The Substitution Effect and the Profit Function in Consumption: expressions from the Marshallian, Hicksian, and Frischian demand functions

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Abstract

In the context of the maximizing behaviour assumption (Becker, 1976), an individual usually maximizes the utility function, minimizes the cost or, finally, can also maximizes the profit function in consumption, with each of these three optimization problems providing a type of demand function: the Marshallian, the Hicksian, and the Frischian. In all three cases, an important concept for both theoretical and empirical reasons is the Substitution Effect (SE), with this measuring the substitution phenomenon in the demanded quantity in function of the price change. In this context, our short paper offers certain alternative theoretical expressions of the Substitution Effect, focusing on the Profit Function in Consumption, thus introducing the inter-temporal context with perfect information.

JEL classification:

Keywords: Demand analysis; Profit Function in Consumption; Substitution Effect; Marshallian, Hicksian and Frischian demand functions

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I. Introduction

Becker (1976) specifies the three foundational assumptions of the economic approach as "maximizing behaviour, market equilibrium, and stable preferences". In the context of the maximizing behaviour assumption, individuals usually maximize the utility function (Primal Optimization Problem), but they can also minimize the cost function (Dual Optimization Problem) or, finally, the consumption profit function. Each of these three optimization problems provide a type of demand function: the Marshallian functions in the case of the Primal, the Hicksian functions in the case of the Dual and, finally, the Frischian functions in the case of the maximization of the Profit Function in Consumption.

In all three cases, an important concept, for both theoretical and empirical reasons, is the Substitution Effect (SE), which can be identified as the variation in the quantity demanded that results from an infinitesimal variation in the price of the corresponding or of a related good, in such a way that the utility or real income remains constant. In this way, the SE offers a real indicator (utility or real income constant) that is very useful in measuring the substitution phenomenon in the demanded quantity as a function of the price change. This indicator has received some attention in the empirical literature (see, for example, Ashenfelter and Heckman, 1974; Manoli and Weber, 2010; Altonji, 1986; Brandt et al., 2013), but few articles have examined its theoretical properties.

In order to partially cover this gap, we offer some alternative theoretical expressions derived from the relationships among the Marshallian, Hicksian, and Frischian demand functions. Specifically, we focus our attention on the Profit Function in Consumption and in its Frischan demand functions, given that a prime objective of individuals throughout the life cycle is to maintain constant their marginal utility of income, obviously, at discounted terms, which, being constant without uncertainty, includes all the known, relevant, extra-current information in the decision process, in this way introducing the inter-temporal context with perfect information in the theoretical formulation of the Substitution Effect.

The paper is organised as follows. In Section 2, we briefly describe the three optimization problems of the individual. In Section 3, we provide alternative theoretical expressions of the Substitution Effect, and Section 4 closes the paper with our main conclusions and the possible extensions of the work.

2. Optimization Problems

The unitary approach to demand analysis studies the individual microeconomic decisions on the basis of the Ordinal Utility Theory, assuming that we do not distinguish between the individual agent and the collective agent (household). This unitary approach allows us to specify observable demand systems, which have provided rich empirical results in many countries.[1] Additionally, this unitary approach constitutes the foundations of the subsequent household approach that have emerged during recent decades, on the basis of the methodological or empirical drawbacks of this initial approach (Molina, 2011). In methodological terms, the traditional assumption that subjective preferences are individual does not fit the normal structure of a household formed by a group of individuals with different preferences. In this context, the unitary approach imposes a series of restrictions on the observed behavior, among which is that this approach does not allow for the establishment of the intra-household distribution of consumption, nor of productive resources and, consequently, of intra-household well-being. In this context, the unitary approach has given way in the literature to a new general approach, the household approach, concerned with analyzing matters related to intra-family negotiations. In accordance with this household approach, the presence of individuals with different preferences is represented by the existence of, at least, two individual functions of utility, one for each spouse. [2]

Having established the importance of the unitary approach from the methodological perspective, in order to understand the household approach, we now describe the three optimization problems that allow us to represent the individual behaviours of agents.

[1] Some examples of empirical applications of demand systems for the case of Spain are, for example, for the case of Spain, Molina, 1994, for food; Molina, 1997, for transport goods; Molina, 1999, for leisure; Molina, 2002, for all consumer goods, and Molina et al. 2015, 2016, for cultural goods.

[2] Some recent examples of empirical applications of the household approach are Andaluz and Molina (2007), Garcia et al. (2007), Molina and Montuenga (2009), Garcia et al (2010, 2011), Molina (2011), Giménez et al. (2012), Andaluz et al. (2013), Gimenez and Molina (2013), Molina (2013, 2015), Molina et al. (2013), Bellido et al. (2016), Giménez and Molina (2016), Andaluz et al. (2017), Campaña et al. (2015 and 2017).

The Primal Decision Problem consists of maximizing the utility function u(**q**) subject to the available income y, with **p** being the price vector:

$$\text{Max } u(\mathbf{q}) \quad \text{s.t } \ y = \mathbf{pq}$$

The maximum first order conditions of this Primal are:

$$u_i(\mathbf{q}) - \lambda\, p_i = 0 \qquad (i = 1, ..., n)$$

$$y - \mathbf{pq} = 0$$

where $u_i(\mathbf{q})$ is the ith good marginal utility and λ is the marginal utility of income. From these conditions, it is easy to derive the Marshallian demand functions $q_i = q_i(\mathbf{p},y)$ $(i = 1, ..., n)$, which can be expressed in terms of expenditure, $c_i = p_i\, q_i = c_i(\mathbf{p},y)$, or in the budgetary distribution, $w_i = p_i\, q_i/y = w_i(\mathbf{p},y)$.

The Dual Decision Problem consists of minimizing the cost, subject to a given utility level:

$$\text{Min } pq \quad \text{s.t } u = u(\mathbf{q})$$

whose interior minimum first order conditions are:

$$p_i - \mu u_i(\mathbf{q}) = 0 \qquad (i = 1, ..., n)$$

$$u - u(\mathbf{q}) = 0$$

where μ is the new Lagrangian parameter in this dual formulation, and from which we can derive the Hicksian demand functions: $q_i = h_i(\mathbf{p}, u)$ $(i = 1, ..., n)$, with an important result being that the first derivative is the Substitution Effect:

$$S_{ij} = \frac{\partial h_i}{\partial p_j} \qquad (i = 1, ..., n)$$

We can see above that there exist different ways of describing consumer behaviours in terms of the Marshallian and Hicksian demand functions from the Primal and Dual, respectively, optimization problems.

We now characterize a new type of demand function, the Frischian function, that is closely related to a new representation of preferences, the Profit Function in Consumption (PFC). This new representation, derived from the isomorphism existing between the consumption theory and the production theory, considers that the consumer uses some inputs, goods, to obtain an output, the utility. The PFC constitutes a Dual representation of preferences, with the property that it perfectly maintains the inter-temporal separability of the utility function. That is to say, the fact that the utility throughout the Life Cycle can be assumed as the inter-temporal sum of the intra-temporal utilities can be extended to the PFC.

Given the Primal Decision Problem:

$$\text{Max } u(\mathbf{q}) \quad \text{s.t } \; y = \mathbf{pq}$$

we can express the first order conditions as:

$$\lambda\, p_i = \frac{p_i}{r} \quad (i = 1, ..., n)$$

$$y = \mathbf{pq}$$

where the Lagrange multiplier λ is the income marginal utility ($\lambda = \frac{du}{dy}$), with its reciprocal, r, being interpreted as the utility marginal cost, $r = \frac{dy}{du}$, or as a utility hypothetical price. Given that u (q) is strictly quasiconcave, the system:

$$u_i(\mathbf{q}) = \lambda p_i = \frac{p_i}{r}$$

could be inverted, obtaining $q_i = f_i(\mathbf{p}, r)$ (i = 1, ..., n). Frisch (1932) used a version of this system in the framework of additive preferences to measure the money marginal utility and, for this, following Browning (1992), we call these the Frischian Demand Functions

According to these functions, the demanded quantities of goods in equilibrium depend on prices and the marginal cost of the utility (inverse of the income marginal utility). The notion is that the consumer is compensated by changes in prices with enough money to keep the marginal utility of income constant at its initial level. This concept is particularly useful in the inter-temporal context, given that a primary objective of individuals during the life cycle is to maintain constant their marginal utility of income, obviously, in discounted terms.[3] That is to say, in the context of perfect information

[3] The inter-generational transmission of socio-economic variables and behaviors has been analyzed, for example, in Molina et al. (2011) for the case of well-being, in Giménez and Molina (2013) for education, in Giménez et al. (2014) and in Giménez et al. (2015) for housework time, in Andaluz et al (2007) and Molina (2014) for the case of altruism.

about the future, the Frischian demands describe the consumption behaviour of individuals in terms of the current prices and on the marginal utility of income r, which, being constant, includes all the known, relevant, extra-current information in the decision process. In other words, the individual must always maintain during his/her life cycle the utility derived from current income, in discounted terms.

Frischian demands are immediately distinguished from the Marshallian demands that relate equilibrium quantities to prices and income, and also from the Hicksian demands, which relate quantities to prices and utility. However, all three types of demand are related. From Frischian demands, we can obtain Marshallian functions by isolating r in the budget constraint in terms of **p** and y, and substituting in $q_i = f_i(\mathbf{p}, r)$:

$$y = \mathbf{pq} = \mathbf{p}f(\mathbf{p}, r) \rightarrow r = r(\mathbf{p}, y) \rightarrow \mathbf{q} = f(\mathbf{p}, r(\mathbf{p}, y)) = \mathbf{q}(\mathbf{p}, y)$$

Similarly, the Frischian demands can be converted into Hicksian ones, expressing r in terms of p and u, and substituting in $q_i = f_i(\mathbf{p}, r)$:

$$y = \mathbf{pq} \rightarrow c(\mathbf{p}, u) = \mathbf{p}f(\mathbf{p}, r) \rightarrow r = r(\mathbf{p}, u) \rightarrow \mathbf{q} = f(\mathbf{p}, r(\mathbf{p}, u)) = \mathbf{h}(\mathbf{p}, u)$$

Knowing that r is the utility marginal cost, we can obtain from the Frischian demands in an alternative and more innovative way. To that end, we consider the isomorphism previously indicated in the production theory and we suppose that the consumer uses certain inputs, goods, to obtain an output, utility, whose price is r. In this exposition, we can define the Profit Function in Consumption as the maximum profit that the agent can attain when "selling" own utility at the price r, given the utility function and good prices. For a general function $u = u(\mathbf{q})$ strictly quasiconcave, the appropriate benefit function will be given by :

$$\pi(\mathbf{p}, r) = \mathrm{Max}_{u,q} \{ r u - \mathbf{pq};\ u = u(\mathbf{q})\}$$

which is continuous, convex, linear homogeneous in **p** and r, increasing in r and decreasing in **p**. An alternative form, frequently used, of expressing the benefit function is in terms of the expenditure function, that is to say:

$$\pi(\mathbf{p}, r) = \mathrm{Max}_{u} \{ r u - c(\mathbf{p}, u)\}$$

From this function and making use of the Hotelling theorem, we can derive the demanded quantities of each good in equilibrium and, additionally, we can also derive the utility in terms of the good prices and the price of utility:

$$-\pi_i(\mathbf{p}, r) = q_i = f_i(\mathbf{p}, r) \quad (i = 1, ..., n)$$

$$\pi_r(\mathbf{p}, r) = u$$

The functions $q_i = f_i(\mathbf{p}, r)$ $(i = 1, ..., n)$ show the quantities of the goods that are going to be demanded to achieve a determined level of income marginal utility (or utility marginal cost), with constant prices, in such a way that the benefit from consumption is the maximum possible.

3. Alternative expressions of the Substitution Effect

We now derive two alternative expressions of the Substitution Effect from the relationships among the Marshallian, Hicksian, and Frischian demand functions.

Proposition 1. *We express the Substitution Effect in terms of the Profit Function in Consumption:*

$$S_{ij} = -\pi_{ij} + \frac{\pi_{ir}\pi_{jr}}{\pi_{rr}}$$

Proof.

From the Slutsky Equation, $S_{ij} = T_{ij} - R_{ij}$, and using the Hotelling theorem:

$$S_{ij} = \frac{\partial q_i}{\partial p_j} + q_j \frac{\partial q_i}{\partial y} = \frac{\partial q_i}{\partial p_j} + \frac{\partial c}{\partial p_j} \frac{\partial q_i}{\partial y}$$

Now assuming that income is equal to expenditure and dividing by ∂u :

$$S_{ij} = \frac{\partial q_i}{\partial p_j} + \frac{\partial(\partial y / \partial u)}{\partial p_j} \frac{\partial q_i}{\partial(\partial y / \partial u)}$$

Given that c(**p**, u) = Max$_r$ { r u - π (**p**, r)} and, therefore, $\frac{\partial y}{\partial u} = r$, knowing that -π_i(**p**, r) = q_i and dividing and multiplying by ∂r :

$$S_{ij} = -\frac{\partial \pi_i}{\partial p_j} + \frac{\partial(\partial y/\partial u)}{\partial p_j}\left(-\frac{\partial \pi_i}{\partial r}\right) = -\pi_{ij} + \frac{\partial(\partial c/\partial p_j)}{\partial u}\left(-\pi_{ir}\right) = -\pi_{ij} = -\pi_{ij} + \frac{\partial q_j}{\partial r}\frac{\partial r}{\partial u}\left(-\pi_{ir}\right)$$

And, finally, given that π (**p**, r) = r u(r) - c(**p**, u) and, consequently:

$$\frac{\partial \pi}{\partial r} = \pi_r = u(r) + r\frac{\partial u}{\partial r} - \frac{\partial c}{\partial r} = u(r) + r\frac{\partial u}{\partial r} - \frac{\partial y}{\partial u}\frac{\partial u}{\partial r} = u$$

then:

$$S_{ij} = -\pi_{ij} + \frac{\partial q_j}{\partial r}\frac{\partial r}{\partial u}\left(-\pi_{ir}\right) = -\pi_{ij} + \left(-\pi_{jr}\right)\frac{\partial r}{\partial \pi_r}\left(-\pi_{ir}\right) = -\pi_{ij} + \frac{\pi_{ir}\pi_{jr}}{\pi_{rr}}$$

Proposition 2. *We express the Substitution Effect in terms of the Income Flexibility concept of Frisch* $\omega = \frac{\partial \log r}{\partial \log y}$*:*

$$S_{ij} = f_{ij} + y\omega^{-1}\frac{\partial q_i}{\partial y}\frac{\partial q_j}{\partial y}$$

Proof.

Our starting point in the previous demonstration, $S_{ij} = -\pi_{ij} + \frac{\pi_{ir}\pi_{jr}}{\pi_{rr}}$, and given that -$\pi_i$(**p**, r) = q_i = f_i(**p**, r):

$$-\pi_{ij} = -\frac{\partial^2 \pi}{\partial p_i \partial p_j} = \frac{\partial f_i}{\partial p_j} = f_{ij}$$

Also, given that: π_i(**p**, r) = -q_i and that π_r(**p**, r) = u:

$$\frac{\pi_{ri}\pi_{rj}}{\pi_{rr}} = \frac{\frac{\partial^2 \pi}{\partial r \partial p_i}\frac{\partial^2 \pi}{\partial r \partial p_j}}{\frac{\partial^2 \pi}{\partial r^2}} = \frac{\left(-\frac{\partial q_i}{\partial r}\right)\left(-\frac{\partial q_j}{\partial r}\right)}{\frac{\partial u}{\partial r}}$$

and now dividing and multiplying by ∂y the three elements, given that $\frac{\partial y}{\partial u} = r$, and then dividing and multiplying by y:

$$\frac{\partial q_i}{\partial y}\frac{\partial y}{\partial r}\frac{\partial q_j}{\partial y}\frac{\partial y}{\partial r}\frac{\partial r}{\partial y}\frac{\partial y}{\partial u} = \frac{\partial q_i}{\partial y}\frac{\partial q_j}{\partial y}\frac{\partial y}{\partial r} r \frac{y}{y} = \frac{\partial q_i}{\partial y}\frac{\partial q_j}{\partial y}\omega^{-1} y$$

Thus:

$$S_{ij} = -\pi_{ij} + \frac{\pi_{ir}\pi_{jr}}{\pi_{rr}} = \frac{\partial q_i}{\partial y}\frac{\partial q_j}{\partial y}\omega^{-1} y$$

4. Conclusions

This paper offers some alternative theoretical expressions of the Substitution Effect derived from the relationships among the Marshallian, Hicksian, and Frischian demand functions. In particular, we use the Profit Function in Consumption as the Income Flexibility of Frisch, in order to derive alternative expressions of the SE.

We work in the context of perfect information about the future, in such a way that the marginal utility of income is constant, thus including all the known, relevant, extra-current information in the decision process. However, it is usual that individuals decide in an environment of uncertainty where individuals will have, as time goes on, new information that they must incorporate to the marginal utility of income, thus modifying it successively. A clear extension of our work is to derive the alternative theoretical expressions of the SE in this new uncertainty context.

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