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Güriş, Burak

Istanbul University

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Assoc. Prof. Dr. Burak Güriş
Istanbul University, Faculty of Economics, Istanbul Turkey
E-mail: bguris@istanbul.edu.tr

Abstract

Traditional unit root tests display a tendency to be nonstationary in the case of structural breaks and nonlinearity. To eliminate this problem this paper proposes a new flexible Fourier form nonlinear unit root test. This test eliminates this problem to add structural breaks and nonlinearity together to the test procedure. In this test procedure, structural breaks are modeled by means of a Fourier function and nonlinear adjustment is modeled by means of an Exponential Smooth Threshold Autoregressive (ESTAR) model. The simulation results indicate that the proposed unit root test is more powerful than the Kruse (2011) and KSS(2003) tests.

JEL classification: C12, C22

Keywords: Flexible Fourier Form, Unit Root Test, Nonlinearity

1.Introduction

Almost all of the empirical studies that use the time series techniques use unit root tests. During the last four decades, an increasing number of studies have developed tests to analyze the order of integration of variables. Unit root tests were first introduced to literature by Dickey and Fuller (1979). The change in general in the testing concept was introduced by Perron (1989). According to Perron (1989), traditional unit root tests will display a tendency not to be stationary in the case of a structural break. After Becker et. al. (2006), the flexible Fourier transformation is used quite frequently in modeling structural breaks in recent years. The main advantage of this approach is that it eliminates the need to determine the number and the type of structural breaks.

Enders and Granger (1998) demonstrate that the standard tests for unit root and cointegration all have lower power in the presence of misspecified dynamics. In the light of this information, it is important to determine not only the structural break but also the type of model of the nonlinear structure. There have been significant developments in nonlinear unit root tests in recent years and various significant tests that make use of various types of models have been developed (Kapetanious *et al.* (2003)(KSS), Sollis (2004, 2009), Kruse (2011)).

Christopoulos and Leon-Ledesma (2010) made a significant contribution to literature by proposing new test procedures that combine Fourier transformation and nonlinearity. This procedure is based upon using the Fourier form in the first stage and the KSS test in the second stage. This allows for modeling both nonlinearity and structural break.

This study proposes a new test procedure combining the Kruse (2011) test developed in the light of the main criticisms to the KSS test with the Fourier transformation. In this test procedure structural breaks are modeled by means of a Fourier function and nonlinear adjustment is modeled by means of an Exponential Smooth Threshold Autoregressive (ESTAR) model as proposed by Kruse (2011).

The proposed unit root test is going to be explained in the second section of the study, the third section is going to focus on the Monte Carlo simulations and measure the critical values, empirical size and the power of the test and the fourth section is going to focus on conclusion.

2. The Flexible Fourier Form Nonlinear Unit Root Test

The recent developments in unit root tests concentrate mostly on using nonlinear model specifications and tests with structural breaks. The structural break tests were first introduced to literature by Perron (1989) and the tests by Zivot and Andrews (1992), Lee and Strazicich (2003, 2004) Carrion-i-Silvestre *et al.* (2009) were developed later to define the history and the number of structural break tests. Prodan (2008) demonstrate that when the breaks are of opposite sign, it can be difficult to estimate the number and the magnitude of multiple breaks.

Becker *et al.* (2004, 2006) propose to use a Fourier series expansion to approximate the unknown number of breaks. In this approach, it is not necessary to assume that the number or the dates of breaks are known a priori. Following these developments, Enders and Lee (2012) suggest a unit root test with a Fourier function in the deterministic term in a Dickey Fuller type regression framework.

Christopoulos and Leon-Ledesma (2010) suggest a unit root test that account jointly for structural breaks and nonlinear adjustment. They modeled structural breaks by means of a Fourier function. They also modeled nonlinear adjustment by means of an ESTAR model proposed by Kapetanious *et al.* (2003).

This study is an extension of the test proposed by Christopoulos and Leon-Ledesma (2010). The Fourier function was used in the first stage for the proposed test following Christopoulos and Leon-Ledesma (2010) to model structural breaks in unknown forms and numbers. In the second test, the unit root was tested by using the Kruse (2011) test developed in the light of the criticisms made for the KSS test.

The test developed by Kruse (2011) is the advanced version of the root test introduced to literature by Kapetanios *et al*. (2003). The test developed by Kruse (2011) examines the nonlinear stationary exponential smooth transition autoregressive (ESTAR) against the null hypothesis of unit root.

The ESTAR model could be shown as follows:

$$y_t = \beta y_{t-1} + \gamma y_{t-1}\left[1 - \exp\left\{-\theta\left(y_{t-1} - c\right)^2\right\}\right] + \varepsilon_t$$

Contrary to Kapetanios *et al.* (2003), the Kruse (2011) study has shown that in real world examples, the possibility of non-zero location parameter ($c \neq 0$) is imminent (Anoruo and Murthy, 2014). Based on this, the equation was changed as follows using the Taylor approximation in the Kruse (2011) study:

$$\Delta y_t = \delta_1 y_{t-1}^3 + \delta_2 y_{t-1}^2 + \sum_{j=1}^{p} \varphi_j \Delta y_{t-j} + \varepsilon_t$$

Kruse (2011) proposes a τ test here to test the null hypothesis of unit root ($H_0: \delta_1 = \delta_2 = 0$) against globally stationary ESTAR process ($H_1: \delta_1 < 0,\ \delta_2 \neq 0$). This test statistics is formulated as follows:

$$\tau = t^2_{\delta_2^{\perp}=0} + 1(\hat{\delta}_1 < 0)t^2_{\delta_1=0}$$

Kruse(2011) show that τ statistic has the following asymptotic distribution which is free of nuisance parameters

$$\tau \Rightarrow a\big(W(r)\big) + B(W(r))$$

where A and B are function of Brownian motion W(r) (for details, see Kruse(2011)).

The test procedure proposed in the study can be shown as follows similar to the study by Christopoulos and Leon-Ledesma (2010).

Step 1: the nonlinear deterministic component is specified in the first stage.

$$y_t = \alpha_0 + \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) + \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right) + v_t$$

k* is the optimal frequency and it will be obtained by assigning values to k changing between 1 to 5, then predicting the equation by using OLS and minimizing the total of the squares of error terms. The error terms of the equation predicted will be obtained.

$$v_t = y_t - \alpha_0 - \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) - \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right)$$

Step 2: The test statistics is calculated predicting the equation below using the error terms obtained in the first stage:

$$\Delta v_t = \delta_1 y_{t-1}^3 + \delta_2 y_{t-1}^2 + \sum_{j=1}^{p} \varphi_j \Delta v_{t-j} + \varepsilon_t$$

Step 3: If the null hypothesis of unit root is rejected, then $H_0: \alpha_1 = \alpha_2 = 0$ against the alternative hypotheses $H_1: \alpha_1 = \alpha_2 \neq 0$ is tested in this step using the F test. If the null hypothesis is rejected, we can conclude that the variable is stationary around a breaking deterministic function. The critical values of this test are tabulated in Becker *et al.* (2006).

3. Monte Carlo Results

The empirical size and power comparison of the critical values for the proposed flexible Fourier form nonlinear unit root test are presented in this section.

3.1. Critical Values

The computed critical values of the Fourier Kruse test statistics are presented in Table 1. They are based on 50,000 replications for T= 50, 100, 250, 500 and k=1, 2, 3, 4, 5. This paper reports the critical values for nominal significance levels of 1, 5 and 10%, respectively.

Table 1: Critical Values for Kruse test with Fourier Aproximation

		Level			Trend		
	k	1%	5%	10%	1%	5%	10%
T = 50	1	20.32	14.72	12.32	24.24	18.38	15.66
	2	16.04	11.46	9.28	22.34	15.62	13.16
	3	14.48	10.14	8.38	19.26	13.96	11.62
	4	13.38	9.56	7.92	17.88	13.1	10.88
	5	13.58	9.58	7.90	17.5	12.6	10.58
T = 100	1	19.46	14.76	12.44	23.78	18.4	15.78
	2	16.06	11.6	9.64	21.42	15.62	13.28
	3	14.74	10.7	8.96	18.62	14.46	12.14
	4	14.26	10.3	8.64	17.96	13.56	11.58
	5	14.26	10.06	8.52	17.8	13.24	11.2
T = 250	1	18.82	14.8	12.52	23.56	18.14	15.74
	2	15.84	11.92	9.98	20.02	15.74	13.5
	3	15.04	11.14	9.28	18.78	14.2	12.32
	4	14.58	10.74	9.1	18.28	13.88	11.96
	5	13.98	10.4	8.8	17.76	13.6	11.52
T = 500	1	19.56	14.86	12.7	23.26	18.14	15.8
	2	16.36	12.01	9.92	20.4	16.08	13.64
	3	14.9	11.24	9.4	19.12	14.6	12.44
	4	14.6	10.94	9.2	18.24	13.9	11.98
	5	14.34	10.92	9.1	17.66	13.76	11.76

3.2. Finite Sample Size

To evaluate the size of the test statistics, we consider the following data generating process (DGP)

$$y_t = \alpha_0 + \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) + \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right) + v_t$$

$$v_t = v_{t-1} + \varepsilon_t$$

where ε_t is a sequence of standard normal errors and k^* stands for optimal frequency. The empirical size is considered for sample sizes $T = 50, 100, 250, 500$, values of k=1,2,3, and $\alpha_1 = \alpha_2 = 1, 0.5, 0.1$ with nominal sizes of 5%.

Table 2 : Empirical Sizes of the Test

$\alpha_1 = \alpha_2$	k=1	k=2	k=3
		T=50	
1	0.042	0.043	0.058
0.5	0.042	0.043	0.058
0.1	0.042	0.043	0.059
		T=100	
1	0.051	0.053	0.053
0.5	0.051	0.054	0.053
0.1	0.051	0.053	0.053
		T=250	
1	0.045	0.047	0.047
0.5	0.045	0.047	0.047
0.1	0.05	0.047	0.047
		T=500	
1	0.05	0.05	0.052
0.5	0.05	0.05	0.052
0.1	0.05	0.05	0.052

The results in Table 2 show that the size of proposed test is close to 5% in all cases with different values of k and α.

3.3. Empirical Power

We next investigate the power properties of the unit root tests against globally stationary process using the following Fourier-ESTAR model as a DGP:

$$y_t = \alpha_0 + \alpha_1 sin\left(\frac{2\pi k^* t}{T}\right) + \alpha_2 cos\left(\frac{2\pi k^* t}{T}\right) + v_t$$

$$\Delta v_t = \phi v_{t-1}(1 - exp\{-\gamma(v_{t-1} - c)^2\}) + \varepsilon_t$$

following Taylor *et al.* (2001) $\phi = -1$ parameter restriction. The location parameter c and smoothness parameter γ are $c = (-10, -5, 0, 5, 10)$, $\gamma = (0.05, 0.1, 1)$, respectively.

Table 3 : Power Analysis of Fourier Kruse, Kruse and KSS Tests

T=50		Fourier Kruse			Kruse			KSS		
γ	c	k=1	k=2	k=3	k=1	k=2	k=3	k=1	k=2	k=3
0.05	-10	0.94	0.98	0.99	0.83	0.83	0.86	0.88	0.89	0.91
	-5	0.6	0.77	0.81	0.44	0.44	0.47	0.49	0.49	0.52
	0	0.35	0.55	0.59	0.28	0.27	0.23	0.31	0.31	0.27
	5	0.59	0.77	0.81	0.46	0.44	0.47	0.5	0.49	0.52
	10	0.94	0.98	0.99	0.84	0.83	0.86	0.9	0.89	0.91
0.1	-10	0.95	0.98	0.99	0.85	0.84	0.87	0.9	0.9	0.92
	-5	0.79	0.9	0.93	0.64	0.64	0.68	0.7	0.7	0.75
	0	0.58	0.77	0.82	0.44	0.43	0.4	0.47	0.47	0.47
	5	0.79	0.9	0.93	0.65	0.65	0.68	0.71	0.72	0.75
	10	0.95	0.98	0.99	0.85	0.84	0.87	0.9	0.9	0.92
1	-10	0.95	0.98	0.99	0.85	0.84	0.87	0.89	0.9	0.92
	-5	0.95	0.98	0.99	0.84	0.84	0.87	0.9	0.91	0.92
	0	0.96	0.99	0.99	0.81	0.81	0.83	0.86	0.86	0.89
	5	0.95	0.98	0.99	0.85	0.84	0.87	0.89	0.91	0.92
	10	0.95	0.98	0.99	0.84	0.84	0.87	0.9	0.9	0.92
T=100										
0.05	-10	0.99	1	0.99	0.99	1	0.99	0.99	1	0.99
	-5	0.9	0.94	0.97	0.88	0.89	0.88	0.78	0.79	0.78
	0	0.81	0.94	0.96	0.76	0.76	0.75	0.79	0.8	0.79
	5	0.89	0.94	0.97	0.87	0.89	0.88	0.79	0.79	0.78
	10	0.99	1	0.99	0.99	1	0.99	0.99	1	0.99
0.1	-10	0.99	1	1	0.99	1	1	0.99	1	1
	-5	0.98	0.99	0.99	0.97	0.97	0.97	0.95	0.95	0.95
	0	0.97	0.99	0.99	0.91	0.92	0.91	0.93	0.94	0.93
	5	0.98	0.99	0.99	0.97	0.97	0.97	0.95	0.95	0.95
	10	0.99	1	1	0.99	1	1	0.99	1	1
1	-10	0.99	1	1	0.99	1	1	0.99	1	1
	-5	0.99	0.99	1	0.99	0.99	1	0.99	0.99	1
	0	0.99	1	1	0.99	0.99	0.99	0.99	0.99	1
	5	0.99	0.99	1	0.99	0.99	1	0.99	0.99	1
	10	0.99	1	1	0.99	1	1	0.99	1	1

Note: T is the sample size.

The results of power experiments are presented in Table 3. A combination of the $c = (-10, -5, 0, 5, 10)$, $\gamma = (0.05, 0.1, 1)$ and k=1,2,3 were used. General outcome obtained from Table 3 is that the Fourier Kruse test is more powerful than the Kruse (2011) and KSS tests for all combinations of parameter values and frequencies. In small samples T=50, the power performance of the proposed test is good.

4. Conclusion

In this study, a new unit root test which can be useful in the presence of unknown number of breaks and nonlinearity was proposed. The finite sample properties of the suggested test via Monte Carlo simulations were examined. It was found that the proposed test has greater power than the Kruse (2011) and KSS tests. Especially for small sample cases, the power and size performance of the proposed test is good. This test eliminates the problems over-acceptance of the null of nonstationarity to add structural breaks and nonlinearity together into the test procedure.

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