group, the EPP-ED, and so other groups. This example appears in Winter (1989), Calvo et al. (1996) and Vidal-Puga (2005).

In these cases, a more detailed mapping of the cooperation structure is needed.

This cooperation description of the players is called a *levels structure.* There are several values in the literature that take into account the levels structure. For the particular case of one single level, Aumann and Drèze (1974) first proposed a value for this class of games. Owen (1977) defined a new value, the *Owen value.* Both values extend the Shapley value (Shapley, (1953b)). Other extensions are provided by Hamiache (2006) and Kamijo (2007). On the other hand, Levy and McLean (1989) proposed a value that is an extension of the weighted Shapley value (Kalai and Samet (1984)).

Winter (1989) defined a value, the *levels structure value,* first suggested by Owen in 1977. This value is an extension of the *Owen value* for several levels.

Calvo et al. (1996) provided a characterization of the *levels structure value* using the principle of *balanced contributions.* This property states that, for any two coalitions that belong to the same coalition at higher levels, the amount that the players in each coalition would gain or lose by the other's coalition withdrawal from the game should be equal.

Nevertheless, when the coalitions represent groups of different size, this symmetry among coalitions may not be always a reasonable requirement for a value. See, for instance, Levy and McLean (1989) or Kalai and Samet (1987).

Vidal-Puga (2006) defined a value, ζ for games with a unique level of cooperation taking into account the asymmetry between the coalitions due to their different size.

In this paper we extend the value ζ for games with levels structure. Moreover we introduce a new property for this kind of games, *balanced per capita contributions,* that is related to another property proposed by Myerson (1980) and also studied by Hart and Mas-Colell (1989), Sánchez (1997) and Calvo and Santos (2000).

The property of *balanced per capita contributions* states that for any two coalitions that belong to the same coalition at higher levels, the *average* amount that the players in each coalition would gain or lose by the other's coalition withdrawal from the game should be equal. The average is taken over the number of single agents in each coalition.

A similar axiom was introduced in Herings et al. (2005) in the context of *cycle-free graph games*.

We also provide a characterization of the new value using this property.

The paper is organized as follows. In Section 2 we introduce the model. In Section 3 we introduce a new value for this class of games. In Section 4 we define the property of *balanced per capita contributions*. Moreover we prove that the new value satisfies this property. In Section 5 we provide a characterization of the value. In Section 6 we compare our results with those presented by Calvo et al. (1996).

2 The model

Let $U = \{1, 2, ...\}$ be a (may be infinite) set of potential players.

A game with transferable utility, TU game, is a pair (N, v), where $N \subset U$ is finite and $v : 2^N \to \mathbb{R}$ satisfies $v(\emptyset) = 0$. We denote by $TU(N)$ the set of TU games with N as player set. If $S \subset N$, we denote by (S, v) the restriction of v to the player set S.

Given $N \subset U$ finite, we call *coalition structure* on N a partition of the player set N, *i.e.* $\mathcal{C} = \{C_1, C_2, ..., C_m\} \subset 2^N$ is a coalition structure if it satisfies $\bigcup_{C_q \in \mathcal{C}} C_q = N$ and $C_q \cap C_r = \emptyset$ when $q \neq r$. We also assume $C_q \neq \emptyset$ for all q. Given $S \subset N$, we denote by $\mathcal{C}_S$ the coalition structure restricted to S, *i.e.* $\mathcal{C}_S = \{C_q \cap S : C_q \in \mathcal{C},\ C_q \cap S \neq \emptyset\}$.

A *levels structure* for N is a sequence $\mathfrak{C} = (\mathcal{C}^0, \mathcal{C}^1, ..., \mathcal{C}^h)$, $h \geq 1$ with $\mathcal{C}^l$ $(0 \leq l \leq h)$ coalition structure on N such that:

1. $\mathcal{C}^0 = \{\{1\}, \{2\}, ..., \{n\}\}$.
2. $\mathcal{C}^h = \{N\}$.

3. If $C_q^l \in \mathcal{C}^l$ with $0 < l \leq h$ then $C_q^l = \bigcup_{S \in \mathcal{Q}} S$ for some $\mathcal{Q} \subset \mathcal{C}^{l-1}$.

We call $\mathcal{C}^l$ the *l-th level* of $\mathfrak{C}$. We say that $\mathfrak{C}$ is a levels structure of *degree* h. Hence, the levels structure $\mathfrak{C}$ has $h+1$ levels.

If $h = 1$, we say that $\mathfrak{C}$ is a *trivial* levels structure.

Given $\mathcal{C}^l \in \mathfrak{C}$, we define $\mathfrak{C}/\mathcal{C}^l$ as the levels structure induced from $\mathfrak{C}$ by considering the coalitions in $\mathcal{C}^l$ as players.

Let LTU be the set of all $(N, v, \mathfrak{C})$ with $(N, v) \in TU(N)$ and $\mathfrak{C}$ levels structure for N. When the levels structure is clear, we may write (N, v) or v instead of $(N, v, \mathfrak{C})$.

Denote $\mathcal{C}^l = \{C_1^l, ..., C_{m(l)}^l\}$ and let $N^l = \{1, ..., m(l)\}$. The *quotient game* $(N^l, v/\mathcal{C}^l, \mathfrak{C}/\mathcal{C}^l)$ is the game in LTU defined on the coalition structure $\mathcal{C}^l$ with characteristic function

$$(v/\mathcal{C}^l)(Q) = v\left(\bigcup_{q \in Q} C_q^l\right)$$

for all $Q \subset N^l$.

A *value* in LTU is a function f that assigns to each $(N, v, \mathfrak{C}) \in LTU$ a vector $f(N, v, \mathfrak{C}) \in \mathbb{R}^N$. As usual, $f_i(N, v, \mathfrak{C})$ represents the payoff received by player $i \in N$.

For any $(N, v, \mathfrak{C}) \in LTU$, a value f is *efficient* if

$$\sum_{i \in N} f_i(N, v, \mathfrak{C}) = v(N).$$

One of the most important values in TU games is the *Shapley value* (Shapley (1953b)). We denote the Shapley value of the TU game (N, v) as $Sh(v) \in \mathbb{R}^N$.

A nonsymmetric generalization of the Shapley value is the *weighted Shapley value* (Shapley (1953a), Kalai and Samet (1987, 1988)). Given a vector of weights $\omega \in \mathbb{R}_{++}^N$, we denote the weighted Shapley value as $Sh^\omega(v) \in \mathbb{R}^N$.

3 A new value

Winter (1989) defined *the levels structure value (LSV)*, that is an extension of the *Owen value* for this kind of games.

One of the properties that satisfies the LSV is *coalitional symmetry* (Winter, 1989, page 229). This property ensures that if two coalitions are symmetric in the quotient game and furthermore they belong to the same coalition in the next level, their members should receive the same aggregate amount. There are several authors in the literature that claim that when the coalitions represent groups of different size, this symmetry may not be a reasonable requirement. See, for instance, Levy and McLean (1989) or Kalai and Samet (1987). In particular, the latter claimed that in this case, it seems reasonable to assign a size-depending weight to each coalition.

The value ζ presented by Vidal-Puga (2006) for TU games with coalition structure takes this idea into account.

Now we extend the value ζ for games with levels structure. The intuitive idea of this value is as follows: In a first stage, we distribute $v(N)$ among the coalitions of the $(h-1)$-*th* level through the weighted Shapley value with weights given by the size of the coalitions. Then, for any $C_q^{h-1} \in \mathcal{C}^{h-1}$, we distribute the payment received by C_q^{h-1} in the first stage among all the coalitions in $\mathcal{C}^{h-2}$ that belong to C_q^{h-1}. Again we do it through the weighted Shapley value. At the last stage, we distribute the payment received by the coalitions in $\mathcal{C}^1$ among the agents.

The formal definition is as follows:

Define the TU game $\left(N, v_N^{Nh}\right)$ as (N, v). Assume we have defined the TU game $(C_p^k, v_{C_p^k}^{Nk})$ for $k > l$ and $p \in N^k$ and moreover, $\sum\limits_{p \in N^k} v_{C_p^k}^{Nk}(C_p^k) = v(N)$. Given $q \in N^l$, we define a new TU game $(C_q^l, v_{C_q^l}^{Nl})$. Take C_s^{l+1} such that $C_q^l \subset C_s^{l+1}$. Let $T \subset C_q^l$. We will define $v_{C_q^l}^{Nl}(T)$. Let $\delta \in \mathbb{R}_{++}^{N^l}$ be defined as $\delta_q = |T|$ and $\delta_r = \left|C_r^l\right|$ for $r \neq q$. We define:

$$v_{C_q^l}^{Nl}(T) = Sh_q^\delta \left(v_{C_s^{l+1}}^{N(l+1)} / \mathcal{C}^l_{(C_s^{l+1} \setminus C_q^l) \cup T} \right)$$

for all $T \subset C_q^l$.

It follows from this definition and the induction hypothesis that

$$\sum_{p \in N^l} v_{C_p^l}^{Nl}(C_p^l) = v(N).$$

Then, we define:

$$\phi_i(N, v, \mathfrak{C}) = v_{\{i\}}^{N0}(\{i\})$$

for all $i \in N$.

If $h = 1$, both LSV and ϕ coincide with Sh. For $h = 2$, the LSV coincides with the *Owen value* and ϕ coincides with ζ.

4 Balanced *per capita* contributions

Myerson (1980) defined the following properties:

Definition 1 *A value f satisfies* Balanced Individual Contributions[1] *(BIC) if and only if*

$$f_i(N, v) - f_i(N \backslash \{j\}, v) = f_j(N, v) - f_j(N \backslash \{i\}, v)$$

for all $i, j \in N$ and all $(N, v) \in TU$.

Definition 2 *Let (N, v) be a TU game. Let $\alpha \in \mathbb{R}^N_{++}$. A value f satisfies* α-Balanced Individual Contributions[2] *(α-BIC) if and only if*

$$\frac{f_i(N, v) - f_i(N \backslash \{j\}, v)}{\alpha_i} = \frac{f_j(N, v) - f_j(N \backslash \{i\}, v)}{\alpha_j}$$

for all $i, j \in N$.

Myerson (1980, Lemma 6) proved that, given $\alpha \in \mathbb{R}^N_{++}$, there exists a unique efficient value satisfying *α-balanced individual contributions.*

Hart and Mas-Colell (1989) showed that this family of values coincides with the family of weighted Shapley values. See also Sánchez (1997) and Calvo and Santos (2000).

Proposition 3 *(Hart and Mas-Colell, 1989, page 604): For any $\alpha \in \mathbb{R}^N_{++}$, Sh^α is the only efficient value that satisfies α-BIC.*

Calvo et al. (1996) extended BIC to the context of games with levels structure as follows:

[1] Myerson called it Balanced Contributions.

[2] Myerson used the equivalent notation λ-*Balanced Contributions* with $\lambda = \frac{1}{\alpha}$, where $\left(\frac{1}{\alpha}\right)_i = \frac{1}{\alpha_i}$.

Definition 4 *A value f satisfies* Balanced Group Contributions[3] (BGC) *if for all $C_q^l, C_r^l \in \mathcal{C}^l$ such that $C_q^l, C_r^l \subset C_k^{l+1} \in \mathcal{C}^{l+1}$ $(l = 0, 1, ..., h-1)$, we have*

$$\sum_{i \in C_q^l} f_i(N, v) - \sum_{i \in C_q^l} f_i(N \backslash C_r^l, v) = \sum_{i \in C_r^l} f_i(N, v) - \sum_{i \in C_r^l} f_i(N \backslash C_q^l, v)$$

for all $(N, v, \mathfrak{C}) \in LTU$.

This property states that for any two coalitions, C_q^l and C_r^l that belong to the same coalitions at higher levels, the contributions of C_q^l to the total payoff of the members in C_r^l must be equal to the contribution of C_r^l to the total payoff of the members in C_q^l.

Calvo et al. (1996) characterized the *levels structure value* with the property of *BGC.*

The equivalent of α-BIC for the context of the TU games with levels structure, with $\alpha_q = \left|C_q^l\right|$ for all $C_q^l \in \mathcal{C}^l$ and all $l = 0, 1, ..., h-1$, is the following:

Definition 5 *A value f satisfies* Balanced Per Capita Contributions (BPCC) *if for all $C_q^l, C_r^l \in \mathcal{C}^l$ such that $C_q^l, C_r^l \subset C_k^{l+1} \in \mathcal{C}^{l+1}$ $(l = 0, 1, ..., h-1)$, we have*

$$\frac{\sum_{i \in C_q^l} f_i(N, v) - \sum_{i \in C_q^l} f_i(N \backslash C_r^l, v)}{\left|C_q^l\right|} = \frac{\sum_{i \in C_r^l} f_i(N, v) - \sum_{i \in C_r^l} f_i(N \backslash C_q^l, v)}{\left|C_r^l\right|}$$

for all $(N, v, \mathfrak{C}) \in LTU$.

This property ensures that for any two coalitions C_q^l and C_r^l that belong to the same coalitions at higher levels, the change *per capita* in the payoffs of the players in C_q^l if C_r^l leaves the game should be equal to the change *per capita* in the payoffs of the players in C_r^l if C_q^l leaves the game.

Proposition 6 *ϕ is efficient and satisfies $BPCC$.*

Proof. *It is straightforward to check that ϕ is efficient.*

Now we prove that ϕ satisfies $BPCC$: Fix $(N, v, \mathfrak{C})$ with $\mathfrak{C} = \{\mathcal{C}^0, ..., \mathcal{C}^h\}$. Given $C_k^{l+1} \in \mathcal{C}^{l+1}$, let $C_q^l, C_r^l \in \mathcal{C}^l$ such that $C_q^l, C_r^l \subset C_k^{l+1}$.

[3] Calvo et al. called it Balanced Contributions.

By definition of ϕ*, we have that*

$$\sum_{i \in C_q^l} \phi_i(N, v) = Sh_q^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1}}^l \right)$$

and

$$\sum_{i \in C_q^l} \phi_i(N \backslash C_r^l, v) = Sh_q^\delta \left(v_{C_k^{l+1} \backslash C_r^l}^{(N \backslash C_r^l)(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_r^l}^l \right).$$

Hence,

$$\begin{aligned} & \sum_{i \in C_q^l} \phi_i(N, v) - \sum_{i \in C_q^l} \phi_i(N \backslash C_r^l, v) \\ = \quad & Sh_q^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1}}^l \right) - Sh_q^\delta \left(v_{C_k^{l+1} \backslash C_r^l}^{(N \backslash C_r^l)(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_r^l}^l \right). \end{aligned}$$

By definition, $v_{C_k^{l+1}}^{N(l+1)}(T) = v_{C_k^{l+1} \backslash C_r^l}^{(N \backslash C_r^l)(l+1)}(T)$ *for all* $T \subset C_k^{l+1} \backslash C_r^l$*, hence we have that* $v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_r^l}^l = v_{C_k^{l+1} \backslash C_r^l}^{(N \backslash C_r^l)(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_r^l}^l$*, and so expression above can be restated as:*

$$\begin{aligned} & \sum_{i \in C_q^l} \phi_i(N, v) - \sum_{i \in C_q^l} \phi_i(N \backslash C_r^l, v) \\ = \quad & Sh_q^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1}}^l \right) - Sh_q^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_r^l}^l \right). \end{aligned}$$

Analogously,

$$\begin{aligned} & \sum_{i \in C_r^l} \phi_i(N, v) - \sum_{i \in C_r^l} \phi_i(N \backslash C_q^l, v) \\ = \quad & Sh_r^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1}}^l \right) - Sh_r^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_q^l}^l \right). \end{aligned}$$

Since Sh^δ *satisfies* δ*-BIC (Proposition 3),*

$$\begin{aligned} & \frac{Sh_q^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1}}^l \right) - Sh_q^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_r^l}^l \right)}{\delta_q} \\ = \quad & \frac{Sh_r^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1}}^l \right) - Sh_r^\delta \left(v_{C_k^{l+1}}^{N(l+1)} / \mathcal{C}_{C_k^{l+1} \backslash C_q^l}^l \right)}{\delta_r}. \end{aligned}$$

Moreover, by definition of δ, $\delta_q = \left|C_q^l\right|$ *and* $\delta_r = \left|C_r^l\right|$, *thus we obtain the result.* ■

5 Characterization

In this Section we provide a characterization of the value ϕ using the property of *Balanced Per Capita Contributions* introduced in the previous section.

Theorem 7 *An efficient value* f *over the set of games with level structure satisfies BPCC if and only if* $f = \phi$.

Proof. *Let* $(N, v, \mathfrak{C}) \in LTU$ *and suppose there exist two efficient values* f^1 *and* f^2 *satisfying BPCC. We will prove that* $f^1_{C_q^l}(N, v) = f^2_{C_q^l}(N, v)$ *for all* $l \in \{0, ..., h\}$ *and* $C_q^l \in \mathcal{C}^l$, *where* $f^x_{C_q^l}(N, v) := \sum_{i \in C_q^l} f_i^x(N, v)$, $x = 1, 2$. *This is enough to prove the result because the* C_q^l *are singletons for* $l = 0$.

Note that by efficiency,

$$\sum_{C_q^l \in \mathcal{C}^l} f^x_{C_q^l}(N, v) = v(N)$$

for $x = 1, 2$.

We will prove the result by induction on the level. Consider level h, $\mathcal{C}^h = \{N\}$. *Since* f^1 *and* f^2 *are efficient, we have that* $f^1_N(v) = f^2_N(v) = v(N)$.

Let us assume that the result holds for level k, $k \geq l$, *i.e.*

$$f^1_{C_q^k}(v) = f^2_{C_q^k}(v)$$

for all $C_q^k \in \mathcal{C}^k$ *with* $l \leq k \leq h$.

Let $C_q^l \in \mathcal{C}^l$. *Denote* $\mathcal{Q}^{l-1} := \{C_r^{l-1} \in \mathcal{C}^{l-1} : C_r^{l-1} \subset C_q^l\}$.

We use an induction argument on the cardinal of $\mathcal{Q}^{l-1}$.

Assume that $\left|\mathcal{Q}^{l-1}\right| = 1$, *say* $\mathcal{Q}^{l-1} = \{C_q^{l-1}\}$. *Hence,* $\mathcal{Q}^{l-1} = \{C_q^l\}$ *and by induction hypothesis:*

$$f^1_{C_q^{l-1}}(v) = f^1_{C_q^l}(v) = f^2_{C_q^l}(v) = f^2_{C_q^{l-1}}(v).$$

Assume that the result holds for $\left|\mathcal{Q}^{l-1}\right| = m - 1$. *Now we prove that it holds for* $\left|\mathcal{Q}^{l-1}\right| = m$.

Suppose that $\left|\mathcal{Q}^{l-1}\right| = m$, *say* $\mathcal{Q}^{l-1} = \{C_1^{l-1}, ..., C_m^{l-1}\}$. *Let* $M = \{1, ..., m\}$.

Let $C_r^{l-1}, C_s^{l-1} \in \mathcal{Q}^{l-1}$. *By BPCC,*

$$\frac{f^1_{C_r^{l-1}}(N,v) - f^1_{C_r^{l-1}}(N \backslash C_s^{l-1}, v)}{|C_r^{l-1}|} = \frac{f^1_{C_s^{l-1}}(N,v) - f^1_{C_s^{l-1}}(N \backslash C_r^{l-1}, v)}{|C_s^{l-1}|} \tag{1}$$

and

$$\frac{f^2_{C_r^{l-1}}(N,v) - f^2_{C_r^{l-1}}(N \backslash C_s^{l-1}, v)}{|C_r^{l-1}|} = \frac{f^2_{C_s^{l-1}}(N,v) - f^2_{C_s^{l-1}}(N \backslash C_r^{l-1}, v)}{|C_s^{l-1}|}. \tag{2}$$

Moreover, by induction hypothesis on $\left|\mathcal{Q}^{l-1}\right|$ *we have that* $f^1_{C_r^{l-1}}(N \backslash C_s^{l-1}, v) = f^2_{C_r^{l-1}}(N \backslash C_s^{l-1}, v)$ *and* $f^1_{C_s^{l-1}}(N \backslash C_r^{l-1}, v) = f^2_{C_s^{l-1}}(N \backslash C_r^{l-1}, v)$.

Taking into account these expressions and operating with (1) and (2), we have that

$$\frac{f^1_{C_r^{l-1}}(N,v) - f^2_{C_r^{l-1}}(N,v)}{|C_r^{l-1}|} = \frac{f^1_{C_s^{l-1}}(N,v) - f^2_{C_s^{l-1}}(N,v)}{|C_s^{l-1}|}$$

and so,

$$f^1_{C_r^{l-1}}(N,v) - f^2_{C_r^{l-1}}(N,v) = \frac{\left|C_r^{l-1}\right|}{\left|C_s^{l-1}\right|} \left[f^1_{C_s^{l-1}}(N,v) - f^2_{C_s^{l-1}}(N,v) \right]. \tag{3}$$

Applying the induction hypothesis on levels, we have that $f^1_{C_q^l}(N,v) = f^2_{C_q^l}(N,v)$. *That is,*

$$\sum_{C_p^{l-1} \in \mathcal{Q}^{l-1}} f^1_{C_p^{l-1}}(N,v) = \sum_{C_p^{l-1} \in \mathcal{Q}^{l-1}} f^2_{C_p^{l-1}}(N,v).$$

Therefore,

$$\begin{aligned} 0 &= \sum_{C_p^{l-1} \in \mathcal{Q}^{l-1}} f^1_{C_p^{l-1}}(N,v) - \sum_{C_p^{l-1} \in \mathcal{Q}^{l-1}} f^2_{C_p^{l-1}}(N,v) \qquad (4) \\ &= f^1_{C_1^{l-1}}(N,v) + ... + f^1_{C_m^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v) - .... - f^2_{C_m^{l-1}}(N,v) \\ &= \left(f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v) \right) + ... + \left(f^1_{C_m^{l-1}}(N,v) - f^2_{C_m^{l-1}}(N,v) \right). \end{aligned}$$

Taking $s = 1$ in equation (3), we deduce that

$$f^1_{C_r^{l-1}}(N,v) - f^2_{C_r^{l-1}}(N,v) = \frac{\left|C_r^{l-1}\right|}{\left|C_1^{l-1}\right|}\left[f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v)\right] \qquad (5)$$

for all $C_r^{l-1} \in \mathcal{Q}^{l-1} \backslash C_1^{l-1}$.

Replacing these expressions in (4),

$$\begin{aligned} 0 &= \left(f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v)\right) + \frac{\left|C_2^{l-1}\right|}{\left|C_1^{l-1}\right|}\left[f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v)\right] \\ &\quad + \ldots\ldots + \frac{\left|C_m^{l-1}\right|}{\left|C_1^{l-1}\right|}\left[f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v)\right] \\ &= \left(f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v)\right)\frac{\left|C_1^{l-1}\right| + \left|C_2^{l-1}\right| + \ldots + \left|C_m^{l-1}\right|}{\left|C_1^{l-1}\right|}. \end{aligned}$$

But by definition,

$$\frac{\left|C_1^{l-1}\right| + \left|C_2^{l-1}\right| + \ldots + \left|C_m^{l-1}\right|}{\left|C_1^{l-1}\right|} > 0,$$

therefore, $f^1_{C_1^{l-1}}(N,v) - f^2_{C_1^{l-1}}(N,v) = 0$ *and so,*

$$f^1_{C_1^{l-1}}(N,v) = f^2_{C_1^{l-1}}(N,v).$$

Replacing this expression in (5) we conclude that

$$f^1_{C_r^{l-1}}(N,v) = f^2_{C_r^{l-1}}(N,v)$$

for all $C_r^{l-1} \in \mathcal{Q}^{l-1}$.

Using the same induction argument for any level t with $0 \leq t \leq l-1$ we obtain that $f^1 = f^2$. ■

6 Concluding remarks

As opposed to Calvo et al. (1996), who characterized the *LSV* using the property of Balanced Group Contributions, we provide a characterization of a new value using its *per capita* version. The advantage of these characterizations is that they do not use properties of additivity nor consistency. Hence, the characterization results still hold for many relevant subfamilies of TU games, such as the family of simple games, totally balanced games, or games with a single coalition structure. In this latter case, the property of balanced *per capita* contributions reduces to two conditions: one of them is the property of Balanced Individual Contributions for the members of the same Coalition (BICC); formally

$$f_i(N,v) - f_i(N\backslash\{j\},v) = f_j(N,v) - f_j(N\backslash\{i\},v)$$

for all i,j that belong to the same coalition C_q. Following Calvo et al. (1996), the Owen value is the only efficient value that satisfies this property and Balanced Individual Contributions in the game between coalitions, formally

$$\sum_{i\in C_q} f_i(N,v) - \sum_{i\in C_q} f_i(N\backslash C_r,v) = \sum_{i\in C_r} f_i(N,v) - \sum_{i\in C_r} f_i(N\backslash C_q,v)$$

for all distinct coalitions C_q, C_r.

As opposed, ζ is the only efficient value satisfying BICC and balanced *per capita* contributions among coalitions, formally

$$\frac{\sum_{i\in C_q} f_i(N,v) - \sum_{i\in C_q} f_i(N\backslash C_r,v)}{|C_q|} = \frac{\sum_{i\in C_r} f_i(N,v) - \sum_{i\in C_r} f_i(N\backslash C_q,v)}{|C_r|}$$

for all distinct coalitions C_q, C_r.

References

[1] Aumann, R. and Drèze, J. (1974) *Cooperative games with coalition structure.* International Journal of Game Theory 3, 217–237.

[2] Calvo, E., Lasaga, J. and Winter, E. (1996) *The principle of balanced contributions and hierarchies of cooperation.* Mathematical Social Sciences 31, 171-182.

[3] Calvo, E. and Santos, J.C. (2000) *Weighted weak semivalues.* International Journal of Game Theory 29, 1-9.

[4] Hamiache G. (2006) *A value for games with coalition structures.* Social Choice and Welfare 26, 93-105.

[5] Hart, S. and Kurz, M. (1983) *Endogenous formation of coalitions.* Econometrica 51 (4), 1047 1064.

[6] Hart, S. and Mas-Colell, A. (1989) *Potencial, value and consistency.* Econometrica 57 (3), 589-614.

[7] Herings, P.J.J., van der Laan, G. and Talman, D. *The component fairness solution for cycle-free graph games.* CentER Discussion paper 2005-127.

[8] Kalai, E., and Samet, D. (1987) *On weighted Shapley values.* International Journal of Game Theory, 16, 205-222.

[9] Kalai, E., and Samet, D. (1988) *Weighted Shapley values.* In: A. Roth, ed., The Shapley value: Essays in honor of L. Shapley (Cambrige), Cambrige University Press, 83-99.

[10] Kamijo Y. (2007) *A collective value: a new interpretation of a value and a coalition structure. W*aseda University. 21 COE-GLOPE Working Paper Series 27.

[11] Levy, A. and McLean, R.P. (1989) *Weighted coalition structure values.* Games and Economic Behavior 1, 234-249.

[12] Myerson, R.B. (1980) *Conference structures and fair allocation rules.* International Journal of Game Theory 9, 169-182.

[13] Owen, G. (1977) *Values of games with a priori unions.* In: Henn, R., Moeschlin, O. (eds.) Essays in mathematical economics and game theory (Berlin) Heidelberg NewYork: Springer, 76-88.

[14] Pérez-Castrillo, D. and Wettstein, D. (2001) *Bidding for the surplus: A non-cooperative approach to the Shapley value.* Journal of Economic Theory 100, 274-294.

[15] Sánchez, F. (1997) *Balanced contributions axiom in the solution of cooperative games.* Games and Economic Behavior 20, 161-168.

[16] Shapley, L. S. (1953a) *Additive and Non-Additive Set Functions.* Ph.D. Dissertation, Princeton University.

[17] Shapley, L. S. (1953b) *A value for n-person games.* In: Huhn, H.W.,Tucker, A.W. (eds.) Contributions to the theory of games II. Annals of Mathematics Studies 28, pp. 307–317. Princeton: Princeton University Press.

[18] Vidal-Puga, J. J. (2005) *Implementation of the levels structure value.* Annals of Operations Research 137(1), 191-209.

[19] Vidal-Puga, J. J. (2006) *The Harsanyi paradox and the "right to talk" in bargaining among coalitions.* EconWPA Working paper number 0501005. Available at http://ideas.repec.org/p/wpa/wuwpga/0501005.html.

[20] Winter, E. (1989) *A value for cooperative games with level structure of cooperation.* International Journal of Game Theory 18, 227-240.