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Coordination Frictions and Economic Growth*

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Abstract

In practice, firms face a mass of scarce innovation projects. They choose a particular research avenue towards which to direct their effort, but do not coordinate these choices. This gives rise to coordination frictions. Our paper develops an expanding-variety endogenous growth model to study the impact of these frictions on the economy. The coordination failure generates a mass of foregone innovation and reduces the economy-wide research intensity. Both of these effects decrease the growth rate. Because of this, the frictions also amplify the fraction of wasteful simultaneous innovation. A numerical exercise suggests that the impact of coordination frictions on both the growth rate and welfare is substantial.

Keywords: Growth, Frictions, Coordination, Simultaneous Innovation, Search for Ideas.

JEL Codes: O30, O31, O32, O33, O40.

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1 Introduction

Innovators have technological access to many distinct research avenues (ideas).[1] However, it is often the case that several firms engage in an innovation race for the exact same idea, i.e. research avenues are scarce. In particular, Lemley (2011) details anecdotal evidence that virtually every major historical innovation (such as the cotton gin, the steam engine, the computer, and the laser) has been simultaneously innovated by several groups of researchers. Perhaps the most famous example is that of the Alexander Bell and Elisha Gray telephone controversy. On February 14, 1876 Bell filed a patent application for the telephone and only hours later Gray submitted a similar application for the exact same innovation. Furthermore, the same empirical regularity is observed for non-major innovations. Cohen and Ishii (2005) find that a positive fraction of patents for the period between 1988 and 1996 were declared in interference.[2] More recent examples of simultaneous innovation include companies such as Siemens, Philips, Google Inc., Microsoft Corporation, and Yahoo! Inc.[3]

Furthermore, coordination of research efforts by firms (firm A directs its effort towards project 1, firm B towards project 2, and so on) is very unlikely in this setting because of two main reasons. First, the size of the "market" for ideas makes coordination very hard to achieve. Second, such coordination requires each firm to know the portfolio of research projects of all of its rivals. This is particularly implausible in the current context given that firms actively employ secrecy as an intellectual property protection mechanism.[4]

[1]For example, during 2015 the U.S. Patent and Trademark Office granted more than quarter of a million patents.

[2]Patents are declared in interference if two innovators file for the same patent within three months of each other (six months for major innovations).

[3]Siemens applied for a patent for a positron emission tomography scanner on April 23, 2013 (application number 13/868,256). Most claims are rejected because Philips (application number 14/009,666 filed on March 29, 2012 and application number 14/378,203 filed on February 25, 2013) had simultaneously made similar innovations. Google Inc. filed a patent application on November 1 2012 (number 13/666,391) for methods, systems, and apparatus that provide content to multiple linked devices. All twelve claims contained in the application are rejected because of simultaneous innovations made by Yahoo! Inc. (application number 13/282,180 with filing date October 26, 2011), Microsoft Corporation (application number 13/164,681 with filing date June 20, 2011), and Comscore Inc. (application number 13/481,474 with filing date May 25, 2012). The information on the patent applications is taken from the U.S. Patent and Trademark Office Patent Application Information Retrieval.

[4]For a survey of the evidence see, for example, Hall *et al.* (2014).

Motivated by these observations, we develop an expanding-variety endogenous growth model that features scarce research avenues and lack of research effort coordination. The paper examines the impact of these coordination frictions on firms' decision to undertake R&D activities as well as their aggregate consequences. We also study the implications of the frictions for the planner's allocation. Furthermore, we gauge the importance of the coordination problems for growth and welfare in a numerical exercise.

In our model, R&D firms direct their research efforts towards a particular project out of an endogenously determined mass of ideas. If innovated, each idea is transformed into one new variety. Firms which secure a patent over a variety produce. We focus on the symmetric equilibrium where firms use identical mixed strategies when directing their R&D efforts, so as to highlight their inability to coordinate. Thus, each idea is innovated by a random number of firms with mean equal to the tightness in the market for ideas (the ratio of firms to ideas). Knowledge is cumulative — each innovated idea allows firms to "stand on the shoulders of giants" and gain technological access to a number of new research projects. This intertemporal spillover effect is the ultimate source of growth in our economy — an expanding mass of ideas permanently alleviates future congestion problems, thus reducing the cost of discovering new varieties. Along the balanced growth path (BGP henceforth), the growth rate of the economy is determined by the growth rate of the mass of ideas, which is in turn endogenously determined by the market tightness and the coordination problems.

The frictions in our model have a direct impact on the growth rate. Firms cannot coordinate their efforts, so they unintentionally gravitate towards the same research projects. This leaves a mass of profitable ideas uninnovated each period. As a result, the growth rate of the decentralized frictional economy (DE henceforth) is lower, as compared to a hypothetical economy in which firms can coordinate their efforts (CE henceforth). At the same time, due to a general equilibrium effect, the frictions amplify the fraction of wasteful simultaneous innovation.[5] Due to the lower growth rate firms discount future profit streams at a lower rate. This increases the value of holding a patent and, in equilibrium, induces more congestion

[5]Since only one firm can obtain a patent over a particular variety, the R&D investment by all other rivals who innovate simultaneously represents wasteful duplication of effort.

in the market for ideas. This higher congestion, in turn translates to a higher fraction of wasteful innovation. Furthermore, for any market tightness, the coordination frictions reduce firms' probability of securing a monopoly position. Given a market tightness, the ratio of innovations to ideas is the same for both the DE and the CE. In the DE, however, there is a mass of foregone innovation. Hence, a lower fraction of these innovations are distinct which leads to a lower number of patents to be distributed among firms. This reduced probability of securing a patent induces firms to decrease their entry into the R&D sector, leaving the DE with a lower R&D intensity (market tightness). As a result, the DE growth rate is decreased even further.

The planner's second-best allocation (SB henceforth) also features positive fractions of foregone innovation and wasteful simultaneous innovation. The planner can choose the mass of R&D entrants, but she cannot assign firms to projects. When the market tightness is low, so is the fraction of wasteful innovation, but many innovations are foregone. When the tightness is high, foregone innovation is low, but many firms make a wasteful duplication of effort. Thus, at the margin the planner chooses a tightness that strikes a balance between these two effects. The SB research intensity may be higher or lower than the one in the first-best allocation (FB henceforth). This is the case because in the FB, the planner can assign firms to projects and as a consequence she does not face the same trade-off. Thus, she sets the first-best tightness to unity.

The frictions in our model impact welfare negatively through two channels: they (i) generate a mass of foregone innovation and (ii) amplify the fraction of wasteful innovation. In the benchmark calibration, eliminating the frictions in the DE leads to a 13% welfare gain (in consumption equivalent terms). The DE growth rate is only 2/3 of the CE one, so the welfare cost of foregone innovation is 10.35%. Coordination problems increase the fraction of wasteful innovation by 8pp (to 39%), which translates to a 2.65% welfare cost. Moreover, if the planner could eliminated the frictions and assign the FB, she would achieve welfare 16.15% higher than that in the SB. However, only 5.66pp of the gain is due to eliminating foregone innovation. This is because of two reasons. First, the SB features a much smaller fraction of foregone innovation than the DE. Second, removing the frictions in the SB reduces the fraction of wasteful duplication of effort from 52% to 0 since the FB does not suffer from the over-investment present in CE.

Relationship to the literature. Our paper models firms' choice of direction for their R&D efforts and the coordination problems inherent in this decision. As such, it is related to a recent literature on economic growth which emphasizes matching and other frictions in the innovation process (see, for example, Perla and Tonetti (2014), Lucas and Moll (2014), Benhabib *et al.* (2014), Chiu *et al.* (2015), and Akcigit *et al.* (2016)). The work here complements that literature by examining a different source of friction. In particular, to the best of my knowledge, this is the first growth paper to emphasize search frictions in the market for ideas which take the form of a coordination failure. Previous growth models have focused instead on a search process which takes the form of arrival rate of innovations, a McCall-type search for innovations, or frictions in the market for innovations.[6]

The theoretical model in this paper differs from the existing literature on economic growth in a number of additional dimensions. First, our analysis emphasizes firms' choice of research avenues by explicitly modeling the mass of available ideas. In particular, we make a distinction between potential innovations (ideas) and actual innovations.[7] Second, our model features a scarce mass of potential research projects such as, for example, Grossman and Helpman (1991) and Klette and Kortum (2004).[8] Unlike those studies, our paper explicitly models the decision of firms to direct their R&D activities and emphasizes the coordination

[6]Papers which feature search as arrival rate of innovations include Aghion and Howitt (1992), Grossman and Helpman (1991), and Klette and Kortum (2004). Kortum (1997), Perla and Tonetti (2014), and Lucas and Moll (2014), among others, feature a McCall-type search for heterogeneous technologies. For papers which focus on frictions in the market for innovations see, for example, Chiu *et al.* (2015) and Akcigit *et al.* (2016). It is worth noting that Chiu *et al.* (2015) and Akcigit *et al.* (2016) do not make a distinction between ideas and innovations. In particular, the market for ideas in our paper (firms searching for a potential R&D project) is different from the "market for ideas" in Chiu *et al.* (2015) and Akcigit *et al.* (2016) where firms search for opportunities to trade the property rights over an innovation.

[7]This is in contrast to the previous literature on economic growth (Jones, 1995, 2002; Jones and Kim, 2014; Chiu *et al.*, 2015; Akcigit *et al.*, 2016; Bloom *et al.*, 2016) which has used ideas and innovations interchangeably.

[8]In contrast, some previous studies (Romer, 1990; Corriveau, 1994, 1998; Kortum, 1997) have examined models which feature an abundance of research avenues, whereas others (Aghion and Howitt, 1992; Segerstrom *et al.*, 1990) have examined models where a single avenue of research is available. For a recent review of the literature see, for example, Aghion *et al.* (2014).

frictions inherent in this problem.[9] Third, in contrast to the previous literature, this paper features an endogenously determined mass of ideas. Fourth, in our paper firms compete for ideas through their choice of research avenue. This competition is different than the competition firms face in the product market or the innovation race which the previous literature has examined.[10]

Within the literature on industrial organization the two closest papers are Kultti *et al.* (2007) and Kultti and Takalo (2008) which also feature search frictions in the market for ideas. In these papers there is the possibility of simultaneous innovation due to a matching technology which is the same as the equilibrium one in our paper. Kultti *et al.* (2007) and Kultti and Takalo (2008) focus on intellectual property rights in a partial equilibrium framework with a fixed mass of ideas and without free entry into the innovation sector. In contrast, our model focuses on a general equilibrium framework with growth, an endogenously determined mass of ideas, and an endogenously determined market tightness through free entry in the R&D sector.

The rest of the paper is organized as follows. Section two introduces the environment and characterizes the decentralized equilibrium. Section three examines the social planner's second-best allocation. Section four highlights the impact of coordination frictions in our model. Section five presents a numerical exercise. Section six concludes.

2 The Economy

The environment is an augmented, discrete time version of the textbook model in Barro and Sala-i Martin (2003) Chapter 6 (BSM henceforth). There are three types of agents — a final good producer, a unit measure of consumers, and a continuum of R&D firms. The only point of departure from BSM is in the R&D sector, so as to emphasize the novel features of

[9] In contrast, these papers do not focus on this decision and assume that firms can either perfectly coordinate their efforts (Grossman and Helpman, 1991) or cannot choose the direction of their research altogether (Klette and Kortum, 2004).

[10] See, for example, Segerstrom *et al.* (1990), Aghion and Howitt (1992), Corriveau (1994), Corriveau (1998), Aghion *et al.* (2005), and Acemoglu and Akcigit (2012)

the model. In particular, R&D projects are scarce and R&D entrants can direct their efforts towards a particular project, but they cannot coordinate their research activities.

2.1 Final Good Sector

The final good is produced by a single price taker, using the following technology

$$Y_t = AL^{1-\lambda} \int_0^{N_t} X_t^{\lambda}(n)dn, \quad 0 < \lambda < 1 \tag{1}$$

where Y_t is output, L is the fixed labor supply of households, N_t is the mass of intermediate varieties, and $X_t(n)$ is the amount of a particular variety n employed in production. The price of the final good is normalized to unity. The final good firm faces a competitive market for labor, which is hired at the wage w_t, and a monopolistically competitive market for varieties, where a unit of each variety n is bought at the price $P_t(n)$. As in BSM, the firm's maximizing behavior yields the wage $w_t = (1-\lambda)Y_t/L$ and the inverse demand function for varieties $P_t(n) = \lambda AL^{1-\lambda}X_t^{\lambda-1}(n)$.

2.2 R&D Sector

The novel features of our model are contained in the R&D sector of the economy. The innovation process has three stages and makes a distinction between potential innovations (ideas) and actual innovations (new varieties). At stage one, firms enter the R&D sector at a cost $\eta > 0$ units of the final good. The mass of R&D entrants is denoted by μ_t and is to be determined in equilibrium. At stage two firms direct their innovative effort towards a particular R&D project from a finite mass ν_t of ideas. The choice is private knowledge and firms cannot coordinate their efforts. To capture this coordination failure, we follow the previous literature on coordination frictions and focus on a symmetric equilibrium where firms use identical mixed strategies.[11] Ideas are identical and, if innovated, transform into exactly one new variety. Innovation takes one period — a firm which enters at time t

[11]See, for example, Julien *et al.* (2000), Burdett *et al.* (2001), and Shimer (2005).

innovates the chosen project at time $t+1$. Thus, the only source of uncertainty in our model is the random realization of firms' equilibrium mixed strategies — some ideas may be innovated by many firms simultaneously, while others may not be innovated at all. Innovators apply for a patent which grants perpetual monopoly rights over the variety. Each innovation is protected by exactly one patent — if several firms simultaneously apply for the same patent, then each has an equal chance of receiving it. Stage three is as in BSM. Patent holders supply their variety in a monopolistically competitive market. Both the average and marginal costs of production are normalized to unity so profits are given by $\pi_t(n) = (P_t(n) - 1)X_t(n)$. Furthermore, the value of holding a monopoly over a variety n at time t, V_t, is given by

$$V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi_i(n) \tag{2}$$

where d_{it} is the stochastic discount factor.

A necessary condition for positive long term growth in our model is that the mass of ideas, ν_t, grows at a positive rate. We follow Kortum (1997) and Romer (1990), among others, and assume that knowledge is cumulative. Patenting an idea at time t allows firms to "stand on the shoulders of giants" and gain access to $M > 1$ new research avenues at $t + 1$. Thus, unlike previous growth models, in ours the mass of ideas is endogenously determined. Once an idea is innovated, it is no longer a potential R&D project and so it is removed from the pool.[12] Thus, the net increase in the pool of ideas from innovating one new variety is $M - 1$. Due to the frictions in our model, there is a chance that an idea is not innovated, i.e. no firm directs its research efforts towards the idea in question. Let us denote this probability by ζ_t, then the law of motion for ideas is given by

$$\nu_{t+1} = \nu_t + (1 - \zeta_t)(M - 1)\nu_t \tag{3}$$

[12]Each innovation is protected by a patent, so no firm has an incentive to imitate at a late date. Thus, the idea no longer represents a profitable R&D project and as a consequence it is no longer in ν_{t+1}.

As each innovated idea is transformed into a new variety, it follows that

$$N_{t+1} = N_t + (1 - \zeta_t)\nu_t \tag{4}$$

2.3 Households

Consumers are endowed with a discount factor β and a per-period utility function $U(C_t) = \ln C_t$. They can save by accumulating assets, which in this economy are claims on intermediate firms' profits. In particular, households have access to a mutual fund that covers all intermediate good firms. Let a_t denote the amount of shares held by the representative household at the beginning of period t. Each period all profits are redistributed as dividends, thus the total assets of the household entering period t are $a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn$. At time t households decide on the shares they would like to hold at $t+1$, a_{t+1}. The mutual fund at that time covers all firms which exist at time $t+1$, N_{t+1}. Hence, the household's budget constraint is given by

$$a_{t+1} \int_0^{N_{t+1}} V_t(n)dn = a_t \int_0^{N_t} (\pi_t(n) + V_t(n))dn + w_t L - C_t \tag{5}$$

The household's first order conditions imply the Euler equation below

$$\frac{1}{C_t} = \frac{\beta}{C_{t+1}} \Big(\int_0^{N_{t+1}} (\pi_{t+1}(n) + V_{t+1}(n))dn \Big) \Big(\int_0^{N_{t+1}} V_t(n)dn \Big)^{-1} \tag{6}$$

The intuition is standard — consumers equate the marginal utility at time t with the discounted marginal utility at time $t+1$, times the gross rate of return on their assets.

2.4 Equilibrium

We restrict the analysis to a set of parameter values which ensures that firms have an incentive to enter the R&D sector, i.e. $\eta \leq (1-\lambda)\beta(\lambda^2 A)^{1/(1-\lambda)} L/[\lambda(M-\beta)]$. The usual profit maximization of intermediate good firms along with the demand function imply that $P_t(n) = 1/\lambda$ and $X := X_t(n) = (\lambda^2 A)^{1/(1-\lambda)} L$. Thus, every intermediate good firm yields the same per period profits of $\pi := \pi_t(n) = X(1-\lambda)/\lambda$. This implies that $V_t := V_t(n) = \sum_{i=t+1}^{\infty} d_{it}\pi$ — every firm is equally valuable. Since each variety carries the same amount of profits, the stage two equilibrium strategy of firms is to direct their R&D effort towards each idea with equal probability.[13] This implies the following equilibrium outcome.

Proposition 1. *The number of firms which direct their R&D effort towards a particular idea follows a Poisson distribution with mean θ_t, where $\theta_t \equiv \mu_t/\nu_t$.*

A proof is in Appendix C. The random realization of firms' equilibrium strategies gives rise to the standard urn-ball matching technology.[14] The ratio of firms to ideas, θ_t, represents the tightness in the market for ideas and captures the level of congestion in the economy. An R&D firm becomes a monopolist with probability $\sum_{m=0}^{\infty} Pr($ exactly m rival firms direct their research effort towards the particular idea$)/(m+1) = \sum_{m=0}^{\infty} e^{-\theta_t}\theta_t^m/(m+1)! = (1-e^{-\theta_t})/\theta_t$. This probability captures the business-stealing effect in the model. An innovator faces the threat that a rival directs its research efforts towards the exact same idea. If that is the case, then the rival has a chance of securing a patent over the innovation, effectively stealing the innovator's monopoly rents. Thus, higher congestion increases the expected number of rivals, which lowers each firm's chance of securing a patent. Given free entry, it follows that

$$\eta = \frac{1-e^{-\theta_t}}{\theta_t} V_t \tag{7}$$

The level of congestion firms are willing to tolerate is governed by the net present value of profits and the entry cost. Higher profits (or lower costs) induce firms to tolerate a lower chance of securing a monopoly position and as a consequence higher tightness. The matching

[13] We follow the literature on coordination frictions (see, for example, Julien *et al.* (2000)) and derive the optimal behavior for firms when there are finite number of ideas. The result is then obtained by taking the limit as $\nu_t \to \infty$, keeping the ratio μ_t/ν_t constant.

[14] See, for example, Wolinsky (1988), Lu and McAfee (1996), Julien *et al.* (2000), and Burdett *et al.* (2001).

technology implies that $\zeta_t = e^{-\theta_t}$. Hence,

$$\nu_{t+1} = \nu_t + (1 - e^{-\theta_t})(M - 1)\nu_t \tag{8}$$

$$N_{t+1} = N_t + (1 - e^{-\theta_t})\nu_t \tag{9}$$

Furthermore, the frictions in our model induce an economy-wide varieties production function (New Varieties $= (1 - e^{-R_t/(\eta\nu_t)})\nu_t$) which is concave in the aggregate research effort, $R_t \equiv \eta\mu_t$. A higher aggregate research effort is associated with higher mass of firms which, in turn, increases the congestion in the market. Thus, the marginal entrant has a higher chance of duplicating an innovation, rather than innovating a distinct new variety. In particular, the higher level of congestion increases the fraction of wasteful duplicative innovation, $\omega \equiv 1 - (1 - e^{-\theta_t})/\theta_t$.[15]

Since all firms receive the same profits, the Euler equation simplifies to

$$V_t = \beta \frac{C_t}{C_{t+1}} \Big(\pi + V_{t+1}\Big) \tag{10}$$

Hence, the stochastic discount factor is $d_{it} = \beta^i C_t / C_{t+i}$. Given consumers' budget constraint, free entry, and the law of motion for varieties it is straightforward to derive the economy-wide resource constraint which takes the usual form — output is distributed towards consumption, production of intermediate inputs, and investment in R&D.

$$Y_t = C_t + N_t X + \mu_t \eta \tag{11}$$

2.5 Balanced Growth Path

Our analysis focuses on the BGP of the economy, where output, consumption, varieties, ideas, and the mass of entrants all grow at constant (but possibly different) rates. Denote

[15] Only one firm can hold a patent over a certain variety. Hence, whenever $m \geq 1$ firms innovate the same idea, $m - 1$ of them make a wasteful duplicative innovation. Each entrant makes an innovation, so the total number of innovations is μ_t. The total number of useful innovations equals the total number of new varieties, $(1 - e^{-\theta_t})\nu_t$. Thus, the fraction of innovations which represent wasteful duplication of effort is simply $1 - (1 - e^{-\theta_t})/\theta_t$.

the growth rate of any variable x along the BGP by g_x. It is straightforward to establish that output, varieties, consumption, entry into R&D, and the stock of ideas all grow at the same rate along the BGP. Namely, $g \equiv g_Y = g_C = g_N = g_\mu = g_\nu = (1 - e^{-\theta})(M - 1)$, where θ is the value of the market tightness along the BGP.[16] As in BSM Y_t, C_t, N_t, and μ_t all grow at the same rate. In our model, the mass of ideas, ν_t, also grows at this rate. In fact, the expansion of ν_t is the ultimate source of growth in the economy. Due to learning, innovation today increases the mass of ideas in the future. This permanently reduces the severity of the coordination problems and subsequently the cost of securing a monopoly position.[17] This lower cost in turn induces higher entry into R&D up to the point where congestion reaches its BGP level. Furthermore, the fraction of foregone innovation, $e^{-\theta}$, directly impacts the growth rate, as only innovated ideas at time t contribute to the expansion of ν_{t+1}.

It is convenient to solve the model by looking at the stable ratios θ, $\frac{\nu}{N}$ and $\frac{C}{N}$. From the law of motion of ideas and varieties, and from $g_N = g_\nu$, it follows that $\frac{\nu}{N} = M - 1$. Next, the resource constraint implies that

$$\frac{C}{N} = \frac{1 + \lambda}{\lambda}\pi - \eta\theta(M - 1) \tag{12}$$

Lastly, we can use the fact that $g_C = g_\nu$, the Euler equation, the law of motion for ν_t, and the free entry condition to find an implicit solution for the market tightness.

$$\eta = \Big(\frac{1 - e^{-\theta}}{\theta}\Big)\frac{\beta\pi}{1 + (1 - e^{-\theta})(M - 1) - \beta} \tag{13}$$

Even though we cannot explicitly solve for θ, it is straightforward to establish that the solution is unique. Intuitively, as θ increases the market for ideas gets more congested and each firm's chance of becoming a monopolist decreases. At the same time, higher market tightness implies a higher growth rate. This, in turn, increases the rate with which firms discount future profit streams and as a consequence decreases the value of holding a patent.

[16] A proof is available upon request.

[17] The average cost of securing a monopoly position is $\eta/Pr(\text{monopoly}) = \eta\theta/(1-e^{-\theta})$, which is decreasing in ν_t.

Both of these effects decrease the incentives to enter the R&D sector when the market tightness is high and vice versa.

Proposition 2. *The equilibrium market tightness, θ, is:*

- *increasing in π and β*
- *decreasing in η and M*

A proof is included in Appendix C. Intuitively, an increase in profits raises the value of being a monopolist, V_t. This increases firms' incentives to innovate, which leads to a higher mass of R&D entrants and subsequently to a higher market tightness. Similarly, a higher entry cost, η, discourages entry into R&D, which decreases the market tightness. An increase in β or a decrease in M both lead to an increase in the effective discount factor, $\beta C_t/C_{t+1}$, along the BGP. Thus, firms value future profits more, which increases the value of a patent, V_t, and ultimately the market tightness.

3 Second-Best Allocation

This section examines the planner's second best allocation — the planner chooses the optimal BGP allocations subject to the coordination frictions in the market for ideas. Without loss of generality, we impose symmetry in the intermediate varieties, i.e. $X_t(n) = X_t(n')$ for any varieties n and n'. Thus, the planner faces the problem of choosing production of varieties, consumption, a mass of varieties, a mass of ideas, and the market tightness in order to maximize welfare subject to the resource constraint, the laws of motion for ideas and varieties, and the coordination frictions.

$$\max_{\{C_t, X_t, \theta_t, N_t, \nu_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$AL^{1-\lambda} N_t X_t^{\lambda} = N_t X_t + C_t + \eta\theta_t\nu_t \tag{14}$$

$$N_{t+1} = N_t + (1 - e^{-\theta_t})\nu_t \tag{15}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\theta_t})(M-1)\nu_t \tag{16}$$

Maximizing with respect to X_t yields the usual solution for varieties $X^* := X_t = (\lambda A)^{1/(1-\lambda)}L$. As in BSM the difference between the planner's solution and the decentralized outcome comes from the monopoly pricing of intermediate goods. Let $\pi^* = X^*(1-\lambda)/\lambda$ denote the implied per period monopoly profits at efficient level of intermediate varieties. Then, the rest of the first order conditions are:

$$[C_t]: \quad \beta\frac{C_t}{C_{t+1}} = \frac{\phi_{t+1}}{\phi_t} \tag{17}$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \phi_{t+1}\pi^* \tag{18}$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1}\Big(e^{-\theta_{t+1}} + (1-e^{-\theta_{t+1}})M\Big) + h_{t+1}(1-e^{-\theta_{t+1}}) - \phi_{t+1}\eta\theta_{t+1} \tag{19}$$

$$[\theta_t]: \quad \eta = e^{-\theta_t}\Big(\frac{h_t}{\phi_t} + \frac{\lambda_t}{\phi_t}(M-1)\Big) \tag{20}$$

where ϕ_t, h_t, λ_t are the multipliers associated with (14), (15), and (16), respectively. From (17) and (18), it follows that

$$\frac{h_t}{\phi_t} = \beta\frac{C_t}{C_{t+1}}\Big(\pi^* + \frac{h_{t+1}}{\phi_{t+1}}\Big) \tag{21}$$

The above equation characterizes the planner's valuation of varieties: the value of a variety equals the discounted sum of per period profits, π^*, and the continuation value h_{t+1}/ϕ_{t+1}. There are only two differences as compared to the DE — the level of profits is higher and the planner chooses a different tightness.

The value of an idea is the discounted sum of several terms.

$$\frac{\lambda_t}{\phi_t} = \beta\frac{C_t}{C_{t+1}}\Big(-\eta\theta_{t+1} + (1-e^{-\theta_{t+1}})\Big(\frac{h_{t+1}}{\phi_{t+1}} + \frac{\lambda_{t+1}}{\phi_{t+1}}(M-1)\Big) + \frac{\lambda_{t+1}}{\phi_{t+1}}\Big) \tag{22}$$

First, there is the dividend, $-\eta\theta_{t+1}$, which represents the average cost of R&D per idea. It captures the intuition that unlike other assets, which carry positive returns, an idea is only valuable if it is innovated. Hence, the planner finds it costly to keep a stock of ideas because it diverts resources away from consumption and into R&D. The second term represents the capital gain from innovation — the probability an idea is innovated, $(1-e^{-\theta_{t+1}})$, times the social benefit from innovating. This benefit is the value of the extra variety, h_{t+1}/ϕ_{t+1}, plus the value of the extra ideas that would be added to the pool because of innovation, $\lambda_{t+1}/\phi_{t+1}(M-1)$. Lastly, the idea carries its continuation value λ_{t+1}/ϕ_{t+1}.

Apart from the monopoly pricing of intermediate goods, there are two additional externalities in the model, which are illustrated in equation (20). First, the congestion externality manifests through the difference in the fraction of socially and privately beneficial innovations. The planner finds the marginal entry beneficial only if the firm is the sole inventor, i.e. with probability $e^{-\theta_t}$. Firms, on the other hand, value entry even if they duplicate an innovation, as long as they receive the patent for it. In particular, due to the business-stealing effect, the probability of a privately beneficial innovation is $(1-e^{-\theta_t})/\theta_t > e^{-\theta_t}$. Hence, the congestion exterlaity induces firms to over-invest in R&D as compared to the SB. This business-stealing effect is similar to the one examined in the previous literature.[18] Unlike in previous papers, in ours the magnitude of the effect is affected by the coordination frictions. Second, there is the learning externality — firms cannot appropriate the benefit of any ideas that come about from their innovations, so they do not value them. The planner, on the other hand, does because they permanently alleviate future coordination problems. Specifically, more innovation today increases the amount of future research avenues, which allows the economy to innovate more varieties without increasing the congestion problems. Thus, the extra ideas permanently reduce the cost of discovering new varieties.[19] As a result the learning externality creates incentives for firms to under-invest as compared to the SB. This externality is similar in spirit to the inter-temporal spillover effects present in previous models.[20] In ours, the externality operates through the market for ideas — the planner values ideas because they alleviate the coordination problems in the economy.

It is straightforward to establish that along the BGP the SB allocations are characterized

[18] See, for example, Corriveau (1994) and Corriveau (1998).

[19] The average cost of discovering one new variety is $\eta/Pr(\text{sole inventor}) = \eta e^{\theta_t}$, which is decreasing in the mass of ideas.

[20] See, for example, Romer (1990), Aghion and Howitt (1992), and Grossman and Helpman (1991).

by[21]

$$\left(\frac{\nu}{N}\right)^{SB} = M - 1 \tag{23}$$

$$\left(\frac{C}{N}\right)^{SB} = \pi^* - \eta\theta^{SB}(M - 1) \tag{24}$$

$$1 + (1 - e^{-\theta^{SB}})(M - 1) = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\theta^{SB}} + (1 - e^{-\theta^{SB}} - \theta^{SB}e^{-\theta^{SB}})(M - 1)\Big) \tag{25}$$

The difference between the SB solution for the market tightness, (25), and the DE one, (13), comes from the aforementioned externalities. To see their impact clearly, let us define the implied rate of return in the DE by

$$r := \frac{C_{t+1}}{\beta C_t} - 1 = \frac{\pi}{\eta}\Big(\frac{1 - e^{-\theta}}{\theta}\Big) \tag{26}$$

which is nothing but the rate of return on a unit investment in R&D — π is the flow of profits and $(1 - e^{-\theta})/\theta$ is the probability of securing a monopoly position. The implied rate of return in the SB represents the social rate of return on a unit of investment on R&D and is defined by

$$r^{SB} := \frac{C^{SB}_{t+1}}{\beta C^{SB}_t} - 1 = e^{-\theta^{SB}}\Big(\frac{\pi^*}{\eta} - \theta^{SB}(M - 1)\Big) + (1 - e^{-\theta^{SB}})(M - 1) \tag{27}$$

First, the planner eliminates the monopoly distortion, so the flow of profits is π^*. Second, she values the marginal innovation only when the firm is the sole inventor, which occurs with probability $e^{-\theta^{SB}}$. In that event, the net return is given by the normalized profits, π^*/η, less the normalized "storage cost" of the new research avenues, $\theta^{SB}(M - 1)$. Third, each innovation increases the mass of ideas, so the permanent decrease in future congestion yields the return of $(1 - e^{-\theta^{SB}})(M - 1)$.

To implement the SB, the planner needs to impose a tax on the entry into R&D. This is because the congestion externality is larger than the learning one, so the over-investment effect of the former dominates the under-investment effect of the latter. In particular, suppose

[21] A proof is available upon request.

that the government imposes a subsidy on the purchases of intermediate varieties at a rate s and a tax on R&D activities at a rate τ. Furthermore, if the government keeps a balanced budget through the means of lump-sum transfers, then the optimal policy is summarized below.

Proposition 3. *The optimal subsidy on the purchase of intermediate varieties is given by $s^* = 1 - \lambda$. The optimal tax rate on R&D entry is given by*

$$\tau^* = \frac{\beta \pi^* (1 - e^{-\theta^{SB}})}{\eta \theta^{SB} (e^{-\theta^{SB}} + (1 - e^{-\theta^{SB}}) M - \beta)} - 1 \tag{28}$$

Furthermore, $\tau^* > 0$ because the magnitude of the congestion externality is larger than that of the learning externality.

A proof is included in Appendix C. Even though it is optimal to impose a tax on R&D spending, it may be the case that the decentralized economy suffers from under-investment, i.e. $\theta < \theta^{SB}$. This is due to the monopoly distortion induced by patents. Whether or not there will be under-investment in equilibrium depends on parameter values.

Proposition 4. *The second best market tightness, θ^*, is:*

- *increasing in π^* and β*
- *decreasing in η and M*

A proof is included Appendix C. Intuitively, an increase in the implied profits, π^*, increases the planner's valuation of each variety and each idea. Hence, each entry is now more beneficial, so the planner increases the market tightness. This increases congestion and as a consequence decreases the value of the marginal entry. The planner increases the tightness until the value of the marginal entry reaches the entry cost η. Similarly, an increase in the entry cost, η, requires the planner to extract more benefit from the marginal entry. Thus, she finds it optimal to reduce the market tightness and decrease congestion. At the same time, an increase in η leads to an increase in the storage cost of ideas and subsequently reduces their value. This induces the planner to decrease the tightness further.

An increase in β increases the discount factor, so the value of varietis and ideas increases because the stream of future profits is now more valuable. This increases the value of the marginal entry and induces the planner to increase the market tightness. An increase in M leads to two opposing effects. First, a higher M is associated with a higher growth rate. This decreases the value of future consumption and induces the planner to set a lower tightness. At the same time, a higher M implies each innovation increases the mass of ideas next period, ν_{t+1}, by a higher amount. Thus, the value of the marginal entry into R&D is higher and the planner has an incentive to set a higher market tightness. At the optimum, the former effect dominates the latter and the planner decreases θ^{SB}.

4 The Impact of Coordination Frictions

4.1 Decentralized Economy

A goal of the analysis is to study the impact of coordination frictions in our economy. To this end we compare the DE's BGP to the BGP of a hypothetical CE. In particular, the only difference between the latter economy and the DE is that firms can coordinate their research efforts at stage two of the innovation process.[22] Let superscript c denote the value of any variable in the CE along the BGP. Evidently, when firms can coordinate their research efforts, all research avenues are undertaken and subsequently all ideas are innovated. At the same time, the CE may feature a positive fraction of wasteful duplication of effort due to the usual "over-grazing" problem.[23] However, this waste, ω^c, is smaller than the one in the DE. Furthermore, this is the case, even though the CE features a higher market tightness.

Proposition 5. *In the coordination economy all ideas are innovated and the growth rate equals $M-1$. Furthermore, $\theta < \theta^c$ and*

$$\omega - \omega^c = \frac{e^{-\theta}(M-1)\eta}{\beta\pi} > 0 \tag{29}$$

[22]The proof of Proposition 5 explicitly defines the process of coordination.

[23]For a survey of the literature see, for example, Reinganum (1989).

A proof is included in Appendix C. Intuitively, when firms can coordinate their R&D activities all ideas are innovated because each of them represents an opportunity to gain a profitable monopoly position. Thus, in the CE there is no foregone innovation. This results in a higher growth rate as compared to the DE. Because of this the foregone innovation in the DE generates a general equilibrium effect which induces firms to tolerate a higher congestion than firms in the CE. In particular, the lower growth rate increases the effective discount factor, which in turn raises the value of holding a patent. Since, in both economies, the probability of making a wasteful innovation is simply the probability of not receiving a patent, it follows that $\omega > \omega^c$. In particular, the level of amplification, $\omega - \omega^c$, equals the difference in the growth rates, $g^c - g = e^{-\theta}(M-1)$, divided by the discounted normalized profits, $\beta\pi/\eta$.

Moreover, $\theta < \theta^c$, even though the DE features a higher fraction of wasteful simultaneous innovation. This is the case because, for a given market tightness, the coordination frictions reduce an entrant's chance of securing a monopoly position. In particular, the probability of securing a patent in the DE for a given tightness $\tilde{\theta}$, $Pr(\text{patent}|\tilde{\theta}) = (1-e^{-\tilde{\theta}})/\tilde{\theta}$, is only a fraction $1-e^{-\tilde{\theta}}$ of the one in the CE, $Pr(\text{patent}|\tilde{\theta})^c = 1/\tilde{\theta}$. As firms cannot coordinate their efforts, in the DE only a fraction $1-e^{-\tilde{\theta}}$ of ideas are patented. Thus, even though the number of patent applications per idea, $\tilde{\theta}$, is the same in both economies, in the DE there are relatively less patents to be distributed among innovators. This decreases each entrant's chance of securing a monopoly position and subsequently reduces the incentives to enter the R&D sector. This is true even though the DE features a higher value of holding a patent. In other words, the decrease in the probability of securing a patent dominates the increase in the net present value of profits, ultimately reducing incentives to enter the R&D sector and decreasing the market tightness. Furthermore, the effect on the market tightness provides an indirect channel through which the presence of foregone innovation reduces the growth rate in the DE. A lower tightness decreases each idea's chance of being innovated which results in a lower aggregate mass of innovation.

The impact of coordination frictions is higher when profits are low, consumers are more inpatient, and the entry cost is high. The next proposition states the result.

Proposition 6. *The fraction of foregone innovation, $e^{-\theta}$, the level of amplification of wasteful innovation, $\omega - \omega^c$, and the amount by which the tightness is reduced, $\theta^c - \theta$ are*

- *decreasing in π and β*
- *increasing in η*

$e^{-\theta}$ and $\omega - \omega^c$ are increasing in M.

A proof is included in Appendix C. The fraction of foregone innovation depends only on the market tightness in the DE. When the tightness is low, the probability that an idea is not matched with any firm is high, which leads to a high fraction of foregone innovation and vice versa. The level of the amplification of wasteful innovation moves in the same direction as $e^{-\theta}$. This is because firms in the DE are willing to tolerate lower probability of securing a monopoly position only due to the higher value of holding a patent induced by $g < g^c$. As the fraction of foregone innovation decreases, the difference in the growth rates decreases as well. This reduces the incentives for firms to tolerate extra congestion, which decreases the amplification.

When the fraction of foregone innovation is low, the incentives for firms in the DE to over-invest (as compared to the CE) induced by the difference in the growth rates is low as well. This generates an upward pressure on the difference in research intensities, $\theta^c - \theta$. At the same time, a smaller fraction of forgone innovation implies that, for a given market tightness, there are relatively more patents to be distributed among firms in the DE. Hence, $Pr(\text{patent}|\tilde{\theta})^c - Pr(\text{patent}|\tilde{\theta})$ decreases, which increases the incentives for firms in the DE (relative to CE) to enter the R&D sector. Consequently, this generates a downward pressure on $\theta^c - \theta$. For a decrease in the fraction of foregone innovation induced by changes in π, β, or η the latter effect dominates the former and $\theta^c - \theta$ decreases.

A decrease in the fraction of foregone innovation due to lower M can lead to either an increase or decrease in $\theta^c - \theta$, depending on the relative size of M. This is because changes in M directly impact the difference in the growth rates and consequently increases the relative

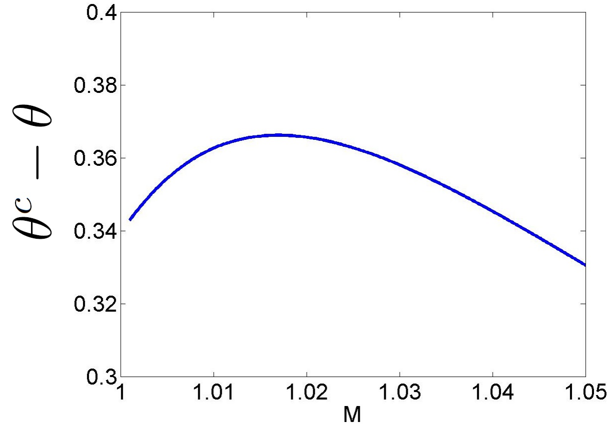


Figure 1: Difference in the Market Tightness

size of the former effect. For low M this increase is small, so $\theta^c - \theta$ moves in the same direction as the fraction of foregone innovation. For large M, however, the increase in the size of the effect in question is large and so $\theta^c - \theta$ is decreasing in M (Figure 1).[24]

4.2 Planner's Allocation

To highlight the impact of coordination frictions in the planner's allocation we compare the BGP in the SB to that in the FB. In the FB the planner can directly assign firms to projects. Thus, it is straightforward to establish that $\theta^{FB} = 1$ and that the FB does not feature any foregone innovation, nor any wasteful duplication of effort.[25] Thus, it is readily observable that the frictions amplify both the fractions of foregone and wasteful innovation. Unlike in the decentralized case, however, the coordination failure does necessarily reduce the research intensity in the economy. This is so because in the SB the planner faces a trade-off when deciding on the market tightness (as depicted in equation (20)). On the one hand, a higher tightness increases congestion and subsequently the cost of wasteful innovation, $\eta \times Pr(\text{duplication of effort}) = \eta(1 - e^{-\theta_t})$. On the other hand, a higher tightness decreases

[24] All parameter values used in Figure 1, except for M are set at their calibrated values detailed in section 5.

[25] Furthermore, $(\nu/N)^{FB} = M - 1$, $(C/N)^{FB} = \pi^* - \eta(M-1)$, and $g^{FB} = M - 1$. A proof is available upon request.

the fraction of foregone innovation. The benefit from this decrease is given by the probability the marginal firm is the sole inventor, $e^{-\theta_t}$, times the the social benefit of the innovation net of the entry cost, η. Thus, the planner chooses θ^{SB} that, at the margin, strikes a balance between these two opposing effects. In the FB, however, she faces no such trade-off so the decision of setting the market tightness is independent of the parameters which govern the welfare costs of wasteful duplication of effort and foregone innovation.

Proposition 7. *The fraction of wasteful (foregone) innovation, ω^{SB} ($e^{-\theta^{SB}}$), is*

- *increasing (decreasing) in π^* and β*
- *decreasing (increasing) in η and M*

The proof is immediate from Proposition 4 and $\omega^{SB} = 1-(1-e^{-\theta^{SB}})/\theta^{SB}$. The intuition behind the result is straightforward. High π^*, β or low η, M induce the planner to set high θ^{SB}. This leads to more congestion, which subsequently increases ω^{SB} and decreases $e^{-\theta^{SB}}$.

5 Numerical Exercise

We gauge the importance of the frictions in our model for growth and welfare through the means of a numerical exercise. Our calibration matches key moments of the U.S. economy and is set at annual frequency. The discount factor, β, is set to 0.95, the productivity parameter, A, and labor supply, L, are both normalized to unity. We set the markup to 17.43% ($\lambda = 0.8516$) to match the average R&D share of non-farm GDP, $\eta\mu_t/Y_t = 3.1194\%$, for the period between 1966 and 2011.[26] To calibrate η and M we use two additional moments. First, we match the average growth rate of non-farm GDP for the same period of 1.7546%. Second, in our model the ratio of patent grants to patent applications is $(1-e^{-\theta})/\theta$.

[26] The data on non-farm GDP is in 2009 chained dollars and taken from NIPA table 1.3.6. The data on nominal R&D expenditures is from NIPA table 5.6.5 and includes private fixed investment in R&D (including software). To obtain the series on real R&D investment, we deflate the nominal series using the implicit GDP price deflator from NIPA table 1.1.9.

Matching this fraction to its empirical counterpart, 0.60957, results in a market tightness $\theta = 1.0876$.[27] Together these two moments yield $\eta = 0.1715$ and $M = 1.0265$.

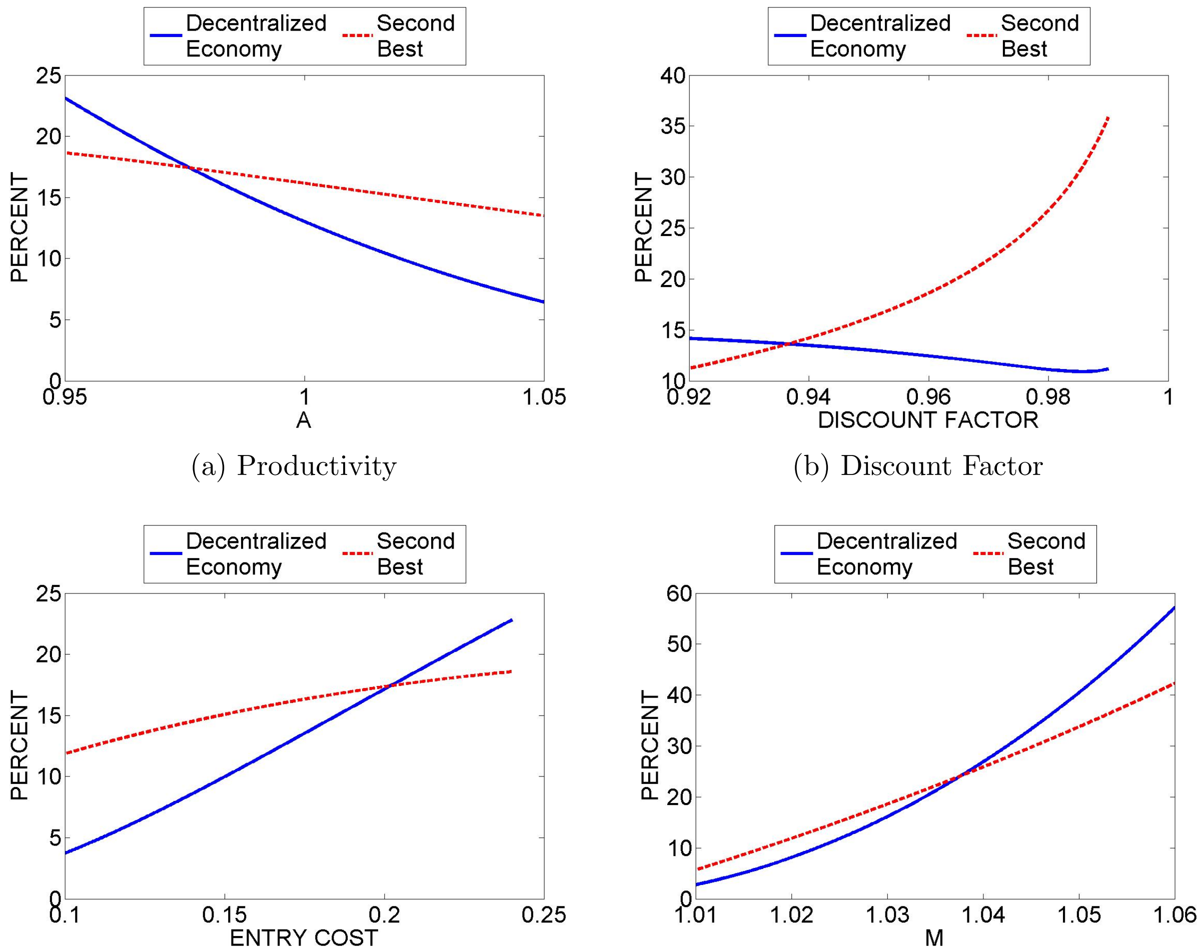
The calibrated DE features a fraction of wasteful innovation $\omega = 39\%$. This is about 25% larger than that in the CE, $\omega^c = 31\%$, even though θ is about 25% smaller than $\theta^c = 1.4491$. At the same time, the DE features a large fraction of research avenues which are not undertaken — 33.7%. This implies that the growth rate is about 2/3 of the CE growth rate $g^c = 2.65\%$. Eliminating the frictions generates a welfare gain of 13% in consumption equivalent terms.[28] About 10.35pp of the gain is due to the increased growth rate and the rest is due to the reduction in the fraction of wasteful innovation.

The DE exhibits too little innovation — the SB market tightness, θ^{SB}, is 1.7154. Thus, in the SB the percentage of innovations which represent a wasteful duplication of effort, ω^{SB}, is 52%. The SB features a fraction of uninnovated research avenues of 18%. While this is still quite sizable, it is about half of that in the DE. As a consequence, the SB growth rate (of 2.17%) is considerably larger than the one in the DE. Eliminating the frictions in the planner's allocation results in a 16.15% welfare gain. Of this 5.6pp is the gain due to eliminating the fraction of foregone innovation and the rest is due to eliminating the fraction of wasteful innovation.

The relative welfare costs of foregone and wasteful innovation are different in the decentralized equilibrium and the planner's allocation. This is the case because of two reasons. First, the planner chooses θ^{SB} which, at the margin, strikes a balance between these two welfare costs. As a result the fraction of foregone innovation in the SB is much smaller. Thus, eliminating this fraction leads to a relatively smaller welfare gain. Second, eliminating the frictions in the DE does not fully eliminate wasteful innovation. In particular, since the CE features $w^c = 31\%$, the reduction in the waste is only 8pp. On the other hand, if the planner could achieve the first-best, then all of the waste would be eliminated, leading to a reduction of 52pp.

[27] The data is taken from the U.S. Patent and Trademark Office. The data on patent grants is by year of application.

[28] A detailed explanation of the welfare calculations is included in Appendix A.



(a) Productivity

(b) Discount Factor

(c) Entry Cost

(d) M

Figure 2: Welfare Gain: Comparative Statics

An increase in productivity (or a decrease in η, M) leads to a decrease in the welfare cost of frictions in both the DE and the panner's allocation (Figure 2).[29] In the DE, this is because high π (or low η, M) leads to a smaller fraction of foregone innovation and a lower level of amplification in the waste. In the planner's allocation, an increase in π^* (or a decrease in η, M) decreases the relative social cost of wasteful innovations. The planner, thus finds it optimal to increase the tightness. As a result the welfare costs of both foregone and wasteful innovation decrease.

An increase in the discount factor has two effects on the welfare gain. The first is the same as that of an increase in productivity. The second effect is directly related to consumers' impatience — high β makes agents more patient and as a result the same reduction in the growth rate has a higher welfare impact. At the extreme, as $\beta \to 1$, any reduction in the growth rate due to frictions generates infinite welfare losses. For the planner's allocation the latter effect always dominates, whereas for the DE it dominates only when β is sufficiently high (Figure 2b).

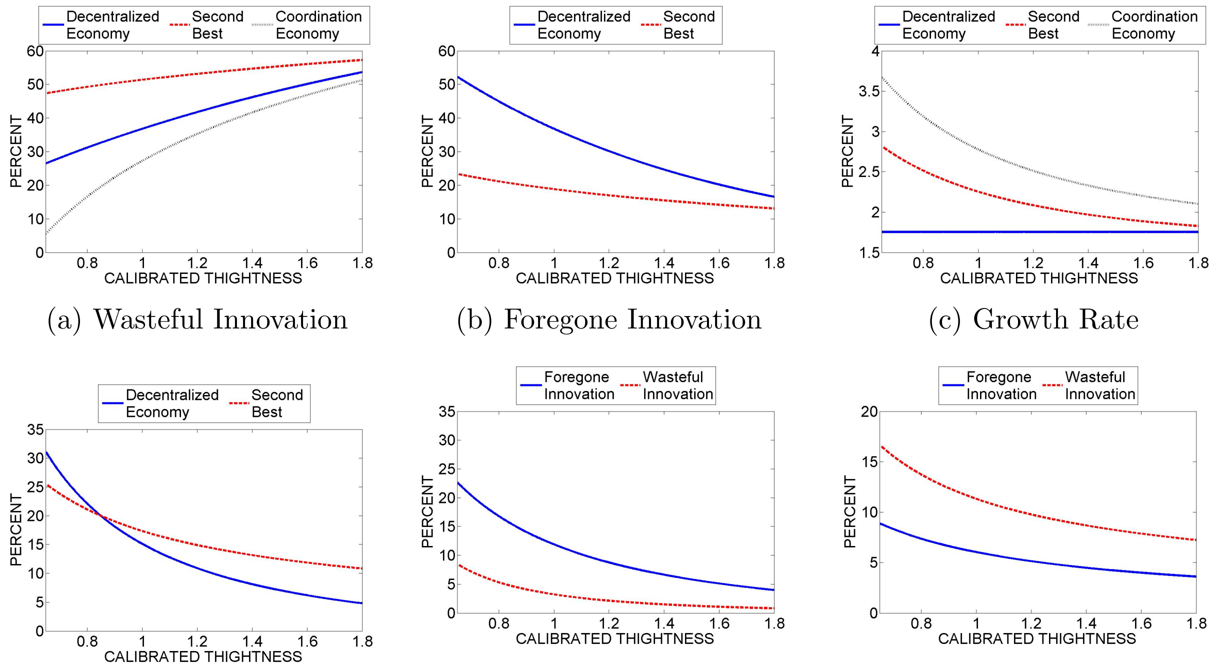
Two potential shortcomings of our calibration strategy are that, in practice, not all firms use patents and not all patents are rejected because of simultaneous innovation.[30] To eliminate these potential issues, we turn to an alternative calibration strategy. Rather than matching the fraction of successful patent applications, we calibrate the market tightness, θ, so as to be consistent with estimates on the return to R&D expenditure from the existing literature.[31] The majority of these estimates suggest the rate of return for the U.S. economy is between 20% to 40%. This yields a market tightness $\theta \in [0.6471, 1.8039]$.

Figure 3 illustrates the quantities of interest for the different values of the calibrated market tightness. The welfare gain is substantial for all considered values — it is at least 4.7% for the decentralized economy and 10.8% for the planner's allocation (Figure 3d). Furthermore, the gain is decreasing in the calibrated value of the tightness. Intuitively,

[29] Unless on the horizontal axis, all parameter are set at their calibrated values. Different levels of productivity capture different levels of π and π^*.

[30] For a recent survey of the literature on the choice between formal and informal intellectual property protection mechanisms see, for example, Hall *et al.* (2014).

[31] For a survey see, for example, Hall *et al.* (2010).



(a) Wasteful Innovation
(b) Foregone Innovation
(c) Growth Rate
(d) Welfare Gain
(e) Welfare Gain: Decentralized Economy
(f) Welfare Gain: Planner's Allocation

Figure 3: Alternative Calibration Values

higher θ implies a lower fraction of foregone innovation, $e^{-\theta}$, which in turn decreases the difference between the CE and DE growth rates, $g^c - g = e^{-\theta}(M-1)$. At the same time, the reduced growth rate gap implies that the amplification in the fraction of wasteful innovation, $(\omega - \omega^c)$ is smaller. Both of these effects serve to mitigate the impact of the coordination frictions and as a consequence the welfare gain from eliminating these frictions. The intuition for the case of the planner's allocation is slightly different and it serves to explain why the welfare gain decreases relatively less than the gain in the decentralized case. Firstly, a higher calibrated market tightness implies a lower parameter value for the entry cost, η, and for the number of new ideas generated per new variety, M. Thus, the planner finds it optimal to set a higher SB market tightness, since innovation is cheaper and the net present value of implied profits higher. However, this increase is relatively smaller than the corresponding increase in θ. Thus, the response in $e^{-\theta^{SB}}$ and the SB growth rate is relatively smaller. At the same time a higher θ^{SB} leads to a larger fraction of wasteful duplication of effort, ω^{SB}.

This effect puts an upward preassure on the welfare gain as θ^{SB} increases. Nonetheless, the welfare cost of wasteful innovation decreases because of the lower entry cost and ratio of ideas to varieties, $(\nu_t/N_t)^{SB}$. The resulting net effect on the welfare gain due to eliminating wasteful innovation is negative. Yet, this effect is smaller than the one in the DE and as a result the welfare gain in question is larger for all considered values of θ.

We explore the robustness of the quantitative results in an extension of our baseline model. In particular, our augmented model features uncertainty in the innovation process and endogenous firm-level research intensity. Upon choosing a direction for their effort, R&D firms decide on an intensity i and incur the cost ϕi ($\phi > 0$). The amount of effort devoted affects their probability of successfully innovating the idea according to $Pr(\text{success}) = 1 - e^{-\gamma i}$ ($\gamma > 0$). The welfare cost of frictions in this extension is virtually the same as in the baseline model, so the results are presented in Appendix B.

The numerical exercise suggests that firms' failure to coordinate is likely to have a substantial impact on welfare. Thus, it may be a worthwhile endeavor to devise and analyze policies aimed at mitigating the coordination problems. One such policy might be for the government to allocate project-specific grants to firms. In our current model, for example, the planner could set a high enough tax rate on R&D investment so that no firm finds it worthwhile to engage in innovative activity. She can then allow firms to apply for a subsidy to work on a specific idea prior to entering the R&D sector. Each firm chooses one project and applies for a subsidy to innovate that particular idea. Since applications are costless, all ideas receive at least one application, so there would be no forgone innovation in equilibrium. The planner can then eliminate wasteful duplication of effort by granting a subsidy to a single firm per idea. A careful examination of such policies and their practical feasibility, however, is left for future work.

6 Conclusion

We develop an expanding-variety endogenous growth model in which firms direct their investment towards a specific research avenue (out of a scarce mass of potential R&D projects), but cannot coordinate their efforts. Due to the coordination frictions, the equilibrium number of firms which innovate the exact same idea is a random variable with mean given by the tightness in the market for ideas. Because of the frictions in our model, a fraction of research avenues remain uninnovated. This foregone innovation reduces the growth rate which in turn generates a general equilibrium effect that amplifies the fraction of wasteful simultaneous innovation. Furthermore, these frictions reduce the equilibrium level of research intensity. In the planner's allocation, the frictions amplify both the fraction of foregone and wasteful innovation. Whether or not they reduce the level of the optimal research intensity, however, depends on parameter values.

Our paper gauges the impact of coordination frictions on the growth rate and welfare. In the benchmark calibration, eliminating the coordination failure in the decentralized economy results in a 13% welfare gain, whereas the gain in the planner's allocation is 16.15%. Furthermore, the majority of the welfare gain is due to eliminating the welfare cost of foregone innovation in the decentralized case and due to eliminating the welfare cost of wasteful simultaneous innovation in the planner's allocation.

References

Acemoglu, D. and Akcigit, U. (2012). Intellectual property rights policy, competition and innovation. *Journal of the European Economic Association*, **10** (1), 1–42.

Aghion, P., Akcigit, U. and Howitt, P. (2014). Chapter 1 - what do we learn from schumpeterian growth theory? In P. Aghion and S. N. Durlauf (eds.), *Handbook of Economic Growth*, *Handbook of Economic Growth*, vol. 2, Elsevier, pp. 515 – 563.

—, Bloom, N., Blundell, R., Griffith, R. and Howitt, P. (2005). Competition and innovation: An inverted-u relationship. *The Quarterly Journal of Economics*, **120** (2), 701–728.

— and Howitt, P. (1992). A model of growth through creative destruction. *Econometrica*, **60** (2).

Akcigit, U., Celik, M. A. and Greenwood, J. (2016). Buy, keep, or sell: Economic growth and the market for ideas. *Econometrica*, **84** (3), 943–984.

Barro, R. J. and Sala-i Martin, X. I. (2003). *Economic Growth*. The MIT Press, 2nd edn.

Benhabib, J., Perla, J. and Tonetti, C. (2014). Catch-up and fall-back through innovation and imitation. *Journal of Economic Growth*, **19** (1), 1–35.

Bloom, N., Jones, C. I., Van Reenen, J. and Webb, M. (2016). Are ideas getting harder to find? *Manuscript, Stanford University, Palo Alto*.

Burdett, K., Shi, S. and Wright, R. (2001). Pricing and matching with frictions. *Journal of Political Economy*, **109** (5), 1060–1085.

Chiu, J., Meh, C. and Wright, R. (2015). *Innovation and growth with financial, and other, frictions*. Working Paper Series 4732, Victoria University of Wellington, School of Economics and Finance.

Cohen, L. R. and Ishii, J. (2005). Competition, innovation and racing for priority at the us patent and trademark office. *USC CLEO Research Paper*, (C05-13), 05–22.

Corriveau, L. (1994). Entrepreneurs, growth and cycles. *Economica*, pp. 1–15.

— (1998). Innovation races, strategic externalities and endogenous growth. *Economica*, **65** (259), 303–325.

Grossman, G. M. and Helpman, E. (1991). Quality ladders in the theory of growth. *The Review of Economic Studies*, **58** (1), 43–61.

Hall, B., Helmers, C., Rogers, M. and Sena, V. (2014). The choice between formal and informal intellectual property: a review. *Journal of Economic Literature*, **52** (2), 375–423.

Hall, B. H., Mairesse, J. and Mohnen, P. (2010). Measuring the returns to r&d. *Handbook of the Economics of Innovation*, **2**, 1033–1082.

Jones, C. I. (1995). R&D-Based Models of Economic Growth. *Journal of Political Economy*, **103** (4), 759–784.

— (2002). Sources of us economic growth in a world of ideas. *The American Economic Review*, **92** (1), 220–239.

— and Kim, J. (2014). *A Schumpeterian model of top income inequality*. Tech. rep., National Bureau of Economic Research.

Julien, B., Kennes, J. and King, I. (2000). Bidding for labor. *Review of Economic Dynamics*, **3** (4), 619–649.

Klette, T. J. and Kortum, S. (2004). Innovating firms and aggregate innovation. *Journal of Political Economy*, **112** (5).

Kortum, S. S. (1997). Research, patenting, and technological change. *Econometrica: Journal of the Econometric Society*, pp. 1389–1419.

Kultti, K. and Takalo, T. (2008). Optimal fragmentation of intellectual property rights. *International Journal of Industrial Organization*, **26** (1), 137–149.

—, — and Toikka, J. (2007). Secrecy versus patenting. *The RAND Journal of Economics*, **38** (1), 22–42.

Lemley, M. A. (2011). Myth of the sole inventor, the. *Mich. L. Rev.*, **110**, 709.

Lu, X. and McAfee, R. P. (1996). The evolutionary stability of auctions over bargaining. *Games and Economic Behavior*, **15** (2), 228–254.

Lucas, R. E. J. and Moll, B. (2014). Knowledge Growth and the Allocation of Time. *Journal of Political Economy*, **122** (1), 1 – 51.

Perla, J. and Tonetti, C. (2014). Equilibrium imitation and growth. *Journal of Political Economy*, **122** (1), 52–76.

Reinganum, J. F. (1989). The timing of innovation: Research, development, and diffusion. *Handbook of industrial organization*, **1**, 849–908.

Romer, P. M. (1990). Endogenous technological change. *Journal of Political Economy*, **98** (5 pt 2).

Segerstrom, P. S., Anant, T. C. and Dinopoulos, E. (1990). A schumpeterian model of the product life cycle. *The American Economic Review*, pp. 1077–1091.

Shimer, R. (2005). The assignment of workers to jobs in an economy with coordination frictions. *Journal of Political Economy*, **113** (5), 996–1025.

Wolinsky, A. (1988). Dynamic markets with competitive bidding. *The Review of Economic Studies*, **55** (1), 71–84.

7 Appendix

7.A Appendix A: Welfare Comparison

We follow Akcigit *et al.* (2016) and compare the welfare difference between any two economies A and B in consumption equivalent terms. In particular, consider the welfare in economy A, W^A, and economy B, W^B, along their BGPs. Suppose at time $t = 0$, both economies start at the same initial position with $N_0^A = N_0^B$. Now, welfare in economy i is given by

$$W^i = \sum_{t=0}^{\infty} \beta^t \ln C_t^i = \ln\left((1+g^i)^{\frac{\beta}{(1-\beta)^2}} C_0^{i\,\frac{1}{1-\beta}} \right) \tag{30}$$

Then, let $\alpha^{A,B}$ measure the fraction with which initial consumption in economy A, C_0^A, must be increased for consumers to have the same welfare as people in economy B. Thus, α is given by

$$\alpha^{A,B} = e^{(1-\beta)(W^B - W^A)} - 1 \tag{31}$$

This measure of welfare is used throughout the text. In particular, the welfare gain from eliminating frictions in the DE is given by $\alpha^{DE,CE}$ and the gain from eliminating frictions in the planner's allocation is given by $\alpha^{SB,FB}$.

We decompose the welfare gain from eliminating frictions into the gain from eliminating foregone innovation and the gain from eliminating wasteful innovation. The welfare gain from eliminating foregone innovation in the DE is given by $\alpha^{DE,DEF}$, where DEF is a hypothetical decentralized economy that features no foregone innovation but the same level of wasteful innovation as the DE. In particular, $g^{DEF} = g^c$, $C_0^{DEF}/N_0 = \pi(1+\lambda)/\lambda - \eta\theta^{DEF}(M-1)$, and $\theta^{DEF} = \beta\pi/(\eta(1+g^{DE}-\beta)$. Thus, the welfare cost of wasteful innovation in the decentralized economy is given by $\alpha^{DE,CE} - \alpha^{DE,DEF}$. Similarly, the welfare cost of foregone innovation in the SB is given by $\alpha^{SB,SBF}$, where SBF is a hypothetical allocation in which the planner can assign firms to projects but has to keep the fraction of wasteful innovation as in the SB. In particular, $g^{SBF} = g^{FB}$, $C_0^{SBF}/N_0 = \pi^* - \eta\theta^{SBF}(M-1)$, and $\theta^{SBF} = \theta^{SB}/(1-e^{-\theta^{SB}})$. Thus, the welfare cost of wasteful duplication of effort in the SB is given by $\alpha^{SB,FB} - \alpha^{SB,SBF}$.

7.B Appendix B: Augmented Model

We explore the robustness of the quantitative results from section five in an extension of our baseline model. The economy in this extension features uncertainty in the innovation process and endogenous research effort intensity. In the interest of consistency, the only difference with the baseline model is in the innovation sector. At stage one firms still enter at a cost $\eta > 0$ and at stage two firms still choose a direction for their R&D effort. However, now at stage three entrants choose a research intensity i which affects their probability of successfully innovating. In particular, the cost of exerting effort i is ϕi and the probability of successfully innovating the chosen project is $1 - e^{-\gamma i}$, where $\phi, \gamma > 0$. Stage four is as in the baseline model.

7.B.1 Decentralized Economy

The final good sector and the final stage of the innovation process are as in the baseline model. Hence, $P_t(n) = 1/\lambda$ and $X_t(n) = X$. At stage three, firms choose effort i that maximizes the expected reward from the R&D stage, $R_t(i) \equiv Pr(\text{patent})V_t - \phi i$. Since $Pr(\text{patent}) = Pr(\text{success})Pr(\text{patent}|\text{success}) = (1 - e^{-\gamma i})Pr(\text{patent}|\text{success})$, it follows that the optimal research effort solves

$$Pr(\text{patent}|\text{success})V_t = \frac{\phi}{\gamma}e^{\gamma j} \tag{32}$$

where j is the level of research effort in a symmetric equilibrium. The second stage is analogous to the one in the baseline model, except now there is a chance firms are not successful in innovating. Let the effective market tightness be denoted by $\tilde{\theta}_t \equiv (1 - e^{-\gamma j})\theta_t$. Then, it is straightforward to establish that the number of firms that successfully innovate a particular idea follows a Poisson distribution with mean $\tilde{\theta}_t$.[32] Thus, the probability of receiving a patent conditional on innovating is given by $Pr(\text{patent}|\text{success}) = (1 - e^{-\tilde{\theta}_t})/\tilde{\theta}_t$. Hence,

$$\frac{1 - e^{-\tilde{\theta}_t}}{\tilde{\theta}_t}V_t = \frac{\phi}{\gamma}e^{\gamma j} \tag{33}$$

Free entry implies that $\eta = R(j)$. Thus,

$$\eta + \phi j = \frac{\phi}{\gamma}(e^{\gamma j} - 1) \tag{34}$$

[32]A proof is available upon request.

which yields an implicit solution for the equilibrium research intensity j.

The laws of motion for varieties and ideas are analogous to the baseline model, with the only exception that now the probability an idea is innovated is given by $1 - e^{-\tilde{\theta}_t}$. Hence, $\nu/N = M - 1$ and $g = (1 - e^{-\tilde{\theta}})(M - 1)$. Furthermore, consumers face the same problem as in the baseline model. Thus, $V_t = \beta\pi/(1 + g - \beta)$. Using the economy's resource constraint and (34) it follows that along the BGP

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \tilde{\theta}(M-1)\frac{\phi}{\gamma}e^{\gamma j} \tag{35}$$

Finally, using (33) and the expression for V_t, it follows that the effective market tightness solves

$$\frac{\beta\pi}{1 + (1 - e^{-\tilde{\theta}})(M-1) - \beta} = \frac{\phi}{\gamma}e^{\gamma j}\frac{\tilde{\theta}}{1 - e^{-\tilde{\theta}}} \tag{36}$$

7.B.2 Coordination Economy

As in the baseline version of the model, the only difference between the DE and the CE is that at stage two of the innovation process — in the CE, a Walrasian auctioneer coordinates firm's research efforts. Thus, $P_t(n) = 1/\lambda$ and $X_t(n) = X$. Next, as in the DE, the optimal research effort in equilibrium solves

$$Pr(\text{patent}|\text{success})V_t^c = \frac{\phi}{\gamma}e^{\gamma j} \tag{37}$$

Then, let us focus on stage two. Whenever there are $\mu_t < \nu_t$ firms in the R&D sector, the auctioneer assigns a unique idea to each firm and $Pr(\text{patent}|\text{success}) = 1$. When $\theta^c \geq 1$, however, the auctioneer distributes firms to ideas as equally as she can, subject to assigning integer number of firms to each research avenue. In the event that $l \geq 1$ firms successfully innovate the same idea, they each receives the patent with probability $1/l$.[33] For example, if $\theta^c = 8.2$, then a fraction 0.2 of ideas are matched with 9 firms and a fraction 0.8 of ideas

[33] This process of coordination is different from the one in the baseline model where innovation is certain, so the auctioneer can effectively assign patents to entrants. This is because firms do not innovate for sure in our augmented model.

are matched with 8 firms. Thus, a fraction 1.8/8.2 of firms face 8 rivals and a fraction 6.4/8.2 face 7. In general, a fraction $\lceil\theta^c\rceil(\theta^c - \lfloor\theta^c\rfloor)/\theta^c$ of firms face $\lfloor\theta^c\rfloor$ rivals and a fraction $\lfloor\theta^c\rfloor(\lceil\theta^c\rceil - \theta^c)/\theta^c$ face $\lfloor\theta^c\rfloor - 1$, where $\lfloor x\rfloor$ is the largest integer less than x and $\lceil x\rceil$ is the smallest integer larger than x. Hence,

$$\begin{aligned}
Pr(\text{patent}|\text{success}) =& \frac{\lfloor\theta^c\rfloor(\lceil\theta^c\rceil - \theta^c)}{\theta^c} \sum_{l=0}^{\lfloor\theta^c\rfloor-1} \binom{\lfloor\theta^c\rfloor - 1}{l} (1 - e^{-\gamma j})^l e^{-\gamma j(\lfloor\theta^c\rfloor-1-l)} \frac{1}{l+1} \\
&+ \frac{\lceil\theta^c\rceil(\theta^c - \lfloor\theta^c\rfloor)}{\theta^c} \sum_{l=0}^{\lfloor\theta^c\rfloor} \binom{\lfloor\theta^c\rfloor}{l} (1 - e^{-\gamma j})^l e^{-\gamma j(\lfloor\theta^c\rfloor-l)} \frac{1}{l+1} \\
=& \frac{\lceil\theta^c\rceil - \theta^c}{(1 - e^{-\gamma j})\theta^c}(1 - e^{-\gamma j\lfloor\theta^c\rfloor}) + \frac{\theta^c - \lfloor\theta^c\rfloor}{(1 - e^{-\gamma j})\theta^c}(1 - e^{-\gamma j\lceil\theta^c\rceil})
\end{aligned} \tag{38}$$

The case relevant for out numerical exercise is $\theta^c \geq 1$, so we restrict our attention to it.

Next, free entry and (37) imply that

$$\eta + \phi j^c = \frac{\phi}{\gamma}(e^{\gamma j^c} - 1) \tag{39}$$

which yields the same equilibrium research effort as in the DE.

The laws of motion for varieties and ideas is the same as in the DE with the exception that now the probability an idea is innovated is given by $(\theta^c - \lfloor\theta^c\rfloor)(1 - e^{-\gamma j^c\lceil\theta^c\rceil}) + (\lceil\theta^c\rceil - \theta^c)(1 - e^{-\gamma j^c\lfloor\theta^c\rfloor})$. The consumer's optimization problem yields $V_t^c = \beta\pi/(1 - \beta + g^c)$, where $g^c = (\theta^c - \lfloor\theta^c\rfloor)(1 - e^{-\gamma j^c\lceil\theta^c\rceil}) + (\lceil\theta^c\rceil - \theta^c)(1 - e^{-\gamma j^c\lfloor\theta^c\rfloor})(M-1)$. Lastly, the resource constraint and the expression for the value of holding a patent yield

$$\left(\frac{C}{N}\right)^c = \frac{1+\lambda}{\lambda}\pi - \theta^c(M-1)(\eta + \phi j^c) \tag{40}$$

$$\left(\frac{\lceil\theta^c\rceil - \theta^c}{(1 - e^{-\gamma j})\theta^c}(1 - e^{-\gamma j\lfloor\theta^c\rfloor}) + \frac{\theta^c - \lfloor\theta^c\rfloor}{(1 - e^{-\gamma j})\theta^c}(1 - e^{-\gamma j\lceil\theta^c\rceil})\right)^{-1} \frac{\phi}{\gamma} e^{\gamma j^c} = \frac{\beta\pi}{1 - \beta + g^c} \tag{41}$$

7.B.3 Second-Best Allocation

Analogously to the baseline model, the planner solves

$$\max_{\{C_t,X_t,\tilde{\theta}_t,N_t,\nu_t,j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$AL^{1-\lambda}N_tX_t^{\lambda} = N_tX_t + C_t + \tilde{\theta}_t\nu_t\frac{\eta+\phi j}{1-e^{-\gamma j}} \tag{42}$$

$$N_{t+1} = N_t + (1-e^{-\tilde{\theta}_t})\nu_t \tag{43}$$

$$\nu_{t+1} = \nu_t + (1-e^{-\tilde{\theta}_t})(M-1)\nu_t \tag{44}$$

The first order condition with respect to X_t yields $X_t = X^*$ as in the baseline model. Furthermore, the first order condition with respect to the research effort, j, yields

$$\eta + \phi j^{SB} = \frac{\phi}{\gamma}(e^{\gamma j^{SB}} - 1) \tag{45}$$

which is the same level of research effort as in the DE. Let $\tilde{\eta} \equiv \phi e^{\gamma j^{SB}}/\gamma$, hence, the rest of the first order conditions are

$$[C_t]: \quad \beta\frac{C_t}{C_{t+1}} = \frac{\tilde{\phi}_{t+1}}{\tilde{\phi}_t} \tag{46}$$

$$[N_{t+1}]: \quad h_t = h_{t+1} + \tilde{\phi}_{t+1}\pi^* \tag{47}$$

$$[\nu_{t+1}]: \quad \lambda_t = \lambda_{t+1}\Big(e^{-\tilde{\theta}_{t+1}} + (1-e^{-\tilde{\theta}_{t+1}})M\Big) + h_{t+1}(1-e^{-\tilde{\theta}_{t+1}}) - \tilde{\phi}_{t+1}\tilde{\eta}\tilde{\theta}_{t+1} \tag{48}$$

$$[\tilde{\theta}_t]: \quad \tilde{\eta} = e^{-\tilde{\theta}_t}\Big(\frac{h_t}{\tilde{\phi}_t} + \frac{\lambda_t}{\tilde{\phi}_t}(M-1)\Big) \tag{49}$$

where $\tilde{\phi}_t$, h_t, and λ_t and the multipliers associated with (42), (43), and (44), respectively. Thus, the planner's problem reduces to the one in the baseline model. Hence,

$$\left(\frac{\nu}{N}\right)^{SB} = M - 1 \tag{50}$$

$$\left(\frac{C}{N}\right)^{SB} = \pi^* - \tilde{\eta}\tilde{\theta}^{SB}(M-1) \tag{51}$$

$$1 + (1 - e^{-\tilde{\theta}^{SB}})(M-1) = \beta\Big(1 + \frac{\pi^*}{\eta}e^{-\tilde{\theta}^{SB}} + (1 - e^{-\tilde{\theta}^{SB}} - \tilde{\theta}^{SB}e^{-\tilde{\theta}^{SB}})(M-1)\Big) \tag{52}$$

7.B.4 First-Best Allocation

Without loss of generality we impose symmetry in the production of varieties and the research effort intensity. Observe that by symmetry the planner assigns the same number of firms per idea. Hence, the probability an idea is innovated is given by $1 - e^{-\gamma\tilde{j}_t}$, where $\tilde{j}_t = j\theta_t$ is the effective research effort per idea. Now, the planner can achieve an additional unit of effective research by either increasing θ_t by $1/j$ units or increasing j by $1/\theta_t$ units. Furthermore, the cost of the former is $\nu_t\phi + \nu_t\eta/j$ units of the final good and the cost of the latter is $\nu_t\phi$. Thus, it is always cheaper to induce higher effective research effort by increasing the research intensity, j. Hence, $\theta^{FB} = 1$. Then, the planner's problem reduces to

$$\max_{\{C_t, X_t, N_t, \nu_t, j\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_t$$

$$AL^{1-\lambda}N_tX_t^{\lambda} = N_tX_t + C_t + \nu_t\eta + \nu_t\phi j \tag{53}$$

$$N_{t+1} = N_t + (1 - e^{-\gamma j})\nu_t \tag{54}$$

$$\nu_{t+1} = \nu_t + (1 - e^{-\gamma j})(M-1)\nu_t \tag{55}$$

The first order condition for X_t implies that the level of intermediate varieties is still given by X^*. Taking the rest of the first order conditions and applying straightforward algebra yields

$$\left(\frac{C}{N}\right)^{FB} = \pi^* - (\eta + \phi j^{FB})(M-1) \quad (56)$$

$$\frac{\phi}{\gamma} e^{\gamma j^{FB}} = \frac{\beta \pi^*}{1-\beta+g^{FB}} + \frac{\beta^2 \pi^* (1-e^{-\gamma j^{FB}})(M-1)}{(1-\beta+g^{FB})(1+g^{FB})(1-\beta)} - \frac{\beta(\eta+\phi j^{FB})(M-1)}{(1+g^{FB})(1-\beta)} \quad (57)$$

where $g^{FB} = (1 - e^{-\gamma j^{FB}})(M-1)$.

7.B.5 Numerical Exercise

As in the baseline case, we calibrate the model at annual frequency, so the discount factor is set at $\beta = 0.95$. Furthermore, we normalize $\gamma = L = A = 1$. To calibrate η, M, and λ we use the same three moments as in the baseline case. In addition, we set the elasticity of firm-level output with respect to R&D investment at 0.05. This value is consistent with most firm-level estimates for the U.S.[34] The elasticity in our model is given by $\gamma j e^{-\gamma j}/(1-e^{-\gamma j})$, hence, the equilibrium research effort of firms is $j = 4.5139$.[35] Setting the R&D share of GDP to its empirical value yields $\lambda = 0.8516$, as in the baseline model. Next, the fraction of patents to patent applications is $(1-e^{-\tilde{\theta}})/\tilde{\theta}$. Matching this expression to its empirical counterpart yields $\tilde{\theta} = 1.0876$. Hence, $\theta = 1.0997$. Lastly, setting $g = (1-e^{-\tilde{\theta}})(M-1) = 1.7546\%$ and using (34), (36) yields $M = 1.0265$, $\eta = 0.1611$, and $\phi = 0.0019$. The resulting welfare cost of coordination frictions in the DE is 12.76% and in the SB is 15.97%.

As in the baseline model, we explore the robustness of our quantitative results by varying the effective market tightness in the interval $\tilde{\theta} \in [0.65, 1.8]$.[36] The magnitude of the welfare costs is virtually the same as in the baseline model for all considered values of $\tilde{\theta}$ (Figure 4).

[34] For a survey see Hall *et al.* (2010).

[35] In our model firm-level output corresponds to $O(\tilde{c}) \equiv (1-e^{-\gamma\tilde{c}/\phi})(1-e^{-\tilde{\theta}})V_t/\tilde{\theta}$, where $\tilde{c} \equiv \phi j$ is the firm's R&D investment.

[36] In our augmented model the two alternative calibration strategy sets the bounds on $\tilde{\theta}$. In particular the return of R&D is now given by $\partial Y_{t+1}/\partial R_t = g\tilde{\theta}e^{-\tilde{\theta}}/((1-e^{-\tilde{\theta}})\eta\mu_t/Y_t)$.

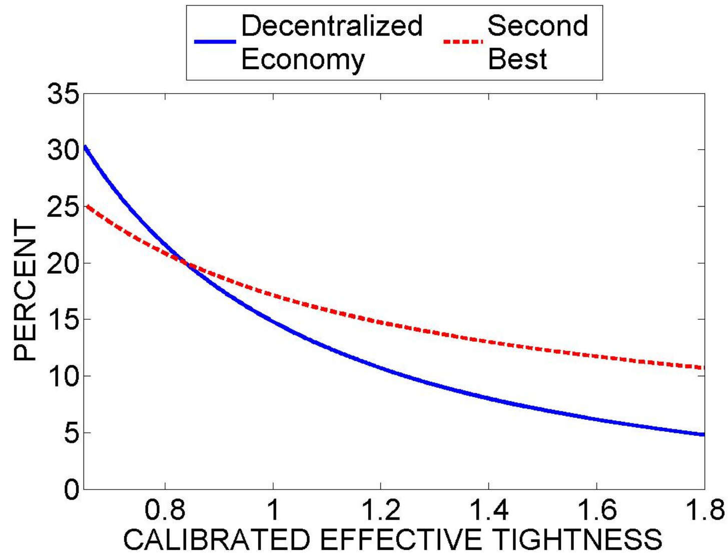


Figure 4: Welfare Gain in the Augmented Model

7.C Appendix C: Proofs Omitted from the Text

Proof of Proposition 1:

Proof. We follow previous literature (see, for example, Julien *et al.* (2000)) and threat the mass of entrants, μ_t and ideas, ν_t, as finite. Then the resulting equilibrium outcome is evaluated at the limit as $\mu_t, \nu_t \to \infty$ (keeping θ_t constant), so as to characterize the behavior in a market with continuum of firms and ideas.

First, by assumption, the firm's probability of securing a monopoly position given that there are exactly n rivals, $Pr(\text{monopoly}|n) = 1/(n+1)$. In a symmetric equilibrium all firms place the same probability s_i of directing their effort towards a particular idea i. Then, the chance that a firm would face exactly n rivals is

$$Pr(n) = \binom{\mu_t - 1}{n} s_i^n (1 - s_i)^{\mu_t - 1 - n}$$

Hence, the probability of securing a monopoly position is given by

$$
\begin{aligned}
Pr(\text{monopoly}) &= \sum_{n=0}^{\mu_t-1} Pr(\text{monopoly}|n)P(n) = \sum_{n=0}^{\mu_t-1} \binom{\mu_t-1}{n} s_i^n(1-s_i)^{\mu_t-1-n}\frac{1}{n+1} = \\
&= \frac{1}{\mu_t}\sum_{n=0}^{\mu_t-1}\binom{\mu_t}{n+1} s_i^n(1-s_i)^{\mu_t-1-n} = \frac{1}{\mu_t s_i}\left(\sum_{n=0}^{\mu_t}\binom{\mu_t}{n} s_i^n(1-s_i)^{\mu_t-n} - (1-s_i)^{\mu_t}\right) = \\
&= \frac{1-(1-s_i)^{\mu_t}}{\mu_t s_i} \qquad (58)
\end{aligned}
$$

Next, we show that $s_k = s_j$ for all $k, j \in \nu_t$. Suppose not. Then, there exists some k, j such that $s_k > s_j$. But for any $i \in \nu_t$, we have that

$$\frac{\partial Pr(\text{monopoly})}{\partial s_i} = \frac{\mu_t^2 s_i(1-s_i)^{\mu_t-1} - \mu_t[1-(1-s_i)^{\mu_t}]}{(\mu_t s_i)^2} \qquad (59)$$

For any $s_i \in (0,1)$, it follows that $Pr(\text{monopoly})$ is decreasing in s_i if and only if $(1-s_i)^{\mu_t-1} < Pr(\text{monopoly})$ which clearly holds since $\mu_t \geq 2$. Now, for $s_i = 1$, we have that $\partial Pr(\text{monopoly})/\partial s_i = -1/\mu_t < 0$. Furthermore, it is easy to see that $\lim_{s_i\to 0} \partial Pr(\text{monopoly})/\partial s_i = -(\mu_t - 1)/2 < 0$. Hence, $Pr(\text{monopoly})$ is decreasing in s_i everywhere in its domain. Then, $s_k > s_j$ implies that $Pr_k(\text{monopoly}) < Pr(\text{monopoly})_j$, which then implies that $Pr_k(\text{monopoly})V_{k,t} < Pr_j(\text{monopoly})V_{j,t}$ since all varieties are equally profitable. Thus, $s_k > s_j$ cannot be an equilibrium. Hence, we must have $s_i = s_j$ for all $i, j \in \nu_t$. Thus, $s_i = 1/\nu_t$.

Then, it follows that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) = \binom{\mu_t}{n}\left(\frac{1}{\nu_t}\right)^n\left(1-\frac{1}{\nu_t}\right)^{\mu_t-n} \qquad (60)$$

Taking the limit as $\mu_t, \nu_t \to \infty$ (keeping the ratio θ_t constant) we get that

$$Pr(i \text{ is matched with exactly } n \text{ firms}) \to \frac{\theta_t^n e^{-\theta_t}}{n!} \qquad (61)$$

■

Proof of Proposition 2

Proof. Totally differentiating both sides of (13) with respect to π yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}\pi} = \frac{\beta}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} > 0 \tag{62}$$

which is positive since $1-e^{-\theta}-\theta e^{-\theta} > 0$. Similarly, totally differentiating (13) with respect to β, η, and M yields

$$\frac{\mathrm{d}\theta}{\mathrm{d}\beta} = \Big[1 + \frac{\pi}{\eta}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big]\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} > 0 \tag{63}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\eta} = -\frac{\beta\pi}{\eta^2}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big]\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} < 0 \tag{64}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}M} = -(1-e^{-\theta})\Big[e^{-\theta}(M-1) + \frac{\beta\pi}{\eta}\Big(\frac{1-e^{-\theta}-\theta e^{-\theta}}{\theta^2}\Big)\Big]^{-1} < 0 \tag{65}$$

■

Proof of Proposition 3:

Proof. First, let us prove the following lemma

Lemma 1. *The magnitude of the congestion externality is larger than that of the learning externality.*

Proof. First, we can decompose the difference between the planner's valuation of the benefit of entry and the firm's valuation of this benefit. At the SB this difference is given by

$$\mathcal{A} + \mathcal{L} + \mathcal{C} = \eta - \Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big)V^{SB} \tag{66}$$

where $\mathcal{A}$, $\mathcal{L}$, and $\mathcal{C}$ denote the appropriability, learning, and congestion externalities; $V^{SB} := \beta\pi/(e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta)$ is the value of having a monopoly position at the second best level of the market tightness. The right hand side of (66) gives the difference between the planner's valuation of the benefit of entry, η, and the firm's, V^{SB} times the probability of securing a patent. Then, one can decompose the sum of the three externalities in the following manner

$$\mathcal{A} := \Big(\Big(\frac{h}{\phi}\Big)^{SB} - V^{SB}\Big)\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) \tag{67}$$

$$\mathcal{L} := \Big(\frac{\lambda}{\phi}\Big)^{SB}\Big(e^{-\theta^{SB}}(M-1)\Big) \tag{68}$$

$$\mathcal{C} := -\Big(\frac{h}{\phi}\Big)^{SB}\left(\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) - e^{-\theta^{SB}}\right) \tag{69}$$

Thus, $\mathcal{A}$ is the measure of how much more would the planner value entry than the firm if the appropriability externality was the only one in the model. $\mathcal{L}$ and $\mathcal{C}$ measure the same difference if the only externality in the model was learning and congestion, respectively.

From equations (68) and (69), it follows that the magnitude of the congestion externality is larger than that of the learning externality if and only if

$$\Big(\frac{h}{\phi}\Big)^{SB}\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) > e^{-\theta^{SB}}\left(\Big(\frac{h}{\phi}\Big)^{SB} + \Big(\frac{\lambda}{\phi}\Big)^{SB}(M-1)\right) \tag{70}$$

From equations (20) and (21), it then follows that (70) holds if and only if

$$\frac{(1-e^{-\theta^{SB}})\beta\pi^*}{\theta^{SB}\eta} > e^{-\theta^{SB}} + (1-e^{-\theta^{SB}})M - \beta \tag{71}$$

Next, from the planner's solution, (25), it follows that $|\mathcal{C}| > \mathcal{L}$ if and only if $\pi^* - \eta\theta^{SB}(M-1) > 0$. But this has to hold, from equation (24), as the SB must feature $C_t > 0$.

■

Now, let us turn back to the problem of implementing the SB. The government imposes a tax on R&D activities at a rate τ and subsidizes the purchase of intermediate varieties at a rate s. Furthermore, it keeps a balanced budget through the means of lump-sum transfers to households in the amount T_t. Thus, the government's budget constraint is given by

$$T_t = \int_0^{N_t} sP_t(n)X_t(n)dn - \tau\eta\mu_t \tag{72}$$

The final good firm chooses labor and intermediate inputs to maximize profits, now given by $Y_t - w_t L - \int_0^{N_t}(1-s)P_t(n)X_t(n)dn$. The first order conditions yield the same labor demand equation as in the DE, $w_t = (1-\lambda)Y_t/L$, and an inverse demand function for intermediaries given by $P_t(n) = \lambda A L^{1-\lambda} X_t^{\lambda-1}(n)/(1-s)$.

At stage three of the innovation process, the monopolist faces an analogous problem as in the DE. The only difference now is in the inverse demand function. Hence, in equilibrium, $P = 1/\lambda$, $X = [A\lambda^2/(1-s)]^{1/(1-\lambda)}L$, $\pi = (1-\lambda)X/\lambda$, $Y_t = [A(\lambda^2/(1-s))^{\lambda}]^{1/(1-\lambda)}LN_t$.

As in the economy without government intervention, all ideas are equally profitable, so the matching technology is as in the DE. The free entry condition is now given by

$$\eta(1+\tau) = \frac{1-e^{-\theta_t}}{\theta_t}V_t \tag{73}$$

where the value of the monopoly position, V_t, is defined as in the DE.

The laws of motion for ideas and varieties, and the Euler equation are as in the DE. Hence, the value of the monopoly position is still given by (10). Furthermore, the resource constraint is still given by (11).

Along the BGP, we still have that $\nu_t/N_t = M-1$, as the laws of motion for ideas and varieties are as in the DE. Thus, from the resource constraint, (11) it follows that

$$\frac{C}{N} = \frac{1-s-\lambda^2}{(1-\lambda)\lambda}\pi - \eta\theta(M-1) \tag{74}$$

Next, (10), the law of motion for ideas, and the free entry condition imply that

$$1 + (1-e^{-\theta})(M-1) = \beta\Big(1 + \frac{\pi}{\eta(1+\tau)}\Big(\frac{1-e^{-\theta}}{\theta}\Big)\Big) \tag{75}$$

Then, setting $s = s^{SB}$ implies that $\pi = \pi^*$ and setting $\tau = \tau^*$ implies that $\theta = \theta^{SB}$. Thus,

$C/N = (C/N)^{SB}$. Furthermore, τ^* is given by

$$\tau^* = \frac{\beta\pi^*(1-e^{-\theta^{SB}})}{\eta\theta^{SB}(e^{-\theta^{SB}}+(1-e^{-\theta^{SB}})M-\beta)} - 1 \tag{76}$$

To see that the optimal tax rate is positive because the congestion externality dominates the learning one, observe that

$$\begin{aligned} -\mathcal{C}-\mathcal{L} = & \Big(\frac{h}{\phi}\Big)^{SB}\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) - e^{-\theta^{SB}}\Bigg(\Big(\frac{h}{\phi}\Big)^{SB} + \Big(\frac{\lambda}{\phi}\Big)^{SB}(M-1)\Bigg) \\ = & \Big(\frac{h}{\phi}\Big)^{SB}\Big(\frac{1-e^{-\theta^{SB}}}{\theta^{SB}}\Big) - \eta \\ = & \eta\tau^* \end{aligned} \tag{77}$$

where the first equality follows from (20) and the second equality from (21) and the fact that the SB growth rate is given by $(1-e^{-\theta^{SB}})(M-1)$. Hence, $|\mathcal{C}| > |\mathcal{L}| \Rightarrow \tau^* > 0$.

■

Proof of Proposition 4

Proof. Totally differentiating (25) with respect to π^*, β, η, and M respectively and applying some algebra yields

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\pi^*} = \frac{\beta e^{-\theta^*}/\eta}{e^{-\theta^*}(M-1)+(1-\beta)\Big(e^{-\theta^*}+(1-e^{-\theta^*})M\Big)} > 0 \tag{78}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\beta} = \frac{1+\pi^* e^{-\theta^*}/\eta + (1-e^{-\theta^*}-\theta^* e^{-\theta^*})(M-1)}{e^{-\theta^*}(M-1)+(1-\beta)\Big(e^{-\theta^*}+(1-e^{-\theta^*})M\Big)} > 0 \tag{79}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}\eta} = -\frac{\beta\pi^* e^{-\theta^*}/\eta^2}{e^{-\theta^*}(M-1)+(1-\beta)\Big(e^{-\theta^*}+(1-e^{-\theta^*})M\Big)} < 0 \tag{80}$$

$$\frac{\mathrm{d}\theta^*}{\mathrm{d}M} = -\frac{(1-\beta)(1-e^{-\theta^*})+\beta\theta^* e^{-\theta^*}}{e^{-\theta^*}(M-1)+(1-\beta)\Big(e^{-\theta^*}+(1-e^{-\theta^*})M\Big)} < 0 \tag{81}$$

■

Proof of Proposition 5:

Proof. First, let us explicitly characterize the environment in the CE. The only difference to the DE is at the second stage in the innovation process. Coordination is achieved through the means of a centralized allocation of firms to ideas. In particular, upon entry, a Walrasian auctioneer directs firms' research efforts and assigns patents in the following way. If $\mu_t \leq v_t$, then each firm is directed towards a distinct project and each firm receives a patent. If $\mu_t > \nu_t$, the auctioneer chooses ν_t firms at random, assigns each a distinct project, and grants each a patent over the corresponding variety. The rest $\mu_t - \nu_t$ firms are randomly assigned a project, but none of them receives a patent.

The assumption we have placed on the parameter vales ensures that firms find all research avenues profitable. Hence, in equilibrium, all ideas are innovated, i.e. $\mu_t \geq \nu_t$, and each firm secures a patent with probability $Pr(\text{monopoly}) = 1/\theta_t$. Hence, the laws of motion for ideas and varieties are given by

$$\nu_{t+1} = M\nu_t \tag{82}$$

$$N_{t+1} = N_t + \nu_t \tag{83}$$

Since the final good sector and the intermediate varieties production technology are as in the DE, it follows that in equilibrium it is still the case that $P_t(n) = 1/\lambda$, $X = (\lambda^2 A)^{1/(1-\lambda)}L$, $Y_t = (\lambda^{2\lambda}A)^{1/(1-\lambda)}LN_t$, $\pi = X(1-\lambda)/\lambda$, $V_t^c = \sum_{i=t+1}^{\infty} d_{it}\pi$. As all ideas are equally productive, the free entry condition is now given by

$$\eta = \frac{1}{\theta_t}V_t^c \tag{84}$$

Moreover, consumers face the same problem as in the DE, so the Euler equation is analogous to (10):

$$V_t^c = \beta\frac{C_t}{C_{t+1}}\left(\pi + V_{t+1}^c\right) \tag{85}$$

Furthermore, the resource constraint is still given by (11).

One can establish in a manner analogous to that in the DE case that have $g_Y = g_C = g_N = g_\mu = g_\nu$. However, now from the law of motion for ideas, it follows that $g_\nu = M - 1$.

Next, using the laws of motion for ideas and varieties, it follows that along the BGP we still have, $\nu/N = M - 1$. Furthermore, from the resource constraint, it follows that

$$\frac{C}{N} = \frac{1+\lambda}{\lambda}\pi - \eta\theta^c(M-1) \tag{86}$$

Lastly, using the free entry condition and the Euler equation, it follows that the market tightness is given by

$$\theta^c = \frac{\beta\pi}{\eta(M-\beta)} \tag{87}$$

Next, we can compare the percent of wasteful innovations in the two economies. In the CE there are μ_t innovations and ν_t of those are beneficial. Hence, $\omega^c = 1 - 1/\theta^c$. Then, it follows that

$$\omega - \omega^c = \frac{\eta(M-\beta)}{\beta\pi} - \frac{\eta(1+g-\beta)}{\beta\pi} = \frac{e^{-\theta}(M-1)\eta}{\beta\pi} > 0 \tag{88}$$

Next, from (87) it follows that

$$\frac{\theta^c}{1-e^{-\theta}} = \frac{\beta\pi}{\eta(M-\beta)(1-e^{-\theta})} > \frac{\beta\pi}{\eta(1+(1-e^{-\theta})(M-1)-\beta)} = \frac{\theta}{1-e^{-\theta}} \tag{89}$$

where the inequality follows because $\beta < 1 \Rightarrow 1 + (1-e^{-\theta})(M-1) - \beta > (M-\beta)(1-e^{-\theta})$. Hence, $\theta^c > \theta$. ■

Proof of Proposition 6:

Proof. The results for the fraction of foregone innovation are immediate from Proposition 2. Next, let us look at the difference in the fraction of wasteful simultaneous innovation.

Totally differentiating equation (29) with respect to π, β, η, and M yields

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}\pi} = -\frac{\omega-\omega^c}{\pi} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}\pi} < 0 \tag{90}$$

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}\beta} = -\frac{\omega-\omega^c}{\beta} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}\beta} < 0 \tag{91}$$

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}\eta} = \frac{\omega-\omega^c}{\eta} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}\eta} > 0 \tag{92}$$

$$\frac{\mathrm{d}(\omega-\omega^c)}{\mathrm{d}M} = \frac{\eta e^{-\theta}}{\beta\pi} - (\omega-\omega^c)\frac{\mathrm{d}\theta}{\mathrm{d}M} > 0 \tag{93}$$

Next, let us look in the difference in the market tightness. Given $\theta^c = \beta\pi/(\eta(M-\beta))$, it follows that

$$\frac{\mathrm{d}\theta^c}{\mathrm{d}\pi} = \frac{\beta}{\eta(M-\beta)} \tag{94}$$

$$\frac{\mathrm{d}\theta^c}{\mathrm{d}\eta} = -\frac{\beta\pi}{\eta^2(M-\beta)} = -\left(\frac{\pi}{\eta}\right)\frac{\mathrm{d}\theta^c}{\mathrm{d}\pi} \tag{95}$$

$$\frac{\mathrm{d}\theta^c}{\mathrm{d}\beta} = \frac{M\pi}{\eta(M-\beta)^2} = \left(\frac{M\pi}{\beta(M-\beta)}\right)\frac{\mathrm{d}\theta^c}{\mathrm{d}\pi} \tag{96}$$

Then, using equations (62) and (94) and straightforward algebra, it follows that

$$\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\pi} < 0 \tag{97}$$

Similarly, equations (64) and (95) imply that

$$\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\eta} = -\left(\frac{\pi}{\eta}\right)\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\pi} > 0 \tag{98}$$

Lastly, equation (63) implies that

$$\begin{aligned}
\frac{\mathrm{d}\theta}{\mathrm{d}\beta} &= \left(\frac{\eta\theta}{\beta(1-e^{-\theta})}\right)\left(1+\frac{\pi}{\eta}\left(\frac{1-e^{-\theta}}{\theta}\right)\right)\frac{\mathrm{d}\theta}{\mathrm{d}\pi} \\
\frac{\mathrm{d}\theta}{\mathrm{d}\beta} &= \left(\frac{\pi}{\beta}+\frac{\pi}{1-\beta+g}\right)\frac{\mathrm{d}\theta}{\mathrm{d}\pi} \\
\frac{\mathrm{d}\theta}{\mathrm{d}\beta} &= \left(\frac{(1+g)\pi}{\beta(1-\beta+g)}\right)\frac{\mathrm{d}\theta}{\mathrm{d}\pi} \qquad (99)
\end{aligned}$$

Since $(1+g)\pi/(\beta(1-\beta+g)) > M\pi/(\beta(M-\beta))$, it follows that

$$\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\beta} < \left(\frac{M\pi}{\beta(M-\beta)}\right)\frac{\mathrm{d}(\theta^c-\theta)}{\mathrm{d}\pi} < 0 \qquad (100)$$

■