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Abstract

We develop a game-theoretic analysis of terrorism that examines the interaction between a terrorist organization and multiple target countries, and considers both pre-emption and deterrence as counter-terrorist policies. The damage from terror includes not only the material cost of fatality, injury and loss of property, but also the resultant fear. The fear-effect leads to different kinds of equilibria and implications for counter-terrorism policies. In particular, the model identifies conditions under which greater pre-emption may be the rational response to an increase in terrorism, i.e., it analyzes the merit of the dictum: "offense is the best defense." Further, it examines the characteristics of cooperative behavior among target countries in dealing with the threat of terrorism.

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1 Introduction

This is the way the world ends,
This is the way the world ends,
This is the way the world ends,
Not with a whimper, but a bang.
- With apologies to T.S. Elliot.

Terrorism has become a global phenomenon and a matter of concern for both developed and developing countries. While there is a large list of terrorist organizations, many of which are 'local' and target one particular country, over the last two decades some terrorist organizations have been targeting multiple countries. Al-Qaeda is the prime example at present.[1] In the 1990s, the Abu Nadal Organization (ANO) carried out activities against twenty countries, including U.S., France, Israel and various Arab countries (U.S. Department of State, 1997).[2]

Multi-country targeting not only involves strategic interdependence between a terrorist organization and individual countries, it implies such interdependence among the target countries themselves. In this paper, we adopt a game-theoretic approach to study this problem, by explicitly formulating the choice problem facing a terrorist organization and its target countries. In the game-theory terminology it can be called a *common-enemy* problem.[3] We consider a two-stage game where, in the first stage, the countries simultaneously decide on their levels of both pre-emption and deterrence, followed in the next stage by the organization deciding on its levels of attack.

Attacks are conceptualized through the notion of a production function relating 'terror input' to potential damage. Damage from terror not only includes direct effects like loss of property and human tragedy in the form of casualties and fatalities, but is also meant to encompass the fear it engenders among the public. In other words, terrorism is viewed as production of *terror*. As recognized since Hobbs, human life is governed by a social contract, which pre-supposes social stability. Terror is aimed at undermining this very sta-

[1]Not only does Al-Qaeda itself targets many countries, it purportedly supports local or regional terrorist organizations, thus creating inter-linkages among the activities of the latter.

[2]Since Abu Nadal's death in 2002 the ANO has remained largely passive.

[3]Mirza and Verdier (2006) also stresses the necessity of such a formulation, but they stop short of setting up a formal analytical model.

bility, leading to significant distortions in social contracts. Recently, Becker and Rubinstein (2005) have argued that small-probability events such as an incident of terror can, via producing fear, have major effects on people's utility and well-being. According to Richman et. al. (2008), *9/11* has had a lingering effect on mental health of Americans in the form of depression, anxiety, drinking etc. Additional empirical evidence that terrorism typically has large effects on economic outcomes is provided by Abadie and Gardeazabal (2003).[4]

A main innovation of this paper is to characterize fear and its implications. More specifically, we differentiate between *weak-marginal-fear* and *strong-marginal-fear*, referring to whether an increase in the deployment of the terror input leads to a decrease, or an increase in its *marginal* effect on expected damage. We consider equilibrium in each of these two cases – as well as in a situation, where, depending on the magnitude of the attacks, there could be a regime switch from weak-marginal-fear to strong-marginal-fear. In the presence of regime switch there could be an equilibrium, which we shall call brink-of-panic (BOP for short), in which the terrorist organization is indifferent between operating in the weak and the strong-marginal fear regime (to be made precise later). Interestingly, the resultant situation is knife-edge in the aggregate in that an arbitrarily small reduction in pre-emption by any target country would lead to a drastic increase in the scale of attack by the Organization with the countries 'plunging' into panic, but it can be sustained as a (self-enforcing) Nash-equilibrium. Furthermore, multiple equilibria may arise.

One of the main predictions of our model is that as a response to an increase in the potency of the organization – interpreted as an increase in terrorism – the target countries scale up their security-deterrence measures, but may either decrease or increase pre-emption, depending on whether the marginal-fear effect is weak or strong. Along the BOP equilibrium 'an increase in terrorism' necessarily leads to an increase in pre-emption, despite the fact that the equilibrium occurs in the weak-marginal-fear region.

Inter alia, our results demonstrate the usefulness of explicitly modeling the terrorist organization as an optimizing agent. In comparison to Sandler and Sequeira (2006) for example, who adopt a reduced-form approach and directly postulate that (a) an increase in security-deterrence by one tar-

[4]In their follow-up work, Abadie and Gardeazabal (2008) argue that mobility of productive capital in an open economy may account for much of the impact of terrorism.

get country induces the terrorist organization to focus more on other target countries and (b) pre-emption measures undertaken by target countries that reduce the *overall* strike capacity of the organization are strategic substitutes, our analysis shows that depending on institutional realities, these may or may not hold, so that imposing these properties by fiat may be misleading.

Another insight is that the equilibrium nature of counter-terrorism policies and how they respond to changes in parameters facing the terrorist organization, depend on the characteristics of fear. In particular, the model sheds light on the merit of the dictum, "offense is the best defense", i.e., whether more offense is the optimal response to a higher threat of terrorism. We find that this is true in the strong marginal-fear, or what we call in section 3, the panic equilibrium, as well as in the BOP equilibrium. But it does not hold in the weak-marginal-fear or the no-panic equilibrium.

In terms of welfare implications for target countries we find that there is a role for cooperation irrespective of whether the marginal fear effect is weak or strong, and whether pre-emption measures are strategic substitutes or complements. Under cooperation the aggregate level of pre-emption is higher compared to the non-cooperative equilibrium, while security-deterrence levels by individual countries are lower. These results hold as long as there are no possible regime switches. If a regime switch from weak to strong-marginal fear is possible however, additional implications follow. In the BOP equilibrium explicit collusion may not be required at all – because the countries coordinate on the appropriate levels of pre-emption anyway so as to avoid panic. Furthermore, the presence of multiple equilibria imply that, even in the absence of explicit collusion, coordination alone can be welfare improving.

We finally relate our paper to the literature, in particular the game theoretic ones.[5] Das and Lahiri (2006) have formulated a non-cooperative game between one terrorist organization and *one* target country, in which the former selects its scale of attack and the latter its security-deterrence level. They do not allow for multiple countries, or pre-emption though. Sandler and Siqueira (2006) do consider interdependence between target countries – in terms of choosing the levels of deterrence and pre-emptive strikes – but the behavior of the terrorist organization remains implicit. Postulating 'reduced-form' externalities between target countries is subject to a 'Lucas-type critique'. An ad hoc analysis of this kind has two problems. First, functions

[5]There is an earlier theoretical literature dealing with hostage scenarios and subsequent negotiations between terrorists and governments.

representing externalities between target countries may change when some parameters facing either these countries or the organization change. Second, the presumed nature of externalities – however intuitive they may be – may not be valid.

The rest of the paper proceeds as follows. In the next section we set up the basic model, and analyze the case where the marginal-fear effect is either uniformly strong, or uniformly weak. In section 3, we examine an integrated model that allows for a regime switch from weak to strong-marginal-fear. Section 4 discusses some extensions of our model and concludes the paper.

2 The Model

2.1 General Features

An economic analysis dealing with a terrorist organization and target countries is built on two maintained hypotheses. (1) Like the countries, the terrorist organization is a rational entity, taking into account benefits and opportunity costs of its actions. (2) Reducing damage from terror is costly for a target country and these costs are taken to consideration while setting counter-terrorism policies; in other words, countries do *not* aim to reduce or eliminate terror at any arbitrarily high cost.

In our model there is one terrorist organization – henceforth called the Organization – and at least two target countries, indexed by $i \in \{1, 2, .., I\}$. We view terrorist acts in terms of a production function. Let x_i denote the level of a composite input being used against country i. We will call it the terror input supplied to country i. The strategy vector of the Organization is $(x_1, \cdots, x_I)$, where $x_i \geq 0$.

The target countries choose security-deterrence, as well as pre-emptive measures, each represented by a univariate index, namely, s_i and z_i respectively. The strategy vector of country i is thus a non-negative vector (s_i, z_i).

The target countries and the Organization play a two-stage game, with the former moving first and simultaneously, choosing the level of their counter-terrorism measures, followed by the latter in stage two, choosing its terror input levels. The nature of timing amounts to commitment on the part of the target countries with respect to counter-terrorism policies.[6] We ignore re-

[6]It will be noted later that in a simultaneous game, optimal pre-emption is always zero.

peated interactions, or negotiations between the Organization and the target countries.

The *expected* damage to a country, D_i, is a function of x_i and s_i, i.e., $D_i = F(s_i, x_i)$. We assume $F(s_i, x_i) : [0, \infty)^2 \to [0, \infty)$, where the function $F(\cdot)$ is adequately differentiable. This function is a product of two functions $P = P(\underset{-}{s_i}, \underset{-}{x_i})$ and $y(\underset{+}{x})$, where P is the probability of success in implementing terrorist activities and y is the potential damage, i.e., the total damage if the activities succeed. The higher the level of security-deterrence, the greater is probability of detection and hence the smaller is the probability of success. It is natural to assume

A1: $F_s(s_i, x_i) < 0, \qquad F_{ss}(s_i, x_i) \geq 0, \qquad F_{sx}(s_i, x_i) \leq 0,$

i.e. the expected damage is decreasing and convex in the level of deterrence and the marginal damage with respect to the terror input is decreasing in the level of deterrence.

In general, x has two opposing effects on $F(x, s)$. First, there is the direct effect through the production function $y(x)$. While, in a narrow sense, $y(x)$ can be thought of as some single index capturing casualties, fatalities and loss of property, we will interpret it more generally. Given that causing fear among the common mass is the hall-mark of terrorism as a form of conflict, we presume that $y(x)$ includes the psychological damage from fear. The contagiousness of fear motivates a further assumption that $y''(x) > 0$.[7] Second, ceteris paribus, a higher level of x_i attracts greater visibility and chance of detection, which has a negative impact on $F(s, x)$. In general, $F_x(s_i, x_i) = P(s_i, x_i)y'(x_i) + P_x(s_i, x_i)y(x_i) \gtrless 0$. But it is natural to assume that the direct effect dominates, so that

A2: $F_x(s_i, x_i) > 0$.[8]

Importantly, observe that $F_{xx} \lesseqgtr 0$. Clearly, the sign and the magnitude

[7] Thus, fear is dependent on the magnitude of terrorist attempts, not on the rate of their success. Alternatively, one can add another tier or mapping on $F(\cdot)$, say $\mathcal{F}[F(\cdot)]$, with $\mathcal{F}' > 0$, $\mathcal{F}'' > 0$, where $\mathcal{F}$ represents both fear and the material damage caused. This would imply $\mathcal{F} = \bar{\mathcal{F}}(s, x)$. Imposing analogous assumptions on the $\bar{\mathcal{F}}$ function would yield the same results.

[8] Otherwise, the Organization would have no incentive to launch any terrorist activity against country i, i.e., the problem of terrorism would not arise.

of F_{xx} depend on the strength of fear relative to the negative visibility effect of an increase in x_i. More precisely, $F_{xx} > 0$ if the marginal fear effect is relatively strong, i.e., if $y''(\cdot)$ is sufficiently large. Otherwise, $F_{xx} < 0$. As one might expect, the qualitative aspects of variations in equilibrium policy level would depend on the sign and magnitude of F_{xx}.

We do not model the long-term goal of the Organization; neither do we take a position on how reasonable it is. Our primitive is that there is a group of sympathizers behind the 'cause' and the Organization is an outgrowth of that. It derives its utility from the expected damage it is able to directly inflict on the target countries, equal to $\Sigma_i D_i$. It is presumed that all countries are targeted equally.[9]

The Organization also cares about the cost of its 'production.' These include the cost of recruiting, financing and training terrorists, cost of equipment and that of building and maintaining infrastructure etc. They are specified by

$$T = C\left(\underset{(+)}{Z} + \alpha, \underset{(+)}{X}\right) + \Sigma_i c(\underset{(+)}{x_i}), \; C_{ZX} > 0, \text{ where } Z \equiv \Sigma_i z_i, \; X \equiv \Sigma_i x_i. \quad (1)$$

The functions $C(\cdot)$ and $c(\cdot)$ respectively are the 'common-resource' and country-specific costs. The former would include, for example, the costs of maintaining infrastructure to train terrorists. Note that Z is the aggregate level of pre-emptive measures undertaken by all target countries. An increase in Z (in terms of bombing terrorist bases, freezing suspect funds etc.) shifts up the Organization's cost function.[10]

A decrease in α reflects either a technology improvement or an exogenous infusion of resources for the Organization. It can be interpreted as the Organization becoming more potent or simply 'an increase in terrorism'.

The positive sign of the cross partial in the $C(\cdot)$ function means that an increase in aggregate pre-emption or a decrease in the potency of the Organization increases its common-resource marginal cost.

We assume increasing marginal country-specific costs i.e.,

A4: $c''(x) > 0$.

[9] In the concluding section we discuss some implications of allowing for asymmetric targeting.

[10] This specification ignores country-specific pre-emptive actions, and thereby enables us to differentiate this policy sharply from security-deterrence which is largely a country-specific policy.

At this point we do not impose any restriction on the sign of C_{XX}, i.e., whether the common-resource marginal cost function is increasing, constant or decreasing. We shall see below that the sign of C_{XX} will determine the nature of the cross effect of security-deterrence choice by one country on the Organization's choice of terror input toward another.

2.2 The Organization's Problem: Stage 2

It seeks to maximize $\Sigma_i D_i - T$ with respect to $(x_1, \cdots, x_I)$, given s_i and z_i. The first-order conditions are:

$$F_x(s_i, x_i) = C_X(Z + \alpha, X) + c'(x_i), \ \forall i. \tag{2}$$

The left-hand and right-hand side expressions respectively indicate the marginal benefit and marginal cost facing the Organization. We impose that

A5: (i) $|c''|$ is sufficiently large, such that $F_{xx}(\cdot) - IC_{XX}(\cdot) - c''(\cdot) < 0$, $F_{xx}(\cdot) - c''(\cdot) < 0$ and (ii) $|F_{xx}(\cdot) - C_{XX}(\cdot) - c''(\cdot)| > (I-1)|C_{XX}(\cdot)|$,

which ensures that the second-order conditions for the Organization's problem are met.

The following proposition summarizes the response of the Organization to changes in deterrence and pre-emptive measures.

Proposition 1 *Let assumptions A1-A5 hold. An increase in the aggregate pre-emption forces the Organization to lower its scale of activities in all target countries, while an increase in security-deterrence by country i induces the Organization to (a) decrease x_i but (b) increase, leave unchanged or decrease x_j $(j \neq i)$ as $C_{XX} \gtreqless 0$.*

Proof: See Appendix.

The first part of Proposition 1 is apparent. It is also intuitive that if country i enhances its security-deterrence measure, the Organization scales x_i down. Aggregate level of terror input, X, tends to fall. This decreases or increases the marginal cost of using common resources as $C_{XX} \gtrless 0$. In turn, it decreases or increases the 'total' marginal cost of x_j – and therefore induces the Organization to step up or reduce x_j – according as $C_{XX} \gtrless 0$. Hence, unlike Sandler and Sequeira (2006), it is not necessary that the Organization would redirect its activities more toward a particular country if it faces more stringent security-deterrence measures in another.

Having noted Proposition 1, we however assume henceforth, for the sake of analytical tractability, the intermediate case of

A6: $C_{XX} = 0$.

This assumption essentially implies no externalities between target countries originating from their choice with regard to security-deterrence. Given A6, we can specify, without loss of generality,

$$\mathcal{C} = (Z + \alpha)(\Sigma_i x_i) + \Sigma_i c(x_i). \tag{3}$$

Accordingly, (2) simplifies to

$$F_x(s_i, x_i) = Z + \alpha + c'(x_i). \tag{4}$$

In keeping with Proposition 1, (4) implicitly yields $x_i = g(s_i, Z + \alpha)$, where

$$\frac{\partial g}{\partial s_i} = \frac{F_{sx}}{c'' - F_{xx}} < 0; \quad \frac{\partial g}{\partial Z} = -\frac{1}{c'' - F_{xx}} < 0. \tag{5}$$

2.3 Counter-Terrorism Measures by Target Countries: Stage 1

The target countries incur the damage from terror, as well as the costs of security-deterrence and pre-emptive measures. Let the latter two be indicated by $\beta_i u(s_i)$ and $\lambda_i v(z_i)$ respectively. The total cost function is given by $\Omega_i(x_i, s_i, z_i) = F(s_i, x_i) + \beta_i u(s_i) + \lambda_i v(z_i)$. We assume

A7: $u',\ v',\ u'',\ v'' > 0$,

i.e. the respective cost functions are convex. A target country's objective is to minimize this cost. Note that the benefit to country i from pre-emption enters through the Organization's choice of x_i. Thus, had we assumed a simultaneous game, it would have meant minimizing Ω_i at given x_i, implying optimal z_i to be zero.

Because of the two-stage nature of the game, the qualitative nature of equilibrium would, in general, depend partly on third-order derivatives. For analytical simplicity as well as ease of interpretation, we assume however that

A8: F_{ss}, F_{sx}, F_{xx}, c'', u'' and v'' are all constant.

In what follows we compare non-cooperative equilibrium with cooperative equilibrium in which the target countries coordinate their counter-terrorism policies. The optimization problems facing the countries in the two scenarios would require various second-order conditions to be met. These are collectively captured in:

A9:

$$(R1) \quad c'' > F_{xx}. \tag{6}$$

$$\begin{aligned}&(R2) \quad \lambda_i v''(c'' - F_{xx})^2 A_i - B_i > 0, \text{ where} \\ &A_i \equiv (F_{ss} + \beta_i u'')(c'' - F_{xx})^2 + (2c'' - F_{xx})F_{sx}^2 \\ &B_i = (c'')^2 F_{sx}^2 - A_i F_{xx}.\end{aligned} \tag{7}$$

$$(R3) \quad \mathbf{K} \equiv \begin{bmatrix} \lambda_1 \tilde{A} v''(c'' - F_{xx})^2 - \tilde{B} & .... & -\tilde{B} \\ -\tilde{B} & .... & -\tilde{B} \\ .... & & \\ .... & & \\ -\tilde{B} & .... & \lambda_I \tilde{A} v''(c'' - F_{xx})^2 - \tilde{B} \end{bmatrix}$$

is positive semi-definite,

$$\text{where } \tilde{A} = \frac{1}{\sum_i 1/A_i}; \; \tilde{B} = (c'')^2 F_{sx}^2 - I\tilde{A}F_{xx}. \tag{8}$$

In this section we assume that F_{xx} is either positive or negative for all $x > 0$; given this, A9 is met if c'', β_i and λ_i are large enough.[11]

We now analyze the non-cooperative Nash behavior of target countries in stage 1. Country i aims to:

$$\min_{s_i, z_i} \Omega_i(s_i, z_1, \cdots, z_I) = F[s_i, g(s_i, Z + \alpha)] + \beta_i u(s_i) + \lambda_i v(z_i) \tag{9}$$

subject to (5). Under our assumption of $C_{XX} = 0$, country i's cost is not affected by any other country's security-deterrence measures. But it is affected (partly) by others' choice of pre-emption levels. This is the basis of

[11] All conditions in A9 need not be binding, e.g., if $F_{xx} \leq 0$, (6) is met for any $c'' > 0$.

externality between the choices of pre-emption, which is not internalized in the non-cooperative equilibrium.[12]

The following first-order conditions hold for country i:

$$\begin{aligned}
&\frac{\partial \Omega_i}{\partial s_i} = F_s(\cdot) + F_x(\cdot)\frac{\partial g}{\partial s_i} + \beta_i u'(s_i) = 0 \\
&\Leftrightarrow \beta_i u'(s_i) = \left[-F_s(\cdot) - \frac{F_x(\cdot)F_{sx}}{c'' - F_{xx}}\right]. \qquad (10) \\
&\frac{\partial \Omega_i}{\partial z_i} = F_x(\cdot)\frac{\partial g}{\partial Z} + \lambda_i v'(z_i) = 0 \\
&\Leftrightarrow \lambda_i v'(z_i) = \left[\frac{F_x(\cdot)}{c'' - F_{xx}}\right]. \qquad (11)
\end{aligned}$$

Remarks:

1. In (10) and (11), the left-hand-side terms are respectively the marginal costs of deterrence and pre-emption, which are increasing in their respective arguments.
2. The marginal benefits are indicated by the terms in the right-hand side. From (10), the marginal benefit of deterrence decreases with deterrence.[13] But, from (11),

   Lemma 1 *The marginal benefit of pre-emption increases or decreases with pre-emption as* $F_{xx} \lesseqgtr 0$.

   This will serve as the key in understanding the role of pre-emptive measures. Suppose $F_{xx} < 0$, i.e., the marginal damage from the terror input is diminishing. It implies that the lower the level of this input, the higher is the marginal damage. Because a higher z_i implies, ceteris paribus, a higher Z and hence a lower level of x_i, it follows that at a higher level of pre-emption, the marginal benefit from pre-emption is higher. Likewise, if $F_{xx} > 0$, the marginal benefit decreases with pre-emption.

   Earlier, we have discussed that the greater the fear effect, the higher is the value of F_{xx}. Hence, a corollary of Lemma 1 is that the marginal benefit of pre-emption increases or decrease with pre-emption, as the marginal fear effect is sufficiently large or small.

[12] Such externality would not arise if all pre-emptive actions were country-specific.

[13] The derivative of the term in the square bracket of (10) with respect to s_i equals $-F_{ss} - c''F_{sx}^2/(c'' - F_{xx})^2$, which is negative.

Eqs. (10) and (11) imply the following rankings of counter-terrorism policies across countries.

Proposition 2 *If $\beta_i = \beta_j$ and $\lambda_i < \lambda_j$, then $s_i = s_j$ and $z_i > z_j$. Whereas if $\beta_i < \beta_j$ and $\lambda_i = \lambda_j$, then $s_i > s_j$ and $z_i < z_j$.*

The following proposition relates the levels of two policies employed by a target country, as well as whether pre-emption levels chosen across countries are strategic substitutes or complements.

Proposition 3 *(A) At any given vector of pre-emption levels chosen by other countries, security-deterrence (s_i) and pre-emption (z_i) chosen by any target country are gross substitutes.*
(B) As long as security-deterrence levels are adjusted optimally, pre-emption levels chosen across the target countries are strategic complements or substitutes as the marginal fear effect is weak or strong enough such that $B_i \gtrless 0$ (where B_i is defined in (R2)).
(C) Given that pre-emption levels by other countries are adjusted optimally, s_i and z_i are net substitutes or complements, as the marginal fear effect is weak or strong enough such that $B_i \gtrless 0$.

Proof. (A) Eq. (10) implicitly defines s_i as a function of $Z + \alpha$, with

$$\frac{ds_i}{d(Z+\alpha)} = \frac{c'' F_{sx}}{A_i} < 0.\text{[14]} \tag{12}$$

Thus, at any given $Z' \equiv Z - z_i$, s_i and z_i are negatively related.

(B) There are two components. By definition, $B_i \gtrless 0$ as $F_{xx} \lesseqgtr (c'')^2 F_{sx}^2 / A_i$. If, in the terror production function $|y''|$ is large enough, i.e., the marginal fear effect is high enough, F_{xx} would be positive and large enough to outweigh $(c'')^2 F_{sx}^2/A_i$. Otherwise, F_{xx} is positive and small enough or negative such that $F_{xx} < (c'')^2 F_{sx}^2/A_i$. This is how the marginal fear effect is related to the sign of B_i.

Next, substitute (12) into (11) to implicitly express z_i as a function of $Z + \alpha$ with

$$\frac{dz_i}{d(Z+\alpha)} = \frac{B_i}{\lambda_i v''(c'' - F_{xx})^2 A_i}. \tag{13}$$

[14] Given A1, A3 and (R1), $A_i > 0$.

Hence $dz_i/dz_j \gtreqless 0$ according as $B_i \gtreqless 0$.

(C) Finally, (12) and (13) imply

$$\frac{ds_i}{dz_i} = \frac{\lambda_i v''(c'' - F_{xx})^2 c'' F_{sx}}{B_i} \lesseqgtr 0 \text{ as } B_i \gtreqless 0. \tag{14}$$

■

It is important to understand why, unlike the case of public-good provision, pre-emption levels may not be strategic substitutes and, why security-deterrence and pre-emption may be *net* substitutes or complements.

Considering the first initially, there are two effects at work. Suppose z_j increases. It tends to increase the aggregate level of pre-emption, thereby shifting up the marginal cost function facing the Organization. It reduces the supply of terror input to any particular country i. This induces country i to reduce its security-deterrence. Since security-deterrence and pre-emptive measures are gross substitutes, a reduction in the former tends to imply a higher level of pre-emption by country i. In brief, because of substitutability between security-deterrence and pre-emption as alternative forms of counter-terrorism policies, pre-emption levels across target countries tend to be strategic complements.[15]

The other effect relates to the interdependence between z_i and z_j at any given vector of s_i's. If the marginal fear effect is weak enough to the extent that $F_{xx} < 0$, then by Lemma 1, the marginal benefit from pre-emption increases with pre-emption. As an increase in z_j tends to increase aggregate pre-emption, the marginal benefit from z_i *increases.* Country i responds by increasing z_i. On this account also pre-emption levels are strategic complements.[16] But, by analogous reasoning, if the marginal fear effect is strong enough so that $F_{xx} > 0$, pre-emption choices across countries would be strategic substitutes at given s_i's.

Combining these two effects yields that pre-emption levels may be strategic complements or substitutes. If $F_{xx} < 0$, the fear effect is weak and works in the same direction as the gross substitutability effect. If $0 < F_{xx} < (c'')^2 F_{sx}^2$, the fear effect works in the opposite direction but is outweighed by

[15]This is captured by the positive term $(c'')^2 F_{sx}^2$ in the definition of B_i. Indeed, if countries were to keep their security-deterrence levels unchanged, this term would not appear in dz_i/dz_j.

[16]Algebraically it is verified by the fact that if $F_{xx} < 0$, the term B_i is unambiguously positive.

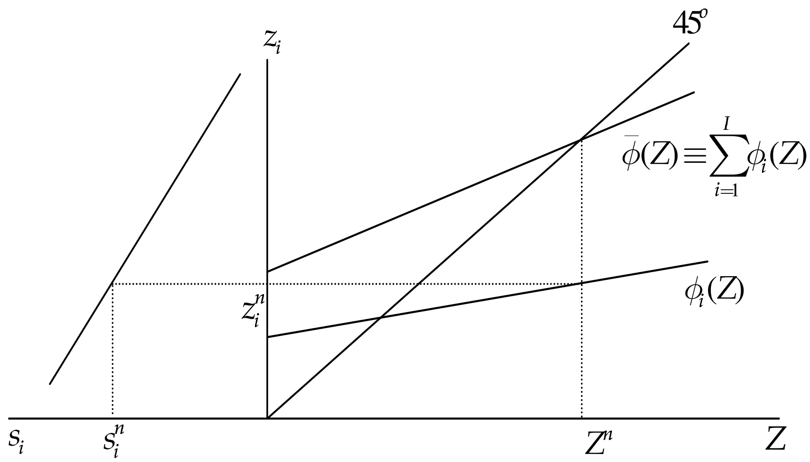

Figure 1: Non-Cooperative Equilibrium: The Case of Weak Marginal Fear

the gross substitutability effect. The net result is that pre-emption choices are strategic complements. It is only when the fear effect is strong enough to outweigh the substitutability effect between deterrence and pre-emption, pre-emption choices are strategic substitutes.

Finally, as detailed above, an increase in aggregate pre-emption, on the one hand, implies a decline in security-deterrence, and, on the other, may induce a particular country to increase or reduce its level of pre-emption measures depending on the strength of the marginal fear effect. This explains why security-deterrence and pre-emption may be net substitutes or complements.

The nature of the overall equilibrium thus depends whether the marginal fear effect is weak or strong.

2.4 Weak Marginal Fear Effect: $\mathbf{B}_i > 0$

The solution is illustrated in Figure 1. Given that $B_i > 0$, (13) implies that $dz_i/dZ > 0$. Hence the $z_i = \phi_i(Z)$ schedule in the right quadrant is upward sloping. The left quadrant shows the negative relationship between s_i and z_i in view of (14).

Summing up $\phi_i(Z)$ curves vertically yields the $\Sigma_i \phi_i(Z) \equiv \bar{\phi}(Z)$ curve. As shown in the Appendix, condition (8) implies $\bar{\phi}'(Z) < 1$. Thus the $\bar{\phi}(Z)$ curve intersects the 45° line. An equilibrium exists and it is unique. The point Z^n

marks the equilibrium level of aggregate pre-emption, where superscript n stands for non-cooperation. The pre-emption level chosen by country i is read off the $\phi_i(Z)$ curve, indicated by z_i^n. The corresponding solution of s_i^n is obtained in the second quadrant.

We now consider the comparative-statics effects of a *decrease* in the parameter α, which poses the question of how target countries react when the Organization becomes more potent. From (13), as α decreases, the $\phi_i(Z)$ and $\bar{\phi}(Z)$ curves shift down. The implication is that each target country *lowers* its level of pre-emptive activity. Furthermore, given that $B_i > 0$ and thus security-deterrence and pre-emption are net substitutes, a lower z_i is associated with a higher s_i. In summary, the target countries 'beef up' their security-deterrence measures but reduce pre-emptive measures.

Intuitively, as α falls, the Organization tends to step up the terror input in each country. To counter this the target countries enhance their security-deterrence levels. Gross substitutability between security-deterrence and pre-emption implies that each country tends to lower the level of pre-emption. In addition, a weak fear effect implies either a decline in the marginal benefit from pre-emption as the Organization increases its terror input, inducing a target country to lower its pre-emptive measure or a weak increase in marginal benefit from pre-emption and hence a small increase in the level of pre-emption, which is outweighed by the decrease in pre-emption due to the gross substitutability effect. The net effect on pre-emption is negative.

2.5 Strong Marginal Fear Effect: $\mathbf{B}_i < 0$

In this case, from (13) and (14) respectively, $dz_i/dZ < 0$ and $ds_i/dz_i > 0$. Figure 2 is the analog of Figure 1. Both the $\phi_i(Z)$ and $\bar{\phi}(Z)$ curves are downward sloping. The intersection of the latter with the 45° line defines equilibrium, which is unique. The locus between s_i and z_i is upward sloping.

Consider now a decrease in α. Both curves in the first quadrant shift to the right, implying an increase in z_i for all i. From the left quadrant, s_i increases. In this case, a higher input level chosen by the Organization in response to a decline in α sufficiently improves the marginal benefit from pre-emption. Thus pre-emption levels increase too.

Combining the cases where the marginal fear effect is weak and where it is strong, we have

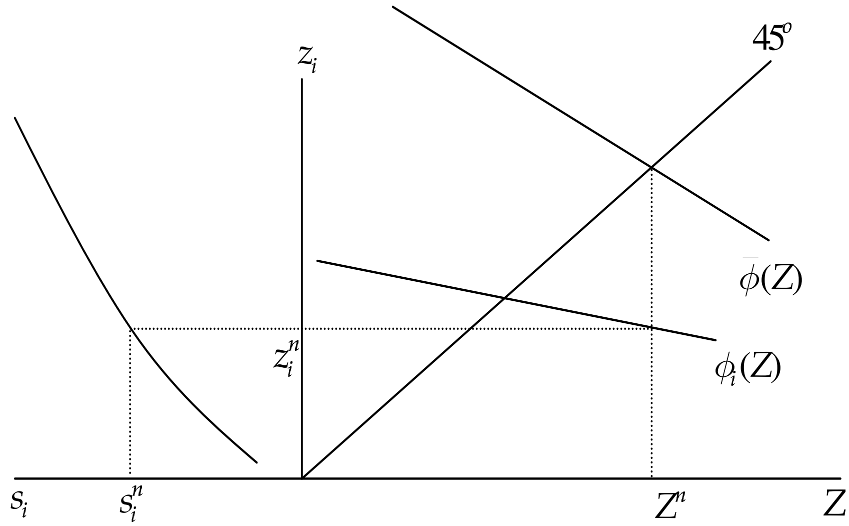

Figure 2: Non-Cooperative Equilibrium: The Case of Strong Marginal Fear

Proposition 4 *Irrespective of whether the marginal fear effect is weak or strong, there is a unique non-cooperative equilibrium. An increase in the potency of the Organization leads each target country to step up its security-deterrence level. Pre-emptive measures are scaled down or up as the marginal fear effect is weak or strong.*

2.6 Cooperation Among All Countries

Countries facing a common terrorist organization may wish to coordinate their policies to deal with the 'common enemy.' What is the nature of the co-operative solution and how does it compare with that under non-cooperation?

Suppose that in stage 1 all target countries collectively decide s_i and z_i by minimizing their joint cost. Further let there be compensatory side payments such that participation in such a grand coalition is not an issue.

In Stage 2 the Organization faces the same optimization problem however and thus it has the same input supply function (5). In Stage 1, $\Sigma_i \Omega_i = \Sigma_i \left[F(s_i, g(s_i, Z)) + \beta_i u(s_i) + \lambda_i v(z_i)\right]$ is minimized with respect to s_i and z_i. Using the partials of the $g(\cdot)$ function, the first-order conditions with respect to s_i and z_i are:

$$F_s(\cdot) + \frac{F_x(\cdot)F_{sx}}{c'' - F_{xx}} + \beta_i u'(s_i) = 0, \tag{15}$$

$$-\sum_{i=1}^{I} \frac{F_x(\cdot)}{c'' - F_{xx}} + \lambda_i v'(z_i) = 0. \tag{16}$$

Eq. (16) captures the internalization of spill-over effects of pre-emption among the target countries. But notice that (15) is same as (10). It is because given that $C_{XX} = 0$, there is no direct cross effect of s_i on country j's cost.[17] This leads us to

Proposition 5 *Full cooperation in terms of jointly setting deterrence and pre-emptive measures is equivalent to partial cooperation in terms of coordinating pre-emptive measures only.*

To compare the cooperative to the non-cooperative solution, substitute the implicit function of s_i in terms of Z derived from (15) into (16) and then differentiate (16) to obtain

$$[\lambda_i \tilde{A} v''(c'' - F_{xx})^2 - \tilde{B}]dz_i - \Sigma_{j\neq i}\tilde{B}dz_j = 0, \tag{17}$$

where $\tilde{A}$ and $\tilde{B}$ are defined earlier in (8).[18] This equation implies

$$z_i = \Phi_i(Z), \text{ with } \Phi_i' = \frac{\tilde{B}}{\lambda_i \tilde{A} v''(c'' - F_{xx})^2}. \tag{18}$$

Based on the function $\Phi_i(Z)$, we obtain the following proposition.

Proposition 6 *Compared to non-cooperation, at the cooperative equilibrium, each country chooses a lower level of security-deterrence ($s_i^c < s_i^n$) and the aggregate level of pre-emptive measures is higher ($Z^c > Z^n$). If countries are symmetric, or they are asymmetric and $B_i > 0$, then $z_i^c > z_i^n$ $\forall i$. Otherwise, $z_i^c > z_i^n$ for some i.*

[17] Hence if pre-emption levels were constant there is no over- or under- provision of deterrence.

[18] Check that, in view of (17), our assumption (R3) in A9 implies that the second-order conditions are being met.

Proof: Let $\bar{\Phi}(Z) \equiv \sum_i \Phi_i(Z)$. Suppose $B_i > 0\ \forall i$. Then, as shown in Appendix 1, $0 < \bar{\Phi}'(Z) = \sum_i \Phi'(Z) < 1$. If $B_i < 0$ for all i, $\bar{\Phi}'(Z) < 0$. Hence, in general, $\bar{\Phi}'(Z) < 1$. In turn, this implies that, if Z^c solves $Z = \bar{\Phi}(Z)$, then, for any particular $Z = Z_0$, $Z^c \gtrless Z_0$ if and only if $\bar{\Phi}(Z_0) \gtrless Z_0$.

Now, from the deterrence-setting rule, which is same under both non-cooperation and cooperation, for any given Z, s_i is same between the two regimes, and, comparing (11) and (16), this implies that z_i is higher under cooperation. That is, $\Phi_i(Z) > \phi_i(Z)$. This implies that for any Z, $\bar{\Phi}(Z) > \bar{\phi}(Z)$. Hence $\bar{\Phi}(Z^n) > \bar{\phi}(Z^n) = Z^n$, implying $Z^c > Z^n$ (since, as already proved, $Z^c > Z$ if and only if $\bar{\Phi}(Z) > Z$). Because the same negative locus between s_i and Z holds both under non-cooperation and cooperation, $Z^c > Z^n \Rightarrow s_i^c < s_i^n$.

Since $Z^c > Z^n$, $z_i^c > z_i^n$ for some i. If all target countries are symmetric, then, trivially, $z_i^c > z_i^n\ \forall i$. Or if $B_i > 0$, as shown in Appendix 1, $\Phi_i'(Z) > 0$. Then $\forall i,\ z_i^c = \Phi_i(Z^c) > \Phi_i(Z^n) > \phi_i(Z^n) = z_i^n$. ■

2.7 Partial Coalition

By cooperation we have so far meant cooperation by *all* target countries. Obviously there are transactions costs in forming such a 'grand coalition' – even if side payments are promised. What if only a proper subset of countries form a coalition?

By definition, such a coalition leaves out other countries who would adjust their strategies non-cooperatively in response to the formation of the coalition. These adjustments may imply qualitatively asymmetric implications towards the member countries in the coalition and the 'outside' countries – even if all countries are symmetric otherwise.

In what follows we consider a comparative-statics analysis of the effects of M out of I target countries forming a coalition, where all countries face symmetric parameters with respect to their cost functions. (We do not undertake a full analysis of alternative coalition formation, deviation from coalition etc.)

While the formal analysis is laid out in the Appendix, the implications are intuitive. At given level of pre-emption chosen by outside countries, those in the M-member coalition internalize the externalities from pre-emption and choose a higher level of pre-emption. Consider the response by outside countries. In case the marginal fear effect is weak (respectively strong), from Proposition 3(B), pre-emption levels are strategic complements (respectively substitutes), which implies that these countries increase (respectively decrease) their pre-emption levels.

Interestingly, regardless of which way the outside countries adjust their pre-emption levels, their response, *per se*, constitutes an additional positive effect on the choice of pre-emption by member countries. When the marginal fear effect is weak and outside countries increase pre-emption, strategic complementarity also implies that the coalition would increase its pre-emption. When the marginal effect is strong and outside countries decrease pre-emption, strategic substitutability implies the same for the coalition.

The overall effect on pre-emption chosen by the coalition is thus unambiguously positive. Further, this effect either supplements the increase in pre-emption by outside countries or outweighs the negative adjustment by these countries. Hence aggregate pre-emption is higher vis-a-vis the non-cooperative situation. Finally, given that the level of security-deterrence chosen by any particular country, inside or outside the coalition, is negatively related to aggregate pre-emption, it follows taht deterrence levels by all countries decrease. The following proposition summarizes these results.

Proposition 7 *A coalition by a proper subset of target countries leads to*
(a) a higher level of pre-emption by member countries in the coalition,
(b) a higher or lower level of pre-emption by outside countries according as the marginal fear effect is weak or strong,
(c) a higher aggregate level of pre-emption, and
(d) a lower level of security deterrence by all countries.

While it is evident that under a grand coalition each country benefits, in the case of a coalition by some countries, a member country may or may not benefit. In case of weak marginal fear effect, outside countries increase their pre-emption levels which benefit the coalition. But if the marginal fear effect is strong, the negative adjustment in pre-emption by outside countries imposes a negative externality on the coalition. It is then possible that this adverse effect outweighs the direct gains from cooperation among the member countries. All else the same, the smaller the size of the coalition, the greater is the number of outside countries and hence the greater is the magnitude of the negative externality effect.

In other words, if the marginal fear effect is strong, then a (partial) coalition may not be sustainable. A sustainable coalition would require a 'critical mass' of countries joining hands together. In what follows, this point is illustrated through a numerical example.

An Example. Consider a simple pre-emption game, in which the target countries choose their pre-emptive activity levels, while security-deterrence measures are given for exogenous reasons. Let $c(x_i)$ and $v(z_i)$ functions be $x_i^2/2$ and $z_i^2/2$ respectively, and $F(x_i) = \epsilon x_i + x_i^2/4$. Hence the total cost facing country i has the expression $\Omega_i = \epsilon x_i + x_i^2/4 + \lambda z_i^2/2$. As $F_{xx} > 0$, there is a possibility of negative welfare effect of coalition. Let Ω^l denote the total cost of a country within the M-member coalition, and Ω^n its total cost in the pre-coalition state. It is worked out in the Appendix that if $\epsilon = 20; \alpha = 2; \lambda = 65; I = 28$ and $M = 3$, then $\Omega^l/\Omega^n = 1.37$, i.e., the total cost under coalition is higher. Hence this coalition does not benefit (even if the damage from terror is less). The same holds if $M = 4$. But for $M = 5$ or higher, $\Omega^l/\Omega^n < 1$. Hence, in this example, '5' is the critical number.

On the other hand, an outside country always benefits, as it enjoys the positive externality from the enhanced level of pre-emption undertaken by the countries in the coalition. In summary,

Proposition 8 *A coalition always benefits the outside countries, whereas it may not benefit itself if the size of the coalition is sufficiently small.*

3 Integrating Weak and Strong Marginal Fear Effects

Thus far we have assumed that the marginal fear effect is either throughout weak or throughout strong irrespective of the level of terrorist activity. It may be more realistic to suppose that the society is resilient and can repair small tears in the social fabric. It is only when the scale of the terror activity crosses a critical level does the strong fear effect 'take over.'

In this scenario it seems apt to interpret the strong marginal fear effect as *panic* and the weak marginal fear effect as *no-panic*, and, these terms will, in fact, be used in the current section. A richer set of possibilities arise when no-panic gives away to panic after a threshold. As discussed in the Introduction, apart from no-panic or panic equilibrium, there may arise *brink-of-panic* (BOP) equilibria. Also, multiple equilibria of two different kinds can arise. First, some parameter configurations support both BOP and panic equilibria. Second, for the same parameter values that permit a symmetric BOP equilibrium, there exist an infinite number of asymmetric

BOP equilibria.

All else the same, the analysis integrating no-panic and panic becomes quite complex however. To keep analytical tractability, we restrict ourselves to a pre-emption game only, and further assume symmetry of the cost function of pre-emption, i.e. $\lambda_i = \lambda$, $\forall i$.[19] As we shall see, cost symmetry does not necessarily imply symmetry of pre-emption choices across countries.

'No panic turning into panic' is modeled by postulating an expected damage function that is initially concave and then convex in x_i – as shown in Figure 3. This is equivalent to the marginal expected damage being initially decreasing, and then increasing in the terror input. With some abuse of notation, we let $F(x)$ denote the expected damage function.

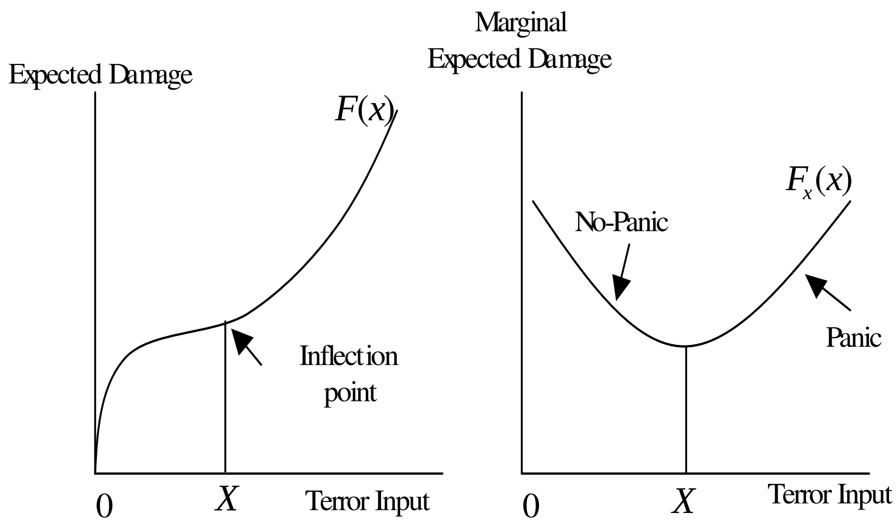

Figure 3: No Panic followed by Panic

3.1 Different Possibilities and Heuristic Arguments

How various possibilities and equilibria may emerge can be understood by focusing on the problem facing the Organization in stage 2. Recall its first-

[19]Looking at pre-emption alone is of independent interest, when, for example, security-deterrence is handled by a different government wing, with deterrence levels impacting on aspects other than terrorist threat. For instance, too high a level of deterrence would adversely affect tourism industry and the trade sector of a country. Thus factors other than security concerns may be more decisive in choosing security-deterrence levels.

order condition (4). The right-hand-side is the marginal cost of x_i and the left-hand-side is the marginal benefit, which, in our revised notation, is $F_x(x)$. Figure 4 shows various possibilities, supposing that Z, the aggregate pre-emption, is already chosen by countries in stage 1.

Clearly, in panel (a), the Organization chooses x^1 along the no-panic portion of the $F_x(x)$ curve. It represents a no-panic equilibrium. In panel (b), the two curves intersect thrice, at x^1, x^2 and x^3, but the second-order condition is fulfilled only at x^1 and x^3. (Multiple intersection does *not* mean multiple equilibria.) Given that area A exceeds area B, the Organization chooses x^1. This corresponds to a no-panic equilibrium. Panel (c) is opposite of panel (b). Area A is less than area B, the Organization chooses x^3 and it corresponds to a panic equilibrium. In panel (d), area A equals area B and the Organization is indifferent between x^1 and x^3. We make an intuitive tie-breaking rule that in case of indifference the Organization does not take trouble of mounting a high level of activity x^3 and selects x^1 instead. This corresponds to a BOP equilibrium. For now we ignore panels (e) and (f).

It follows that, for the same F_x schedule, no-panic, panic or BOP equilibrium will occur in stage 1 depending on whether the aggregate pre-emption, Z, is high enough, low enough or assumes a critical (intermediate) value.

Turning now to stage 1, the choice of Z would critically depend upon the shift parameter λ of the marginal cost function of pre-emption. If it is smaller than a critical value, say λ_1, the target countries would all choose high enough z_i such that the no-panic equilibrium will prevail. Similarly, there would exist another critical value, say λ_3, so that for $\lambda > \lambda_3$ there is a panic equilibrium.

The BOP equilibrium is associated with a unique level of aggregate pre-emption, which we will denote as $\tilde{Z}$. What may not be apparent is that this equilibrium would exist for a range of parameter values, in particular an interval of intermediate values of λ, not just at a single value. This is because the marginal damage from terror to country i, at given z_j, $j \neq i$, is *discontinuous* with respect to z_i and the BOP equilibrium occurs at this point of discontinuity. Note that, as long as the sum total of pre-emption by other countries ($j \neq i$) is not high enough, so that at $z_i = 0$ the Organization selects x^3, a positive but sufficiently small z_i induces the Organization to reduce x_i along the panic portion of the F_x curve. But once z_i exceeds a critical value, the Organization 'jumps down' to a point such as x^1 and accordingly there is a discrete decline in the *marginal* damage from terror. The upshot is that the BOP equilibrium, which is 'knife-edge' in terms of the scale of terrorist

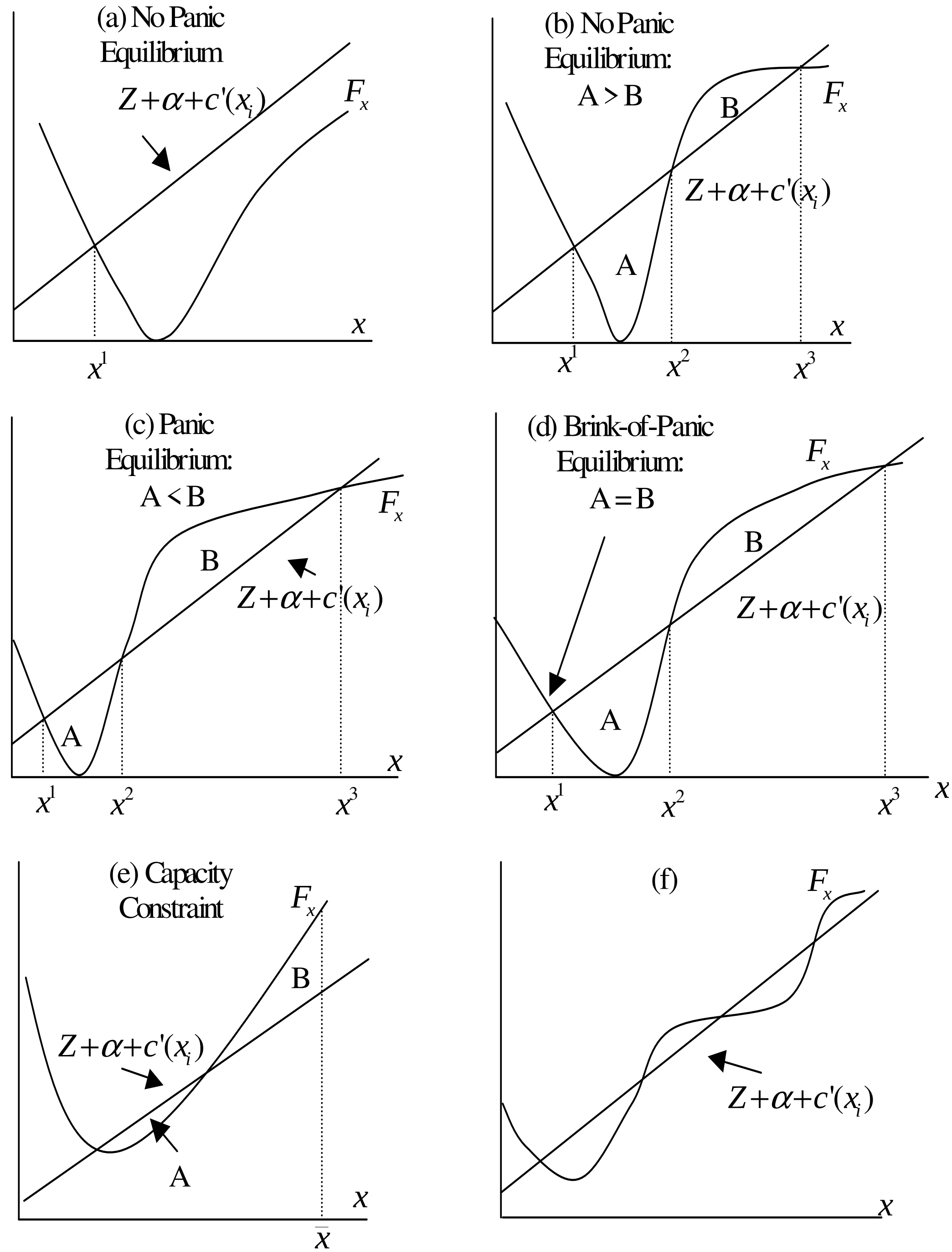

Figure 4: Various Possibilities

activity and scale of fear, would hold for a parameter range having a positive measure.

It is also noteworthy that a downward shift of the marginal cost curve facing the Organization (via a decrease in α) implies a higher $\tilde{Z}$. This means that along the BOP equilibria, an increase in the potency of the Organization invites a higher level of aggregate pre-emption – opposite of the effect of a decrease in α on pre-emption in the no-panic case. This is because averting panic is the primary objective of target countries in the BOP equilibrium.

3.2 Formal Analysis

We shall work with specific functional forms of the cost and the expected damage functions. Let

A10: $c(x_i) = x_i^2/2$ and $v(z_i) = z_i^2/2$, and

A11: $F(x)$ satisfy
(a) $F(x) : [0, \infty) \to [0, \infty)$ is thrice differentiable.
(b) $\exists X > 0$, such that $F(x) = f(x)\ \forall x \leq X$, where $f(x) = ax - bx^2/2$, $a > \alpha$, $b > 0$ and $X = a/b$.
(c) For $x \geq X$, $F(x) = \gamma h(x)$ such that (i) $f'(X) = \gamma h'(X) = 0$, (ii) for $x > X$, $h''(x) > 0 > h'''(x)$ and (iii) $\lim_{x\to\infty} h''(x) = 0$.[20]
(d) $(1 - \gamma h''(x^3(Z))h''(x^3(Z)) + \gamma h'(x^3(Z))h'''(x^3(Z)) > 0\ \forall Z$.

Thus $f(x)$ and $\gamma h(x)$ respectively capture the damage before and after panic takes effect, with γ parametrizing the strength of the panic effect. We call γ the 'panic coefficient.' Figure 5 illustrates the $F_x(\cdot)$ function and the marginal cost function facing the Organization.

In view of A10, the Organization's first-order condition for choosing x_i is:

$$F_x(x_i) = Z + \alpha + x_i, \quad \forall i. \tag{19}$$

Thus x_i depends on aggregate pre-emption, not its distribution, implying that the Organization chooses the same level of attack against all countries, i.e., $x_i = x$.[21] The area A (respectively B) is that between the $F_x(\cdot)$ and $Z + \alpha + x_i$ curves over the interval (x^1, x^2) (respectively (x^2, x^3)).

[20]An example of $h(x)$ satisfying 4(c) is $h(x) = \frac{2}{3}(x - X)^{1.5}$.
[21]This holds even if the cost functions of pre-emptive measures are asymmetric.

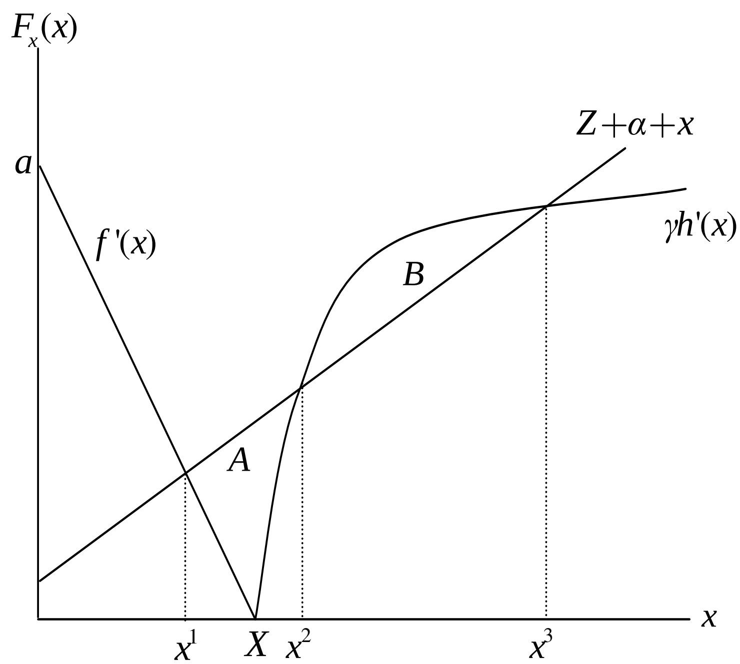

Figure 5: The Marginal Damage Function

The analysis of various cases begins now. For a given Z, if γ is small enough, there is no intersection of the Organization's marginal cost curve with the $\gamma h'(x)$ function. For a single, critical value of γ, the former will be tangential to the latter and for higher values, there will be two intersections (since $h'''(x) < 0$, there cannot be more than two intersections with the $\gamma h'(x)$ curve).

To begin with, suppose $Z = 0$. Consider that value of γ, say γ_0, such that the area A equals area B. Let x_0^1, x_0^2 and x_0^3 denote three levels of x_i at which the F_x curve intersects the marginal cost line $Z+\alpha+x_i$. The following equation defines γ_0:

$$\int_{x_0^1}^{X} [\alpha+x-f'(x)]dx + \int_{X}^{x_0^2} [\alpha+x-\gamma h'(x)]dx = \int_{x_0^2}^{x_0^3} [\gamma h'(x)-\alpha-x]dx. \quad (20)$$

Clearly, for $\gamma < \gamma_0$, area A exceeds area B for any $Z > 0$. Consequently, a no-panic equilibrium is the only possibility for any $\lambda > 0$.

Now consider the case where $\gamma > \gamma_0$. There must exist a critical, positive Z, say $\tilde{Z}$, such that area $A =$ area B, so that the Organization is indifferent between choosing x^1 and x^3. The equation below solves $\tilde{Z}$:

$$\int_{x^1(Z)}^{X} [Z + \alpha + x - f'(x)]dx + \int_{X}^{x^2(Z)} [Z + \alpha + x - \gamma h'(x)]dx = \int_{x^2(Z)}^{x^3(Z)} [\gamma h'(x) - Z - \alpha - x]dx. \tag{21}$$

It follows that no-panic, BOP or panic equilibrium occurs respectively as $Z \gtreqless \tilde{Z}$. Further, since a change in λ affects the choice of x_i only through its impact on Z, and, $\gamma h'(x)$ is increasing in γ without bound at any given x, we have

Lemma 2 *$\tilde{Z}$ is independent of λ, decreases with α, and increases without bound with respect to γ.*

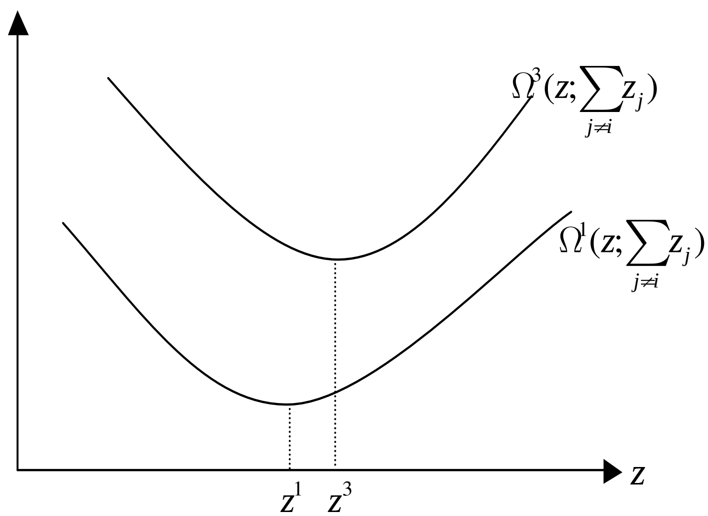

Figure 6: Ω^1 and Ω^3 Functions

We now turn to stage 1 of the game. Because the Organization chooses either $x^1(Z)$ or $x^3(Z)$, the total cost function facing any target country can take either of two forms:

$$\Omega(z_i; Z', \lambda) = \begin{cases} \Omega^1(z_i; Z', \lambda) = F(x^1(z_i + Z')) + \frac{\lambda z_i^2}{2}, \text{ if } z_i + Z' \leq \tilde{Z} \\ \Omega^3(z_i; Z', \lambda) = F(x^3(z_i + Z')) + \frac{\lambda z_i^2}{2}, \text{ otherwise,} \end{cases} \tag{22}$$

where Z' is the sum total of pre-emption used by other countries.[22]

Clearly $\Omega^1(z_i;\cdot) < \Omega^3(z_i;\cdot)$. Also, $z^1(Z',\lambda) < z^3(Z',\lambda)$, where $z^k(Z',\lambda)$ minimizes $\Omega^k(z_i;Z')$, $k=1,\ 3$ at a given Z'.[23] Figure 6 illustrates this. The total cost incurred by country i is read off the $\Omega^1(\cdot)$ or the $\Omega^3(\cdot)$ function according as $z_i \gtrless \tilde{Z} - Z'$.

We are in a position to formally define various equilibria.

Definition. A pre-emption vector $(z_1^1, \cdots, z_I^1)$ is said to constitute a *no-panic equilibrium* if, for $Z' = \Sigma_{j\neq i} z_j^1$, (a) $z_i^1 = \text{argmin}_{z_i}\ \Omega^1(z_i;Z',\lambda)\ \forall i$, (b) $\Sigma_i z_i^1 > \tilde{Z}$ and (c) $\Omega^1(z_i^1, Z', \lambda) \leq \min_{z_i<\tilde{Z}-Z'} \Omega^3(z_i;Z',\lambda)\ \forall i$.

Definition. A pre-emption vector $(\tilde{z}_1, \cdots, \tilde{z}_I)$ is said to constitute a *brink-of-panic equilibrium* if for $Z'' = \Sigma_{j\neq i}\tilde{z}_j$ (a) $\tilde{z}_i = \text{argmin}_{z_i\geq\tilde{z}_i}\ \Omega^1(z_i;Z'',\lambda)\ \forall i$, (b) $\Sigma_j \tilde{z}_j = \tilde{Z}$ and (c) $\Omega^1(\tilde{z}_i, Z'', \lambda) \leq \min_{z_i<\tilde{z}_i} \Omega^3(z_i, Z'', \lambda)\ \forall i$.

Definition. A pre-emption vector $(z_1^3, \cdots, z_I^3)$ constitutes a *panic equilibrium* if for $Z''' = \Sigma_{j\neq i} z_j^3$, (a) $z_i^3 = \text{argmin}\ \Omega^3(z_i, Z''', \lambda)\ \forall i$, (b) $\Sigma_j z_j^3 < \tilde{Z}$ and (c) $\min_{z_i\geq\tilde{Z}-Z'''} \Omega^1(z_i, Z''', \lambda) \geq \Omega^3(z_i^3, Z''', \lambda)\ \forall i$.

These definitions are general, independent of the functional forms we assume. Part (a) says that the equilibrium z_i is the optimal point on the respective Ω^k function in its domain. Part (b) specifies whether $Z \lesseqqgtr \tilde{Z}$. Part (c) is an incentive-compatibility constraint saying that it does not pay to a target country to change z_i so as to jump to a point on the alternative cost function. The BOP equilibrium is shown in Figure 7, while the other two kinds of equilibria can be shown in similar diagrams.[24]

Various equilibria in the (γ, λ) space are characterized in Figure 8 and we have

Proposition 9 *Let assumptions A1, A4, A10 and A11 hold. As illustrated in Figure 8, depending on the paramaters γ and λ, various equilibria arise.*

Proof: See the Appendix.

[22]We explicitly write the parameter λ in the Ω function, since, as we shall see, various cases depend on the value of this parameter in a systematic way. Other parameters matter too, but we avoid making them explicit for the sake of notational ease.

[23]The proof follows from the fact that $f'' < 0 < h''$ and $\gamma h'(x^3(z)) > f'(x^1(z))$.

[24]There are two panels, depending on the magnitude of $\tilde{Z}/I$ relative to z^3 that minimizes the Ω^3 function.

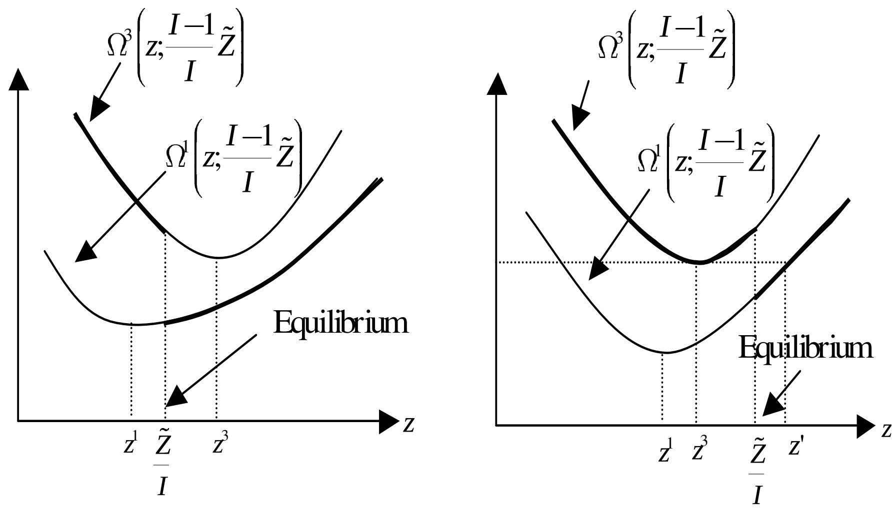

Figure 7: BOP Equilibrium

Remarks

1. As discussed earlier, BOP or panic equilibria arise only when $\gamma > \gamma_0$.
2. For a given $\gamma \in (\gamma_0, \gamma_1)$, no panic (respectively panic) equilibrium obtains if λ is relatively small (respectively large). For intermediate values of λ, the BOP equilibrium obtains.
3. If, for any given γ, λ lies in-between λ_3 and λ_2 or in-between λ'_3 and λ'_2, there can be two kinds of equilibria: panic and BOP. Multiple equilibria essentially arise out of a coordination failure.[25] In the BOP equilibrium, every country expects the others to supply $\widetilde{Z}/I$, so that supplying the same amount is optimal for it. In the panic equilibrium, however, every country expects the other to supply a lower level of pre-emption at $z^3(\lambda)$. Thus panic-prevention is too costly, and all countries prefer to supply $z^3(\lambda)$ instead.
4. We see that if $\gamma > \gamma_1$, there is no no-panic equilibrium (which is intuitive), and at any BOP equilibrium, the scale of attack is zero. This will be explained below.

Notice further that, if it exists, a no-panic equilibrium is unique and symmetric. This follows from the cost function of pre-emption across countries being symmetric and the marginal benefit and the marginal cost of

[25] If there were a single target country then $\lambda_2 = \lambda_3$ and multiple equilibria cannot arise.

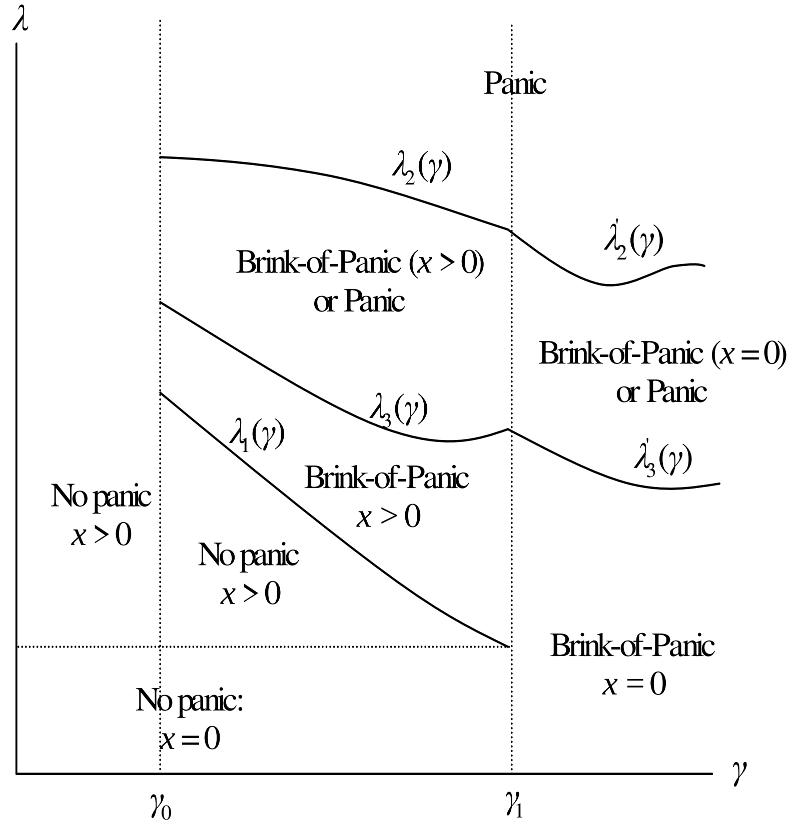

Figure 8: Various Equilibria in the (γ, λ) Space

pre-emption being respectively decreasing and increasing in the level of pre-emption. The same holds for panic equilibrium by virtue of A11(d), ensuring that the second-order condition is met.

Given that a panic equilibrium may co-exist with a BOP equilibrium for some parameter values, it is natural to ask why the same is not true for the no-panic equilibrium. The reason is that both panic and BOP equilibria entail relatively "little" aggregate pre-emption. For a panic equilibrium this statement is almost tautological. Even for a BOP equilibrium, pre-emption is 'just sufficient' to prevent panic from setting in. However, no-panic equilibrium is associated with a relatively high level of pre-emption. Therefore, if there is a no-panic equilibrium for some parameter configuration, other equilibria cannot arise for those parameter values.

We now focus on the BOP equilibria, which is novel. For each symmetric

BOP equilibrium (given that it exists), we find that there are an infinite number of asymmetric BOP equilibria.[26] To see this, start with a symmetric BOP equilibrium in which each country 'supplies' pre-emption equal to $\tilde{Z}/I$. Corresponding to this set of strategies, consider another in which country i's level of pre-emption is equal to $z_i' = \tilde{Z}/I + \epsilon_i$, where (i) ϵ_i is small and (ii) $\Sigma\epsilon_i = 0$. (ii) implies that $\Sigma z_i' = \tilde{Z}$. Notice from Figure 7 that at the symmetric BOP equilibrium, Ω^1 is increasing in z_i. Hence for ϵ_i sufficiently small, Ω^1 is increasing in z_i' whether $\epsilon \gtreqless 0$. Choosing $z_i' = \tilde{Z}/I + \epsilon_i$ is thus preferred to any $z_i > z_i'$. In conjuction with Figure 8, it is also seen that as long as λ is either in the interior of (λ_1, λ_2) or strictly less than λ_2', and ϵ_i is small enough, we have $\min_{z_i < z_i'} \Omega^3(z_i, \Sigma_{j \neq i} z_j') > \Omega^1(z_i', \Sigma_{j \neq i} z_j')$. Hence z_i' is also preferred to any $z_i < z_i'$. Thus any country i supplying the pre-emption level of z_i' constitutes a BOP equilibrium.

Intuitively, a BOP equilibrium is intrinsically tied to the panic threat. Averting this threat is of paramount importance as long as the cost of pre-emption is not too high. If one of the countries lowers the level of pre-emption by a small amount compared to an existing symmetric BOP equilibrium, the other countries have an incentive to make up this short-fall.

We next turn to some comparartive statics. An increase in the potency of the Organization (a decrease in α) leads to a decline or an increase in aggregate pre-emption according as it is a no-panic or panic equilibrium. This is consistent with our analysis in the previous section. Interestingly, if it is a BOP equilibrium, a higher α implies a higher $\tilde{Z}$, meaning a *greater* level of aggregate pre-emption – which is opposite of the response along the no-panic equilibrium – even though in this equilibrium too the Organization operates on the no-panic segment of the marginal damage curve. It is because the sole objective of the countries in the BOP equilibrium is to avert panic.

Consider an increase in γ, the panic coefficient. Along the BOP equilibrium, an increase γ implies a *decrease* in the level of terror input employed by the Organization. Intuitively, the greater the coefficient of panic, the higher is the level of pre-emption required to avert panic and thus the *smaller* is the scale of attack or terror input used by the Organization. Hence if γ is sufficiently large, the marginal cost line $\tilde{Z} + \alpha + x$ will intersect the $f'(x)$ line at point a or above it in Figure 5. This explains why in Figure 8, $x_i = 0$ in

[26]This is a situation of multiple equilibria in a different dimension compared to the coexistence of BOP and panic equilibria.

the BOP equilibrium when $\gamma > \gamma_1$.[27]

Proposition 10 *Let assumptions A1, A4, A10 and A11 hold.*
(a) Whenever it exists, the no-panic equilibrium is unique and symmetric. A similar statement holds for the panic equilibrium.
(b) Associated with each symmetric BOP equilibrium are infinitely many asymmetric BOP equilibria.
(c) In a BOP equilibrium an increase in the potency of the Organization leads to a higher level of aggregate pre-emption. Moreover, an increase in the panic coefficient implies a higher level aggregate pre-emption, as well as a lower scale of attack by the Organization.

3.3 Cooperation

The analysis of cooperative behavior for the no-panic (respectively panic) equilibrium is quite similar to that for the weak-marginal-fear (respectively strong-marginal-fear) case analyzed earlier. Consequently, we focus on the BOP equilibrium, and, in this case, several new insights on the nature of cooperation emerge.

First, there exist parameter values (namely, λ not too small) such that the aggregate pre-emption level, $\tilde{Z}$, will be the first best anyway. Thus, the standard argument for cooperation – that it helps internalize some externality – does not apply. The intuition hinges on that in the first-best outcome the countries are constrained to supply 'enough' pre-emption, so as to avoid panic. But this is exactly the *symmetric* BOP equilibrium. Second, there is a rationale for cooperation however, if the BOP equilibrium involves an *asymmetric* allocation of pre-emption among the target countries. Third, there is a further case for coordination if there is a panic equilibrium, even when a BOP equilibrium co-exists. This is because in this case a symmetric BOP equilibrium Pareto dominates the panic one.

The last two arguments, which *favor* cooperation, are, interestingly, based on *coordination* on the appropriate non-cooperative equilibrium, not on explicit *collusion*. Even in the absence of explicit collusion, the presence of multiple equilibria implies that cooperation can help to coordinate on the welfare maximizing equilibrium.

[27] This possibility would not have risen if $f(x)$ satisfied the Inada condition at $x = 0$.

We then turn to the formal analysis. In order to focus on the case of interest let $\gamma_1 > \gamma > \gamma_0$, so that there is some role for pre-emption.[28] Under cooperation the countries jointly solve $\min_{z_1,\cdots,z_I} IF(x^*(\Sigma_i z_i)) + \lambda(\Sigma_i z_i^2)/2$, where $x^*(\cdot)$ solves the Organization's optimization problem. We first look for solution assuming that it operates in the no-panic region, i.e. $x^*(\Sigma_i z_i) = x^1(\Sigma_i z_i)$. From the first-order conditions it follows that any solution must be symmetric. Consequently, the symmetric solution is:

$$z^C(\lambda) = \frac{I(a + b\alpha)}{\lambda(1+b)^2 - bI^2}. \tag{23}$$

It is easy to see that $z^C(\lambda) > z^*(\lambda)$, where recall that $z^*(\lambda)$ is the symmetric solution to minimizing $\Omega^1(z_i, \sum_{j\neq i} z_j)$. This is intuitive as the cooperation internalizes the externalities generated by pre-emption.

However, note that the outcome where every country supplies $z^C(\lambda)$ may not be the first best, because if $Iz^C(\lambda) < \tilde{Z}$ panic will be triggered. In that case the first best, after taking the possibility of panic into account, involves every country supplying $\tilde{Z}/I$, so as to avoid panic. If, however, $Iz^C(\lambda) \geq \tilde{Z}$, then the first best involves every country supplying $z^C(\lambda)$. Given that $z^C(\lambda)$ is decreasing in λ, the first best involves every country supplying $\tilde{Z}/I$ if and only if $\lambda > \lambda_C$, where λ_C solves $z^C(\lambda) = \tilde{Z}/I$. Given that $\lambda_C > \lambda_1$ (see (23) and (A.10)), we focus on two cases:

Case 1. $\lambda_0 \leq \lambda \leq \min\{\lambda_2, \lambda_C\}$, so that the first-best outcome involves every country supplying a pre-emption level of $z^c(\lambda)$, ensuring that panic is prevented. In this case collusion, internalizing the externalities, would increase welfare irrespective of the nature of equilibrium.

Case 2. $\lambda_c < \lambda \leq \lambda_2$.[29] In this case the first best involves all the countries supplying $\tilde{Z}/I$, so that the first best outcome coincides with the symmetric BOP equilibrium. If, moreover, the non-cooperative equilibrium is *symmetric*, then the cooperative and non-cooperative outcome is the same, so that there is no role for cooperation. In case it is *asymmetric*, however, cooperation can improve aggregate utility of the target countries by improving the allocation of pre-emption. This follows since, with convex and symmetric costs of pre-emption, efficiency involves a symmetric allocation of pre-emption among countries.

[28] All the results go through for $\gamma \geq \gamma_1$ if we replace λ_i by λ_i'.

[29] The cases where $\lambda_2 < \lambda < \lambda_C$, or where $\lambda > \max\{\lambda_C, \lambda_2\}$ yield no new insights and hence igonored.

Continuing with Case (2), if $\lambda_3 < \lambda \leq \lambda_2$, then there are both panic and BOP equilibria. It is straightforward to show that the countries prefer the *symmetric* BOP equilibrium over the panic equilibrium. This follows since

$$\Omega^1\left[\frac{\tilde{Z}}{I}, \frac{(I-1)\tilde{Z}}{I}, \lambda\right] \leq \Omega^3\left[z^3\left(\frac{(I-1)\tilde{Z}}{I}, \lambda\right), \frac{(I-1)\tilde{Z}}{I}, \lambda\right]$$
$$< \Omega^3[z^3((I-1)z^3, \lambda), (I-1)z^3, \lambda],$$

where the first inequality follows because the outcome where every country supplies $\tilde{Z}/I$ constitutes a BOP equilibrium, and the last inequality from the fact that for all $\lambda \geq \lambda_3$, $\tilde{Z}/I > z^3[(I-1)\tilde{Z}/I, \lambda]$. Consequently, coordination may help the countries reach the symmetric BOP equilibrium, which welfare dominates the panic equilibrium.

Summarizing the above discussion we have our next result.

Proposition 11 *Let assumptions A1, A4, A10 and A11 hold. Further, suppose that* $\gamma_0 < \gamma < \gamma_1$ *and* $\lambda_C < \lambda \leq \lambda_2$.
(i) A symmetric BOP equilibrium exists, which, moreover, leads to the first best outcome.
(ii) If the equilibrium is an asymmetric BOP one, then coordinating to obtain the symmetric BOP equilibrium is welfare improving.
(iii) If $\lambda_3 < \lambda \leq \lambda_2$ *and there is a panic equilibrium, then coordinating to obtain symmetric BOP equilibrium is welfare improving.*

For $\gamma \geq \gamma_1$, very similar results go through.

3.4 Generalizations and Other Possibilities

The preceding analysis has assumed specific forms of the pre-emption cost function, the cost function of producing terrorism, as well as the damage function. But many of them were used for the sake of technical convenience only. For instance, the linear segment of $F(x)$ in the no panic zone, combined with linear cost function of the Organization, yielded a closed-form solution of x_i in this zone – as well as the possibility that a sufficiently high level of aggregate pre-emption totally eliminate terrorist attacks. Any general downward sloping $F_x(x)$ function over $x \leq X$ would have served the purpose. The assumptions on $F_x(x)$ over the panic zone implied strict concavity of the marginal damage function and hence a unique panic equilibria. The main insights and particularly how the BOP equilibrium emerges would hold as long as the marginal damage function is downward sloping initially, and, strictly concave when upward sloping.

Consider now panel (e) in Figure 4. The marginal damage function is not concave in its upward sloping range and thus, in the panic region, the Organization would choose any arbitrarily high level of x_i. This is obviously not a plausible outcome. A natural assumption which eliminates this possibility would be the existence of an exogenous capacity constraint on the terrorist organization, say at $\bar{x}$ (as shown in Figure 4).[30] Indeed, the analysis of this case is simpler – and it does not hamper any of the results we have obtained – because the panic equilibrium holds at $\bar{x}$ and it is robust to a marginal change in Z as long as $Z < \tilde{Z}$.

Finally, in a scenario such as panel (f) there are many intersections of the marginal damage function (in its upward range) with the marginal cost function. Multiple panic equilibria is an additional possibility. But, as long as the Organization's optimal x_i declines with aggregate pre-emption (which is quite plausible), the BOP and no-panic equilibria would arise the same way as before.

4 Concluding Remarks and Extensions

Some terrorist organizations operate globally and target many countries. We have developed a game-theoretic model incorporating the behavior of target countries and the terrorist organization explicitly. The damage from terror is defined in terms of the panic or fear it engenders and we have modeled the implications of a regime swtich from no-panic to panic beyond a threshold attack level. Both pre-emption and deterrence are considered as counter-terrorism policy instruments. Our analysis allows us to ask several policy questions of interest – in particular, if more offense is the best response to more potential terror. Different kinds of equilibria emerge, including one which is rather non-standard, namely, the brink-of-panic equilibrium.

One can consider many relevant extensions of the problem we have taken up, such as more than one terrorist organization, various forms of informational asymmetries, learning and reputation building, strategic negotiations

[30]We can think of this constraint as arising from an absolute limit on the funds, or volunteers available to the terrorist organization.

etc. In closing, we briefly comment on two direct extensions of our model, both relating to asymmetry in targeting by the Organization.

This paper has assumed that the Organization targets all countries equally. What happens when there is differential targeting for extraneous reasons? This can be modeled by attaching differential weights, say μ_i, in the objective function of the Organization to the expected damage caused to a country. (These weights may be seen as hatred parameters.) Its objective function can be set up as

$$\underset{x_i}{\text{Maximize}}\ \ \Sigma_i \mu_i F(s_i, x_i) - (Z + \alpha)(\Sigma_i x_i) - \Sigma_i c(x_i),\ \Sigma_i \mu_i = 1.$$

To focus on difference in targeting, let us assume that the countries are symmetric otherwise, i.e., $\beta_i = \beta$ and $\lambda_i = \lambda$.

Consider first the case where the marginal fear effect is either uniformly strong, or uniformy weak. Straightforward calculations show that if $\mu_i > \mu_j$, then $s_i > s_j$, that is, a relatively high-target country will choose a relatively high security-deterrence levels. How do the pre-emptive measures compare? A relatively more targeted country chooses a relatively *lower* (respectively higher) level of pre-emption, if the marginal fear effect is weak (respectively strong). This is because pre-emption measures are strategic substitutes (complements) if the marginal fear effect is weak (strong). Such asymmetric targeting may have interesting implications for the case of no panic yielding to panic. In reference to the BOP equilibrium in particular, we conjecture the possibility that countries facing relatively greater targeting supply the total level of pre-emption that averts panic, while the other countries free-ride.

Instead of targeting being exogenous, it may be a reaction to pre-emption levels chosen by target countries. Broadly speaking, this would have the following implications. First, all else the same, such 'endogenous targetting' implies a higher marginal cost of pre-emption activity and this would tend to reduce the level of pre-emption. Second, insofar as targeting is a function of the relative magnitude of pre-emption it generates complementarity in pre-emptive activity: an increase in pre-emption by country i makes the Organization target country j less than before, inducing the latter to increase its pre-emptive measures. Third, cooperation among target countries would contain the element of minimizing the relative effect on targeting through a choice of relatively more symmetric levels of pre-emption than the intrinsic source of asymmetry among target countries would have otherwise implied.

Appendix

Proof of Proposition 1

For any square matrix M, let $|M|$ denote the determinant of M. Totally differentiating the first order condition for the organization, i.e. system (2), with respect to $x_1, \cdots, x_I$ and s_j

$$\frac{dx_i}{ds_j} = \frac{|N_{ij}|}{|N|} \tag{A.1}$$

where $|N|$ is the Hessian of the system (2), where note that the i-th row and j-th column of N equals $F_{xx}(x_i, s_i) - C_{XX} - c''(x_i)$ if $i = j$, and C_{XX} otherwise. Further, N_{ij} is the matrix where the i-th column of N is replaced by the column C_j, where C_j has zero in every element, except the j-th row which is equal to $-F_{xs}(x_j, s_j)$.

For $i = j$, straightforward manipulations yield that the numerator of (A.1) equals $(-F_{xs}(x_i, s_i))|N^{ii}|$, where N^{ii} is the N matrix with the i-th row and column removed. Given A5, the second order condition is satisfied and thus $|N^{ii}|/|N| < 0$. As $F_{xs} < 0$, it follows that $dx_i/ds_i < 0$.

In case of $i \neq j$, the numerator of (A.1) equals

$$(-F_{xs}(x_i, s_j))C_{XX}(Z + \alpha, X)Y,$$

where

$$Y = \sum_{1 \le k \le I,\, k \neq j} F_{xx}(x_k, s_k) - c''(x_k)$$

Note that Y has $(I - 1)$ terms. Since, from A5, $F_{xx}(x_l, s_l) - c''(x_l) < 0$, Y is positive for I odd, whereas its negative for I even. Further, recall that $F_{xs} < 0$. Next consider the denominator. Given that the second-order condition is met, the denominator is positive for I even, and negative for I odd. Hence the sign of dx_i/ds_j is the same as that of C_{XX}.

Proof that the Slope of the $\bar{\phi}'(Z) < 1$ if $\mathbf{B}_i > 0$

We begin with the second-order condition under cooperation. This requires $\mathbf{K}$ to be positive semi-definite, implying that $|\mathbf{K}| > 0$. Using (18),

$$|\mathbf{K}| = \begin{vmatrix} \frac{\tilde{B}}{\Phi_1'(Z)} - \tilde{B} & -\tilde{B} & ... & -\tilde{B} \\ -\tilde{B} & \frac{\tilde{B}}{\Phi_2'(Z)} - \tilde{B} & ... & -\tilde{B} \\ .... & & & \\ .... & & & \\ -\tilde{B} & -\tilde{B} & ... & \frac{\tilde{B}}{\Phi_I'(Z)} - \tilde{B} \end{vmatrix} = (\tilde{B})^I \frac{1 - \sum_{i=1} \Phi_i'(Z)}{\Phi_1'(Z) \cdots \Phi_I'(Z)}. \tag{A.2}$$

By their definitions,

$$\frac{\tilde{B}}{\tilde{A}} = \sum_i \frac{B_i}{A_i}.$$

This, together with $B_i > 0 \ \forall i$ implies

$$\tilde{B} > 0; \quad \Phi_i'(Z) = \frac{\sum_i \frac{B_i}{A_i}}{\lambda_i v''(c'' - F_{xx})^2} > \frac{\frac{B_i}{A_i}}{\lambda_i v''(c'' - F_{xx})^2} = \phi_i'(Z) > 0. \tag{A.3}$$

Using these and in view of (A.2), $|\mathbf{K}| > 0$ implies

$$0 < \Sigma_i \phi_i'(Z) < \Sigma_i \Phi_i'(Z) < 1. \tag{A.4}$$

Partial Coalition: Proof of Proposition 7, and Proof of Proposition 8 by Example

Let z_o and z_m respectively be the pre-emption level chosen by an outside country and a member country in the coalition, with z_o^l and z_m^l denoting their equilibrium levels. Let z_o^n and z_m^n denote their equilibrium levels of pre-emption in the pre-coalition state. Given symmetry, $z_o^n = z_m^n$. Finally, let Z^l and Z^n be the aggregate pre-emption in the coalition and pre-coalition states.

Pre-coalition and coalition equilibria can be conveniently depicted in the (z_o, Z) space. Consider first the behavior of an outside country. It is characterized by eqs. (10) and (11) both before and after the coalition. After

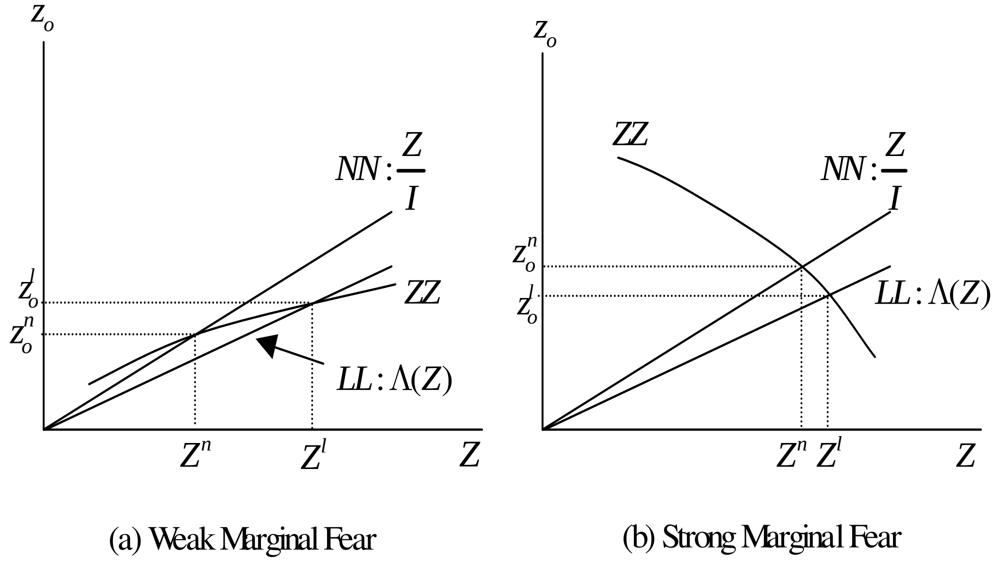

Figure 9: Coalition

implicitly solving for s_o, these two conditions imply that the relationship between z_o and Z is same as the $\phi_i(Z)$ function. This is depicted in Figure 9 by the ZZ curve. It is upward-sloping or downward-sloping as the marginal fear effect is weak or strong. Next, under no-coalition, symmetry implies $z_o = Z/I$. This is the NN line. The intersection of ZZ and NN solve z_o^n and Z^n.[31]

When there is a coalition, $z_m \neq z_o$, even though countries are symmetric, and therefore $z_o \neq Z/I$. The first-order conditions with respect to s_m and z_m are given by (10) and

$$-M\frac{F_x(\cdot)}{c'' - F_{xx}} + \lambda v'(z_m) = 0. \tag{A.5}$$

This equation is same as (16), except M substituting I.

However, in view of (11) and (A.5), the marginal benefit from pre-emption

[31]The condition (8) implies that in panel (a) the NN curve steeper than the ZZ curve.

to one member country is M times that to an outside country.[32] Hence

$$M = \frac{v'(z_m^l)}{v'(z_o^l)}. \tag{A.6}$$

It implicitly defines a function $z_m \equiv h(z_o)$. Thus we can express $Z = Mh(z_o) + (I - M)z_o \equiv Z(z_o)$. Note that $Z(z_o) > Iz_o$ (because cooperation implies a higher level of aggregate premption by member countries) and $Z'(z_o) > 0$. The inverse of this function, $\Lambda(Z)$, is the LL curve. (It may not be a straight line.)

The intersection of ZZ and LL defines z_o^l and Z^l. We are now able to compare. Aggregate pre-emption is higher in the presence of a coalition, but the response of outside countries with respect to pre-emption depends on whether the marginal fear effect is weak or strong. If it is weak, pre-emption choices are strategic complements and thus outside countries step up their pre-emption levels. If it is strong, there is strategic substitutability and the outside countries step down their pre-emption levels.

Furthermore, since s_i is 1:1 and negatively related to Z and the relationship is the same under coalition or no coalition, $Z^l > Z^n$ implies $s_i^l < s_i^n$. Hence Proposition 7 is proved.

Next we work out the example given in the text, and, this proves Proposition 8. In stage 2 the Organization's problem yields

$$x_i = g(Z + \alpha) = 2[\epsilon - (Z + \alpha)]. \tag{A.7}$$

Under no coalition, this leads to the equilibrium pre-emption by any single country and the equilibrium level of terror input to any single country equal to

$$z^n = \frac{4\epsilon - 2\alpha}{2I + \lambda}; \quad x^n = \frac{2(\lambda\epsilon - \lambda\alpha - 2I\epsilon)}{2I + \lambda}.^{33}$$

Given these expressions and the parametric values specified in the text, we compute the total cost, Ω^n, facing any country in the no-coalition equilibrium.

[32] This holds after solving out s_o or s_m, because (a) the first-order condition with respect to security-deterrence, i.e., (10), holds whether there is a coalition or not, (b) it is the same equation for a member country and an outside country and (c) the level of security-deterrence is dependent of aggregate pre-emption Z, not z_i.

[33] We impose that $\epsilon > \alpha$ and λ are high enough such that both z^n and x^n are positive.

In the coalition case, the first-order conditions for optimal z are:

$$-2\left(\epsilon+\frac{g(\cdot)}{2}\right)+\lambda z_o^l=0; \quad -2M\left(\epsilon+\frac{g(\cdot)}{2}\right)+\lambda z_m^l=0,$$

respectively for an outside country and a member country, obtained from (11) and (A.5) respectively. These equations lead to:

$$z_o^l=\frac{4\epsilon-2\alpha}{\lambda+2(I+M^2-M)}; \quad z_m^l=\frac{M(4\epsilon-2\alpha)}{\lambda+2(I+M^2-M)}.$$

Note that $z_m^l=Mz_o^l$ and thus $Z^l=(I+M^2-M)z_o^l$. Once we know z_o^l, x^l is determined from (A.7) and we are in a position to compute Ω^l, the total cost incurred by a member country in the coalition. The text discusses whether the ratio $\Omega^l/\Omega^n \gtrless 1$ corresponding to different values of M, the size of the coalition.

Proof of Proposition 9

No-Panic Equilibrium

It is already proved in the text that if $\gamma\le\gamma_0$, then no-panic equilibrium is the only possibility. Note that γ_0 is the solution of the equation $\tilde{Z}(\gamma)=0$.

Consider now the case of $\gamma>\gamma_0$. The Organization's first-order condition (19) reads as $a-bx_i=Z+\alpha+x_i$. Thus if $Z>a-\alpha$, then $x_i=0$ (terrorism is completely stopped). Otherwise,

$$x_i^1=\frac{a-\alpha-Z}{1+b}. \tag{A.8}$$

Turning to stage 1, the Nash first-order condition with respect to pre-emption for any particular country given by

$$\frac{f'[x^1(Z)]}{1-f''}=\frac{a+b(Z+\alpha)}{(1+b)^2}=\lambda z_i. \tag{A.9}$$

It implicitly defines $z_i=z^1(Z',\lambda)$, where Z' denotes the sum-total of pre-emption undertaken by *other* countries. Eq. (A.9) implies symmetry: $z_i=z$. The reduced-form solution has the expression:

$$\bar{z}^1(\lambda)=\begin{cases}\frac{a-\alpha}{I}\ (\Rightarrow x=0), & \text{if } \lambda\le\frac{a(1+b)I}{(a-\alpha)(1+b)^2},\\ \frac{a+b\alpha}{\lambda(1+b)^2-bI}\ (\Rightarrow x>0), & \text{otherwise.}\end{cases} \tag{A.10}$$

Now define γ_1 as the solution to $\tilde{Z}(\gamma) = I\bar{z}^1(\gamma) = a - \alpha$. It represents that high value of the panic coefficient such that if countries prevent panic, they have to use so high a level of pre-emption that the Organization is forced to choose $x_i = 0$. We have $\gamma_1 > \gamma_0$ as $\tilde{Z}$ is increasing in γ.

Consider $\gamma \in (\gamma_0,\ \gamma_1)$. No-panic equilibrium prevails if and only if λ is small enough such that $I\bar{z}^1(\lambda) > \tilde{Z}(\gamma)$. Thus we set up the equation

$$\tilde{Z}(\gamma) = \frac{a + b\alpha}{\lambda(1+b)^2 - bI}, \tag{A.11}$$

which implicitly defines a negative relation between λ and γ, say, $\lambda_1(\gamma)$. No-panic equilibrium occurs for $\lambda \leq \lambda_1(\gamma)$.

Next, if $\gamma > \gamma_1$, then $\tilde{Z}(\gamma) > a - \alpha$. It is easy to see that there cannot exist any no-panic equilibrium. It is because no-panic intervention would require $z^1 > \tilde{Z}/I > (a-\alpha)/I$. But at such high intervention $x_i = 0$. Hence any single country can unilaterally reduce z_i by a sufficiently small amount – and hence save on costs of pre-emption – while still ensuring aggregate Z to be no less than $\tilde{Z}$ and $x_i = 0$.

As shown in Figure 8, no-panic equilibrium occurs to the left of γ_0, and, between γ_0 and γ_1 as long as λ is below the $\lambda_1(\gamma)$ curve.

Next, we consider BOP and panic equilibria in the case where $\gamma \in (\gamma_0, \gamma_1)$, followed by the remaining case of $\gamma > \gamma_1$.

BOP and Panic Equilibria when $\gamma \in (\gamma_0, \gamma_1)$

Consider first the BOP equilibrium, where $Z = \tilde{Z}$. As will be discussed later, countries may or may not choose the same level of pre-emption. But for now, we focus on *symmetric* BOP equilibrium and the case where $\gamma \in (\gamma_0, \gamma_1)$. We argue below that, for any given γ in this range, $\exists\ \lambda_2$, higher than λ_1 such that the BOP equilibrium exists if and only if $\lambda \in (\lambda_1,\ \lambda_2)$.

Analogous to $z^1(Z', \lambda)$, define, from now on, $z^3(Z', \lambda) \equiv \underset{z_i}{\operatorname{argmax}}\ \Omega^3(z_i; Z', \lambda)$. While the function $z^1(Z', \lambda)$ is derived from (A.9), $z^3(Z', \lambda)$ solves the first-order condition,

$$\frac{\gamma h'(x^3(z + Z'))}{1 - \gamma h''(x^3(z + Z'))} = \lambda z. \tag{A.12}$$

Given A11(e), the l.h.s., equal to the marginal benefit from pre-emption, is decreasing in z_i. Thus $z^3(Z', \lambda)$ is well defined. Furthermore, (i) both

$z^1(Z', \lambda)$ and $z^3(Z', \lambda)$ decline in λ and (ii) $z^3(Z', \lambda) > z^1(Z', \lambda)$.[34]

Now let $Z' = \tilde{Z}(I-1)/I$. One immediate implication is that for λ greater than, but close to λ_1, a BOP equilibrium exists. The argument is simple. For $\lambda = \lambda_1$, in view of (A.11), $z^1(Z', \lambda) = \tilde{Z}/I$. Thus $\tilde{Z}/I < z^3(Z', \lambda)$. Hence the U-shape (convexity) of $\Omega^3(z, \cdot)$ implies that it is negatively sloped at $\tilde{Z}/I$. Consider any λ, say $\lambda_1 + \epsilon$, close to but greater than λ_1 (see the left hand panel of Figure 7). Because $\partial z^1/\partial\lambda < 0$, $z^1(Z', \lambda + \epsilon) < \tilde{Z}/I$. It follows that if country i chooses any z higher than $\tilde{Z}/I$, its cost will increase along Ω^1. Thus it has no incentive to choose any z higher than $\tilde{Z}/I$. Further, by virtue of continuity, $\tilde{Z}/I < z^3(\tilde{Z}(I-1)/I, \lambda)$, which implies that if country i chooses any z less than $\tilde{Z}/I$, its cost, along Ω^3, will also increase. Thus all the conditions of a BOP equilibrium are satisfied. This is illustrated in the left-hand panel of Figure 7, which features that $z^1(\tilde{Z}(I-1)/I, \lambda) < \tilde{Z}/I < z^3(\tilde{Z}(I-1)/I, \lambda)$.

As λ gradually increases, both $z^1(\tilde{Z}(I-1)/I, \lambda)$ and $z^3(\tilde{Z}(I-1)/I, \lambda)$ continue to decline. At a critical value, say λ', $z^3(Z', \lambda) = \tilde{Z}/I$. From the structure of BOP equilibrium shown in the left-hand panel of Figure 7, it follows that a symmetric BOP equilibrium exists at this value of λ and by continuity it will hold in a neighborhood to the right of λ'. This is illustrated in the right-hand panel of Figure 7. If a country chooses a pre-emption level higher than $\tilde{Z}/I$, the cost along the Ω^1 curve is higher; if it chooses a level lower than $\tilde{Z}/I$, the minimum cost along Ω^3 is higher too.

The right panel of Figure 7 is indicative that the BOP equilibrium holds up to that value of λ, say λ_2, such that the cost of $z_i = \tilde{Z}/I$ along $\Omega^1(\cdot)$ is equal to the minimum cost along $\Omega^3(\cdot)$. This is illustrated in Figure 10. Formally,

Definition. Let λ_2 is such that for $Z' = \tilde{Z}(I-1)/I$,

$$\Omega^3\left[z^3\left(Z', \lambda_2\right), Z', \lambda_2\right] = \Omega^1\left(\frac{\tilde{Z}}{I}, Z', \lambda_2\right).$$

We now establish that λ_2 exists and it is unique. Define

$$D_1(\lambda) \equiv \Omega^3(z^3(Z', \lambda), Z', \lambda) - \Omega^1(\tilde{Z}/I, Z', \lambda),$$

[34]The proof follows from the fact that given $f'' < 0$, $h'' > 0$ and $\gamma h'(x^3(z)) > f'(x^1(z))$, we have that $\frac{\gamma h'(x^3(z+Z'))}{1-\gamma h''(x^3(z+Z'))} > \frac{f'(x^1(z+Z'))}{1-f''(x^1(z+Z'))}$, $\forall z$.

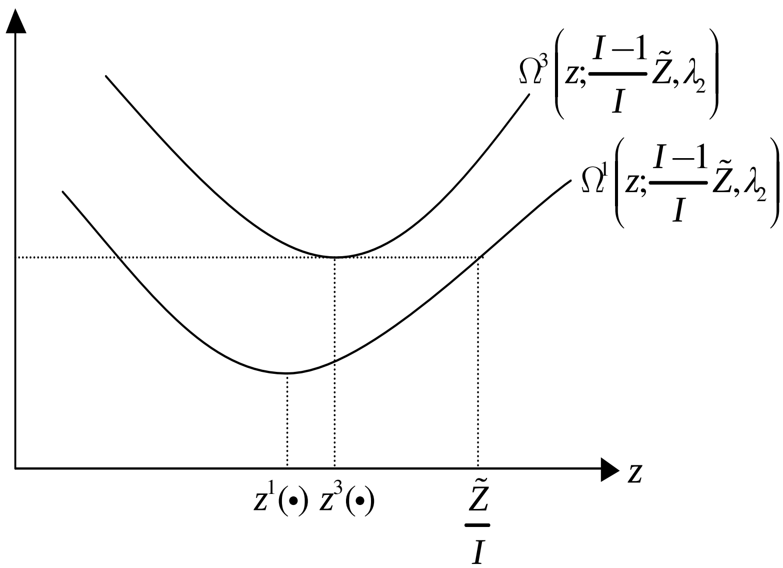

Figure 10: Definition of λ_2

where $Z' = (I-1)\tilde{Z}/I$. At $\lambda = \lambda_1$, for which the no-panic equilibrium exists, $D_1(\lambda) > 0$. As $\lambda \to \infty$, we have $\Omega^1(\tilde{Z}/I, \tilde{Z}(I-1)/I, \lambda) \to \infty$, whereas $\Omega^3(z^3(Z', \lambda), \tilde{Z}(I-1)/I, \lambda)$ is bounded above by $\gamma h[x^3(0)]$. Hence $D_1(\lambda) < 0$, as $\lambda \to \infty$. By virtue of continuity, there exists λ such that $D_1(\lambda) = 0$. This defines λ_2. Hence it exists and exceeds λ_1.

It remains to show that λ_2 is unique. By the envelope theorem,

$$\frac{d\Omega^3[z^3(Z', \lambda), Z', \lambda]}{d\lambda} = \frac{[z^3(Z', \lambda)]^2}{2}.$$

Further, $d\Omega^1(\tilde{Z}/I, Z', \lambda)/d\lambda = (\tilde{Z}/I)^2/2$. Hence,

$$\frac{dD_1}{d\lambda} = \frac{[z^3(Z', \lambda)]^2 - (\tilde{Z}/I)^2}{2}.$$

Because $z^3(Z', \lambda)$ is monotonically decreasing in λ, so does $dD_1/d\lambda$, i.e., D_1 is strictly concave in λ. It is shown in the text that $z^3(Z', \lambda_1) < \tilde{Z}/I$. This implies, $\frac{dD_1}{d\lambda}\Big|_{\lambda_1} = \frac{[z^3(Z', \lambda_1)]^2 - (\tilde{Z}/I)^2}{2} > 0$. Further, given that $z^3(Z', \lambda)$ is decreasing in λ, $D_1(\lambda)$ as a function of λ is concave over the interval (λ_1, ∞), implying that $D_1(\lambda) = 0$ holds exactly at one value of λ. This proves that λ_2 is unique.

Since $dD_1/d\lambda < 0$ and $D_1 = 0$ at λ_2, it follows that for $\lambda \in (\lambda_1, \lambda_2)$, $D_1 > 0$ and thus BOP equilibrium holds.

The equation $D_1(\lambda) = 0$ implicitly defines λ_2 as a function of γ. At any given λ, an increase in γ, ceteris paribus, tends to increase the damage from panic and thus favors BOP equilibrium. But $\tilde{Z}$ rises too, implying an increase in the cost of maintaining the BOP equilibrium. Hence the 'scope' of BOP may increase or decrease. This translates into λ_2 being increasing or decreasing with respect to γ.

We now characterize the parameter zone such that a panic equilibrium will exist. This equilibrium is characterized by the first-order condition (A.12). It implies a symmetric solution, determined by:

$$\frac{\gamma h'(x^3(I\bar{z}^3(\lambda))}{1-\gamma h''(x^3(\bar{z}^3(\lambda)))} = \lambda \bar{z}^3(\lambda). \tag{A.13}$$

Given A11(e), the l.h.s. is declining in z_i. Thus $\bar{z}^3(\lambda)$ is well defined and decreasing in λ. Clearly, as $\lambda \to \infty$, $\bar{z}^3(\lambda) \to 0$. For $Z' = (I-1)\bar{z}^3(\lambda)$, let us define D_2 and λ_3 such that

$$D_2(\lambda) \equiv \Omega^3[\bar{z}^3(\lambda), Z', \lambda] - \Omega^1[\tilde{Z} - Z', Z', \lambda]; \quad D_2(\lambda_3) = 0.^{35} \tag{A.14}$$

A panic equilibrium exists if and only if $D_2(\lambda) \leq 0$. We now prove that $D_2(\lambda) \leq 0$ if and only if $\lambda > \lambda_3$.

Straightforward differentiation and the application of the envelope theorem yield that for $Z' = (I-1)\bar{z}^3(\lambda)$,

$$\begin{aligned}
\frac{d\Omega^3(z^3(\lambda), Z', \lambda)}{d\lambda} &= -(I-1)\frac{\gamma h'(x^3(I\bar{z}^3))}{1-\gamma h''(x^3(I\bar{z}^3))}\frac{d\bar{z}^3}{d\lambda} + \frac{(\bar{z}^3)^2}{2} \\
&= -\lambda(I-1)\bar{z}^3\frac{d\bar{z}^3}{d\lambda} + \frac{(\bar{z}^3)^2}{2}. \\
\frac{d\Omega^1(\tilde{Z}-Z', Z', \lambda)}{d\lambda} &= -\lambda(\tilde{Z}-Z')(I-1)\frac{d\bar{z}^3}{d\lambda} + \frac{(\tilde{Z}-Z')^2}{2}.
\end{aligned}$$

Hence

$$\frac{dD_2(\lambda)}{d\lambda} = \lambda\tilde{Z}\frac{d\bar{z}^3}{d\lambda} + \left[\frac{(\bar{z}^3)^2}{2} - \frac{(\tilde{Z}-Z')^2}{2}\right] < 0, \tag{A.15a}$$

[35] The critical value λ_3 is analogous to λ_2, except that the aggregate level of pre-emption used by other countries is $(I-1)\bar{z}^3(\lambda)$ rather than $(I-1)\tilde{Z}/I$.

where the last inequality follows since $d\bar{z}^3/d\lambda < 0$ and $\tilde{Z}/I > \bar{z}^3$.

Next, note that as $\lambda \to \infty$, $z^3 \to 0$ (i.e. the individual and aggregate pre-emption is zero) and thus $\Omega^3[\bar{z}^3(\lambda), Z', \lambda] \to \gamma h(x^3(0))$, which is finite.[36] But $\Omega^1[\tilde{Z} - Z', Z', \lambda] \to \infty$ since the cost of maintaining panic preventing pre-emption becomes arbitrarily large. Thus

$$\lim_{\lambda \to \infty} D_2(\lambda) < 0. \tag{A.15b}$$

Also note that at $\lambda = \lambda_1$, given that a no-panic equilibrium exists, a panic equilibrium cannot exist. [37] Hence

$$D_2(\lambda_1) > 0. \tag{A.15c}$$

The signs in (A.15a)-(A.15c) prove (i) the existence and uniqueness of λ_3, (ii) for $\lambda > \lambda_3$, panic equilibrium holds (as $D_2(\lambda) < 0$) and (iii) $\lambda_3 > \lambda_1$.

The equation $D_2(\lambda) = 0$ implies λ_3 as a function of γ. However, just as λ_2, λ_3 may increase or decrease with γ for analogous reasons.

We next prove that $\lambda_3 < \lambda_2$, which implies that if $\lambda \in (\lambda_3, \lambda_2)$, there can be either BOP or panic equilibria. Define, a function

$$V(Z', \lambda) = \Omega^3[z^3(Z', \lambda), Z', \lambda] - \Omega^1(\tilde{Z} - Z', Z', \lambda). \tag{A.16}$$

Note that (i) λ_2 solves $V(Z', \lambda)|_{Z'=(I-1)\tilde{Z}/I} = 0$ and (ii) λ_3 solves $V(Z', \lambda)|_{Z'=(I-1)z^3(Z', \lambda)} = 0$.

Differentiating eq. (A.16) and using the first-order condition for optimal pre-emption along the panic equilibrium, we find

$$\frac{dV(Z', \lambda)}{dZ'} = -\frac{\gamma h'}{1 - \gamma h''}\left(1 + \frac{\partial z^3}{\partial Z'}\right) + \lambda z^3 \frac{\partial z^3}{\partial Z'} + \lambda(\tilde{Z} - Z') = \lambda(\tilde{Z} - Z' - z^3), \tag{A.17}$$

where the last equality follows from equation (28). For any $Z' \in [(I-1)z^3(Z', \lambda), (I-1)\tilde{Z}/I)]$, $\tilde{Z} - Z' - z^3 > 0$, implying $dV(\cdot)/dZ' > 0$. Hence $V((I-1)\tilde{Z}/I, \lambda) > V((I-1)z^3(Z', \lambda), \lambda)$. In turn, this implies $\lambda_2 > \lambda_3$.

[36] Zero pre-emption does not imply zero cost of terrorist activity for the Organization; hence $x^3(0)$ is finite.

[37] Suppose not. Then from the first-order conditions we have that $z^1 \leq \bar{z}^3$, which is a contradiction.

BOP and Panic Equilibria when $\gamma > \gamma_1$

As discussed earlier, a no-panic equilibrium does not exist in this range. At the BOP equilibrium $F(\cdot) = 0$ as $x_i = 0$. Hence, $\Omega^1 = \lambda(\tilde{Z}/I)^2/2$. This equilibrium holds if

$$D_3(\lambda) \equiv \Omega^3 \left[\bar{z}^3(\lambda), \frac{I-1}{I}\tilde{Z}, \lambda\right] - \lambda\frac{(\tilde{Z}/I)^2}{2} \geq 0.$$

It is straightforward to establish that D_3 is increasing and concave in λ. Moreover, as the Organization chooses x^3 and thus $F = \gamma h(\cdot)$ is bounded away from zero, for λ sufficiently small, $D_3 > 0$. Since $\gamma h(\cdot)$ is also bounded from above (at $Z = 0$), for large enough λ, $D_3 < 0$. Thus there must exist a unique λ_2' at which $D_3(\lambda) = 0$, and, if $\lambda \leq \lambda_2'$, then $D_3 \geq 0$ and thus BOP equilibrium prevails.

Panic equilibrium holds if, for $Z' = (I-1)\bar{z}^3(\lambda)$, $D_4(\lambda) \equiv \Omega^3[\bar{z}^3(\lambda), Z', \lambda] - \lambda\frac{(\tilde{Z}-Z')^2}{2} \leq 0$. Similar reasoning as for D_3 implies that D_4 is positive for small enough λ and negative for large enough λ. Furthermore, $dD_4/d\lambda < 0$, the proof of which is analogous to establishing $dD_2/d\lambda < 0$. Thus a unique value of λ, say λ_3' exists at which $D_4 = 0$ and $D_4 \leq 0$ for $\lambda > \lambda_3'$; panic equilibrium holds in this interval of λ.

That $\lambda_3' < \lambda_2'$ can be proved following the method used earlier to prove $\lambda_3 < \lambda_2$. Thus, BOP or panic equilibrium can occur when $\lambda \in (\lambda_3',\ \lambda_2')$.

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